TOWARDS M-MATRIX: PART I

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0.1 PREFACE

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. The approximate localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family and Kalevi and Ritva Tikkanen and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 45 lonely years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During the last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss my work. Pertti Kärkkäinen is my old physicist friend and has provided continued economic support for a long time. I have also had stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Tommi Ullgren has provided both economic support and encouragement during years. Pekka Rapinoja has offered his help in this respect and I am especially grateful to him for my Python skills.

During the last five years I have had inspiring discussions with many people in Finland interested in TGD. We have had video discussions with Sini Kunnas and had podcast discussions with Marko Manninen related to the TGD based view of physics and consciousness. Marko has also helped in the practical issues related to computers and quite recently he has done a lot of testing of chatGPT helping me to get an overall view of what it is. The discussions in a Zoom group involving Marko Manninen, Tuomas Sorakivi and Rode Majakka have given me the valuable opportunity to clarify my thoughts.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation in CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. I am grateful to Mark McWilliams, Paul Kirsch, Gary Ehlenberg, and Ulla Matfolk and many others for providing links to possibly interesting websites and articles. We have collaborated with Peter Gariaev and Reza Rastmanesh. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the public through the iron wall of academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as an individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without direct support from power holders- even in archives like arXiv.org.

Situation changed as Andrew Adamatsky proposed the writing of a book about TGD when I had already gotten used to the thought that my work would not be published during my lifetime. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loophole. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy.

And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his 73th birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During the period when the right wing held political power this trend was steadily strengthening and the situation is the same as I am writing this. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Karkkila, August 30, 2023, Finland

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Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K4].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A63]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of

electromagnetic fields are nonvanishing. The correlations functions for weak fields are non-vanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A25] [B29, B22, B23]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B19]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A51, A62, A40, A58].

The identification of the space-time as a sub-manifold [A52, A79] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

[1]There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.1.5 Construction of scattering amplitudes

Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A71, A86, A97]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K75]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their $M^8 - H$ duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p = 3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M^8_c$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since M^8-H duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K70].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of ”complex”. Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \rightarrow f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} . \quad (1.1.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \to Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \to 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \to SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.

2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.

4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.

5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same.

The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of M^8-H duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4 \times CP_2$?

1. The selection of $SU(3) \subset G_2$ for ordinary M^8-H duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3 \subset M^8_c$ and its image $H^3 \subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3) \subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of M^8-H duality as something more fundamental than the ordinary M^8-H duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3 \times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1) \times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1 \oplus 3 \oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3 \oplus \overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8 \oplus 3 \oplus \overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3) \times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x) \times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that

Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K101].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K89, K90, K87]) support the view that dark matter might be a key player in living matter.

Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K116]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A63]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   M^8-H duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L70].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L53]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t-m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

1.2 Bird's Eye of View about the Topics of "Towards M-matrix"

This book is devoted to a detailed representation of quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory.

Number theoretic vision leads to the notion of adelic physics fusing real physics with p-adic physics as physics of cognition. It also leads to M^8-H duality raising classical number fields in central role and reducing the dynamics of space-time surfaces in $M^4 \times CP_2$ determined by action principle and subject to infinite number of analogs of gauge conditions to purely algebraic dynamics in M^8. Twistor lift of TGD is a further central notion.

The most important guiding principle is quantum classical correspondence, whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type II_1 about which the spinors of the world of classical worlds represent a canonical example.

1.2.1 Zero energy ontology

1. The new view about energy and time finding a justification in the framework of zero energy ontology (ZEO) means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action. ZEO has led to a new view about quantum measurement theory extending it to a theory of consciousness solving the basic paradox of quantum measurement theory in its standard form.

2. Classical theory is in a well-defined sense exact part of quantum TGD. Action principle should assign to a given 3-surface unique space-time surface analogous to Bohr orbit. In zero energy ontology (ZEO) 3-surface is identified as a disjoint pair of 3-surfaces with members located at the opposite boundaries of causal diamond (CD) being analogous to initial and final states

of a unique classical time evolution represented by preferred extremals. What the action principle is and what *preferred* does mean? During years I have considered several answers to these questions.

For a long time action was identified as 4-D Kähler action but the emergence of the twistor lift of TGD changed this view. 4-D space-time surface is replaced with the analog of its 6-D twistor-space represented as 6-D surfac having the structure of S^2 bundle with base space identifiable as 4-D space-time surface. Twistor structure of this 6-surface is induced from the 12-D Cartesian product of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having Kähler structure only for M^4 and CP_2. This allows to define 6-D Kähler action whose dimensionally reduced extremals induce of twistor structure to the 6-D surface. Quantum criticality suggests that all preferred extremals are minimal surfaces apart from 2-D singular surfaces identifiable as string world sheets and partonic 2-surfaces. The reason is that the dynamics in this case is independent of coupling parameters (Kähler coupling strength).

The dimensionally reduced action is sum of Kähler action and volume term having interpretation in terms of cosmological constant. Minimal surfaces are extremals of both volume term and Kähler action separately. Therefore all extremals of Kähler action with non-vanishing Kähler form are also minimal surfaces so that no changes emerge. Therefore I have kept the old chapters studying extremals of Kähler action as such.

3. The differences between the Kähler action with volume term and mere Kähler action emerge only in the vacuum sector. For non-vanishing value of cosmological constant the vacuum extremals with vanishing induced Kähler form are not possible but one can consider the possibility that the dynamically determined cosmological constant [L70] can vanish at the limiting situation when the space-time surfaces have infinite size. The emerging huge vacuum degeneracy and the failure of the classical determinism in the conventional sense, would have strong implications.

   One would have near vacuum extremals of Kähler action a strongly interacting theory defined by volume action with a small cosmological constant with large quantum fluctuations characterizing quantum criticality playing a key role. Vacuum degeneracy implies spin glass degeneracy in 4-D sense. Whether this nearly vacuum degeneracy is a fundamental characteristic of TGD Universe in long length scales, remains an open question.

1.2.2 Quantum classical correspondence

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

   The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.

3. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

   Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and Z^0 fields are ideal for communication purposes and charged W MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy W MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

1.2.3 Physics as infinite-dimensional geometry in the "world of classical worlds"

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. WCW must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type II_1 having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

1.2.4 Physics as a generalized number theory

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics. This led eventually to what I call adelic physics [L53, L54]. One of the outcomes was a proposal for a number theoretical interpretation for the hierarchy of Planck constants: the integer defining effective Planck constant $h_{eff} = n \times h_0$ would correspond to the dimension of the extension of rationals defining the adele.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I christened quaternionic sub-manifolds of complexified octonionic imbedding space M^8_c.

$M^8 - H$ duality leads to a concrete proposal stating that space-time surfaces in 16-D M^8_c consist of regions for which either real or imaginary part of a complexified-octonion valued polynomial (additional imaginary unit i commutes with octonion units) vanishes. Imaginary and real part refer now to complexified quaternions $o_c = q_{1,c} + J_4 q_{2,c}$ so that 2×4 conditions give 8-D complexified space-time surface. 4-D space-time surfaces in M^8 could correspond to projections of these with respect to M^8, that is time coordinate would be real and remaining 7 coordinates imaginary.

The development of ideas involved a rather strange quirk, which I noticed while doing the updating in 2019.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-

manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

Calibrated k-manifolds are minimal surfaces in their homology class. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and one expects that they are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces play key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality would be realized as decoupling of the two parts of action. Could all preferred extremals be regarded as calibrated in some generalized sense.

   If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

1.2.5 Towards M-matrix or towards S-matrix?

S-matrix codes the predictions of quantum field theory and the challenge is to construct the analogy or generalization of S-matrix.

1. In ZEO one is forced to challenge the usual notion of S-matrix. Ordinary S-matrix is between ordinary quantum states associated with time=constant snapshot of time evolution S-matrix. Now these states are replaced by zero energy states formed by these pairs with members at boundaries of CD.

   The first proposal was that S-matrix is replaced with M-matrix between zero energy states and identifiable as time-like entanglement coefficients between positive and negative energy parts of zero energy states assignable to the past and future boundaries of 4-surfaces inside causal diamond defined as intersection of future and past directed light-cones.

   M-matrix would be a product of diagonal density matrix and unitary S-matrix and there are reasons to believe that S-matrix is universal. Generalized Feynman rules based on the generalization of Feynman diagrams obtained by replacing lines with light-like 3-surfaces and vertices with 2-D surfaces at which the lines meet.

   In M-matrix approach without any constraints the state would be superposition of pairs of states with S-matrix defining entanglement coefficients. This zero energy state with sum over states associated with all CDs. The square root of density matrix could take care of the normalization: without it the state has infinite norm. For hyper-finite factors this state could be normalized to unity and one could also require that the normal unitary conditions hold true when one fixes the boundaries of CD and looks for the scattering rates for fixed states at the passive boundary of CD. This should give S-matrix components from given initial state at passive boundary of CD to states and the active boundary of CD.

It is however far from clear what unitary time evolution following preparation of initial state could mean in this picture. It seems that the standard view about quantum measurement requires that the second boundary of CD - the passive bound - and states at it must be regarded as fixed and that unitary evolution affects only the active boundary and states at it.

Remark: After the emergence of ZEO the name of this chapter has fluctuated between "T"owards S-matrix and "T"owards M-matrix. This reflects my fluctuating views about what the counterpart of S-matrix could be in ZEO.

2. Later it turned out that the generalization of quantum measurement theory to a theory of consciousness indeed requires a more conservative view. Observer, conscious entity, or self corresponds to a sequence of unitary time evolutions followed by state function reductions for which the active boundary of CD shifts farther away from the passive boundary, which remains unchanged.

   The states at active boundary are changed by unitary time evolution implying also time delocalization of the active boundary in the moduli space of CDs with fixed passive boundary. The state function reduction induces localization in this moduli space and is analogous to weak measurement. The localization means also time localization since the temporal distance between the tips of CD is fixed. Eventually all observables are measured in the sense that there are no state function reductions not affective the states at passive boundary. The roles of passive and active boundary are changed. One can say that self dies and reincarnates as self living in opposite direction of time since its is the former passive boundary which shifts farther away from former active boundary. The distance between the tips can also increase in statistical sense only.

   S-matrix would be associated with the unitary evolution assignable to the active boundary of CD and involving shift of this boundary farther away from the passive boundary.

1.2.6 Organization of "Towards M-matrix: Part I"

The book contains 2 parts.

1. The 1st part of this book has title "The recent view about field equations" and describes what is known about preferred extremals and the vision about physics as spinor geometry of "world of classical worlds" (WCW).
   (a) In the first two chapters the preferred extremals of Kähler action are studied. The twistor lift adds to this action a volume term but since the non-vacuum extremals of Kähler action are minimal surfaces, I have kept the old chapters talking about extremals of Kähler action as such.
   (b) In two chapters quantum TGD as WCW geometry and spinor structure are discussed. General coordinate invariance and generalized super-conformal symmetries - the latter present only for 4-dimensional space-time surfaces and for 4-D Minkowski space - define the basic symmetries of quantum TGD.
   (c) Occam's razor is the basic weapon used in the attempts to debunk a new theory and this argument is discussed in the last chapter of the first part of the book.
2. The 2nd part of part I is devoted to the challenge of constructing the scattering amplitudes. The dream of finding analogs of Feynman rules has turned out to be unreachable with my analytic skills, and the work is concentrated on identifying the basic principles.
   (a) Symmetries determine the predictions of quantum theory to a very high degree. General Coordinate Invariance and huge super-symplectic symmetries making possible the existence of the Kähler geometry of WCW are considered in the first chapter.
   (b) The construction of M-matrix and S-matrix in ZEO are considered in the second chapter. The third chapter is devoted to the attempts to guess what scattering amplitudes should look like.

1.3 Sources

The eight online books about TGD [K123, K117, K94, K81, K23, K76, K54, K104] and nine online books about TGD inspired theory of consciousness and quantum biology [K111, K18, K86, K17, K50, K63, K66, K103, K109] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 PART I: THE RECENT VIEW ABOUT CLASSICAL TGD

Basic extremals of the Kähler action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the geometry of the "world of classical worlds" (WCW) and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to associative and co-associative regions. The reduction of the classical theory to the level of the Kähler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [?] It is not clear whether criticality and associativeity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

   Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. In ZEO one can also consider the possibility that there is no selection of preferred extremal at all! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. Actually all the discussions of this chapter are about extremals in general so that the attribute "preferred" is not relevant for them.

3. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.
4. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy. Coupling constant evolution would be replaced by effective discrete evolution with respect to p-adic length scale and angle variable defined by the phases appearing in the algebraic extension of p-adic numbers in question.
5. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with Kähler-Dirac action defines classical theory: this is in complete accordance with the proposed definition of the WCW spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2, gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.
2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.
3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extemals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariace from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

About Identification of the Preferred extremals of Kähler Action

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [?]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

WCW Spinor Structure

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of

WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the WCW spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the WCW and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the WCW are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2. Kähler-Dirac equation for induced spinor fields

Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

   The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

awcwspin

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Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

This chapter represents the updated version of the construction providing a solution to the problems of the previous construction. The basic formulas remain as such but the expressions for WCW super-Hamiltonians defining WCW Hamiltonians (and matrix elements of WCW metric) as their anticommutator are replaced with those following from the dynamics of the Kähler-Dirac action.

Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occarm's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Käction in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent embedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic

2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At embedding space level the modes of embedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at embedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

1.4.2 PART II: GENERAL THEORY

Construction of Quantum Theory: Symmetries

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. In fact, the general definition of geometry is as a structure characterized by given symmetries. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" (WCW) identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

1. Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the WCW forms what I have used to call super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

4. Kähler-Dirac equation (or Kähler-Dirac equation) gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved

fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the Kähler-Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

A crucial feature of the Kähler-Dirac equation is the localization of the modes to 2-D surfaces with vanishing induced W fields (this in generic situation and for all modes but covariantly constant right-handed neutrino): this is needed in order to have modes with well-defined em charge. Also Z^0 fields can be vanish and is expected to do so - at least above weak scale. This implies that all elementary particles are string like objects in very concrete sense.

2. p-adic physics and p-adic variants of basic symmetries

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

3. Hierarchy of Planck constants and dark matter hierarchy

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzle leads to a profound generalization of quantum TGD through a generalization of the notion of embedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the embedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups. The extension of embedding space can be seen as a formal tool allowing an elegant description of the multi-sheetednes due to the non-determinism of Kähler action. At the space-like ends the sheets fuse together so that a singular covering is in question.

The original intuition was the the space-time would be n-sheeted for $h_{eff} = n$. Quantum criticality expected on basis of the vacuum degeneracy of Kähler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have n conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by n. This defines infinite number of inclusion hierarchies $.. \subset C(n_1) \subset C(n_3)...$ such that n_{i+1} divides n_i. These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

4. Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids - ends of string world sheets - can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number

theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the Kähler-Dirac gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4, one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4. One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H. so that TGD could be seen as a generalized number theory.

Zero Energy Ontology

Zero energy ontology (ZEO) has become gradually one of the corner stones of quantum TGD. This motivates the collection of material related to ZEO in a single chapter providing an overall view about the development of ideas. The sections are independent and reflect different views about ZEO.

The following gives a brief summary of the most recent view (2021) of ZEO.

1. The notion of a causal diamond (CD) is a central concept. Its little cousin "cd" can be identified as a union of two half-cones of M^4 glued together along their bottoms (3-D balls). The half-cones are mirror images of each other. CD=$cd \times CP_2$ is the Cartesian product of cd with CP_2 and obtained by replacing the points of cd with CP_2. The notion of CD emerges naturally in the number theoretic vision of TGD (adelic physics via the $M^8 - H$ duality.

2. In ZEO, quantum states are not 3-dimensional if the determinism does not fail as it actually does, but superpositions of 4-dimensional deterministic time evolutions connecting ordinary 3-dimensional states. For the strongest form of holography implied by general coordinate invariance (GCI), the time evolutions are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution.

   The failure of determinism probably implies that a given 3-surface at the passive boundary of CD (PB) corresponds to a finite number of 4-D minimal surfaces and that the minimal surface can be regarded as an analog of soap film spanned by a frame having fixed parts at the boundaries of CD and dynamically generated parts in the interior of CD. The frame can be identified as a topological analog of a Feynman diagram.

3. Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced by a new superposition. The classical determinism of individual time evolution is not violated. This solves the basic paradox of quantum measurement theory. There are two kinds of SFRs: BSFRs (counterparts of ordinary SFRs) changing the arrow of time (AT) and SSFRs (analogs of "weak" measurements) preserving the arrow of time that give rise to an analog of the Zeno effect. The findings of Minev et al provide strong support for ZEO.

New result about causal diamonds from the TGD view point of view

This chapter was inspired by two interesting results related to the notion of causal diamond (CD) playing a central role in quantum TGD. One interpretation is as a quantization volume and the second interpretation is as a geometric representation of the perceptive field of conscious entity. CDs can be said to define the backbone of the "world of classical worlds" (WCW) central for quantum TGD.

For these reasons it is interesting to ask the precise mathematical definition of the moduli space of CDs. TGD suggests a definition as the semidirect product $D \rtimes P/SO(3)$ of scaling group and Poincare group divided by $SO(3)$ subgroup leaving the CD invariant: this gives 8-D space. The definition that inspired this article is based on conformal group and gives also 8-D space $SO(2,4)/SO(1,3) \times SO(1,1)$. The metric signature is $(4,4)$ for both spaces and they could be identical. These definitions are compared and one can consider the conditions under which both identification can give rise to representations of the Poincare group as expected with the scaling group reduced to a discrete subgroup.

Second result relates to the finding that special conformal transformations in the time direction defined by CD leave CD invariant. The corresponding hyperbolic flows correspond to a motion with constant acceleration to which the so-called Unruh effect is associated. One can consider an SL(2,R) algebra assignable to a conformal quantum mechanics and assign a hyperbolic time evolution operator to this flow. The conformal 2-point functions associated with this operator correspond to thermal partition functions with thermal mass defined by the temperature which is essentially the inverse of the CD scale.

Holography does not allow us to consider these flows for the space-time surfaces insid CD but the action of the hyperbolic evolution operator on quantum states at the boundaries of CD is well-defined. This also raises interesting questions related to TGD inspired consciousness, where subsequent scalings of CD in state function reductions (SFRs) give rise to the correlation of subjective time and geometric time defined as the distance between the tips of CD. The SFRs associated with the hyperbolic time evolution operator would not affect CD and would correspond to "timeless" state of consciousness. One cannot avoid reconsidering the details of "small" SSFRs defining the subjective time flow correlating with the flow of geometric time assigned with the increase of CD.

Number theoretic vision, Hyper-finite Factors and S-matrix

During years the basic mathematical and conceptual building bricks of quantum TGD have become rather obvious. The basic goal is the construction of scattering amplitudes.

1. Zero Energy Ontology (ZEO) forces to generalize the notion of S-matrix by introducing M-matrix as a matrix characterizing the entanglement between pairs of states forming zero energy states.

2. Second building brick consists of various hierarchies and connections between them. There is the hierarchy of quantum criticalities for super-symplectic algebra and its Yangian extension acting as a spectrum generating algebra. This hierarchy is closely related to the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchies of criticalities correspond also to fractal hierarchies of breakings of super-symplectic gauge conformal symmetry: only the sub-algebra isomorphic to the original gauge algebra acts as gauge algebra after the breaking. At each step one criticality is reduced and the number of physical degrees of freedom increases.

   There is a natural connection between these hierarchies with the hierarchies of hyperfinite factors of type II_1 (HFFs) and their inclusions providing a description for the notion of measurement resolution.

3. Number theoretic realized as adelic physics fusing real number based physics as a correlate of sensory experience and p-adic physics as correlate of cognition involves several elements: $M^8 - H$ duality, hierarchy of effective Planck constants $h_{eff} = nh_0$ with n identified as a dimension of extension of rationals, cognitive representations characterized by extensions of rationals, and p-adic length scale hypothesis.

The identification of the TGD counterpart of S-matrix is the key topic of this chapter. What this matrix actually means is far from obvious.

1. One can characterize zero energy state by a "square root" of density matrix which is product of hermitian matrix and unitary matrix: I have called this matrix M-matrix. The unitary matrix related to the M-matrix could relate closely to the S-matrix assigned with particle reactions.

2. One can assign the analog of unitary S-matrix to "small" state function reductions (SSFRs) defining the TGD counterparts of "weak" measurements. The states at the passive boundary PB are unaffected, which has interpretation as the TGD counterpart of Zeno effect. This S-matrix could relate to the evolution of self as a conscious entity and to its cognitive time evolution.

3. One can also assign an S-matrix like entity to "big" SFRs (BSFRs) in which the arrow of time changes. This S-matrix would be the counterpart of the ordinary S-matrix and should closely relate to the M-matrix.
4. I have also introduced the notion of U-matrix, which would be defined between zero energy states without fixing states at the passive boundary essential for fixing the arrow of time. This notion has remained somewhat misty and it seems that it is not needed since the matrices assigned SSFRs and BSFRs indeed are between zero energy states.

The construction of these matrices is discussed at the general level.

Philosophy of Adelic Physics

The p-adic aspects of Topological Geometrodynamics (TGD) will be discussed. Introduction gives a short summary about classical and quantum TGD. This is needed since the p-adic ideas are inspired by TGD based view about physics.

p-Adic mass calculations relying on p-adic generalization of thermodynamics and super-symplectic and super-conformal symmetries are summarized. Number theoretical existence constrains lead to highly non-trivial and successful physical predictions. The notion of canonical identification mapping p-adic mass squared to real mass squared emerges, and is expected to be a key player of adelic physics allowing to map various invariants from p-adics to reals and vice versa.

A view about p-adicization and adelization of real number based physics is proposed. The proposal is a fusion of real physics and various p-adic physics to single coherent whole achieved by a generalization of number concept by fusing reals and extensions of p-adic numbers induced by given extension of rationals to a larger structure and having the extension of rationals as their intersection.

The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far from obvious and various constraints lead to the idea of number theoretic universality (NTU) and finite measurement resolution realized in terms of number theory. An attractive manner to overcome the problems in case of symmetric spaces relies on the replacement of angle variables and their hyperbolic analogs with their exponentials identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

Also the understanding of the correspondence between real and p-adic physics at various levels - space-time level, embedding space level, and level of "world of classical worlds" (WCW) - is a challenge. The gigantic isometry group of WCW and the maximal isometry group of embedding space give hopes about a resolution of the problems. Strong form of holography (SH) allows a non-local correspondence between real and p-adic space-time surfaces induced by algebraic continuation from common string world sheets and partonic 2-surfaces. Also local correspondence seems intuitively plausible and is based on number theoretic discretization as intersection of real and p-adic surfaces providing automatically finite "cognitive" resolution. he existence p-adic variants of Kähler geometry of WCW is a challenge, and NTU might allow to realize it.

I will also sum up the role of p-adic physics in TGD inspired theory of consciousness. Negentropic entanglement (NE) characterized by number theoretical entanglement negentropy (NEN) plays a key role. Negentropy Maximization Principle (NMP) forces the generation of NE. The interpretation is in terms of evolution as increase of negentropy resources.

Negentropy Maximization Principle

In TGD Universe the moments of consciousness are associated with quantum jumps between quantum histories. The proposal is that the dynamics of consciousness is governed by Negentropy Maximization Principle (NMP), which states the information content of conscious experience is maximal. The formulation of NMP is the basic topic of this chapter.

NMP codes for the dynamics of standard state function reduction and states that the state function reduction process following U-process gives rise to a maximal reduction of entanglement entropy at each step. In the generic case this implies at each step a decomposition of the system to unique unentangled subsystems and the process repeats itself for these subsystems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states. The natural assumption is that self loses consciousness when it entangles via bound state entanglement.

There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement (NE), which is not bound state entanglement in standard sense since the condition that state function reduction leads to an eigenstate of density matrix requires the final state density matrix to be a projection operator.

NE might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble. For the generation of NE the outcome of the reduction is not random: the prediction is that second law is not a universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and p-adic worlds, this suggests that life resides in this intersection. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism. A natural assumption is that self experiences expansion of consciousness as it entangles in this manner. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

There are two options to consider. Strong form of NMP, which would demand maximal negentropy gain: this would not allow morally responsible free will if ethics is defined in terms of evolution as increase of NE resources. Weak form of NMP would allow self to choose also lower-dimensional sub-space of the projector defining the final state sub-space for strong form of NMP. Weak form turns out to have several highly desirable consequences: it favours dimensions of final state space coming as powers of prime, and in particular dimensions which are primes near powers of prime: as a special case, p-adic length scale hypothesis follows. Weak form of NMP allows also quantum computations, which halt unlike strong form of NMP.

Besides number theoretic negentropies there are also other new elements as compared to the earlier formulation of NMP.

1. ZEO modifies dramatically the formulation of NMP since U-matrix acts between zero energy states and can be regarded as a collection of orthonormal M-matrices, which generalize the ordinary S-matrix and define what might be called a complex square root of density matrix so that kind of a square root of thermodynamics at single particle level justifying also p-adic mass calculations based on p-adic thermodynamics is in question.

2. The hierarchy of Planck constants labelling a hierarchy of quantum criticalities is a further new element having important implications for conciousness and biology.

3. Hyper-finite factors of type II_1 represent an additional technical complication requiring separate treatment of NMP taking into account finite measurement resolution realized in terms of inclusions of these factors.

NMP has wide range of important implications.

1. In particular, one must give up the standard view about second law and replace it with NMP taking into account the hierarchy of CDs assigned with ZEO and dark matter hierarchy labelled by the values of Planck constants, as well as the effects due to NE. The breaking of second law in standard sense is expected to take place and be crucial for the understanding of evolution.

2. Self hierarchy having the hierarchy of CDs as embedding space correlate leads naturally to a description of the contents of consciousness analogous to thermodynamics except that the entropy is replaced with negentropy.

3. In the case of living matter NMP allows to understand the origin of metabolism. NMP demands that self generates somehow negentropy: otherwise a state function reduction to tjhe opposite boundary of CD takes place and means death and re-incarnation of self. Metabolism as gathering of nutrients, which by definition carry NE is the manner to avoid this fate. This

leads to a vision about the role of NE in the generation of sensory qualia and a connection with metabolism. Metabolites would carry NE and each metabolite would correspond to a particular qualia (not only energy but also other quantum numbers would correspond to metabolites). That primary qualia would be associated with nutrient flow is not actually surprising!

4. NE leads to a vision about cognition. Negentropically entangled state consisting of a superposition of pairs can be interpreted as a conscious abstraction or rule: negentropically entangled Schrödinger cat knows that it is better to keep the bottle closed.

5. NMP implies continual generation of NE. One might refer to this ever expanding universal library as "Akaschic records". NE could be experienced directly during the repeated state function reductions to the passive boundary of CD - that is during the life cycle of sub-self defining the mental image. Another, less feasible option is that interaction free measurement is required to assign to NE conscious experience. As mentioned, qualia characterizing the metabolite carrying the NE could characterize this conscious experience.

6. A connection with fuzzy qubits and quantum groups with NE is highly suggestive. The implications are highly non-trivial also for quantum computation allowed by weak form of NMP since NE is by definition stable and lasts the lifetime of self in question.

What Scattering Amplitudes Should Look Like?

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyperfinite factors of type II_1 meant a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the ZEO applied already earlier in TGD inspired cosmology. ZEO motivated the replacement of the term "S-matrix" with "M-matrix".

The general mathematical concepts are not enough to get to the level of concrete scattering amplitudes. The notion of preferred extremal inspiring the notion of generalized Feynman diagram is central in bringing in this concretia. The very notion of preferred extremals means that ordinary Feynman diagrams providing a visualization of path integral are not in question. Generalized Feynman diagrams have 4-D Euclidian space-time regions (wormhole contacts) as lines, and light-like partonic orbits of 2-surfaces as 3-D lines. String world sheets carrying fermions are also present and have 1-D boundaries at the light-like orbits of partonic 2-surfaces carrying fermion number and light-like 8-momenta suggesting strongly 8-D generalization of twistor approach.

The resulting objects could be indeed seen as generalizations of twistor diagrams rather than Feynman diagrams. The preferred extremal property strongly encourages the old and forgotten TGD inspried idea as sequences of algebraic operations with product and co-product representing 3-vertices. The sequences connect given states at the opposite boundaries of CD and have minimal length. The algebraic structure in question would be the Yangian of the super-symplectic algebra with generators identified as super-symplectic charges assignable to strings connecting partonic 2-surfaces.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction requires the introduction of generalization of twistor approach to 8-D context.

TGD View about Coupling Constant Evolution?

New results related to the TGD view about coupling constant evolution are discussed. The results emerge from the discussion of the recent claim of Atyiah that fine structure constant could be understood purely mathematically. The new view allows to understand the recently introduced

TGD based construction of scattering amplitudes based on the analog of micro-canonical ensemble as a cognitive representation for the much more complex construction of full scattering amplitudes using real numbers rather than p-adic number fields. This construction utilizes number theoretic discretization of space-time surface inducing that of "world of classical worlds" (WCW) and makes possible adelization of quantum TGD.

The understanding of coupling constant evolution has been one of most longstanding problems of TGD and I have made several proposals during years.

Could number theoretical constraints fix the evolution? Adelization suffers from serious number theoretical problem due to the fact that the action exponentials do not in general exist p-adically for given adele. The solution of the problem turned out to be trivial. The exponentials disappear from the scattering amplitudes! Contrary to the first beliefs, adelization does not therefore seem to determine coupling constant evolution.

TGD view about cosmological constant turned out to be the solution of the problem. The formulation of the twistor lift of Kähler action led to a rather detailed view about the interpretation of cosmological constant as an approximate parameterization of the dimensionally reduced 6-D Kähler action (or energy) allowing also to understand how it can decrease so fast as a function of p-adic length scale. In particular, a dynamical mechanism for the dimensional reduction of 6-D Kähler action giving rise to the induction of the twistor structure and predicting this evolution emerges.

In standard QFT view about coupling constant evolution ultraviolet cutoff length serves as the evolution parameter. TGD is however free of infinities and there is no cutoff parameter. It turned out cosmological constant replaces this parameter and coupling constant evolution is induced by that for cosmological constant from the condition that the twistor lift of the action is not affected by small enough modifications of the moduli of the induced twistor structure. The moduli space for them corresponds to rotation group $SO(3)$. This leads to explicit evolution equations for α_K, which can be studied numerically.

The approach is also related to the view about coupling constant evolution based on the inclusions of hyper-finite factors of type II_1, and it is proposed that Galois group replaces discrete subgroup of $SU(2)$ leaving invariant the algebras of observables of the factors appearing in the inclusion.

Some Questions about Coupling Constant Evolution

In this chapter questions related to the hierarchy of Planck constants and p-adic coupling constant evolution (CCE) in the TGD framework are considered.

1. Is p-adic length scale hypothesis (PLS) correct in this recent form and can one deduce this hypothesis or its generalization from the basic physics of TGD defined by Kähler function of the "world of classical worlds" (WCW)? The fact, that the scaling of the roots of polynomial does not affect the algebraic properties of the extension strongly suggests that p-adic prime does not depend on purely algebraic properties of EQ. In particular, the proposed identification of p as a ramified prime of EQ could be wrong.

   Number theoretical universality suggests the formula $exp(\Delta K) = p^n$, where ΔK is the contribution to Kähler function of WCW for a given space-time surface inside causal diamond (CD).

2. The understanding of p-adic length scale evolution is also a problem. The "dark" CCE would be $\alpha_K = g_K^2/2h_{eff} = g_K^2/2nh_0$, and the PLS evolution $g_K^2(k) = g_K^2(max)/k$ should define independent evolutions since scalings commute with number theory. The total evolution $\alpha_K = \alpha_K(max)/nk$ would induce also the evolution of other coupling strengths if the coupling strenghts are related to α_K by Möbius transformation as suggested.

3. The formula $h_{eff} = nh_0$ involves the minimal value h_0. How could one determine it? p-Adic mass calculations for $h_{eff} = h$ lead to the conclusion that the CP_2 scale R is roughly $10^{7.5}$ times longer than Planck length l_P. Classical argument however suggests $R \simeq l_P$. If one assumes $h_{eff} = h_0$ in the p-adic mass calculations, this is indeed the case for $h/h_0 = (R(CP_2)/l_P)^2$. This ratio follows from number theoretic arguments as $h/h_0 = n_0 = (7!)^2$. This gives $\alpha_K = n_0/kn$, and perturbation theory can converge even for $n = 1$ for sufficiently long p-adic length scales. Gauge coupling strengths are predicted to be practically zero at

gravitational flux tubes so that only gravitational interaction is effectively present. This conforms with the view about dark matter.

4. Nottale hypothesis predicts gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ ($\beta_0 = v_0/c$ is velocity parameter), which has gigantic values. Gravitational fine structure constant is given by $\alpha_{gr} = \beta_0/4\pi$. Kepler's law $\beta^2 = GM/r = r_S/2r$ suggests length scale evolution $\beta^2 = xr_S/2L_N = \beta_{0,max}^2/N^2$, where x is proportionality constant, which can be fixed.

   Phase transitions changing β_0 are possible at $L_N/a_{gr} = N^2$ and these scales correspond to radii for the gravitational analogs of the Bohr orbits of hydrogen. p-Adic length scale hierarchy is replaced by that for the radii of Bohr orbits. The simplest option is that β_0 obeys a CCE induced by α_K.

   This picture conforms with the existing applications and makes it possible to understand the value of β_0 for the solar system, and is consistent with the application to the superfluid fountain effect.

The Recent View about SUSY in TGD Universe

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge would appear as space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with the precise understanding of quantum criticality and understand the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials. The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is indeed bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma_s^\alpha = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

Super-Dirac action vanishes on-mass-shell. The proposed construction relying on ZEO allows however to get scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation is however needed and makes possible to express the derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution conforms with the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

Could ZEO provide a new approach to the quantization of fermions?

The exact details of the quantization of fermions have remained open in TGD framework. The basic problem is the possibility of divergences coming from anti-commutators of fermions expected to involve delta functions in the continuum case. In standard framework normal ordering saves from these divergences for the "free" part of the action but higher order terms give the usual divergences of quantum field theories. In supersymmetric theories the normal ordering divergences however cancel.

In TGD the bosonic divergenges are absent due to the generalization of the notion of point-like particle to 3-surface. In fermionic sector normal ordering divergences cancel in unique number theoretic discretization based on what I call cognitive representations but in continuum case the situation is unclear.

Induction procedure plays a key role in the construction of classical TGD. The longstanding question has been whether the induction of spinor structure could be generalized to the induction of second quantization of free fermions at the level of 8-D embedding space to the level of space-time. The problem is that the anticommutators are 8-D delta functions in continuum case and could induce rather horrible divergences. It will be found that zero energy ontology (ZEO) and new view about space-time and particles allow to modify the standard quantization procedure by making modified Dirac action bi-local so that one gets rid of divergences. Also the multi-local Yangian algebras proposed on basis of physical intuition to be central in TGD emerge automatically.

Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory

The understanding of the unitarity of the S-matrix has remained a major challenge of Topological Geometrodynamics (TGD) for 4 decades. It has become clear that some basic principle is still lacking. Assigning S-matrix to a unitary evolution works in non-relativistic theory but fails already in the generic quantum field theory (QFT). The solution of the problem turned out to be extremely simple. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space would replace unitary S-matrix.

An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of Teichmueller matrix. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong from of holography (SH), the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmueller condition guaranteeing non-negative probabilities.

Equivalence Principle generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

Breakthrough in understanding of M^8-H duality

A critical re-examination of M^8-H duality is discussed. M^8-H duality is one of the cornerstones of Topological Geometrodynamics (TGD). The original version of M^8-H duality assumed that space-time surfaces in M^8 can be identified as associative or co-associative surfaces. If the surface has associative tangent or normal space and contains a complex or co-complex surface, it can be mapped to a 4-surface in $H=M^4\times CP_2$.

Later emerged the idea that octonionic analyticity realized in terms of real polynomials P algebraically continued to polynomials of complexified octonion could fulfill the dream. The vanishing of the real part $Re_Q(P)$ (imaginary part $Im_Q(P)$) in the quaternionic sense would give rise to an associative (co-associative) space-time surface.

The realization of the general coordinate invariance motivated the notion of strong form of holography (SH) in H allowing realization of a weaker form of M^8-H duality by assuming that associativity/co-associativity conditions are needed only at 2-D string world sheet and partonic 2-surfaces and possibly also at their light-like 3-orbits.

The outcome of the re-examination yielded both positive and negative surprises.

1. Although no interesting associative space-time surfaces are possible, every distribution of normal associative planes (co-associativity) is integrable.

2. Another positive surprise was that Minkowski signature is the only possible option. Equivalently, the image of M^4 as real co-associative subspace of O_c (complex valued octonion norm squared is real valued for them) by an element of local G_2 or rather, its subgroup $SU(3)$, gives a real co-associative space-time surface.

3. The conjecture based on naive dimensional counting, which was not correct, was that the polynomials P determine these 4-D surfaces as roots of $Re_Q(P)$. The normal spaces of these surfaces possess a fixed 2-D commuting sub-manifold or possibly their distribution allowing the mapping to H by M^8-H duality as a whole.

   If this conjecture were correct, strong form of holography (SH) would not be needed and would be replaced with extremely powerful number theoretic holography determining space-time surface from its roots and selection of real subspace of O_c characterizing the state of motion of a particle. erate

4. The concrete calculation of the octonion polynomial was the most recent step - carried already earlier [L49, L50, L51] but without realizing the implications of the extremely simple outcome. The imaginary part of the polynomial is proportional to the imaginary part of octonion itself. It turned out that the roots $P=0$ of the octonion polynomial P are 12-D complex surfaces

in O_c rather than being discrete set of points defined as zeros $X = 0, Y = 0$ of two complex functions of 2 complex arguments. The analogs of branes are in question. Already earlier 6-D real branes assignable to the roots of the real polynomial P at the light-like boundary of 8-D light-cone were discovered: also their complex continuations are 12-D [L81, L91].

5. P has quaternionic de-composition $P = Re_Q(P) + I_4 Im_Q(P)$ to real and imaginary parts in a quaternionic sense. The naive expectation was that the condition $X = 0$ implies that the resulting surface is a 4-D complex surface X^4_c with a 4-D real projection X^4_r, which could be co-associative.

   The expectation was wrong! The equations $X = 0$ and $Y = 0$ involve the same(!) complex argument o_c^2 as a complex analog for the Lorentz invariant distance squared from the tip of the light-cone. This implies a cold shower. Without any additional conditions, $X = 0$ conditions have as solutions 7-D complex mass shells H^7_c determined by the roots of P. The explanation comes from the symmetries of the octonionic polynomial.

   There are solutions $X = 0$ and $Y = 0$ only if the two polynomials considered have a common a_c^2 as a root! Also now the solutions are complex mass shells H^7_c.

How could one obtain 4-D surfaces X^4_c as sub-manifolds of H^7_c? One should pose a condition eliminating 4 complex coordinates: after that a projection to M^4 would produce a real 4-surface X^4 .

1. The key observation is that G_2 acts as the automorphism group of octonions respects the co-associativity of the 4-D real sub-basis of octonions. Therefore a local G_2 gauge transformation applied to a 4-D co-associative sub-space M^4 gives a co-associative four-surface as a real projection. Octonion analyticity would correspond to G_2 gauge transformation: this would realize the original idea about octonion analyticity.

2. A co-associative X^4_c satisfying also the conditions posed by the existence of $M^8 - H$ duality is obtained by acting with a local SU_3 transformation g to a co-associative plane $M^4 \subset M^8_c$. If the image point $g(p)$ is invariant under $U(2)$, the transformation corresponds to a local CP_2 element and the map defines $M^8 - H$ duality even if the co-associativity in geometric sense were not satisfied.

   The co-associativity of the plane M^4 is preserved in the map because G_2 acts as an automorphism group of the octonions. If this map also preserves the value of 4-D complex mass squared, one can require that the intersections of X^4_c with H^7_c correspond to 3-D complex mass shells. One obtains holography with mass shells defined by the roots of P giving boundary data. The condition H images are analogous to Bohr orbits, corresponds to number theoretic holography.

   The group $SU(3)$ has interpretation as a Kac-Moody type analog of color group and the map defining space-time surface. This picture conforms with the H-picture in which gluon gauge potentials are identified as color gauge potentials. Note that at QFT limit the gauge potentials are replaced by their sums over parallel space-time sheets to give gauge fields as the space-time sheets are approximated with a single region of Minkowski space.

3. Octonionic Dirac equation as analog of momentum space variant of ordinary Dirac equation forces the interpretation of M^8 as an analog of momentum space and Uncertainty Principle forces to modify the map $M^4 \subset M^8 \to M^4 \subset H$ from an identification to an almost inversion. The octonionic Dirac equation reduces to the mass shell condition $m^2 = r_n$, where r_n is a root of the polynomial P defining the 4-surface but only in the co-associative case.

This picture combined with zero energy ontology leads also to a view about quantum TGD at the level of M^8. A local $SU(3)$ element defining 4-surface in M^8, which suggests a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by P. The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

New findings related to the number theoretical view of TGD

The geometric vision of TGD is rather well-understood but there is still a lot of fog in the number theoretic vision.

1. There are uncertainties related to the interpretation of the 4-surfaces in M^8 what the analogy with space-time surface in $H = M^4 \times CP_2$ time evolution of 3-surface in H could mean physically?
2. The detailed realization of $M^8 - H$ duality involves uncertainties: in particular, how the complexification of M^8 to M_c^8 can be consistent with the reality of $M^4 \subset H$.
3. The formulation of the number theoretic holography with dynamics based on associativity involves open questions. The polynomial P determining the 4-surface in M^8 doesn't fix the 3-surfaces at mass shells corresponding to its roots. Quantum classical correspondence suggests the coding of fermionic momenta to the geometric properties of 3-D surfaces: how could this be achieved?
4. How unique is the choice of 3-D surfaces at the mass shells $H_m^3 \subset M^4 \subset M^8$ and whether a strong form of holography as almost $2 \rightarrow 4$ holography could be realized and make this choice highly unique.

These and many other questions motivated this article and led to the observation that the model geometries used in the classification of 3-manifolds seem to be rather closely related to the known space-time surfaces extremizing practically any general coordinate invariant action constructible in terms of the induced geometry.

The 4-surfaces in M^8 would define coupling constant evolutions for quantum states as analogs of and mappable to time evolutions at the level of H and obeying conservation laws associated with the dual conformal invariance analogous to that in twistor approach.

The momenta of fundamental fermions in the quantum state would be coded by the cusp singularities of 3-surfaces at the mass shells of M^8 and also its image in H provided by $M^8 - H$ duality. One can consider the possibility of $2 \rightarrow 3$ holography in which the boundaries of fundamental region of H^3/Γ is 2-D hyperbolic space H^2/Γ so that TGD could to high degree reduce to algebraic geometry.

Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole

The theoretical framework behind TGD involves several different strands and the goal is to unify them to a single coherent whole. TGD involves number theoretic and geometric visions about physics and $M^8 - H$ duality, analogous to Langlands duality, is proposed to unify them. Also quantum classical correspondence (QCC) is a central aspect of TGD. One should understand both the $M^8 - H$ duality and QCC at the level of detail.

The following mathematical notions are expected to be of relevance for this goal.

1. Von Neumann algebras, call them M, in particular hyperfinite factors of type II_1 (HFFs), are in a central role. Both the geometric and number theoretic side, QCC could mathematically correspond to the relationship between M and its commutant M'.

   For instance, symplectic transformations leave induced Kähler form invariant and various fluxes of Kähler form are symplectic invariants and correspond to classical physics commuting with quantum physics coded by the super symplectic algebra (SSA). On the number theoretic side, the Galois invariants assignable to the polynomials determining space-time surfaces are analogous classical invariants.
2. The generalization of ordinary arithmetics to quantum arithmetics obtained by replacing $+$ and $\times$ with $\oplus$ and $\otimes$ allows us to replace the notions of finite and p-adic number fields with their quantum variants. The same applies to various algebras.
3. Number theoretic vision leads to adelic physics involving a fusion of various p-adic physics and real physics and to hierarchies of extensions of rationals involving hierarchies of Galois groups involving inclusions of normal subgroups. The notion of adele can be generalized by replacing various p-adic number fields with the p-adic representations of various algebras.

4. The physical interpretation of the notion of infinite prime has remained elusive although a formal interpretation in terms of a repeated quantization of a supersymmetric arithmetic QFT is highly suggestive. One can also generalize infinite primes to their quantum variants. The proposal is that the hierarchy of infinite primes generalizes the notion of adele.

The formulation of physics as Kähler geometry of the "world of classical worlds" (WCW) involves f 3 kinds of algebras A; supersymplectic isometries SSA acting on $\delta M^4_+ \times CP_2$, affine algebras Aff acting on light-like partonic orbits, and isometries I of light-cone boundary δM^4_+, allowing hierarchies A_n.

The braided Galois group algebras at the number theory side and algebras $\{A_n\}$ at the geometric side define excellent candidates for inclusion hierarchies of HFFs. $M^8 - H$ duality suggests that n corresponds to the degree $n$of the polynomial P defining space-time surface and that the n roots of P correspond to n braid strands at H side. Braided Galois group would act in A_n and hierarchies of Galois groups would induce hierarchies of inclusions of HFFs. The ramified primes of P would correspond to physically preferred p-adic primes in the adelic structure formed by p-adic variants of A_n with $+$ and $\times$ replaced with $\oplus$ and $\otimes$.

Homology of "world of classical worlds" in relation to Floer homology and quantum homology

One of the mathematical challenges of TGD is the construction of the homology of "world of classical worlds" (WCW). The generalization of Floer homology looks rather obvious in the zero ontology (ZEO) based view about quantum TGD. ZEO, the notion of preferred extremal (PE), and the intuitive connection between the failure of strict non-determinism and criticality are essential elements. The homology group is defined in terms of the free group formed by preferred extremals $PE(X^3, Y^3)$ for which X^3 is a stable maximum of Kähler function K associated with the passive boundary of CD and Y^3 associated with the passive boundary is a more general critical point.

The identification of PEs as minimal surfaces with lower-dimensional singularities as loci of instabilities implying non-determinism allows to assign to the set $PE(X^3, Y^3_i)$ numbers $n(X^3, Y^3_i \rightarrow Y^3_j)$ as the number of instabilities of singularities leading from Y^3_i to Y^3_j and define the analog of criticality index (number of negative eigenvalues of Hessian of function at critical point) as number $n(X^3, Y^3_i) = \sum_j n(X^3, Y^3_i \rightarrow Y^3_j)$. The differential d defining WCW homology is defined in terms of $n(X^3, Y^3_i \rightarrow Y^3_j)$ for pairs Y^3_i, Y^3_j such that $n(X^3, Y^3_j) - n(X^3, Y^3_i) = 1$ is satisfied.

Part I

THE RECENT VIEW ABOUT FIELD EQUATIONS

Chapter 2

Basic Extremals of the Kähler Action

2.1 Introduction

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable. The progress made during next five years led to a detailed understanding of quantum TGD at the fundamental parton level and this provides considerable additional insights concerning the interpretation of field equations.

2.1.1 About The Notion Of Preferred Extremal

The notion of preferred extremal has been central in classical TGD although the known solutions could be preferred or not: the main challenge has been to understand what "preferred" could mean.

In zero energy ontology (ZEO) one can also consider the releaving possibility that all extremals are preferred ones! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. The non-determinism of Kähler action suggests that "preferred" could be obsolete in given length scale resolution.

Actually all the discussions of this chapter are about known extremals in general so that the attribute "preferred" is not relevant for them.

2.1.2 Beltrami Fields And Extremals

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that preferred extremals satisfy the condition. The vanishing of the Lorentz 4-force in turn implies a local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. If Kähler action is defined by CP_2 Kähler form alone, the condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteeing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

An important outcome is the notion of Hamilton-Jacobi structure meaning dual slicings of M^4 projection of preferred extremals to string world sheets and partonic 2-surfaces. The necessity of this slicing was discovered years later from number theoretic compactification and is now a key element of quantum TGD allowing to deduce Equivalence Principle in its stringy form from quantum TGD and formulate and understand quantum TGD in terms of Kähler-Dirac action assignable to Kähler action. The conservation of Noether charges associated with Kähler-Dirac action requires the vanishing of the second second variation of Kähler action for preferred extremals. Preferred extremals would thus define space-time representation for quantum criticality. Infinite-dimensional variant for the hierarchy of criticalities analogous to the hierarchy assigned to the extrema of potential function with levels labeled by the rank of the matrix defined by the second derivatives of the potential function in catastrophe theory would suggest itself.

A natural interpretation for deformations would be as conformal gauge symmetries due to the non-determinism of Kähler action. They would transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They would preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

2.1.3 In What Sense Field Equations Could Mimic Dissipative Dynamics?

By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. The nontrivial question is what this means in TGD framework.

1. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property of Kähler action abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context. The general solution ansatz discussed in the last section of the chapter assumes that all conserved isometry currents are proportional to instanton current so that various charges are conserved separately for all flow lines: this means esssentially the integrability of the theory. This ansatz is forced by the hypothesis that TGD reduces to almost topological QFT and this idea. The basic consequence is that dissipation is impossible classically.

2. A more radical view inspired by zero energy ontology is that the light-like 3-surfaces and corresponding space-time regions with Euclidian signature defining generalized Feynman diagrams provide a space-time representation of dissipative dynamics just as they provide this representation in quantum field theory. Minkowskian regions would represent empty space so that the vanishing of Lorentz 4-force and absence of dissipation would be natural. This would mean very precise particle field duality and the topological pattern associated with the generalized Feynman diagram would represent dissipation. One could also interprete dissipation as transfer of energy between sheets of the many-sheeted space time and thus as an essentially topological phenomenon. This option seems to be the only viable one.

2.1.4 The Dimension Of CP_2 Projection As Classifier For The Fundamental Phases Of Matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. The natural guess would be that this phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. It seems however that preferred extremals can correspond only to small perturbations of these extremals resulting by topological condensation of CP_2 type vacuum extremals and through topological condensation to larger space-time sheets. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase could be seen as the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

The original proposal was that $D(CP_2) = 4$ phase is completely chaotic. This is not true if the reduction to almost topological QFT takes place. This phase must correspond to Maxwellian phase with a vanishing Kähler current as concluded already earlier. Various isometry currents are however proportional to the instanton current and conserved along the flow lines of the instanton current whose flow parameter extends to a global coordinate. Hence a completely chaotic phase is not in question even in this case.

2.1.5 Specific Extremals Of Kähler Action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynman diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

3. There are also various non-vacuum extremals.

   (a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

(b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

(c) In the so called Maxwell phase, ordinary Maxwell equations for the induced Kähler field would be satisfied in an excellent approximation. It is however far from clear whether this kind of extremals exist. Their non-existence would actually simplify the theory enormously since all extremals would have quantal character. The recent view indeed is that Maxwell phase makes sense only as as genuinely many-sheeted structure and solutions of Maxwell's equation appear only at the level of effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Gauge potentials in effective space-time are determined in the same way. Since the gauge potentials sum up, it is possible to understand how field configurations of Maxwell's theory emerge at this limit.

2.1.6 The Weak Form Of Electric-Magnetic Duality And Modification Of Kähler Action

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

Generalized Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector and there are reasons to hope that TGD is completely integrable theory.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L17].

2.2 General Considerations

The solution families of field equations studied in this chapter were found already during eighties. The physical interpretation turned out to be the really tough problem. What is the principle selecting preferred extremals of Kähler action as analogs of Bohr orbits assigning to 3-surface X^3 a unique space-time surface $X^4(X^3)$? Does Equivalence Principle hold true and if so, in what sense? These have been the key questions. The realization that light-like 3-surfaces X_l^3 associated with the light-like wormhole throats at which the signature of the induced metric changes from Minkowskian to Euclidian led to the formulation of quantum TGD in terms of second quantized induced spinor fields at these surfaces. Together with the notion of number theoretical compactification this approach allowed to identify the conditions characterizing the preferred extremals. What is remarkable that these conditions are consistent with what is known about extremals.

Also a connection with string models emerges and partial understanding of the space-time realization of Equivalence Principle suggests itself. However, much more general argument allows to understand how GRT space-time appears from the many-sheeted space-time of TGD (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) as effective concept [K121]: this more general view is not in conflict with the much earlier proposal discussed below.

In this section the theoretical background behind field equations is briefly summarized. I will not repeat the discussion of previous two chapters [**?**, K48] summarizing the general vision about many-sheeted space-time, and consideration will be restricted to those aspects of vision leading to direct predictions about the properties of preferred extremals of Kähler action.

2.2.1 Number Theoretical Compactification And $M^8 - H$ Duality

The notion of hyper-quaternionic and octonionic manifold makes sense but it not plausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Situation changes if H is replaced with hyper-octonionic M^8. Suppose that $X^4 \subset M^8$ consists of hyper-quaternionic and co-hyper-quaternionic regions. The basic observation is that the hyper-quaternionic sub-spaces of M^8 with a fixed hyper-complex structure (containing in their tangent space a fixed hyper-complex subspace M^2 or at least one of the light-like lines of M^2) are labeled by points of CP_2. Hence each hyper-quaternionic and co-hyper-quaternionic four-surface of M^8 defines a 4-surface of $M^4 \times CP_2$. One can loosely say that the number-theoretic analog of spontaneous compactification occurs: this of course has nothing to do with dynamics.

This picture was still too naïve and it became clear that not all known extremals of Kähler action contain fixed $M^2 \subset M^4$ or light-like line of M^2 in their tangent space.

1. The first option represents the minimal form of number theoretical compactification. M^8 is interpreted as the tangent space of H. Only the 4-D tangent spaces of light-like 3-surfaces X^3_l (wormhole throats or boundaries) are assumed to be hyper-quaternionic or co-hyper-quaternionic and contain fixed M^2 or its light-like line in their tangent space. Hyper-quaternionic regions would naturally correspond to space-time regions with Minkowskian signature of the induced metric and their co-counterparts to the regions for which the signature is Euclidian. What is of special importance is that this assumption solves the problem of identifying the boundary conditions fixing the preferred extremals of Kähler action since in the generic case the intersection of M^2 with the 3-D tangent space of X^3_l is 1-dimensional. The surfaces $X^4(X^3_l) \subset M^8$ would be hyper-quaternionic or co-hyper-quaternionic but would not allow a local mapping between the 4-surfaces of M^8 and H.
2. One can also consider a more local map of $X^4(X^3_l) \subset H$ to $X^4(X^3_l) \subset M^8$. The idea is to allow $M^2 \subset M^4 \subset M^8$ to vary from point to point so that $S^2 = SO(3)/SO(2)$ characterizes the local choice of M^2 in the interior of X^4. This leads to a quite nice view about strong geometric form of $M^8 - H$ duality in which M^8 is interpreted as tangent space of H and $X^4(X^3_l) \subset M^8$ has interpretation as tangent for a curve defined by light-like 3-surfaces at X^3_l and represented by $X^4(X^3_l) \subset H$. Space-time surfaces $X^4(X^3_l) \subset M^8$ consisting of hyper-quaternionic and co-hyper-quaternionic regions would naturally represent a preferred extremal of E^4 Kähler action. The value of the action would be same as CP_2 Kähler action. $M^8 - H$ duality would apply also at the induced spinor field and at the level of WCW . The possibility to assign $M^2(x) \subset M^4$ to each point of M^4 projection $P_{M^4}(X^4(X^3_l))$ is consistent with what is known about extremals of Kähler action with only one exception: CP_2 type vacuum extremals. In this case M^2 can be assigned to the normal space.
3. Strong form of $M^8 - H$ duality satisfies all the needed constraints if it represents Kähler isometry between $X^4(X^3_l) \subset M^8$ and $X^4(X^3_l) \subset H$. This implies that light-like 3-surface is mapped to light-like 3-surface and induced metrics and Kähler forms are identical so that also Kähler action and field equations are identical. The only differences appear at the level of induced spinor fields at the light-like boundaries since due to the fact that gauge potentials are not identical.
4. The map of $X^3_l \subset H \rightarrow X^3_l \subset M^8$ would be crucial for the realization of the number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as those preferred coordinates in which the points of embedding space are rational/algebraic. Thus the point of $X^4 \subset H$ is algebraic if it is mapped to algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactification could thus be motivated by the number theoretical universality.
5. The possibility to use either M^8 or H picture might be extremely useful for calculational purposes. In particular, M^8 picture based on $SO(4)$ gluons rather than $SU(3)$ gluons could perturbative description of low energy hadron physics. The strong $SO(4)$ symmetry of low energy hadron physics can be indeed seen direct experimental support for the $M^8 - H$ duality.

Number theoretical compactification has quite deep implications for quantum TGD and is actually responsible for most of the progress in the understanding of the mathematical structure of quantum TGD. A very powerful prediction is that preferred extremals should allow slicings to either stringy world sheets or dual partonic 2-surfaces as well as slicing by light-like 3-surfaces. Both predictions are consistent with what is known about extremals.

1. If the distribution of planes $M^2(x)$ is integrable, it is possible to slice $X^4(X^3)$ to a union of 2-dimensional surfaces having interpretation as string world sheets and dual 2-dimensional copies of partonic surfaces X^2. This decomposition defining 2+2 Kaluza-Klein type structure could realize quantum gravitational holography and might allow to understand Equivalence Principle at space-time level in the sense that dimensional reduction defined by the integral of Kähler action over the 2-dimensional space labeling stringy world sheets gives rise to the analog of stringy action and one obtains string model like description of quantum TGD as dual for a description based on light-like partonic 3-surfaces. String tension is not however equal to the inverse of gravitational constant as one might naïvely expect but the connection is more delicate. As already mentioned, TGD-GRT connection and EP can be understood at general level only from very general arguments [K121].
2. Second implication is the slicing of $X^4(X^3_l)$ to light-like 3-surfaces Y^3_l "parallel" to X^3_l. Also this slicing realizes quantum gravitational holography if one requires General Coordinate Invariance in the sense that the Dirac determinant differs for two 3-surfaces Y^3_l in the slicing only by an exponent of a real part of a holomorphic function of WCW complex coordinates giving no contribution to the Kähler metric.
3. The square of the Dirac determinant would be equal to the modulus squared for the exponent of vacuum functional and would be formally defined as the product of conformal weights assignable to the modes of the Dirac operator at string world sheets at the ends of strings at partonic 2-surfaces defining the ends of Y^3_l. The detailed definition requires to specify what one means with the conformal weights assignable with the modes of the Kähler-Dirac operator.
4. The localization of the modes of Kähler-Dirac operator to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) [K127] following from the condition that electromagnetic charges of the modes is well-defined is very strong restriction and reduces Dirac determinant to a product of Dirac determinants assignable with these 2-surfaces.

2.2.2 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries. The natural identification would be as conformal symmetries. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

For instance, the natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??**in the appendix of this book).

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).
2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.
3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.
4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

The basic question is whether number theoretic view about preferred extremals imply absolute minimization or something analogous to it.

1. The number theoretic conditions defining preferred extremals are purely algebraic and make sense also p-adically and this is enough since p-adic variants of field equations make sense although the notion of Kähler action does not make sense as integral. Despite this the identification of the vacuum functional as exponent of Kähler function as Dirac determinant allows to define the exponent of Kähler function as a p-adic number [K127].
2. The general objection against all extremization principles is that they do not make sense p-adically since p-adic numbers are not well-ordered.
3. These observations do not encourage the idea about equivalence of the two approaches. On the other hand, real and p-adic sectors are related by algebraic continuation and it could be quite enough if the equivalence were true in real context alone.

The finite-dimensional analogy allows to compare absolute minimization and criticality with each other.

1. Absolute minimization would select the branch of Thom's catastrophe surface with the smallest value of potential function for given values of control variables. In general this value would not correspond to criticality since absolute minimization says nothing about the values of control variables (zero modes).

2. Criticality forces the space-time surface to belong to the bifurcation set and thus fixes the values of control variables, that is the interior of 3-surface assignable to the partonic 2-surface, and realized holography. If the catastrophe has more than $N = 3$ sheets, several preferred extremals are possible for given values of control variables fixing $X^3(X^2)$ unless one assumes that absolute minimization or some other criterion is applied in the bifurcation set. In this sense absolute minimization might make sense in the real context and if the selection is between finite number of alternatives is in question, it should be possible carry out the selection in number theoretically universal manner.

It must be emphasized that there are several proposals for what preferred extremal property could mean. For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of embedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of embedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K15] defining also this kind of slicing and the approaches could be equivalent. A further approach is based on the identification of preferred extremal property as quantum criticality [K15].

2.2.3 Can One Determine Experimentally The Shape Of The Space-Time Surface?

The question "Can one determine experimentally the shape of the space-time surface?" does not relate directly to the topic of this chapter in technical sense, and the only excuse for its inclusion is the title of this section plus the fact that the general conceptual framework behind quantum TGD assumes an affirmative answer to this question. If physics were purely classical physics, operationalism in the strong sense of the word would require that one can experimentally determine the shape of the space-time as a surface of the embedding space with arbitrary accuracy by measuring suitable classical observables. In quantum physics situation is considerably more complex and quantum effects are both a blessing and a curse.

Measuring classically the shape of the space-time surface

Consider first the purely classical situation to see what is involved.

1. All classical gauge fields are expressible in terms of CP_2 coordinates and their space-time gradients so that the measurement of four field quantities with some finite resolution in some space-time volume could in principle give enough information to deduce the remaining field quantities. The requirement that space-time surface corresponds to an extremal of Kähler action gives a further strong consistency constraint and one can in principle test whether this constraint is satisfied. A highly over-determined system is in question.

2. The freedom to choose the space-time coordinates freely causes complications and it seems that one must be able to determine also the distances between the points at which the field quantities are determined. At purely classical Riemannian level this boils down to the measurement of the induced metric defining classical gravitational field. In macroscopic length scales one could base the approach to iterative procedure in which one starts from the assumption that the coordinates used are Minkowski coordinates and gravitational corrections are very weak.

3. The measurement of induced Kähler form in some space-time volume determines space-time surface only modulo canonical transformations of CP_2 and isometries of the embedding space. If one measures classical electromagnetic field, which is not canonical invariant in general case, with some precision, one can determine to what kind of surface space-time region corresponds apart from the action of the isometries of H.

Quantum measurement of the shape of the space-time surface

In practice the measurement of the shape of the space-time surface is necessarily a bootstrap procedure based on the model for space-time region and on the requirement of internal consistency. Many-sheeted space-time and quantum phenomena produce considerable complications but also provide universal measurement standards.

Consider first how quantum effects could help to measure classical fields and distances.

1. The measurement of distances by measuring first induced metric at each point of space-time sheet is rather unpractical procedure. Many-sheeted space-time however comes in rescue here. p-Adic length scale hypothesis provides a hierarchy of natural length scales and one can use p-adic length and time scales as natural units of length and time: space-time sheets serve as meter sticks. For instance, length measurement reduces in principle to a finite number of operations using various space-time sheets with standardized lengths given by p-adic length scales. Also various transition frequencies and corresponding wavelengths provide universal time and length units. Atomic clock provides a standard example of this kind of time unit. A highly nontrivial implication is the possibility to deduce the composition of distant star from its spectral lines. Without p-adic length scale hypothesis the scales for the mass spectra of the elementary particles would be variable and atomic spectra would vary from point to point in TGD universe.

   Do the p-adic length scales correspond to the length units of the induced metric or of M^4_+ metric? If the topological condensation a meter stick space-time sheet at a larger space-time sheet does not stretch the meter stick but only bends it, the length topologically condensed meter stick in the induced metric equals to its original length measured using M^4_+ metric.

2. If superconducting order parameters are expressible in terms of the CP_2 coordinates (there is evidence for this, see the chapter "Macroscopic quantum phenomena and CP_2 geometry"), one might determine directly the CP_2 coordinates as functions of Minkowski coordinates and this would allow to estimate all classical fields directly and thus to deduce strong consistency constraints.

3. At quantum level only the fluxes of the classical fields through surface areas with some minimum size determined by the length scale resolution can be measured. In case of magnetic fields the quantization of the magnetic flux simplifies the situation dramatically. Topological field quantization quite generally modifies the measurement of continuous field variables to the measurement of fluxes. Interestingly, the construction of WCW geometry uses as WCW coordinates various electric and magnetic fluxes over 2-dimensional cross sections of 3-surface.

Quantum effects introduce also difficulties and restrictions.

1. Canonical transformations localized with respect to the boundary of the light cone or more general light like surfaces act as isometries of WCW and one can determine the space-time surface only modulo these isometries. Even more, only the values of the non-quantum fluctuating zero modes characterizing the shape and size of the space-time surface are measurable with arbitrary precision in quantum theory. At the level of conscious experience quantum fluctuating degrees of freedom correspond to sensory qualia like color having no classical geometric content.

2. Space-time surface is replaced by a new one in each quantum jump (or rather the superposition of perceptively equivalent space-time surfaces). Only in the approximation that the change of the space-time region in single quantum jump is negligible, the measurement of the shape of space-time surface makes sense. The physical criterion for this is that dissipation is negligible. The change of the space-time region in single quantum jump can indeed be negligible if the measurement is performed with a finite resolution.

3. Conscious experience of self is an average over quantum jumps defining moments of consciousness. In particular, only the average increment of the zero modes is experienced and this means that one cannot fix the space-time surface apart from canonical transformation affecting the zero modes. Again the notion of measurement resolution comes in rescue.

4. The possibility of coherent states of photons and gravitons brings in a further quantum complication since the effective classical em and gravitational fields are superpositions of classical

field and the order parameter describing the coherent state. In principle the extremely strong constraints between the classical field quantities allow to measure both the order parameters of the coherent phases and classical fields.

Quantum holography and the shape of the space-time surface

If the Dirac determinant asssognable to the mass squared eigenvalue spectrum of the Kähler-Dirac operator $D_K(X^2)$ equals to the exponent of Kähler action of a preferred extremal, it is fair to say that a lot of information about the shape of the space-time surface is coded to physical observables, which eigenvalues indeed represent. Quantum gravitational holography due to the Bohr orbit like character of space-time surface reduces the amount of information needed. Only a finite number of eigenvalues is involved and the eigen modes are associated with the 3-D light-like wormhole throats rather than with the space-time surface itself. If the eigenvalues were known or could be measured with infinite accuracy, one could in principle fix the boundary conditions at X_l^3 and solve field equations determining the preferred extremal of Kähler action.

What is of course needed is the complete knowledge of the light-like 3-surfaces X_l^3. Needless to say, in practice a complete knowledge of X_l^3 is impossible since measurement resolution is finite. The notion number theoretic braid provides a precise realization for the finite measurement accuracy at space-time level. At the level of WCW spinors fields (world of classical worlds) just the fact that the number of eigenvalues is finite is correlate for the finite measurement accuracy. Furthermore, quantum states are actually quantum superpositions of 3-surfaces, which means that one can only speak about quantum average space-time surface for which the phase factors coding for the quantum numbers of elementary particles assigned to the strands of number theoretic braids are stationary so that correlation of classical gauge charges with quantum gauge charges is obtained.

2.3 The Vanishing Of Super-Conformal Charges As Gauge Conditions Selecting Preferred Extremals Of Kähler Action

Classical TGD [K15] involves several key questions waiting for clearcut answers.

1. The notion of preferred extremal emerges naturally in positive energy ontology, where Kähler metric assigns a unique (apart from gauge symmetries) preferred extremal to given 3-surface at M^4 time= constant section of embedding space $H = M^4 \times CP_2$. This would quantize the initial values of the time derivatives of embedding coordinates and this could correspond to the Bohr orbitology in quantum mechanics.
2. In zero energy ontology (ZEO) initial conditions are replaced by boundary conditions. One fixes only the 3-surfaces at the opposite boundaries of CD and in an ideal situation there would exist a unique space-time surface connecting them. One must however notice that the existence of light-like wormhole throat orbits at which the signature of the induced metric changes ($det(g_4) = 0$) its signature might change the situation. Does the attribute "preferred" become obsolete and does one lose the beautiful Bohr orbitology, which looks intuitively compelling and would realize quantum classical correspondence?
3. Intuitively it has become clear that the generalization of super-conformal symmetries by replacing 2-D manifold with metrically 2-D but topologically 3-D light-like boundary of causal diamond makes sense. Generalized super-conformal symmetries should apply also to the wormhole throat orbits which are also metrically 2-D and for which conformal symmetries respect $detg(g_4) = 0$ condition. Quantum classical correspondence demands that the generalized super-confornal invariance has a classical counterpart. How could this classical counterpart be realized?
4. Holography is one key aspect of TGD and mean that 3-surfaces dictate everything. In positive energy ontology the content of this statement would be rather obvious and reduce to Bohr orbitology but in ZEO situation is different. On the other hand, TGD strongly suggests strong form of holography based stating that partonic 2-surfaces (the ends of wormhole throat orbits

at boundaries of CD) and tangent space data at them code for quantum physics of TGD. General coordinate invariance would be realized in strong sense: one could formulate the theory either in terms of space-like 3-surfaces at the ends of CD or in terms of light-like wormhole throat orbits. This would realize Bohr orbitology also in ZEO by reducing the boundary conditions to those at partonic 2-surfaces. How to realize this explicitly at the level of field equations? This has been the challenge.

Answering questions is extremely useful activity. During last years my friend Hamed has posed continually questions related to the basic TGD. At this time Hamed asked about the derivation of field equations of TGD. In "simple" field theories involving some polynomial non-linearities the deduction of field equations is of course totally trivial process but in the extremely non-linear geometric framework of TGD situation is quite different.

While answering the questions I ended up with the following question. Could one assume that the variations at the light-like boundaries of CD vanish for all conformal variations, which are not isometries. For isometries the contributions from the ends of CD cancel each other automatically so that the corresponding variations need not vanish separately at boundaries of CD! This is extremely simple and profound fact. This would be nothing but the realisation of the analogs of conformal symmetries classically and give precise content for the notion of preferred external, Bohr orbitology, and strong form of holography. And the condition makes sense only in ZEO!

I attach below the answers to the questions of Hamed almost as such apart from slight editing and little additions, re-organization, and correction of typos.

2.3.1 Field Equations For Kähler Action

My friend Hamed made some questions relating to the derivation of field equations for the extremals of Kähler action, which led to the recent progress. I comment first these questions since they lead naturally to the basic new idea.

The addition of the volume term implied by the twistor lift of TGD [L147, L148], having interpretation in terms of cosmological constant, adds to the field equations only a term proportional to $D_\beta g^{alpha\beta}\partial_\beta h^k$. There are excellent reasons to believe that solutions of field equations representing preferred extremals as analogs of Bohr orbits are actually minimal surfaces except at singular surfaces of dimension $D > 4$. One might speak of 4-D analogies of soap films spanned by frames.

The physical interpretation of the canonical momentum current

Hamed asked about the physical meaning of $T^n_k \equiv \partial L/\partial(\partial_n h^k)$ - normal components of canonical momentum labelled by the label k of embedding space coordinates - it is good to start from the physical meaning of a more general vector field

$$T^\alpha_k \equiv \frac{\partial L}{\partial(\partial_\alpha h^k)}$$

with both embedding space indices k and space-time indices α - canonical momentum currents. L refers to Kähler action.

1. One can start from the analogy with Newton's equations derived from action principle (Lagrangian). Now the analogs are the partial derivatives $\partial L/\partial(dx^k/dt)$. For a particle in potential one obtains just the momentum. Therefore the term canonical momentum current/density: one has kind of momentum current for each embedding space coordinate.
2. By contracting with Killing vector fields of the embedding space isometries j^k_A (Poincare and color) one indeed obtains conserved currents associated with isometries by Noether's theorem:

$$j^{A\alpha} = T^\alpha_k j^{Ak} \quad .$$

By field equations the divergences of these currents vanish and one obtains conserved charged-classical four-momentum and color charges:

$$D_\alpha T^{A\alpha} = 0 \quad .$$

The field equations are essentially hydrodynamical and replace Einstein's equations $T^{alpha\beta} = kG^{\alpha\beta}$. The conditions $D_\beta G^{\alpha\beta} = 0$, stating the vanishing of the covariant divergence of Einstein's tensor $G^{\alpha\beta}$, is the counterpart for the conservation of the isometry currents, but do not give rise to conserved charges.

3. The normal component of the conserved current must vanish at boundaries with one time-like direction if one has such:

$$T^{An} = 0.$$

Now one has wormhole throat orbits which are not genuine boundaries albeit analogous to them and one must be very careful. The quantity T^n_k determines the values of normal components of currents and must vanish at possible space-like boundaries.

Note that in TGD field equations reduce to the conservation of isometry currents as in hydrodynamics where basic equations are just conservation laws.

The basic steps in the derivation of field equations

First a general recipe for deriving field equations from Kähler action - or any action as a matter of fact.

1. At the first step one writes an expression of the variation of the Kähler action as sum of variations with respect to the induced metric g and induced Kähler form J. The partial derivatives in question are energy momentum tensor and contravariant Kähler form.
2. After this the variations of g and J are expressed in terms of variations of embedding space coordinates, which are the primary dynamical variables.
3. The integral defining the variation can be decomposed to a total divergence plus a term vanishing for extremals for all variations: this gives the field equations. Total divergence term gives a boundary term and it vanishes by boundary conditions if the boundaries in question have time-like direction.

   If the boundary is space-like, the situation is more delicate in TGD framework: this will be considered in the sequel. In TGD situation is also delicate also because the light-like 3-surfaces which are common boundaries of regions with Minkowskian or Euclidian signature of the induced metric are not ordinary topological boundaries. Therefore a careful treatment of both cases is required in order to not to miss important physics.

Expressing this summary more explicitly, the variation of the Kahler action with respect to the gradients of the embedding space coordinates reduces to the integral of

$$T^\alpha_k \partial_\alpha \delta h^k + \frac{\partial K}{\partial h^k} \delta h^k \quad .$$

The latter term comes only from the dependence of the embedding space metric and Kähler form on embedding space coordinates. One can use a simple trick. Assume that they do not depend at all on embedding space coordinates, derive field equations, and replaced partial derivatives by covariant derivatives at the end. Covariant derivative means covariance with respect to both space-time and embedding space vector indices for the tensorial quantities involved. The trick works because embedding space metric and Kähler form are covariantly constant quantities.

The integral of the first term $T^\alpha_k \partial_\alpha \delta h^k$ decomposes to two parts.

1. The first term, whose vanishing gives rise to field equations, is integral of

$$D_\alpha T^\alpha_k \delta h^k \quad .$$

2. The second term is integral of

$$\partial_\alpha (T^\alpha_k \delta h^k) \quad .$$

   This term reduces as a total divergence to a 3-D surface integral over the boundary of the region of fixed signature of the induced metric consisting of the ends of CD and wormhole throat orbits (boundary of region with fixed signature of induced metric). This term vanishes if the normal components T^n_k of canonical momentum currents vanishes at the boundary like region.

In the sequel the boundary terms are discussed explicitly and it will be found that their treatment indeed involves highly non-trivial physics.

Complex isometry charges and twistorialization

TGD space-time contains regions of both Minkowskian and Euclidian signature of metric. This has some highly non-trivial consequences.

1. Should one assume that $\sqrt{det(g_4)}$ is imaginary in Minkowskian and real in Euclidian region? For Kähler action this is sensible and Euclidian region would give a real negative contribution giving rise to exponent of Kähler function of WCW ("world of classical worlds") making the functional integral convergent. Minkowskian regions would give imaginary contribution to the exponent causing interference effects absolutely essential in quantum field theory. This contribution would correspond to Morse function for WCW .

   The implication would be that the classical four-momenta in Euclidian/Minkowskian regions are imaginary/real. What could the interpretation be? Should one accept as a fact that four-momenta are complex.

2. Twistor approach to TGD is now in quite good shape [K116]. $M^4 \times CP_2$ is the unique choice is one requires that the Cartesian factors allow twistor space with Kähler structure [A63] and classical TGD allows twistor formulation.

   In the recent formulation the fundamental fermions are assumed to propagate with light-like momenta along wormhole throats. At gauge theory limit particles must have massless or massive four-momenta. One can however also consider the possibility of complex massless momenta and in the standard twistor approach on mass shell massless particles appearing in graphs indeed have complex momenta. These complex momenta should by quantum classical correspondence correspond directly to classical complex momenta.

3. A funny question popping in mind is whether the massivation of particles could be such that the momenta remain massless in complex sense! The complex variant of light-likeness condition would be

   $$p_{re}^2 = p_{Im}^2 \quad , \quad p_{re} \cdot p_{Im} = 0 \quad .$$

   Could one interpret p_{Im}^2 as the mass squared of the particle? Or could p_{Im}^2 code for the decay width of an unstable particle? This option does not look feasible.

4. The complex momenta could provide an elegant 4-D space-time level representation for the isometry quantum numbers at the level of embedding space. The ground states of the super-conformal representations have as building bricks the spinor harmonics of the embedding space which correspond to the analogs of massless particles in 8-D sense [K65]. Indeed, the condition giving mass squared eigenvalues for the spinor harmonics is just massless condition in $M^4 \times CP_2$.

   At the space-time level these conditions must be replaced by 4-D conditions and complex masslessness would be the elegant manner to realizes this. Also the massivation of massless states by p-adic thermodynamics could have similar description.

   This interpretation would also conform with $M^8 - M^4 \times CP_2$ duality [K125] at the level of momentum space.

2.3.2 Boundary Conditions At Boundaries Of CD

In positive energy ontology one would formulate boundary conditions as initial conditions by fixing both the 3-surface and associated canonical momentum densities at either end of CD (positions and momenta of particles in mechanics). This would bring asymmetry between boundaries of CD. In ZEO the basic boundary condition is that space-time surfaces have as their ends the members of pairs of surfaces at the ends of CD. Besides this one can have additional boundary conditions and the notion of preferred extremal suggests this.

Do boundary conditions realize quantum classical correspondence?

In TGD framework one must carefully consider the boundary conditions at the boundaries of CDs. What is clear that the time-like boundary contributions from the boundaries of CD to the variation must vanish.

1. This is true if the variations are assumed to vanish at the ends of CD. This might be however too strong a condition.
2. One cannot demand the vanishing of T^t_k (t refers to time coordinate as normal coordinate) since this would give only vacuum extremals. One could however require quantum classical correspondence for any Cartan sub-algebra of isometries whose elements define maximal set of isometry generators. The eigenvalues of quantal variants of isometry charge assignable to second quantized induced spinors at the ends of space-time surface are equal to the classical charges. Is this actually a formulation of Equivalence Principle, is not quite clear to me.

Do boundary conditions realize preferred extremal property as a choice of conformal gauge?

While writing this a completely new idea popped to my mind. What if one poses the vanishing of the boundary terms at boundaries of CDs as additional boundary conditions for *all* variations *except isometries* ? Of perhaps for all conformal variations (conformal in TGD sense)? This would *not* imply vanishing of isometry charges since the variations coming from the opposite ends of CD cancel each other! It soon became clear that this would allow to meet all the challenges listed in the beginning!

1. These conditions would realize Bohr orbitology also to ZEO approach and define what "preferred extremal" means.
2. The conditions would be very much like super-Virasoro conditions stating that the superconformal generators with non-vanishing conformal weight annihilate states or create zero norm states but no conditions are posed on generators with vanishing conformal weight (now isometries). One could indeed assume only deformations, which are local isometries assignable to the generalised conformal algebra of the $\delta M^4_+/-\times CP_2$. For arbitrary variations one would not require the vanishing. This could be the long sought for precise formulation of superconformal invariance at the level of classical field equations!

   It is enough co consider the weaker conditions that the conformal charges defined as integrals of corresponding Noether currents vanish. These conditions would be direct equivalents of quantal conditions.
3. The natural interpretation would be as a fixing of conformal gauge. This fixing would be motivated by the fact that WCW Kähler metric must possess isometries associated with the conformal algebra and can depend only on the tangent data at partonic 2-surfaces as became clear already for more than two decades ago. An alternative, non-practical option would be to allow all 3-surfaces at the ends of CD: this would lead to the problem of eliminating the analog of the volume of gauge group from the functional integral.
4. The conditions would also define precisely the notion of holography and its reduction to strong form of holography in which partonic 2-surfaces and their tangent space data code for the dynamics.

Needless to say, the modification of this approach could make sense also at partonic orbits.

2.3.3 Boundary Conditions At Parton Orbits

The contributions from the orbits of wormhole throats are singular since the contravariant form of the induced metric develops components which are infinite ($det(g_4) = 0$). The contributions are real at Euclidian side of throat orbit and imaginary at the Minkowskian side so that they must be treated as independently.

Conformal gauge choice, preferred extremal property, hierarchy of Planck constants, and TGD as almost topological QFT

The generalization of the boundary conditions as a classical realization conformal gauge invariance is natural.

1. One can consider the possibility that under rather general conditions the normal components $T^n_k\sqrt{det(g_4)}$ approach to zero at partonic orbits since $det(g_4)$ is vanishing. Note however the appearance of contravariant appearing twice as index raising operator in Kähler action. If so, the vanishing of $T^n_k\sqrt{det(g_4)}$ need not fix completely the "boundary" conditions. In fact, I assign to the wormhole throat orbits conformal gauge symmetries so that just this is expected on physical grounds.
2. Generalized conformal invariance would suggest that the variations defined as integrals of $T^n_k\sqrt{det(g_4)}\delta h^k$ vanish in a non-trivial manner for the conformal algebra associated with the light-like wormhole throats with deformations respecting $det(g_4) = 0$ condition. Also the variations defined by infinitesimal isometries (zero conformal weight sector) should vanish since otherwise one would lose the conservation laws for isometry charges. The conditions for isometries might reduce to $T^n_k\sqrt{det(g_4)} \rightarrow 0$ at partonic orbits. Also now the interpretation would be in terms of fixing of conformal gauge.
3. Even $T^n_k\sqrt{g} = 0$ condition need not fix the partonic orbit completely. The Gribov ambiguity meaning that gauge conditions do not fix uniquely the gauge potential could have counterpart in TGD framework. It could be that there are several conformally non-equivalent space-time surfaces connecting 3-surfaces at the opposite ends of CD.

   If so, the boundary values at wormhole throats orbits could matter to some degree: very natural in boundary value problem thinking but new in initial value thinking. This would conform with the non-determinism of Kähler action implying criticality and the possibility that the 3-surfaces at the ends of CD are connected by several space-time surfaces which are physically non-equivalent.
4. The hierarchy of Planck [K43] constants assigned to dark matter, quantum criticality and even criticality indeed relies on the assumption that $h_{eff} = n \times h$ corresponds to n-fold coverings having n space-time sheets which coincide at the ends of CD and that conformal symmetries act on the sheets as gauge symmetries. One would have as Gribov copies n conformal equivalence classes of wormhole throat orbits and corresponding space-time surfaces. Depending on whether one fixes the conformal gauge one has n equivalence classes of space-time surfaces or just one representative from each conformal equivalent class.
5. There is also the question about the correspondence with the weak form of electric magnetic duality [K15]. This duality plus the condition that $j^\alpha A_\alpha = 0$ in the interior of space-time surface imply the reduction of Kähler action to Chern-Simons terms. This would suggest that the boundary variation of the Kähler action reduces to that for Chern-Simons action which is indeed well-defined for light-like 3-surfaces.

   If so, the gauge fixing would reduce to variational equations for Chern-Simons action! A weaker condition is that classical conformal charges vanish. This would give a nice connection to the vision about TGD as almost topological QFT. In TGD framework these conditions do not imply the vanishing of Kähler form at boundaries. The conditions are satisfied if the CP_2 projection of the partonic orbit is 2-D: the reason is that Chern-Simons term vanishes identically in this case.

Fractal hierarchy of conformal symmetry breakings

A further intuitively natural hypothesis is that there is a fractal hierarchy of breakings of conformal symmetry.

1. Only the generators of conformal sub-algebra with conformal weight multiple of n act as gauge symmetries. This would give infinite hierarchies of breakings of conformal symmetry interpreted in terms of criticality: in the hierarchy n_i divides n_{i+1}.

   Similar degeneracy would be associated with both the parton orbits and the space-like ends at CD boundaries and I have considered the possibility that the integer n appearing in h_{eff}

has decomposition $n = n_1 n_2$ corresponding to the degeneracies associated with the two kinds of boundaries. Alternatively, one could have just $n = n_1 = n_2$ from the condition that the two conformal symmetries are 3-dimensional manifestations of single 4-D analog of conformal symmetry.

2. In the symmetry breaking $n_i \rightarrow n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Could one require that they are conserved that is the contributions of the boundary terms at the ends of CD cancel each other? If so, one would have dynamical conformal symmetry.

What could the proper interpretation of the conformal hierarchies $n_i \rightarrow n_{i+1}$?

1. Could one interpret the hierarchy in terms of increasing measurement resolution? Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and the conformal hierarchies would correspond to an inclusion hierarchies for hyper-finite factors of type II_1 [K126]. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

   This interpretation might make sense for the symplectic algebra of $\delta M^4_\pm \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

2. Suppose that the Kähler action has vanishing variation under deformations defined by the broken conformal symmetries so that the corresponding conformal charges As a consequence, Kähler function would be critical with respect to the corresponding variations. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N} = 4$ symmetric gauge theories.

   In this kind of situation one could consider the interpretation in terms of criticality: the lower the criticality, the larger then value of h_{eff} and h and the higher the resolution.

3. n gives also the number of space-time sheets in the singular covering. Could the interpretation be in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

As should have become clear, the derivation of field equations in TGD framework is not just an application of a formal recipe as in field theories and a lot of non-trivial physics is involved!

2.4 General View About Field Equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. This condition implies that covariant divergence of energy momentum tensor vanishes and in General Relativity context this leads to Einstein's equations. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics. There are however could reasons to keep the identification of preferred extremely property open.

2.4.1 Field Equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &- j^\alpha J^k_{\ l}\partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu^{\ \beta} - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \tag{2.4.1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &- j^\alpha(J_\alpha^{\ \beta}h^k_{\ \beta} + J^k_{\ l}\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \tag{2.4.2}$$

$H^k_{\alpha\beta}$ denotes the components of the

second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} = T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha^{\ \beta}\partial_\beta h^k + J^k_{\ l})\partial_\alpha h^k) = 0 \ . \tag{2.4.3}$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} = 0 \tag{2.4.4}$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K48]. Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K127] led to the conditions

$$g_{ni} = 0 \ , \ J_{ni} = 0 \ . \tag{2.4.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X^3_l). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$\begin{array}{llll} J_{ni} = 0 \ , & g_{ni} = 0 \ , & J_{ir} = 0 \ , & g_{ir} = 0 \ , \\ J^{nk} = 0 \ \ k \neq r \ , & g^{nk} = 0 \ \ k \neq r \ , & J^{nr}\sqrt{g_4} \neq 0 \ , & g^{nr}\sqrt{g_4} \neq 0 \ . \end{array} \tag{2.4.6}$$

The interpretation of this conditions is rather transparent.

1. The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X_l^3 and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.
2. Third and fourth condition state that the induced Kähler field at X_l^3 is purely magnetic and that the metric of x_l^3 reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the Kähler-Dirac operator is considered [K127].
3. The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X_l^3.
4. The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X_l^3)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X_l^3)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y_l^3.
5. The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

2.4.2 Topologization And Light-Likeness Of The Kähler Current As Alternative ways To Guarantee Vanishing Of Lorentz 4-Force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

Topologization of the Kähler current for $D_{CP_2} = 3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2} = 2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi \times j_I^\alpha = \psi \times \epsilon^{\alpha\beta\gamma\delta} J_{\beta\gamma} A_\delta \ . \tag{2.4.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2} < 4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2} < 4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi \times j_I^\alpha J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta} J_{\mu\nu} A_\delta J_{\alpha\beta} \ . \end{aligned} \tag{2.4.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2} < 4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \tag{2.4.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

Topologization of the Kähler current for $D_{CP_2} = 3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E}\times\overline{A} + \phi\overline{B} \ , \quad \rho_I = \overline{B}\cdot\overline{A} \ . \tag{2.4.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned}\nabla\times\overline{B} - \partial_t\overline{E} &= \overline{j} = \psi\overline{j}_I = \psi\left(\phi\overline{B} + \overline{E}\times\overline{A}\right) \ , \\ \nabla\cdot E &= \rho = \psi\rho_I \ .\end{aligned} \tag{2.4.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla\times\overline{B} = \alpha\overline{B} \ , \quad \alpha = \psi\phi \ . \tag{2.4.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha = \phi = constant$, which allows an explicit solution [B11].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I\overline{E} + \overline{j}\times\overline{B} = \psi\overline{B}\cdot\overline{AE} + \psi\left(\overline{E}\times\overline{A} + \phi\overline{B}\right)\times\overline{B} = 0 \ . \tag{2.4.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j}\cdot\overline{E} = \psi\overline{B}\cdot\overline{E} + \psi\left(\overline{E}\times\overline{A} + \phi\overline{B}\right)\cdot\overline{E} = 0 \ . \tag{2.4.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho = \psi\rho_I = \psi B\cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

1. The $\overline{E} \times \overline{A}$ term contributing besides $\phi \overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla \Phi - \partial_t \overline{A} = \beta \overline{A} \tag{2.4.15}$$

   This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinatesand$\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

2. The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} = \beta \overline{B} \tag{2.4.16}$$

   The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B \cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A}, \overline{B}) \rightarrow_{\nabla \times} (\overline{B}, \overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the embedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B59].

In hydrodynamics the role of the magnetic field is taken by the velocity field. This raises the idea that the incompressible flow could occur along the field lines of some natural vector field. The considerations of the last section show that the instanton current defines a universal candidate as far as the general solution of the field equations is considered. All conserved currents defined by the isometry charges would be parallel to the instanton current: one can say each flow line of instanton current is a carrier of conserved quantum numbers. Perhaps even the flow lines of an incompressible hydrodynamic flow could in reasonable approximation correspond to those of instanton current.

The conservation laws are satisfied for each flow line separately and therefore it seems that one cannot have the analog of viscous hydrodynamic flow in this framework. One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Does something go badly wrong?

The following argument suggests a way out of the problem. Dissipation is certaily due to the quantum jumps at scales below that associated with causal diamond (CD) associated with the observer and is thus assignable to sub-CDs. The quantum jumps for sub-CDs would eventually lead to a thermal ensemble of sub-CDs.

The usual description of dissipation in terms of viscocity and similar parameters emerges at the GRT-QFT limit of TGD replacing in long length scales the many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) with a piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. This lumping of space-time sheets means that induced gauge fields and gravitational fields from various space-time sheet sum up and become random (by central limit theorems). Thus locally the dynamics is dissipation free for individual space-time sheets and dissipation emerges at the level of GRT space-time carrying effective metric and effective gauge fields.

The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds contact. Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta} A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta} J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

1. The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.
2. This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler current is non-vanishing. The condition that the field lines of the magnetic field allow a global

coordinate requires $\overline{B}\cdot\nabla\times\overline{B}=0$. The condition is not satisfied by Beltrami fields with $\alpha\neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi)=0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X)=1$ and $dA(X;)=0$ hold true. The latter condition states the vanishing of the cross product $X\times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. *Stability of the Beltrami flow and contact structures*

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B34], and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B34] !

Beltrami flows associated with Euler equations are known to be unstable [B34]. Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2}=4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

2.4.3 How To Satisfy Field Equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 . \qquad (2.4.17)$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

String model as a starting point

String model serves as a starting point.

1. In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u,v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.

2. For Euclidian minimal surfaces (u, v) is replaced by complex coordinates $(w, \overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \tag{2.4.18}$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla \times X \cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E} \times \overline{B}$ such that the coordinates $u = t - z$ and $v = t + z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t + z, S^- \equiv v = t - z, w = x + iy, \overline{w} = x - iy)$.
2. In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_\pm)^2 = 0 \quad .$$

The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_{\pm}$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t - z \rightarrow v = t + z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t-z, t+z, x, y) \rightarrow S^-(t-z, t+z, x, y) = S^+(t+z, t-z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

3. The coordinates $(S_{\pm}, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\ &+ g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\ &+ g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \ . \end{aligned} \tag{2.4.19}$$

Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

4. The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w\partial_{\overline{w}}K \ , \quad g_{+-} = \partial_{S^+}\partial_{S^-}K \ , \\ g_{w\pm} &= \partial_w\partial_{S^\pm}K \ , \quad g_{\overline{w}\pm} = \partial_{\overline{w}}\partial_{S^\pm}K \ . \end{aligned} \tag{2.4.20}$$

for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$K = w_0\overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \tag{2.4.21}$$

The Christoffel symbols satisfy the conditions

$$\left\{ {k \atop w\ \overline{w}} \right\} = 0 \ , \quad \left\{ {k \atop +-} \right\} = 0 \ . \tag{2.4.22}$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

5. The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y_l^3 labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y_l^3 is implied. This is what number theoretic compactification and $M^8 - H$ duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K48, K107].

Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M_+^4 case.

$$s_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \ , \quad s_{ws} = \partial_w \partial_s K \ , \quad s_{\overline{w}s} = \partial_{\overline{w}} \partial_s K \ . \tag{2.4.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{{}^{\ k}_{\xi\ \overline{\xi}}\}$.

$$\{{}^{\ k}_{\xi\ \overline{\xi}}\} \ = \ 0 \ . \tag{2.4.24}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \overline{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \overline{\xi}, s)$ and some coordinate of M_+^4, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta (J^{\alpha\beta} \sqrt{g}) \ &= \ 0 \text{ for } \ \alpha \in \{\xi, \overline{\xi}, s\} \ , \\ g^{4i} \neq 0 \ . \end{aligned} \tag{2.4.25}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

A solution ansatz yielding light-like current in $D_{CP_2} = 3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi, \overline{\xi}, s, S^-)$.

1. The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

$$S^+ = S^+(s) \ , \quad w = w(\xi, s) \ . \tag{2.4.26}$$

   Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

2. Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

3. By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \overline{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

$$g_{ss} = 0 \ , \quad g_{\xi s} = 0 \ , \quad g_{\overline{\xi}s} = 0 \ . \tag{2.4.27}$$

   Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

4. The condition guaranteeing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi, s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s\overline{w}(\xi, s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\overline{\xi}} &= m_{+w}\partial_{\overline{\xi}} w(\overline{\xi})\partial_s S^+(s) \ . \end{aligned} \tag{2.4.28}$$

   Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

5. The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} & j^\xi\sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\overline{\xi}}\sqrt{g} = \partial_\beta(J^{\overline{\xi}\beta}\sqrt{g}) = 0 \ , \\ & j^+\sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \tag{2.4.29}$$

   Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^+ = \partial_\beta(J^{+\beta}\sqrt{g}) \ = \ 0 \tag{2.4.30}$$

   is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

1. The light-like character of the Kähler current brings in mind CP_2 type extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\overline{\xi}}, J^{\xi -}, J^{\overline{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

2. Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If preferred extremals correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

3. The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \overline{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

4. Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y_l^3 of $X^4(X_l^3)$ "parallel" to X_l^3 requires only that gauge currents are parallel to Y_l^3 and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

1. Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

2. The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z, s) \ , \quad w = w(\xi, s) \ . \tag{2.4.31}$$

   If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

3. The components of the induced metric are

$$\begin{array}{ll} g_{ZZ} = m_{ZZ} + m_{TT}\partial_Z T\partial_s T \ , & g_{Zs} = m_{TT}\partial_Z T\partial_s T \ , \\ g_{ss} = s_{ss} + m_{TT}\partial_s T\partial_s T \ , & g_{w\overline{w}} = s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w\partial_{\overline{\xi}}\overline{w} \ , \\ g_{s\xi} = s_{s\xi} \ , & g_{s\overline{\xi}} = s_{s\overline{\xi}} \ . \end{array} \tag{2.4.32}$$

Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} = 0 \tag{2.4.33}$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{array}{rcl} \partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) & = & 0 \ , \\ \partial_\xi(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi}Z}\sqrt{g}) & = & 0 \ . \end{array} \tag{2.4.34}$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M^4_+ Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{array}{l} \{ {}^{\ k}_{T\ w} \} = 0 \ , \quad \{ {}^{\ k}_{T\ \overline{w}} \} = 0 \ , \\ \{ {}^{\ k}_{Z\ w} \} = 0 \ , \quad \{ {}^{\ k}_{Z\ \overline{w}} \} = 0 \end{array} \tag{2.4.35}$$

hold true. The conditions are equivalent with the conditions

$$\{ {}^{\ k}_{\pm\ w} \} = 0 \ , \quad \{ {}^{\ k}_{\pm\ \overline{w}} \} = 0 \ . \tag{2.4.36}$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s, Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

$D_{CP_2} = 4$ case

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

1. Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.
2. One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1, \xi^2) \ , \quad w = w(\xi^1, \xi^2) \ , \quad S^- = constant \ . \qquad (2.4.37)$$

The induced metric does possesses only components of type $g_{i\bar{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \qquad (2.4.38)$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1, ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\{ {}^{k}_{w+} \} = 0 \ , \quad \{ {}^{k}_{\overline{w}+} \} = 0 \ , \quad \{ {}^{k}_{++} \} = 0 \ . \qquad (2.4.39)$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \quad = \quad 0 \ . \qquad (2.4.40)$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \quad = \quad a_k \xi^k + \overline{a}_k \overline{\xi}^k \ . \qquad (2.4.41)$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j (J^{j\bar{i}} \sqrt{g}) = 0 \ , \quad \partial_{\bar{j}} (J^{\bar{j}i} \sqrt{g}) = 0 \ , \qquad (2.4.42)$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i}S^+\partial_{\xi^j}S^- + m_{+-}\partial_{\xi^i}S^-\partial_{\xi^j}S^+)$ are non-vanishing.

1. The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.
2. The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s,r), S^-(s,r)) \quad , \quad w = w(\xi) \quad . \tag{2.4.43}$$

   This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time. Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

$D_{CP_2} = 2$ case

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

1. String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the embedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves

moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

2. Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s,r) = (s(u,v), r(u,v)) \quad , \quad \xi = constant \quad , \tag{2.4.44}$$

and corresponds to a vanishing Kähler current.

3. Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B} \cdot \overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s,r) = (s,r)\,(u,v,w,\overline{w}) \quad , \quad \xi = constant \quad . \tag{2.4.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

1. Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w,\overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.

2. The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \quad , \quad w = w(\xi) \quad , \quad S^- = s^- \quad , \quad S^+ = s^+ + f(\xi,\overline{\xi}) \quad .$$

Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi \partial_+ h^k$ and $\partial_{\overline{\xi}} \partial_+ h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

Could $D_{CP_2} = 2 \rightarrow 3$ transition occur in rotating magnetic systems?

I have studied the embeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K118].

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U \equiv cos(\theta), \Phi)$ and let (t, ρ, ϕ, z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4 \subset M^4_+ \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{aligned} U &= U(\rho) \ , \qquad \Phi = n\phi + kz \ , \\ J_{\rho\phi} &= n\partial_\rho U \ , \quad J_{\rho z} = k\partial_\rho U \ . \end{aligned} \tag{2.4.46}$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \rightarrow 3$. This could help to understand various strange effects related to the rotating magnetic systems [K118]. For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts/flux tubes and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

2.4.4 $D_{CP_2} = 3$ Phase Allows Infinite Number Of Topological Charges Characterizing The Linking Of Magnetic Field Lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i\int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B40]. Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r - r')}{|r - r'|^3} \times B(r') \right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

2.4.5 Preferred Extremal Property And The Topologization/Light-Likeness Of Kähler Current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

1. Zero energy ontology challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.

2. One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic

configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X^3_l. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y^3_l varies. This conforms nicely with the notion of quantum gravitational holography.

3. The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.

4. One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X^3_l)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

1. $M^8 - H$ duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

2. One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X^3_l so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

3. What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM^4_- of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM^4_+? An identification consistent with the fractal structure of zero energy ontology and TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the

asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [K8].

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

2.4.6 Generalized Beltrami Fields And Biological Systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y_l^3 associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

1. Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.
2. Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B59] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

$D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ corresponds to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A}\cdot\overline{B} \neq 0$ and Kähler current $\overline{E}\times\overline{A}+\phi\overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [?]. For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?

Alain Connes wrote [A29] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

1. The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.
2. If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

1. The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.
2. Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.
3. If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing by a continuous Schrödinger time evolution.
4. The condition that the modes of the induced spinor field have well-defined em charge implies that CP_2 projection for the region of space-time in which induced spinor field is non-vanishing is 2-dimensional. In the generic case a localization to 2-surfaces - string world sheets and

possibly partonic 2-surface. At light-like 3-surfaces this implies that modes are localized at 1-D curves so that the hydrodynamic picture is realized [K127].

3. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

1. For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition $D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.
2. For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.
3. $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D embedding space which defines one sector of the generalized embedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

2.4.7 About Small Perturbations Of Field Equations

The study of small perturbations of the known solutions of field equations is a standard manner to get information about the properties of the solutions, their stability in particular. Fourier expansion is the standard manner to do the perturbation theory. In the recent case an appropriate modification of this ansatz might make sense if the solution in question is representable as a map $M^4_+ \to CP_2$, and the perturbations are rapidly varying when compared to the components of the induced metric and Kähler form so that one can make adiabatic approximation and approximate them as being effectively constant. Presumably also restrictions on directions of wave 4-vectors $k_\mu = (\omega, \overline{k}))$ are necessary so that the direction of wave vector adapts to the slowly varying background as in ray optics. Also Hamilton Jacobi structure is expected to modify the most straightforward approach. The four CP_2 coordinates are the dynamical variables so that the situation is relatively simple.

A completely different approach is inspired by the physical picture. In this approach one glues CP_2 type vacuum extremal to a known extremal and tries to deduce the behavior of the deformed extremal in the vicinity of wormhole throat by posing the general conditions on the slicing by light-like 3-surfaces Y^3_l. This approach is not followed now.

Generalized plane waves

Individual plane waves are geometrically very special since they represent a deformation of the space-time surface depending on single coordinate only. Despite this one might hope that plane waves or their appropriate modifications allowing to algebraize the treatment of small perturbations could give useful information also now.

1. Lorentz invariance plus the translational invariance due to the assumption that the induced metric and Kähler form are approximately constant encourage to think that the coordinates reduce Minkowski coordinates locally with the orientation of the local Minkowski frame depending slowly on space-time position. Hamilton Jacobi $(S^+, S^-, w, \overline{w})$ are a good candidate for this kind of coordinates. The properties of the Hamilton Jacobi structure and of the solution ansatz suggest that excitations are generalized plane waves in longitudinal degrees

of freedom only so that four-momentum would be replaced by the longitudinal momentum. In transverse degrees of freedom one might expect that holomorphic plane-waves $exp(ik_T w)$, where k_T is transverse momentum, make algebraization possible.

For time-like longitudinal momenta one can choose the local M^4 coordinates in such a way that longitudinal momentum reduces to $(\omega_0, 0)$, where ω_0 plays the role of rest mass and is analogous to the plasma frequency serving as an infrared cutoff for plasma waves. In these coordinates the simplest candidates for excitations with time-like momentum would be of form $\Delta s^k = \epsilon a^k exp(i\omega_0 u)$, where s^k are some real coordinates for CP_2, a^k are Fourier coefficients, and time-like coordinate is defined as $u = S^+ + S^-$. The excitations moving with light velocity correspond to $\omega_0 = 0$, and one must treat this case separately using plane wave $exp(i\omega S^{\pm})$, where ω has continuum of values.

2. It is possible that only some preferred CP_2 coordinates are excited in longitudinal degrees of freedom. For $D_{CP_2} = 3$ ansatz the simplest option is that the complex CP_2 coordinate ξ depends analytically on w and the longitudinal CP_2 coordinate s obeys the plane wave ansatz. $\xi(w) = a \times exp(ik_T w)$, where k_T is transverse momentum allows the algebraization of the solution ansatz also in the transversal degrees of freedom so that a dispersion relation results. For imaginary values of k_T and ω the equations are real.

2. General form for the second variation of the field equations

For time-like four-momentum the second variation of field equations contains three kinds of terms. There are terms quadratic in ω_0 and coming from the second derivatives of the deformation, terms proportional to $i\omega_0$ coming from the variation with respect to the derivatives of CP_2 coordinates, and terms which do not depend on ω_0 and come from the variations of metric and Kähler form with respect to the CP_2 coordinates.

In standard perturbation theory the terms proportional to $i\omega_0$ would have interpretation as analogs of dissipative terms. This forces to assume that ω_0 is complex: note that in purely imaginary ω_0 the equations are real. The basic assumption is that Kähler action is able to mimic dissipation despite the fact that energy and momentum are conserved quantities. The vanishing of the Lorentz force has an interpretation as the vanishing of the dissipative effects. This would suggest that the terms proportional to $i\omega_0$ vanish for the perturbations of the solution preserving the non-dissipative character of the asymptotic solutions. This might quite well result from the vanishing of the contractions with the deformation of the energy momentum tensor with the second fundamental form and of energy momentum tensor with the deformation of the second fundamental form coming from first derivatives.

Physical intuition would suggest that dissipation-less propagation is possible only along special directions. Thus the vanishing of the linear terms should occur only for special directions of the longitudinal momentum vector, say for light-like four-momenta in the direction of coordinate lines of S^+ or S^-. Quite generally, the sub-space of allowed four-momenta is expected to depend on position since the components of metric and Kähler form are slowly varying. This dependence is completely analogous with that appearing in the Hamilton Jacobi (ray-optics) approach to the approximate treatment of wave equations and makes sense if the phase of the plane wave varies rapidly as compared to the variation of CP_2 coordinates for the unperturbed solution.

Complex values of ω_0 are also possible, and would allow to deduce important information about the rate at which small deviations from asymptotia vanish as well as about instabilities of the asymptotic solutions. In particular, for imaginary values of ω_0 one obtains completely well-defined solution ansatz representing exponentially decaying or increasing perturbation.

High energy limit

One can gain valuable information by studying the perturbations at the limit of very large four-momentum. At this limit the terms which are quadratic in the components of momentum dominate and come from the second derivatives of the CP_2 coordinates appearing in the second fundamental form. The resulting equations reduce for all CP_2 coordinates to the same condition

$$T^{\alpha\beta} k_\alpha k_\beta = 0 \ .$$

This condition is generalization of masslessness condition with metric replaced by the energy momentum tensor, which means that light velocity is replaced by an effective light velocity. In fact, energy momentum tensor effectively replaces metric also in the modified Dirac equation whose form is dictated by super symmetry. Light-like four momentum is a rather general solution to the condition and corresponds to $\omega_0 = 0$ case.

Reduction of the dispersion relation to the graph of swallowtail catastrophe

Also the general structure of the equations for small perturbations allows to deduce highly non-trivial conclusions about the character of perturbations.

1. The equations for four CP_2 coordinates are simultaneously satisfied if the determinant associated with the equations vanishes. This condition defines a 3-dimensional surface in the 4-dimensional space defined by ω_0 and coordinates of 3-space playing the role of slowly varying control parameters. 4×4 determinant results and corresponds to a polynomial which is of order $d = 8$ in ω_0. If the determinant is real, the polynomial can depend on ω_0^2 only so that a fourth order polynomial in $w = \omega_0^2$ results.
2. Only complex roots are possible in the case that the terms linear in $i\omega_0$ are non-vanishing. One might hope that the linear term vanishes for certain choices of the direction of slowly varying four-momentum vector $k^\mu(x)$ at least. For purely imaginary values of ω_0 the equations determinant are real always. Hence catastrophe theoretic description applies in this case at least, and the so called swallow tail [A50] with three control parameters applies to the situation.
3. The general form of the vanishing determinant is

$$D(w, a, b, c) = w^4 - ew^3 - cw^2 - bw - a \ .$$

The transition from the oscillatory to purely dissipative case changes only the sign of w. By the shift $w = \hat{w} + e/4$ the determinant reduces to the canonical form

$$D(\hat{w}, a, b, c) = \hat{w}^4 - c\hat{w}^2 - b\hat{w} - a$$

of the swallowtail catastrophe. This catastrophe has three control variables, which basically correspond to the spatial 3-coordinates on which the induced metric and Kähler form depend. The variation of these coefficients at the space-time sheet of course covers only a finite region of the parameter space of the swallowtail catastrophe. The number of real roots for $w = \omega_0^2$ is four, two, or none since complex roots appear in complex conjugate pairs for a real polynomial. The general shape of the region of 3-space is that for a portion of swallow tail catastrophe (see **Fig. 2.1**).
4. The dispersion relation for the "rest mass" ω_0 (decay rate for the imaginary value of ω_0) has at most four real branches, which conforms with the fact that there are four dynamical variables. In real case ω_0 is analogous to plasma frequency acting as an infrared cutoff for the frequencies of plasma excitations. To get some grasp on the situation notice that for $a = 0$ the swallowtail reduces to $\hat{w} = 0$ and

$$\hat{w}^3 - c\hat{w} - b = 0 \ ,$$

which represents the cusp catastrophe easy to illustrate in 3-dimensional space. Cusp (see **Fig. 2.2**) in turn reduces for $b = 0$ to $\hat{w} = 0$ and fold catastrophe $\hat{w} = \pm\sqrt{c}$. Thus the catastrophe surface becomes 4-sheeted for $c \geq 0$ for sufficiently small values of the parameters a and b. The possibility of negative values of $\hat{w}$ in principle allows $\omega^2 = \hat{w} + e/4 < 0$ solutions identifiable as exponentially decaying or amplified perturbations. At the high frequency limit the 4 branches degenerate to a single branch $T^{\alpha\beta}k_\alpha k_\beta = 0$, which as a special case gives light-like four-momenta corresponding to $\omega_0 = 0$ and the origin of the swallowtail catastrophe.
5. It is quite possible that the imaginary terms proportional to $i\omega_0$ cannot be neglected in the time-like case. The interpretation would be as dissipative effects. If these effects are not too large, an approximate description in terms of butterfly catastrophe makes still sense.

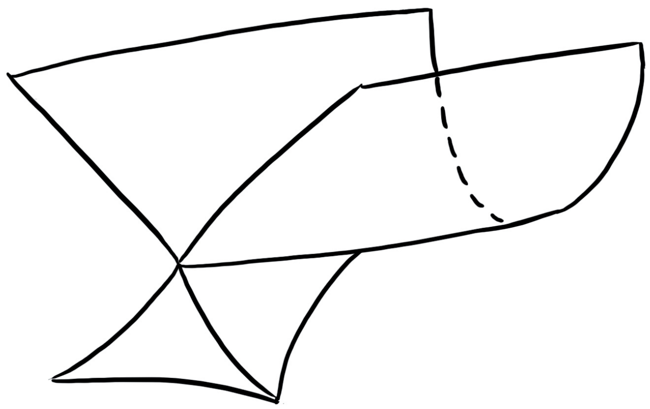

Figure 2.1: The projection of the bifurcation set of the swallowtail catastrophe to the 3-dimensional space of control variables. The potential function has four extrema in the interior of the swallowtail bounded by the triangles, no extrema in the valley above the swallowtail, and 2 extrema elsewhere.

Note however that the second variation contains besides gravitational terms potentially large dissipative terms coming from the variation of the induced Kähler form and from the variation of CP_2 Christoffel symbols.

6. Additional complications are encountered at the points, where the induced Kähler field vanishes since the second variation vanishes identically at these points. By the arguments represented earlier, these points quite generally represent instabilities.

2.5 Vacuum Extremals

Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance

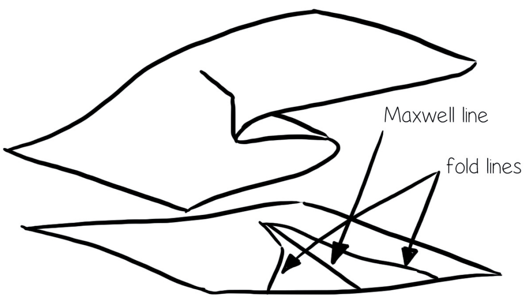

Figure 2.2: Cusp catastrophe. Vertical direction corresponds to the behavior variable and orthogonal directions to control variables.

in TGD universe. Vacuum extremals are not gravitational vacua and they are indeed fundamental in TGD inspired cosmology.

2.5.1 CP_2 Type Extremals

CP_2 type vacuum extremals

These extremals correspond to various isometric embeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the embedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \tag{2.5.1}$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1,2,3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the p-adic QFT limit of TGD. Furthermore, it has turned out that Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD and appears both at the level of embedding space and space-time surfaces.

The action for all extremals is same and given by the Kähler action for the embedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} \ . \tag{2.5.2}$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action.

The absolute minimization of Kähler action was the original suggestion for what preferred extremal property could mean, and suggested that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. The same conclusion however follows also from the mere vacuum degeneracy of Käbler action. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. This identification seems reasonable: the 4-D lines of generalized Feynman graphs [K46] would be regions with Euclidian signature of induced metric and identifiable as deformations of CP_2 type vacuum extremals, and even TGD counterparts of blackholes would be analogous to lines of Feynman diagrams. Their M^4 projection would be of course arbitrarily of macroscopic size. The nice generalization of the area law for the entropy of black hole [K47] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere zitterbewegung so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A58]. A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

Are CP_2 type non-vacuum extremals possible?

The isometric embeddings of CP_2 are all vacuum extremals so that these extremals as such cannot correspond to physical particles. One obtains however non-vacuum extremals as deformations of these solutions. There are several types of deformations leading to non-vacuum solutions. In order to describe some of them, recall the expressions of metric and Kähler form of CP_2 in the coordinates (r, Θ, Ψ, Φ) [A51] are given by

$$\begin{aligned}
\frac{ds^2}{R^2} &= \frac{dr^2}{(1+r^2)^2} + \frac{r}{2(1+r^2))^2}(d\Psi + cos(\Theta)d\Phi)^2 \\
&+ \frac{r^2}{(4(1+r^2)}(d\Theta^2 + sin^2\Theta d\Phi^2) \ , \\
J &= \frac{r}{(1+r^2)}dr \wedge (d\Psi + cos(\Theta)d\Phi) \\
&- \frac{r^2}{(2(1+r^2)}sin(\Theta)d\Theta \wedge d\Phi \ . \qquad (2.5.3)
\end{aligned}$$

The scaling of the line element is defined so that πR is the length of the CP_2 geodesic line. Note that Φ and Ψ appear as "cyclic" coordinates in metric and Kähler form: this feature plays important role in the solution ansatze to be described.

Let $M^4 = M^2 \times E^2$ denote the decomposition of M^4 to a product of 2-dimensional Minkowski space and 2-dimensional Euclidian plane. This decomposition corresponds physically to the decomposition of momentum degrees of freedom for massless particle: E^2 corresponds to polarization degrees of freedom.

There are several types of non-vacuum extremals.

"Virtual particle" extremals: the mass spectrum is continuous (also Euclidian momenta are allowed) but these extremals reduce to vacuum extremals in the massless limit.

2. Massless extremals.

Consider first an example of virtual particle extremal. The simplest extremal of this type is obtained in the following form

$$m^k = a^k\Psi + b^k\Phi \ . \qquad (2.5.4)$$

Here a^k and b^k are some constant quantities. Field equations are equivalent to the conditions expressing four-momentum conservation and are identically satisfied the reason being that induced metric and Kähler form do not depend on the coordinates Ψ and Φ.

Extremal describes 3-surface, which moves with constant velocity in M^4. Four-momentum of the solution can be both space and time like. In the massless limit solution however reduces to a vacuum extremal. Therefore the interpretation as an off mass shell massless particle seems appropriate.

Massless extremals are obtained from the following solution ansatz.

$$\begin{aligned} m^0 &= m^3 = a\Psi + b\Phi \ , \\ (m^1, m^2) &= (m^1(r,\Theta), m^2(r,\Theta)) \ . \end{aligned} \tag{2.5.5}$$

Only E^2 degrees of freedom contribute to the induced metric and the line element is obtained from

$$ds^2 = ds^2_{CP_2} - (dm^1)^2 - (dm^2)^2 \ . \tag{2.5.6}$$

Field equations reduce to conservation condition for the componenents of four-momentum in E^2 plane. By their cyclicity the coordinates Ψ and Φ disappear from field equations and one obtains essentially current conservation condition for two-dimensional field theory defined in space spanned by the coordinates r and Θ.

$$\begin{aligned} (J^i_a)_{,i} &= 0 \ , \\ J^i_a &= T^{ij} f^a_{,j} \sqrt{g} \ . \end{aligned} \tag{2.5.7}$$

Here the index i and a refer to r and Θ and to E^2 coordinates m^1 and m^2 respectively. T^{ij} denotes the canonical energy momentum tensor associated with Kähler action. One can express the components of T^{ij} in terms of induced metric and CP_2 metric in the following form

$$T^{ij} = (-g^{ik}g^{jl} + g^{ij}g^{kl}/2)s_{kl} \ . \tag{2.5.8}$$

This expression holds true for all components of the energy momentum tensor.

Since field equations are essentially two-dimensional conservation conditions they imply that components of momentum currents can be regarded as vector fields of some canonical transformations

$$J^i_a = \varepsilon^{ij} H^a_{,j} \ , \tag{2.5.9}$$

where ε^{ij} denotes two-dimensional constant symplectic form. An open problem is whether one could solve field equations exactly and whether there exists some nonlinear superposition principle for the solutions of these equations. Solutions are massless since transversal momentum densities vanish identically.

Consider as a special case the solution obtained by assuming that one E^2 coordinate is constant and second coordinate is function $f(r)$ of the variable r only. Field equations reduce to the following form

$$f_{,r} = \pm \frac{k}{(1+r^2)^{1/3}} \sqrt{r^2 - k^2(1+r^2)^{4/3}} \ . \tag{2.5.10}$$

The solution is well defined only for sufficiently small values of the parameter k appearing as integration constant and becomes ill defined at two singular values of the variable r. Boundary conditions are identically satisfied at the singular values of r since the radial component of induced metric diverges at these values of r. The result leads to suspect that the generation of boundary components dynamically is a general phenomenon so that all non-vacuum solutions have boundary components in accordance with basic ideas of TGD.

$CP_2\#CP_2\#...\#CP_2$: s as generalized Feynman graphs

There are reasons to believe that point like particles might be identified as CP_2 type extremals in TGD approach. Also the geometric counterparts of the massless on mass shell particles and virtual particles have been identified. It is natural to extend this idea to the level of particle interactions: the lines of Feynman diagrams of quantum field theory are thickened to four-manifolds, which are in a good approximation CP_2 type vacuum extremals. This would mean that generalized Feynman graphs are essentially connected sums of CP_2: s (see **Fig. 2.3**): $X^4 = CP_2\#CP_2....\#CP_2$).

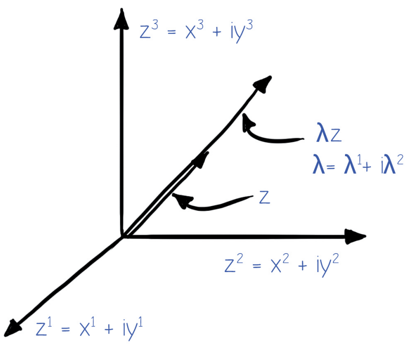

Figure 2.3: Topological sum of CP_2: s as Feynman graph with lines thickened to four-manifolds

Unfortunately, this picture seems to be oversimplified. First, it is questionable whether the cross sections for the scattering of CP_2 type extremals have anything to do with the cross sections associated with the standard gauge interactions. A naïve geometric argument suggests that the cross section should reflect the geometric size of the scattered objects and therefore be of the order of CP_2 radius for topologically non-condensed CP_2 type extremals. The observed cross sections would result at the first level of condensation, where particles are effectively replaced by surfaces with size of order Compton length. Secondly, the $h_{vac} = -D$ rule, considered in the previous chapter, suggests that only real particles correspond to the CP_2 type extremals whereas virtual particles in general correspond to the vacuum extremals with a vanishing Kähler action. The reason is that the negative exponent of the Kähler action reduces the contribution of the CP_2 type extremals to the functional integral very effectively. Therefore the exchanges of CP_2 type extremals are suppressed by the negative exponent of the Kähler action very effectively so that geometric scattering cross section is obtained.

2.5.2 Vacuum Extremals With Vanishing Kähler Field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Legendre manifold. The condition expressing Legendre manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k \ . \tag{2.5.11}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \tag{2.5.12}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the configuration space metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D_{CP_2} = 3$ for CP_2 type extremals, $D_{CP_2} = 2$ for string like objects, $D_{CP_2} = 1$ for membranes and $D_{CP_2} = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \to CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \to D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

2.6 Non-Vacuum Extremals

2.6.1 Cosmic Strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the absolute minimization of the Kähler action is global rather than a local principle. A more general approach gives up absolute minimization as definition of preferred extremal property and there are indeed several proposals for what preferred extremal property could mean. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \ , \tag{2.6.1}$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

2.6.2 Massless Extremals

Massless extremals (or topological light rays) are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$\begin{array}{ll} s^k = f^k(u,v) \ , & \\ u = p\cdot m \ , & v = \varepsilon\cdot m \ , \\ p\cdot\varepsilon = 0 \ , & p^2 = 0 \ . \end{array}$$

CP_2 coordinates are arbitrary functions of $p \cdot m$ and $\varepsilon \cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2 + m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$\begin{array}{l} H(p\cdot m, \varepsilon\cdot m, \varepsilon_1\cdot m) = 0 \ , \\ \quad \varepsilon\cdot p = 0 \ , \quad \varepsilon_1\cdot p = 0 \ , \quad \varepsilon\cdot\varepsilon_1 = 0 \ . \end{array} \tag{2.6.2}$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many ways to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta} J^k_{\ l} \partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a way to satisfy all boundary conditions but it is not clear whether there are any other ways to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamcis this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are bounded this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

2.6.3 Does GRT really allow gravitational radiation: could cosmological constant save the situation?

In Facebook discussion Nils Grebäck mentioned Weyl tensor and I learned something that I should have noticed long time ago. Wikipedia article (see `http://tinyurl.com/y7fsnzk8`) lists the basic properties of Weyl tensor as the traceless part of curvature tensor, call it R. Weyl tensor C is vanishing for conformally flat space-times. In dimensions D=2,3 Weyl tensor vanishes identically so that they are always conformally flat: this obviously makes the dimension $D = 3$ for space very special. Interestingly, one can have non-flat space-times with nonvanishing Weyl tensor but the vanishing Schouten/Ricci/Einstein tensor and thus also with vanishing energy momentum tensor.

The rest of curvature tensor R can be expressed in terms of so called Kulkarni-Nomizu product $P \cdot g$ of Schouten tensor P and metric tensor g: $R = C + P \cdot g$, which can be also transformed to a definition of Weyl tensor using the definition of curvature tensor in terms of Christoffel symbols as the fundamental definition. Kulkarni-Nomizu product $\cdot$ is defined as tensor product of two 2-tensors with symmetrization with respect to first and second index pairs plus antisymmetrization with respect to second and fourth indices.

Schouten tensor P is expressible as a combination of Ricci tensor Ric defined by the trace of R with respect to the first two indices and metric tensor g multiplied by curvature scalar s (rather than R in order to use index free notation without confusion with the curvature tensor). The expression reads as

$$P = \frac{1}{D-2}\left[Ric - \frac{s}{2(D-1)}g\right] \quad .$$

Note that the coefficients of Ric and g differ from those for Einstein tensor. Ricci tensor and Einstein tensor are proportional to energy momentum tensor by Einstein equations relate to the part.

Weyl tensor is assigned with gravitational radiation in GRT. What I see as a serious interpretational problem is that by Einstein's equations gravitational radiation would carry no energy and momentum in absence of matter. One could argue that there are no free gravitons in GRT if this interpretation is adopted! This could be seen as a further argument against GRT besides the problems with the notions of energy and momentum: I had not realized this earlier.

Interestingly, in TGD framework so called massless extremals (MEs) [K15, K9, K82] are four-surfaces, which are extremals of Kähler action, have Weyl tensor equal to curvature tensor and therefore would have interpretation in terms of gravitons. Now these extremals are however non-vacuum extremals.

1. Massless extremals correspond to graphs of possibly multi-valued maps from M^4 to CP_2. CP_2 coordinates are arbitrary functions of variables $u = k \cot m$ and $w = \epsilon \cdot m$. k is light-like wave vector and ϵ space-like polarization vector orthogonal to k so that the interpretation in terms of massless particle with polarization is possible. ME describes in the most general case a wave packet preserving its shape and propagating with maximal signal velocity along a kind of tube analogous to wave guide so that they are ideal for precisely targeted communications and central in TGD inspired quantum biology. MEs do not have Maxwellian counterparts. For instance, MEs can carry light-like gauge currents parallel to them: this is not possible in Maxwell's theory.

2. I have discussed a generalization of this solution ansatz so that the directions defined by light-like vector k and polarization vector ϵ orthogonal to it are not constant anymore but define a slicing of M^4 by orthogonal curved surfaces (analogs of string world sheets and space-like surfaces orthogonal to them). MEs in their simplest form at least are minimal surfaces and actually extremals of practically any general coordinate invariance action principle. For instance, this is the case if the volume term suggested by the twistor lift of Kähler action [K46] and identifiable in terms of cosmological constant is added to Kähler action.

3. MEs carry non-trivial induced gauge fields and gravitational fields identified in terms of the induced metric. I have identified them as correlates for particles, which correspond to pairs of wormhole contacts between two space-times such that at least one of them is ME. MEs would accompany to both gravitational radiation and other forms or radiation classically and serve as their correlates. For massless extremals the metric tensor is of form

$$g = m + a\epsilon \otimes \epsilon + bk \otimes k + c(\epsilon \otimes kv + k \otimes \epsilon) \ ,$$

   where m is the metric of empty Minkowski space. The curvature tensor is necessarily quadrilinear in polarization vector ϵ and light-like wave vector k (light-like ifor both M^4 and ME metric) and from the general expression of Weyl tensor C in terms of R and g it is equal to curvature tensor: $C = R$.

   Hence the interpretation as graviton solution conforms with the GRT interpretation. Now however the energy momentum tensor for the induced Kähler form is non-vanishing and bilinear in velocity vector k and the interpretational problem is avoided.

What is interesting that also at GRT limit cosmological constant saves gravitons from reducing to vacuum solutions. The deviation of the energy density given by cosmological term from that for Minkowski metric is identifiable as gravitonic energy density. The mysterious cosmological constant would be necessary for making gravitons non-vacuum solutions. The value of graviton amplitude would be determined by the continuity conditions for Einstein's equations with cosmological term. The p-adic evolution of cosmological term predicted by TGD is however difficult to understand in GRT framework.

2.6.4 Gravitational memory effect and quantum criticality of TGD

Gary Ehlenberg sent an interesting post about the gravitational memory effect (see this and this).

Classical gravitational waves would leave a memory of its propagation to the metric of space-time affecting distances between mass points. The computations are done by treating Einstein's theory as a field theory in the background defined by the energy momentum tensor of matter and calculations are carried out only in the lowest non-trivial order.

There are two kinds of effects: the linear memory effect occurs for instance when a planet moves along non-closed hyperbolic orbit around a star and involves only the energy momentum tensor of the system. The non-linear memory effect also involves the energy momentum tensor of gravitational radiation as a source added to the energy momentum tensor of matter.

The effect is accumulative and involves integration over the history of the matter source over the entire past. The reason why the memory effect is non-vanishing is basically that the source of the gravitational radiation is quadratic in metric. In Maxwellian electrodynamics the source does not have this property.

I have never thought of the memory effect. The formula used to estimate the effect is however highly interesting.

1. In the formula for the non-linear memory effect, that is for the action of d'Alembert operator acting on the radiation contribution to the metric, the source term is obtained by adding to the energy momentum tensor of the matter, the energy momentum tensor of the gravitational radiation.

2. This formula can be iterated and if the limit as a fixed point exists, the energy momentum tensor of the gravitational radiation produced by the total energy momentum tensor, including also the radiative contribution, should vanish. This brings in mind fractals and criticality.

   One of the basic facts about iteration for polynomials is that it need not always converge. Limit cycles typically emerge. In more complex situations also objects known as strange attractors can appear. Does the same problem occur now, when the situation is much much more complex?

3. What is interesting is that gravitational wave solutions indeed have vanishing energy momentum tensors. This is problematic if one considers them as radiation in empty space. In the presence of matter, this might be true only for very special background metrics as a sum of matter part and radiation part: just these gravitationally critical fixed point metrics.

Could the fixed point property of these metrics (matter plus gravitational radiation) be used to gain information of the total metric as sum of matter and gravitational parts?

4. As a matter of fact, all solutions of non-linear field theories are constructed by similar iteration and the radiative contribution in a given order is determined by the contribution in lower orders.

   Under what conditions can one assume convergence of the perturbation series, that is fixed point property? Are limit cycles and chaotic attractors, and only a specialist knows what, unavoidable? Could this fixed point property have some physical relevance? Could the fixed points correspond in quantum field theory context to fixed points of the renormalization group and lead to quantization of coupling constants?

Does the fixed point property have a TGD counterpart?

1. In the TGD, framework Einstein's equations are expected only at the QFT limit at which space-time sheets are replaced with a single region of M^4 carrying gauge fields and gravitational fields, which are sums of the induced fields associated with space-time sheets. What happens at the level of the basic TGD?

   What is intriguing, is that quantum criticality is the basic principle of TGD and fixes discrete coupling constant evolution: could the quantum criticality realize itself also as gravitational criticality in the above sense? And even the idea that perturbation series can converge only at critical points and becomes actually trivial?

2. What does the year 2023 version of classical TGD say about the situation? In TGD, space-time surfaces obey almost deterministic holography required by general coordinate invariance [L170, L141]. Holography follows from the general coordinate invariance and implies that path integral trivializes to sum over the analogs of Bohr orbits of particles represented as 3-D surfaces. This states quantum criticality and fixed point property: radiative contributions vanish. This also implies a number theoretic view of coupling constant evolution based on number theoretic vision about physics.

   M^8-H duality [L109, L110, L168] implies that the space-time regions defined by Bohr orbits are extremely simple and form an evolutionary hierarchy characterized by extensions of rationals associated with polynomials characterizing the counterparts of space-time surfaces as 4-surfaces in complexified M^8 mapped to space-time surfaces in $M^4\times CP_2$ by M^8-H duality.

   There is also universality: the Bohr orbits in H are minimal surfaces [L146], which satisfy a 4-D generalization of 2-D holomorphy and are independent of the action principle as long as it is general coordinate invariant and constructible in terms of the induced geometry. The only dependence on coupling constants comes from singularities at which minimal surface property fails. Also classical conserved quantities depend on coupling constants.

3. The so called "massless extremals" (MEs) represent radiation with very special properties such as precisely targeted propagation with light velocity, absence of dispersion of wave packed, and restricted linear superposition for massless modes propagating in the direction of ME. They are analogous to laser beams, Bohr orbits for radiation fields. The gauge currents associated with MEs are light-like and Lorentz 4-force vanishes.

4. Could the Einstein tensor of ME vanish? The energy momentum tensor expressed in terms of Einstein tensor involves a dimensional parameter and measures the breaking of scale invariance. MEs are conformally invariant objects: does this imply the vanishing of the Einstein tensor? Note however that the energy momentum tensor assignable to the induced gauge fields is non-vanishing: however, its scale covariance is an inherent property of gauge fields so that it need not vanish.

2.6.5 Generalization Of The Solution Ansatz Defining Massless Extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

Local light cone coordinates

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.
2. With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{2.6.3}$$

   If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.
3. This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k \cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \ , \ (\nabla_3 f)^2 = 1 \ ,$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \ .$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many ways.

Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

1. Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.
2. The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement
$$\epsilon_i \to \hat{\epsilon}_i = \epsilon_i + k\nabla S_+ \ ,$$
where k is any function. With the choice
$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$
one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.
3. The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition
$$k = k(S^+)$$
is satisfiedin this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.
4. The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions
$$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \ ,$$
$$\nabla S^+ \cdot \nabla E^i = 0 \ , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ .$$
The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \to -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

1. Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.
2. Quite a general family of MEs corresponds to the solution family of the field equations having the general form
$$s^k = f^k(S^+, E) \ ,$$
where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

3. The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then "free" solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.
4. For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.
5. One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the embedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 = 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \quad . \tag{2.6.4}$$

$g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} = s_{kl}\partial_1 s^k \partial_+ s^k \quad . \tag{2.6.5}$$

The diagonalization of the metric seems to be a general aspect of preferred extremals. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

1. The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.
2. Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal symmetries just like the boundary of the embedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

3. These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the absolute minimization of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of embedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

2.6.6 Maxwell Phase

"Maxwell phase" corresponds to small deformations of the M^4 type vacuum extremals. Since energy momentum tensor is quadratic in Kähler field the term proportional to the contraction of the energy momentum tensor with second fundamental form drops from field equations and one obtains in lowest order the following field equations

$$j^\alpha J^k_{\ l} s^l_{,\alpha} = 0 \ . \tag{2.6.6}$$

These equations are satisfied if Maxwell's equations

$$j^\alpha = 0 \tag{2.6.7}$$

hold true. Massless extremals and Maxwell phase clearly exclude each other and it seems that they must corresponds to different space-time sheets.

The explicit construction of these extremals reduces to the task of finding an embedding for an arbitrary free Maxwell field to H. One can also allow source terms corresponding to the presence of the point like charges: these should correspond to the regions of the space-time, where the flat space-time approximation of the space-time fails. The regions where the approximation defining the Maxwell phase fails might correspond to a topologically condensed CP_2 type extremals, for example. As a consequence, Kähler field is superposition of radiation type Kähler field and of Coulomb term. A second possibility is the generation of "hole" with similar Coulombic Kähler field.

An important property of the Maxwell phase (also of massless extremals) is its approximate canonical invariance. Canonical transformations do not spoil the extremal property of the four-surface in the approximation used, since it corresponds to a mere $U(1)$ gauge transformation. This implies the counterpart of the vacuum degeneracy, that is, the existence of an enormous number of four-surfaces with very nearly the same action. Also there is an approximate $Diff(M^4_+)$ invariance.

The canonical degeneracy has some very interesting consequences concerning the understanding of the electro-weak symmetry breaking and color confinement. Kähler field is canonical invariant and satisfies Maxwells equations. This is in accordance with the identification of Kähler field as $U(1)$ part of the electro-weak gauge field. Electromagnetic gauge field is a superposition of Kähler field and Z^0 field$\gamma = 3J - sin^2(\theta_W)Z^0/2$ so that also electromagnetic gauge field is long ranged assuming that Z^0 and W^+ fields are short ranged. These fields are not canonical invariants and their behavior seems to be essentially random, which implies short range correlations and the consequent massivation.

There is an objection against this argument. For the known $D < 4$ solutions of field equations weak fields are not random at all. These situations could represent asymptotic configurations assignable to space-time sheets. This conforms with the interpretation that weak gauge fields are essentially massless within the asymptotic space-time sheets representing weak bosons. Gauge fields are however transferred between space-time sheets through # contacts modellable as pieces of CP_2 type extremals having $D_{CP_2} = 4$. In contrast to Kähler and color gauge fluxes, weak gauge fluxes are not conserved in the Euclidian time evolution between the 3-D causal horizons separating the Euclidian # contact from space-time sheets with Minkowskian signature. This non-conservation implying the loss of coherence in the transfer of fields between space-time sheets is a plausible mechanism for the loss of correlations and massivation of the weak gauge fields.

Classical gluon fields are proportional to Kähler field and to the Hamiltonians associated with the color isometry generators.

$$g^A_{\alpha\beta} = kH^A J_{\alpha\beta} \ . \tag{2.6.8}$$

This implies that the direction of gluon fields in color algebra is random. One can always perform a canonical transformation, which reduces to a global color rotation in some arbitrary small region of space-time and reduces to identity outside this region. The proportionality of a gluon field to Kähler form implies that there is a classical long range correlation in X^4 degrees of freedom: in this sense classical gluon fields differ from massive electro-weak fields in Maxwell phase.

2.6.7 Stationary, Spherically Symmetric Extremals

The stationary, spherically symmetric extremals of the Kähler action imbeddable in $M^4 \times S^2$, where S^2 is geodesic sphere, are the simplest extremals, which one can study as models for the space-time surrounding a topologically condensed particle, say CP_2 type vacuum extremal. In the region near the particle the spherical symmetry is an unrealistic assumption since it excludes the presence of magnetic fields needed to cancel the total Kähler action. The stationarity is also unrealistic assumption since zitterbewegung seems to provide a necessary mechanism for generating Kähler magnetic field and for satisfying boundary conditions. Also the imbeddability to $M^4 \times S^2$ implies unrealistic relationship between Z^0 and photon charges.

According to the general wisdom, the generation of a Kähler electric field must take place in order to minimize the action and it indeed turns out that the extremal is characterized by essentially $1/r^2$ Kähler electric field. The necessary presence of a hole or of a topologically condensed object is also demonstrated: it is impossible to find extremals well defined in the region surrounding the origin. It is impossible to satisfy boundary conditions at a hole: this is in accordance with the idea that Euclidian region corresponding to a CP_2 type extremal performing zitterbewegung is generated. In case of CP_2 type extremal radius is of the order of the Compton length of the particle and in case of a "hole" of the order of Planck length. The value of the vacuum frequency ω is of order of particle mass whereas for macroscopic vacuum extremals it must be of the order of $1/R$. This does not lead to a contradiction if the concept of a many-sheeted space-time is accepted.

The Poincare energy of the exterior region is considerably smaller than the gravitational mass; this conforms with the interpretation that gravitational mass is sum of absolute values of positive and negative inertial masses associated with matter and negative energy antimatter. It is quite possible that classical considerations cannot provide much understanding concerning the inertial masses of topologically condensed particles. Electro-weak gauge forces are considerably weaker than the gravitational force at large distances, when the value of the frequency parameter ω is of order $1/R$. Both these desirable properties fail to be true if CP_2 radius is of order Planck length as believed earlier.

In light of the general ideas about topological condensation it is clear that in planetary length scales these kind of extremals cannot provide a realistic description of space-time. Indeed, spherically symmetric extremals predict a wrong rate for the precession of the perihelion of Mercury. Scwhartschild and Reissner-Nordström metric do this and indeed allow embedding as vacuum extremals for which the inertial masses of positive energy matter and negative energy antimatter sum up to zero.

This does not yet resolve the interpretational challenge due to the unavoidable long range color and weak gauge fields. A dark matter hierarchy giving rise to a hierarchy of color and electro-weak physics characterized by increasing values of weak and confinement scales explains these fields. # contacts involve a pair of causal horizons at which the Euclidian metric signature of # contact transforms to Minkowskian one. These causal horizons have interpretation as partons so that # contact can be regarded as a bound state of partons bound together by a gravitational instanton (CP_2 type extremal). # contacts provide basic example of dark matter creating long ranged weak fields.

An important result is the correlation between the sign of the vacuum frequency ω and that of the Kähler charge, which is of opposite sign for fermions and anti-fermions. This suggests an explanation for matter-antimatter asymmetry. Matter and antimatter condense stably on disjoint regions of the space-time surface at different space-time sheets. Stable antimatter could correspond to negative time orientation and negative energy. This leads to a model for the primordial generation of matter as spontaneous generation of zero energy # contacts between space-time sheets of opposite time orientations. If CP conjugation is not exact symmetry, # contacts and their CP conjugates are created with slightly different rates and this gives rise to CP asymmetry at each of the two space-time sheets involved. After the splitting of # contacts and subsequent annihilation of particles and antiparticles at each space-time sheet, the two space-time sheets contain only positive energy matter and negative energy antimatter.

General solution ansatz

The general form of the solution ansatz is obtained by assuming that the space-time surface in question is a sub-manifold of $M^4 \times S^2$, where S^2 is the homologically non-trivial geodesic sphere of CP_2. S^2 is most conveniently realized as $r = \infty$ surface of CP_2, for which all values of the coordinate Ψ correspond to same point of CP_2 so that one can use Θ and Φ as the coordinates of S^2.

The solution ansatz is given by the expression

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t \ , \\ m^0 &= \lambda t \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{2.6.9}$$

The induced metric is given by the expression

$$ds^2 = \left[\lambda^2 - \frac{R^2}{4}\omega^2(1-u^2)\right] dt^2 - (1 + \frac{R^2}{4}\theta_{,r}^2)dr^2 - r^2 d\Omega^2 \ . \tag{2.6.10}$$

The value of the parameter λ is fixed by the condition $g_{tt}(\infty) = 1$:

$$\lambda^2 - \frac{R^2}{4}\omega^2(1-u(\infty)^2) = 1 \ . \tag{2.6.11}$$

From the condition $e^0 \wedge e^3 = 0$ the non-vanishing components of the induced Kähler field are given by the expression

$$J_{tr} = \frac{\omega}{4} u_{,r} \ . \tag{2.6.12}$$

Geodesic sphere property implies that Z^0 and photon fields are proportional to Kähler field:

$$\begin{aligned} \gamma &= (3 - p/2)J \ , \\ Z^0 &= J \ . \end{aligned} \tag{2.6.13}$$

From this formula one obtains the expressions

$$\begin{aligned} Q_{em} &= \frac{(3-p/2)}{4\pi\alpha_{em}} Q_K \ , \quad Q_Z = \frac{1}{4\pi\alpha_Z} Q \ , \\ Q &\equiv \frac{J_{tr} 4\pi r^2}{\sqrt{-g_{rr}g_{tt}}} \ . \end{aligned} \tag{2.6.14}$$

for the electromagnetic and Z^0 charges of the solution using e and g_Z as unit.

Field equations can be written as conditions for energy momentum conservation (two equations is in principle all what is needed in the case of geodesic sphere). Energy conservation holds identically true and conservation of momentum, say, in z-direction gives the equation

$$(T^{rr}z_{,r})_{,r} + (T^{\theta\theta}z_{,\theta})_{,\theta} = 0 \ . \tag{2.6.15}$$

Using the explicit expressions for the components of the energy momentum tensor

$$\begin{aligned} T^{rr} &= g^{rr}L/2 \ , \\ T^{\theta\theta} &= -g^{\theta\theta}L/2 \ , \\ L &= g^{tt}g^{rr}(J_{tr})^2\sqrt{g}/2 \ , \end{aligned} \tag{2.6.16}$$

and the following notations

$$\begin{aligned} A &= g^{tt}g^{rr}r^2\sqrt{-g_{tt}g_{rr}} \ , \\ X &\equiv (J_{tr})^2 \ , \end{aligned} \tag{2.6.17}$$

the field equations reduce to the following form

$$(g^{rr}AX)_{,r} - \frac{2AX}{r} = 0 \ . \tag{2.6.18}$$

In the approximation $g^{rr} = 1$ this equation can be readily integrated to give $AX = C/r^2$. Integrating Eq. (3.2.7), one obtains integral equation for X

$$J_{tr} = \frac{q}{r_c}(|g_{rr}|^3 g_{tt})^{1/4} exp(\int_{r_c}^{r} dr\frac{g_{rr}}{r})\frac{1}{r} \ , \tag{2.6.19}$$

where q is integration constant, which is related to the charge parameter of the long range Kähler electric field associated with the solution. r_c denotes the critical radius at which the solution ceases to be well defined.

The inspection of this formula shows that J_{tr} behaves essentially as $1/r^2$ Coulomb field. This behavior doesn't depend on the detailed properties of the solution ansatz (for example the imbeddability to $M^4 \times S^2$): stationarity and spherical symmetry is what matters only. The compactness of CP_2 means that stationary, spherically symmetric solution is not possible in the region containing origin. This is in concordance with the idea that either a hole surrounds the origin or there is a topologically condensed CP_2 type extremal performing zitterbewegung near the origin and making the solution non-stationary and breaking spherical symmetry.

Second integration gives the following integral equation for CP_2 coordinate $u = cos(\Theta)$

$$u(r) = u_0 + \frac{4q}{\omega}\int_{r_c}^{r}(-g_{rr}^3 g_{tt})^{1/4}\frac{1}{r}exp(\int_{r_c}^{r} dr\frac{g_{rr}}{r}) \ . \tag{2.6.20}$$

Here u_0 denotes the value of the coordinate u at $r = r_0$.

The form of the field equation suggests a natural iterative procedure for the numerical construction of the solution for large values of r.

$$u_n(r) = T_{n-1} \ , \tag{2.6.21}$$

where T_{n-1} is evaluated using the induced metric associated with u_{n-1}. The physical content of the approximation procedure is clear: estimate the gravitational effects using lower order solution since these are expected to be small.

A more convenient manner to solve u is based on Taylor expansion around the point $V \equiv 1/r = 0$. The coefficients appearing in the power series expansion $u = \sum_n u_n A^n V^n : \ A = q/\omega$ can be solved by calculating successive derivatives of the integral equation for u.

The lowest order solution is simply

$$u_0 = u_\infty \ , \tag{2.6.22}$$

and the corresponding metric is flat metric. In the first order one obtains for $u(r)$ the expression

$$u = u_\infty - \frac{4q}{\omega r} \ , \tag{2.6.23}$$

which expresses the fact that Kähler field behaves essentially as $1/r^2$ Coulomb field. The behavior of u as a function of r is identical with that obtained for the embedding of the Reissner-Nordström solution.

To study the properties of the solution we fix the signs of the parameters in the following manner:

$$u_\infty < 0 \ , \quad q < 0 \ , \quad \omega > 0 \tag{2.6.24}$$

(reasons become clear later).

Concerning the behavior of the solution one can consider two different cases.

1. The condition $g_{tt} > 0$ hold true for all values of Θ. In this case u decreases and the rate of decrease gets faster for small values of r. This means that in the lowest order the solution becomes certainly ill defined at a critical radius $r = r_c$ given by the the condition $u = 1$: the reason is that u cannot get values large than one. The expression of the critical radius is given by

$$\begin{aligned} r_c &\geq \frac{4q}{(|u_\infty| + 1)\omega} \\ &= \frac{4\alpha Q_{em}}{(3 - p/2)} \frac{1}{(|u_\infty| + 1)\omega} \ . \end{aligned} \tag{2.6.25}$$

   The presence of the critical radius for the actual solution is also a necessity as the inspection of the expression for J_{tr} shows: $\partial_r \theta$ grows near the origin without bound and $u = 1$ is reached at some finite value of r. Boundary conditions require that the quantity $X = T^{rr}\sqrt{g}$ vanishes at critical radius (no momentum flows through the boundary). Substituting the expression of J_{tr} from the field equation to T^{rr} the expression for X reduces to a form, from which it is clear that X cannot vanish. The cautious conclusion is that boundary conditions cannot be satisfied and the underlying reason is probably the stationarity and spherical symmetry of the solution. Physical intuition suggests that that CP_2 type extremal performing zitterbewegung is needed to satisfy the boundary conditions.

2. g_{tt} vanishes for some value of Θ. In this case the radial derivative of u together with g_{tt} can become zero for some value of $r = r_c$. Boundary conditions can be satisfied only provided $r_c = 0$. Thus it seems that for the values of ω satisfying the condition $\omega^2 = \frac{4\lambda^2}{R^2 sin^2(\Theta_0)}$ it might be possible to find a globally defined solution. The study of differential equation for u however shows that the ansatz doesn't work. The conclusion is that although the boundary is generated it is not possible to satisfy boundary conditions.

A direct calculation of the coefficients u_n from power series expansion gives the following third order polynomial approximation for u ($V = 1/r$)

$$\begin{aligned} u &= \sum_n u_n A^n V^n \ , \\ u_0 &= u_\infty (< 0) \ , \quad u_1 = 1 \ , \\ u_2 &= K|u_\infty| \ , \quad u_3 = K(1 + 4K|u_\infty|) \ , \\ A &\equiv \frac{4q}{\omega} \ , \quad K \equiv \omega^2 \frac{R^2}{4} \ . \end{aligned} \tag{2.6.26}$$

The coefficients u_2 and u_3 are indeed positive which means that the value of the critical radius gets larger at least in these orders.

Solution contains three parameters: Kähler electric flux $Q = 4\pi q$, parameter ωR and parameter u_∞. The latter parameters can be regarded as parameters describing the properties of a flat vacuum extremal (lowest order solution) to which particle like solution is glued and are analogous to the parameters describing symmetry broken vacuum in gauge theories.

Solution is not a realistic model for topological condensation

The solution does not provide realistic model for topological condensation although it gives indirect support for some essential assumptions of TGD based description of Higgs mechanism.

1. When the value of ω is of the order of CP_2 mass the solution could be interpreted as the "exterior metric" of a "hole".
   i) The radius of the hole is of the order of CP_2 length and its mass is of the order of CP_2 mass.
   ii) Kähler electric field is generated and charge renormalization takes place classically at CP_2 length scales as is clear from the expression of $Q(r)$: $Q(r) \propto (\frac{-g_{rr}}{g_{tt}})^{1/4}$ and charge increases at short distances.
   iii) The existence of the critical radius is unavoidable but boundary conditions cannot be satisfied. The failure to satisfy boundary conditions might be related to stationarity or to the absence of magnetic field. The motion of the boundary component with velocity of light might be the only manner to satisfy boundary conditions. Second possibility is the breaking of spherical symmetry by the generation of a static magnetic field.
   iv) The absence of the Kähler magnetic field implies that the Kähler action has an infinite magnitude and the probability of the configuration is zero. A more realistic solution ansatz would break spherical symmetry containing dipole type magnetic field in the nearby region of the hole. The motion of the boundary with a velocity of light could serves as an alternative mechanism for the generation of magnetic field. The third possibility, supported by physical intuition, is that one must give up "hole" type extremal totally.
2. For sufficiently large values of r and for small values of ω (of the order of elementary particle mass scale), the solution might provide an approximate description for the region surrounding elementary particle. Although it is not possible to satisfy boundary conditions the order of magnitude estimate for the size of critical radius ($r_c \simeq \alpha/\omega$) should hold true for more realistic solutions, too. The order of magnitude for the critical radius is smaller than Compton length or larger if the vacuum parameter ω is larger than the mass of the particle. In macroscopic length scales the value of ω is of order $1/R$. This does not lead to a contradiction if the many-sheeted space-time concept is accepted so that $\omega < m$ corresponds to elementary particle space-time sheet. An unrealistic feature of the solution is that the relationship between Z^0 and em charges is not correct: Z^0 charge should be very small in these length scales.

Exterior solution cannot be identified as a counterpart of Schwartshild solution

The first thing, which comes into mind is to ask whether one might identify exterior solution as the TGD counterpart of the Schwartshild solution. The identification of gravitational mass as absolute value of inertial mass which is negative for antimatter implies that vacuum extremals are vacua only with respect to the inertial four-momentum and have a non-vanishing gravitational four-momentum. Hence, in the approximation that the net density of inertial mass vanishes, vacuum extremals provide the proper manner to model matter, and the identification of the ansatz for a spherically symmetric extremal as the counterpart of Scwhartschild metric is certainly not possible. It is however useful to show explicitly that the identification is indeed unrealistic. The solution is consistent with Equivalence Principle but the electro-weak gauge forces are considerably weaker than gravitational forces. A wrong perihelion shift is also predicted so that the identification as an exterior metric of macroscopic objects is out of question.

1. Is Equivalence Principle respected?

The following calculation demonstrates that Equivalence Principle might not be satisfied for the solution ansatz (which need not actually define a preferred extremal!).

The gravitational mass of the solution is determined from the asymptotic behavior of g_{tt} and is given by

$$M_{gr} = \frac{R^2}{G}\omega q u_\infty \ , \tag{2.6.27}$$

and is proportional to the Kähler charge q of the solution.

One can estimate the gravitational mass density also by applying Newtonian approximation to the time component of the metric $g_{tt} = 1 - 2\Phi_{gr}$. One obtains Φ_{gr} corresponds in the lowest order approximation to a solution of Einstein's equations with the source consisting of a mass point at origin and the energy density of the Kähler electric field. The effective value of gravitational constant is however $G_{eg} = 8R^2\alpha_K$. Thus the only sensible interpretation is that the density of Kähler (inertial) energy is only a fraction $G/G_{eq} \equiv \epsilon \simeq .22 \times 10^{-6}$ of the density of gravitational mass. Hence the densities of positive energy matter and negative energy antimatter cancel each other in a good approximation.

The work with cosmic strings lead to a possible interpretation of the solution as a space-time sheet containing topologically condensed magnetic flux tube idealizable as a point. The negative Kähler electric action must cancel the positive Kähler magnetic action. The resulting structure in turn can condense to a vacuum extremal and Schwartshild metric is a good approximation for the metric.

One can estimate the contribution of the exterior region ($r > r_c$) to the inertial mass of the system and Equivalence principle requires this to be a fraction of order ϵ about the gravitational mass unless the region $r < r_c$ contains negative inertial mass density, which is of course quite possible. Approximating the metric with a flat metric and using first order approximation for $u(r)$ the energy reduces just to the standard Coulomb energy of charged sphere with radius r_c

$$\begin{aligned} M_I(ext) &= \frac{1}{32\pi\alpha_K}\int_{r>r_c} E^2\sqrt{g}d^3x \\ &\simeq \frac{\lambda q^2}{2\alpha_K r_c} \ , \\ \lambda &= \sqrt{1+\frac{R^2}{4}\omega^2(1-u_\infty^2)}\ (>1) \ . \end{aligned} \tag{2.6.28}$$

Approximating the metric with flat metric the contribution of the region $r > r_c$ to the energy of the solution is given by

$$M_I(ext) = \frac{1}{8\alpha_K}\lambda q\omega(1+|u_\infty|) \ . \tag{2.6.29}$$

The contribution is proportional to Kähler charge as expected. The ratio of external inertial and gravitational masses is given by the expression

$$\begin{aligned} \frac{M_I(ext)}{M_{gr}} &= \frac{G}{4R^2\alpha_K}x \ , \\ x &= \frac{(1+|u_\infty|)}{|u_\infty|} > 1 \ . \end{aligned} \tag{2.6.30}$$

In the approximation used the ratio of external inertial and gravitational masses is of order 10^{-6} for $R \sim 10^4\sqrt{G}$ implied by the p-adic length scale hypothesis and for $x \sim 1$. The result conforms with the above discussed interpretation.

The result forces to challenge the underlying implicit assumptions behind the calculation.

1. Many-sheeted space-time means that single space-time sheet need not be a good approximation for astrophysical systems. The GRT limit of TGD can be interpreted as obtained by lumping many-sheeted space-time time to Minkowski space with effective metric defined

as sum M^4 metric and sum of deviations from M^4 metric for various space-time sheets involved [K121]. This effective metric should correspond to that of General Relativity and Einstein's equations reflect the underlying Poincare invariance. Gravitational and cosmological constants follow as predictions and EP is satisfied.

2. The systems considered need not be preferred extremals of Kähler action so that one cannot take the results obtained too seriously. For vacuum extremals one does not encounter this problem at all and it could be that vacuum extremals with induced metric identified as GRT metric are a good approximation in astrophysical systems. This requires that single-sheetedness is a good approximation. TGD based single-sheeted models for astrophysical and cosmological systems rely on this assumption.

2. Z^0 and electromagnetic forces are much weaker than gravitational force

The extremal in question carries Kähler charge and therefore also Z^0 and electromagnetic charge. This implies long range gauge interactions, which ought to be weaker than gravitational interaction in the astrophysical scales. This is indeed the case as the following argument shows.

Expressing the Kähler charge using Planck mass as unit and using the relationships between gauge fields one obtains a direct measure for the strength of the Z^0 force as compared with the strength of gravitational force.

$$Q_Z \equiv \varepsilon_Z M_{gr}\sqrt{G} \ . \tag{2.6.31}$$

The value of the parameter ε_Z should be smaller than one. A transparent form for this condition is obtained, when one writes $\Phi = \omega t = \Omega m^0 : \Omega = \lambda\omega$:

$$\varepsilon_Z = \frac{\alpha_K}{\alpha_Z}\frac{1}{\pi(1+|u_\infty|)\Omega R}\sqrt{\frac{G}{R}} \ . \tag{2.6.32}$$

The order of magnitude is determined by the values of the parameters $\sqrt{\frac{G}{R^2}} \sim 10^{-4}$ and ΩR. Global Minkowskian signature of the induced metric implies the condition $\Omega R < 2$ for the allowed values of the parameter ΩR. In macroscopic length scales one has $\Omega R \sim 1$ so that Z^0 force is by a factor of order 10^{-4} weaker than gravitational force. In elementary particle length scales with $\omega \sim m$ situation is completely different as expected.

3. The shift of the perihelion is predicted incorrectly

The g_{rr} component of Reissner-Nordström and TGD metrics are given by the expressions

$$g_{rr} = -\frac{1}{(1-\frac{2GM}{r})} \ , \tag{2.6.33}$$

and

$$g_{rr} \simeq 1 - \frac{\frac{Rq}{\omega^2}}{\left[1-(u_\infty - \frac{4q}{\omega r})^2\right] r^4} \ , \tag{2.6.34}$$

respectively. For reasonable values of q, ω and u_∞ the this terms is extremely small as compared with $1/r$ term so that these expressions differ by $1/r$ term.

The absence of the $1/r$ term from g_{rr}-component of the metric predicts that the shift of the perihelion for elliptic plane orbits is about 2/3 times that predicted by GRT so that the identification as a metric associated with objects of a planetary scale leads to an experimental contradiction. Reissner-Nordström solutions are obtained as vacuum extremals so that standard predictions of GRT are obtained for the planetary motion.

One might hope that the generalization of the form of the spherically symmetric ansatz by introducing the same modification as needed for the embedding of Reissner-Nordstrm̈ metric might help. The modification would read as

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t + f(r) \ , \\ m^0 &= \lambda t + h(r) \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{2.6.35}$$

The vanishing of the g_{tr} component of the metric gives the condition

$$\lambda \partial_r h - \frac{R^2}{4} sin^2(\Theta)\omega \partial_r f = 0 \ . \tag{2.6.36}$$

The expression for the radial component of the metric transforms to

$$g_{rr} \simeq \partial_r h^2 - 1 - \frac{R^2}{4}(\partial_r \Theta)^2 - \frac{R^2}{4} sin^2(\Theta)\partial_r f^2 \ , \tag{2.6.37}$$

Essentially the same perihelion shift as for Schwartschild metric is obtained if g_{rr} approaches asymptotically to its expression for Schwartschild metric. This is guaranteed if the following conditions hold true:

$$f(r)_{r\rightarrow\infty} \rightarrow \omega r \ , \ \Lambda^2 - 1 = \frac{R^2\omega^2}{4} sin^2(\Theta_\infty) \ll \frac{2GM}{\langle r \rangle} \ . \tag{2.6.38}$$

In the second equation $\langle r \rangle$ corresponds to the average radius of the planetary orbit.

The field equations for this ansatz can be written as conditions for energy momentum and color charge conservation. Two equations are enough to determine the functions $\Theta(r)$ and $f(r)$. The equation for momentum conservation is same as before. Second field equation corresponds to the conserved isometry current associated with the color isometry $\Phi \rightarrow \Phi + \epsilon$ and gives equation for f.

$$[T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g}]_{,r} = 0 \ . \tag{2.6.39}$$

The conservation laws associated with other infinitesimal $SU(2)$ rotations of S^2_I should be satisfied identically. This equation can be readily integrated to give

$$T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g_{tt}g_{rr}} = \frac{C}{r^2} \ . \tag{2.6.40}$$

Unfortunately, the result is inconsistent with the $1/r^4$ behavior of T^{rr} and $f \rightarrow \omega r$ implies by correct red shift.

It seems that the only possible way out of the difficulty is to replace spherical symmetry with a symmetry with respect to the rotations around z-axis. The simplest modification of the solution ansatz is as follows:

$$m^0 = \lambda t + h(\rho) \ , \quad \Phi = \omega t + k\rho \quad .$$

Thanks to the linear dependence of Φ on ρ, the conservation laws for momentum and color isospin reduce to the same condition. The ansatz induces a small breaking of spherical symmetry by adding to $g_{\rho\rho}$ the term

$$(\partial_\rho h)^2 - \frac{R^2}{4} sin^2(\Theta)k^2 \ .$$

One might hope that in the plane $\theta = \pi/2$, where $r = \rho$ holds true, the ansatz could behave like Schwartschild metric if the conditions discussed above are posed (including the condition $k = \omega$). The breaking of the spherical symmetry in the planetary system would be coded already to the gravitational field of Sun.

Also the study of the embeddings of Reissner-Nordström metric as vacuum extremals and the investigation of spherically symmetric (inertial) vacuum extremals for which gravitational four-momentum is conserved [K121] leads to the conclusion that the loss of spherical symmetry due to rotation is inevitable characteristic of realistic solutions.

2.6.8 Maxwell Hydrodynamics As A Toy Model For TGD

The field equations of TGD are extremely non-linear and all known solutions have been discovered by symmetry arguments. Chern-Simons term plays essential role also in the construction of solutions of field equations and at partonic level defines braiding for light-like partonic 3-surfaces expected to play key role in the construction of S-matrix. The inspiration for this section came from Terence Tao's blog posting *2006 ICM: Etienne Ghys, "Knots and dynamics"* [A92] giving an elegant summary about amazing mathematical results related to knots, links, braids and hydrodynamical flows in dimension $D = 3$. Posting tells about really amazing mathematical results related to knots.

Chern-Simons term as helicity invariant

Tao mentions helicity as an invariant of fluid flow. Chern-Simons action defined by the induced Kähler gauge potential for light-like 3-surfaces has interpretation as helicity when Kähler gauge potential is identified as fluid velocity. This flow can be continued to the interior of space-time sheet. Also the dual of the induced Kähler form defines a flow at the light-like partonic surfaces but not in the interior of space-time sheet. The lines of this flow can be interpreted as magnetic field lines. This flow is incompressible and represents a conserved charge (Kähler magnetic flux).

The question is which of these flows should define number theoretical braids. Perhaps both of them can appear in the definition of S-matrix and correspond to different kinds of partonic matter (electric/magnetic charges, quarks/leptons?, ...). Second kind of matter could not flow in the interior of space-time sheet. Or could interpretation in terms of electric magnetic duality make sense?

Helicity is not gauge invariant and this is as it must be in TGD framework since CP_2 symplectic transformations induce $U(1)$ gauge transformation, which deforms space-time surface an modifies induced metric as well as classical electroweak fields defined by induced spinor connection. Gauge degeneracy is transformed to spin glass degeneracy.

Maxwell hydrodynamics

In TGD Maxwell's equations are replaced with field equations which express conservation laws and are thus hydrodynamical in character. With this background the idea that the analogy between gauge theory and hydrodynamics might be applied also in the reverse direction is natural. Hence one might ask what kind of relativistic hydrodynamics results if assumes that the action principle is Maxwell action for the four-velocity u^α with the constraint term saying that light velocity is maximal signal velocity.

1. For massive particles the length of four-velocity equals to 1: $u^\alpha u_\alpha = 1$. In massless case one has $u^\alpha u_\alpha = 0$. Geometrically this means that one has sigma model with target space which is 3-D Lobatschevski space or at light-cone boundary. This condition means the addition of constraint term

$$\lambda(u^\alpha u_\alpha - \epsilon) \tag{2.6.41}$$

to the Maxwell action. $\epsilon = 1/0$ holds for massive/massless flow. In the following the notation of electrodynamics is used to make easier the comparison with electrodynamics.

2. The constraint term destroys gauge invariance by allowing to express A^0 in terms of A^i but in general the constraint is not equivalent to a choice of gauge in electrodynamics since the solutions to the field equations with constraint term are not solutions of field equations without it. One obtains field equations for an effectively massive em field with Lagrange multiplier λ having interpretation as photon mass depending on space-time point:

$$\begin{aligned} j^\alpha &= \partial_\beta F^{\alpha\beta} = \lambda A^\alpha \ , \\ A^\alpha &\equiv u^\alpha \ , \ F^{\alpha\beta} = \partial^\beta A^\alpha - \partial^\alpha A^\beta \ . \end{aligned} \tag{2.6.42}$$

3. In electrodynamic context the natural interpretation would be in terms of spontaneous massivation of photon and seems to occur for both values of ϵ. The analog of em current given by λA^α is in general non-vanishing and conserved. This conservation law is quite strong additional constraint on the hydrodynamics. What is interesting is that breaking of gauge invariance does not lead to a loss of charge conservation.
4. One can solve λ by contracting the equations with A_α to obtain

$$\lambda = j^\alpha A_\alpha$$

for $\epsilon = 1$. For $\epsilon = 0$ one obtains

$$j^\alpha A_\alpha = 0$$

stating that the field does not dissipate energy: λ can be however non-vanishing unless field equations imply $j^\alpha = 0$. One can say that for $\epsilon = 0$ spontaneous massivation can occur. For $\epsilon = 1$ massivation is present from the beginning and dissipation rate determines photon mass: a natural interpretation for $\epsilon = 1$ would be in terms of thermal massivation of photon. Non-tachyonicity fixes the sign of the dissipation term so that the thermodynamical arrow of time is fixed by causality.
5. For $\epsilon = 0$ massless plane wave solutions are possible and one has

$$\partial_\alpha \partial_\beta A^\beta = \lambda A_\alpha \ .$$

$\lambda = 0$ is obtained in Lorentz gauge which is consistent with the condition $\epsilon = 0$. Also superpositions of plane waves with same polarization and direction of propagation are solutions of field equations: these solutions represent dispersionless precisely targeted pulses. For superpositions of plane waves λ with 4-momenta, which are not all parallel λ is non-vanishing so that non-linear self interactions due to the constraint can be said to induce massivation. In asymptotic states for which gauge symmetry is not broken one expects a decomposition of solutions to regions of space-time carrying this kind of pulses, which brings in mind final states of particle reactions containing free photons with fixed polarizations.
6. Gradient flows satisfying the conditions

$$A_\alpha = \partial_\alpha \Phi \ , \ A^\alpha A_\alpha = \epsilon \tag{2.6.43}$$

give rise to identically vanishing hydrodynamical gauge fields and $\lambda = 0$ holds true. These solutions are vacua since energy momentum tensor vanishes identically. There is huge number of this kind of solutions and spin glass degeneracy suggests itself. Small deformations of these vacuum flows are expected to give rise to non-vacuum flows.
7. The counterparts of charged solutions are of special interest. For $\epsilon = 0$ the solution $(u^0, u^r) = (Q/r)(1,1)$ is a solution of field equations outside origin and corresponds to electric field of a point charge Q. In fact, for $\epsilon = 0$ any ansatz $(u^0, u^r) = f(r)(1,1)$ satisfies field equations for a suitable choice of $\lambda(r)$ since the ratio of equations associate with j^0 and j^r gives an equation which is trivially satisfied. For $\epsilon = 1$ the ansatz $(u^0, u^r) = (cosh(u), sinh(u))$ expressing solution in terms of hyperbolic angle linearizes the field equation obtained by dividing the equations for j^0 and j^r to eliminate λ. The resulting equation is

$$\partial_r^2 u + \frac{2\partial_r u}{r} = 0$$

for ordinary Coulomb potential and one obtains $(u^0, u^r) = (cosh(u_0 + k/r), sinh(u_0 + k/r))$. The charge of the solution at the limit $r \to \infty$ approaches to the value $Q = sinh(u_0)k$ and diverges at the limit $r \to 0$. The charge increases exponentially as a function of $1/r$ near origin rather than logarithmically as in QED and the interpretation in terms of thermal screening suggests itself. Hyperbolic ansatz might simplify considerably the field equations also in the general case.

Similarities with TGD

There are strong similarities with TGD which suggests that the proposed model might provide a toy model for the dynamics defined by Kähler action.

1. Also in TGD field equations are essentially hydrodynamical equations stating the conservation of various isometry charges. Gauge invariance is broken for the induced Kähler field although Kähler charge is conserved. There is huge vacuum degeneracy corresponding to vanishing of induced Kähler field and the interpretation is in terms of spin glass degeneracy.
2. Also in TGD dissipation rate vanishes for the known solutions of field equations and a possible interpretation is as space-time correlates for asymptotic non-dissipating self organization patterns.
3. In TGD framework massless extremals represent the analogs for superpositions of plane waves with fixed polarization and propagation direction and representing targeted and dispersionless propagation of signal. Gauge currents are light-like and non-vanishing for these solutions. The decomposition of space-time surface to space-time sheets representing particles is much more general counterpart for the asymptotic solutions of Maxwell hydrodynamics with vanishing λ.
4. In TGD framework one can consider the possibility that the four-velocity assignable to a macroscopic quantum phase is proportional to the induced Kähler gauge potential. In this kind of situation one could speak of a quantal variant of Maxwell hydrodynamics, at least for light-like partonic 3-surfaces. For instance, the condition

$$D^\alpha D_\alpha \Psi = 0 \ \ , \ \ D_\alpha \Psi = (\partial_\alpha - iq_K A_\alpha)\Psi$$

for the order parameter of the quantum phase corresponds at classical level to the condition $p^\alpha = q_K Q^\alpha + l^\alpha$, where q_K is Kähler charge of fermion and l^α is a light-like vector field naturally assignable to the partonic boundary component. This gives $u^\alpha = (q_K Q^\alpha + l^\alpha)/m$, $m^2 = p^\alpha p_\alpha$, which is somewhat more general condition. The expressibility of u^α in terms of the vector fields provided by the induced geometry is very natural.

The value ϵ depends on space-time region and it would seem that also $\epsilon = -1$ is possible meaning tachyonicity and breaking of causality. Kähler gauge potential could however have a time-like pure gauge component in M^4 possibly saving the situation. The construction of quantum TGD at parton level indeed forces to assume that Kähler gauge potential has Lorentz invariant M^4 component $A_a = constant$ in the direction of the light-cone proper time coordinate axis a. Note that the decomposition of WCW to sectors consisting of space-time sheets inside future or past light-cone of M^4 is an essential element of the construction of WCW geometry and does not imply breaking of Poincare invariance. Without this component $u_\alpha u^\alpha$ could certainly be negative. The contribution of M^4 component could prevent this for preferred extremals.

If TGD is taken seriously, these similarities force to ask whether Maxwell hydrodynamics might be interpreted as a nonlinear variant of electrodynamics. Probably not: in TGD em field is proportional to the induced Kähler form only in special cases and is in general non-vanishing also for vacuum extremals.

Chapter 3

About Identification of the Preferred extremals of Kähler Action

3.1 Introduction

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K116]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

3.1.1 Preferred Extremals As Critical Extremals

The study of the Kähler-Dirac equation leads to a detailed view about criticality. Quantum criticality [D7] fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The discovery that the hierarchy of Planck constants realized in terms of singular covering spaces of $CD \times CP_2$ can be understood in terms of the extremely non-linear dynamics of Kähler action implying 1-to-many correspondence between canonical momentum densities and time derivatives of the embedding space coordinates led to a further very concrete understanding of the criticality at space-time level and its relationship to zero energy ontology [K57].

Criticality is accompanied by conformal invariance and this leads to the proposal that critical deformations correspond to Kac-Moody type conformal algebra respecting the light-likeness of the partonic orbits and acting trivially at partonic 2-surfaces. Sub-algebras of conformal algebras with conformal weights divisible by integer n would act as gauge symmetries and these algebras would form an inclusion hierarchy defining hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

3.1.2 Construction Of Preferred Extremals

There has been considerable progress in the understanding of both preferred extremals and Kähler-Dirac equation.

1. For preferred extremals the generalization of conformal invariance to 4-D situation is very attractive idea and leads to concrete conditions formally similar to those encountered in string model [K15]. In particular, Einstein's equations with cosmological constant would solve consistency conditions and field equations would reduce to a purely algebraic statements analogous to those appearing in equations for minimal surfaces if one assumes that space-time surface has Hermitian structure or its Minkowskian variant Hamilton-Jacobi structure (Appendix).

2. The older approach based on basic heuristics for massless equations, on effective 3-dimensionality, weak form of electric magnetic duality, and Beltrami flows is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic embedding space [K107].

The basic step of progress was the realization that the known extremals of Kähler action - certainly limiting cases of more general extremals - can be deformed to more general extremals having interpretation as preferred extremals.

1. The generalization boils down to the condition that field equations reduce to the condition that the traces $Tr(TH^k)$ for the product of energy momentum tensor and second fundamental form vanish. In string models energy momentum tensor corresponds to metric and one obtains minimal surface equations. The equations reduce to purely algebraic conditions stating that T and H^k have no common components. Complex structure of string world sheet makes this possible.

   Stringy conditions for metric stating $g_{zz} = g_{\overline{z}\overline{z}} = 0$ generalize. The condition that field equations reduce to $Tr(TH^k) = 0$ requires that the terms involving Kähler gauge current in field equations vanish. This is achieved if Einstein's equations hold true (one can consider also more general way to satisfy the conditions). The conditions guaranteeing the vanishing of the trace in turn boil down to the existence of Hermitian structure in the case of Euclidian signature and to the existence of its analog - Hamilton-Jacobi structure - for Minkowskian signature (Appendix). These conditions state that certain components of the induced metric vanish in complex coordinates or Hamilton-Jacobi coordinates.

2. In string model the replacement of the embedding space coordinate variables with quantized ones allows to interpret the conditions on metric as Virasoro conditions. In the recent case a generalization of classical Virasoro conditions to four-dimensional ones would be in question. An interesting question is whether quantization of these conditions could make sense also in TGD framework at least as a useful trick to deduce information about quantum states in WCW degrees of freedom.

3. The interpretation of the extended algebra as Yangian [A25] [B22] suggested previously [K116] to act as a generalization of conformal algebra in TGD Universe is attractive. There is also the conjecture that preferred extremals could be interpreted as quaternionic of co-quaternionic 4-surface of the octonionic embedding space with octonionic representation of the gamma matrices defining the notion of tangent space quanternionicity.

3.2 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B5] was proposed first by Olive and Montonen and is central in $\mathcal{N}=4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K29] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2,-1,-1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed

solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current. Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

3.2.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the embedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \qquad (3.2.1)$$

A more general form of this duality is suggested by the considerations of [K57] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \qquad (3.2.2)$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \qquad (3.2.3)$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_ZF_Z}{\hbar} = 2R_{03} \ . \end{aligned} \qquad (3.2.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{3.2.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{3.2.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi} \times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{3.2.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.
2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K85] supports this interpretation.
3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of

course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{3.2.8}$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{3.2.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{3.2.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the

fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K92]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.
2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K121]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.
3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.
4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

3.2.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum

numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L \nu_R$. $\nu_L \overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D4] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [**?**] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies ZEO. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes

in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K70] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K71] .

3.2.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j^{\alpha}_K A_{\alpha}$ plus and integral of the boundary term $J^{n\beta}A_{\beta}\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_{\beta}\sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $h \to n \times h$ would effectively describe this. Boundary conditions would however give $1/n$ factor so that $\hbar$ would disappear from the Kähler function! It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute “almost” would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K15]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.
2. The original naïve conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \ . \tag{3.2.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.
3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.
4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{3.2.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} = 0 \ . \tag{3.2.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B11] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K15] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha\phi$ and $j_I^\alpha\phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha\phi) \quad = \quad 0 \ . \tag{3.2.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

3.3 An attempt to understand preferred extremals of Kähler action

Preferred extremal of Kähler action is one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what "preferred" really means. For instance, the conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [L45]. The problem is however how to assign a complex coordinate with the string world sheet having Minkowskian signature of metric. One can hope that the understanding of preferred extremals could allow to identify two preferred complex coordinates whose existence is also suggested by number theoretical vision giving preferred role for the rational points of partonic 2-surfaces in preferred coordinates. The best one could hope is a general solution of field equations in accordance with the hints that TGD is integrable quantum theory.

3.3.1 What "preferred" could mean?

The first question is what preferred extremal could mean.

1. In positive energy ontology preferred extremal would be a space-time surface assignable to given 3-surface and unique in the ideal situation: since one cannot pose conditions to the normal derivatives of embedding space coordinates at 3-surface, there is infinity of extremals. Some additional conditions are required and space-time surface would be analogous to Bohr orbit: hence the attribute "preferred". The problem would be to understand what "preferred" could mean. The non-determinism of Kähler action however destroyed this dream in its original form and led to zero energy ontology (ZEO).

2. In ZEO one considers extremals as space-time surfaces connecting two space-like 3-surfaces at the boundaries. One might hope that these 4-surfaces are unique. The non-determinism of Kähler action suggests that this is not the case. At least there is conformal invariance respecting the light-likeness of the 3-D parton orbits at which the signature of the induced metric changes: the conformal transformations would leave the space-like 3-D ends or at least partonic 2-surfaces invariant. This non-determinism would correspond to quantum criticality.

3. Effective 2-dimensionality follows from strong form of general coordinate invariance (GCI) stating that light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface are equivalent physically: partonic 2-surfaces and their 4-D tangent space data would determine everything. One can however worry about how effective 2-dimensionality relates to the fact that the modes of the induced spinor field are localized at string world sheets and partonic 2-surface. Are the tangent space data equivalent with the data characterizing string world sheets as surfaces carrying vanishing electroweak fields?

   There is however a problem: the hierarchy of Planck constants (dark matter) requires that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom so that either space-like or light-like surfaces do not seem to be quite enough.

   Should one then include also the light-like partonic orbits to the what one calls 3-surface? The resulting connected 3-surfaces would define analogs of Wilson loops. Could the conformal equivalence class of the preferred extremal be unique without any additional conditions? If so, one could get rid of the attribute "preferred". The fractal character of the many-sheeted space-time however suggests that one can have this kind of uniqueness only in given length scale resolution and that "radiative corrections" due to the non-determinism are always present.

These considerations show that the notion of preferred extremal is still far from being precisely defined and it is not even clear whether the attribute "preferred" is needed. If not then the question is what are the extremals of Kähler action.

3.3.2 What is known about extremals?

A lot is is known about properties of extremals and just by trying to integrate all this understanding, one might gain new visions. The problem is that all these arguments are heuristic and rely heavily on physical intuition. The following considerations relate to the space-time regions having Minkowskian signature of the induced metric. The attempt to generalize the construction also to Euclidian regions could be very rewarding. Only a humble attempt to combine various ideas to a more coherent picture is in question.

The core observations and visions are following.

1. Hamilton-Jacobi coordinates for M^4 (discussed in this chapter) define natural preferred coordinates for Minkowskian space-time sheet and might allow to identify string world sheets for X^4 as those for M^4. Hamilton-Jacobi coordinates consist of light-like coordinate m and its dual defining local 2-plane $M^2 \subset M^4$ and complex transversal complex coordinates $(w, \overline{w})$ for a plane E^2_x orthogonal to M^2_x at each point of M^4. Clearly, hyper-complex analyticity and complex analyticity are in question.
2. Space-time sheets allow a slicing by string world sheets (partonic 2-surfaces) labelled by partonic 2-surfaces (string world sheets).
3. The quaternionic planes of octonion space containing preferred hyper-complex plane are labelled by CP_2, which might be called CP_2^{mod} [K107]. The identification $CP_2 = CP_2^{mod}$ motivates the notion of $M^8 - -M^4 \times CP_2$ duality [K28]. It also inspires a concrete solution ansatz assuming the equivalence of two different identifications of the quaternionic tangent space of the space-time sheet and implying that string world sheets can be regarded as strings in the 6-D coset space $G_2/SU(3)$. The group G_2 of octonion automorphisms has already earlier appeared in TGD framework.
4. The duality between partonic 2-surfaces and string world sheets in turn suggests that the $CP_2 = CP_2^{mod}$ conditions reduce to string model for partonic 2-surfaces in $CP_2 = SU(3)/U(2)$. String model in both cases could mean just hypercomplex/complex analyticity for the coordinates of the coset space as functions of hyper-complex/complex coordinate of string world sheet/partonic 2-surface.

The considerations of this section lead to a revival of an old very ambitious and very romantic number theoretic idea.

1. To begin with express octonions in the form $o = q_1 + Iq_2$, where q_i is quaternion and I is an octonionic imaginary unit in the complement of fixed a quaternionic sub-space of octonions. Map preferred coordinates of $H = M^4 \times CP_2$ to octonionic coordinate, form an arbitrary octonion analytic function having expansion with real Taylor or Laurent coefficients to avoid problems due to non-commutativity and non-associativity. Map the outcome to a point of H to get a map $H \rightarrow H$. This procedure is nothing but a generalization of Wick rotation to get an 8-D generalization of analytic map.
2. Identify the preferred extremals of Kähler action as surfaces obtained by requiring the vanishing of the imaginary part of an octonion analytic function. Partonic 2-surfaces and string world sheets would correspond to commutative sub-manifolds of the space-time surface and of embedding space and would emerge naturally. The ends of braid strands at partonic 2-surface would naturally correspond to the poles of the octonion analytic functions. This would mean a huge generalization of conformal invariance of string models to octonionic conformal invariance and an exact solution of the field equations of TGD and presumably of quantum TGD itself.

3.3.3 Basic ideas about preferred extremals

The slicing of the space-time sheet by partonic 2-surfaces and string world sheets

The basic vision is that space-time sheets are sliced by partonic 2-surfaces and string world sheets. The challenge is to formulate this more precisely at the level of the preferred extremals of Kähler action.

1. Almost topological QFT property means that the Kähler action reduces to Chern-Simons terms assignable to 3-surfaces. This is guaranteed by the vanishing of the Coulomb term in the action density implied automatically if conserved Kähler current is proportional to the instanton current with proportionality coefficient some scalar function.
2. The field equations reduce to the conservation of isometry currents. An attractive ansatz is that the flow lines of these currents define global coordinates. This means that these currents are Beltrami flows [B11] so that corresponding 1-forms J satisfy the condition $J \wedge dJ = 0$. These conditions are satisfied if

$$J = \Phi \nabla \Psi$$

hold true for conserved currents. From this one obtains that Ψ defines global coordinate varying along flow lines of J.
3. A possible interpretation is in terms of local polarization and momentum directions defined by the scalar functions involved and natural additional conditions are that the gradients of Ψ and Φ are orthogonal:

$$\nabla \Phi \cdot \nabla \Psi = 0 \ ,$$

and that the Ψ satisfies massless d'Alembert equation

$$\nabla^2 \Psi = 0$$

as a consequence of current conservation. If Ψ defines a light-like vector field - in other words

$$\nabla \Psi \cdot \nabla \Psi = 0 \ ,$$

the light-like dual of Φ -call it Φ_c- defines a light-like like coordinate and Φ and Φ_c defines a light-like plane at each point of space-time sheet.

If also Φ satisfies d'Alembert equation

$$\nabla^2 \Phi = 0 \ ,$$

also the current

$$K = \Psi \nabla \Phi$$

is conserved and its flow lines define a global coordinate in the polarization plane orthogonal to time-lik plane defined by local light-like momentum direction.

If Φ allows a continuation to an analytic function of the transversal complex coordinate, one obtains a coordinatization of space-time surface by Ψ and its dual (defining hyper-complex coordinate) and $w, \overline{w}$. Complex analyticity and its hyper-complex variant would allow to provide space-time surface with four coordinates very much analogous with Hamilton-Jacobi coordinates of M^4.

This would mean a decomposition of the tangent space of space-time surface to orthogonal planes defined by light-like momentum and plane orthogonal to it. If the flow lines of J defined Beltrami flow it seems that the distribution of momentum planes is integrable.
4. General arguments suggest that the space-time sheets allow a slicing by string world sheets parametrized by partonic 2-surfaces or vice versa. This would mean a intimate connection with the mathematics of string models. The two complex coordinates assignable to the Yangian of affine algebra would naturally relate to string world sheets and partonic 2-surfaces and the highly non-trivial challenge is to identify them appropriately.

Hamilton-Jacobi coordinates for M^4

The earlier attempts to construct preferred extremals [K15] led to the realization that so called Hamilton-Jacobi coordinates (m, w) for M^4 define its slicing by string world sheets parametrized by partonic 2-surfaces. m would be pair of light-like conjugate coordinates associated with an integrable distribution of planes M^2 and w would define a complex coordinate for the integrable distribution of 2-planes E^2 orthogonal to M^2. There is a great temptation to assume that these coordinates define preferred coordinates for M^4.

1. The slicing is very much analogous to that for space-time sheets and the natural question is how these slicings relate. What is of special interest is that the momentum plane M^2 can be defined by massless momentum. The scaling of this vector does not matter so that these planes are labelled by points z of sphere S^2 telling the direction of the line $M^2 \cap E^3$, when one assigns rest frame and therefore S^2 with the preferred time coordinate defined by the line connecting the tips of CD. This direction vector can be mapped to a twistor consisting of a spinor and its conjugate. The complex scalings of the twistor $(u, \overline{u}) \rightarrow \lambda u, \overline{u}/\lambda)$ define the same plane. Projective twistor like entities defining CP_1 having only one complex component instead of three are in question. This complex number defines with certain prerequisites a local coordinate for space-time sheet and together with the complex coordinate of E^2 could serve as a pair of complex coordinates (z, w) for space-time sheet. This brings strongly in mind the two complex coordinates appearing in the expansion of the generators of quantum Yangian of quantum affine algebra [L45].
2. The coordinate Ψ appearing in Beltrami flow defines the light-like vector field defining M^2 distribution. Its hyper-complex conjugate would define Ψ_c and conjugate light-like direction. An attractive possibility is that Φ allows analytic continuation to a holomorphic function of w. In this manner one would have four coordinates for M^4 also for space-time sheet.
3. The general vision is that at each point of space-time surface one can decompose the tangent space to $M^2(x) \subset M^4 = M^2_x \times E^2_x$ representing momentum plane and polarization plane $E^2 \subset E^2_x \times T(CP_2)$. The moduli space of planes $E^2 \subset E^6$ is 8-dimensional and parametrized by $SO(6)/SO(2) \times SO(4)$ for a given E^2_x. How can one achieve this selection and what conditions it must satisfy? Certainly the choice must be integrable but this is not the only condition.

Space-time surfaces as associative/co-associative surfaces

The idea that number theory determines classical dynamics in terms of associativity condition means that space-time surfaces are in some sense quaternionic surfaces of an octonionic space-time. It took several trials before the recent form of this hypothesis was achieved.

1. Octonionic structure is defined in terms of the octonionic representaton of gamma matrices of the embedding space existing only in dimension $D = 8$ since octonion units are in one-one correspondence with tangent vectors of the tangent space. Octonionic real unit corresponds to a preferred time axes (and rest frame) identified naturally as that connecting the tips of CD. What modified gamma matrices mean depends on variational principle for space-time surface. For volume action one would obtain induced gamma matrices. For Kähler action one obtains something different. In particular, the modified gamma matrices do not define vector basis identical with tangent vector basis of space-time surface.
2. Quaternionicity means that the modified gamma matrices defined as contractions of gamma matrices of H with canonical momentum densities for Kähler action span quaternionic sub-space of the octonionic tangent space [K127, K96]. A further condition is that each quaternionic space defined in this manner contains a preferred hyper-complex subspace of octonions.
3. The sub-space defined by the modified gamma matrices does not co-incide with the tangent space of space-time surface in general so that the interpretation of this condition is far from obvious. The canonical momentum densities need not define four independent vectors at given point. For instance, for massless extremals these densities are proportional to light-like vector so that the situation is degenerate and the space in question reduces to 2-D hyper-complex sub-space since light-like vector defines plane M^2.

The obvious questions are following.

1. Does the analog of tangent space defined by the octonionic modified gammas contain the local tangent space $M^2 \subset M^4$ for preferred extremals? For massless extremals [K15] this condition would be true. The orthogonal decomposition $T(X^4) = M^2 \oplus_\perp E^2$ can be defined at each point if this is true. For massless extremals also the functions Ψ and Φ can be identified.

2. One should answer also the following delicate question. Can M^2 really depend on point x of space-time? CP_2 as a moduli space of quaternionic planes emerges naturally if M^2 is *same* everywhere. It however seems that one should allow an integrable distribution of M^2_x such that M^2_x is same for all points of a given partonic 2-surface.

   How could one speak about fixed CP_2 (the embedding space) at the entire space-time sheet even when M^2_x varies?

   (a) Note first that G_2 (see `http://tinyurl.com/y9rrs7un`) defines the Lie group of octonionic automorphisms and G_2 action is needed to change the preferred hyper-octonionic sub-space. Various $SU(3)$ subgroups of G_2 are related by G_2 automorphism. Clearly, one must assign to each point of a string world sheet in the slicing parameterizing the partonic 2-surfaces an element of G_2. One would have Minkowskian string model with G_2 as a target space. As a matter fact, this string model is defined in the target space $G_2/SU(3)$ having dimension $D = 6$ since $SU(3)$ automorphisms leave given $SU(3)$ invariant.

   (b) This would allow to identify at each point of the string world sheet standard quaternionic basis - say in terms of complexified basis vectors consisting of two hyper-complex units and octonionic unit q_1 with "color isospin" $I_3 = 1/2$ and "color hypercharge" $Y = -1/3$ and its conjugate $\overline{q}_1$ with opposite color isospin and hypercharge.

   (c) The CP_2 point assigned with the quaternionic basis would correspond to the $SU(3)$ rotation needed to rotate the standard basis to this basis and would actually correspond to the first row of $SU(3)$ rotation matrix. Hyper-complex analyticity is the basic property of the solutions of the field equations representing Minkowskian string world sheets. Also now the same assumption is highly natural. In the case of string models in Minkowski space, the reduction of the induced metric to standard form implies Virasoro conditions and similar conditions are expected also now. There is no need to introduce action principle -just the hyper-complex analycitity is enough-since Kähler action already defines it.

3. The WZW model (see `http://tinyurl.com/ydxcvfhv`) inspired approach to the situation would be following. The parameterization corresponds to a map $g : X^2 \rightarrow G_2$ for which g defines a flat G_2 connection at string world sheet. WZW type action would give rise to this kind of situation. The transition $G_2 \rightarrow G_2/SU(3)$ would require that one gauges $SU(3)$ degrees of freedom by bringing in $SU(3)$ connection. Similar procedure for $CP_2 = SU(3)/U(2)$ would bring in $SU(3)$ valued chiral field and $U(2)$ gauge field. Instead of introducing these connections one can simply introduce $G_2/SU(3)$ and $SU(3)/U(2)$ valued chiral fields. What this observation suggests that this ansatz indeed predicts gluons and electroweak gauge bosons assignable to string like objects so that the mathematical picture would be consistent with physical intuition.

The two interpretations of CP_2

An old observation very relevant for what I have called $M^8 - H$ duality [K28] is that the moduli space of quaternionic sub-spaces of octonionic space (identifiable as M^8) containing preferred hyper-complex plane is CP_2. Or equivalently, the space of two planes whose addition extends hyper-complex plane to some quaternionic subspace can be parametrized by CP_2. This CP_2 can be called it CP_2^{mod} to avoid confusion. In the recent case this would mean that the space $E^2(x) \subset E^2_x \times T(CP_2)$ is represented by a point of CP_2^{mod}. On the other hand, the embedding of space-time surface to H defines a point of "real" CP_2. This gives two different CP_2s.

1. The highly suggestive idea is that the identification $CP_2^{mod} = CP_2$ (apart from isometry) is crucial for the construction of preferred extremals. Indeed, the projection of the space-time point to CP_2 would fix the local polarization plane completely. This condition for $E^2(x)$ would be purely local and depend on the values of CP_2 coordinates only. Second condition

for $E^2(x)$ would involve the gradients of embedding space coordinates including those of CP_2 coordinates.

2. The conditions that the planes M_x^2 form an integrable distribution at space-like level and that M_x^2 is determined by the modified gamma matrices. The integrability of this distribution for M^4 could imply the integrability for X^2. X^4 would differ from M^4 only by a deformation in degrees of freedom transversal to the string world sheets defined by the distribution of M^2s.

   Does this mean that one can begin from vacuum extremal with constant values of CP_2 coordinates and makes them non-constant but allows to depend only on transversal degrees of freedom? This condition is too strong even for simplest massless extremals for which CP_2 coordinates depend on transversal coordinates defined by $\epsilon \cdot m$ and $\epsilon \cdot k$. One could however allow dependence of CP_2 coordinates on light-like M^4 coordinate since the modification of the induced metric is light-like so that light-like coordinate remains light-like coordinate in this modification of the metric.

   Therefore, if one generalizes directly what is known about massless extremals, the most general dependence of CP_2 points on the light-like coordinates assignable to the distribution of M_x^2 would be dependence on either of the light-like coordinates of Hamilton-Jacobi coordinates but not both.

3.3.4 What could be the construction recipe for the preferred extremals assuming $CP_2 = CP_2^{mod}$ identification?

The crucial condition is that the planes $E^2(x)$ determined by the point of $CP_2 = CP_2^{mod}$ identification and by the tangent space of $E_x^2 \times CP_2$ are same. The challenge is to transform this condition to an explicit form. $CP_2 = CP_2^{mod}$ identification should be general coordinate invariant. This requires that also the representation of E^2 as (e^2, e^3) plane is general coordinate invariant suggesting that the use of preferred CP_2 coordinates - presumably complex Eguchi-Hanson coordinates - could make life easy. Preferred coordinates are also suggested by number theoretical vision. A careful consideration of the situation would be required.

The modified gamma matrices define a quaternionic sub-space analogous to tangent space of X^4 but not in general identical with the tangent space: this would be the case only if the action were 4-volume. I will use the notation $T_x^m(X^4)$ about the modified tangent space and call the vectors of $T_x^m(X^4)$ modified tangent vectors. I hope that this would not cause confusion.

$CP_2 = CP_2^{mod}$ condition

Quaternionic property of the counterpart of $T_x^m(X^4)$ allows an explicit formulation using the tangent vectors of $T_x^m(X^4)$.

1. The unit vector pair (e_2, e_3) should correspond to a unique tangent vector of H defined by the coordinate differentials dh^k in some natural coordinates used. Complex Eguchi-Hanson coordinates [L1] are a natural candidate for CP_2 and require complexified octonionic imaginary units. If octonionic units correspond to the tangent vector basis of H uniquely, this is possible.

2. The pair (e_2, e_3) as also its complexification $(q_1 = e_2 + ie_3, \overline{q}_1 = e_2 - ie_3)$ is expressible as a linear combination of octonionic units $I_2, ...I_7$ should be mapped to a point of $CP_2^{mod} = CP_2$ in canonical manner. This mapping is what should be expressed explicitly. One should express given (e_2, e_3) in terms of $SU(3)$ rotation applied to a standard vector. After that one should define the corresponding CP_2 point by the bundle projection $SU(3) \rightarrow CP_2$.

3. The tangent vector pair

$$(\partial_w h^k, \partial_{\overline{w}} h^k)$$

   defines second representation of the tangent space of $E^2(x)$. This pair should be equivalent with the pair $(q_1, \overline{q}_1)$. Here one must be however very cautious with the choice of coordinates. If the choice of w is unique apart from constant the gradients should be unique. One can use also real coordinates (x, y) instead of $(w = x + iy, \overline{w} = x - iy)$ and the pair (e_2, e_3). One can project the tangent vector pair to the standard vielbein basis which must correspond to the octonionic basis

$$(\partial_x h^k, \partial_y h^k) \rightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A) e_A) \leftrightarrow (e_2, e_3) \ ,$$

where the e_A denote the octonion units in 1-1 correspondence with vielbein vectors. This expression can be compared to the expression of (e_2, e_3) derived from the knowledge of CP_2 projection.

Formulation of quaternionicity condition in terms of octonionic structure constants

One can consider also a formulation of the quaternionic tangent planes in terms of (e_2, e_3) expressed in terms of octonionic units deducible from the condition that unit vectors obey quaternionic algebra. The expressions for octonionic (see `http://tinyurl.com/5m5lqr`) *resp.* quaternionic (see `http://tinyurl.com/3rr79p9`) structure constants can be found at [A16] *resp.* [A17].

1. The ansatz is

$$\begin{aligned}
\{E_k\} &= \{1, I_1, E_2, E_3\} \ , \\
E_2 &= E_{2k} e^k \equiv \sum_{k=2}^{7} E_{2k} e^k \ \ , \ \ E_3 = E_{3k} e^k \equiv \sum_{k=2}^{7} E_{3k} e^k \ , \\
|E_2| &= 1 \ , \ |E_3| = 1 \ .
\end{aligned} \tag{3.3.1}$$

2. The multiplication table for octonionic units expressible in terms of octonionic triangle (see `http://tinyurl.com/5m5lqr`) [A16] gives

$$f^{1kl} E_{2k} = E_{3l} \ \ , \ \ f^{1kl} E_{3k} = -E_{2l} \ \ , \ \ f^{klr} E_{2k} E_{3l} = \delta_1^r \ . \tag{3.3.2}$$

   Here the indices are raised by unit metric so that there is no difference between lower and upper indices. Summation convention is assumed. Also the contribution of the real unit is present in the structure constants of third equation but this contribution must vanish.

3. The conditions are linear and quadratic in the coefficients E_{2k} and E_{3k} and are expected to allow an explicit solution. The first two conditions define homogenous equations which must allow solution. The coefficient matrix acting on (E_2, E_3) is of the form

$$\left(\begin{array}{cc} f_1 & 1 \\ -1 & f_1 \end{array} \right) \ ,$$

   where 1 denotes unit matrix. The vanishing of the determinant of this matrix should be due to the highly symmetric properties of the structure constants. In fact the equations can be written as eigen conditions

$$f_1 \circ (E_2 \pm i E_3) = \mp i (E_2 \pm i E_3) \ ,$$

   and one can say that the structure constants are eigenstates of the hermitian operator defined by I_1 analogous to color hyper charge. Both values of color hyper charged are obtained.

Explicit expression for the $CP_2 = CP_2^{mod}$ conditions

The symmetry under $SU(3)$ allows to construct the solutions of the above equations directly.

1. One can introduce complexified basis of octonion units transforming like $(1, 1, 3, \overline{3})$ under SU(3). Note the analogy of triplet with color triplet of quarks. One can write complexified basis as $(1, e_1, (q_1, q_2, q_3), (\overline{q}_1 \overline{q}_2, \overline{q}_3))$. The expressions for complexified basis elements are

$$(q_1, q_2, q_3) = \frac{1}{\sqrt{2}} (e_2 + i e_3, e_4 + i e_5, e_6 + i e_7) \ .$$

These options can be seen to be possible by studying octonionic triangle in which all lines containing 3 units defined associative triple: any pair of octonion units at this kind of line can be used to form pair of complexified unit and its conjugate. In the tangent space of $M^4 \times CP_2$ the basis vectors q_1, and q_2 are mixtures of E^2_x and CP_2 tangent vectors. q_3 involves only CP_2 tangent vectors and there is a temptation to interpret it as the analog of the quark having no color isospin.

2. The quaternionic basis is real and must transform like $(1, 1, q_1, \overline{q}_1)$, where q_1 is any quark in the triplet and $\overline{q}_1$ its conjugate in antitriplet. Having fixed some basis one can perform SU(3) rotations to get a new basis. The action of the rotation is by 3×3 special unitary matrix. The over all phases of its rows do not matter since they induce only a rotation in (e_2, e_3) plane not affecting the plane itself. The action of $SU(3)$ on q_1 is simply the action of its first row on (q_1, q_2, q_3) triplet:

$$\begin{aligned} q_1 &\rightarrow (Uq)_1 = U_{11}q_1 + U_{12}q_2 + U_{13}q_3 \equiv z_1q_1 + z_2q_2 + z_3q_3 \\ &= z_1(e_2 + ie_3) + z_2(e_4 + ie_5) + z_3(e_6 + ie_7) \ . \end{aligned} \tag{3.3.3}$$

The triplets (z_1, z_2, z_3) defining a complex unit vector and point of S^5. Since overall phase does not matter a point of CP_2 is in question. The new real octonion units are given by the formulas

$$\begin{aligned} e_2 &\rightarrow Re(z_1)e_2 + Re(z_2)e_4 + Re(z_3)e_6 - Im(z_1)e_3 - Im(z_2)e_5 - Im(z_3)e_7 \ , \\ e_3 &\rightarrow Im(z_1)e_2 + Im(z_2)e_4 + Im(z_3)e_6 + Re(z_1)e_3 + Re(z_2)e_5 + Re(z_3)e_7 \ . \end{aligned} \tag{3.3.4}$$

For instance the CP_2 coordinates corresponding to the coordinate patch (z_1, z_2, z_3) with $z_3 \neq 0$ are obtained as $(\xi_1, \xi_2) = (z_1/z_3, z_2/z_3)$.

Using these expressions the equations expressing the conjecture $CP_2 = CP_2^{mod}$ equivalence can be expressed explicitly as first order differential equations. The conditions state the equivalence

$$(e_2, e_3) \leftrightarrow (\partial_x h^k e^A_k e_A, \partial_y h^k e^A_k e_A) \ , \tag{3.3.5}$$

where e_A denote octonion units. The comparison of two pairs of vectors requires normalization of the tangent vectors on the right hand side to unit vectors so that one takes unit vector in the direction of the tangent vector. After this the vectors can be equated. This allows to expresses the contractions of the partial derivatives with vielbein vectors with the 6 components of e_2 and e_3. Each condition gives 6+6 first order partial differential equations which are non-linear by the presence of the overal normalization factor for the right hand side. The equations are invariant under scalings of (x, y). The very special form of these equations suggests that some symmetry is involved.

It must be emphasized that these equations make sense only in preferred coordinates: ordinary Minkowski coordinates and Hamilton-Jacobi coordinates for M^4 and Eguchi-Hanson complex coordinates in which $SU(2) \times U(1)$ is represented linearly for CP_2. These coordinates are preferred because they carry deep physical meaning.

Does TGD boil down to two string models?

It is good to look what have we obtained. Besides Hamilton-Jacobi conditions, and $CP_2 = CP_2^{mod}$ conditions one has what one might call string model with 6-dimensional $G_2/SU(3)$ as targent space. The orbit of string in $G_2/SU(3)$ allows to deduce the G_2 rotation identifiable as a point of $G_2/SU(3)$ defining what one means with standard quaternionic plane at given point of string world sheet. The hypothesis is that hyper-complex analyticity solves these equations.

The conjectured electric-magnetic duality implies duality between string world sheet and partonic 2-surfaces central for the proposed mathematical applications of TGD [K60, K61, K105, K69]. This duality suggests that the solutions to the $CP_2 = CP_2^{mod}$ conditions could reduce to holomorphy with respect to the coordinate w for partonic 2-surface plus the analogs of Virasoro conditions. The dependence on light-like coordinate would appear as a parametric dependence.

If this were the case, TGD would reduce at least partially to what might be regarded as dual string models in $G_2/SU(3)$ and $SU(3)/U(2)$ and also to string model in M^4 and X^4! In the previous arguments one ends up to string models in moduli spaces of string world sheets and partonic 2-surfaces. TGD seems to yield an inflation of string models! This not actually surprising since the slicing of space-time sheets by string world sheets and partonic 2-surfaces implies automatically various kinds of maps having interpretation in terms of string orbits.

3.4 In What Sense TGD Could Be An Integrable Theory?

During years evidence supporting the idea that TGD could be an integrable theory in some sense has accumulated. The challenge is to show that various ideas about what integrability means form pieces of a bigger coherent picture. Of course, some of the ideas are doomed to be only partially correct or simply wrong. Since it is not possible to know beforehand what ideas are wrong and what are right the situation is very much like in experimental physics and it is easy to claim (and has been and will be claimed) that all this argumentation is useless speculation. This is the price that must be paid for real thinking.

Integrable theories allow to solve nonlinear classical dynamics in terms of scattering data for a linear system. In TGD framework this translates to quantum classical correspondence. The solutions of Kähler-Dirac equation define the scattering data. This data should define a real analytic function whose octonionic extension defines the space-time surface as a surface for which its imaginary part in the representation as bi-quaternion vanishes. There are excellent hopes about this thanks to the reduction of the Kähler-Dirac equation to geometric optics.

In the following I will first discuss briefly what integrability means in (quantum) field theories, list some bits of evidence for integrability in TGD framework, discuss once again the question whether the different pieces of evidence are consistent with other and what one really means with various notions. An an outcome I represent what I regard as a more coherent view about integrability of TGD. The notion of octonion analyticity developed in the previous section is essential for the for what follows.

3.4.1 What Integrable Theories Are?

The following is an attempt to get some bird's eye of view about the landscape of integrable theories.

Examples of integrable theories

Integrable theories are typically non-linear 1+1-dimensional (quantum) field theories. Solitons and various other particle like structures are the characteristic phenomenon in these theories. Scattering matrix is trivial in the sense that the particles go through each other in the scattering and suffer only a phase change. In particular, momenta are conserved. Korteveg- de Vries equation (see `http://tinyurl.com/3cyt8hk`) [B3] was motivated by the attempt to explain the experimentally discovered shallow water wave preserving its shape and moving with a constant velocity. Sine-Gordon equation (see `http://tinyurl.com/yafl243x`) [B8] describes geometrically constant curvature surfaces and defines a Lorentz invariant non-linear field theory in 1+1-dimensional space-time, which can be applied to Josephson junctions (in TGD inspired quantum biology it is encountered in the model of nerve pulse [K92]). Non- linear Schrödinger equation (see `http://tinyurl.com/y88efbo7`) [B6] having applications to optics and water waves represents a further example. All these equations have various variants.

From TGD point of view conformal field theories represent an especially interesting example of integrable theories. (Super-)conformal invariance is the basic underlying symmetry and by its infinite-dimensional character implies infinite number of conserved quantities. The construction of the theory reduces to the construction of the representations of (super-)conformal algebra. One can solve 2-point functions exactly and characterize them in terms of (possibly anomalous) scaling dimensions of conformal fields involved and the coefficients appearing in 3-point functions can be solved in terms of fusion rules leading to an associative algebra for conformal fields. The basic applications are to 2-dimensional critical thermodynamical systems whose scaling invariance generalizes to conformal invariance. String models represent second application in which a collection of super-conformal field theories associated with various genera of 2-surface is needed to describe loop corrections to the scattering amplitudes. Also moduli spaces of conformal equivalence classes become important.

Topological quantum field theories (see `http://tinyurl.com/lsvx7g3`) are also examples of integrable theories. Because of its independence on the metric Chern-Simons action (see `http://tinyurl.com/ydgsqm2c`) is in 3-D case the unique action defining a topological quantum field theory. The calculations of knot invariants (for TGD approach see [K60]), topological invariants of 3-manifolds and 4-manifolds, and topological quantum computation (see `http://tinyurl.com/dkpo4y`) (for a model of DNA as topological quantum computer see [K5]) represent applications of this approach. TGD as almost topological QFT means that the Kähler action for preferred extremals reduces to a surface term by the vanishing of Coulomb term in action and by the weak form of electric-magnetic duality reduces to Chern-Simons action. Both Euclidian and Minkowskian regions give this kind of contribution.

$\mathcal{N} = 4$ SYM is the a four-dimensional and very nearly realistic candidate for an integral quantum field theory. The observation that twistor amplitudes allow also a dual of the 4-D conformal symmetry motivates the extension of this symmetry to its infinite-dimensional Yangian variant [A25]. Also the enormous progress in the construction of scattering amplitudes suggests integrability. In TGD framework Yangian symmetry would emerge naturally by extending the symplectic variant of Kac-Moody algebra from light-cone boundary to the interior of causal diamond and the Kac-Moody algebra from light-like 3-surface representing wormhole throats at which the signature of the induced metric changes to the space-time interior [K116].

About mathematical methods

The mathematical methods used in integrable theories are rather refined and have contributed to the development of the modern mathematical physics. Mention only quantum groups, conformal algebras, and Yangian algebras.

The basic element of integrability is the possibility to transform the non-linear classical problem for which the interaction is characterized by a potential function or its analog to a linear scattering problem depending on time. For instance, for the ordinary Schrödinger function one can solve potential once single solution of the equation is known. This does not work in practice. One can however gather information about the asymptotic states in scattering to deduce the potential. One cannot do without information about bound state energies too.

In TGD framework asymptotic states correspond to partonic 2-surfaces at the two light-like boundaries of CD (more precisely: the largest CD involved and defining the IR resolution for momenta). From the scattering data coding information about scattering for various values of energy of the incoming particle one deduced the potential function or its analog.

1. The basic tool is inverse scattering transform known as Gelfand-Marchenko-Levitan (GML) transform (see `http://tinyurl.com/y9f7ybln`) described in simple terms in [B9].

   (a) In 1+1 dimensional case the S-matrix characterizing scattering is very simple since the only thing that can take place in scattering is reflection or transmission. Therefore the S-matrix elements describe either of these processes and by unitarity the sum of corresponding probabilities equals to 1. The particle can arrive to the potential either from left or right and is characterized by a momentum. The transmission coefficient can have a pole meaning complex (imaginary in the simplest case) wave vector serving as a signal for the formation of a bound state or resonance. The scattering data are represented by the reflection and transmission coefficients as function of time.

   (b) One can deduce an integral equation for a propagator like function $K(t, x)$ describing how delta pulse moving with light velocity is scattered from the potential and is expressible in terms of time integral over scattering data with contributions from both scattering states and bound states. The derivation of GML transform [B9] uses time reversal and time translational invariance and causality defined in terms of light velocity. After some tricks one obtains the integral equation as well as an expression for the time independent potential as $V(x) = K(x,x)$. The argument can be generalized to more complex problems to deduce the GML transform.

2. The so called Lax pair (see http://tinyurl.com/yc93nw53) is one manner to describe integrable systems [B4]. Lax pair consists of two operators L and M. One studies what might be identified as "energy" eigenstates satisfying $L(x,t)\Psi = \lambda\Psi$. λ does not depend on time and one can say that the dynamics is associated with x coordinate whereas as t is time coordinate parametrizing different variants of eigenvalue problem with the same spectrum for L. The operator $M(t)$ does not depend on x at all and the independence of λ on time implies the condition

$$\partial_t L = [L, M] \ .$$

   This equation is analogous to a quantum mechanical evolution equation for an operator induced by time dependent "Hamiltonian" M and gives the non-linear classical evolution equation when the commutator on the right hand side is a multiplicative operator (so that it does not involve differential operators acting on the coordinate x). Non-linear classical dynamics for the time dependent potential emerges as an integrability condition.

   One could say that $M(t)$ introduces the time evolution of $L(t,x)$ as an automorphism which depends on time and therefore does not affect the spectrum. One has $L(t,x) = U(t)L(0,x)U^{-1}(t)$ with $dU(t)/dt = M(t)U(t)$. The time evolution of the analog of the quantum state is given by a similar equation.

3. A more refined view about Lax pair is based on the observation that the above equation can be generalized so that M depends also on x. The generalization of the basic equation for $M(x,t)$ reads as

$$\partial_t L - \partial_x M - [L, M] = 0 \ .$$

   The condition has interpretation as a vanishing of the curvature of a gauge potential having components $A_x = L, A_t = M$. This generalization allows a beautiful geometric formulation of the integrability conditions and extends the applicability of the inverse scattering transform. The monodromy of the flat connection becomes important in this approach. Flat connections in moduli spaces are indeed important in topological quantum field theories and in conformal field theories.

4. There is also a connection with the so called Riemann-Hilbert problem (see http://tinyurl.com/ybay4qjg) [A19]. The monodromies of the flat connection define monodromy group and Riemann-Hilbert problem concerns the existence of linear differential equations having a given monodromy group. Monodromy group emerges in the analytic continuation of an analytic function and the action of the element of the monodromy group tells what happens for the resulting many-valued analytic function as one turns around a singularity once ("mono-"). The linear equations obviously relate to the linear scattering problem. The flat connection (M, L) in turn defines the monodromy group. What is needed is that the functions involved are analytic functions of (t, x) replaced with a complex or hyper-complex variable. Again Wick rotation is involved. Similar approach generalizes also to higher dimensional moduli spaces with complex structures.

   In TGD framework the effective 2-dimensionality raises the hope that this kind of mathematical apparatus could be used. An interesting possibility is that finite measurement resolution could be realized in terms of a gauge group or Kac-Moody type group represented by trivial gauge potential defining a monodromy group for n-point functions. Monodromy invariance would hold for the full n-point functions constructed in terms of analytic n-point functions and their conjugates. The ends of braid strands are natural candidates for the singularities around which monodromies are defined.

3.4.2 Why TGD Could Be Integrable Theory In Some Sense?

There are many indications that TGD could be an integrable theory in some sense. The challenge is to see which ideas are consistent with each other and to build a coherent picture where everything finds its own place.

1. 2-dimensionality or at least effective 2-dimensionality seems to be a prerequisite for integrability. Effective 2-dimensionality is suggested by the strong form of General Coordinate Invariance implying also holography and generalized conformal invariance predicting infinite number of conservation laws. The dual roles of partonic 2-surfaces and string world sheets supports a four-dimensional generalization of conformal invariance. Twistor considerations [K116, L31] indeed suggest that Yangian invariance and Kac-Moody invariances combine to a 4-D analog of conformal invariance induced by 2-dimensional one by algebraic continuation.
2. Octonionic representation of embedding space Clifford algebra and the identification of the space-time surfaces as quaternionic space-time surfaces would define a number theoretically natural generalization of conformal invariance. The reason for using gamma matrix representation is that vector field representation for octonionic units does not exist. The problem concerns the precise meaning of the octonionic representation of gamma matrices.

   Space-time surfaces could be quaternionic also in the sense that conformal invariance is analytically continued from string curve to 8-D space by octonion real-analyticity. The question is whether the Clifford algebra based notion of tangent space quaternionicity is equivalent with octonionic real-analyticity based notion of quaternionicity.

   The notions of co-associativity and co-quaternionicity make also sense and one must consider seriously the possibility that associativity-co-associativity dichotomy corresponds to Minkowskian-Euclidian dichotomy.
3. Field equations define hydrodynamic Beltrami flows satisfying integrability conditions of form $J \wedge dJ = 0$.
   (a) One can assign local momentum and polarization directions to the preferred extremals and this gives a decomposition of Minkowskian space-time regions to massless quanta analogous to the 1+1-dimensional decomposition to solitons. The linear superposition of modes with 4-momenta with different directions possible for free Maxwell action does not look plausible for the preferred extremals of Kähler action. This rather quantal and solitonic character is in accordance with the quantum classical correspondence giving very concrete connection between quantal and classical particle pictures. For 4-D volume action one does not obtain this kind of decomposition. In 2-D case volume action gives superposition of solutions with different polarization directions so that the situation is nearer to that for free Maxwell action and is not like soliton decomposition.
   (b) Beltrami property in strong sense allows to identify 4 preferred coordinates for the space-time surface in terms of corresponding Beltrami flows. This is possible also in Euclidian regions using two complex coordinates instead of hyper-complex coordinate and complex coordinate. The assumption that isometry currents are parallel to the same light-like Beltrami flow implies hydrodynamic character of the field equations in the sense that one can say that each flow line is analogous to particle carrying some quantum numbers. This property is not true for all extremals (say cosmic strings).
   (c) The tangent bundle theoretic view about integrability is that one can find a Lie algebra of vector fields in some manifold spanning the tangent space of a lower-dimensional manifolds and is expressed in terms of Frobenius theorem (see `http://tinyurl.com/of6vfz5`) [A6]. The gradients of scalar functions defining Beltrami flows appearing in the ansatz for preferred exremals would define these vector fields and the slicing. Partonic 2-surfaces would correspond to two complex conjugate vector fields (local polarization direction) and string world sheets to light-like vector field and its dual (light-like momentum directions). This slicing generalizes to the Euclidian regions.
4. Infinite number of conservation laws is the signature of integrability. Classical field equations follow from the condition that the vector field defined by Kähler-Dirac gamma matrices has vanishing divergence and can be identified an integrability condition for the Kähler-Dirac equation guaranteeing also the conservation of super currents so that one obtains an infinite number of conserved charges.
5. Quantum criticality is a further signal of integrability. 2-D conformal field theories describe critical systems so that the natural guess is that quantum criticality in TGD framework relates to the generalization of conformal invariance and to integrability. Quantum criticality implies that Kähler coupling strength is analogous to critical temperature. This condition

does affects classical field equations only via boundary conditions expressed as weak form of electric magnetic duality at the wormhole throats at which the signature of the metric changes.

For finite-dimensional systems the vanishing of the determinant of the matrix defined by the second derivatives of potential is similar signature and applies in catastrophe theory. Therefore the existence of vanishing second variations of Kähler action should characterize criticality and define a property of preferred extremals. The vanishing of second variations indeed leads to an infinite number of conserved currents [K15] following the conditions that the deformation of Kähler-Dirac gamma matrix is also divergenceless and that the Kähler-Dirac equation associated with it is satisfied.

3.4.3 Could TGD Be An Integrable Theory?

Consider first the abstraction of integrability in TGD framework. Quantum classical correspondence could be seen as a correspondence between linear quantum dynamics and non-linear classical dynamics. Integrability would realize this correspondence. In integrable models such as Sine-Gordon equation particle interactions are described by potential in 1+1 dimensions. This too primitive for the purposes of TGD. The vertices of generalized Feynman diagrams take care of this. At lines one has free particle dynamics so that the situation could be much simpler than in integrable models if one restricts the considerations to the lines or Minkowskian space-time regions surrounding them.

The non-linear dynamics for the space-time sheets representing incoming lines of generalized Feynman diagram should be obtainable from the linear dynamics for the induced spinor fields defined by Kähler-Dirac operator. There are two options.

1. Strong form of the quantum classical correspondence states that each solution for the linear dynamics of spinor fields corresponds to space-time sheet. This is analogous to solving the potential function in terms of a single solution of Schrödinger equation. Coupling of space-time geometry to quantum numbers via measurement interaction term is a proposal for realizing this option. It is however the quantum numbers of positive/negative energy parts of zero energy state which would be visible in the classical dynamics rather than those of induced spinor field modes.
2. Only overall dynamics characterized by scattering data- the counterpart of S-matrix for the Kähler-Dirac operator- is mapped to the geometry of the space-time sheet. This is much more abstract realization of quantum classical correspondence.
3. Can these two approaches be equivalent? This might be the case since quantum numbers of the state are not those of the modes of induced spinor fields.

What the scattering data could be for the induced spinor field satisfying Kähler-Dirac equation?

1. If the solution of field equation has hydrodynamic character, the solutions of the Kähler-Dirac equation can be localized to light-like Beltrami flow lines of hydrodynamic flow. These correspond to basic solutions and the general solution is a superposition of these. There is no dispersion and the dynamics is that of geometric optics at the basic level. This means geometric optics like character of the spinor dynamics.

   Solutions of the Kähler-Dirac equation are completely analogous to the pulse solutions defining the fundamental solution for the wave equation in the argument leading from wave equation with external time independent potential to Marchenko-Gelfand-Levitan equation allowing to identify potential in terms of scattering data. There is however no potential present now since the interactions are described by the vertices of Feynman diagram where the particle lines meet. Note that particle like regions are Euclidian and that this picture applies only to the Minkowskian exteriors of particles.
2. Partonic 2-surfaces at the ends of the line of generalized Feynman diagram are connected by flow lines. Partonic 2-surfaces at which the signature of the induced metric changes are in a special position. Only the imaginary part of the bi-quaternionic value of the octonion valued map is non-vanishing at these surfaces which can be said to be co-complex 2-surfaces. By

geometric optics behavior the scattering data correspond to a diffeomorphism mapping initial partonic 2-surface to the final one in some preferred complex coordinates common to both ends of the line.

3. What could be these preferred coordinates? Complex coordinates for S^2 at light-cone boundary define natural complex coordinates for the partonic 2-surface. With these coordinates the diffeomorphism defining scattering data is diffeomorphism of S^2. Suppose that this map is real analytic so that maps "real axis" of S^2 to itself. This map would be same as the map defining the octonionic real analyticity as algebraic extension of the complex real analytic map. By octonionic analyticity one can make large number of alternative choices for the coordinates of partonic 2-surface.

4. There can be non-uniqueness due to the possibility of $G_2/SU(3)$ valued map characterizing the local octonionic units. The proposal is that the choice of octonionic imaginary units can depend on the point of string like orbit: this would give string model in $G_2/SU(3)$. Conformal invariance for this string model would imply analyticity and helps considerably but would not probably fix the situation completely since the element of the coset space would constant at the partonic 2-surfaces at the ends of CD. One can of course ask whether the $G_2/SU(3)$ element could be constant for each propagator line and would change only at the 2-D vertices?

This would be the inverse scattering problem formulated in the spirit of TGD. There could be also dependence of space-time surface on quantum numbers of quantum states but not on individual solution for the induced spinor field since the scattering data of this solution would be purely geometric.

3.5 Do Geometric Invariants Of Preferred Extremals Define Topological Invariants Of Space-time Surface And Code For Quantumphysics?

The recent progress in the understanding of preferred extremals [K15] led to a reduction of the field equations to conditions stating for Euclidian signature the existence of Kähler metric. The resulting conditions are a direct generalization of corresponding conditions emerging for the string world sheet and stating that the 2-metric has only non-diagonal components in complex/hypercomplex coordinates. Also energy momentum of Kähler action and has this characteristic (1, 1) tensor structure. In Minkowskian signature one obtains the analog of 4-D complex structure combining hyper-complex structure and 2-D complex structure.

The construction lead also to the understanding of how Einstein's equations with cosmological term follow as a consistency condition guaranteeing that the covariant divergence of the Maxwell's energy momentum tensor assignable to Kähler action vanishes. This gives $T = kG + \Lambda g$. By taking trace a further condition follows from the vanishing trace of T:

$$R = \frac{4\Lambda}{k} . \tag{3.5.1}$$

That any preferred extremal should have a constant Ricci scalar proportional to cosmological constant is very strong prediction. Note that the accelerating expansion of the Universe would support positive value of Λ. Note however that both Λ and $k \propto 1/G$ are both parameters characterizing one particular preferred extremal. One could of course argue that the dynamics allowing only constant curvature space-times is too simple. The point is however that particle can topologically condense on several space-time sheets meaning effective superposition of various classical fields defined by induced metric and spinor connection.

The following considerations demonstrate that preferred extremals can be seen as canonical representatives for the constant curvature manifolds playing central role in Thurston's geometrization theorem (see `http://tinyurl.com/y8bbzlnr`) [A22] known also as hyperbolization theorem implying that geometric invariants of space-time surfaces transform to topological invariants. The generalization of the notion of Ricci flow to Maxwell flow in the space of metrics and further to

Kähler flow for preferred extremals in turn gives a rather detailed vision about how preferred extremals organize to one-parameter orbits. It is quite possible that Kähler flow is actually discrete. The natural interpretation is in terms of dissipation and self organization.

Quantum classical correspondence suggests that this line of thought could be continued even further: could the geometric invariants of the preferred extremals could code not only for space-time topology but also for quantum physics? How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge of quantum TGD. Could the correlation functions be reduced to statistical geometric invariants of preferred extemals? The latest (means the end of 2012) and perhaps the most powerful idea hitherto about coupling constant evolution is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

3.5.1 Preferred Extremals Of Kähler Action As Manifolds With Constant Ricci Scalar Whose Geometric Invariants Are Topological-Invariants

An old conjecture inspired by the preferred extremal property is that the geometric invariants of space-time surface serve as topological invariants. The reduction of Kähler action to 3-D Chern-Simons terms (see `http://tinyurl.com/ybp86sho`) [K15] gives support for this conjecture as a classical counterpart for the view about TGD as almost topological QFT. The following arguments give a more precise content to this conjecture in terms of existing mathematics.

1. It is not possible to represent the scaling of the induced metric as a deformation of the space-time surface preserving the preferred extremal property since the scale of CP_2 breaks scale invariance. Therefore the curvature scalar cannot be chosen to be equal to one numerically. Therefore also the parameter $R = 4\Lambda/k$ and also Λ and k separately characterize the equivalence class of preferred extremals as is also physically clear.

   Also the volume of the space-time sheet closed inside causal diamond CD remains constant along the orbits of the flow and thus characterizes the space-time surface. Λ and even $k \propto 1/G$ can indeed depend on space-time sheet and p-adic length scale hypothesis suggests a discrete spectrum for Λ/k expressible in terms of p-adic length scales: $\Lambda/k \propto 1/L_p^2$ with $p \simeq 2^k$ favored by p-adic length scale hypothesis. During cosmic evolution the p-adic length scale would increase gradually. This would resolve the problem posed by cosmological constant in GRT based theories.

2. One could also see the preferred extremals as 4-D counterparts of constant curvature 3-manifolds in the topology of 3-manifolds. An interesting possibility raised by the observed negative value of Λ is that most 4-surfaces are constant negative curvature 4-manifolds. By a general theorem coset spaces (see `http://tinyurl.com/y8d3udpr`) H^4/Γ, where $H^4 = SO(1,4)/SO(4)$ is hyperboloid of M^5 and Γ a torsion free discrete subgroup of $SO(1,4)$ [A10]. It is not clear to me, whether the constant value of Ricci scalar implies constant sectional curvatures and therefore hyperbolic space property. It could happen that the space of spaces with constant Ricci curvature contain a hyperbolic manifold as an especially symmetric representative. In any case, the geometric invariants of hyperbolic metric are topological invariants.

   By Mostow rigidity theorem (see `http://tinyurl.com/yacbu8sk`) [A15] finite-volume hyperbolic manifold is unique for $D > 2$ and determined by the fundamental group of the manifold. Since the orbits under the Kähler flow preserve the curvature scalar the manifolds at the orbit must represent different embeddings of one and hyperbolic 4-manifold. In 2-D case the moduli space for hyperbolic metric for a given genus $g > 0$ is defined by Teichmueller parameters and has dimension $6(g-1)$. Obviously the exceptional character of $D = 2$ case relates to conformal invariance. Note that the moduli space in question (see `http://tinyurl.com/ybowqm5v`) plays a key role in p-adic mass calculations [K26].

In the recent case Mostow rigidity theorem could hold true for the Euclidian regions and maybe generalize also to Minkowskian regions. If so then both "topological" and "geometro" in "Topological GeometroDynamics" would be fully justified. The fact that geometric invariants become topological invariants also conforms with "TGD as almost topological QFT" and allows the notion of scale to find its place in topology. Also the dream about exact solvability of the theory would be realized in rather convincing manner.

These conjectures are the main result independent of whether the generalization of the Ricci flow discussed in the sequel exists as a continuous flow or possibly discrete sequence of iterates in the space of preferred extremals of Kähler action. My sincere hope is that the reader could grasp how far reaching these result really are.

3.5.2 Is There A Connection Between Preferred Extremals And AdS$_4$/CFT Correspondence?

The preferred extremals satisfy Einstein Maxwell equations with a cosmological constant and have negative scalar curvature for negative value of Λ. 4-D space-times with hyperbolic metric provide canonical representation for a large class of four-manifolds and an interesting question is whether these spaces are obtained as preferred extremals and/or vacuum extremals.

4-D hyperbolic space with Minkowski signature is locally isometric with AdS$_4$. This suggests at connection with AdS$_4$/CFT correspondence of M-theory. The boundary of AdS would be now replaced with 3-D light-like orbit of partonic 2-surface at which the signature of the induced metric changes. The metric 2-dimensionality of the light-like surface makes possible generalization of 2-D conformal invariance with the light-like coordinate taking the role of complex coordinate at light-like boundary. AdS could represent a special case of a more general family of space-time surfaces with constant Ricci scalar satistying Einstein-Maxwell equations and generalizing the AdS$_4$/CFT correspondence. There is however a strong objection from cosmology: the accelerated expansion of the Universe requires positive value of Λ and favors De Sitter Space dS_4 instead of AdS_4.

These observations provide motivations for finding whether AdS$_4$ and/or dS_4 allows an embedding as a vacuum extremal to $M^4\times S^2\subset M^4\times CP_2$, where S^2 is a homologically trivial geodesic sphere of CP_2. It is easy to guess the general form of the embedding by writing the line elements of, M^4, S^2, and AdS$_4$.

1. The line element of M^4 in spherical Minkowski coordinates (m, r_M, θ, ϕ) reads as

$$ds^2 = dm^2 - dr_M^2 - r_M^2 d\Omega^2 \ . \tag{3.5.2}$$

2. Also the line element of S^2 is familiar:

$$ds^2 = -R^2(d\Theta^2 + sin^2(\theta)d\Phi^2) \ . \tag{3.5.3}$$

3. By visiting in Wikipedia (see `http://tinyurl.com/y9hw95ql`) one learns that in spherical coordinate the line element of AdS$_4$/dS_4 is given by

$$\begin{aligned} ds^2 &= A(r)dt^2 - \frac{1}{A(r)}dr^2 - r^2 d\Omega^2 \ , \\ A(r) &= 1+\epsilon y^2 \ , \ y = \frac{r}{r_0} \ , \\ \epsilon &= 1 \text{ for } AdS_4 \ , \ \epsilon = -1 \text{ for } dS_4 \ . \end{aligned} \tag{3.5.4}$$

4. From these formulas it is easy to see that the ansatz is of the same general form as for the embedding of Schwartschild-Nordstöm metric:

$$\begin{aligned} &m = \Lambda t + h(y) \ , \quad r_M = r \ , \\ &\Theta = s(y) \ , \qquad \Phi = \omega(t+f(y)) \ . \end{aligned} \tag{3.5.5}$$

The non-trivial conditions on the components of the induced metric are given by

$$\begin{aligned}
g_{tt} &= \Lambda^2 - x^2 sin^2(\Theta) = A(r) \ , \\
g_{tr} &= \frac{1}{r_0}\left[\Lambda \frac{dh}{dy} - x^2 sin^2(\theta)\frac{df}{dr}\right] = 0 \ , \\
g_{rr} &= \frac{1}{r_0^2}\left[(\frac{dh}{dy})^2 - 1 - x^2 sin^2(\theta)(\frac{df}{dy})^2 - R^2(\frac{d\Theta}{dy})^2\right] = -\frac{1}{A(r)} \ , \\
x &= R\omega \ .
\end{aligned} \tag{3.5.6}$$

By some simple algebraic manipulations one can derive expressions for $sin(\Theta)$, df/dr and dh/dr.

1. For $\Theta(r)$ the equation for g_{tt} gives the expression

$$\begin{aligned}
sin(\Theta) &= \pm\frac{P^{1/2}}{x} \ , \\
P &= \Lambda^2 - A = \Lambda^2 - 1 - \epsilon y^2 \ .
\end{aligned} \tag{3.5.7}$$

   The condition $0 \leq sin^2(\Theta) \leq 1$ gives the conditions

$$\begin{array}{rcll}
(\Lambda^2 - x^2 - 1)^{1/2} \leq & y & \leq (\Lambda^2 - 1)^{1/2} & \text{for } \epsilon = 1 \ (AdS_4) \ , \\
(-\Lambda^2 + 1)^{1/2} \leq & y & \leq (x^2 + 1 - \Lambda^2)^{1/2} & \text{for } \epsilon = -1 \ (dS_4) \ .
\end{array} \tag{3.5.8}$$

   Only a spherical shell is possible in both cases. The model for the final state of star considered in [K121] predicted similar layer layer like structure and inspired the proposal that stars quite generally have an onion-like structure with radii of various shells characterize by p-adic length scale hypothesis and thus coming in some powers of $\sqrt{2}$. This brings in mind also Titius-Bode law.

2. From the vanishing of g_{tr} one obtains

$$\frac{dh}{dy} = \frac{P}{\Lambda}\frac{df}{dy} \ . \tag{3.5.9}$$

3. The condition for g_{rr} gives

$$(\frac{df}{dy})^2 = \frac{r_0^2}{AP}[A^{-1} - R^2(\frac{d\Theta}{dy})^2] \ . \tag{3.5.10}$$

   Clearly, the right-hand side is positive if $P \geq 0$ holds true and $Rd\Theta/dy$ is small. One can express $d\Theta/dy$ using chain rule as

$$(\frac{d\Theta}{dy})^2 = \frac{x^2y^2}{P(P-x^2)} \ . \tag{3.5.11}$$

   One obtains

$$(\frac{df}{dy})^2 = \Lambda r_0^2\frac{y^2}{AP}\left[\frac{1}{1+y^2} - x^2(\frac{R}{r_0})^2\frac{1}{P(P-x^2)}\right] \ . \tag{3.5.12}$$

   The right hand side of this equation is non-negative for certain range of parameters and variable y. Note that for $r_0 \gg R$ the second term on the right hand side can be neglected. In this case it is easy to integrate $f(y)$.

The conclusion is that both AdS_4 and dS^4 allow a local embedding as a vacuum extremal. Whether also an embedding as a non-vacuum preferred extremal to $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere is possible, is an interesting question.

3.5.3 Generalizing Ricci Flow To Maxwell Flow For 4-Geometries And Kähler Flow For Space-Time Surfaces

The notion of Ricci flow has played a key part in the geometrization of topological invariants of Riemann manifolds. I certainly did not have this in mind when I choose to call my unification attempt "Topological Geometrodynamics" but this title strongly suggests that a suitable generalization of Ricci flow could play a key role in the understanding of also TGD.

Ricci flow and Maxwell flow for 4-geometries

The observation about constancy of 4-D curvature scalar for preferred extremals inspires a generalization of the well-known volume preserving Ricci flow (see http://tinyurl.com/2cwlzh9l) [A18] introduced by Richard Hamilton. Ricci flow is defined in the space of Riemann metrics as

$$\frac{dg_{\alpha\beta}}{dt} = -2R_{\alpha\beta} + 2\frac{R_{avg}}{D}g_{\alpha\beta} \quad . \tag{3.5.13}$$

Here R_{avg} denotes the average of the scalar curvature, and D is the dimension of the Riemann manifold. The flow is volume preserving in average sense as one easily checks ($\langle g^{\alpha\beta}dg_{\alpha\beta}/dt\rangle = 0$). The volume preserving property of this flow allows to intuitively understand that the volume of a 3-manifold in the asymptotic metric defined by the Ricci flow is topological invariant. The fixed points of the flow serve as canonical representatives for the topological equivalence classes of 3-manifolds. These 3-manifolds (for instance hyperbolic 3-manifolds with constant sectional curvatures) are highly symmetric. This is easy to understand since the flow is dissipative and destroys all details from the metric.

What happens in the recent case? The first thing to do is to consider what might be called Maxwell flow in the space of all 4-D Riemann manifolds allowing Maxwell field.

1. First of all, the vanishing of the trace of Maxwell's energy momentum tensor codes for the volume preserving character of the flow defined as

$$\frac{dg_{\alpha\beta}}{dt} = T_{\alpha\beta} \quad . \tag{3.5.14}$$

   Taking covariant divergence on both sides and assuming that d/dt and D_α commute, one obtains that $T^{\alpha\beta}$ is divergenceless.

   This is true if one assumes Einstein's equations with cosmological term. This gives

$$\frac{dg_{\alpha\beta}}{dt} = kG_{\alpha\beta} + \Lambda g_{\alpha\beta} = kR_{\alpha\beta} + (-\frac{kR}{2} + \Lambda)g_{\alpha\beta} \quad . \tag{3.5.15}$$

   The trace of this equation gives that the curvature scalar is constant. Note that the value of the Kähler coupling strength plays a highly non-trivial role in these equations and it is quite possible that solutions exist only for some critical values of α_K. Quantum criticality should fix the allow value triplets (G, Λ, α_K) apart from overall scaling

$$(G, \Lambda, \alpha_K) \rightarrow (xG, \Lambda/x, x\alpha_K \quad .$$

   Fixing the value of G fixes the values remaining parameters at critical points. The rescaling of the parameter t induces a scaling by x.

2. By taking trace one obtains the already mentioned condition fixing the curvature to be constant, and one can write

$$\frac{dg_{\alpha\beta}}{dt} = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \quad . \tag{3.5.16}$$

Note that in the recent case $R_{avg} = R$ holds true since curvature scalar is constant. The fixed points of the flow would be Einstein manifolds (see http://tinyurl.com/ybrnakuu) [A4, A66] satisfying

$$R_{\alpha\beta} = \frac{\Lambda}{k} g_{\alpha\beta} \tag{3.5.17}$$

.

3. It is by no means obvious that continuous flow is possible. The condition that Einstein-Maxwell equations are satisfied might pick up from a completely general Maxwell flow a discrete subset as solutions of Einstein-Maxwell equations with a cosmological term. If so, one could assign to this subset a sequence of values t_n of the flow parameter t.

4. I do not know whether 3-dimensionality is somehow absolutely essential for getting the topological classification of closed 3-manifolds using Ricci flow. This ignorance allows me to pose some innocent questions. Could one have a canonical representation of 4-geometries as spaces with constant Ricci scalar? Could one select one particular Einstein space in the class four-metrics and could the ratio Λ/k represent topological invariant if one normalizes metric or curvature scalar suitably. In the 3-dimensional case curvature scalar is normalized to unity. In the recent case this normalization would give $k = 4\Lambda$ in turn giving $R_{\alpha\beta} = g_{\alpha\beta}/4$. Does this mean that there is only single fixed point in local sense, analogous to black hole toward which all geometries are driven by the Maxwell flow? Does this imply that only the 4-volume of the original space would serve as a topological invariant?

Maxwell flow for space-time surfaces

One can consider Maxwell flow for space-time surfaces too. In this case Kähler flow would be the appropriate term and provides families of preferred extremals. Since space-time surfaces inside CD are the basic physical objects are in TGD framework, a possible interpretation of these families would be as flows describing physical dissipation as a four-dimensional phenomenon polishing details from the space-time surface interpreted as an analog of Bohr orbit.

1. The flow is now induced by a vector field $j^k(x,t)$ of the space-time surface having values in the tangent bundle of embedding space $M^4 \times CP_2$. In the most general case one has Kähler flow without the Einstein equations. This flow would be defined in the space of all space-time surfaces or possibly in the space of all extremals. The flow equations reduce to

$$h_{kl} D_\alpha j^k(x,t) D_\beta h^l = \frac{1}{2} T_{\alpha\beta} \ . \tag{3.5.18}$$

The left hand side is the projection of the covariant gradient $D_\alpha j^k(x,t)$ of the flow vector field $j^k(x,t)$ to the tangent space of the space-time surface. D_{alpha} is covariant derivative taking into account that j^k is embedding space vector field. For a fixed point space-time surface this projection must vanish assuming that this space-time surface reachable. A good guess for the asymptotia is that the divergence of Maxwell energy momentum tensor vanishes and that Einstein's equations with cosmological constant are well-defined.

Asymptotes corresponds to vacuum extremals. In Euclidian regions CP_2 type vacuum extremals and in Minkowskian regions to any space-time surface in any 6-D sub-manifold $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähler form. Symplectic transformations of CP_2 combined with diffeomorphisms of M^4 give new Lagrangian manifolds. One would expect that vacuum extremals are approached but never reached at second extreme for the flow.

If one assumes Einstein's equations with a cosmological term, allowed vacuum extremals must be Einstein manifolds. For CP_2 type vacuum extremals this is the case. It is quite possible that these fixed points do not actually exist in Minkowskian sector, and could be replaced with more complex asymptotic behavior such as limit, chaos, or strange attractor.

2. The flow could be also restricted to the space of preferred extremals. Assuming that Einstein Maxwell equations indeed hold true, the flow equations reduce to

$$h_{kl}D_\alpha j^k(x,t)\partial_\beta h^l = \frac{1}{2}(kR_{\alpha\beta} - \Lambda g_{\alpha\beta}) \ . \tag{3.5.19}$$

   Preferred extremals would correspond to a fixed sub-manifold of the general flow in the space of all 4-surfaces.

3. One can also consider a situation in which $j^k(x,t)$ is replaced with $j^k(h,t)$ defining a flow in the entire embedding space. This assumption is probably too restrictive. In this case the equations reduce to

$$(D_r j_l(x,t) + D_l j_r)\partial_\alpha h^r \partial_\beta h^l = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \tag{3.5.20}$$

   Here D_r denotes covariant derivative. Asymptotia is achieved if the tensor $D_k j_l + D_k j_l$ becomes orthogonal to the space-time surface. Note for that Killing vector fields of H the left hand side vanishes identically. Killing vector fields are indeed symmetries of also asymptotic states.

It must be made clear that the existence of a continuous flow in the space of preferred extremals might be too strong a condition. Already the restriction of the general Maxwell flow in the space of metrics to solutions of Einstein-Maxwell equations with cosmological term might lead to discretization, and the assumption about reprentability as 4-surface in $M^4 \times CP_2$ would give a further condition reducing the number of solutions. On the other hand, one might consiser a possibility of a continuous flow in the space of constant Ricci scalar metrics with a fixed 4-volume and having hyperbolic spaces as the most symmetric representative.

Dissipation, self organization, transition to chaos, and coupling constant evolution

A beautiful connection with concepts like dissipation, self-organization, transition to chaos, and coupling constant evolution suggests itself.

1. It is not at all clear whether the vacuum extremal limits of the preferred extremals can correspond to Einstein spaces except in special cases such as CP_2 type vacuum extremals isometric with CP_2. The imbeddability condition however defines a constraint force which might well force asymptotically more complex situations such as limit cycles and strange attractors. In ordinary dissipative dynamics an external energy feed is essential prerequisite for this kind of non-trivial self-organization patterns.

   In the recent case the external energy feed could be replaced by the constraint forces due to the imbeddability condition. It is not too difficult to imagine that the flow (if it exists!) could define something analogous to a transition to chaos taking place in a stepwise manner for critical values of the parameter t. Alternatively, these discrete values could correspond to those values of t for which the preferred extremal property holds true for a general Maxwell flow in the space of 4-metrics. Therefore the preferred extremals of Kähler action could emerge as one-parameter (possibly discrete) families describing dissipation and self-organization at the level of space-time dynamics.

2. For instance, one can consider the possibility that in some situations Einstein's equations split into two mutually consistent equations of which only the first one is independent

$$\begin{aligned} xJ^{\alpha}{}_{\nu}J^{\nu\beta} &= R^{\alpha\beta} \ , \\ L_K &= xJ^{\alpha}{}_{\nu}J^{\nu\beta} = 4\Lambda \ , \\ x &= \frac{1}{16\pi\alpha_K} \ . \end{aligned} \tag{3.5.21}$$

   Note that the first equation indeed gives the second one by tracing. This happens for CP_2 type vacuum extremals.

Kähler action density would reduce to cosmological constant which should have a continuous spectrum if this happens always. A more plausible alternative is that this holds true only asymptotically. In this case the flow equation could not lead arbitrary near to vacuum extremal, and one can think of situation in which $L_K = 4\Lambda$ defines an analog of limiting cycle or perhaps even strange attractor. In any case, the assumption would allow to deduce the asymptotic value of the action density which is of utmost importance from calculational point of view: action would be simply $S_K = 4\Lambda V_4$ and one could also say that one has minimal surface with Λ taking the role of string tension.

3. One of the key ideas of TGD is quantum criticality implying that Kähler coupling strength is analogous to critical temperature. Second key idea is that p-adic coupling constant evolution represents discretized version of continuous coupling constant evolution so that each p-adic prime would correspond a fixed point of ordinary coupling constant evolution in the sense that the 4-volume characterized by the p-adic length scale remains constant. The invariance of the geometric and thus geometric parameters of hyperbolic 4-manifold under the Kähler flow would conform with the interpretation as a flow preserving scale assignable to a given p-adic prime. The continuous evolution in question (if possible at all!) might correspond to a fixed p-adic prime. Also the hierarchy of Planck constants relates to this picture naturally. Planck constant $\hbar_{eff} = n\hbar$ corresponds to a multi-furcation generating n-sheeted structure and certainly affecting the fundamental group.
4. One can of course question the assumption that a continuous flow exists. The property of being a solution of Einstein-Maxwell equations, imbeddability property, and preferred extremal property might allow allow only discrete sequences of space-time surfaces perhaps interpretable as orbit of an iterated map leading gradually to a fractal limit. This kind of discrete sequence might be also be selected as preferred extremals from the orbit of Maxwell flow without assuming Einstein-Maxwell equations. Perhaps the discrete p-adic coupling constant evolution could be seen in this manner and be regarded as an iteration so that the connection with fractality would become obvious too.

Does a 4-D counterpart of thermodynamics make sense?

The interpretation of the Kähler flow in terms of dissipation, the constancy of R, and almost constancy of L_K suggest an interpretation in terms of 4-D variant of thermodynamics natural in zero energy ontology (ZEO), where physical states are analogs for pairs of initial and final states of quantum event are quantum superpositions of classical time evolutions. Quantum theory becomes a "square root" of thermodynamics so that 4-D analog of thermodynamics might even replace ordinary thermodynamics as a fundamental description. If so this 4-D thermodynamics should be qualitatively consistent with the ordinary 3-D thermodynamics.

1. The first naïve guess would be the interpretation of the action density L_K as an analog of energy density $e = E/V_3$ and that of R as the analog to entropy density $s = S/V_3$. The asymptotic states would be analogs of thermodynamical equilibria having constant values of L_K and R.
2. Apart from an overall sign factor ϵ to be discussed, the analog of the first law $de = Tds - pdV/V$ would be

$$dL_K = kdR + \Lambda \frac{dV_4}{V_4} \quad .$$

   One would have the correspondences $S \rightarrow \epsilon RV_4$, $e \rightarrow \epsilon L_K$ and $k \rightarrow T$, $p \rightarrow -\Lambda$. $k \propto 1/G$ indeed appears formally in the role of temperature in Einstein's action defining a formal partition function via its exponent. The analog of second law would state the increase of the magnitude of ϵRV_4 during the Kähler flow.
3. One must be very careful with the signs and discuss Euclidian and Minkowskian regions separately. Concerning purely thermodynamic aspects at the level of vacuum functional Euclidian regions are those which matter.
   (a) For CP_2 type vacuum extremals $L_K \propto E^2 + B^2$, $R = \Lambda/k$, and Λ are positive. In thermodynamical analogy for $\epsilon = 1$ this would mean that pressure is negative.

(b) In Minkowskian regions the value of $R = \Lambda/k$ is negative for $\Lambda < 0$ suggested by the large abundance of 4-manifolds allowing hyperbolic metric and also by cosmological considerations. The asymptotic formula $L_K = 4\Lambda$ considered above suggests that also Kähler action is negative in Minkowskian regions for magnetic flux tubes dominating in TGD inspired cosmology: the reason is that the magnetic contribution to the action density $L_K \propto E^2 - B^2$ dominates.

Consider now in more detail the 4-D thermodynamics interpretation in Euclidian and Minkowskian regions assuming that the evolution by quantum jumps has Kähler flow as a space-time correlate.

1. In Euclidian regions the choice $\epsilon = 1$ seems to be more reasonable one. In Euclidian regions $-\Lambda$ as the analog of pressure would be negative, and asymptotically (that is for CP_2 type vacuum extremals) its value would be proportional to $\Lambda \propto 1/GR^2$, where R denotes CP_2 radius defined by the length of its geodesic circle.

   A possible interpretation for negative pressure is in terms of string tension effectively inducing negative pressure (note that the solutions of the Kähler-Dirac equation indeed assign a string to the wormhole contact). The analog of the second law would require the increase of RV_4 in quantum jumps. The magnitudes of L_K, R, V_4 and Λ would be reduced and approach their asymptotic values. In particular, V_4 would approach asymptotically the volume of CP_2.

2. In Minkowskian regions Kähler action contributes to the vacuum functional a phase factor analogous to an imaginary exponent of action serving in the role of Morse function so that thermodynamics interpretation can be questioned. Despite this one can check whether thermodynamic interpretation can be considered. The choice $\epsilon = -1$ seems to be the correct choice now. $-\Lambda$ would be analogous to a negative pressure whose gradually decreases. In 3-D thermodynamics it is natural to assign negative pressure to the magnetic flux tube like structures as their effective string tension defined by the density of magnetic energy per unit length. $-R \geq 0$ would entropy and $-L_K \geq 0$ would be the analog of energy density.

   $R = \Lambda/k$ and the reduction of Λ during cosmic evolution by quantum jumps suggests that the larger the volume of CD and thus of (at least) Minkowskian space-time sheet the smaller the negative value of Λ.

   Assume the recent view about state function reduction explaining how the arrow of geometric time is induced by the quantum jump sequence defining experienced time [K8]. According to this view zero energy states are quantum superpositions over CDs of various size scales but with common tip, which can correspond to either the upper or lower light-like boundary of CD. The sequence of quantum jumps the gradual increase of the average size of CD in the quantum superposition and therefore that of average value of V_4. On the other hand, a gradual decrease of both $-L_K$ and $-R$ looks physically very natural. If Kähler flow describes the effect of dissipation by quantum jumps in ZEO then the space-time surfaces would gradually approach nearly vacuum extremals with constant value of entropy density $-R$ but gradually increasing 4-volume so that the analog of second law stating the increase of $-RV_4$ would hold true.

3. The interpretation of $-R > 0$ as negentropy density assignable to entanglement is also possible and is consistent with the interpretation in terms of second law. This interpretation would only change the sign factor ϵ in the proposed formula. Otherwise the above arguments would remain as such.

3.5.4 Could Correlation Functions, S-Matrix, And Coupling Constant Evolution Be Coded The Statistical Properties Of Preferred Extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. The general structure of U-matrix is however understood [K75]. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW .

This might of course be too optimistic guess. If a sub-algebra of symplectic algebra acts as gauge symmmetries of the preferred extremals in the sense that corresponding Noether charges vanish, it can quite well be that correlations functions correspond to averages for extremals belonging to single conformal equivalence class.

1. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

2. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

3. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

   Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root" of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

4. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix constructible as inner products of M-matrices associated with CDs with various size scales [K75].
5. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

1. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

   Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.
2. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \to \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.
3. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly

QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

3.6 About Deformations Of Known Extremals Of Kähler Action

I have done a considerable amount of speculative guesswork to identify what I have used to call preferred extremals of Kähler action. The difficulty is that the mathematical problem at hand is extremely non-linear and that I do not know about existing mathematical literature relevant to the situation. One must proceed by trying to guess the general constraints on the preferred extremals which look physically and mathematically plausible. The hope is that this net of constraints could eventually chrystallize to Eureka! Certainly the recent speculative picture involves also wrong guesses. The need to find explicit ansatz for the deformations of known extremals based on some common principles has become pressing. The following considerations represent an attempt to combine the existing information to achieve this.

3.6.1 What Might Be The Common Features Of The Deformations Of Known Extremals

The dream is to discover the deformations of all known extremals by guessing what is common to all of them. One might hope that the following list summarizes at least some common features.

Effective three-dimensionality at the level of action

1. Holography realized as effective 3-dimensionality also at the level of action requires that it reduces to 3-dimensional effective boundary terms. This is achieved if the contraction $j^\alpha A_\alpha$ vanishes. This is true if j^α vanishes or is light-like, or if it is proportional to instanton current in which case current conservation requires that CP_2 projection of the space-time surface is 3-dimensional. The first two options for j have a realization for known extremals. The status of the third option - proportionality to instanton current - has remained unclear.
2. As I started to work again with the problem, I realized that instanton current could be replaced with a more general current $j = *B \wedge J$ or concretely: $j^\alpha = \epsilon^{\alpha\beta\gamma\delta} B_\beta J_{\gamma\delta}$, where B is vector field and CP_2 projection is 3-dimensional, which it must be in any case. The contractions of j appearing in field equations vanish automatically with this ansatz.
3. Almost topological QFT property in turn requires the reduction of effective boundary terms to Chern-Simons terms: this is achieved by boundary conditions expressing weak form of electric magnetic duality. If one generalizes the weak form of electric-magnetic duality to $J = \Phi * J$ one has $B = d\Phi$ and j has a vanishing divergence for 3-D CP_2 projection. This is clearly a more general solution ansatz than the one based on proportionality of j with instanton current and would reduce the field equations in concise notation to $Tr(TH^k) = 0$.
4. Any of the alternative properties of the Kähler current implies that the field equations reduce to $Tr(TH^k) = 0$, where T and H^k are shorthands for Maxwellian energy momentum tensor and second fundamental form and the product of tensors is obvious generalization of matrix product involving index contraction.

Could Einstein's equations emerge dynamically?

For j^α satisfying one of the three conditions, the field equations have the same form as the equations for minimal surfaces except that the metric g is replaced with Maxwell energy momentum tensor T.

1. This raises the question about dynamical generation of small cosmological constant Λ: $T = \Lambda g$ would reduce equations to those for minimal surfaces. For $T = \Lambda g$ Kähler-Dirac gamma matrices would reduce to induced gamma matrices and the Kähler-Dirac operator would be

proportional to ordinary Dirac operator defined by the induced gamma matrices. One can also consider weak form for $T = \Lambda g$ obtained by restricting the consideration to a sub-space of tangent space so that space-time surface is only "partially" minimal surface but this option is not so elegant although necessary for other than CP_2 type vacuum extremals.

2. What is remarkable is that $T = \Lambda g$ implies that the divergence of T which in the general case equals to $j^\beta J^\alpha_\beta$ vanishes. This is guaranteed by one of the conditions for the Kähler current. Since also Einstein tensor has a vanishing divergence, one can ask whether the condition to $T = \kappa G + \Lambda g$ could the general condition. This would give Einstein's equations with cosmological term besides the generalization of the minimal surface equations. GRT would emerge dynamically from the non-linear Maxwell's theory although in slightly different sense as conjectured [K121] ! Note that the expression for G involves also second derivatives of the embedding space coordinates so that actually a partial differential equation is in question. If field equations reduce to purely algebraic ones, as the basic conjecture states, it is possible to have $Tr(GH^k) = 0$ and $Tr(gH^k) = 0$ separately so that also minimal surface equations would hold true.

   What is amusing that the first guess for the action of TGD was curvature scalar. It gave analogs of Einstein's equations as a definition of conserved four-momentum currents. The recent proposal would give the analog of ordinary Einstein equations as a dynamical constraint relating Maxwellian energy momentum tensor to Einstein tensor and metric.

3. Minimal surface property is physically extremely nice since field equations can be interpreted as a non-linear generalization of massless wave equation: something very natural for non-linear variant of Maxwell action. The theory would be also very "stringy" although the fundamental action would not be space-time volume. This can however hold true only for Euclidian signature. Note that for CP_2 type vacuum extremals Einstein tensor is proportional to metric so that for them the two options are equivalent. For their small deformations situation changes and it might happen that the presence of G is necessary. The GRT limit of TGD discussed in [K121] [L15] indeed suggests that CP_2 type solutions satisfy Einstein's equations with large cosmological constant and that the small observed value of the cosmological constant is due to averaging and small volume fraction of regions of Euclidian signature (lines of generalized Feynman diagrams).

4. For massless extremals and their deformations $T = \Lambda g$ cannot hold true. The reason is that for massless extremals energy momentum tensor has component T^{vv} which actually quite essential for field equations since one has $H^k_{vv} = 0$. Hence for massless extremals and their deformations $T = \Lambda g$ cannot hold true if the induced metric has Hamilton-Jacobi structure meaning that g^{uu} and g^{vv} vanish. A more general relationship of form $T = \kappa G + \Lambda G$ can however be consistent with non-vanishing T^{vv} but require that deformation has at most 3-D CP_2 projection (CP_2 coordinates do not depend on v).

5. The non-determinism of vacuum extremals suggest for their non-vacuum deformations a conflict with the conservation laws. In, also massless extremals are characterized by a non-determinism with respect to the light-like coordinate but like-likeness saves the situation. This suggests that the transformation of a properly chosen time coordinate of vacuum extremal to a light-like coordinate in the induced metric combined with Einstein's equations in the induced metric of the deformation could allow to handle the non-determinism.

Are complex structure of CP_2 and Hamilton-Jacobi structure of M^4 respected by the deformations?

The complex structure of CP_2 and Hamilton-Jacobi structure of M^4 could be central for the understanding of the preferred extremal property algebraically.

1. There are reasons to believe that the Hermitian structure of the induced metric ((1, 1) structure in complex coordinates) for the deformations of CP_2 type vacuum extremals could be crucial property of the preferred extremals. Also the presence of light-like direction is also an essential elements and 3-dimensionality of M^4 projection could be essential. Hence a good guess is that allowed deformations of CP_2 type vacuum extremals are such that (2, 0) and (0, 2) components the induced metric and/or of the energy momentum tensor vanish. This gives rise to the conditions implying Virasoro conditions in string models in quantization:

$$g_{\xi^i\xi^j} = 0 \ , \quad g_{\overline{\xi}^i\overline{\xi}^j} = 0 \ , \quad i,j = 1,2 \ . \tag{3.6.1}$$

Holomorphisms of CP_2 preserve the complex structure and Virasoro conditions are expected to generalize to 4-dimensional conditions involving two complex coordinates. This means that the generators have two integer valued indices but otherwise obey an algebra very similar to the Virasoro algebra. Also the super-conformal variant of this algebra is expected to make sense.

These Virasoro conditions apply in the coordinate space for CP_2 type vacuum extremals. One expects similar conditions hold true also in field space, that is for M^4 coordinates.

2. The integrable decomposition $M^4(m) = M^2(m)+E^2(m)$ of M^4 tangent space to longitudinal and transversal parts (non-physical and physical polarizations) - Hamilton-Jacobi structure- could be a very general property of preferred extremals and very natural since non-linear Maxwellian electrodynamics is in question. This decomposition led rather early to the introduction of the analog of complex structure in terms of what I called Hamilton-Jacobi coordinates $(u, v, w, \overline{w})$ for M^4. (u, v) defines a pair of light-like coordinates for the local longitudinal space $M^2(m)$ and $(w, \overline{w})$ complex coordinates for $E^2(m)$. The metric would not contain any cross terms between $M^2(m)$ and $E^2(m)$: $g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$.

   A good guess is that the deformations of massless extremals respect this structure. This condition gives rise to the analog of the constraints leading to Virasoro conditions stating the vanishing of the non-allowed components of the induced metric. $g_{uu} = g_{vv} = g_{ww} = g_{\overline{w}\overline{w}} = g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$. Again the generators of the algebra would involve two integers and the structure is that of Virasoro algebra and also generalization to super algebra is expected to make sense. The moduli space of Hamilton-Jacobi structures would be part of the moduli space of the preferred extremals and analogous to the space of all possible choices of complex coordinates. The analogs of infinitesimal holomorphic transformations would preserve the modular parameters and give rise to a 4-dimensional Minkowskian analog of Virasoro algebra. The conformal algebra acting on CP_2 coordinates acts in field degrees of freedom for Minkowskian signature.

Field equations as purely algebraic conditions

If the proposed picture is correct, field equations would reduce basically to purely algebraically conditions stating that the Maxwellian energy momentum tensor has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1, 1) and second fundamental form H^k a tensor of type (2, 0) and (0, 2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

3.6.2 What Small Deformations Of CP_2 Type Vacuum Extremals Could Be?

I was led to these arguments when I tried find preferred extremals of Kähler action, which would have 4-D CP_2 and M^4 projections - the Maxwell phase analogous to the solutions of Maxwell's equations that I conjectured long time ago. It however turned out that the dimensions of the projections can be $(D_{M^4} \leq 3, D_{CP_2} = 4)$ or $(D_{M^4} = 4, D_{CP_2} \leq 3)$. What happens is essentially breakdown of linear superposition so that locally one can have superposition of modes which have 4-D wave vectors in the same direction. This is actually very much like quantization of radiation field to photons now represented as separate space-time sheets and one can say that Maxwellian superposition corresponds to union of separate photonic space-time sheets in TGD.

Approximate linear superposition of fields is fundamental in standard physics framework and is replaced in TGD with a linear superposition of effects of classical fields on a test particle topologically condensed simultaneously to several space-time sheets. One can say that linear superposition is replaced with a disjoint union of space-time sheets. In the following I shall restrict the consideration to the deformations of CP_2 type vacuum extremals.

Solution ansatz

I proceed by the following arguments to the ansatz.

1. Effective 3-dimensionality for action (holography) requires that action decomposes to vanishing $j^\alpha A_\alpha$ term + total divergence giving 3-D "boundary" terms. The first term certainly vanishes (giving effective 3-dimensionality) for

$$D_\beta J^{\alpha\beta} = j^\alpha = 0 \ .$$

   Empty space Maxwell equations, something extremely natural. Also for the proposed GRT limit these equations are true.

2. How to obtain empty space Maxwell equations $j^\alpha = 0$? The answer is simple: assume self duality or its slight modification:

$$J = *J$$

   holding for CP_2 type vacuum extremals or a more general condition

$$J = k * J \ ,$$

   In the simplest situation k is some constant not far from unity. * is Hodge dual involving 4-D permutation symbol. $k = \textit{constant}$ requires that the determinant of the induced metric is apart from constant equal to that of CP_2 metric. It does not require that the induced metric is proportional to the CP_2 metric, which is not possible since M^4 contribution to metric has Minkowskian signature and cannot be therefore proportional to CP_2 metric.

   One can consider also a more general situation in which k is scalar function as a generalization of the weak electric-magnetic duality. In this case the Kähler current is non-vanishing but divergenceless. This also guarantees the reduction to $Tr(TH^k) = 0$. In this case however the proportionality of the metric determinant to that for CP_2 metric is not needed. This solution ansatz becomes therefore more general.

3. Field equations reduce with these assumptions to equations differing from minimal surfaces equations only in that metric g is replaced by Maxwellian energy momentum tensor T. Schematically:

$$Tr(TH^k) = 0 \ ,$$

   where T is the Maxwellian energy momentum tensor and H^k is the second fundamental form - asymmetric 2-tensor defined by covariant derivative of gradients of embedding space coordinates.

How to satisfy the condition $Tr(TH^k) = 0$?

It would be nice to have minimal surface equations since they are the non-linear generalization of massless wave equations. It would be also nice to have the vanishing of the terms involving Kähler current in field equations as a consequence of this condition. Indeed, $T = \kappa G + \Lambda g$ implies this. In the case of CP_2 vacuum extremals one cannot distinguish between these options since CP_2 itself is constant curvature space with $G \propto g$. Furthermore, if G and g have similar tensor structure the algebraic field equations for G and g are satisfied separately so that one obtains minimal surface property also now. In the following minimal surface option is considered.

1. The first opton is achieved if one has

$$T = \Lambda g \ .$$

Maxwell energy momentum tensor would be proportional to the metric! One would have dynamically generated cosmological constant! This begins to look really interesting since it appeared also at the proposed GRT limit of TGD [L15] (see `http://tinyurl.com/hzkldnb`). Note that here also non-constant value of Λ can be considered and would correspond to a situation in which k is scalar function: in this case the the determinant condition can be dropped and one obtains just the minimal surface equations.

2. Very schematically and forgetting indices and being sloppy with signs, the expression for T reads as

$$T = JJ - g/4Tr(JJ) \ .$$

Note that the product of tensors is obtained by generalizing matrix product. This should be proportional to metric.

Self duality implies that $Tr(JJ)$ is just the instanton density and does not depend on metric and is constant.

For CP_2 type vacuum extremals one obtains

$$T = -g + g = 0 \ .$$

Cosmological constant would vanish in this case.

3. Could it happen that for deformations a small value of cosmological constant is generated? The condition would reduce to

$$JJ = (\Lambda - 1)g \ .$$

Λ must relate to the value of parameter k appearing in the generalized self-duality condition. For the most general ansatz Λ would not be constant anymore.

This would generalize the defining condition for Kähler form

$$JJ = -g \ \ (i^2 = -1 \ \ geometrically)$$

stating that the square of Kähler form is the negative of metric. The only modification would be that index raising is carried out by using the induced metric containing also M^4 contribution rather than CP_2 metric.

4. Explicitly:

$$J_{\alpha\mu}J^{\mu}_{\ \beta} = (\Lambda - 1)g_{\alpha\beta} \ .$$

Cosmological constant would measure the breaking of Kähler structure. By writing $g = s+m$ and defining index raising of tensors using CP_2 metric and their product accordingly, this condition can be also written as

$$Jm = (\Lambda - 1)mJ \ .$$

If the parameter k is constant, the determinant of the induced metric must be proportional to the CP_2 metric. If k is scalar function, this condition can be dropped. Cosmological constant would not be constant anymore but the dependence on k would drop out from the field equations and one would hope of obtaining minimal surface equations also now. It however seems that the dimension of M^4 projection cannot be four. For 4-D M^4 projection the contribution of the M^2 part of the M^4 metric gives a non-holomorphic contribution to CP_2 metric and this spoils the field equations.

For $T = \kappa G + \Lambda g$ option the value of the cosmological constant is large - just as it is for the proposed GRT limit of TGD [K121] [L15]. The interpretation in this case is that the average value of cosmological constant is small since the portion of space-time volume containing generalized Feynman diagrams is very small.

More detailed ansatz for the deformations of CP_2 type vacuum extremals

One can develop the ansatz to a more detailed form. The most obvious guess is that the induced metric is apart from constant conformal factor the metric of CP_2. This would guarantee self-duality apart from constant factor and $j^\alpha = 0$. Metric would be in complex CP_2 coordinates tensor of type (1, 1) whereas CP_2 Riemann connection would have only purely holomorphic or anti-holomorphic indices. Therefore CP_2 contributions in $Tr(TH^k)$ would vanish identically. M^4 degrees of freedom however bring in difficulty. The M^4 contribution to the induced metric should be proportional to CP_2 metric and this is impossible due to the different signatures. The M^4 contribution to the induced metric breaks its Kähler property but would preserve Hermitian structure.

A more realistic guess based on the attempt to construct deformations of CP_2 type vacuum extremals is following.

1. Physical intuition suggests that M^4 coordinates can be chosen so that one has integrable decomposition to longitudinal degrees of freedom parametrized by two light-like coordinates u and v and to transversal polarization degrees of freedom parametrized by complex coordinate w and its conjugate. M^4 metric would reduce in these coordinates to a direct sum of longitudinal and transverse parts. I have called these coordinates Hamilton-Jacobi coordinates.
2. w would be holomorphic function of CP_2 coordinates and therefore satisfy the analog of massless wave equation. This would give hopes about rather general solution ansatz. u and v cannot be holomorphic functions of CP_2 coordinates. Unless wither u or v is constant, the induced metric would receive contributions of type (2, 0) and (0, 2) coming from u and v which would break Kähler structure and complex structure. These contributions would give no-vanishing contribution to all minimal surface equations. Therefore either u or v is constant: the coordinate line for non-constant coordinate -say u- would be analogous to the M^4 projection of CP_2 type vacuum extremal.
3. With these assumptions the induced metric would remain $(1, 1)$ tensor and one might hope that $Tr(TH^k)$ contractions vanishes for all variables except u because the there are no common index pairs (this if non-vanishing Christoffel symbols for H involve only holomorphic or anti-holomorphic indices in CP_2 coordinates). For u one would obtain massless wave equation expressing the minimal surface property.
4. If the value of k is constant the determinant of the induced metric must be proportional to the determinant of CP_2 metric. The induced metric would contain only the contribution from the transversal degrees of freedom besides CP_2 contribution. Minkowski contribution has however rank 2 as CP_2 tensor and cannot be proportional to CP_2 metric. It is however enough that its determinant is proportional to the determinant of CP_2 metric with constant proportionality coefficient. This condition gives an additional non-linear condition to the solution. One would have wave equation for u (also w and its conjugate satisfy massless wave equation) and determinant condition as an additional condition.

   The determinant condition reduces by the linearity of determinant with respect to its rows to sum of conditions involved 0, 1, 2 rows replaced by the transversal M^4 contribution to metric given if M^4 metric decomposes to direct sum of longitudinal and transversal parts. Derivatives with respect to derivative with respect to particular CP_2 complex coordinate appear linearly in this expression they can depend on u via the dependence of transversal metric components on u. The challenge is to show that this equation has (or does not have) non-trivial solutions.
5. If the value of k is scalar function the situation changes and one has only the minimal surface equations and Virasoro conditions.

What makes the ansatz attractive is that special solutions of Maxwell empty space equations are in question, equations reduces to non-linear generalizations of Euclidian massless wave equations, and possibly space-time dependent cosmological constant pops up dynamically. These properties are true also for the GRT limit of TGD [L15] (see `http://tinyurl.com/hzkldnb`).

3.6.3 Hamilton-Jacobi Conditions In Minkowskian Signature

The maximally optimistic guess is that the basic properties of the deformations of CP_2 type vacuum extremals generalize to the deformations of other known extremals such as massless extremals, vacuum extremals with 2-D CP_2 projection which is Lagrangian manifold, and cosmic strings characterized by Minkowskian signature of the induced metric. These properties would be following.

1. The recomposition of M^4 tangent space to longitudinal and transversal parts giving Hamilton-Jacobi structure. The longitudinal part has hypercomplex structure but the second light-like coordinate is constant: this plays a crucial role in guaranteeing the vanishing of contractions in $Tr(TH^k)$. It is the algebraic properties of g and T which are crucial. T can however have light-like component T^{vv}. For the deformations of CP_2 type vacuum extremals $(1,1)$ structure is enough and is guaranteed if second light-like coordinate of M^4 is constant whereas w is holomorphic function of CP_2 coordinates.
2. What could happen in the case of massless extremals? Now one has 2-D CP_2 projection in the initial situation and CP_2 coordinates depend on light-like coordinate u and single real transversal coordinate. The generalization would be obvious: dependence on single light-like coordinate u and holomorphic dependence on w for complex CP_2 coordinates. The constraint is $T = \Lambda g$ cannot hold true since T^{vv} is non-vanishing (and light-like). This property restricted to transversal degrees of freedom could reduce the field equations to minimal surface equations in transversal degrees of freedom. The transversal part of energy momentum tensor would be proportional to metric and hence covariantly constant. Gauge current would remain light-like but would not be given by $j = *d\phi \wedge J$. $T = \kappa G + \Lambda g$ seems to define the attractive option.

It therefore seems that the essential ingredient could be the condition

$$T = \kappa G + \lambda g \ ,$$

which has structure (1, 1) in both $M^2(m)$ and $E^2(m)$ degrees of freedom apart from the presence of T^{vv} component with deformations having no dependence on v. If the second fundamental form has (2, 0)+(0, 2) structure, the minimal surface equations are satisfied provided Kähler current satisfies on of the proposed three conditions and if G and g have similar tensor structure.

One can actually pose the conditions of metric as complete analogs of stringy constraints leading to Virasoro conditions in quantization to give

$$g_{uu} = 0 \ , \quad g_{vv} = 0 \ , \quad g_{ww} = 0 \ , \quad g_{\overline{w}\overline{w}} = 0 \ . \tag{3.6.2}$$

This brings in mind the generalization of Virasoro algebra to four-dimensional algebra for which an identification in terms of non-local Yangian symmetry [A25] [B29, B22, B23] has been proposed [K116]. The number of conditions is four and the same as the number of independent field equations. One can consider similar conditions also for the energy momentum tensor T but allowing non-vanishing component T^{vv} if deformations has no v-dependence. This would solve the field equations if the gauge current vanishes or is light-like. On this case the number of equations is 8. First order differential equations are in question and they can be also interpreted as conditions fixing the coordinates used since there is infinite number of ways to choose the Hamilton-Jacobi coordinates.

One can can try to apply the physical intuition about general solutions of field equations in the linear case by writing the solution as a superposition of left and right propagating solutions:

$$\xi^k = f^k_+(u,w) + f^k_+(v,w) \ . \tag{3.6.3}$$

This could guarantee that second fundamental form is of form (2, 0)+(0, 2) in both M^2 and E^2 part of the tangent space and these terms if $Tr(TH^k)$ vanish identically. The remaining terms involve contractions of T^{uw}, $T^{u\overline{w}}$ and T^{vw}, $T^{v\overline{w}}$ with second fundamental form. Also these terms should sum up to zero or vanish separately. Second fundamental form has components coming from f^k_+ and f^k_-

Second fundamental form H^k has as basic building bricks terms $\hat{H}^k$ given by

$$\hat{H}^k_{\alpha\beta} \ = \ \partial_\alpha\partial_\beta h^k + \left\{ {k \atop l \ m} \right\} \partial_\alpha h^l \partial_\beta h^m \ . \tag{3.6.4}$$

For the proposed ansatz the first terms give vanishing contribution to H^k_{uv}. The terms containing Christoffel symbols however give a non-vanishing contribution and one can allow only f^k_+ or f^k_- as in the case of massless extremals. This reduces the dimension of CP_2 projection to $D = 3$.

What about the condition for Kähler current? Kähler form has components of type $J_{w\overline{w}}$ whose contravariant counterpart gives rise to space-like current component. J_{uw} and $J_{u\overline{w}}$ give rise to light-like currents components. The condition would state that the $J^{w\overline{w}}$ is covariantly constant. Solutions would be characterized by a constant Kähler magnetic field. Also electric field is represent. The interpretation both radiation and magnetic flux tube makes sense.

3.6.4 Deformations Of Cosmic Strings

In the physical applications it has been assumed that the thickening of cosmic strings to Kähler magnetic flux tubes takes place. One indeed expects that the proposed construction generalizes also to the case of cosmic strings having the decomposition $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 a complex homologically non-trivial sub-manifold of CP_2. Now the starting point structure is Hamilton-Jacobi structure for $M^2_m \times Y^2$ defining the coordinate space.

1. The deformation should increase the dimension of either CP_2 or M^4 projection or both. How this thickening could take place? What comes in mind that the string orbits X^2 can be interpreted as a distribution of longitudinal spaces $M^2(x)$ so that for the deformation w coordinate becomes a holomorphic function of the natural Y^2 complex coordinate so that M^4 projection becomes 4-D but CP_2 projection remains 2-D. The new contribution to the X^2 part of the induced metric is vanishing and the contribution to the Y^2 part is of type $(1,1)$ and the ansatz $T = \kappa G + \Lambda g$ might be needed as a generalization of the minimal surface equations The ratio of κ and G would be determined from the form of the Maxwellian energy momentum tensor and be fixed at the limit of undeformed cosmic strong to $T = (ag(Y^2) - bg(Y^2)$. The value of cosmological constant is now large, and overall consistency suggests that $T = \kappa G + \Lambda g$ is the correct option also for the CP_2 type vacuum extremals.
2. One could also imagine that remaining CP_2 coordinates could depend on the complex coordinate of Y^2 so that also CP_2 projection would become 4-dimensional. The induced metric would receive holomorphic contributions in Y^2 part. As a matter fact, this option is already implied by the assumption that Y^2 is a complex surface of CP_2.

3.6.5 Deformations Of Vacuum Extremals?

What about the deformations of vacuum extremals representable as maps from M^4 to CP_2?

1. The basic challenge is the non-determinism of the vacuum extremals. One should perform the deformation so that conservation laws are satisfied. For massless extremals there is also non-determinism but it is associated with the light-like coordinate so that there are no problems with the conservation laws. This would suggest that a properly chosen time coordinate consistent with Hamilton-Jacobi decomposition becomes light-like coordinate in the induced metric. This poses a conditions on the induced metric.
2. Physical intuition suggests that one cannot require $T = \Lambda g$ since this would mean that the rank of T is maximal whereas the original situation corresponds to the vanishing of T. For small deformations rank two for T looks more natural and one could think that T is proportional to a projection of metric to a 2-D subspace. The vision about the long length scale limit of TGD is that Einstein's equations are satisfied and this would suggest $T = kG$ or $T = \kappa G + \Lambda g$. The rank of T could be smaller than four for this ansatz and this conditions binds together the values of κ and G.
3. These extremals have CP_2 projection which in the generic case is 2-D Lagrangian sub-manifold Y^2. Again one could assume Hamilton-Jacobi coordinates for X^4. For CP_2 one could assume Darboux coordinates (P_i, Q_i), $i = 1, 2$, in which one has $A = P_i dQ^i$, and that $Y^2 \subset CP_2$ corresponds to $Q_i = constant$. In principle P_i would depend on arbitrary manner on M^4 coordinates. It might be more convenient to use as coordinates (u, v) for M^2 and (P_1, P_2) for Y^2. This covers also the situation when M^4 projection is not 4-D. By its 2-dimensionality

Y^2 allows always a complex structure defined by its induced metric: this complex structure is not consistent with the complex structure of CP_2 (Y^2 is not complex sub-manifold).

Using Hamilton-Jacobi coordinates the pre-image of a given point of Y^2 is a 2-dimensional sub-manifold X^2 of X^4 and defines also 2-D sub-manifold of M^4. The following picture suggests itself. The projection of X^2 to M^4 can be seen for a suitable choice of Hamilton-Jacobi coordinates as an analog of Lagrangian sub-manifold in M^4 that is as surface for which v and $Im(w)$ vary and u and $Re(w)$ are constant. X^2 would be obtained by allowing u and $Re(w)$ to vary: as a matter fact, (P_1, P_2) and $(u, Re(w))$ would be related to each other. The induced metric should be consistent with this picture. This would requires $g_{uRe(w)} = 0$.

For the deformations Q_1 and Q_2 would become non-constant and they should depend on the second light-like coordinate v only so that only g_{uu} and g_{uw} and $gu\overline{w}$ $g_{w,w}$ and $g_{\overline{w},\overline{w}}$ receive contributions which vanish. This would give rise to the analogs of Virasoro conditions guaranteeing that T is a tensor of form $(1,1)$ in both M^2 and E^2 indices and that there are no cross components in the induced metric. A more general formulation states that energy momentum tensor satisfies these conditions. The conditions on T might be equivalent with the conditions for g and G separately.

4. Einstein's equations provide an attractive manner to achieve the vanishing of effective 3-dimensionality of the action. Einstein equations would be second order differential equations and the idea that a deformation of vacuum extremal is in question suggests that the dynamics associated with them is in directions transversal to Y^2 so that only the deformation is dictated partially by Einstein's equations.

5. Lagrangian manifolds do not involve complex structure in any obvious manner. One could however ask whether the deformations could involve complex structure in a natural manner in CP_2 degrees of freedom so that the vanishing of g_{ww} would be guaranteed by holomorphy of CP_2 complex coordinate as function of w.

   One should get the complex structure in some natural manner: in other words, the complex structure should relate to the geometry of CP_2 somehow. The complex coordinate defined by say $z = P_1 + iQ^1$ for the deformation suggests itself. This would suggest that at the limit when one puts $Q_1 = 0$ one obtains $P_1 = P_1(Re(w))$ for the vacuum extremals and the deformation could be seen as an analytic continuation of real function to region of complex plane. This is in spirit with the algebraic approach. The vanishing of Kähler current requires that the Kähler magnetic field is covariantly constant: $D_z J^{z\overline{z}} = 0$ and $D_{\overline{z}} J^{z\overline{z}} = 0$.

6. One could consider the possibility that the resulting 3-D sub-manifold of CP_2 can be regarded as contact manifold with induced Kähler form non-vanishing in 2-D section with natural complex coordinates. The third coordinate variable- call it s- of the contact manifold and second coordinate of its transversal section would depend on time space-time coordinates for vacuum extremals. The coordinate associated with the transversal section would be continued to a complex coordinate which is holomorphic function of w and u.

7. The resulting thickened magnetic flux tubes could be seen as another representation of Kähler magnetic flux tubes: at this time as deformations of vacuum flux tubes rather than cosmic strings. For this ansatz it is however difficult to imagine deformations carrying Kähler electric field.

3.6.6 About The Interpretation Of The Generalized Conformal Algebras

The long-standing challenge has been finding of the direct connection between the super-conformal symmetries assumed in the construction of the geometry of the "world of classical worlds" (WCW) and possible conformal symmetries of field equations. 4-dimensionality and Minkowskian signature have been the basic problems. The recent construction provides new insights to this problem.

1. In the case of string models the quantization of the Fourier coefficients of coordinate variables of the target space gives rise to Kac-Moody type algebra and Virasoro algebra generators are quadratic in these. Also now Kac-Moody type algebra is expected. If one were to perform a quantization of the coefficients in Laurents series for complex CP_2 coordinates, one would obtain interpretation in terms of $su(3) = u(2) + t$ decomposition, where t corresponds to CP_3:

the oscillator operators would correspond to generators in t and their commutator would give generators in $u(2)$. SU(3)/SU(2) coset representation for Kac-Moody algebra would be in question. Kac-Moody algebra would be associated with the generators in both M^4 and CP_2 degrees of freedom. This kind of Kac-Moody algebra appears in quantum TGD.

2. The constraints on induced metric imply a very close resemblance with string models and a generalization of Virasoro algebra emerges. An interesting question is how the two algebras acting on coordinate and field degrees of freedom relate to the super-conformal algebras defined by the symplectic group of $\delta M^4_+ \times CP_2$ acting on space-like 3-surfaces at boundaries of CD and to the Kac-Moody algebras acting on light-like 3-surfaces. It has been conjectured that these algebras allow a continuation to the interior of space-time surface made possible by its slicing by 2-surfaces parametrized by 2-surfaces. The proposed construction indeed provides this kind of slicings in both M^4 and CP_2 factor.

3. In the recent case, the algebras defined by the Fourier coefficients of field variables would be Kac-Moody algebras. Virasoro algebra acting on preferred coordinates would be expressed in terms of the Kac-Moody algebra in the standard Sugawara construction applied in string models. The algebra acting on field space would be analogous to the conformal algebra assignable to the symplectic algebra so that also symplectic algebra is present. Stringy pragmatist could imagine quantization of symplectic algebra by replacing CP_2 coordinates in the expressions of Hamiltonians with oscillator operators. This description would be counterpart for the construction of spinor harmonics in WCW and might provide some useful insights.

4. For given type of space-time surface either CP_2 or M^4 corresponds to Kac-Moody algebra but not both. From the point of view of quantum TGD it looks as that something were missing. An analogous problem was encountered at GRT limit of TGD [L15]. When Euclidian space-time regions are allowed Einstein-Maxwell action is able to mimic standard model with a surprising accuracy but there is a problem: one obtains either color charges or M^4 charges but not both. Perhaps it is not enough to consider either CP_2 type vacuum extremal or its exterior but both to describe particle: this would give the direct product of the Minkowskian and Euclidian algebras acting on tensor product. This does not however seem to be consistent with the idea that the two descriptions are duality related (the analog of T-duality).

3.7 About TGD counterparts of classical field configurations in Maxwell's theory

Classical physics is an exact part of TGD so that the study of extremals of dimensionally reduces 6-D Kähler action can provide a lot of intuition about quantum TGD and see how quantum-classical correspondence is realized. In the following I will try to develop further understanding about TGD counterparts of the simplest field configurations in Maxwell's theory.

In the sequel CP_2 type extremals will be considered from the point of view of quantum criticality and the view about string world sheets, their lightlike boundaries as carriers of fermion number, and the ends as point like particles as singularities acting as sources for minimal surfaces satisfying non-linear generalization of d'Alembert equation.

I will also discuss the delicacies associated with M^4 Kähler structure and its connection with what I call Hamilton-Jacobi structure and with M^8 approach based on classical number fields. I will argue that the breaking of CP symmetry associated with M^4 Kähler structure is small without any additional assumptions: this is in contrast with the earlier view.

The difference between TGD and Maxwell's theory and consider the TGD counterparts of simple em field configurations will be also discussed. Topological field quantization provides a geometric view about formation of atoms as bound states based on flux tubes as correlates for binding, and allows to identify space-time correlates for second quantization. These considerations force to take seriously the possibility that preferred extremals besides being minimal surfaces also possess generalized holomorphy reducing field equations to purely algebraic conditions and that minimal surfaces without this property are not preferred extremals. If so, at microscopic level only CP_2 type extremals, massless extremals, and string like objects and their deformations would exist as preferred extremals and serve as building bricks for the counterparts of Maxwellian field configurations and the counterparts of Maxwellian field configurations such as Coulomb potential would emerge only at the QFT limit.

3.7.1 About differences between Maxwell's ED and TGD

TGD differs from Maxwell's theory in several important aspects.

1. The TGD counterparts of classical electroweak gauge potentials are induced from component of spinor connection of CP_2. Classical color gauge potentials corresponds to the projections of Killing vector fields of color isometries.

2. Also M^4 has Kähler potential, which is induced to space-time surface and gives rise to an additional $U(1)$ force. The couplings of M^4 gauge potential to quarks and leptons are of same sign whereas the couplings of CP_2 Kähler potential to B and L are of opposite sign so that the contributions to 6-D Kähler action reduce to separate terms without interference term.

   Coupling to induced M^4 Kähler potential implies CP breaking. This could explain the small CP breaking in hadronic systems and also matter antimatter asymmetry in which there are opposite matter-antimatter asymmetries inside cosmic strings and their exteriors respectively. A priori it is however not obvious that the CP breaking is small.

3. General coordinate invariance implies that there are only 4 local field like degrees of freedom so that for extremals with 4-D M^4 projection corresponding to GRT space-time both metric, electroweak and color gauge potentials can be expressed in terms four CP_2 coordinates and their gradients. Preferred extremal property realized as minimal surface condition means that field equations are satisfied separately for the 4-D Kähler and volume action reduces the degrees of freedom further.

   If the CP_2 part of Kähler form is non-vanishing, minimal surface conditions can be guaranteed by a generalization of holomorphy realizing quantum criticality (satisfied by known extremals). One can say that there is no dependence on coupling parameters. If CP_2 part of Kähler form vanishes identically, the minimal surface condition need not be guaranteed by holomorphy. It is not at all clear whether quantum criticality and preferred extremal property allow this kind of extremals.

4. Supersymplectic symmetries act as isometries of "world of classical worlds" (WCW). In a well-defined sense supersymplectic symmetry generalizes 2-D conformal invariance to 4-D context. The key observation here is that light-like 3-surfaces are metrically 2-D and therefore allow extended conformal invariance.

   Preferred extremal property realizing quantum criticality boils down to a condition that sub-algebra of SSA and its commutator with SSA annihilate physical states and that corresponding Noether charges vanish. These conditions could be equivalent with minimal surface property. This implies that the set of possible field patterns is extremely restricted and one might talk about "archetypal" field patterns analogous to partial waves or plane waves in Maxwell's theory.

5. Linear superposition of the archetypal field patterns is not possible. TGD however implies the notion of many-sheeted space-time and each sheet can carry its own field pattern. A test particle which is space-time surface itself touches all these sheets and experiences the sum of the effects caused by fields at various sheets. Effects are superposed rather than fields and this is enough. This means weakening of the superposition principle of Maxwell's theory and the linear superposition of fields at same space-time sheet is replaced with set theoretic union of space-time sheets carrying the field patterns whose effects superpose.

   This observation is also essential in the construction of QFT limit of TGD. The gauge potentials in standard model and gravitational field in general relativity are superpositions of those associated with space-time sheets idealized with slightly curved piece of Minkowski space M^4.

6. An important implication is that each system has field identity - field body or magnetic body (MB). In Maxwell's theory superposition of fields coming from different sources leads to a loss of information since one does not anymore now which part of field came from particular source. In TGD this information loss does not happen and this is essential for TGD inspired quantum biology.

Remark: An interesting algebraic analog is the notion of co-algebra. Co-product is analogous to reversal of product AB= C in the sense that it assigns to C and a linear combination of products $\sum A_i \otimes B_i$ such that $A_i B_i = C$. Quantum groups and co-algebras are indeed important in TGD and it might be that there is a relationship. In TGD inspired quantum biology magnetic body plays a key role as an intentional agent receiving sensory data from biological body and using it as motor instrument.

7. I have already earlier considered a space-time correlate for second quantization in terms of sheets of covering for $h_{eff} = nh_0$. In [L71] it is proposed that n factorizes as $n = n_1 n_2$ such that n_1 (n_2) is the number sheets for space-time surface as covering of CP_2 (M^4). One could have quantum mechanical linear superposition of space-time sheets, each with a particular field pattern. This kind state would correspond to single particle state created by quantum field in QFT limit. For instance, one could have spherical harmonic for orientations of magnetic flux tube or electric flux tube.

   One could also have superposition of configurations containing several space-time sheets simultaneously as analogs of many-boson states. Many-sheeted space-time would correspond to this kind many-boson states. Second quantization in quantum field theory (QFT) could be seen as an algebraic description of many-sheetedness having no obvious classical correlate in classical QFT.

8. Flux tubes should be somehow different for gravitational fields, em fields, and also weak and color gauge fields. The value of $n = n_1 n_2$ [L71] for gravitational flux tubes is very large by Nottale formula $\hbar_{eff} = \hbar_{gr} = GMm/v_0$. The value of n_2 for gravitational flux tubes is $n_2 \sim 10^7$ if one accepts the formula $G = R^2/n_2\hbar$. For em fields much smaller values of n and therefore of n_2 are suggestive. There the value of n measuring in adelic physics algebraic complexity and evolutionary level would distinguish between gravitational and em flux tubes.

   Large value of n would mean quantum coherence in long scales. For gravitation this makes sense since screening is absent unlike for gauge interactions. Note that the large value of $h_{eff} = h_{gr}$ implies that $\alpha_{em} = e^2/4\pi\hbar_{eff}$ is extremely small for gravitational flux tubes so that they would indeed be gravitational in an excellent approximation.

   n would be the dimension of extension of rationals involved and n_2 would be the number space-time sheets as covering of M^4. If this picture is correct, gravitation would correspond to much larger algebraic complexity and much larger value of Planck constant. This conforms with the intuition that gravitation plays essential role in the quantum physics of living matter.

   There are also other number theoretic characteristics such as ramified primes of the extension identifiable as preferred p-adic primes in turn characterizing elementary particle. Also flux tubes mediating weak and strong interactions should allow characterization in terms of number theoretic parameters. There are arguments that in atomic physics one has $h = 6h_0$. Since the quantum coherence scale of hadrons is smaller than atomic scale, one can ask whether one could have $h_{eff} < h$.

3.7.2 CP_2 type extremals as ultimate sources of fields and singularities

CP_2 type extremals have Euclidian signature of induced metric and therefore represent the most radical deviation from Maxwell's ED, gauge theories, and GRT. CP_2 type extremal with light-like geodesic as M^4 projection represents a model for wormhole contact. The light-like orbit of partonic 2-surface correspond to boundary between wormhole contact and Minkowskian region and is associated with both throats of wormhole contact. The throats of wormhole contact can carry part of a boundary of string world sheet connecting the partonic orbits associated with different particles. These light-like lines can carry fermion number and would correspond to lines of TGD counterparts of twistor diagrams.

These world lines would correspond to singularities for the minimal surface equations analogous to sources of massless vector fields carrying charge [L70, L79]. These singularities would serve as ultimate sources of classical em fields. Various currents would consist of wormhole throat pairs representing elementary particle and carrying charges at the partonic orbits. Two-sheetedness is essential and could be interpreted in terms of a double covering formed by space-time sheet glued along their common boundary. This necessary since space-time sheet has a finite size being not continuable beyond certain minimal size as preferred extremal since some of the real coordinates would become complex.

Quantum criticality for CP_2 type extremals

TGD predicts a hierarchy of quantum criticalities. The increase in criticality means that some space-time sheets for space-time surface regarded as a covering with sheets related by Galois group of extension of rationals degenerate to single sheet. The action of Galois group would reduce to that for its subgroup.

This is analogous to the degeneration of some roots of polynomial to single root and in M^8 representation space-time sheets are indeed quite concretely roots of octonionic polymomial defined by vanishing of real or imaginary part in the decomposition $o = q_1 + iq_2$ of octonion to a sum quaternionic real and imaginary parts.

The hierarchy of criticalities is closely related to the hierarchy of Planck constants $h_{eff}/h_0 = n = n_1n_2$, where n_1 corresponds to number of sheets as covering over CP_2 and n_2 as covering over M^4. One can also consider special cases in which M^4 projection has dimension $D < 4$. The proposal is that n corresponds to the dimension of Galois group for extension of rationals defining the level of dark matter hierarchy. If n is prime, one has either $n_1 = 1$ or $n_2 = 1$.

It seems that the range of n_2 is rather limited since the expression for Newton's constant as $G = R^2/n_2\hbar$ varies in rather narrow range. If the covering has symmetries assignable to some discrete subgroup of $SU(3)$ acting as isometries of CP_2 this could be understood. The increase of criticality could mean that n_1 or n_2 or both are reduced.

What is the position of CP_2 type extremals in the hierarchies of Planck constants and quantum criticalities?

1. Consider first n_2. CP_2 type extremal have 1-D geodesic line as M^4 projection. The light-like geodesic as 1-D structure could be interpreted as covering for which two geodesic lines along the orbits of opposite throats of wormhole contact form a kind of time loop. In this case one would have $n_2 = 2$ and one could have $n = 2p$, p prime.

   In this sense CP_2 type extremal or at least its core would be maximally critical. Deformations replacing the light-like geodesic as projection with higher-D region of M^4 presumably reduce criticality and one has $n_2 > 2$ is obtained. Whether this is possible inside wormhole contact is not clear. One can imagine that as one approaches partonic 2-surface, the criticality and degeneration increase in CP_2 degrees of freedom step by step and reach maximum in its core. This would be like realization of Thom's catastrophe involving parts with various degrees of criticalities.

   At the flux tubes mediating gravitational interaction $n_2 \sim 10^7$ would hold true in the exterior of associated CP_2 type extremals. This would suggests that CP_2 type extremals have maximal criticality in M^4 degrees of freedom and M^4 covering reduces to 2-fold covering for wormhole contacts.

2. What about criticality as n_1-fold covering of CP_2. This covering corresponds to a situation in which CP_2 coordinates as field in M^4 have given values of CP_2 coordinates n_1 times. A lattice like structure formed by n_1 wormhole contacts is suggestive. n_1 can be arbitrary large in principle and the gravitational Planck constant $h_{gr}/h_0 = n_1n_2$ would correspond to this situation. Singularities would now correspond to a degeneration of some wormhole contacts to single wormhole contact and could have interpretation in terms of fusion of particles to single particle. One might perhaps interpret elementary particle reaction vertices as catastrophes.

Wormhole contacts can be regarded as CP_2 type extremals having two holes corresponding to the 3-D orbits of wormhole contacts. Mathematician would probably speak of a blow up. CP_2 type extremals is glued to surrounding Minkowskian space-time sheets at the 3-D boundaries of these holes. At the orbit of partonic 2-surface the induced 4-metric degenerates to 3-D metric and 4-D tangent space becomes metrically 3-D. Light-likeness of the M^4 projection would correspond to this. For CP_2 type extremal 3 space-like M^4 directions of Minkowskian region would transmute to CP_2 directions at the light-like geodesic and time direction would become light-like. This is like graph of function for which tangent becomes vertical. For deformations of CP_2 type extremals this process could take place in several steps, one dimension in given step. This process could take place inside CP_2 or outside it depending on which order the transmutation of dimensions takes place.

3.7.3 Delicacies associated with M^4 Kähler structure

Twistor lift forces to assume that also M^4 possesses the analog of Kähler form, and Minkowskian signature does not prevent this [K19]. M^4 Kähler structure breaks CP symmetry and provides a very attractive manner to break CP symmetry and explain generation of matter antimatter symmetry and CP breaking in hadron physics. The CP breaking is very small characterized by a dimensionless number of order 10^{-9} identifiable as photon/baryon ratio. Can one understand the smallness of CP breaking in TGD framework?

Hamilton-Jacobi structure

Hamilton-Jacobi structure [L164] can be seen as a generalization of complex structure and involves a local but integrable selection of subspaces of various dimension for the tangent space of M^4. Integrability means that the selected subspaces are tangent spaces of a sub-manifold of M^4. M^8-H duality allows to interpret this selection as being induced by a global selection of a hierarchy of real, complex, and quaternionic subspaces associated with octonionic structure mapped to M^4 in such a way that this global selection becomes local at the level of H.

1. The 4-D analog of conformal invariance is due to very special conformal properties of light-like 3-surfaces and light-cone boundary of M^4. This raises hopes about construction of general solution families by utilizing the generalized form of conformal invariance. Massless extremals (MEs) in fact define extremely general solution family of this kind and involve light-like direction vector k and polarization vector ϵ orthogonal to it defining decomposition $M^4 = M^2 \times E^2$. I have proposed that this decomposition generalizes to local but integrable decomposition so that the distributions for M^2 and E^2 integrate to string world sheets and partonic 2-surfaces.
2. One can have decomposition $M^4 = M^2 \times E^2$ such that one has Minkowskian analog of conformal symmetry in M^2. This decomposition is defined by the vectors k and ϵ. An unproven conjecture is that these vectors can depend on point and the proposed Hamilton-Jacobi structure would mean a *local* decomposition of tangent space of M^4 , which is integrable meaning that local M^2s integrate to string world sheet in M^4 and local E^2s integrate to closed 2-surface as special case corresponds to partonic 2-surface. Generalizing the terminology, one could talk about family of partonic surfaces. These decompositions could define families of exremals.

   An integrable decomposition of M^4 to string world sheets and partonic 2-surfaces would characterize the preferred extremals with 4-D M^4 projection. Integrable distribution would mean assignment of partonic 2-surface to each point of string world sheet and vice versa.
3. M^4 Kähler form defines unique decomposition $M^2 \times E^2$. This is however not consistent Lorentz invariance. To cure this problem one must allow moduli space for M^4 Kähler forms such that one can assign to each Hamilton-Jacobi structure M^4 Kähler form defining the corresponding integrable surfaces in terms of light-like vector and polarization vector whose directions depend on point of M^4.

   This looks strange since the very idea is that the embedding space if unique. However, this local decomposition could be secondary being associated only with $H = M^4 \times CP_2$ and emerge in $M^8 - H$ duality mapping of space-time surfaces $X^4 \subset M^8$ to surfaces in $M^4 \times CP_2$. There is a moduli space for octonion structures in M^8 defined as a choice of preferred time axis M^1 (rest system), preferred M^2 defining hypercomplex place and preferred direction (light-like vector), and quaternionic plane $M^2 \times E^2$ (also polarization direction is included). Lorentz boosts mixing the real and imaginary octonion coordinates and changing the direction of time axis give rise to octonion structures not equivalent with the original one.

   Thus the choice $M^1 \subset M^2 \subset M^4 = M^2 \times E^2 \subset M^8$ is involved with the definition of octonion structure and quaternionion structure. The image of this decomposition under M^8-H duality mapping quaternionic tangent space of $X^4 \subset M^8$ containing M^1 and M^2 as sub-spaces would be such that the image of $M^1 \subset M^2 \subset M^2 \times E^2$ depends on point of $M^4 \subset H$ in integrable manner so that Hamilton-Jacobi structure in H is obtained.

Also CP_2 allows the analog of Hamilton-Jacobi structure as a local decomposition integrating to a family of geodesic spheres S^2_I as analog of partonic 2-surfaces with complex structure and having at each point as a fiber different S^2_I - these spheres necessary intersect at single point. This decomposition could correspond to the 4-D complex structure of CP_2 and complex coordinates of CP_2 would serve as coordinates for the two geodesic spheres.

Could one imagine decompositions in which fiber is 2-D Lagrangian manifold - say S^2_{II} - with vanishing induced Kähler form and not possessing induced complex structure? S^2_{II} does not have complex structure as induced complex structure and is therefore analogous to M^2. S^2_{II} coordinates would be functions of string world sheet coordinates (in special as analytic in hypercomplex sense and describing wave propagating with light-velocity). S^2_I coordinates would be analytic functions of complex coordinates of partonic 2-surface.

CP breaking and M^4 Kähler structure

The CP breaking induce by M^4 Kähler structure should be small. Is this automatically true or must one make some assumptions to achieve this.

Could one guarantee this by brute force by assuming M^4 and CP_2 parts of Kähler action to have different normalizations. The proposal for the length scale evolution of cosmological constant however relies on almost cancellation M^4 induced Kähler forms of M^4 and CP_2 parts due to the fact that the induced forms differ from each other by a rotation of the twistor sphere S^2. The S^2 part $M^4 \times S^2$ Kähler for can have opposite with respect to $T(CP_2) = SU(3)/U(1) \times U(1)$ Kähler so that for trivial rotation the forms cancel completely. If the normalizations of Kähler actions differ this cannot happen at the level of 4-D Kähler action.

To make progress, it is useful to look at the situation more concretely.

1. Kähler action is dimensionless. The square of Kähler form is metric so that $J_{kl}J^{kl}$ is dimensionless. One must include to the 4-D Kähler action a dimensional factor $1/L^4$ to make it dimensionless. The natural choice for L is as the radius R of CP_2 geodesic sphere to radius of twistor spheres for M^4 and CP_2. Note however that there is numerical constant involved and if it is changed there must be a compensating change of Kähler coupling strength. Therefore M^4 contribution to action is proportional to the volume of M^4 region using R^4 as unit. This contribution is very large for macroscopic regions of M^4 unless self-duality of M^4 Kähler form would not cause cancellation ($E^2 - B^2 = 0$).

2. What about energy density? The naïve expectation based on Maxwell's theory is that the energy density assignable to M^4 Kähler form is by self-duality proportional to $E^2 + B^2 = 2E^2$ and non-vanishing. By naïve order of magnitude estimate using Maxwellian formula for the energy of this kind extremal is proportional to Vol_3/R^4 and very large. Does this exclude these extremals or should one assume that they have very small volume? For macroscopic lengths of one should assume extremely thin MEs with thickness smaller than R. Could one have 2-fold covering formed by gluing to copies of very thin MEs together along their boundaries. This does not look feasible.

   Luckily, the Maxwellian intuition fails in TGD framework. The Noether currents associated in presence of M^4 Kähler action involve also a term coming from the variation of the induced M^4 Kähler form. This term guarantees that canonical momentum currents as H-vector fields are orthogonal to the space-time surface. In the case of CP_2 type extremals this causes the cancellation of the canonical momentum currents associated with Kähler action and corresponding contributions to conserved charges. The complete symmetry between M^4 and CP_2 and also physical intuition demanding that canonically imbedded M^4 os vacuum require that cancellation takes place also for M^4 part so that only the term corresponding to cosmological constant remains.

M^4 Kähler form and CP breaking for various kinds of extremals

I have considered already earlier the proposal that CP breaking is due to M^4 Kähler form [K19]. CP breaking is very small and the proposal inspired by the Cartesian product structure of the embedding space and its twistor bundle and also by the similar decomposition of $T(M^4) = M^4 \times S^2$ was that the coefficient of M^4 part of Kähler action can be chosen to be much smaller than the coefficient of CP_2 part. The proposed mechanism giving rise to p-adic length scale evolution of cosmological constant however requires that the coefficients of are identical. Luckily, the CP breaking term is automatically very small as the following arguments based on the examination of various kinds of extremals demonstrate.

1. For CP_2 type extremals with light-like M^4 geodesics as M^4 projection the induced M^4 Kähler form vanishes so that there is no CP breaking. For small deformations CP_2 type extremals thickening the M^4 projection the induced M^4 Kähler form is non-vanishing. An attractive hypothesis is that the small CP breaking parameter quantifies the order of magnitude of the induced M^4 Kähler form. This picture could allow to understand CP breaking of hadrons.
2. Canonically imbedded M^4 is a minimal surface. A small breaking of CP symmetry is generated in small deformations of M^4. In particular, for massless extremals (MEs) having 4-D M^4 projection the action associated with M^4 part of Kähler action vanishes at the M^4 limit when the local polarization vector characterizing ME approaches zero. The small CP breaking is characterized by the size of the polarization vector ϵ giving a contribution to the induced metric. This conforms with the perturbative CP breaking.
3. String like objects of type $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 is 2-surface in CP_2. The M^4 projection contains only electric part but no magnetic part. The M^4 part of action is proportional to the volume Y^2 and therefore very small. This in turn guarantees smallness of CP breaking effects.
   (a) If Y^2 is homologically non-trivial (magnetic flux tube carries monopole flux), CP_2 part of action is large since action density is proportional $1/\sqrt{det(g_2)}$ for Y^2 and therefore large. The thickening of the flux tube however reduces the value of the action by flux conservation as discussed already earlier.
   M^4 and CP_2 contributions to the actions are of opposite sign but M^4 contribution os however very small as compared to CP_2 contribution. One can look the situation in $M^2 \times S^2$ coordinates. The transverse deformation would correspond to the dependence of E^2 coordinates on S^2 coordinates. The induced Kähler form would give a contribution to the S^2 part of induced Kähler form whose size would characterize CP breaking.
   (b) Y^2 can be also homologically trivial. In particular, for $Y^2 = S^2_{II}$ the CP_2 contribution to the total Kähler action vanishes and only the small M^4 contribution proportional to the area of Y^2 remains.

3.7.4 About TGD counterparts for the simplest classical field patterns

What could be the TGD counterparts of typical configurations of classical fields? Since minimal surface equation is a nonlinear generalization of massless field equations, one can hope that the simplest solutions of Maxwell's equations have TGD analogs. The strong non-linearity poses a strong constraint, which can be solved if the extremal allows generalization of holomorphic structure so that field equations are trivially true since they involve in complex coordinates a contraction of tensors of type (1,1) with tensors of type (2,0) or (0,2). It is not clear whether minimal surface property reducing to holomorphy is equivalent with preferred extremal property.

Can one have the basic field patterns such as multipoles as structures with 4-D M^4 projection or could it be that flux tube picture based on spherical harmonics for the orientation of flux tube is all that one can have? Same question can be made for radiation fields having MEs as archetypal representatives in TGD framework. What about the possible consistency problems produced by M^4 Kähler form breaking Lorentz invariance?

I have considered these questions already earlier. The following approach is just making questions and guesses possibly helping to develop general ideas about the correspondence.

1. In QFT approach one expresses fields as superpositions of partial waves, which are indeed very simple field patterns and the coefficients in the superposition become oscillator operators. What could be the analogs of partial waves in TGD? Simultaneous extremals of Kähler action and volume strongly suggest themselves as carriers of field archetypes but the non-linearity of field equations does not support the idea that partial waves could be realized at classical level as extremals with 4-D M^4 projection. A more plausible option is that they correspond

to spherical harmonics for the orientation of flux tube carrying say electric flux. Could the flux tubes of various kinds serve as building of all classical fields?

2. String-like objects $X^2 \times Y^2 \subset M^4 \times CP_2$, where string world sheet X^2 is minimal surface and Y^2 is sub-manifold of CP_2 and their deformations in M^4 degrees of freedom transversal to X^2 and depending on the coordinates Y^2 are certainly good candidates for archetypal field configurations.

   Y^2 can be homologically trivial and could correspond to Lagrangian sub-manifold. Y^2 can also carry homology charge n identifiable as Kähler magnetic charge and correspond to complex sub-manifold of CP_2 with complex structure induced from that of CP_2.

The simplest option corresponds to geodesic sphere $Y^2 = S^2$. There are two geodesic spheres in CP_2 and they correspond to simplest string like objects.

1. S^2_I has Kähler magnetic charge of one unit and the cosmic and its deformations carry monopole flux. These field configurations are not possible in Maxwell's electrodynamics and the proposal is that they appear in all length scales. The model for the formation of galaxies solving also the problem of galactic dark matter relies on long cosmic strings. They are proposed to appear also in biology.

2. S^2_{II} is homologically trivial so that magnetic flux over it vanishes although magnetic field is non-vanishing. Note that although the Kähler magnetic field is vanishing, the electromagnetic ordinary magnetic field is non-vanishing because em field is a combination of Kähler form and component of CP_2 curvature form with vanishing weak isospin. The total flux of ordinary magnetic field over S^2_{II} vanishes whereas electric flux can be non-vanishing.

Coulomb fields

By the vanishing of magnetic flux flux tubes for S^2_{II} cannot represent ordinary magnetic field. They can however serve as radial flux tubes carrying electromagnetic flux. Magnetic flux tubes indeed allow time dependent deformations for which the phase angles of CP_2 coordinates depend linearly of M^4 time coordinate. This would give rise to an archetypal flux tube representation of the electric field created by point charge. Also gravitational flux tubes should correspond to this kind flux tubes emanating radially from the source.

Charge quantization suggests that these flux tubes carry unit charge. In the case of charged elementary particle there would be only single flux tubes but there would be wave function for its orientation having no angular dependence. In principle, this wave function can any spherical harmonic.

Does the orientation angle dependence of flux distribution have any counterpart in Maxwell's theory. One would have the analog of $1/r$ Coulomb potential with the modulus squared of spherical harmonic Y_{lm} modulating it. Could one consider the possibility that in atoms the spherical harmonics for excited states correspond to this kind of distribution for the electric flux coming from nucleus. The probability amplitude for electrons touching the flux tube would inherit this distribution.

For many particle system with large em charge there would be large number of radial flux tubes and the approximation of electric field with Coulomb field becomes natural. In the case of atoms this limit is achieved for large enough nuclear charges. This does not exclude the possibility of having space-time surfaces carrying Coulomb potential in Maxwellian sense: in this case however the field equations cannot solved by holomorphy and quantum criticality might exclude these configurations.

What about gravitation? The notion of gravitational Planck constant requires that Planck mass replaced in TGD framework by CP_2 mass defining the unit of gravitational flux - $h_{gr}0GMm/v_0$ cannot be smaller than h_0. What happens in systems possessing mass smaller than CP_2 mass? Are gravitational flux tubes absent. Is gravitational interaction absent in this kind of systems or is its description analogous to string model description meaning that $h_{gr} = h_0$ for masses smaller than CP_2 mass?

Magnetic fields

As such S^2_{II} flux tubes cannot serve as counterparts of ordinary magnetic fields. The flux tubes have now boundary and the current at boundary creates the magnetic field inside the tube. This would mean cutting of a disk D^2 from S^2_{II} so that the net magnetic flux becomes non-vanishing.

The assumption has been that genuine boundaries are not possible since conservation laws very probably prevent them (the normal components of canonical momentum currents should vanish at boundaries but this is not possible). This requires that this flux tube must be glued along the boundary of $D^2 \times D^1$ to surrounding space-time surface X^4, which has a similar hole. At the boundary of this hole the space-time surface must turn to the direction of CP_2 meaning that the dimension of M^4 projection is reduced to $D = 2$. Algebraic geometer would talk about blow-up.

Ordinary multipole magnetic field could correspond to spherical harmonic for the orientation of this kind flux tubes. They could also carry electric flux but the em charge could be fractionized. These flux tubes might relate to anyons carrying fractional em charge. Also the fractional charges of quarks could classically correspond to flux tubes mediating both color magnetic field and em flux. The spherical harmonic in question corresponds to that associated with electron in atoms.

Magnetic and electric fields associated with straight current wire

Magnetic and electric fields associated with straight current wire need not allow representation as archetypes since they are obviously macroscopic entities.

1. Is the magnetic field associated with straight current wire representable in terms of extremal with 4-D M^4 projection. The magnetic field lines rotate around the current and it is does not seem natural to model it the field in terms of flux tubes. Forget the presence of M^4 Kähler form. One can imbed this kind of magnetic field as a surface with 4-D M^4 projection and possessing cylindrical symmetry. Line current would correspond to a source of the magnetic field and could be realized as a flux tube carrying em current and topologically condensed to the space-time sheet in question.

   The embedding however fails at certain critical radius and the assumption is that no boundaries are allowed by conservation laws. Should one glue the structure to the surrounding space-time surface at this radius. In Maxwell's theory one would have surface current in direction opposite to the source cancelling the magnetic field outside. Could this current have interpretation as a return current?

   One can also imagine glueing its copy to it along the boundary at critical radius. It would seem that the magnetic fields must have same direction at the boundary and therefore also in interior.

2. What about current ring? Separation of variables is essential for the simplest embeddings implying a reduction of partial different equations to differential equation. There is rather small number of coordinates system in E^3 in which Laplacian allows separation of variables. The metric is diagonal in these coordinates. One example is toroidal coordinates assignable with a current ring having toroidal geometry. This would allow a construction of minimal surface solution in some finite volume. Minimal surface property would *not* reduce to complex analyticity for these extremals and they would be naturally associated $M^4 \times S^2_{II}$.

Remark: This kind of extremals are not holomorphic and could be excluded by quantum criticality and preferred extremal property. GRT space-time would be idealization making sense only at the QFT limit of TGD.

Time dependent fields

What about time dependent fields such as the field created by oscillating dipole and radiation fields? One can imagine quantal and classical option.

1. The simplest possibility is reduction to quantum description at single particle level. The dipole current corresponds to a wave function for the source particle system consisting of systems with opposite total charge.

Spherical harmonics representing multipoles would induce wave function for the orientations of MEs (topological light ray) carrying radial wave. This is certainly the most natural options as far radiation field at large distances from sources is considered. One can also have second quantization in the proposed sense giving rise to multi-photon states and one can also define coherent states.

One should also understand time dependent fields near sources having also non-radiative part. This requires a model for source such as oscillating dipole. The simplest possibility is that in the case of dipole there are charges of opposite sign with oscillating distance creating Coulomb fields represented in the proposed manner. It is however not obvious that preferred extremals of this kind exist.

2. One can consider also classical description. The model of elementary particle as consisting of two wormhole contacts, whose throats effectively serve as end of monopole flux tubes at the two sheets involved suggests a possible model. If the wormhole contacts carry opposite em charges realized in terms of fermion and antifermions an oscillating dipole could correspond to flux tube whose length oscillates. This means generation of radiation and for elementary particles this would suggest instability against decay. One can however consider excitation which decay to ground states - say for hadrons. For scaled up variants of this structure this would not mean instability although energy is lost and the system must end up to non-oscillating state.

   One possibility is that there are two charges at different space-time sheets connected by wormhole contacts and oscillating by their mutual interaction in harmonic oscillator state. Ground state would be stable and have not dipole moment.

Effectively 2-D systems

In classical electrodynamics effectively 2-D systems are very special in that they allow conformal invariance assignable to 2-D Laplacian.

1. Since minimal surface equation is generalization of massless d'Alambertian and since field equations are trivially true for analytic solutions, one can hope that the basic solutions of 4-D d'Alembertian generalize in TGD framework. This would conform with the universality of quantum criticality meaning that coupling parameters disappear from field equations. Conformal invariance or its generalization would mean huge variety of field patterns. This suggests that effectively 2-D systems serve as basic building bricks of more complex field configurations. Flux tubes of various kinds would represent basic examples of this kind of surfaces. Also the magnetic end electric fields associated with straight current wire would serve as an example.

2. Are there preferred extremals analogous to the solutions of field equations of general relativity in faraway regions, where they become simple and might allow an analog in TGD framework? If our mathematical models reflect the preferred extremals as archetypal structures, this could be the case.

   Forget for a moment the technicalities related to M^4 Kähler form. One can construct a spherically symmetric ansatz in $M^4 \times S^2_{II}$ as a minimal surface for which Φ depends linearly on time t and u is function of r. The ansatz reduces to a highly non-linear differential equation for u. In this case hyper-complex analyticity is obviously not satisfied. This ansatz could give the analog of Schwartschild metric giving also the electric field of point charge appearing as source of the non-linear variant of d'Alembertian. It is however far from clear whether this kind extremals is allowed as preferred extremals.

   Under which conditions spherically symmetric ansatz is consistent with M^4 Kähler form? Obviously, the M^4 Kähler form must be spherically symmetric as also the Hamilton-Jacobi structure it. Suppose local Hamilton-Jacobi structures for which M^2s integrate to t, r coordinate planes and E^2s integrate to (θ, ϕ) sphere are allowed and that M^4 Kähler form defines this decomposition. In this case there are hopes that consistency conditions can be satisfied. Note however that M^4 Kähler form defines in this case orthogonal magnetic and electric monopole fields defining an analog of instanton. Can one really allow this or should one exclude the time line with $r = 0$?

Similar M^4 Kähler structure can be associated with cylindrical coordinates and other separable coordinates system. M^4 Kähler structure would define Hamilton-Jacobi structure.

3.8 Minimal surfaces and TGD

The twistor lift of TGD [K116, L55, L72] meant a revolution in the understanding of TGD and led to a new view about what preferred extremal property means physically and why it is needed.

1. The construction of twistor lift of TGD replaces space-time surfaces with 6-D surfaces but requires that they are dynamically effectively 4-D as the analogs of twistor space having the structure of S^2 bundle with space-time surface as the base. This requires dimensional reduction making S^2 fiber of the twistor space non-dynamical.

   One can say that twistor structure is induced from that for 12-D product of the geometric 6-D twistor spaces of M^4 and CP_2. The condition that 6-D Kähler action exists requires that the twistor spaces of M^4 and CP_2 have Kähler structure. This condition allows only $H = M^4 \times CP_2$ [A63]. The condition that one obtains standard model symmetries leads to the same conclusion.

2. The dimensionally reduced Kähler action decomposes to a sum of 4-D Kähler action and volume term. The interaction is as analog of Maxwell action plus action of point-like particle replaced with 3-D surface. The coefficient of the volume term has an interpretation as cosmological constant having a discrete spectrum [L79]. The natural proposal it that it depends on p-adic length scale approaching zero in long length scales. This solves the cosmological constant problem.

3. I had actually known for decades that all non-vacuum extremals of 4-D Kähler action are minimal surfaces thus minimizing the space-time volume in the induced metric. This is because the field equations for Kähler action for known non-vacuum extremals were reduced essentially to algebraic conditions realizing holomorphy. Also so called CP_2 type vacuum extremals of 4-D Kähler action are minimal surfaces. This finding conforms with the fact that in $M^8 - H$ duality [L48] one has regard field equations as purely algebraic conditions at M^8 side of the duality.

   This inspired the proposal that preferred extremal property of space-time surface is realized by requiring that space-time surfaces as base spaces of these 6-D twistor spaces are quite generally minimal surfaces, and therefore represent a non-linear geometrization for the notion of massless field in accordance with conformal invariance forced by quantum criticality.

   Also a more general proposal that space-time contains regions inside which there is an exchange of canonical momenta between Kähler action and volume term was considered. Minimal surface regions would correspond to incoming particles and non-minimal ones to interaction regions.

   Later this proposal was simplified by requiring that interaction regions are 2-D string world sheets as singularities: this implied that string world sheets required by general considerations [K127] indeed emerge from 4-D action. This could happen also at the 1-D boundaries of string world sheets at 3-D light-like boundaries between Minkowskian and Euclidian regions behaving like ordinary point-like particles and carrying fermion number, and in the most general case also at these 3-D light-like 3-surfaces.

3.8.1 Space-time surfaces as singular minimal surfaces

From the physics point this is not surprising since minimal surface equations are the geometric analog for massless field equations.

1. The boundary value problem in TGD is analogous to that defining soap films spanned by frames: space-time surface is thus like a 4-D soap film. Space-time surface has 3-D ends at the opposite boundaries of causal diamond of M^4 with points replaced with CP_2: I call this 8-D object just causal diamond (CD). Geometrically CD brings in mind big-bang followed by big crunch.

These 3-D ends are like the frame of a soap film. This and the Minkowskian signature guarantees the existence of minimal surface extremals. Otherwise one would expect that the non-compactness does not allow minimal surfaces as non-self-intersecting surfaces.

2. Space-time is a 4-surface in 8-D $H = M^4 \times CP_2$ and is a minimal surface, which can have 2-D or 1-D singularities identifiable as string world sheets having 1-D singularities as light-like orbits - they could be geodesics of space-time surface.

   Remark: I considered in [L66] the possibility that the minimal surface property could fail only at the reaction vertices associated with partonic 2-surfaces defining the ends of string world sheet boundaries. This condition however seems to be too strong. It is essential that the singular surface defines a sub-manifold giving deltafunction like contribution to the action density and that one can assign conserved quantities to this surface. This requires that the singular contributions to energy momentum tensor and canonical momentum currents as spacetime vectors are parallel to the singular surface. Singular points do not satisfy this condition.

   String boundaries represent orbits of fundamental point-like fermions located at 3-D light-like surfaces which represent orbits of partonic 2-surfaces. String world sheets are minimal surfaces and correspond to stringy objects associated with say hadrons. There are also degrees of freedom associated with space-time interior. One have objects of various dimension which all are minimal surfaces. Modified Dirac equation extends the field equations to supersymmetric system and assigns fermionic degrees of freedom to these minimal surfaces of varying dimension.

   From the physics point of view, the singular surfaces are analogous to carriers of currents acting as point- and string-like sources of massless field equations.

3. Geometrically string world sheets are analogous to folds of paper sheet. Space-time surfaces are extremals of an action which is sum of volume term having interpretation in terms of cosmological constant and what I call Kähler action - analogous to Maxwell action. Outside singularities one has minimal surfaces stationary with respect to variations of both volume term and Kähler action - note the analogy with free massless field. At singularities there is an exchange of conserved quantities between volume and Kähler degrees of freedom analogous to the interaction of charged particle with electromagnetic field. One can see TGD as a generalization of a dynamics of point-like particle coupled to Maxwell field by making particle 3-D surface.

4. The condition that the exchange of conserved charges such as four-momentum is restricted to lower-D surfaces realizes preferred extremal property as a consequence of quantum criticality demanding a universal dynamics independent of coupling parameters [L79]. Indeed, outside the singularities the minimal surfaces dynamics has no explicit dependence on coupling constants provided local minimal surface property guarantees also the local stationarity of Kähler action.

   Preferred extremal property has also other formulations. What is essential is the generalization of super-conformal symmetry playing key role in super string models and in the theory of 2-D critical systems so that field equations reduce to purely algebraic conditions just like for analytic functions in 2-D space providing solutions of Laplace equations.

5. TGD provides a large number of specific examples about closed minimal surfaces [K9]. Cosmic strings are objects, which are Cartesian products of minimal surfaces (string world sheets) in M^4 and of complex algebraic curves (2-D surfaces). Both are minimal surfaces and extremize also Kähler action. These algebraic surfaces are non-contractible and characterized by homology charge having interpretation as Kähler magnetic charge. These surfaces are genuine minima just like the geodesics at torus.

   CP_2 contains two kinds of geodesic spheres, which are trivially minimal surfaces. The reason is that the second fundamental form defining as its trace the analogs of external curvatures in the normal space of the surfaces vanishes identically. The geodesic sphere of the first kind is non-contractible minimal surface and absolute minimum. Geodesic spheres of second kind is contractible and one has Minimax type situation.

   These geodesic spheres are analogous to 2-planes in flat 3-space with vanishing external curvatures. For a generic minimal surface in 3-space the principal curvatures are non-vanishing and sum up to zero. This implies that minimal surfaces look locally like saddles. For 2-plane the curvatures vanish identically so that saddle is not formed.

3.8.2 Kähler action as Morse function in the space of minimal 4-surfaces

It was found that surface volume could define a Morse function in the space of surfaces. What about the situation in TGD, where volume is replaced with action which is sum of volume term and Kähler action [L72, L70, L79]?

Morse function interpretation could appear in two ways. The first possibility is that the action defines an analog of Morse function in the space of 4-surfaces connecting given 3-surfaces at the boundaries of CD. Could it be that there is large number of preferred extremals connecting given 3-surfaces at the boundaries of CD? This would serve as analogy for the existence of infinite number of closed surfaces in the case of compact embedding space. The fact that preferred extremals extremize almost everywhere two different actions suggests that this is not the case but one must consider also this option.

1. The simplest realization of general coordinate invariance would allow only single preferred extremal but I have considered also the option for which one has several preferred extremals. In this case one encounters problem with the definition of Kähler function which would become many-valued unless one is ready to replace 3-surfaces with its covering so that each preferred extremal associated with the given 3-surface gives rise to its own 3-surface in the covering space. Note that analogy with the definition of covering space of say circle by replacing points with the set of homologically equivalence classes of closed paths at given point (rotating arbitrary number of times around circle).
2. Number theoretic vision [K125, K46] suggests that these possibly existing different preferred extremals are analogous to same algebraic computation but performed in different ways or theorem proved in different ways. There is always the shortest manner to do the computation and an attractive idea is that the physical predictions of TGD do not depend on what preferred extremal is chosen.
3. An interesting question is whether the "drum theorem" could generalize to TGD framework. If there exists infinite series of preferred extremals which are singular minimal surfaces, the volume of space-time surface for surfaces in the series would depend only on the volume of the CD containing it. The analogy with the high frequencies and drum suggests that the surfaces in the series have more and more local details. In number theoretic vision this would correspond to emergence of more and more un-necessary pieces to the computation. One cannot exclude the possibility that these details are analogs for what is called loop corrections in quantum field theory.
4. If the action defines Morse action, the preferred extremals give information about its topology. Note however that the requirement that one has extremum of both volume term and Kähler action almost everywhere is an extremely strong additional condition and corresponds physically to quantum criticality.

   Remark: The original assumption was that the space-time surface decomposes to critical regions which are minimal surfaces locally and to non-critical regions inside which there is flow of canonical momentum currents between volume and Kähler degrees of freedom. The stronger hypothesis is that this flow occurs at 2-D and 1-D surfaces only.

3.8.3 Kähler function as Morse function in the space of 3-surfaces

The notion of Morse function can make sense also in the space of 3-surfaces - the world of classical worlds which in zero energy ontology consists of pairs of 3-surfaces at opposite boundaries of CD connected by preferred extremal of Kähler action [K29, K96, L72, L70]. Kähler action for the preferred extremal is assumed to define Kähler function defining Kähler metric of WCW via its second derivates $\partial_K \partial_{\overline{L}} K$. Could Kähler function define a Morse function?

1. First of all, Morse function must be a genuine function. For general Kähler metric this is not the case. Rather, Kähler function K is a section in a $U(1)$ bundle consisting of patches transforming by real part of a complex gradient as one moves between the patches of the

bundle. A good example is CP_2, which has non-trivial topology, and which decomposes to 3 coordinate patches such that Kähler functions in overlapping patches are related bythe analog of U(1) gauge transformation.

Kähler action for preferred extremal associated with given 3-surface is however uniquely defined unless one includes Chern-Simons term which changes in U(1) gauge transformation for Kähler gauge potential of CP_2.

2. What could one conclude about the topology of WCW if the action for preferred extremal defines a Morse function as a functional of 3-surface? This function cannot have saddle points: in a region of WCW around saddle point the WCW metric depending on the second derivatives of Morse function would not be positive definite, and this is excluded by the positivity of Hilbert space inner product defined by the Kähler metric essential for the unitarity of the theory. This would suggest that the space of 3-surfaces has very simple topology if Kähler function.

   This is too hasty conclusion! WCW metric is expected to depend also on zero modes, which do not contribute to the WCW line element. What suggests itself is bundle structure. Zero modes define the base space and dynamical degrees of freedom contributing to WCW line element as fiber. The space of zero modes can be topologically complex.

There is a fascinating open problem related to the metric of WCW.

1. The conjecture is that WCW metric possess the symplectic symmetries of $\Delta M^4_+ \times CP_2$ as isometries. In infinite dimensional case the existence of Riemann/Kähler geometry is not at all obvious as the work of Dan Freed demonstrated in the case of loops spaces [A44], and the maximal group of isometries would guarantee the existence of WCW Kähler geometry. Geometry would be determined by symmetries alone and all points of the space would be metrically equivalent. WCW would be an infinite-dimensional analog of symmetric space.

2. Isometry group property does not require that symplectic symmetries leave Kähler action, and even less volume term for preferred extremal, invariant. Just the opposite: if the action would remain invariant, Kähler function and Kähler metric would be trivial!

3. The condition for the existence of symplectic isometries must fix the ratio of the coefficients of Kähler action and volume term highly uniquely. The physical interpretation is in terms of quantum criticality realized mathematically in terms of the symplectic symmetry serving as analog of ordinary conformal symmetry characterizing 2-D critical systems. Note that at classical level quantum criticality realized as minimal surface property says nothing outside singular surfaces since the field equations in this regions are algebraic. At singularities the situation changes. Note also that the minimal surface property is a geometric analog of masslessness which in turn is a correlate of criticality.

4. Twistor lift of TGD [?]eads to a proposal for the spectra of Kähler coupling strength and cosmological constant allowed by quantum criticality [L70]. What is surprising that cosmological constant identified as the coefficient of the volume term takes the role of cutoff mass in coupling constant evolution in TGD framework. Coupling constant evolution discretizes in accordance with quantum criticality which must give rise to infinite-D group of WCW isometries. There is also a connection with number theoretic vision in which coupling constant evolution has interpretation in terms of extensions of rationals [K125, L53, L48].

3.8.4 Kähler calibrations: an idea before its time?

While updating book introductions I was surprised to find that I had talked about so called calibrations of sub-manifolds as something potentially important for TGD and later forgotten the whole idea! A closer examination however demonstrated that I had ended up with the analog of this notion completely independently later as the idea that preferred extremals are minimal surfaces apart form 2-D singular surfaces, where there would be exchange of Noether charges between Kähler and volume degrees of freedom.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann

manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

Calibrated k-manifolds are minimal surfaces in their homology class, in other words their volume is minimal. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces and representing fundamental fermions play a key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality outside singular surfaces would be realized as decoupling of the two parts of the action. May be all preferred extremals be regarded as calibrated in generalized sense.

   If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

   It is interesting to recall that the original proposal for the preferred extremals as absolute minima of Kähler action has transformed during years to a proposal that they are absolute minima of volume action within given homology class and having fixed ends at the boundaries of CD.

4. The experience with CP_2 would suggest that the Kähler structure of M^4 defining the counterpart of form ϕ is unique. There is however infinite number of different closed self-dual Kähler forms of M^4 defining what I have called Hamilton-Jacobi structures. These forms can have subgroups of Poincare group as symmetries. For instance, magnetic flux tubes correspond to given cylindrically symmetry Kähler form. The problem disappears as one realizes that Kähler structures characterize families of preferred extremals rather than M^4 itself.

If the notion of calibration indeed generalizes, one ends up with the same outcome - preferred extremals as minimal surfaces with 2-D string world sheets and partonic 2-surfaces as singularities - from many different directions.

1. Quantum criticality requires that dynamics does not depend on coupling parameters so that extremals must be separately extremals of both volume term and Kähler action and therefore minimal surfaces for which these degrees of freedom decouple except at singular 2-surfaces, where the necessary transfer of Noether charges between two degrees of freedom takes place at these. One ends up with string picture but strings alone are of course not enough. For instance, the dynamical string tension is determined by the dynamics for the twistor lift.

2. Almost topological QFT picture implies the same outcome: topological QFT property fails only at the string world sheets.

3. Discrete coupling constant evolution, vanishing of loop corrections, and number theoretical condition that scattering amplitudes make sense also in p-adic number fields, requires a representation of scattering amplitudes as sum over resonances realized in terms of string world sheets.

4. In the standard QFT picture about scattering incoming states are solutions of free massless field equations and interaction regions the fields have currents as sources. This picture is realized by the twistor lift of TGD in which the volume action corresponds to geodesic length and Kähler action to Maxwell action and coupling corresponds to a transfer of Noether charges between volume and Kähler degrees of freedom. Massless modes are represented by minimal surfaces arriving inside causal diamond (CD) and minimal surface property fails in the scattering region consisting of string world sheets.

5. Twistor lift forces M^4 to have generalize Kähler form and this in turn strongly suggests a generalization of the notion of calibration. At physics side the implication is the understanding of CP breaking and matter anti-matter asymmetry.

6. $M^8 - H$ duality requires that the dynamics of space-time surfaces in H is equivalent with the algebraic dynamics in M^8. The effective reduction to almost topological dynamics implied by the minimal surface property implies this. String world sheets (partonic 2-surfaces) in H would be images of complex (co-complex sub-manifolds) of $X^4 \subset M^8$ in H. This should allows to understand why the partial derivatives of embedding space coordinates can be discontinuous at these edges/folds but there is no flow between interior and singular surface implying that string world sheets are minimal surfaces (so that one has conformal invariance).

The analogy with foams in 3-D space deserves to be noticed.

1. Foams can be modelled as 2-D minimal surfaces with edges meeting at vertices. TGD space-time could be seen as a dynamically generated foam in 4-D many-sheeted space-time consisting of 2-D minimal surfaces such that also the 4-D complement is a minimal surface. The counterparts for vertices would be light-like curves at light like orbits of partonic 2-surfaces from which several string world sheets can emanate.

2. Can one imagine something more analogous to the usual 3-D foam? Could the light-like orbits of partonic 2-surfaces define an analog of ordinary foam? Could also partonic 2-surfaces have edges consisting of 2-D minimal surfaces joined along edges representing strings connecting fermions inside partonic 2-surface?

   For years ago I proposed what I called as symplectic QFT (SQFT) as an analog of conformal QFT and as part of quantum TGD [K22]. SQFT would have symplectic transformations as symmetries, and provide a description for the symplectic dynamics of partonic 2-surfaces. SQFT involves an analog of triangulation at partonic 2-surfaces and Kähler magnetic fluxes associated with them serve as observables. The problem was how to fix this kind of network. Partonic foam could serve as a concrete physical realization for the symplectic network and have fundamental fermions at vertices. The edges at partonic 2-surfaces would be space-like geodesics. The outcome would be a calibration involving objects of all dimensions $0 \leq D \leq 4$ - a physical analog of homology theory.

3.9 Are space-time boundaries possible in the TGD framework?

One of the key ideas of TGD from the very beginning was that the space-time surface has boundaries and we see them directly as boundaries of physical objects.

It however turned out that it is not at all clear whether the boundary conditions stating that no isometry currents flow out of the boundary, can be satisfied. Therefore the cautious conclusion was that perhaps the boundaries are only apparent. For instance, the space-time regions correspond to maps $M^4 \rightarrow CP_2$, which are many-valued and have as turning points, which have 3-D projections to M^4. The boundary surfaces between regions with Minkowskian and Euclidean signatures of the induced metric seem to be unavoidable, at least those assignable to deformations of CP_2 type extremals assignable to wormhole contacts.

There are good reasons to expect that the possible boundaries are light-like and possibly also satisfy the $det(g_4) = 0$ condition and I have considered the boundary conditions but have not been able to make definite conclusions about how they could be realized.

1. The action principle defining space-times as 4-surfaces in $H = M^4 \times CP_2$ as preferred extremals contains a 4-D volume term and the Kähler action plus possible boundary term if boundaries are possible at all. This action would give rise to a boundary term representing a normal flow of isometry currents through the boundary. These currents should vanish.
2. There could also be a 3-D boundary part in the action but if the boundary is light-like, it cannot depend on the induced metric. The Chern-Simons term for the Kähler action is the natural choice. Twistor lift suggests that it is present also in M^4 degrees of freedom. Topological field theories utilizing Chern-Simons type actions are standard in condensed matter physics, in particular in the description of anyonic systems, so that the proposal is not so radical as one might think. One might even argue that in anyonic systems, the fundamental dynamics of the space-time surface is not masked by the information loss caused by the approximations leading to the field theory limit of TGD.

   Boundary conditions would state that the normal components of the isometry currents are equal to the divergences of Chern-Simons currents and in this way guarantee conservation laws. In CP_2 degrees of freedom the conditions would be for color currents and in M^4 degrees of freedom for 4-momentum currents.
3. This picture would conform with the general view of TGD. In zero energy ontology (ZEO) [L91, L138] phase transitions would be induced by macroscopic quantum jumps at the level of the magnetic body (MB) of the system. In ZEO, they would have as geometric correlates classical deterministic time evolutions of space-time surface leading from the initial to the final state [L77]. The findings of Minev et al provide [L77] lend support for this picture.

3.9.1 Light-like 3-surfaces from $det(g_4) = 0$ condition

How the light-like 3- surfaces could be realized?

1. A very general condition considered already earlier is the condition $det(g_4) = 0$ at the light-like 4-surface. This condition means that the tangent space of X^4 becomes metrically 3-D and the tangent space of X^3 becomes metrically 2-D. In the local light-like coordinates, $(u, v, W, \overline{W})$ $guv = g_{vu}$) would vanish (g_{uu} and g_{vv} vanish by definition.

   Could $det(g_4) = 0$ and $det(g_3) = 0$ condition implied by it allow a universal solution of the boundary conditions? Could the vanishing of these dimensional quantities be enough for the extended conformal invariance?
2. 3-surfaces with $det(g_4) = 0$ could represent boundaries between space-time regions with Minkowskian and Euclidean signatures or genuine boundaries of Minkowskian regions.

   A highly attractive option is that what we identify the boundaries of physical objects are indeed genuine space-time boundaries so that we would directly see the space-time topology. This was the original vision. Later I became cautious with this interpretation since it seemed difficult to realize, or rather to understand, the boundary conditions.

   The proposal that the outer boundaries of different phases and even molecules make sense and correspond to 3-D membrane like entities [L146], served as a partial inspiration for this article but this proposal is not equivalent with the proposal that light-like boundaries defining genuine space-time boundaries can carry isometry charges and fermions.
3. How does this relate to $M^8 - H$ duality [L109, L110]? At the level of rational polynomials P determined 4-surfaces at the level of M^8 as their "roots" and the roots are mass shells. The points of M^4 have interpretation as momenta and would have values, which are algebraic integers in the extension of rationals defined by P.

   Nothing prevents from posing the additional condition that the region of $H^3 \subset M^4 \subset M^8$ is finite and has a boundary. For instance, fundamental regions of tessellations defining hyperbolic manifolds (one of them appears in the model of the genetic code [L133]) could be considered. $M^8 - H$ duality would give rise to holography associating to these 3-surfaces space-time surfaces in H as minimal surfaces with singularities as 4-D analogies to soap films with frames.

The generalization of the Fermi torus and its boundary (usually called Fermi sphere) as the counterpart of unit cell for a condensed matter cubic lattice to a fundamental region of a tessellation of hyperbolic space H^3 acting is discussed is discussed in [L148]. The number of tessellations is infinite and the properties of the hyperbolic manifolds of the "unit cells" are fascinating. For instance, their volumes define topological invariants and hyperbolic volumes for knot complements serve as knot invariants.

This picture resonates with an old guiding vision about TGD as an almost topological quantum field theory (QFT) [K57, K9, K129], which I have even regarded as a third strand in the 3-braid formed by the basic ideas of TGD based on geometry-number theory-topology trinity.

1. Kähler Chern-Simons form, also identifiable as a boundary term to which the instanton density of Kähler form reduces, defines an analog of topological QFT.
2. In the recent case the metric is however present via boundary conditions and in the dynamics in the interior of the space-time surface. However, the preferred extremal property essential for geometry-number theory duality transforms geometric invariants to topological invariants. Minimal surface property means that the dynamics of volume and Kähler action decouple outside the singularities, where minimal surface property fails. Coupling constants are present in the dynamics only at these lower-D singularities defining the analogs of frames of a 4-D soap film.

   Singularities also include string worlds sheets and partonic 2-surfaces. Partonic two-surfaces play the role of topological vertices and string world sheets couple partonic 2-orbits to a network. It is indeed known that the volume of a minimal surface can be regarded as a homological invariant.
3. If the 3-surfaces assignable to the mass shells H^3 define unit cells of hyperbolic tessellations and therefore hyperbolic manifolds, they also define topological invariants. Whether also string world sheets could define topological invariants is an interesting question.

3.9.2 Can one allow macroscopic Euclidean space-time regions

Euclidean space-time regions are not allowed in General Relativity. Can one allow them in TGD?

1. CP_2 extremals with a Euclidean induced metric and serving as correlates of elementary particles are basic pieces of TGD vision. The quantum numbers of fundamental fermions would reside at the light-like orbit of 2-D wormhole throat forming a boundary between Minkowskian space-time sheet and Euclidean wormhole contact- parton as I have called it. More precisely, fermionic quantum numbers would flow at the 1-D ends of 2-D string world sheets connecting the orbits of partonic 2-surfaces. The signature of the 4-metric would change at it.
2. It is difficult to invent any mathematical reason for excluding even macroscopic surfaces with Euclidean signature or even deformations of CP_2 type extremals with a macroscopic size. The simplest deformation of Minkowski space is to a flat Euclidean space as a warping of the canonical embedding $M^4 \subset M^4 \times S^1$ changing its signature.
3. I have wondered whether space-time sheets with an Euclidean signature could give rise to black-hole like entities. One possibility is that the TGD variants of blackhole-like objects have a space-time sheet which has, besides the counterpart of the ordinary horizon, an additional inner horizon at which the signature changes to the Euclidean one. This could take place already at Schwarzschild radius if g_{rr} component of the metric does not change its sign.

3.9.3 But are the normal components of isometry currents finite?

Whether this scenario works depends on whether the normal components for the isometry currents are finite.

1. $det(g_4) = 0$ condition gives boundaries of Euclidean and Minkowskian regions as 3-D light-like minimal surfaces. There would be no scales in accordance with generalized conformal invariance. g_{uv} in light-cone coordinates for M^2 vanishes and implies the vanishing of $det(g_4)$ and light-likeness of the 3-surface.

   What is important is that the formation of these regions would be unavoidable and they would be stable against perturbations.

2. $g^{uv}\sqrt{|g_4|}$ is finite if $det(g_4) = 0$ condition is satisfied, otherwise it diverges. The terms $g^{ui}\partial_i h^k\sqrt{|g_4|}$ must be finite. $g^{ui} = cof(g_{iu})/det(g_4)$ is finite since $g_{uv}g_{vu}$ in the cofactor cancels it from the determinant in the expression of g^{ui}. The presence of $\sqrt{|g_4|}$ implies that the these contributions to the boundary conditions vanish. Therefore only the condition boundary condition for g^{uv} remains.

3. If also Kähler action is present, the conditions are modified by replacing $T^{uk} = g^{u\alpha}\partial_\alpha h^k\sqrt{|g_4|}$ with a more general expression containing also the contribution of Kähler action. I have discussed the details of the variational problem in [K15, K9].

   The Kähler contribution involves the analogy of Maxwell's energy momentum tensor, which comes from the variation of the induced metric and involves sum of terms proportional to $J_{\alpha\mu}J_\mu^{beta}$ and $g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu}$.

   In the first term, the dangerous index raisings by g^{uv} appear 3 times. The most dangerous term is given by $J^{uv}J_v^v\sqrt{|g|} = g^{u\mu}g^{v\nu}J_{\alpha\beta}g^{vu}J_{vu}\sqrt{|g|}$. The divergent part is $g^{uv}g^{vu}J_{uv}g^{vu}J_{vu}\sqrt{|g|}$. The diverging g^{uv} appears 3 times and $J_{uv} = 0$ condition eliminates two of these. $g^{vu}\sqrt{|g|}$ is finite by $\sqrt{|g|} = 0$ condition. $J_{uv} = 0$ guarantees also the finiteness of the most dangerous part in $g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu}\sqrt{|g|}$.

   There is also an additional term coming from the variation of the induced Kähler form. This to the normal component of the isometry current is proportional to the quantity $J^{n\alpha}J_l^k\partial_\beta h^l\sqrt{|g|}$. Also now, the most singular term in $J^{u\beta} = g^{u\mu}g^{\beta\nu}J_{\mu\nu}$ corresponds to J^{uv} giving $g^{uv}g^{vu}J^{uv}\sqrt{|g|}$. This term is finite by $J_{uv} = 0$ condition.

   Therefore the boundary conditions are well-defined but only because $det(g_4) = 0$ condition is assumed.

4. Twistor lift strongly suggests that the assignment of the analogy of Kähler action also to M^4 and also this would contribute. All terms are finite if $det(g_4) = 0$ condition is satisfied.

5. The isometry currents in the normal direction must be equal to the divergences of the corresponding currents assignable to the Chern-Simons action at the boundary so that the flow of isometry charges to the boundary would go to the Chern-Simons isometry charges at the boundary.

   If the Chern-Simons term is absent, one expects that the boundary condition reduces to $\partial_v h^k = 0$. This would make X^3 2-dimensional so that Chern-Simons term is necessary. Note that light-likeness does not force the M^4 projection to be light-like so that the expansion of X^2 need not take with light-velocity. If CP_2 complex coordinates are holomorphic functions of W depending also on $U = v$ as a parameter, extended conformal invariance is obtained.

3.9.4 $det(g_4) = 0$ condition as a realization of quantum criticality

Quantum criticality is the basic dynamical principle of quantum TGD. What led to its discovery was the question "How to make TGD unique?". TGD has a single coupling constant, Kähler couplings strength, which is analogous to a critical temperature. The idea was obvious: require quantum criticality. This predicts a spectrum of critical values for the Kähler coupling strength. Quantum criticality would make the TGD Universe maximally complex. Concerning living matter, quantum critical dynamics is ideal since it makes the system maximally sensitive and maximallt reactive.

Concerning the realization of quantum criticality, it became gradually clear that the conformal invariance accompanying 2-D criticality, must be generalized. This led to the proposal that super symplectic symmetries, extended isometries and conformal symmetries of the metrically 2-D boundary of lightcone of M^4, and the extension of the Kac-Moody symmetries associated with the light-like boundaries of deformed CP_2 type extremals should act as symmetries of TGD extending the conformal symmetries of 2-D conformal symmetries. These huge infinite-D symmetries are also required by the existence of the Kähler geometry of WCW [K57, K29, K96] [L141, L157].

However, the question whether light-like boundaries of 3-surfaces with scale larger than CP_2 are possible, remained an open question. On the basis of preceding arguments, the answer seems to be affirmative and one can ask for the implications.

1. At M^8 level, the concrete realization of holography would involve two ingredients. The intersections of the space-time surface with the mass shells H^3 with mass squared value determined as the roots of polynomials P and the tlight-like 3-surfaces as $det(g_4) = 0$ surfaces as boundaries (genuine or between Minkowskian and Euclidean regions) associated by $M^8 - H$ duality to 4-surface of M^8 having associative normal space, which contains commutative 2-D subspace at each point. This would make possible both holography and $M^8 - H$ duality.

   Note that the identification of the algebraic geometric characteristics of the counterpart of $det(g_4) = 0$ surface at the level of H remains still open.

   Since holography determines the dynamics in the interior of the space-time surface from the boundary conditions, the classical dynamics can be said to be critical also in the interior.

2. Quantum criticality means ability to self-organize. Number theoretical evolution allows us to identify evolution as an increase of the algebraic complexity. The increase of the degree n of polynomial P serves as a measure for this. $n = h_{eff}/h_0$ also serves as a measure for the scale of quantum coherence, and dark matter as phases of matter would be characterized by the value of n.

3. The 3-D boundaries would be places where quantum criticality prevails. Therefore they would be ideal seats for the development of life. The proposal that the phase boundaries between water and ice serve as seats for the evolution of prebiotic life, is discussed from the point of TGD based view of quantum gravitation involving huge value of gravitational Planck constant $\hbar_{eff} = \hbar_{gr} = GMm/v_0$ making possible quantum coherence in astrophysical scales [L150]. Density fluctuations would play an essential role, and this would mean that the volume enclosed by the 2-D M^4 projection of the space-time boundary would fluctuate. Note that these fluctuations are possible also at the level of the field body and magnetic body.

4. It has been said that boundaries, where the nervous system is located, distinguishes living systems from inanimate ones. One might even say that holography based on $det(g_4) = 0$ condition realizes nervous systems in a universal manner.

5. I have considered several variants for the holography in the TGD framework, in particular strong form of holography (SH). SH would mean that either the light-like 3-surfaces or the 3-surfaces at the ends of the causal diamond (CD) determine the space-time surface so that the 2-D intersections of the 3-D ends of the space-time surface with its light-like boundaries would determine the physics.

   This condition is perhaps too strong but a fascinating, weaker, possibility is that the internal consistency requires that the intersections of the 3-surface with the mass shells H^3 are identifiable as fundamental domains for the coset spaces $SO(1,3)/\Gamma$ defining tessellations of H^3 and hyperbolic manifolds. This would conform nicely with the TGD inspired model of genetic code [L133].

Chapter 4

WCW Spinor Structure

4.1 Introduction

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

4.1.1 Basic Principles

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. [B51] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. Contrary to the long held belief it seems that covariantly constant right handed neutrino does not generate . The differences between quarks and leptons result from the different couplings to

the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the embedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \ .$$

where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

5. TGD as a generalized number theory vision leads to the understanding of how the second quantization of the induced spinor fields should be carried out and space-time conformal symmetries allow to explicitly solve the Dirac equation associated with the Kähler-Dirac action in the interior and at the 3-D light like causal determinants. An essentially new element is the notion of number theoretic braid forced by the fact that the Kähler-Dirac operator allows only finite number of generalized eigen modes so that the number of fermionic oscillator operators is finite. As a consequence, anti-commutation relations can be satisfied only for a finite set of points defined by the number theoretic braid, which is uniquely identifiable. The interpretation is in terms of finite measurement resolution. The finite Clifford algebra spanned by the fermionic oscillator operators is interpreted as the factor space $\mathcal{M}/\mathcal{N}$ of infinite hyper-finite factors of type II_1 defined by WCW Clifford algebra $\mathcal{N}$ and included Clifford algebra $\mathcal{M} \subset \mathcal{N}$ interpreted as the characterizer of the finite measurement resolution. Note that the finite number of eigenvalues guarantees that Dirac determinant identified as the exponent of Kähler function is finite. Finite number of eigenvalues is also essential for number theoretic universality.

Identification of WCW gamma matrices as super Hamiltonians and expression of WCW Kähler metric

The basic super-algebra corresponds to the fermionic oscillator operators and can be regarded as a generalization $\mathcal{N}$ super algebras by replacing $\mathcal{N}$ with the number of solutions of the Kähler-Dirac equation which can be infinite. This leads to QFT SUSY limit of TGD different in many respects crucially from standard SUSYs.

WCW gamma matrices are identified as super generators of super-symplectic and are expressible in terms of these oscillator operators. In the original proposal super-symplectic and super charges were assumed to be expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory.

It took quite long time to realize that it is possible to second quantize induced spinor fields by using just the standard canonical quantization. The only new element is the replacement of the ordinary gamma matrices with K-D gamma matrices identified as canonical momentum currents contracted with the embedding space gamma matrices. This allows to deduce super-generators of super-symplectic algebra as Noether supercharges assignable to the fermionic strings connecting partonic 2-surfaces. Their anti-commutators giving the matrix elements of WCW Kähler metric can be deduced explicitly. This is a decisive calculational advantage since the formal expression of the matrix elements in terms of second derivatives of Kähler function is not possible to calculate with the recent understanding. WCW gamma matrices provide also a natural identification for the counterparts of fermionic oscillator operators creating physical states.

One can also deduce the fermionic Hamiltonians as conserved Noether charges. The expressions for Hamiltonians generalized the earlier expressions as Hamiltonian fluxes in the sense that the embedding space Hamiltonian is replaced with the corresponding fermionic Noether charge. This replacement is analogous to a transition from field theory to string models requiring the replacement of points of partonic 2-surfaces with stringy curves connecting the points of two partonic 2-surfaces. One can consider also several strings emanating from a given partonic 2-surface. This leads to an extension of the super-symplectic algebra to a Yangian, whose generators are multi-local (multi-stringy) operators. This picture does not mean loss of effective 2-dimensionality implied by strong form of general coordinate invariance but allows genuine generalization of super-conformal invariance in 4-D context.

4.1.2 Kähler-Dirac Action

Supersymmetry fixes the interior part of Kähler-Dirac uniquely. The K-D gamma matrices are contractions of the canonical momentum currents of Kähler action with the embedding space gamma matrices and this gives field equations consistent with hermitian conjugation. The modes of K-D equation must be restricted to 2-D string world sheets with vanishing induced W boson fields in order that they have a well-defined em charge. It is not yet clear whether this restriction is part of variational principle or whether it is a property of spinor modes. For the latter option modes one can have 4-D modes if the space-time surface has CP_2 projection carrying vanishing W gauge potentials. Also covariantly constant right-handed neutrino defines this kind of mode.

The boundary terms of Kähler action and Kähler-Dirac action

A long standing question has been whether Kähler action could contain Chern-Simons term cancelling the Chern-Simons contribution of Kähler action at space-time interior at partonic orbit reducing to Chern-Simons terms so that only the contribution at space-like ends of space-time surface at the boundaries of causal diamond (CD) remains. This is however not necessary and super-symmetry would require Chern-Simons-Dirac term as boundary term in Dirac action. This however has unphysical implications since C-S-D Dirac operator acts on CP_2 coordinates only.

The intuitive expectation is that fermionic propagators assignable to string boundaries at light-like partonic orbits are needed in the construction of the scattering amplitudes. These boundaries can be locally space-like or light-like. One could add 1-D massles Dirac action with gamma matrices defined in the induced metric, which is by supersymmetry accompanied by the action defined by geodesic length, which however vanishes for light-like curves. Massless Dirac equation at the boundary of string world sheet fixes the boundary conditions for the spinor modes at the string world sheet. This option seems to be the most plausible at this moment.

Kähler-Dirac equation for induced spinor fields

It has become clear that Kähler-Dirac action with induced spinor fields localized at string world sheets carrying vanishing classical W fields, and the light-like boundaries of the string world sheets at light-like orbits of partonic 2-surfaces carrying massless Dirac operator for induced gamma matrices is the most natural looking option.

The light-like momentum associated with the boundary is a light-like curve of imbedding space and defines light-like 8-momentum, whose M^4 projection is in general time-like. This leads to an 8-D generalization of twistor formalism. The squares of the M^4 and CP_2 parts of the 8-momentum could be identified as mass squared for the embedding space spinor mode assignable to the ground state of super-symplectic representation. This would realize quantum classical correspondence for fermions. The four-momentum assignable to fermion line would have identification as gravitational four-momentum and that associated with the mode of embedding space spinor field as inertial four-momentum.

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vanish at these surfaces.
2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.
3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry.This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

Quantum criticality and K-D action

A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. The recent formulation of quantum criticality states the existence of hierarchy of sub-algebras of super-symplectic algebras isomorphic with the original algebra. The conformal weights of given sub-algebra are n-multiples of those of the full algebra. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter. These sub-algebras correspond to a hierarchy of breakings of super-symplectic gauge symmetry to a sub-algebra. Accordingly the super-symplectic Noether charges of the sub-algebra annihilate physical states and the corresponding classical Noether charges vanish for Kähler action at the ends of space-time surfaces. This defines the notion of preferred extremal. These sub-algebras form an inclusion hierarchy defining a hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

Quantum criticality implies that second variation of Kähler action vanishes for critical deformations defined by the sub-algebra and vanishing of the corresponding Noether charges and super-charges for physical stats. It is not quite clear whether the charges corresponding to broken super-symplectic symmetries are conserved. If this is the case, Kähler action is invariant under brokent symplectic transformations although the second variation is non-vanishing so these deformations contribute to Kähler metric and are thus quantum fluctuating dynamical degrees of freedom.

Quantum classical correspondence

Quantum classical correspondence (QCC) requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question mark is how to realize this coupling.

1. As already described, the massless Dirac equation for induced gamma matrices at the boundary of string world sheets gives as solutions for which local 8-momentum is light-like. The M^4 part of this momentum is in general time-like and can be identified as the 8-momentum of incoming fermion assignable to an embedding space spinor mode. The interpretation is as equivalence of gravitational and inertial masses.

2. QCC can be realized at the level of WCW Dirac operator and Kähler-Dirac operator contains only interior term. The vanishing of the normal component of fermion current replaces Chern-Simons Dirac operator at various boundary like surfaces. I have proposed that WCW spinor fields with given quantum charges in Cartan algebra are superpositions of space-time surfaces with same classical charges. A stronger form of QCC at the level of WCW would be that classical correlation functions for various geometric observables are identical with quantal correlation functions.

   QCC could be realized at the level of WCW by putting it in by hand. One can of course consider also the possibility that the equality of quantal and classical Cartan charges is realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system with Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD in zero energy ontology (ZEO) can be regarded as square root of thermodynamics, the procedure looks logically sound.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L17].

4.2 WCW Spinor Structure: General Definition

The basic problem in constructing WCW spinor structure is clearly the construction of the explicit representation for the gamma matrices of WCW . One should be able to identify the space, where these gamma matrices act as well as the counterparts of the "free" gamma matrices, in terms of which the gamma matrices would be representable using generalized vielbein coefficients.

4.2.1 Defining Relations For Gamma Matrices

The ordinary definition of the gamma matrix algebra is in terms of the anti-commutators

$$\{\gamma_A, \gamma_B\} = 2g_{AB} \quad .$$

This definition served implicitly also as a basic definition of the gamma matrix algebra in TGD context until the difficulties related to the understanding of WCW d'Alembertian defined in terms of the square of the Dirac operator forced to reconsider the definition. If WCW allows Kähler structure, the most general definition allows to replace the metric any covariantly constant Hermitian form. In particular, g_{AB} can be replaced with

$$\{\Gamma_A^\dagger, \Gamma_B\} = iJ_{AB} \quad , \qquad (4.2.1)$$

where J_{AB} denotes the matrix element of the Kähler form of WCW . The reason is that gamma matrices carry fermion number and are non-hermitian in all coordinate systems. This definition is numerically equivalent with the standard one in the complex coordinates but in arbitrary coordinates situation is different since in general coordinates iJ_{kl} is a nontrivial positive square root of g_{kl}. The realization of this delicacy is necessary in order to understand how the square of WCW Dirac operator comes out correctly. Obviously, what one must do is the equivalent of replacing $D^2 = (\Gamma^k D_k)^2$ with $D\hat{D}$ with $\hat{D}$ defined as

$$\hat{D} = iJ^{kl}\Gamma_l^\dagger D_k \quad .$$

4.2.2 General Vielbein Representations

There are two ideas, which make the solution of the problem obvious.

1. Since the classical time development in bosonic degrees of freedom (induced gauge fields) is coded into the geometry of WCW it seems natural to expect that same applies in the case of the spinor structure. The time development of the induced spinor fields dictated by TGD counterpart of the massless Dirac action should be coded into the definition of the WCW spinor structure. This leads to the challenge of defining what classical spinor field means.
2. Since classical scalar field in WCW corresponds to second quantized boson fields of the embedding space same correspondence should apply in the case of the fermions, too. The spinor fields of WCW should correspond to second quantized fermion field of the embedding space and the space of the configuration space spinors should be more or less identical with the Fock space of the second quantized fermion field of embedding space or $X^4(X^3)$. Since classical spinor fields at space-time surface are obtained by restricting the spinor structure to the space-time surface, one might consider the possibility that life is really simple: the second quantized spinor field corresponds to the free spinor field of the embedding space satisfying the counterpart of the massless Dirac equation and more or less standard anti-commutation relations. Unfortunately life is not *so* simple as the construction of WCW spinor structure demonstrates: second quantization must be performed for induced spinor fields.

It is relatively simple to fill in the details once these basic ideas are accepted.

1. The only natural candidate for the second quantized spinor field is just the on X^4. Since this field is free field, one can indeed perform second quantization and construct fermionic oscillator operator algebra with unique anti-commutation relations. The space of WCW spinors can be identified as the associated with these oscillator operators. This space depends on 3-surface and strictly speaking one should speak of the Fock bundle having WCW as its base space.
2. The gamma matrices of WCW (or rather fermionic Kac Moody generators) are representable as super positions of the fermionic oscillator algebra generators:

$$\begin{aligned} \Gamma_A^+ &= E_A^n a_n^\dagger \\ \Gamma_A^- &= \bar{E}_A^n a_n \\ iJ_{A\bar{B}} &= \sum_n E_A^n \bar{E}_B^n \end{aligned} \tag{4.2.2}$$

where E_A^n are the vielbein coefficients. Induced spinor fields can possess zero modes and there is no oscillator operators associated with these modes. Since oscillator operators are spin 1/2 objects, WCW gamma matrices are analogous to spin 3/2 spinor fields (in a very general sense). Therefore the generalized vielbein and WCW metric is analogous to the pair of spin 3/2 and spin 2 fields encountered in super gravitation! Notice that the contractions $j^{Ak}\Gamma_k$ of the complexified gamma matrices with the isometry generators are genuine spin 1/2 objects labeled by the quantum numbers labeling isometry generators. In particular, in CP_2 degrees of freedom these fermions are color octets.
3. A further great idea inspired by the symplectic and Kähler structures of WCW is that configuration gamma matrices are actually generators of super-symplectic symmetries. This simplifies enormously the construction allows to deduce explicit formulas for the gamma matrices.

4.2.3 Inner Product For WCW Spinor Fields

The conjugation operation for WCW spinor s corresponds to the standard $ket \rightarrow bra$ operation for the states of the Fock space:

$$\begin{aligned} \Psi &\leftrightarrow |\Psi\rangle \\ \bar{\Psi} &\leftrightarrow \langle\Psi| \end{aligned} \tag{4.2.3}$$

The inner product for WCW spinor s at a given point of WCW is just the standard Fock space inner product, which is unitary.

$$\bar{\Psi}_1(X^3)\Psi_2(X^3) \quad = \quad \langle \Psi_1|\Psi_2\rangle_{|X^3} \tag{4.2.4}$$

WCW inner product for two WCW spinor fields is obtained as the integral of the Fock space inner product over the whole WCW using the vacuum functional $exp(K)$ as a weight factor

$$\langle \Psi_1|\Psi_2\rangle \quad = \quad \int \langle \Psi_1|\Psi_2\rangle_{|X^3} exp(K)\sqrt{G}dX^3 \tag{4.2.5}$$

This inner product is obviously unitary. A modified form of the inner product is obtained by including the factor $exp(K/2)$ in the definition of the spinor field. In fact, the construction of the central extension for the isometry algebra leads automatically to the appearance of this factor in vacuum spinor field.

The inner product differs from the standard inner product for, say, Minkowski space spinors in that integration is over the entire WCW rather than over a time= constant slice of the WCW . Also the presence of the vacuum functional makes it different from the finite dimensional inner product. These are not un-physical features. The point is that (apart from classical non-determinism forcing to generalized the concept of 3-surface) Diff^4 invariance dictates the behavior of WCW spinor field completely: it is determined form its values at the moment of the big bang. Therefore there is no need to postulate any Dirac equation to determine the behavior and therefore no need to use the inner product derived from dynamics.

4.2.4 Holonomy Group Of The Vielbein Connection

Generalized vielbein allows huge gauge symmetry. An important constraint on physical observables is that they do not depend at all on the gauge chosen to represent the gamma matrices. This is indeed achieved using vielbein connection, which is now quadratic in fermionic oscillator operators. The holonomy group of the vielbein connection is the WCW counterpart of the electro-weak gauge group and its algebra is expected to have same general structure as the algebra of the WCW isometries. In particular, the generators of this algebra should be labeled by conformal weights like the elements of Kac Moody algebras. In present case however conformal weights are complex as the construction of WCW geometry demonstrates.

4.2.5 Realization Of WCW Gamma Matrices In Terms Of Super Symmetry Generators

In string models super symmetry generators behave effectively as gamma matrices and it is very tempting to assume that WCW gamma matrices can be regarded as generators of the symplectic algebra extended to super-symplectic Kac Moody type algebra. The experience with string models suggests also that radial Virasoro algebra extends to Super Virasoro algebra. There are good reasons to expect that WCW Dirac operator and its square give automatically a realization of this algebra. It this is indeed the case, then WCW spinor structure as well as Dirac equation reduces to mere group theory.

One can actually guess the general form of the super-symplectic algebra. The form is a direct generalization of the ordinary super Kac Moody algebra. The complexified super generators S_A are identifiable as WCW gamma matrices:

$$\Gamma_A \quad = \quad S_A \ . \tag{4.2.6}$$

The anti-commutators $\{\Gamma_A^\dagger, \Gamma_B\}_+ = i2J_{A,B}$ define a Hermitian matrix, which is proportional to the Kähler form of the configuration space rather than metric as usually. Only in complex coordinates the anti-commutators equal to the metric numerically. This is, apart from the multiplicative constant n, is expressible as the Poisson bracket of the WCW Hamiltonians H_A and H_B. Therefore one should be able to identify super generators $S_A(r_M)$ for each values of r_M as the counterparts of fluxes. The anti-commutators between the super generators S_A and their Hermitian conjugates should read as

$$\{S_A, S_B^\dagger\}_+ \quad = \quad iQ_m(H_{[A,B]}) \ . \tag{4.2.7}$$

and should be induced directly from the anti-commutation relations of free second quantized spinor fields of the embedding space restricted to the light cone boundary.

The commutation relations between s and super generators follow solely from the transformation properties of the super generators under symplectic transformations, which are same as for the Hamiltonians themselves

$$\{H_{Am}, S_{Bn}\}_- \quad = \quad S_{[Am,Bn]} \ , \tag{4.2.8}$$

and are of the same form as in the case of Super-Kac-Moody algebra.

The task is to derive an explicit representation for the super generators S_A in both cases. For obvious reason the spinor fields restricted to the 3-surfaces on the light cone boundary $\delta M^4_+ \times CP_2$ can be used. Leptonic/quark like oscillator operators are used to construct Ramond/NS type algebra.

What is then the strategy that one should follow?

1. WCW Hamiltonians correspond to either magnetic or electric flux Hamiltonians and the conjecture is that these representations are equivalent. It turns out that this electric-magnetic duality generalizes to the level of super charges. It also turns out that quark representation is the only possible option whereas leptonic super charges super-symmetrize the ordinary function algebra of the light cone boundary.
2. The simplest option would be that second quantized embedding space spinors could be used in the definition of super charges. This turns out to not work and one must second quantize the induced spinor fields.
3. The task is to identify a super-symmetric variational principle for the induced spinors: ordinary Dirac action does not work. It turns out that in the most plausible scenario the Kähler-Dirac action varied with respect to *both* embedding space coordinates and spinor fields is the fundamental action principle. The c-number parts of the conserved symplectic charges associated with this action give rise to bosonic conserved charges defining WCW Hamiltonians. The second quantization of the spinor fields reduces to the requirement that super charges and Hamiltonians generate super-symplectic algebra determining the anti-commutation relations for the induced spinor fields.

4.2.6 Central Extension As Symplectic Extension At WCW Level

The earlier attempts to understand the emergence of central extension of super-symplectic algebra were based on the notion of symplectic extension. This general view is not given up although it seems that this abstract approach is not very practical. Symplectic extension emerged originally in the attempts to construct formal expression for the WCW Dirac equation. The rather obvious idea was that the Dirac equation reduces to super Virasoro conditions with Super Virasoro generators involving the Dirac operator of the embedding space. The basic difficulty was the necessity to assign to the gamma matrices of the embedding space fermion number. In the recent formulation the Dirac operator of H does not appear in in the Super Virasoro conditions so that this problem disappears.

The proposal that Super Virasoro conditions should replaced with conditions stating that the commutator of super-symplectic and super Kac-Moody algebras annihilates physical states, looks rather feasible. One could call these conditions as WCW Dirac equation but at this moment I feel that this would be just play with words and mask the group theoretical content of these conditions. In any case, the formulas for the symplectic extension and action of isometry generators on WCW spinor deserve to be summarized.

Symplectic extension

The Abelian extension of the super-symplectic algebra is obtained by an extremely simple trick. Replace the ordinary derivatives appearing in the definition of, say spinorial isometry generator, by the covariant derivatives defined by a coupling to a multiple of the Kähler potential.

$$\begin{aligned} j^{Ak}\partial_k &\rightarrow j^{Ak}D_k \ , \\ D_k &= \partial_k + ikA_k/2 \ . \end{aligned} \tag{4.2.9}$$

where A_k denotes Kähler potential. The reality of the parameter k is dictated by the Hermiticity requirement and also by the requirement that Abelian extension reduces to the standard form in Cartan algebra. k is expected to be integer also by the requirement that covariant derivative corresponds to connection (quantization of magnetic charge).

The commutation relations for the centrally extended generators J^A read:

$$[J^A, J^B] = J^{[A,B]} + ikj^{Ak}J_{kl}j^{Bl} \equiv J^{[A,B]} + ikJ_{AB} \ . \tag{4.2.10}$$

Since Kähler form defines symplectic structure in WCW one can express Abelian extension term as a Poisson bracket of two Hamiltonians

$$J_{AB} \equiv j^{Ak}J_{kl}j^{Bl} = \{H^A, H^B\} \ . \tag{4.2.11}$$

Notice that Poisson bracket is well defined also when Kähler form is degenerate.

The extension indeed has acceptable properties:

1. Jacobi-identities reduce to the form

$$\sum_{cyclic} H^{[A,[B,C]]} = 0 \ , \tag{4.2.12}$$

and therefore to the Jacobi identities of the original Lie- algebra in Hamiltonian representation.

2. In the Cartan algebra Abelian extension reduces to a constant term since the Poisson bracket for two commuting generators must be a multiple of a unit matrix. This feature is clearly crucial for the non-triviality of the Abelian extension and is encountered already at the level of ordinary (q,p) Poisson algebra: although the differential operators ∂_p and ∂_q commute the Poisson bracket of the corresponding Hamiltonians p and q is nontrivial: $\{p,q\} = 1$. Therefore the extension term commutes with the generators of the Cartan subalgebra. Extension is also local $U(1)$ extension since Poisson algebra differs from the Lie-algebra of the vector fields in that it contains constant Hamiltonian ("1" in the commutator), which commutes with all other Hamiltonians and corresponds to a vanishing vector field.

3. For the generators not belonging to Cartan sub-algebra of CH isometries Abelian extension term is not annihilated by the generators of the original algebra and in this respect the extension differs from the standard central extension for the loop algebras. It must be however emphasized that for the super-symplectic algebra generators correspond to products of δM^4_+ and CP_2 Hamiltonians and this means that generators of say δM^4_+-local $SU(3)$ Cartan algebra are non-commuting and the commutator is completely analogous to central extension term since it is symmetric with respect to $SU(3)$ generators.

4. The proposed method yields a trivial extension in the case of Diff^4. The reason is the (four-dimensional!) Diff degeneracy of the Kähler form. Abelian extension term is given by the contraction of the Diff^4 generators with the Kähler potential

$$j^{Ak}J_{kl}j^{Bl} = 0 \ , \tag{4.2.13}$$

which vanishes identically by the Diff degeneracy of the Kähler form. Therefore neither 3- or 4-dimensional Diff invariance is not expected to cause any difficulties. Recall that 4-dimensional Diff degeneracy is what is needed to eliminate time like vibrational excitations from the spectrum of the theory. By the way, the fact that the loop space metric is not Diff degenerate makes understandable the emergence of Diff anomalies in string models [B51, B44] .

5. The extension is trivial also for the other zero norm generators of the tangent space algebra, in particular for the $k_2 = Im(k) = 0$ symplectic generators possible present so that these generators indeed act as genuine $U(1)$ transformations.
6. Concerning the solution of WCW Dirac equation the maximum of Kähler function is expected to be special, much like origin of Minkowski space and symmetric space property suggests that the construction of solutions reduces to this point. At this point the generators and Hamiltonians of the algebra h in the defining Cartan decomposition $g = h + t$ should vanish. h corresponds to integer values of $k_1 = Re(k)$ for Cartan algebra of super-symplectic algebra and integer valued conformal weights n for Super Kac-Moody algebra. The algebra reduces at the maximum to an exceptionally simple form since only central extension contributes to the metric and Kähler form. In the ideal case the elements of the metric and Kähler form could be even diagonal. The degeneracy of the metric might of course pose additional complications.

Super symplectic action on WCW spinor s

The generators of symplectic transformations are obtained in the spinor representation of the isometry group of WCW by the following formal construction. Take isometry generator in the spinor representation and add to the covariant derivative D_k defined by vielbein connection the coupling to the multiple of the Kähler potential: $D_k \rightarrow D_k + ikAk/2$.

$$\begin{aligned} J^A &= j^{Ak}D_k + D_l j_k \Sigma^{kl}/2 \ , \\ &\rightarrow \ \hat{J}^A = j^{Ak}(D_k + ikA_k/2) + D_l j_k^A \Sigma^{kl}/2 \ , \end{aligned} \tag{4.2.14}$$

This induces the required central term to the commutation relations. Introduce complex coordinates and define bosonic creation and annihilation operators as $(1,0)$ and $(0,1)$ parts of the modified isometry generators

$$\begin{aligned} B_A^\dagger &= J_+^A = j^{Ak}(D_k + ... \ , \\ B_A &= J_-^A = j^{A\bar{k}}(D_{\bar{k}} + ... \ . \end{aligned} \tag{4.2.15}$$

where "k" refers now to complex coordinates and "$\bar{k}$" to their conjugates.

Fermionic generators are obtained as the contractions of the complexified gamma matrices with the isometry generators

$$\begin{aligned} \Gamma_A^\dagger &= j^{Ak}\Gamma_k \ , \\ \Gamma_A &= j^{A\bar{k}}\Gamma_{\bar{k}} \ . \end{aligned} \tag{4.2.16}$$

Notice that the bosonic Cartan algebra generators obey standard oscillator algebra commutation relations and annihilate fermionic Cartan algebra generators. Hermiticity condition holds in the sense that creation type generators are hermitian conjugates of the annihilation operator type generators. There are two kinds of representations depending on whether one uses leptonic or quark like oscillator operators to construct the gammas. These will be assumed to correspond to Ramond and NS type generators with the radial plane waves being labeled by integer and half odd integer indices respectively.

The non-vanishing commutators between the Cartan algebra bosonic generators are given by the matrix elements of the Kähler form in the basis of formed by the isometry generators

$$[B_A^\dagger, B_B] = J(j^{A\dagger}, j^B) \equiv J_{\bar{A}B} \ . \tag{4.2.17}$$

and are isometry invariant quantities. The commutators between local $SU(3)$ generators not belonging to Cartan algebra are just those of the local gauge algebra with Abelian extension term added.

The anti-commutators between the fermionic generators are given by the elements of the metric (as opposed to Kähler form in the case of bosonic generators) in the basis formed by the isometry generators

$$\{\Gamma_A\dagger, \Gamma_B\} = 2g(j^{A\dagger}, j^B) \equiv 2g_{\bar{A}B} \ . \tag{4.2.18}$$

and are invariant under isometries. Numerically the commutators and anti-commutators differ only the presence of the imaginary unit and the scale factor R relating the metric and Kähler form to each other (the factor R is same for CP_2 metric and Kähler form).

The commutators between bosonic and fermionic generators are given by

$$[B_A, \Gamma_B] = \Gamma_{[A,B]} \ . \tag{4.2.19}$$

The presence of vielbein and rotation terms in the representation of the isometry generators is essential for obtaining these nice commutations relations. The commutators vanish identically for Cartan algebra generators. From the commutation relations it is clear that Super Kac Moody algebra structure is directly related to the Kähler structure of WCW : the anti-commutator of fermionic generators is proportional to the metric and the commutator of the bosonic generators is proportional to the Kähler form. It is this algebra, which should generate the solutions of the field equations of the theory.

The vielbein and rotational parts of the bosonic isometry generators are quadratic in the fermionic oscillator operators and this suggests the interpretation as the fermionic contribution to the isometry currents. This means that the action of the bosonic generators is essentially non-perturbative since it creates fermion anti-fermion pairs besides exciting bosonic degrees of freedom.

4.2.7 WCW Clifford Algebra As AHyper-Finite Factor Of Type II_1

The naïve expectation is that the trace of the unit matrix associated with the Clifford algebra spanned by WCW sigma matrices is infinite and thus defines an excellent candidate for a source of divergences in perturbation theory. This potential source of infinities remained un-noticed until it became clear that there is a connection with von Neumann algebras [A69]. In fact, for a separable Hilbert space defines a standard representation for so called [A56]. This guarantees that the trace of the unit matrix equals to unity and there is no danger about divergences.

Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A56].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac based on the notion of delta function, plus the emergence of Feynman graphs, the possibility to formulate the notion of delta function rigorously in terms of distributions, and the emergence of path integral approach meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A90, A47] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A38, A59] relate closely to type II_1 factors. In topological quantum computation [B38] based on braid groups [A100] modular S-matrices they play an especially important role.

Clifford algebra of WCW as von Neumann algebra

The Clifford algebra of WCW provides a school example of a hyper-finite factor of type II_1, which means that fermionic sector does not produce divergence problems. Super-symmetry means that also "orbital" degrees of freedom corresponding to the deformations of 3-surface define similar factor. The general theory of hyper-finite factors of type II_1 is very rich and leads to rather detailed understanding of the general structure of S-matrix in TGD framework. For instance, there is a unitary evolution operator intrinsic to the von Neumann algebra defining in a natural manner single particle time evolution. Also a connection with 3-dimensional topological quantum field theories and knot theory, conformal field theories, braid groups, quantum groups, and quantum counterparts of quaternionic and octonionic division algebras emerges naturally. These aspects are discussed in detail in [K126].

4.3 Under What Conditions Electric Charge Is Conserved For The Kähler-Dirac Equation?

One might think that talking about the conservation of electric charge at 21st century is a waste of time. In TGD framework this is certainly not the case.

1. In quantum field theories there are two ways to define em charge: as electric flux over 2-D surface sufficiently far from the source region or in the case of spinor field quantum mechanically as combination of fermion number and vectorial isospin. The latter definition is quantum mechanically more appropriate.
2. There is however a problem. In standard approach to gauge theory Dirac equation in presence of charged classical gauge fields does not conserve electric charge as quantum number: electron

is transformed to neutrino and vice versa. Quantization solves the problem since the non-conservation can be interpreted in terms of emission of gauge bosons. In TGD framework this does not work since one does not have path integral quantization anymore. Preferred extremals carry classical gauge fields and the question whether em charge is conserved arises. Heuristic picture suggests that em charge must be conserved.

It seems that one should pose the well-definedness of spinorial em charge as an additional condition. Well-definedness of em charge is not the only problem. How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? Does this require that classical weak fields vanish in the regions where the modes of induced spinor fields are non-vanishing?

This condition might be one of the conditions defining what it is to be a preferred extremal/solution of Kähler Dirac equation. It is not however trivial whether this kind of additional condition can be posed unless it follows automatically from the recent formulation for Kähler action and Kähler Dirac action. The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string/parton picture part of TGD. The vanishing of classical weak fields has also number theoretic interpretation: space-time surfaces would have quaternionic (hyper-complex) tangent space and the 2-surfaces carrying spinor fields complex (hyper-complex) tangent space.

4.3.1 Conservation Of EM Charge For Kähler Dirac Equation

What does the conservation of em charge imply in the case of the Kähler-Dirac equation? The obvious guess that the em charged part of the Kähler-Dirac operator must annihilate the solutions, turns out to be correct as the following argument demonstrates.

1. Em charge as coupling matrix can be defined as a linear combination $Q = aI+bI_3$, $I_3 = J_{kl}\Sigma^{kl}$, where I is unit matrix and I_3 vectorial isospin matrix, J_{kl} is the Kähler form of CP_2, Σ^{kl} denotes sigma matrices, and a and b are numerical constants different for quarks and leptons. Q is covariantly constant in $M^4 \times CP_2$ and its covariant derivatives at space-time surface are also well-defined and vanish.

2. The modes of the Kähler-Dirac equation should be eigen modes of Q. This is the case if the Kähler-Dirac operator D commutes with Q. The covariant constancy of Q can be used to derive the condition

$$\begin{aligned} [D,Q]\,\Psi &= D_1\Psi = 0 \ , \\ D &= \hat{\Gamma}^\mu D_\mu \ , \ \ D_1 = [D,Q] = \hat{\Gamma}_1^\mu D_\mu \ , \ \ \hat{\Gamma}_1^\mu = \left[\hat{\Gamma}^\mu, Q\right] \ . \end{aligned} \tag{4.3.1}$$

   Covariant constancy of J is absolutely essential: without it the resulting conditions would not be so simple.

   It is easy to find that also $[D_1,Q]\Psi = 0$ and its higher iterates $[D_n,Q]\Psi = 0$, $D_n = [D_{n-1},Q]$ must be true. The solutions of the Kähler-Dirac equation would have an additional symmetry.

3. The commutator $D_1 = [D,Q]$ reduces to a sum of terms involving the commutators of the vectorial isospin $I_3 = J_{kl}\Sigma^{kl}$ with the CP_2 part of the gamma matrices:

$$D_1 = [Q,D] = [I_3,\Gamma_r]\partial_\mu s^r T^{\alpha\mu} D_\alpha \ . \tag{4.3.2}$$

   In standard complex coordinates in which $U(2)$ acts linearly the complexified gamma matrices can be chosen to be eigenstates of vectorial isospin. Only the charged flat space complexified gamma matrices Γ^A denoted by Γ^+ and Γ^- possessing charges +1 and -1 contribute to the right hand side. Therefore the additional Dirac equation $D_1\Psi = 0$ states

$$\begin{aligned} D_1\Psi &= [Q,D]\Psi = I_3(A)e_{Ar}\Gamma^A\partial_\mu s^r T^{\alpha\mu}D_\alpha\Psi \\ &= (e_{+r}\Gamma^+ - e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu}D_\alpha\Psi = 0 \ . \end{aligned} \tag{4.3.3}$$

The next condition is

$$D_2\Psi = [Q,D]\Psi = (e_{+r}\Gamma^+ + e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu} D_\alpha \Psi = 0 \ . \quad (4.3.4)$$

Only the relative sign of the two terms has changed. The remaining conditions give nothing new.

4. These equations imply two separate equations for the two charged gamma matrices

$$\begin{aligned} D_+\Psi &= T_+^\alpha \Gamma^+ D_\alpha \Psi = 0 \ , \\ D_-\Psi &= T_-^\alpha \Gamma^- D_\alpha \Psi = 0 \ , \\ T_\pm^\alpha &= e_{\pm r}\partial_\mu s^r T^{\alpha\mu} \ . \end{aligned} \quad (4.3.5)$$

These conditions state what one might have expected: the charged part of the Kähler-Dirac operator annihilates separately the solutions. The reason is that the classical W fields are proportional to $e_{r\pm}$.

The above equations can be generalized to define a decomposition of the energy momentum tensor to charged and neutral components in terms of vierbein projections. The equations state that the analogs of the Kähler-Dirac equation defined by charged components of the energy momentum tensor are satisfied separately.

5. In complex coordinates one expects that the two equations are complex conjugates of each other for Euclidian signature. For the Minkowskian signature an analogous condition should hold true. The dynamics enters the game in an essential manner: whether the equations can be satisfied depends on the coefficients a and b in the expression $T = aG + bg$ implied by Einstein's equations in turn guaranteeing that the solution ansatz generalizing minimal surface solutions holds true [K15].

6. As a result one obtains three separate Dirac equations corresponding to the neutral part $D_0\Psi = 0$ and charged parts $D_\pm\Psi = 0$ of the Kähler-Dirac equation. By acting on the equations with these Dirac operators one obtains also that the commutators $[D_+, D_-]$, $[D_0, D_\pm]$ and also higher commutators obtained from these annihilate the induced spinor field model. Therefore entire -possibly- infinite-dimensional algebra would annihilate the induced spinor fields. In string model the counterpart of Dirac equation when quantized gives rise to Super-Virasoro conditions. This analogy would suggest that Kähler-Dirac equation gives rise to the analog of Super-Virasoro conditions in 4-D case. But what the higher conditions mean? Could they relate to the proposed generalization to Yangian algebra [A25] [B29, B22, B23]? Obviously these conditions resemble structurally Virasoro conditions $L_n|phys\rangle = 0$ and their supersymmetric generalizations, and might indeed correspond to a generalization of these conditions just as the field equations for preferred extremals could correspond to the Virasoro conditions if one takes seriously the analogy with the quantized string.

What could this additional symmetry mean from the point of view of the solutions of the Kähler-Dirac equation? The field equations for the preferred extremals of Kähler action reduce to purely algebraic conditions in the same manner as the field equations for the minimal surfaces in string model. Could this happen also for the Kähler-Dirac equation and could the condition on charged part of the Dirac operator help to achieve this?

This argument was very general and one can ask for simple ways to realize these conditions. Obviously the vanishing of classical W fields in the region where the spinor mode is non-vanishing defines this kind of condition.

4.3.2 About The Solutions Of Kähler Dirac Equation For Known Extremals

To gain perpective consider first Dirac equation in in H. Quite generally, one can construct the solutions of the ordinary Dirac equation in H from covariantly constant right-handed neutrino spinor playing the role of fermionic vacuum annihilated by the second half of complexified gamma matrices. Dirac equation reduces to Laplace equation for a scalar function and solution can be constructed from this "vacuum" by multiplying with the spherical harmonics of CP_2 and applying Dirac operator [K65]. Similar construction works quite generally thanks to the existence of covariantly constant right handed neutrino spinor. Spinor harmonics of CP_2 are only replaced with those of space-time surface possessing either hermitian structure or Hamilton-Jacobi structure (corresponding to Euclidian and Minkowskian signatures of the induced metric [K15, K127]). What is remarkable is that these solutions possess well-defined em charge although classical W boson fields are present.

This in sense that H d'Alembertian commutes with em charge matrix defined as a linear combination of unit matrix and the covariantly constant matrix $J^{kl}\Sigma_{kl}$ since the commutators of the covariant derivatives give constant Ricci scalar and $J^{kl}\Sigma_{kl}$ term to the d'Alembertian besides scalar d'Alembertian commuting with em charge. Dirac operator itself does not commute with em charge matrix since gamma matrices not commute with em charge matrix.

Consider now Kähler Dirac operator. The square of Kähler Dirac operator contains commutator of covariant derivatives which contains contraction $[\Gamma^\mu, \Gamma^\nu] F^{weak}_{\mu\nu}$ which is quadratic in sigma matrices of $M^4 \times CP_2$ and does not reduce to a constant term commuting which em charge matrix. Therefore additional condition is required even if one is satisfies with the commutativity of d'Alembertian with em charge. Stronger condition would be commutativity with the Kähler Dirac operator and this will be considered in the following.

To see what happens one must consider space-time regions with Minkowskian and Euclidian signature. What will be assumed is the existence of Hamilton-Jacobi structure [K15] meaning complex structure in Euclidian signature and hyper-complex plus complex structure in Minkowskian signature. The goal is to get insights about what the condition that spinor modes have a well-defined em charge eigenvalue requires. Or more concretely: is the localization at string world sheets guaranteeing well-defined value of em charge predicted by Kähler Dirac operator or must one introduce this condition separately? One can also ask whether this condition reduces to commutativity/co-commutativity in number theoretic vision.

1. CP_2 type vacuum extremals serve as a convenient test case for the Euclidian signature. In this case the Kähler-Dirac equation reduces to the massless ordinary Dirac equation in CP_2 allowing only covariantly constant right-handed neutrino as solution. Only part of CP_2 so that one give up the constraint that the solution is defined in the entire CP_2. In this case holomorphic solution ansatz obtained by assuming that solutions depend on the coordinates ξ^i, $i = 1, 2$ but not on their conjugates and that the gamma matrices $\Gamma^{\bar{i}}$, $i = 1, 2$, annihilate the solutions, works. The solutions ansatz and its conjugate are of exactly the same form as in case string models where one considers string world sheets instead of CP_2 region.

   The solutions are not restricted to 2-D string world sheets and it is not clear whether one can assign to them a well-defined em charge in any sense. Note that for massless Dirac equation in H one obtains all CP_2 harmonics as solutions, and it is possible to talk about em charge of the solution although solution itself is not restricted to 2-D surface of CP_2.

2. For massless extremals and a very wide class of solutions produced by Hamilton-Jacobi structure - perhaps all solutions representable locally as graphs for map $M^4 \to CP_2$ - canonical momentum densities are light-like and solutions are hyper-holomorphic in the coordinates associated with with string world sheet and annihilated by the conjugate gamma and arbitrary functions in transversal coordinates. This allows localization to string world sheets. The localization is now strictly dynamical and implied by the properties of Kähler Dirac operator.

3. For string like objects one obtains massless Dirac equation in $X^2 \times Y^2 \subset M^4 \times Y^2$, Y^2 a complex 2-surface in CP_2. Homologically trivial geodesic sphere corresponds to the simplest choice for Y^2. Modified Dirac operator reduces to a sum of massless Dirac operators associated with X^2 and Y^2. The most general solutions would have Y^2 mass. Holomorphic solutions reduces to product of hyper-holomorphic and holomorphic solutions and massless 2-D Dirac equation is satisfied in both factors.

   For instance, for S^2 a geodesic sphere and $X^2 = M^2$ one obtains M^2 massivation with mass squared spectrum given by Laplace operator for S^2. Conformal and hyper-conformal symmetries are lost, and one might argue that this is quite not what one wants. One must be however resist the temptation to make too hasty conclusions since the massivation of string like objects is expected to take place. The question is whether it takes place already at the level of fundamental spinor fields or only at the level of elementary particles constructed as many-fermion states of them as twistor Grassmann approach assuming massless M^4 propagators for the fundamental fermions strongly suggests [K116].

4. For vacuum extremals the Kähler Dirac operator vanishes identically so that it does not make sense to speak about solutions.

What can one conclude from these observations?

1. The localization of solutions to 2-D string world sheets follows from Kähler Dirac equation only for the Minkowskian regions representable as graphs of map $M^4 \to CP_2$ locally. For string like objects and deformations of CP_2 type vacuum extremals this is not expected to take place.
2. It is not clear whether one can speak about well-defined em charge for the holomorphic spinors annihilated by the conjugate gamma matrices or their conjugates. As noticed, for embedding space spinor harmonics this is however possible.
3. Strong form of conformal symmetry and the condition that em charge is well-defined for the nodes suggests that the localization at 2-D surfaces at which the charged parts of induced electroweak gauge fields vanish must be assumed as an additional condition. Number theoretic vision would suggest that these surfaces correspond to 2-D commutative or co-commutative surfaces. The string world sheets inside space-time surfaces would not emerge from theory but would be defined as basic geometric objects.

   This kind of condition would also allow duals of string worlds sheets as partonic 2-surfaces identified number theoretically as co-commutative surfaces. Commutativity and co-commutativity would become essential elemenents of the number theoretical vision.
4. The localization of solutions of the Kähler-Dirac action at string world sheets and partonic 2-surfaces as a constraint would mean induction procedure for Kähler-Dirac matrices from SX^4 to X^2 - that is projection. The resulting em neutral gamma matrices would correspond to tangent vectors of the string world sheet. The vanishing of the projections of charged parts of energy momentum currents would define these surfaces. The conditions would apply both in Minkowskian and Euclidian regions. An alternative interpretation would be number theoretical: these surface would be commutative or co-commutative.

4.3.3 Concrete Realization Of The Conditions Guaranteeing Well-Defined Em Charge

Well-definedness of the em charge is the fundamental condiiton on spinor modes. Physical intuition suggests that also classical Z^0 field should vanish - at least in scales longer than weak scale. Above the condition guaranteeing vanishing of em charge has been discussed at very general level. It has however turned out that one can understand situation by simply posing the simplest condition that one can imagine: the vanishing of classical W and possibly also Z^0 fields inducing mixing of different charge states.

1. Induced W fields mean that the modes of Kähler-Dirac equation do not in general have well-defined em charge. The problem disappears if the induced W gauge fields vanish. This does not yet guarantee that couplings to classical gauge fields are physical in long scales. Also classical Z^0 field should vanish so that the couplings would be purely vectorial. Vectoriality might be true in long enough scales only. If W and Z^0 fields vanish in all scales then electroweak forces are due to the exchanges of corresponding gauge bosons described as string like objects in TGD and represent non-trivial space-time geometry and topology at microscopic scale.
2. The conditions solve also another long-standing interpretational problem. Color rotations induce rotations in electroweak-holonomy group so that the vanishing of all induced weak fields also guarantees that color rotations do not spoil the property of spinor modes to be eigenstates of em charge.

One can study the conditions quite concretely by using the formulas for the components of spinor curvature [L1] (`http://tinyurl.com/z86o5qk`).

1. The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \quad (4.3.6)$$

   $R_{01} = R_{23}$ and $R_{03} = -R_{31}$ combine to form purely left handed classical W boson fields and Z^0 field corresponds to $Z^0 = 2R_{03}$.
   Kähler form is given by

$$J = 2(e^0\wedge e^3 + e^1\wedge e^2) \ . \quad (4.3.7)$$

2. The vanishing of classical weak fields is guaranteed by the conditions

$$\begin{array}{c} e^0\wedge e^1 - e^2\wedge e^3 = 0 \ , \\ e^0\wedge e^2 - e^3\wedge e^1 \ , \\ 4e^0\wedge e^3 + 2e^1\wedge e^2 \ . \end{array} \quad (4.3.8)$$

3. There are many ways to satisfy these conditions. For instance, the condition $e^1 = a\times e^0$ and $e^2 = -a\times e^3$ with arbitrary a which can depend on position guarantees the vanishing of classical W fields. The CP_2 projection of the tangent space of the region carrying the spinor mode must be 2-D.

   Also classical Z^0 vanishes if $a^2 = 2$ holds true. This guarantees that the couplings of induced gauge potential are purely vectorial. One can consider other alternaties. For instance, one could require that only classical Z^0 field or induced Kähler form is non-vanishing and deduce similar condition.

4. The vanishing of the weak part of induced gauge field implies that the CP_2 projection of the region carrying spinor mode is 2-D. Therefore the condition that the modes of induced spinor field are restricted to 2-surfaces carrying no weak fields sheets guarantees well-definedness of em charge and vanishing of classical weak couplings. This condition does not imply string world sheets in the general case since the CP_2 projection of the space-time sheet can be 2-D.

How string world sheets could emerge?

1. Additional consistency condition to neutrality of string world sheets is that Kähler-Dirac gamma matrices have no components orthogonal to the 2-surface in question. Hence various fermionic would flow along string world sheet.
2. If the Kähler-Dirac gamma matrices at string world sheet are expressible in terms of two non-vanishing gamma matrices parallel to string world sheet and sheet and thus define an integrable distribution of tangent vectors, this is achieved. What is important that modified gamma matrices can indeed span lower than 4-D space and often do so as already described. Induced gamma matrices defined always 4-D space so that the restriction of the modes to string world sheets is not possible.
3. String models suggest that string world sheets are minimal surfaces of space-time surface or of embedding space but it might not be necessary to pose this condition separately.

In the proposed scenario string world sheets emerge rather than being postulated from beginning.

1. The vanishing conditions for induced weak fields allow also 4-D spinor modes if they are true for entire spatime surface. This is true if the space-time surface has 2-D projection. One can expect that the space-time surface has foliation by string world sheets and the general solution of K-D equation is continuous superposition of the 2-D modes in this case and discrete one in the generic case.

2. If the CP_2 projection of space-time surface is homologically non-trivial geodesic sphere S^2, the field equations reduce to those in $M^4 \times S^2$ since the second fundamental form for S^2 is vanishing. It is possible to have geodesic sphere for which induced gauge field has only em component?
3. If the CP_2 projection is complex manifold as it is for string like objects, the vanishing of weak fields might be also achieved.
4. Does the phase of cosmic strings assumed to dominate primordial cosmology correspond to this phase with no classical weak fields? During radiation dominated phase 4-D string like objects would transform to string world sheets.Kind of dimensional transmutation would occur.

Right-handed neutrino has exceptional role in K-D action.

1. Electroweak gauge potentials do not couple to ν_R at all. Therefore the vanishing of W fields is un-necessary if the induced gamma matrices do not mix right handed neutrino with left-handed one. This is guaranteed if M^4 and CP_2 parts of Kähler-Dirac operator annihilate separately right-handed neutrino spinor mode. Also ν_R modes can be interpreted as continuous superpositions of 2-D modes and this allows to define overlap integrals for them and induced spinor fields needed to define WCW gamma matrices and super-generators.
2. For covariantly constant right-handed neutrino mode defining a generator of super-symmetries is certainly a solution of K-D. Whether more general solutions of K-D exist remains to be checked out.

4.3.4 Connection With Number Theoretic Vision?

The interesting potential connection of the Hamilton-Jacobi vision to the number theoretic vision about field equations has been already mentioned.

1. The vision that associativity/co-associativity defines the dynamics of space-time surfaces boils down to M^8-H duality stating that space-time surfaces can be regarded as associative/co-associative surfaces either in M^8 or H [K107, K125]. Associativity reduces to hyper-quaternionicity implying that the tangent/normal space of space-time surface at each point contains preferred sub-space $M^2(x) \subset M^8$ and these sub-spaces forma an integrable distribution. An analogous condition is involved with the definition of Hamilton-Jacobi structure.
2. The octonionic representation of the tangent space of M^8 and H effectively replaces $SO(7,1)$ as tangent space group with its octonionic analog obtained by the replacement of sigma matrices with their octonionic counterparts defined by anti-commutators of gamma matrices. By non-associativity the resulting algebra is not ordinary Lie-algebra and exponentiates to a non-associative analog of Lie group. The original wrong belief was that the reduction takes place to the group G_2 of octonionic automorphisms acting as a subgroup of $SO(7)$. One can ask whether the conditions on the charged part of energy momentum tensor could relate to the reduction of $SO(7,1)$
3. What puts bells ringing is that the Kähler-Dirac equation for the octonionic representation of gamma matrices allows the conservation of electromagnetic charge in the proposed sense. The reason is that the left handed sigma matrices (W charges are left-handed) in the octonionic representation of gamma matrices vanish identically! What remains are vectorial=right-handed em and Z^0 charge which becomes proportional to em charge since its left-handed part vanishes. All spinor modes have a well-defined em charge in the octonionic sense defined by replacing embedding space spinor locally by its octonionic variant? Maybe this could explain why H spinor modes can have well-defined em charge contrary to the naïve expectations.
4. The non-associativity of the octonionic spinors is however a problem. Even non-commutativity poses problems - also at space-time level if one assumes quaternion-real analyticity for the spinor modes. Could one assume commutativity or co-commutativity for the induced spinor modes? This would mean restriction to associative or co-associative 2-surfaces and (hyper-)holomorphic depends on its (hyper-)complex coordinate. The outcome would be a localization to a hyper-commutative of commutative 2-surface, string world sheet or partonic 2-surface.

5. These conditions could also be interpreted by saying that for the Kähler Dirac operator the octonionic induced spinors assumed to be commutative/co-commutative are equivalent with ordinary induced spinors. The well-definedness of em charge for ordinary spinors would correspond to commutativity/co-commutativity for octonionic spinors. Even the Dirac equations based on induced and Kähler-Dirac gamma matrices could be equivalent since it is essentially holomorphy which matters.

To sum up, these considerations inspire to ask whether the associativity/co-associativity of the space-time surface is equivalent with the reduction of the field equations to stringy field equations stating that certain components of the induced metric in complex/Hamilton-Jacobi coordinates vanish in turn guaranteeing that field equations reduce to algebraic identifies following from the fact that energy momentum tensor and second fundamental form have no common components? Commutativity/co-commutativity would characterize fermionic dynamics and would have physical representation as possibility to have em charge eigenspinors. This should be the case if one requires that the two solution ansätze are equivalent.

4.4 Representation Of WCW Metric As Anti-Commutators Of Gamma Matrices Identified As Symplectic Super-Charges

WCW gamma matrices identified as symplectic super Noether charges suggest an elegant representation of WCW metric and Kähler form, which seems to be more practical than the representations in terms of Kähler function or representations guessed by symmetry arguments.

This representation is equivalent with the somewhat dubious representation obtained using symmetry arguments - that is by assuming that the half Poisson brackets of embedding space Hamiltonians defining Kähler form and metric can be lifted to the level of WCW, if the conformal gauge conditions hold true for the spinorial conformal algebra, which is the TGD counterpart of the standard Kac-Moody type algebra of the ordinary strings models. For symplectic algebra the hierarchy of breakings of super-conformal gauge symmetry is possible but not for the standard conformal algebras associated with spinor modes at string world sheets.

4.4.1 Expression For WCW Kähler Metric As Anticommutators As Symplectic Super Charges

During years I have considered several variants for the representation of symplectic Hamiltonians and WCW gamma matrices and each of these proposals have had some weakness. The key question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians.

The original approach based on flux Hamiltonians did not use Noether currents.

1. Magnetic flux Hamiltonians do not refer to the space-time dynamics and imply genuine rather than only effective 2-dimensionality, which is more than one wants. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed, effective 2-dimensionality might be achieved.

   The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians. It seems that this challenge leads to ad hoc constructions.

2. For the purposes of generalization it is useful to give the expression of flux Hamiltonian. Apart from normalization factors one would have

$$Q(H_A) = \int_{X^2} H_A J_{\mu\nu} dx^\mu \wedge dx^\nu \quad .$$

   Here A is a label for the Hamiltonian of $\delta M^4_\pm \times CP_2$ decomposing to product of $\delta M^4_\pm$ and CP_2 Hamiltonians with the first one decomposing to a product of function of the radial light-like coordinate r_M and Hamiltonian depending on S^2 coordinates. It is natural to assume that

Hamiltonians have well- defined $SO(3)$ and $SU(3)$ quantum numbers. This expressions serves as a natural starting point also in the new approach based on Noether charges.

The approach identifying the Hamiltonians as symplectic Noether charges is extremely natural from physics point of view but the fact that it leads to 3-D expressions involving the induced metric led to the conclusion that it cannot work. In hindsight this conclusion seems wrong: I had not yet realized how profound that basic formulas of physics really are. If the generalization of AdS/CFT duality works, Kähler action can be expressed as a sum of string area actions for string world sheets with string area in the effective metric given as the anti-commutator of the Kähler-Dirac gamma matrices for the string world sheet so that also now a reduction of dimension takes place. This is easy to understand if the classical Noether charges vanish for a sub-algebra of symplectic algebra for preferred extremals.

1. If all end points for strings are possible, the recipe for constructing super-conformal generators would be simple. The embedding space Hamiltonian H_A appearing in the expression of the flux Hamiltonian given above would be replaced by the corresponding symplectic quantum Noether charge $Q(H_A)$ associated with the string defined as 1-D integral along the string. By replacing Ψ or its conjugate with a mode of the induced spinor field labeled by electroweak quantum numbers and conformal weight nm one would obtain corresponding super-charged identifiable as WCW gamma matrices. The anti-commutators of the super-charges would give rise to the elements of WCW metric labelled by conformal weights n_1, n_2 not present in the naïve guess for the metric. If one assumes that the fermionic super-conformal symmetries act as gauge symmetries only $n_i = 0$ gives a non-vanishing matrix element.

   Clearly, one would have weaker form of effective 2-dimensionality in the sense that Hamiltonian would be functional of the string emanating from the partonic 2-surface. The quantum Hamiltonian would also carry information about the presence of other wormhole contacts- at least one- when wormhole throats carry Kähler magnetic monopole flux. If only discrete set for the end points for strings is possible one has discrete sum making possible easy p-adicization. It might happen that integrability conditions for the tangent spaces of string world sheets having vanishing W boson fields do not allow all possible strings.

2. The super charges obtained in this manner are not however entirely satisfactory. The problem is that they involve only single string emanating from the partonic 2-surface. The intuitive expectation is that there can be an arbitrarily large number of strings: as the number of strings is increased the resolution improves. Somehow the super-conformal algebra defined by Hamiltonians and super-Hamiltonians should generalize to allow tensor products of the strings providing more physical information about the 3-surface.

3. Here the idea of Yangian symmetry [K116] suggests itself strongly. The notion of Yangian emerges from twistor Grassmann approach and should have a natural place in TGD. In Yangian algebra one has besides product also co-product, which is in some sense "time-reversal" of the product. What is essential is that Yangian algebra is also multi-local.

   The Yangian extension of the super-conformal algebra would be multi-local with respect to the points of partonic surface (or multi-stringy) defining the end points of string. The basic formulas would be schematically

$$O_1^A = f^A_{BC} T^B \otimes T^B \quad ,$$

   where a summation of B, C occurs and f^A_{BC} are the structure constants of the algebra. The operation can be iterated and gives a hierarchy of n-local operators. In the recent case the operators are n-local symplectic super-charges with unit fermion number and symplectic Noether charges with a vanishing fermion number. It would be natural to assume that also the n-local gamma matrix like entities contribute via their anti-commutators to WCW metric and give multi-local information about the partonic 2-surface and 3-surface.

   The operation generating the algebra well-defined if one an assumes that the second quantization of induced spinor fields is carried out using the standard canonical quantization. One could even assume that the points involved belong to different partonic 2-surfaces belonging even at opposite boundaries of CD. The operation is also well-defined if one assumes that induced spinor fields at different space-time points at boundaries of CD always anti-commute. This could make sense at boundary of CD but lead to problems with embedding space-causality if assumed for the spinor modes at opposite boundaries of CD.

4.4.2 Handful Of Problems With A Common Resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose Kähler-Dirac action as their solution.

Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of Kähler-Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K29, K107].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

Super-symmetry forces Kähler-Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k{}_\alpha} L_K \ . \end{aligned} \tag{4.4.1}$$

Here T^α_k is canonical momentum current of Kähler action. If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{4.4.2}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^{\alpha} = \overline{\nu_R} T^{\alpha}_l \Gamma^l \Psi \tag{4.4.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_{\alpha} J^{\alpha k} = \overline{\nu_R} \Gamma^k T^{\alpha}_l \Gamma^l D_{\alpha} \Psi \ . \tag{4.4.4}$$

The requirement that this current vanishes is guaranteed if one assumes that Kähler-Dirac equation

$$\begin{aligned} \hat{\Gamma}^{\alpha} D_{\alpha} \Psi &= 0 \ , \\ \hat{\Gamma}^{\alpha} &= T^{\alpha}_l \Gamma^l \ . \end{aligned} \tag{4.4.5}$$

This equation must be derivable from a Kähler-Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^{\alpha} D_{\alpha} \Psi \ . \tag{4.4.6}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with Kähler-Dirac gamma matrices and the requirement

$$D_{\mu} \hat{\Gamma}^{\mu} = 0 \tag{4.4.7}$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

As a matter fact, any mode of Kähler-Dirac equation contracted with second quantized induced spinor field or its conjugate defines a conserved super charge. Also super-symplectic Noether charges and their super counterparts can be assigned to symplectic generators as Noether charges but they need not be conserved.

Second quantization of the K-D action

Second quantization of Kähler-Dirac action is crucial for the construction of the Kähler metric of world of classical worlds as anti-commutators of gamma matrices identified as super-symplectic Noether charges. To get a unique result, the anti-commutation relations must be fixed uniquely. This has turned out to be far from trivial.

1. *Canonical quantization works after all*

The canonical manner to second quantize fermions identifies spinorial canonical momentum densities and their conjugates as $\overline{\Pi} = \partial L_{K_D}/\partial_{\Psi} = \overline{\Psi}\Gamma^t$ and their conjugates. The vanishing of Γ^t at points, where the induced Kähler form J vanishes can cause problems since anti-commutation relations are not internally consistent anymore. This led me to give up the canonical quantization and to consider various alternatives consistent with the possibility that J vanishes. They were admittedly somewhat ad hoc. Correct (anti-)commutation relations for various fermionic Noether currents seem however to fix the anti-commutation relations to the standard ones. It seems that it is better to be conservative: the canonical method is heavily tested and turned out to work quite nicely.

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Consider first the 4-D situation without the localization to 2-D string world sheets. The canonical anti-commutation relations would state $\{\overline{\Pi}, \Psi\} = \delta^3(x, y)$ at the space-like boundaries of the string world sheet at either boundary of CD. At points where J and thus T^t vanishes, canonical momentum density vanishes identically and the equation seems to be inconsistent.

If fermions are localized at string world sheets assumed to always carry a non-vanishing J at their boundaries at the ends of space-time surfaces, the situation changes since Γ^t is non-vanishing. The localization to string world sheets, which are not vacua saves the situation. The problem is that the limit when string approaches vacuum could be very singular and discontinuous. In the case of elementary particle strings are associated with flux tubes carrying monopole fluxes so that the problem disappears.

It is better to formulate the anti-commutation relations for the modes of the induced spinor field. By starting from

$$\{\overline{\Pi}(x), \Psi(y)\} = \delta^1(x, y) \tag{4.4.8}$$

and contracting with $\Psi(x)$ and $\Pi(y)$ and integrating, one obtains using orthonormality of the modes of Ψ the result

$$\{b_m^\dagger, b_n\} = \gamma^0 \delta_{m,n} \tag{4.4.9}$$

holding for the nodes with non-vanishing norm. At the limit $J \to 0$ there are no modes with non-vanishing norm so that one avoids the conflict between the two sides of the equation.

The proposed anti-commutator would realize the idea that the fermions are massive. The following alternative starts from the assumption of 8-D light-likeness.

2. Does one obtain the analogy of SUSY algebra? In super Poincare algebra anti-commutators of super-generators give translation generator: anti-commutators are proportional to $p^k\sigma_k$. Could it be possible to have an anti-commutator proportional to the contraction of Dirac operator $p^k\sigma_k$ of 4-momentum with quaternionic sigma matrices having or 8-momentum with octonionic 8-matrices?

This would give good hopes that the GRT limit of TGD with many-sheeted space-time replaced with a slightly curved region of M^4 in long length scales has large $\mathcal{N}$ SUSY as an approximate symmetry: $\mathcal{N}$ would correspond to the maximal number of oscillator operators assignable to the partonic 2-surface. If conformal invariance is exact, it is just the number of fermion states for single generation in standard model.

1. The first promising sign is that the action principle indeed assigns a conserved light-like 8-momentum to each fermion line at partonic 2-surface. Therefore octonionic representation of sigma matrices makes sense and the generalization of standard twistorialization of four-momentum also. 8-momentum can be characterized by a pair of octonionic 2-spinors $(\lambda, \overline{\lambda})$ such that one has $\lambda\ \overline{\lambda}) = p^k\sigma_k$.
2. Since fermion line as string boundary is 1-D curve, the corresponding octonionic sub-spaces is just 1-D complex ray in octonion space and imaginary axes is defined by the associated imaginary octonion unit. Non-associativity and non-commutativity play no role and it is as if one had light like momentum in say z-direction.
3. One can select the ininitial values of spinor modes at the ends of fermion lines in such a way that they have well-defined spin and electroweak spin and one can also form linear

superpositions of the spin states. One can also assume that the 8-D algebraic variant of Dirac equation correlating M^4 and CP_2 spins is satisfied.

One can introduce oscillator operators $b^\dagger_{m,\alpha}$ and $b_{n,\alpha}$ with α denoting the spin. The motivation for why electroweak spin is not included as an index is due to the correlation between spin and electroweak spin. Dirac equation at fermion line implies a complete correlation between directions of spin and electroweak spin: if the directions are same for leptons (convention only), they are opposite for antileptons and for quarks since the product of them defines embedding space chirality which distinguishes between quarks and leptons. Instead of introducing electroweak isospin as an additional correlated index one can introduce 4 kinds of oscillator operators: leptonic and quark-like and fermionic and antifermionic.

4. For definiteness one can consider only fermions in leptonic sector. In hope of getting the analog of SUSY algebra one could modify the fermionic anti-commutation relations such that one has

$$\{b^\dagger_{m,\alpha}, b_{n,\beta}\} = \pm i\epsilon_{\alpha\beta}\delta_{m,n} \quad . \tag{4.4.10}$$

Here α is spin label and ϵ is the standard antisymmetric tensor assigned to twistors. The anti-commutator is clearly symmetric also now. The anti-commmutation relations with different signs $\pm$ at the right-hand side distinguish between quarks and leptons and also between fermions and anti-fermions. $\pm = 1$ could be the convention for fermions in lepton sector.

5. One wants combinations of oscillator operators for which one obtains anti-commutators having interpretation in terms of translation generators representing in terms of 8-momentum. The guess would be that the oscillator operators are given by

$$B^\dagger_n = b^\dagger_{m,\alpha}\lambda^\alpha \quad , \quad B_n = \overline{\lambda}^\alpha b_{m,\alpha} \quad . \tag{4.4.11}$$

The anti-commutator would in this case be given by

$$\begin{aligned} &\{B^\dagger_m, B_n\} = i\overline{\lambda}^\alpha\epsilon_{\alpha\beta}\lambda^\beta\delta_{m,n} \\ &= Tr(p^k\sigma_k)\delta_{m,n} = 2p^0\delta_{m,n} \quad . \end{aligned} \tag{4.4.12}$$

The inner product is positive for positive value of energy p^0. This form of anti-commutator obviously breaks Lorentz invariance and this us due the number theoretic selection of preferred time direction as that for real octonion unit. Lorentz invariance is saved by the fact that there is a moduli space for the choices of the quaternion units parameterized by Lorentz boosts for CD.

The anti-commutator vanishes for covariantly constant antineutrino so that it does not generate sparticle states. Only fermions with non-vanishing four-momentum do so and the resulting algebra is very much like that associated with a unitary representation of super Poincare algebra.

6. The recipe gives one helicity state for lepton in given mode m (conformal weight). One has also antilepton with opposite helicity with $\pm = -1$ in the formula defining the anti-commutator. In the similar manner one obtains quarks and antiquarks.

7. Contrary to the hopes, one did not obtain the anti-commutator $p^k\sigma_k$ but $Tr(p^0\sigma_0)$. $2p^0$ is however analogous to the action of Dirac operator $p^k\sigma_k$ to a massless spinor mode with "wrong" helicity giving $2p^0\sigma^0$. Massless modes with wrong helicity are expected to appear in the fermionic propagator lines in TGD variant of twistor approach. Hence one might hope that the resulting algebra is consistent with SUSY limit.

The presence of 8-momentum at each fermion line would allow also to consider the introduction of anti-commutators of form $p^k(8)\sigma_k$ directly making $\mathcal{N} = 8$ SUSY at parton level manifest.

This expression restricts for time-like M^4 momenta always to quaternion and one obtains just the standard picture.

8. Only the fermionic states with vanishing conformal weight seem to be realized if the conformal symmetries associated with the spinor modes are realized as gauge symmetries. Super-generators would correspond to the fermions of single generation standard model: 4+4 =8 states altogether. Interestingly, $\mathcal{N} = 8$ correspond to the maximal SUSY for super-gravity. Right-handed neutrino would obviously generate the least broken SUSY. Also now mixing of M^4 helicities induces massivation and symmetry breaking so that even this SUSY is broken. One must however distinguish this SUSY from the super-symplectic conformal symmetry. The space in which SUSY would be realized would be partonic 2-surfaces and this distinguishes it from the usual SUSY. Also the conservation of fermion number and absence of Majorana spinors is an important distinction.

3. What about quantum deformations of the fermionic oscillator algebra?

Quantum deformation introducing braid statistics is of considerable interest. Quantum deformations are essentially 2-D phenomenon, and the experimental fact that it indeed occurs gives a further strong support for the localization of spinors at string world sheets. If the existence of anyonic phases is taken completely seriously, it supports the existence of the hierarchy of Planck constants and TGD view about dark matter. Note that the localization also at partonic 2-surfaces cannot be excluded yet.

I have wondered whether quantum deformation could relate to the hierarchy of Planck constants in the sense that $n = h_{eff}/h$ corresponds to the value of deformation parameter $q = exp(i2\pi/n)$.

A q-deformation of Clifford algebra of WCW gamma matrices is required. Clifford algebra is characterized in terms of anti-commutators replaced now by q-anticommutators. The natural identification of gamma matrices is as complexified gamma matrices. For q-deformation q-anti-commutators would define WCW Kähler metric. The commutators of the supergenerators should still give anti-symmetric sigma matrices. The q-anticommutation relations should be same in the entire sector of WCW considered and be induced from the q-anticommutation relations for the oscillator operators of induced spinor fields at string world sheets, and reflect the fact that permutation group has braid group as covering group in 2-D case so that braid statistics becomes possible.

In [A57] (`http://tinyurl.com/y9e6pg4d`) the q-deformations of Clifford algebras are discussed, and this discussion seems to apply in TGD framework.

1. It is assumed that a Lie-algebra g has action in the Clifford algebra. The q-deformations of Clifford algebra is required to be consistent with the q-deformation of the universal enveloping algebra Ug.
2. The simplest situation corresponds to group $su(2)$ so that Clifford algebra elements are labelled by spin $\pm 1/2$. In this case the q-anticommutor for creation operators for spin up states reduces to an anti-commutator giving q-deformation I_q of unit matrix but for the spin down states one has genuine q-anti-commutator containing besides I_q also number operator for spin up states at the right hand side.
3. The undeformed anti-commutation relations can be witten as

$$P^{+kl}_{ij} a_k a_l = 0 \ , \quad P^{+kl}_{ij} a^\dagger_k a^\dagger_l = 0 \ , \quad a^i a^\dagger_j + P^{ih}_{jk} a^\dagger_h a^k = \delta^i_j 1 \ . \tag{4.4.13}$$

Here $P^{kl}_{ij} = \delta^i_l \delta^j_k$ is the permutator and $P^{+kl}_{ij} = (1+P)/2$ is projector. The q-deformation reduces to a replacement of the permutator and projector with q-permutator P_q and q-projector and P^+_q, which are both fixed by the quantum group.

4. Also the condition that deformed algebra has same Poincare series as the original one is posed. This says that the representation content is not changed that is the dimensions of summands in a representation as direct sum of graded sub-spaces are same for algebra and its q-deformation. If one has quantum group in a strict sense of the word (quasi-triangularity

(genuine braid group) rather that triangularity requiring that the square of the deformed permutator P_q is unit matrix, one can have two situations.

(a) $g = sl(N)$ (special linear group such as $SL(2,F)$, $F = R,C$) or $g = Sp(N = 2n)$ (symplectic group such as $Sp(2) = SL(2,R)$), which is subgroup of $sl(N)$. Creation (annihilation-) operators must form the N-dimensional defining representation of g.

(b) $g = sl(N)$ and one has direct sum of M N-dimensional defining representations of g. The M copies of representation are ordered so that they can be identified as strands of braid so that the deformation makes sense at the space-like ends of string world sheet naturally. q-projector is proportional to so called universal R-matrix.

5. It is also shown that q-deformed oscillator operators can be expressed as polynomials of the ordinary ones.

The following argument suggest that the g must correspond to the minimal choices $sl(2,R)$ (or $su(2)$) in TGD framework.

1. The q-Clifford algebra structure of WCW should be induced from that for the fermionic oscillator algebra. g cannot correspond to $su(2)_{spin} \times su(2)_{ew}$ since spin and weak isospin label fermionic oscillator operators beside conformal weights but must relate closely to this group. The physical reason is that the separate conservation of quark and lepton numbers and light-likeness in 8-D sense imply correlations between the components of the spinors and reduce g.

2. For a given H-chirality (quark/ lepton) 8-D light-likeness forced by massless Dirac equation at the light-like boundary of the string world sheet at parton orbit implies correlation between M^4 and CP_2 chiralities. Hence there are 4+4 spinor components corresponding to fermions and antifermions with physical (creation oeprators) and unphysical (annihilation operators) polarizations. This allows two creation operators with given H-chirality (quark or lepton) and fermion number. Same holds true for antifermions. By fermion number conservation one obtains a reduction to $SU(2)$ doublets and the quantum group would be $sl(2) = sp(2)$ for which "special linear" implies "symplectic".

4.5 Quantum Criticality And Kähler-Dirac Action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The belief has been that the existence of conserved current for Kähler-Dirac equation are possible if Kähler action is critical for the 3-surface in question in the sense that the deformation in question corresponds to vanishing of second variation of Kähler action. The vanishing of the second variation states that the deformation of the Kähler-Dirac gamma matrix is divergence free just like the Kähler-Dirac gamma matrix itself and is therefore very natural.

2-D conformal invariance accompanies 2-D criticality and allows to satisfy these conditions for spinor modes localized at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This localization is in the generic case forced by the conditions that em charge is well-defined for the spinor modes: this requires that classical W fields vanish and also the vanishing of classical Z^0 field is natural -at least above weak scale. Only 2 Kähler-Dirac gamma matrices can be non-vanishing and this is possible only for Kähler-Dirac action.

4.5.1 What Quantum Criticality Could Mean?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

1. What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the embedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which

in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

2. At more technical level one would expect criticality to correspond to deformations of a given preferred extremal defining a vanishing second variation of Kähler Kähler function or Kähler action.
   (a) For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.
   (b) Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A50]. Cusp catastrophe (see `http://tinyurl.com/yddpfdgo`) [A3] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.
3. Quantum criticality makes sense also for Kähler action.
   (a) Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K43] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.
   (b) Also now one expects a hierarchy of criticalitiesandsince criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.
   (c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?
4. I have discussed what criticality could mean for Kähler-Dirac action [K127].
   (a) I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.
   (b) The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since embedding

space coordinates appear as parameters in Kähler-Dirac action. Kähler-Dirac equation is satisfied if the first variation of the canonical momentum densities contracted with the embedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. One obtains conserved fermion current associated with deformations only if the deformation of the Kähler-Dirac gamma matrix is divergenceless just like the Kähler-Dirac gamma matrix itself. This conditions requires the vanishing of the second variation of Kähler action.

(c) It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the Kähler-Dirac gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor.

This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as a matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied without the restriction to 2-surface.

The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

4.5.2 Quantum Criticality And Fermionic Representation Of Conserved Charges Associated With Second Variations Of Kähler Action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been however slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. The problem is that the embedding space coordinates are in the role of classical external fields and induces spinor fields are second quantized so that it is not at all clear whether one obtains conserved charges.

What does the conservation of the fermionic Noether current require?

The obvious anser to the question of the title is that the conservation of the fermionic current requires the vanishing of the first variation of Kähler-Dirac action with respect to embedding space coordinates. This is certainly true but need not mean vanishing of the second variation of Kähler action as thought first. Hence fermionic conserved currents might be obtained for much more general variations than critical ones.

1. The Kähler-Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the Kähler-Dirac action under this deformation vanishes.

   The vanishing of the first variation for the Kähler-Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the Kähler-Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J_k^\alpha \Psi \ , \\ J_k^\alpha &= \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h_\beta^l}\delta h_\beta^k + \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h^l}\delta h^l \ . \end{aligned} \tag{4.5.1}$$

Here h_β^k denote partial derivative of the embedding space coordinates with respect to space-time coordinates. ΔS_D vanishes if this term vanishes:

$$D_\alpha J_k^\alpha = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J_k^α does not define conserved classical charge in the general case.

2. This condition is however un-necessarily strong. It is enough that that the deformation of Dirac operator anihilates the spinor mode, which can also change in the deformation. It must be possible to compensate the change of the covariant derivative in the deformation by a gauge transformation which requires that deformations act as gauge transformations on induce gauge potentials. This gives additional constraint and strongly suggests Kac-Moody type algebra for the deformations. Conformal transformations would satisfy this constraint and are suggested by quantum criticality.

3. It is essential that the Kähler-Dirac equation holds true so that the Kähler-Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the Kähler-Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D}\times \Gamma^k J_k^\alpha \Psi \ . \tag{4.5.2}$$

Here $1/D$ is the inverse of the Kähler-Dirac operator defining the counterpart of the fermionic propagator.

4. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{4.5.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the Kähler-Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing Kähler-Dirac gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha\delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha\Psi \ . \tag{4.5.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra.

5. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the Kähler-Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.
6. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

It is far from obvious that the criticality conditions or even the weaker conditions guaranteing the existence of (say) isometry charges can be satisfied. It seems that the restriction of spinor modes to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - implied by the condition that em charge is well-define for them, is the manner to achieve this. The reason is that conformal invariance allows complexification of the Kähler-Dirac gamma matrices and allows to construct spinor modes as holomorphic modes and their conjugates. Holomorphy reduces K-D equation to algebraic condition that Γ^z annihilates the spinor mode. If this is true also the deformation of Γ^z then the existince of conserved current follows. It is essential that only two Kähler-Dirac gamma matrices are non-vanishing and this is possible only for Kähler-Dirac action.

About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding second order charges for Kähler action are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates.
2. Contrary to the original conclusion, the corresponding fermionic charges expressible as fermionic bilinears are first order in deformation and do not vanish! Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities and for Kähler-Dirac action as quantal charges.

Critical manifold is infinite-dimensional for Kähler action

Some examples might help to understand what is involved.

1. The action defined by four-volume gives a first glimpse about what one can expect. In this case Kähler-Dirac gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.
2. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical embedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is

one-dimensional, the second variation contains a non-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs_k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

3. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

4. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical embedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

This leads to the conjecture that the critical deformations correspond to sub-algebras of super-conformal algebras with conformal weights coming as integer multiples of fixed integer m. One would have infinite hierarchy of breakings of conformal symmetry labelled by m. The super-conformal algebras would be effectively m-dimensional. Since all commutators with the critical sub-algebra would create zero energy states. In ordinary conformal field theory one have maximal criticality corresponding to $m = 1$.

Critical super-algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the WCW metric.

   The original expectation was that critical deformations correspond to zero modes but this interpretation need not be correct since critical deformations can leave 3-surface invariant but affect corresponding preferred extremal: this would conform with the non-deterministic character of the dynamics which is indeed the basic signature of criticality. Rather, critical deformations are limiting cases of ordinary deformations acting in quantum fluctuating degrees of freedom.

   This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4 and that Kähler action has fourth order terms as first non-vanishing terms in perturbative expansion (for Kähler-Dirac the expansion is quadratic in deformation).

   Therefore the super-conformal algebra associated with the critical deformations has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

Connection with quantum criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question how to make TGD unique if Kähler function for WCW is defined by the Kähler action for a preferred extremal assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obviou idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength is analogous to critical temperature so that TGD would be more or less uniquely defined.

To understand the delicacies it is convenient to consider various variations of Kähler action first.

1. The variation can leave 3-surface invariant but modify space-time surface such that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface and perhaps also at light-like 3-surfaces. In this case the correspondence between X^3 and $X^4(X^3)$ would not be unique and one would have non-deterministic dynamics characteristic for critical systems. This criticality would correspond to criticality of Kähler action at X^3. Note that the original working hypothesis was that $X^4(X^3)$ is unique. The failure of the strict classical determinism implying spin glass type vacuum degeneracy indeed suggets that this is the case.

2. The variation could act on zero modes which do not affect Kähler metric which corresponds to (1, 1) part of Hessian in complex coordinates for WCW . Only the zero modes characterizing 3-surface appearing as parameters in the metric WCW would be affected and the result would be a generalization of conformal transformation. Kähler function would change but only due to the change in zero modes. These transformations do not seem to correspond to critical transformations since Kähler function changes.

3. The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would of course affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at X^3 meaning that (1, 1) part of Hessian is degenerate. This could mean that in the vicinity of X^3 the Kähler form has non-definite signature: physically this is unacceptable since inner product in Hilbert space would not be positive definite.

Critical transformations might relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

1. The critical deformations leave 3-surface X^3 invariant as do also the transformations of H associated with X^3. If H affects $X^4(X^3)$ and corresponds to critical transformations then critical transformation would extend WCW to a bundle for which 3-surfaces would be base points and preferred extremals $X^4(X^3)$ would define the fiber. Gauge invariance with respect to H would generalize the assumption that $X^4(X^3)$ is unique.

2. Critical deformations could correspond to H or sub-group of H (which dependes on X^3). For other 3-surfaces than X^3 the action of H is non-trivial as the case of $CP_2 = SU(3)/U(2)$ makes easy to understand.

3. A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of $\delta M^4 \times CP_2$ and acting as diffeomorphisms for the light-like radial coordinate of δM^4_+. The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies of a given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1. For $m > 1$ one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic

generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. Quantum criticality realized as the vanishing of the second variation gives hopes about a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to deformations of four-surface and second for the Kähler function itself with respect to deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

1. The criticality for preferred extremals would make 4-D criticality a property of all physical systems.
2. The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler metric defined by (1, 1) part of Hessian in complex coordinates must be positive definite. Thus criticality might imply problems.

   This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no zero modes causing troubles in Gaussian approximation to functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function.
3. The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW . These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interaction seems to be that these degrees of freedom correspond to deformations in zero modes.

Both the super-symmetry of D_K and conservation Dirac Noether currents for Kähler-Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.
2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$.

   The formulation of quantal version of Equivalence Principle (EP) in string picture demonstrates that the conservation of of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires that the deformation of the Kähler-Dirac equation obtained by replacing Kähler-Dirac gamma matrices with their deformations is also satisfied. Holomorphy can guarantee this. The original wrong conclusion was that this condition is equivalent with much stronger condition stating the vanishing of the second variation of Kähler action, which it is not. There is analogy for this: massless Dirac equation does not imply the vanishing of four-momentum.

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the embedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the Kähler-Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K43] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. Does this criticality have anything to do with the criticality against the phase transitions changing the value of Planck constant? If the geodesic sphere S^2_I for which induced Kähler form vanishes corresponds to the back of the CP_2 book (as one expects), this could be the case. The homologically non-trivial geodesic sphere S^12_{II} is as far as possible from vacuum extremals. If it corresponds to the back of CP_2 book, cosmic strings would be quantum critical with respect to phase transition changing Planck constant. They cannot however correspond to preferred extremals.

4.5.3 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean

fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [B7] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

4.5.4 Quantum Criticality And Electroweak Symmetries

In the following quantum criticali and electroweak symmetries are discussed for Kähler-Dirac action.

What does one mean with quantum criticality?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

1. What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the embedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

2. At more technical level one would expect criticality to corresponds to deformations of a given preferred extremal defining a vanishing second variation of Kähler Kähler function or Kähler action.

   (a) For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

(b) Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A50]. Cusp catastrophe (see `http://tinyurl.com/yddpfdgo`) [A3] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

3. Quantum criticality makes sense also for Kähler action.

   (a) Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K43] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

   (b) Also now one expects a hierarchy of criticalitiesandsince criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

   (c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. I have discussed what criticality could mean for Kähler-Dirac action [K127].

   (a) I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

   (b) The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since embedding space coordinates appear as parameters in Kähler-Dirac action. The existence of conserved currents does not actually require the vanishing of the second variation of Kähler action as claimed earlier. It is enough that the first variation of the canonical momentum densities contracted with the embedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. Hence conserved currents are obtained also outside the quantum criticality.

   (c) It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generaic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for currents associated with the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor. This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteeing vanishing of classical W fields can be satisfied.

In the following these arguments are formulated more precisely. The unexpected result is that critical deformations induce conformal scalings of the modified metric and electro-weak gauge transformations of the induced spinor connection at X^2. Therefore holomorphy brings in the Kac-Moody symmetries associated with isometries of H (gravitation and color gauge group) and quantum criticality those associated with the holonomies of H (electro-weak-gauge group) as additional symmetries.

The variation of modes of the induced spinor field in a variation of space-time surface respecting the preferred extremal property

Consider first the variation of the induced spinor field in a variation of space-time surface respecting the preferred extremal property. The deformation must be such that the deformed Kähler-Dirac operator D annihilates the modified mode. By writing explicitly the variation of the Kähler-Dirac action (the action vanishes by Kähler-Dirac equation) one obtains deformations and requiring its vanishing one obtains

$$\delta\Psi = D^{-1}(\delta D)\Psi \ . \tag{4.5.5}$$

D^{-1} is the inverse of the Kähler-Dirac operator defining the analog of Dirac propagator and δD defines vertex completely analogous to $\gamma^k\delta A_k$ in gauge theory context. The functional integral over preferred extremals can be carried out perturbatively by expressing δD in terms of δh^k and one obtains stringy perturbation theory around X^2 associated with the preferred extremal defining maximum of Kähler function in Euclidian region and extremum of Kähler action in Minkowskian region (stationary phase approximation).

What one obtains is stringy perturbation theory for calculating n-points functions for fermions at the ends of braid strands located at partonic 2-surfaces and representing intersections of string world sheets and partonic 2-surfaces at the light-like boundaries of CDs. δD- or more precisely, its partial derivatives with respect to functional integration variables - appear atthe vertices located anywhere in the interior of X^2 with outgoing fermions at braid ends. Bosonic propagators are replaced with correlation functions for δh^k. Fermionic propagator is defined by D^{-1}.

After 35 years or hard work this provides for the first time a reasonably explicit formula for the N-point functions of fermions. This is enough since by bosonic emergence these N-point functions define the basic building blocks of the scattering amplitudes. Note that bosonic emergence states that bosons corresponds to wormhole contacts with fermion and anti-fermion at the opposite wormhole throats.

What critical modes could mean for the induced spinor fields?

What critical modes could mean for the induced spinor fields at string world sheets and partonic 2-surfaces. The problematic part seems to be the variation of the Kähler-Dirac operator since it involves gradient. One cannot require that covariant derivative remains invariant since this would require that the components of the induced spinor connection remain invariant and this is quite too restrictive condition. Right handed neutrino solutions de-localized into entire X^2 are however an exception since they have no electro-weak gauge couplings and in this case the condition is obvious: Kähler-Dirac gamma matrices suffer a local scaling for critical deformations:

$$\delta\Gamma^\mu = \Lambda(x)\Gamma^\mu \ . \tag{4.5.6}$$

This guarantees that the Kähler-Dirac operator D is mapped to ΛD and still annihilates the modes of ν_R labelled by conformal weight, which thus remain unchanged.

What is the situation for the 2-D modes located at string world sheets? The condition is obvious. Ψ suffers an electro-weak gauge transformation as does also the induced spinor connection so that D_μ is not affected at all. Criticality condition states that the deformation of the space-time surfaces induces a conformal scaling of Γ^μ at X^2. It might be possible to continue this conformal scaling of the entire space-time sheet but this might be not necessary and this would mean that all critical deformations induced conformal transformations of the effective metric of the space-time surface defined by $\{\Gamma^\mu, \Gamma^\nu\} = 2G^{\mu\nu}$. Thus it seems that effective metric is indeed central concept (recall that if the conjectured quaternionic structure is associated with the effective metric, it might be possible to avoid problem related to the Minkowskian signature in an elegant manner).

In fact, one can consider even more general action of critical deformation: the modes of the induced spinor field would be mixed together in the infinitesimal deformation besides infinitesimal electroweak gauge transformation, which is same for all modes. This would extend electroweak gauge symmetry. Kähler-Dirac equation holds true also for these deformations. One might wonder whether the conjectured dynamically generated gauge symmetries assignable to finite measurement resolution could be generated in this manner.

The infinitesimal generator of a critical deformation J_M can be expressed as tensor product of matrix A_M acting in the space of zero modes and of a generator of infinitesimal electro-weak gauge transformation $T_M(x)$ acting in the same manner on all modes: $J_M = A_M \otimes T_M(x)$. A_M is a spatially constant matrix and $T_M(x)$ decomposes to a direct sum of left- and right-handed $SU(2)\times U(1)$ Lie-algebra generators. Left-handed Lie-algebra generator can be regarded as a quaternion and right handed as a complex number. One can speak of a direct sum of left-handed local quaternion $q_{M,L}$ and right-handed local complex number $c_{M,R}$. The commutator $[J_M, J_N]$ is given by $[J_M, J_N] = [A_M, A_N] \otimes \{T_M(x), T_N(x)\} + \{A_M, A_N\} \otimes [T_M(x), T_N(x)]$. One has $\{T_M(x), T_N(x)\} = \{q_{M,L}(x), q_{N,L}(x)\} \oplus \{c_{M,R}(x), c_{N,R}(x)\}$ and $[T_M(x), T_N(x)] = [q_{M,L}(x), q_{N,L}(x)]$. The commutators make sense also for more general gauge group but quaternion/complex number property might have some deeper role.

Thus the critical deformations would induce conformal scalings of the effective metric and dynamical electro-weak gauge transformations. Electro-weak gauge symmetry would be a dynamical symmetry restricted to string world sheets and partonic 2-surfaces rather than acting at the entire space-time surface. For 4-D de-localized right-handed neutrino modes the conformal scalings of the effective metric are analogous to the conformal transformations of M^4 for $\mathcal{N} = 4$ SYMs. Also ordinary conformal symmetries of M^4 could be present for string world sheets and could act as symmetries of generalized Feynman graphs since even virtual wormhole throats are massless. An interesting question is whether the conformal invariance associated with the effective metric is the analog of dual conformal invariance in $\mathcal{N} = 4$ theories.

Critical deformations of space-time surface are accompanied by conserved fermionic currents. By using standard Noetherian formulas one can write

$$J_i^\mu = \overline{\Psi}\Gamma^\mu \delta_i \Psi + \delta_i \overline{\Psi}\Gamma^\mu \Psi \ . \tag{4.5.7}$$

Here $\delta\Psi_i$ denotes derivative of the variation with respect to a group parameter labeled by i. Since $\delta\Psi_i$ reduces to an infinitesimal gauge transformation of Ψ induced by deformation, these currents are the analogs of gauge currents. The integrals of these currents along the braid strands at the ends of string world sheets define the analogs of gauge charges. The interpretation as Kac-Moody charges is also very attractive and I have proposed that the 2-D Hodge duals of gauge potentials could be identified as Kac-Moody currents. If so, the 2-D Hodge duals of J would define the quantum analogs of dynamical electro-weak gauge fields and Kac-Moody charge could be also seen as non-integral phase factor associated with the braid strand in Abelian approximation (the interpretation in terms of finite measurement resolution is discussed earlier).

One can also define super currents by replacing $\overline{\Psi}$ or Ψ by a particular mode of the induced spinor field as well as c-number valued currents by performing the replacement for both $\overline{\Psi}$ or Ψ. As expected, one obtains a super-conformal algebra with all modes of induced spinor fields acting as generators of super-symmetries restricted to 2-D surfaces. The number of the charges which do not annihilate physical states as also the effective number of fermionic modes could be finite and this would suggest that the integer $\mathcal{N}$ for the supersymmetry in question is finite. This would conform with the earlier proposal inspired by the notion of finite measurement resolution implying the replacement of the partonic 2-surfaces with collections of braid ends.

Note that Kac-Moody charges might be associated with "long" braid strands connecting different wormhole throats as well as short braid strands connecting opposite throats of wormhole contacts. Both kinds of charges would appear in the theory.

What is the interpretation of the critical deformations?

Critical deformations bring in an additional gauge symmetry. Certainly not all possible gauge transformations are induced by the deformations of preferred extremals and a good guess is that they correspond to holomorphic gauge group elements as in theories with Kac-Moody symmetry. What is the physical character of this dynamical gauge symmetry?

1. Do the gauge charges vanish? Do they annihilate the physical states? Do only their positive energy parts annihilate the states so that one has a situation characteristic for the representation of Kac-Moody algebras. Or could some of these charges be analogous to the gauge charges associated with the constant gauge transformations in gauge theories and be therefore non-vanishing in the absence of confinement. Now one has electro-weak gauge charges and these should be non-vanishing. Can one assign them to deformations with a vanishing conformal weight and the remaining deformations to those with non-vanishing conformal weight and acting like Kac-Moody generators on the physical states?

2. The simplest option is that the critical Kac-Moody charges/gauge charges with non-vanishing positive conformal weight annihilate the physical states. Critical degrees of freedom would not disappear but make their presence known via the states labelled by different gauge charges assignable to critical deformations with vanishing conformal weight. Note that constant gauge transformations can be said to break the gauge symmetry also in the ordinary gauge theories unless one has confinement.

3. The hierarchy of quantum criticalities suggests however entire hierarchy of electro-weak Kac-Moody algebras. Does this mean a hierarchy of electro-weak symmetries breakings in which the number of Kac-Moody generators not annihilating the physical states gradually increases as also modes with a higher value of positive conformal weight fail to annihilate the physical state?

   The only manner to have a hierarchy of algebras is by assuming that only the generators satisfying $n \ mod \ N = 0$ define the sub-Kac-Moody algebra annihilating the physical states so that the generators with $n \ mod \ N \neq 0$ would define the analogs of gauge charges. I have suggested for long time ago the relevance of kind of fractal hierarchy of Kac-Moody and Super-Virasoro algebras for TGD but failed to imagine any concrete realization.

   A stronger condition would be that the algebra reduces to a finite dimensional algebra in the sense that the actions of generators Q_n and Q_{n+kN} are identical. This would correspond to periodic boundary conditions in the space of conformal weights. The notion of finite measurement resolution suggests that the number of independent fermionic oscillator operators is proportional to the number of braid ends so that an effective reduction to a finite algebra is expected.

   Whatever the correct interpretation is, this would obviously refine the usual view about electro-weak symmetry breaking.

These arguments suggests the following overall view. The holomorphy of spinor modes gives rise to Kac-Moody algebra defined by isometries and includes besides Minkowskian generators associated with gravitation also SU(3) generators associated with color symmetries. Vanishing second variations in turn define electro-weak Kac-Moody type algebra.

Note that criticality suggests that one must perform functional integral over WCW by decomposing it to an integral over zero modes for which deformations of X^4 induce only an electro-weak gauge transformation of the induced spinor field and to an integral over moduli corresponding to the remaining degrees of freedom.

4.5.5 The Emergence Of Yangian Symmetry And Gauge Potentials As Duals Of Kac-Moody Currents

Yangian symmetry plays a key role in $\mathcal{N} = 4$ super-symmetric gauge theories. What is special in Yangian symmetry is that the algebra contains also multi-local generators. In TGD framework multi-locality would naturally correspond to that with respect to partonic 2-surfaces and string world sheets and the proposal has been that the Super-Kac-Moody algebras assignable to string worlds sheets could generalize to Yangian.

Witten has written a beautiful exposition of Yangian algebras [B22]. Yangian is generated by two kinds of generators J^A and Q^A by a repeated formation of commutators. The number of commutations tells the integer characterizing the multi-locality and provides the Yangian algebra with grading by natural numbers. Witten describes a 2-dimensional QFT like situation in which one has 2-D situation and Kac-Moody currents assignable to real axis define the Kac-Moody charges as integrals in the usual manner. It is also assumed that the gauge potentials defined by the 1-form associated with the Kac-Moody current define a flat connection:

$$\partial_\mu j^A_\nu - \partial_\nu j^A_\nu + [j^A_\mu, j^A_\nu] = 0 \ . \tag{4.5.8}$$

This condition guarantees that the generators of Yangian are conserved charges. One can however consider alternative ways to obtain the conservation.

1. The generators of first kind - call them J^A - are just the conserved Kac-Moody charges. The formula is given by

$$J_A = \int_{-\infty}^{\infty} dx j^{A0}(x,t) \ . \tag{4.5.9}$$

2. The generators of second kind contain bi-local part. They are convolutions of generators of first kind associated with different points of string described as real axis. In the basic formula one has integration over the point of real axis.

$$Q^A = f^A_{BC} \int_{-\infty}^{\infty} dx \int_{x}^{\infty} dy j^{B0}(x,t) j^{C0}(y,t) - 2\int_{-\infty}^{\infty} j^A_x dx \ . \tag{4.5.10}$$

These charges are indeed conserved if the curvature form is vanishing as a little calculation shows.

How to generalize this to the recent context?

1. The Kac-Moody charges would be associated with the braid strands connecting two partonic 2-surfaces - Strands would be located either at the space-like 3-surfaces at the ends of the space-time surface or at light-like 3-surfaces connecting the ends. Kähler-Dirac equation would define Super-Kac-Moody charges as standard Noether charges. Super charges would be obtained by replacing the second quantized spinor field or its conjugate in the fermionic bilinear by particular mode of the spinor field. By replacing both spinor field and its conjugate by its mode one would obtain a conserved c-number charge corresponding to an anti-commutator of two fermionic super-charges. The convolution involving double integral is however not number theoretically attractive whereas single 1-D integrals might make sense.

2. An encouraging observation is that the Hodge dual of the Kac-Moody current defines the analog of gauge potential and exponents of the conserved Kac-Moody charges could be identified as analogs for the non-integrable phase factors for the components of this gauge potential. This identification is precise only in the approximation that generators commute since only in this case the ordered integral $P(exp(i\int Adx))$ reduces to $P(exp(i\int Adx))$.Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization implying that Abelian approximation works. This conforms with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

This would make possible a direct identification of Kac-Moody symmetries in terms of gauge symmetries. For isometries one would obtain color gauge potentials and the analogs of gauge potentials for graviton field (in TGD framework the contraction with M^4 vierbein would transform tensor field to 4 vector fields). For Kac-Moody generators corresponding to holonomies one would obtain electroweak gauge potentials. Note that super-charges would give rise to a collection of spartners of gauge potentials automatically. One would obtain a badly broken SUSY with very large value of $\mathcal{N}$ defined by the number of spinor modes as indeed speculated earlier [?].

3. The condition that the gauge field defined by 1-forms associated with the Kac-Moody currents are trivial looks unphysical since it would give rise to the analog of topological QFT with gauge potentials defined by the Kac-Moody charges. For the duals of Kac-Moody currents defining gauge potentials only covariant divergence vanishes implying that curvature form is

$$F_{\alpha\beta} = \epsilon_{\alpha\beta}[j_\mu, j^\mu] \ , \tag{4.5.11}$$

so that the situation does not reduce to topological QFT unless the induced metric is diagonal. This is not the case in general for string world sheets.

4. It seems however that there is no need to assume that j_μ defines a flat connection. Witten mentions that although the discretization in the definition of J^A does not seem to be possible, it makes sense for Q^A in the case of $G = SU(N)$ for any representation of G. For general G and its general representation there exists no satisfactory definition of Q. For certain representations, such as the fundamental representation of $SU(N)$, the definition of Q^A is especially simple. One just takes the bi-local part of the previous formula:

$$Q^A = f^A_{BC} \sum_{i<j} J^B_i J^C_j \ . \tag{4.5.12}$$

What is remarkable that in this formula the summation need not refer to a discretized point of braid but to braid strands ordered by the label i by requiring that they form a connected polygon. Therefore the definition of J^A could be just as above.

5. This brings strongly in mind the interpretation in terms of twistor diagrams. Yangian would be identified as the algebra generated by the logarithms of non-integrable phase factors in Abelian approximation assigned with pairs of partonic 2-surfaces defined in terms of Kac-Moody currents assigned with the Kähler-Dirac action. Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization. This would fit nicely with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

   The resulting algebra satisfies the basic commutation relations

$$\left[J^A, J^B\right] = f^{AB}_C J^C \ , \ \left[J^A, Q^B\right] = f^{AB}_C Q^C \ . \tag{4.5.13}$$

plus the rather complex Serre relations described in [B22].

4.6 Kähler-Dirac Equation And Super-Symmetries

The previous considerations concerning super-conformal symmetries and space-time SUSY have been based on general arguments. The new vision about preferred extremals and Kähler-Dirac equation however leads to a rather detailed understanding of super-conformal symmetries at the level of field equations and is bound to modify the existing vision about super-conformal symmetries.

Whether TGD predicts some variant of space-time SUSY or not has been a long-standing issue: the reason is that TGD does not allow Majorana spinors since fermion number conservation is exact. The more precise formulation of field equations made possible by the realization that spinor modes are localized at string world sheets allows to conclude that the analog of broken $\mathcal{N} = 8$ SUSY is predicted at parton level and that right-handed neutrino generates the minimally broken $\mathcal{N} = 2$ sub-SUSY.

One important outcome of criticality is the identification of gauge potentials as duals of Kac-Moody currents at the boundaries of string world sheets: quantum gauge potentials are defined only where they are needed that is string curves defining the non-integrable phase factors. This gives also rise to the realization of the conjectured Yangian in terms of the Kac-Moody charges and commutators in accordance with the earlier conjecture.

4.6.1 Super-Conformal Symmetries

It is good to summarize first the basic ideas about Super-Virasoro representations. TGD allows two kinds of super-conformal symmetries.

1. The first super-conformal symmetry is associated with $\delta M^4_\pm \times CP_2$ and corresponds to symplectic symmetries of $\delta M^4_\pm \times CP_2$. The reason for extension of conformal symmetries is metric 2-dimensionality of the light-like boundary $\delta M^4_\pm$ defining upper/lower boundary of causal diamond (CD). This super-conformal symmetry is something new and corresponds to replacing finite-dimensional Lie-group G for Kac-Moody symmetry with infinite-dimensional symplectic group. The light-like radial coordinate of $\delta M^4_\pm$ takes the role of the real part of complex coordinate z for ordinary conformal symmetry. Together with complex coordinate of S^2 it defines 3-D restriction of Hamilton-Jacobi variant of 4-D super-conformal symmetries. One can continue the conformal symmetries from light-cone boundary to CD by forming a slicing by parallel copies of $\delta M^4_\pm$. There are two possible slicings corresponding to the choices δM^4_+ and δM^4_- assignable to the upper and lower boundaries of CD. These two choices correspond to two arrows of geometric time for the basis of zero energy states in ZEO.

2. Super-symplectic degrees of freedom determine the electroweak and color quantum numbers of elementary particles. Bosonic emergence implies that ground states assignable to partonic 2-surfaces correspond to partial waves in $\delta M^4_\pm$ and one obtains color partial waves in particular. These partial waves correspond to the solutions for the Dirac equation in embedding space and the correlation between color and electroweak quantum numbers is not quite correct. Super-Kac-Moody generators give the compensating color for massless states obtained from tachyonic ground states guaranteeing that standard correlation is obtained. Super-symplectic degrees are therefore directly visible in particle spectrum. One can say that at the point-like limit the WCW spinors reduce to tensor products of embedding space spinors assignable to the center of mass degrees of freedom for the partonic 2-surfaces defining wormhole throats.

   I have proposed a physical interpretation of super-symplectic vibrational degrees of freedom in terms of degrees of freedom assignable to non-perturbative QCD. These degrees of freedom would be responsible for most of the baryon masses but their theoretical understanding is lacking in QCD framework.

3. The second super-conformal symmetry is assigned light-like 3-surfaces and to the isometries and holonomies of the embedding space and is analogous to the super-Kac-Moody symmetry of string models. Kac-Moody symmetries could be assigned to the light-like deformations of light-like 3-surfaces. Isometries give tensor factor $E^2 \times SU(3)$ and holonomies factor $SU(2)_L \times U(1)$. Altogether one has 5 tensor factors to super-conformal algebra. That the number is just five is essential for the success p-adic mass calculations [K76, K65].

   The construction of solutions of the Kähler-Dirac equation suggests strongly that the fermionic representation of the Super-Kac-Moody algebra can be assigned as conserved charges associated with the space-like braid strands at both the 3-D space-like ends of space-time surfaces and with the light-like (or space-like with a small deformation) associated with the light-like 3-surfaces. The extension to Yangian algebra involving higher multi-linears of super-Kac Moody generators is also highly suggestive. These charges would be non-local and assignable to several wormhole contacts simultaneously. The ends of braids would correspond points of partonic 2-surfaces defining a discretization of the partonic 2-surface having interpretation in terms of finite measurement resolution.

   These symmetries would correspond to electroweak and strong gauge fields and to gravitation. The duals of the currents giving rise to Kac-Moody charges would define the counterparts of gauge potentials and the conserved Kac-Moody charges would define the counterparts of non-integrable phase factors in gauge theories. The higher Yangian charges would define generalization of non-integrable phase factors. This would suggest a rather direct connection with the twistorial program for calculating the scattering amplitudes implies also by zero energy ontology.

Quantization recipes have worked in the case of super-string models and one can ask whether the application of quantization to the coefficients of powers of complex coordinates or Hamilton-Jacobi coordinates could lead to the understanding of the 4-D variants of the conformal symmetries and give detailed information about the representations of the Kac-Moody algebra too.

4.6.2 WCW Geometry And Super-Conformal Symmetries

The vision about the geometry of WCW has been roughly the following and the recent steps of progress induce to it only small modifications if any.

1. Kähler geometry is forced by the condition that hermitian conjugation allows geometrization. Kähler function is given by the Kähler action coming from space-time regions with Euclidian signature of the induced metric identifiable as lines of generalized Feynman diagrams. Minkowskian regions give imaginary contribution identifiable as the analog of Morse function and implying interference effects and stationary phase approximation. The vision about quantum TGD as almost topological QFT inspires the proposal that Kähler action reduces to 3-D terms reducing to Chern-Simons terms by the weak form of electric-magnetic duality. The recent proposal for preferred extremals is consistent with this property realizing also holography implied by general coordinate invariance. Strong form of general coordinate invariance implying effective 2-dimensionality in turn suggests that Kähler action is expressible string world sheets and possibly also areas of partonic 2-surfaces.
2. The complexified gamma matrices of WCW come as hermitian conjugate pairs and anti-commute to the Kähler metric of WCW . Also bosonic generators of symplectic transformations of $\delta M^4_\pm \times CP_2$ a assumed to act as isometries of WCW geometry can be complexified and appear as similar pairs. The action of isometry generators co-incides with that of symplectic generators at partonic 2-surfaces and string world sheets but elsewhere inside the space-time surface it is expected to be deformed from the symplectic action. The super-conformal transformations of $\delta M^4_\pm \times CP_2$ acting on the light-like radial coordinate of $\delta M^4_\pm$ act as gauge symmetries of the geometry meaning that the corresponding WCW vector fields have zero norm.
3. WCW geometry has also zero modes which by definition do not contribute to WCW metric expect possibly by the dependence of the elements of WCW metric on zero modes through a conformal factor. In particular, induced CP_2 Kähler form and its analog for sphere $r_M = constant$ of light cone boundary are symplectic invariants, and one can define an infinite number of zero modes as invariants defined by Kähler fluxes over partonic 2-surfaces and string world sheets. This requires however the slicing of CD parallel copies of δM^4_+ or δM^4_-. The physical interpretation of these non-quantum fluctuating degrees of freedom is as classical variables necessary for the interpretation of quantum measurement theory. Classical variable would metaphorically correspond the position of the pointer of the measurement instrument.
4. The construction receives a strong philosophical inspiration from the geometry of loop spaces. Loop spaces allow a unique Kähler geometry with maximal isometry group identifiable as Kac-Moody group. The reason is that otherwise Riemann connection does not exist. The only problem is that curvature scalar diverges since the Riemann tensor is by constant curvature property proportional to the metric. In 3-D case one would have union of constant curvature spaces labelled by zero modes and the situation is expected to be even more restrictive. The conjecture indeed is that WCW geometry exists only for $H = M^4 \times CP_2$: infinite-D Kähler geometric existence and therefore physics would be unique. One can also hope that Ricci scalar is finite and therefore zero by the constant curvature property so that Einstein's equations are satisfied.
5. The matrix elements of WCW Kähler metric are given in terms of the anti-commutators of the fermionic Noether super-charges associated with symplectic isometry currents. A given mode

of induced spinor field characterized by embedding space chirality (quark or lepton), by spin and weak spin plus conformal weight n. If the super-conformal transformations for string modes act gauge transformations only the spinor modes with vanishing conformal weight correspond to non-zero modes of the WCW metric and the situation reduces essentially to the analog of $\mathcal{N}=8$ SUSY.

The WCW Hamiltonians generating symplectic isometries correspond to the Hamiltonians spanning the symplectic group of $\delta M^4_\pm \times CP_2$. One can say that the space of quantum fluctuating degrees of freedom is this symplectic group of $\delta M^4_\pm \times CP_2$ or its subgroup or coset space: this must have very deep implications for the structure of the quantum TGD.

An interesting possibility is that the radial conformal weights of the symplectic algebra are linear combinations of the zeros of Riemann Zeta with integer coefficients. Also this option allows to realize the hierarchy of super-symplectic conformal symmetry breakings in terms of sub-algebras isomorphic to the entire super-symplectic algebra. WCW would have fractal structure corresponding to a hierarchy of quantum criticalities.

6. The localization of the induced spinors to string world sheets means that the super-symplectic Noether charges are associated with strings connecting partonic 2-surfaces. The physically obvious fact that given partonic surface can be accompanied by an arbitrary number of strings, forces a generalization of the super-symplectic algebra to a Yangian containing infinite number of n-local variants of various super-symplectic Noether charges. For instance, four -momentum is accompanied by multi-stringy variants involving four-momentum P_0^A and angular momentum generators. At the first level of the hierarchy one has $P_1^A = f^A_{BC} P_0^B \otimes J^C$. This hierarchy might play crucial role in understanding of the four-momenta of bound states.

7. Zero energy ontology brings in additional delicacies. Basic objects are now unions of partonic 2-surfaces at the ends of CD. One can generalize the expressions for the isometry generators in a straightforward manner by requiring that given isometry restricts to a symplectic transformation at partonic 2-surfaces and string world sheets.

8. One could criticize the effective metric 2-dimensionality forced by the general consistency arguments as something non-physical. The WCW Hamiltonians are expressed using only the data at partonic 2-surfaces and string string world sheets: this includes also 4-D tangent space data via the weak form of electric-magnetic duality so that one has only effective 2-dimensionality. Obviously WCW geometry must huge large gauge symmetries besides zero modes. The hierarchy of super-symplectic symmetries indeed represent gauge symmetries of this kind.

   Effective 2-dimensionality realizing strong form of holography in turn is induced by the strong form of general coordinate invariance. Light-like 3-surfaces at which the signature of the induced metric changes must be equivalent with the 3-D space-like ends of space-time surfaces at the light-boundaries of space-time surfaces as far as WCW geometry is considered. This requires that the data from their 2-D intersections defining partonic 2-surfaces should dictate the WCW geometry. Note however that Super-Kac-Moody charges giving information about the interiors of 3-surfaces appear in the construction of the physical states.

4.6.3 The Relationship Between Inertial Gravitational Masses

The relationship between inertial and gravitational masses and Equivalence Principle have been on of the longstanding problems in TGD. Not surprisingly, the realization how GRT space-time relates to the many-sheeted space-time of TGD finally allowed to solve the problem.

ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

1. At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.
2. The most recent view (2014) about understanding how EP emerges in TGD is described in [K121] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).
3. The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the Kähler-Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD spacetime and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore: this approach is not promising.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

Constraints from p-adic mass calculations and ZEO

A further important physical input comes from p-adic thermodynamics forming a core element of p-adic mass calculations.

1. The first thing that one can get worried about relates to the extension of conformal symmetries. If the conformal symmetries generalize to $D = 4$, how can one take seriously the results of p-adic mass calculations based on 2-D conformal invariance? There is no reason to worry. The reduction of the conformal invariance to 2-D one for the preferred extremals takes care of this problem. This however requires that the fermionic contributions assignable to string world sheets and/or partonic 2-surfaces - Super- Kac-Moody contributions - should dictate the elementary particle masses. For hadrons also symplectic contributions should be present. This is a valuable hint in attempts to identify the mathematical structure in more detail.

2. ZEO suggests that all particles, even virtual ones correspond to massless wormhole throats carrying fermions. As a consequence, twistor approach would work and the kinematical constraints to vertices would allow the cancellation of divergences. This would suggest that the p-adic thermal expectation value is for the longitudinal M^2 momentum squared (the definition of CD selects $M^1 \subset M^2 \subset M^4$ as also does number theoretic vision). Also propagator would be determined by M^2 momentum. Lorentz invariance would be obtained by integration of the moduli for CD including also Lorentz boosts of CD.

3. In the original approach one allows states with arbitrary large values of L_0 as physical states. Usually one would require that L_0 annihilates the states. In the calculations however mass squared was assumed to be proportional L_0 apart from vacuum contribution. This is a questionable assumption. ZEO suggests that total mass squared vanishes and that one can decompose mass squared to a sum of longitudinal and transversal parts. If one can do the same decomposition to longitudinal and transverse parts also for the Super Virasoro algebra then one can calculate longitudinal mass squared as a p-adic thermal expectation in the transversal super-Virasoro algebra and only states with $L_0 = 0$ would contribute and one would have conformal invariance in the standard sense.

4. In the original approach the assumption motivated by Lorentz invariance has been that mass squared is replaced with conformal weight in thermodynamics, and that one first calculates the thermal average of the conformal weight and then equates it with mass squared. This assumption is somewhat ad hoc. ZEO however suggests an alternative interpretation in which one has zero energy states for which longitudinal mass squared of positive energy state derive from p-adic thermodynamics. Thermodynamics - or rather, its square root - would become part of quantum theory in ZEO. M-matrix is indeed product of hermitian square root of density matrix multiplied by unitary S-matrix and defines the entanglement coefficients between positive and negative energy parts of zero energy state.
5. The crucial constraint is that the number of super-conformal tensor factors is $N = 5$: this suggests that thermodynamics applied in Super-Kac-Moody degrees of freedom assignable to string world sheets is enough, when one is interested in the masses of fermions and gauge bosons. Super-symplectic degrees of freedom can also contribute and determine the dominant contribution to baryon masses. Should also this contribution obey p-adic thermodynamics in the case when it is present? Or does the very fact that this contribution need not be present mean that it is not thermal? The symplectic contribution should correspond to hadronic p-adic length prime rather the one assignable to (say) u quark. Hadronic p-adic mass squared and partonic p-adic mass squared cannot be summed since primes are different. If one accepts the basic rules [K78], longitudinal energy and momentum are additive as indeed assumed in perturbative QCD.
6. Calculations work if the vacuum expectation value of the mass squared must be assumed to be tachyonic. There are two options depending on whether one whether p-adic thermodynamics gives total mass squared or longitudinal mass squared.
   (a) One could argue that the total mass squared has naturally tachyonic ground state expectation since for massless extremals longitudinal momentum is light-like and transversal momentum squared is necessary present and non-vanishing by the localization to topological light ray of finite thickness of order p-adic length scale. Transversal degrees of freedom would be modeled with a particle in a box.
   (b) If longitudinal mass squared is what is calculated, the condition would require that transversal momentum squared is negative so that instead of plane wave like behavior exponential damping would be required. This would conform with the localization in transversal degrees of freedom.

4.6.4 Realization Of Space-Time SUSY In TGD

The generators of super-conformal algebras are obtained by taking fermionic currents for second quantized fermions and replacing either fermion field or its conjugate with its particular mode. The resulting super currents are conserved and define super charges. By replacing both fermion and its conjugate with modes one obtains c-number valued currents. In this manner one also obtains the analogs of super-Poincare generators labelled by the conformal weight and other spin quantum numbers as Noether charges so that space-time SUSY is suggestive.

The super-conformal invariance in spinor modes is expected to be gauge symmetry so that only the generators with vanishing string world sheet conformal weight create physical states. This would leave only the conformal quantum numbers characterizing super-symplectic generators (radial conformal weight included) under consideration and the hierarchy of its sub-algebras acting as gauge symmetries giving rise to a hierarchy of criticalities having interpretation in terms of dark matter.

As found in the earlier section, the proposed anti-commutation relations for fermionic oscillator operators at the ends of string world sheets can be formulated so that they are analogous to those for Super Poincare algebra. The reason is that field equations assign a conserved 8-momentum to the light-like geodesic line defining the boundary of string at the orbit of partonic 2-surface. Octonionic representation of sigma matrices making possible generalization of twistor formalism to 8-D context is also essential. As a matter, the final justification for the analog of space-time came from the generalization of twistor approach to 8-D context.

By counting the number of spin and weak isospin components of embedding space spinors satisfying massless algebraic Dirac equation one finds that broken $\mathcal{N} = 8$ SUSY is the expected space-time SUSY. $\mathcal{N} = 2$ SUSY assignable to right-handed neutrino is the least broken sub-SUSY and one is forced to consider the possibility that spartners correspond to dark matter with $h_{eff} = n \times h$ and therefore remaining undetected in recent particle physics experiments.

Super-space viz. Grassmann algebra valued fields

Standard SUSY induces super-space extending space-time by adding anti-commuting coordinates as a formal tool. Many mathematicians are not enthusiastic about this approach because of the purely formal nature of anti-commuting coordinates. Also I regard them as a non-sense geometrically and there is actually no need to introduce them as the following little argument shows.

Grassmann parameters (anti-commuting theta parameters) are generators of Grassmann algebra and the natural object replacing super-space is this Grassmann algebra with coefficients of Grassmann algebra basis appearing as ordinary real or complex coordinates. This is just an ordinary space with additional algebraic structure: the mysterious anti-commuting coordinates are not needed. To me this notion is one of the conceptual monsters created by the over-pragmatic thinking of theoreticians.

This allows allows to replace field space with super field space, which is completely well-defined object mathematically, and leave space-time untouched. Linear field space is simply replaced with its Grassmann algebra. For non-linear field space this replacement does not work. This allows to formulate the notion of linear super-field just in the same manner as it is done usually.

The generators of super-symmetries in super-space formulation reduce to super translations, which anti-commute to translations. The super generators Q_α and $\overline{Q}_{\dot{\beta}}$ of super Poincare algebra are Weyl spinors commuting with momenta and anti-commuting to momenta:

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^{\mu}_{\alpha\ \dot{beta}} P_\mu \ . \tag{4.6.1}$$

One particular representation of super generators acting on super fields is given by

$$\begin{aligned} D_\alpha &= i\frac{\partial}{\partial_{\theta_\alpha}} \ , \\ D_{\dot{\alpha}} &= i\frac{\partial}{\partial_{\overline{\theta}}{}_{\dot{alpha}}} + \theta^\beta \sigma^\mu_{\beta\dot{\alpha}} \partial_\mu \end{aligned} \tag{4.6.2}$$

Here the index raising for 2-spinors is carried out using antisymmetric 2-tensor $\epsilon^{\alpha\beta}$. Super-space interpretation is not necessary since one can interpret this action as an action on Grassmann algebra valued field mixing components with different fermion numbers.

Chiral superfields are defined as fields annihilated by $D_{\dot{\alpha}}$. Chiral fields are of form $\Psi(x^\mu + i\overline{\theta}\sigma^\mu\theta, \theta)$. The dependence on $\overline{\theta}_{\dot{\alpha}}$ comes only from its presence in the translated Minkowski coordinate annihilated by $D_{\dot{\alpha}}$. Super-space enthusiast would say that by a translation of M^4 coordinates chiral fields reduce to fields, which depend on θ only.

The space of fermionic Fock states at partonic 2-surface as TGD counterpart of chiral super field

As already noticed, another manner to realize SUSY in terms of representations the super algebra of conserved super-charges. In TGD framework these super charges are naturally associated with the modified Dirac equation, and anti-commuting coordinates and super-fields do not appear anywhere. One can however ask whether one could identify a mathematical structure replacing the notion of chiral super field.

In [?] it was proposed that generalized chiral super-fields could effectively replace induced spinor fields and that second quantized fermionic oscillator operators define the analog of SUSY algebra. One would have $\mathcal{N} = \infty$ if all the conformal excitations of the induced spinor field restricted on 2-surface are present. For right-handed neutrino the modes are labeled by two integers and de-localized to the interior of Euclidian or Minkowskian regions of space-time sheet.

The obvious guess is that chiral super-field generalizes to the field having as its components many-fermions states at partonic 2-surfaces with theta parameters and their conjugates in one-one correspondence with fermionic creation operators and their hermitian conjugates.

1. Fermionic creation operators - in classical theory corresponding anti-commuting Grassmann parameters - replace theta parameters. Theta parameters and their conjugates are not in one-one correspondence with spinor components but with the fermionic creation operators and their hermitian conjugates. One can say that the super-field in question is defined in the "world of classical worlds" (WCW) rather than in space-time. Fermionic Fock state at the partonic 2-surface is the value of the chiral super field at particular point of WCW .
2. The matrix defined by the $\sigma^\mu\partial_\mu$ is replaced with a matrix defined by the Kähler-Dirac operator D between spinor modes acting in the solution space of the Kähler-Dirac equation. Since Kähler-Dirac operator annihilates the modes of the induced spinor field, super covariant derivatives reduce to ordinary derivatives with respect the theta parameters labeling the modes. Hence the chiral super field is a field that depends on θ_m or conjugates $\overline{\theta}_m$ only. In second quantization the modes of the chiral super-field are many-fermion states assigned to partonic 2-surfaces and string world sheets. Note that this is the only possibility since the notion of super-coordinate does not make sense now.
3. It would seem that the notion of super-field does not bring anything new. This is not the case. First of all, the spinor fields are restricted to 2-surfaces. Second point is that one cannot assign to the fermions of the many-fermion states separate non-parallel or even parallel four-momenta. The many-fermion state behaves like elementary particle. This has non-trivial implications for propagators and a simple argument [**?**] leads to the proposal that propagator for N-fermion partonic state is proportional to $1/p^N$. This would mean that only the states with fermion number equal to 1 or 2 behave like ordinary elementary particles.

4.6.5 Comparison Of TGD And Stringy Views About Super-Conformal Symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

1. In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K29]. This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X^3_l and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.
2. A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K27].
3. The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and

allowing only ground state weight 0 an 1/2 disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

4. If Kähler action defines the Kähler-Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

5. What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D \ mod \ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super

Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{array}{r}\{G_m, G_n^\dagger\} = 2L_{m+n} + \frac{c}{3}(m^2 - \frac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} = 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} = 0 \ . \end{array} \tag{4.6.3}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n < 0$ and $G_n^\dagger$, $n > 0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

1. Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K29]. Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

2. The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K127].

   One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

3. The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M_\pm^4 \times CP_2$. Clearly, wormhole throats X_l^3 would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

4. An alternative identification for TGD parts of conformal fields is inspired by $M^8 - H$ duality. Conformal fields would be fields in WCW . The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

4.7 Still about induced spinor fields and TGD counterpart for Higgs

The understanding of the modified Dirac equation and of the possible classical counterpart of Higgs field in TGD framework is not completely satisfactory. The emergence of twistor lift of Kähler action [K46] [L31] inspired a fresh approach to the problem and it turned out that a very nice understanding of the situation emerges.

More precise formulation of the Dirac equation for the induced spinor fields is the first challenge. The well-definedness of em charge has turned out to be very powerful guideline in the understanding of the details of fermionic dynamics. Although induced spinor fields have also a part assignable space-time interior, the spinor modes at string world sheets determine the fermionic dynamics in accordance with strong form of holography (SH).

The well-definedness of em charged is guaranteed if induced spinors are associated with 2-D string world sheets with vanishing classical W boson fields. It turned out that an alternative manner to satisfy the condition is to assume that induced spinors at the boundaries of string world sheets are neutrino-like and that these string world sheets carry only classical W fields. Dirac action contains 4-D interior term and 2-D term assignable to string world sheets. Strong form of holography (SH) allows to interpret 4-D spinor modes as continuations of those assignable to string world sheets so that spinors at 2-D string world sheets determine quantum dynamics.

Twistor lift combined with this picture allows to formulate the Dirac action in more detail. Well-definedness of em charge implies that charged particles are associated with string world sheets assignable to the magnetic flux tubes assignable to homologically non-trivial geodesic sphere and neutrinos with those associated with homologically trivial geodesic sphere. This explains why neutrinos are so light and why dark energy density corresponds to neutrino mass scale, and provides also a new insight about color confinement.

A further important result is that the formalism works only for embedding space dimension $D = 8$. This is due the fact that the number of vector components is the same as the number of spinor components of fixed chirality for $D = 8$ and corresponds directly to the octonionic triality.

p-Adic thermodynamics predicts elementary particle masses in excellent accuracy without Higgs vacuum expectation: the problem is to understand fermionic Higgs couplings. The observation that CP_2 part of the modified gamma matrices gives rise to a term mixing M^4 chiralities contain derivative allows to understand the mass-proportionality of the Higgs-fermion couplings at QFT limit.

4.7.1 More precise view about modified Dirac equation

Consistency conditions demand that modified Dirac equation with modified gamma matrices Γ^α defined as contractions $\Gamma^\alpha = T^{\alpha k}\gamma_k$ of canonical momentum currents $T^{\alpha k}$ associated with the bosonic action with embedding space gamma matrices γ_k [K127, K96]. The Dirac operator is not hermitian in the sense that the conjugation for the Dirac equation for Ψ does not give Dirac equation for $\overline{\Psi}$ unless the modified gamma matrices have vanishing covariant divergence as vector at space-time surface. This says that classical field equations are satisfied. This consistency condition holds true also for spinor modes possibly localized at string world sheets to which one can perhaps assign area action plus topological action defined by Kähler magnetic flux. The interpretation is in terms of super-conformal invariance.

The challenge is to formulate this picture more precisely and here I have not achieved a satisfactory formulation. The question has been whether interior spinor field Ψ are present at all, whether only Ψ is present and somehow becomes singular at string world sheets, or whether both stringy spinors Ψ_s and interior spinors Ψ are present. Both Ψ and Ψ_s could be present and Ψ_s could serve as source for interior spinors with the same H-chirality.

The strong form of holography (SH) suggests that interior spinor modes Ψ_n are obtained as continuations of the stringy spinor modes $\Psi_{s,n}$ and one has $\Psi = \Psi_s$ at string world sheets. Dirac action would thus have a term localized at strong world sheets and bosonic action would contain similar term by the requirement of super-conformal symmetry. Can one realize this intuition?

1. Suppose that Dirac action has interior and stringy parts. For the twistor lift of TGD [L31] the interior part with gamma matrices given by the modified gamma matrices associated

with the sum of Kähler action and volume action proportional to cosmological constant Λ. The variation with respect to the interior spinor field Ψ gives modified Dirac equation in the interior with source term from the string world sheet. The H-chiralites of Ψ and Psi_s would be same. Quark like and leptonic H-chiralities have different couplings to Kähler gauge potential and mathematical consistency strongly encourages this.

What is important is that the string world sheet part, which is bilinear in interior and string world sheet spinor fields Ψ and Ψ_s and otherwise has the same form as Dirac action. The natural assumption is that the stringy Dirac action corresponds to the modified gamma matrices assignable to area action.

2. String world sheet must be minimal surface: otherwise hermiticity is lost. This can be achieved either by adding to the Kähler action string world sheet area term. Whatever the correct option is, quantum criticality should determine the value of string tension. The first string model inspired guess is that the string tension is proportional to gravitational constant $1/G = 1/l_P^2$ defining the radius fo M^4 twistor sphere or to $1/R^2$, R CP_2 radius. This would however allow only strings not much longer than l_P or R. A more natural estimate is that string tension is proportional to the cosmological constant Λ and depends on p-adic length scale as $1/p$ so that the tension becomes small in long length scales. Since Λ coupling contant type parameter, this estimate looks rather reasonable.

3. The variation of stringy Dirac action with action density

$$L = [\overline{\Psi}_s D_s^{\rightarrow}\Psi - \overline{\Psi}_s D_s^{\leftarrow}\Psi]\sqrt{g_2} + h.c. \tag{4.7.1}$$

with respect to stringy spinor field Ψ_s gives for Ψ Dirac equation $D_s\Psi = 0$ if there are no Lagrange multiplier terms (see below). The variation in interior gives $D\Psi = S = D_s\Psi_s$, where the source term S is located at string world sheets. Ψ satisfies at string world sheet the analog of 2-D massless Dirac equation associated with the induced metric. This is just what stringy picture suggests.

The stringy source term for D equals to $D_s\Psi_s$ localized at string world sheets: the construction of solutions would require the construction of propagator for D, and this does not look an attractive idea. For $D_s\Psi_s = 0$ the source term vanishes. Holomorphy for Ψ_s indeed implies $D_s\Psi = 0$.

4. $\Psi_s = \Psi$ would realize SH as a continuation of Ψ_s from string world sheet to Ψ in the interior. Could one introduce Lagrange multiplier term

$$L_1 = \overline{\Lambda}(\Psi - \Psi_s) + h.c.$$

to realize $\Psi_s = \Psi$? Lagrange multiplier spinor field Λ would serve a source in the Dirac equation for $\Psi = \Psi_s$ and Ψ should be constructed at string world sheet in terms of stringy fermionic propagator with Λ as source. The solution for Ψ_s would require the construction of 2-D stringy propagator for Ψ_s but in principle this is not a problem since the modes can be solved by holomorphy in hypercomplex stringy coordinate. The problem of this option is that the H-chiralities of Λ and Ψ would be opposite and the coupling of opposite H-chiralities is not in spirit with H-chirality conservation.

A possible cure is to replace the Lagrange multiplier term with

$$L_1 = \overline{\Lambda}^k\gamma_k(\Psi - \Psi_s) + h.c. \ . \tag{4.7.2}$$

The variation with respect to the spin 3/2 field Λ^k would give 8 conditions - just the number of spinor components for given H-chirality - forcing $\Psi = \Psi_s$! $D = 8$ would be in crucial role! In other embedding space dimensions the number of conditions would be too high or too low. One would however obtain

$$D_s\Psi = D_s\Psi_s = \Lambda^k\gamma_k \ . \tag{4.7.3}$$

One could of course solve Ψ at string world sheet from $\Lambda^k\gamma_k$ by constructing the 2-D propagator associated with D_s. Conformal symmetry for the modes however implies $D_s\Psi = 0$ so that one has actually $\Lambda^k = 0$ and Λ^k remains mere formal tool to realize the constraint $\Psi = \Psi_s$ in mathematically rigorous manner for embedding space dimension $D = 8$. This is a new very powerful argument in favor of TGD.

5. At the string world sheets Ψ would be annihilated both by D and D_s. The simplest possibility is that the actions of D and D_s are proportional to each other at string world sheets. This poses conditions on string world sheets, which might force the CP_2 projection of string world sheet to belong to a geodesic sphere or circle of CP_2. The idea that string world sheets and also 3-D surfaces with special role in TGD could correspond to singular manifolds at which trigonometric functions representing CP_2 coordinates tend to go outside their allowed value range supports this picture. This will be discussed below.
   (a) For the geodesic sphere of type II induced Kähler form vanishes so that the action of 4-D Dirac massless operator would be determined by the volume term (cosmological constant). Could the action of D reduce to that of D_s at string world sheets? Does this require a reduction of the metric to an orthogonal direct sum from string world sheet tangent space and normal space and that also normal part of D annihilates the spinors at the string world sheet? The modes of Ψ at string world sheets are locally constant with respect to normal coordinates.
   (b) For the geodesic sphere of type I induced Kähler form is non-vanishing and brings an additional term to D coming from CP_2 degrees of freedom. This might lead to trouble since the gamma matrix structures of D and D_s would be different. One could however add to string world sheet bosonic action a topological term as Kähler magnetic flux. Although its contribution to the field equations is trivial, the contribution to the modified gamma matrices is non-vanishing and equal to the contraction $J^{\alpha k}\gamma_k$ of half projection of the Kähler form with CP_2 gamma matrices. The presence of this term could allow the reduction of $D\Psi_s = 0$ and $D_s\Psi_s = 0$ to each other also in this case.

4.7.2 A more detailed view about string world sheets

In TGD framework gauge fields are induced and what typically occurs for the space-time surfaces is that they tend to "go out" from CP_2. Could various lower-D surfaces of space-time surface correspond to sub-manifolds of space-time surface?

1. To get a concrete idea about the situation it is best to look what happens in the case of sphere $S^2 = CP_1$. In the case of sphere S^2 the Kähler form vanishes at South and North poles. Here the dimension is reduced by 2 since all values of ϕ correspond to the same point. $sin(\Theta)$ equals to 1 at equator - geodesic circle - and here Kähler form is non-vanishing. Here dimension is reduced by 1 unit. This picture conforms with the expectations in the case of CP_2 These two situations correspond to 1-D and 2-D geodesic sub-manifolds.
2. CP_2 coordinates can be represented as cosines or sines of angles and the modules of cosine or sine tends to become larger than 1 (see `http://tinyurl.com/z3coqau`). In Eguchi-Hanson coordinates (r, Θ, Φ, Ψ) the coordinates r and Θ give rise to this kind of trigonometric coordinates. For the two cyclic angle coordinates (Φ, Ψ) one does not encounter this problem.
3. In the case of CP_2 only geodesic sub-manifolds with dimensions $D = 0, 1, 2$ are possible. 1-D geodesic submanifolds carry vanishing induce spinor curvature. The impossibility of 3-D geodesic sub-manifolds would suggest that 3-D surfaces are not important. CP_2 has two geodesic spheres: S^2_I is homologically non-trivial and S^2_{II} homologically trivial (see `http://tinyurl.com/z3coqau`).
   (a) Let us consider S^2_I first. CP_2 has 3 poles, which obviously relates to SU(3), and in Eguchi Hanson coordinates (r, θ, Φ, Ψ) the surface $r = \infty$ is one of them and corresponds - not to a 3-sphere - but homologically non-trivial geodesic 2- sphere, which is complex sub-manifold and orbits of $SU(2) \times U(1)$ subgroup. Various values of the coordinate Ψ correspond to same point as those of Φ at the poles of S^2. The Kähler form J and classical Z^0 and γ fields are non-vanishing whereas W gauge fields vanish leaving only induced γ

and Z^0 field as one learns by studying the detailed expressions for the curvature of spinor curvature and vierbein of CP_2.
String world sheet could have thus projection to S^2_I but both γ and Z^0 would be vanishing except perhaps at the boundaries of string world sheet, where Z^0 would naturally vanish in the picture provided by standard model. One can criticize the presence of Z^0 field since it would give a parity breaking term to the modified Dirac operator. SH would suggest that the reduction to electromagnetism at string boundaries might make sense as counterpart for standard model picture. Note that the original vision was that besides induced Kähler form and em field also Z^0 field could vanish at string world sheets.

(b) The homologically trivial geodesic sphere S^2_{II} is the orbit of $SO(3)$ subgroup and not a complex manifold. By looking the standard example about S^2_I, one finds that the both J, Z_0, and γ vanish and only the W components of spinor connection are non-vanishing. In this case the notion of em charge would not be well-defined for S^2_{II} without additional conditions. Partonic 2-surfaces, their light-like orbits, and boundaries of string world sheets could do so since string world sheets have 1-D intersection with with the orbits. This picture would make sense for the minimal surfaces replacing vacuum extremals in the case of twistor lift of TGD.
Since em fields are not present, the presence of classical W fields need not cause problems. The absence of classical em fields however suggests that the modes of induced spinor fields at boundaries of string worlds sheets must be em neutral and represent therefore neutrinos. The safest but probably too strong option would be right-handed neutrino having no coupling spinor connection but coupling to the CP_2 gamma matrices transforming it to left handed neutrino. Recall that ν_R represents a candidate for super-symmetry.
Neither charged leptons nor quarks would be allowed at string boundaries and classical W gauge potentials should vanish at the boundaries if also left-handed neutrinos are allowed: this can be achieved in suitable gauge. Quarks and charged leptons could reside only at string world sheets assignable to monopole flux tubes. This could relate to color confinement and also to the widely different mass scales of neutrinos and other fermions as will be found.

To sum up, the new result is that the distinction between neutrinos and other fermions could be understood in terms of the condition that em charge is well-defined. What looked originally a problem of TGD turns out to be a powerful predictive tool.

4.7.3 Classical Higgs field again

A motivation for returning back to Higgs field comes from the twistor lift of Kähler action.

1. The twistor lift of TGD [K46] [L31] brings in cosmological constant as the coefficient of volume term resulting in dimensional reduction of 6-D Kähler action for twistor space of space-time surface realized as surface in the product of twistor space of M^4 and CP_2. The radius of the sphere of M^4 twistor bundle corresponds to Planck length. Volume term is extremely small but removes the huge vacuum degeneracy of Kähler action. Vacuum extremals are replaced by 4-D minimal surfaces and modified Dirac equation is just the analog of massless Dirac equation in complete analogy with string models.
2. The well-definedness and conservation of fermionic em charges and SH demand the localization of fermions to string world sheets. The earlier picture assumed only em fields at string world sheets. More precise picture allows also W fields.
3. The first guess is that string world sheets are minimal surfaces and this is supported by the previous considerations demanding also string area term and Kähler magnetic flux tube. Here gravitational constant assignable to M^4 twistor space would be the first guess for the string tension.

What one can say about the possible existence of classical Higgs field?

1. TGD predicts both Higgs type particles and gauge bosons as bound states of fermions and antifermions and they differ only in that their polarization are in M^4 resp. CP_2 tangent space. p-adic thermodynamics [K65] gives excellent predictions for elementary particle masses

in TGD framework. Higgs vacuum expectation is not needed to predict fermion or boson masses. Standard model gives only a parametrization of these masses by assuming that Higgs couplings to fermions are proportional to their masses, it does not predict them.

The experimental fact is however that the couplings of Higgs are proportional to fermion masses and TGD should be able to predict this and there is a general argument for the proportonality, which however should be deduced from basic TGD. Can one achieve this?

2. Can one imagine any candidate for the classical Higgs field? There is no covariantly constant vector field in CP_2, whose space-time projection could define a candidate for classical Higgs field. This led years ago before the model for how bosons emerge from fermions to the wrong conclusion that TGD does not predict Higgs.

   The first guess for the possibly existing classical counterpart of Higgs field would be as CP_2 part for the divergence of the space-time vector defined modified gamma matrices expressible in terms of canonical momentum currents having natural interpretation as a generalization of force for point like objects to that for extended objects. Higgs field in this sense would however vanish by above consistency conditions and would not couple to spinors at all.

   Classical Higgs field should have only CP_2 part being CP_2 vector. What would be also troublesome that this proposale for classical Higgs field would involve second derivatives of embedding space coordinates. Hence it seems that there is no hope about geometrization of classical Higgs fields.

3. The contribution of the induced Kähler form gives to the modified gamma matrices a term expressible solely in terms of CP_2 gamma matrices. This term appears in modified Dirac equation and mixes M^4 chiralities - a signal for the massivation. This term is analogous to Higgs term expect that it contains covariant derivative.

   The question that I have not posed hitherto is whether this term could at QFT limit of TGD give rise to vacuum expectation of Higgs. The crucial observation is that the presence of derivative, which in quantum theory corresponds roughly to mass proportionality of chirality mixing coupling at QFT limit. This could explain why the coupling of Higgs field to fermions is proportional to the mass of the fermion at QFT limit!

4. For S^2_{II} type string world sheets assignable to neutrinos the contribution to the chirality mixing coupling should be of order of neutrino mass. The coefficient $1/L^4$ of the volume term defining cosmological constant [L31] separates out as over all factor in massless Dirac equation and the parameter characterizing the mass scale causing the mixing is of order $m = \omega_1\omega_2 R$. Here ω_1 characterizes the scale of gradient for CP_2 coordinates. The simplest minimal surface is that for which CP_2 projection is geodesic line with $\Phi = \omega_1 t$. ω_2 characterizes the scale of the gradient of spinor mode.

   Assuming $\omega_1 = \omega_2 \equiv \omega$ the scale m is of order neutrino mass $m_\nu \simeq .1$ eV from the condition $m \sim \omega^2 R \sim m_\nu$. This gives the estimate $\omega \sim \sqrt{m_{CP_2} m_\nu} \sim 10^2 m_p$ from $m_{CP_2} \sim 10^{-4} m_P$, which is weak mass scale and therefore perfectly sensible. The reduction $\Delta c/c$ of the light velocity from maximal signal velocity due the replacement $g_{tt} = 1 - R^2\omega^2$ is $\Delta c/c \sim 10^{-34}$ and thus completely negligible. This estimate does not make sense for charged fermions, which correspond to S^2_I type string world sheets.

   A possible problem is that if the value of the cosmological constant Λ evolves as $1/p$ as function of the length mass scale the mass scale of neutrinos should increase in short scales. This looks strange unless the mass scale remains below the cosmic temperature so that neutrinos would be always effectively massless.

5. For S^2_I type string world sheets assignable to charged fermions Kähler action dominates and the mass scales are expected to be higher than for neutrinos. For S^2_I type strings the modified gamma matrices contain also Kähler term and a rough estimate is that the ratio of two contributions is the ratio of the energy density of Kähler action to vacuum energy density. As Kähler energy density exceeds the value corresponding to vacuum energy density $1/L^4$, $L \sim 40$ μm, Kähler action density begins to dominate over dark energy density.

To sum up, this picture suggest that the large difference between the mass scales of neutrinos and em charged fermions is due to the fact that neutrinos are associated with string world sheet of type II and em charged fermions with string world sheets of type I. Both strings world sheets would be accompanied by flux tubes but for charged particles the flux tubes would carry Kähler magnetic flux. Cosmological constant forced by twistor lift would make neutrinos massive and allow to understand neutrino mass scale.

Chapter 5

Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

5.1 Introduction

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry [A44]. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

The basic idea is that WCW is union of symmetric spaces G/H labelled by zero modes which do not contribute to the WCW metric. There have been many open questions and it seems the details of the ealier approach [?]ust be modified at the level of detailed identifications and interpretations.

1. A longstanding question has been whether one could assign Equivalence Principle (EP) with the coset representation formed by the super-Virasoro representation assigned to G and H in such a way that the four-momenta associated with the representations and identified as inertial and gravitational four-momenta would be identical. This does not seem to be the case. The recent view will be that EP reduces to the view that the classical four-momentum associated with Kähler action is equivalent with that assignable to Kähler-Dirac action supersymmetrically related to Kähler action: quantum classical correspondence (QCC) would be in question. Also strong form of general coordinate invariance implying strong form of holography in turn implying that the super-symplectic representations assignable to space-like and light-like 3-surfaces are equivalent could imply EP with gravitational and inertial four-momenta assigned to these two representations.

   At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least.

2. The detailed identification of groups G and H and corresponding algebras has been a long-standing problem. Symplectic algebra associated with$\delta M^4_\pm \times CP2$ ($\delta M^4_\pm$ is light-cone boundary - or more precisely, with the boundary of causal diamond (CD) defined as Cartesian product of CP_2 with intersection of future and past direct light cones of M^4 has Kac-Moody

type structure with light-like radial coordinate replacing complex coordinate z. Virasoro algebra would correspond to radial diffeomorphisms. I have also introduced Kac-Moody algebra assigned to the isometries and localized with respect to internal coordinates of the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and which serve as natural correlates for elementary particles (in very general sense!). This kind of localization by force could be however argued to be rather ad hoc as opposed to the inherent localization of the symplectic algebra containing the symplectic algebra of isometries as sub-algebra. It turns out that one obtains direct sum of representations of symplectic algebra and Kac-Moody algebra of isometries naturally as required by the success of p-adic mass calculations.

3. The dynamics of Kähler action is not visible in the earlier construction. The construction also expressed WCW Hamiltonians as 2-D integrals over partonic 2-surfaces. Although strong form of general coordinate invariance (GCI) implies strong form of holography meaning that partonic 2-surfaces and their 4-D tangent space data should code for quantum physics, this kind of outcome seems too strong. The progress in the understanding of the solutions of Kähler-Dirac equation led however to the conclusion that spinor modes other than right-handed neutrino are localized at string world sheets with strings connecting different partonic 2-surfaces. This leads to a modification of earlier construction in which WCW super-Hamiltonians are essentially integrals with integrand identified as a Noether super current for the deformations in G Each spinor mode gives rise to super current and the modes of right-handed neutrino and other fermions differ in an essential ways. Right-handed neutrino would correspond to symplectic algebra and other modes to the Kac-Moody algebra and one obtains the crucial 5 tensor factors of Super Virasoro required by p-adic mass calculations.

   The matrix elements of WCW metric between Killing vectors are expressible as anti-commutators of super-Hamiltonians identifiable as contractions of WCW gamma matrices with these vectors and give Poisson brackets of corresponding Hamiltonians. The anti-commutation relates of induced spinor fields are dictated by this condition. Everything is 3-dimensional although one expects that symplectic transformations localized within interior of X^3 act as gauge symmetries so that in this sense effective 2-dimensionality is achieved. The components of WCW metric are labelled by standard model quantum numbers so that the connection with physics is extremely intimate.

4. An open question in the earlier visions was whether finite measurement resolution is realized as discretization at the level of fundamental dynamics. This would mean that only certain string world sheets from the slicing by string world sheets and partonic 2-surfaces are possible. The requirement that anti-commutations are consistent suggests that string world sheets correspond to surfaces for which Kähler magnetic field is constant along string in well defined sense ($J_{\mu\nu}\epsilon^{\mu\nu}g^{1/2}$ remains constant along string). It however turns that by a suitable choice of coordinates of 3-surface one can guarantee that this quantity is constant so that no additional constraint results.

5. Quantum criticality is one of the basic notions of quantum TGD and its relationship to coset construction has remained unclear. In this chapter the concrete realization of criticality in terms of symmetry breaking hierarchy of Super Virasoro algebra acting on symplectic and Kac-Moody algebras. Also a connection with finite measurement resolution - second key notion of TGD - emerges naturally.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L17].

5.2 WCW As A Union Of Homogenous Or Symmetric Spaces

The physical interpretation and detailed mathematical understanding of super-conformal symmetries has developed rather slowly and has involved several side tracks. In the following I try to summarize the basic picture with minimal amount of formulas with the understanding that the statement "Noether charge associated with geometrically realized Kac-Moody symmetry" is enough for the reader to write down the needed formula explicitly. Formula oriented reader might deny the value of the approach giving weight to principles. My personal experience is that piles of formulas too often hide the lack of real understanding.

In the following the vision about WCW as union of coset spaces is discussed in more detail.

5.2.1 Basic Vision

The basic view about coset space construction for WCW has not changed.

1. The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A8] (`http://tinyurl.com/y7u2t8jo`) means at Lie algebra level the decomposition $g = h \oplus t$ to sub-Lie-algebra h and its complement t such that $[h,t] \subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t,t] \subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A21] (`http://tinyurl.com/ycouv7uh`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

   An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t,t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

2. The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW . The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW . In particular, Hamiltonians must be represented in completely new manner. The key idea is to construct WCW Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and embedding space coordinates are treated purely classically.

3. It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

   CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r = 0$ is left invariant by $U(2)$ but for $r > 0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW .

   How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

4. The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h + t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

   For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

   For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

5.2.2 Equivalence Principle And WCW

5.2.3 Equivakence Principle At Quantum And Classical Level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K121].

   A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even

for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.
3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.
4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore: this idea is however not promising.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

5.2.4 Criticism Of The Earlier Construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

1. Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

   One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that embedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

2. The fact that the dynamics of Kähler action and Kähler-Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K127] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the Kähler-Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

5.2.5 Is WCW Homogenous Or Symmetric Space?

A key question is whether WCW can be symmetric space [A21] (`http://tinyurl.com/y8ojglkb`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$. The latter condition is the difficult one.

1. δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

2. For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$ and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

   Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

3. In the case of symplectic algebra one can construct the embedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level embedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

4. A more promising involution is the inversion $r_M \to 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

   $r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by $/r_M$-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in poin-twise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

5. One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a

preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW .

For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t,t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

5.2.6 Symplectic And Kac-Moody Algebras As Basic Building Bricks

5.3 Updated View About Kähler Geometry Of WCW

During last years the understanding of the mathematical aspects of TGD and of its connection with the experimental world has developed rapidly.

TGD differs in several respects from quantum field theories and string models. The basic mathematical difference is that the mathematically poorly defined notion of path integral is replaced with the mathematically well-defined notion of functional integral defined by the Kähler function defining Kähler metric for WCW ("world of classical worlds"). Apart from quantum jump, quantum TGD is essentially theory of classical WCW spinor fields with WCW spinors represented as fermionic Fock states. One can say that Einstein's geometrization of physics program is generalized to the level of quantum theory.

It has been clear from the beginning that the gigantic super-conformal symmetries generalizing ordinary super-conformal symmetries are crucial for the existence of WCW Kähler metric. The detailed identification of Kähler function and WCW Kähler metric has however turned out to be a difficult problem. It is now clear that WCW geometry can be understood in terms of the analog of AdS/CFT duality between fermionic and space-time degrees of freedom (or between Minkowskian and Euclidian space-time regions) allowing to express Kähler metric either in terms of Kähler function or in terms of anti-commutators of WCW gamma matrices identifiable as super-conformal Noether super-charges for the symplectic algebra assignable to $\delta M^4_\pm \times CP_2$. The string model type description of gravitation emerges and also the TGD based view about dark matter becomes more precise. String tension is however dynamical rather than pregiven and the hierarchy of Planck constants is necessary in order to understand the formation of gravitationally bound states. Also the proposal that sparticles correspond to dark matter becomes much stronger: sparticles actually are dark variants of particles.

A crucial element of the construction is the assumption that super-symplectic and other super-conformal symmetries having the same structure as 2-D super-conformal groups can be seen a broken gauge symmetries such that sub-algebra with conformal weights coming as n-ples of those for full algebra act as gauge symmetries. In particular, the Noether charges of this algebra vanish for preferred extremals- this would realize the strong form of holography implied by strong form of General Coordinate Invariance. This gives rise to an infinite number of hierarchies of conformal gauge symmetry breakings with levels labelled by integers $n(i)$ such that $n(i)$ divides $n(i+1)$ interpreted as hierarchies of dark matter with levels labelled by the value of Planck constant $h_{eff} = n \times h$. These hierarchies define also hierarchies of quantum criticalities, and are proposed to give rise to inclusion hierarchies of hyperfinite factors of II_1 having interpretation in terms of finite cognitive resolution with inclusions being characterized by the integers $n(+1)/n(i)$.

These hierarchies are fundamental for the understanding of living matter. Living matter is fighting in order to stay at criticality and uses metabolic energy and homeostasis to achieve this. In the biological death of the system (self) a phase transition increasing h_{eff} finally takes place. The sub-selves of self experienced by self as mental images however die and are reborn at opposite boundary of the corresponding causal diamond (CD) and they genuinely evolve so that self can be said to become wiser even without dying! The purpose of this fighting against criticality would thus allow a possibility for sub-selves to evolve via subsequent re-incarnations. One interesting prediction is the possibility of time reversed mental images. The challenge is to understand what they do mean at the level of conscious experience.

5.3.1 Kähler Function, Kähler Action, And Connection With String Models

The definition of Kähler function in terms of Kähler action is possible because space-time regions can have also Euclidian signature of induced metric. Euclidian regions with 4-D CP_2 projection - wormhole contacts - are identified as lines of generalized Feynman diagrams - space-time correlates for basic building bricks of elementary particles. Kähler action from Minkowskian regions is imaginary and gives to the functional integrand a phase factor crucial for quantum field theoretic interpretation. The basic challenges are the precise specification of Kähler function of "world of classical worlds" (WCW) and Kähler metric.

There are two approaches concerning the definition of Kähler metric: the conjecture analogous to AdS/CFT duality is that these approaches are mathematically equivalent.

1. The Kähler function defining Kähler metric can be identified as Kähler action for space-time regions with Euclidian signature for a preferred extremal containing 3-surface as the ends of the space-time surfaces inside causal diamond (CD). Minkowskian space-time regions give to Kähler action an imaginary contribution interpreted as the counterpart of quantum field theoretic action. The exponent of Kähler function gives rise to a mathematically well-defined functional integral in WCW . WCW metric is dictated by the Euclidian regions of space-time with 4-D CP_2 projection.

   The basic question concerns the attribute "preferred". Physically the preferred extremal is analogous to Bohr orbit. What is the mathematical meaning of preferred extremal of Kähler action? The latest step of progress is the realization that the vanishing of generalized conformal charges for the ends of the space-time surface fixes the preferred extremals to high extent and is nothing but classical counterpart for generalized Virasoro and Kac-Moody conditions.

2. Fermions are also needed. The well-definedness of electromagnetic charge led to the hypothesis that spinors are restricted at string world sheets. One could also consider associativity as basic contraint to fermionic dynamics combined with the requirement that octonionic representation for gamma matrices is equivalent with the ordinary one. The conjecture is that this leads to the same outcome. This point is highly non-trivial and will be discussed below separately.

3. Second manner to define Kähler metric is as anticommutators of WCW gamma matrices identified as super-symplectic Noether charges for the Dirac action for induced spinors with string tension proportional to the inverse of Newton's constant. These charges are associated with the 1-D space-like ends of string world sheets connecting the wormhole throats. WCW metric contains contributions from the spinor modes associated with various string world sheets connecting the partonic 2-surfaces associated with the 3-surface.

   It is clear that the information carried by WCW metric about 3-surface is rather limited and that the larger the number of string world sheets, the larger the information. This conforms with strong form of holography and the notion of measurement resolution as a property of quantums state. Duality clearly means that Kähler function is determined either by space-time dynamics inside Euclidian wormhole contacts or by the dynamics of fermionic strings in Minkowskian regions outside wormhole contacts. This duality brings strongly in mind AdS/CFT duality. One could also speak about fermionic emergence since Kähler function is dictated by the Kähler metric part from a real part of gradient of holomorphic function: a possible identification of the exponent of Kähler function is as Dirac determinant.

5.3.2 Symmetries of WCW

Towards the end of year 2023 a dramatic progress in the understanding of WCW geometry took place and the following piece of text summarizes the findings. It turned that the original intuitive picture was surprisingly near to what now looks the correct view.

The situation before 2023

WCW geometry exists only if it has maximal isometries. I have proposed that WCW could be regarded as a union of generalized symmetric spaces labelled by zero modes which do not contribute to the metric. The induced Kähler field is invariant under symplectic transformations of CP_2 and would therefore define zero mode degrees of freedom if one assumes that WCW metric has symplectic transformations as isometries. In particular, Kähler magnetic fluxes would define zero modes and are quantized closed 2-surfaces. The induced metric appearing in Kähler action is however not zero mode degree of freedom. If the action contains volume term, the assumption about union of symmetric spaces is not well-motivated.

Symplectic transformations are not the only candidates for the isometries of WCW. The basic picture about what these maximal isometries could be, is partially inspired by string models.

1. A weaker proposal is that the symplectomorphisms of H define only symplectomorphisms of WCW. Extended conformal symmetries define also a candidate for isometry group. Remarkably, light-like boundary has an infinite-dimensional group of isometries which are in 1-1 correspondence with conformal symmetries of $S^2 \subset S^2 \times R_+ = \delta M^4_+$.
2. Extended Kac Moody symmetries induced by isometries of δM^4_+ are also natural candidates for isometries. The motivation for the proposal comes from physical intuition deriving from string models. Note they do not include Poincare symmetries, which act naturally as isometries in the moduli space of causal diamonds (CDs) forming the "spine" of WCW.
3. The light-like orbits of partonic 2-surfaces might allow separate symmetry algebras. One must however notice that there is exchange of charges between interior degrees of freedom and partonic 2-surfaces. The essential point is that one can assign to these surface conserved charges when the dual light-like coordinate defines time coordinate. This picture also assumes a slicing of space-time surface by by the partonic orbits for which partonic orbits associated with wormrhole throats and boundaries of the space-time surface would be special. This slicing would correspond to Hamilton-Jacobi structure.
4. Fractal hierarchy of symmetry algebras with conformal weights, which are non-negative integer multiples of fundamental conformal weights, is essential and distinguishes TGD from string models. Gauge conditions are true only the isomorphic subalgebra and its commutator with the entire algebra and the maximal gauge symmetry to a dynamical symmetry with generators having conformal weights below maximal value. This view also conforms with p-adic mass calculations.
5. The realization of the symmetries for 3-surfaces at the boundaries of CD and for light-like orbits of partonic 2-surfaces is known. The problem is how to extend the symmetries to the interior of the space-time surface. It is natural to expect that the symmetries at partonic orbits and light-cone boundary extend to the same symmetries.

Realization Of Super-Conformal Symmetries

The detailed realization of various super-conformal symmetries has been also a long standing problem.

1. Super-conformal symmetry requires that Dirac action for string world sheets is accompanied by string world sheet area as part of bosonic action. String world sheets are implied and can be present only in Minkowskian regions if one demands that octonionic and ordinary representations of induced spinor structure are equivalent (this requires vanishing of induced spinor curvature to achieve associativity in turn implying that CP_2 projection is 1-D). Note that 1-dimensionality of CP_2 projection is symplectically invariant property. Kähler action is not invariant under symplectic transformations. This is necessary for having non-trivial Kähler metric. Whether WCW really possesses super-symplectic isometries remains an open problem.
2. Super-conformal symmetry also demands that Kähler action is accompanied by what I call Kähler-Dirac action with gamma matrices defined by the contractions of the canonical momentum currents with embedding space-gamma matrices. Both the well-definedness of em charge and equivalence of octonionic spinor dynamics with ordinary one require the restriction of spinor modes to string world sheets with light-like boundaries at wormhole throats. K-D action with the localization of induced spinors at string world sheets is certainly the minimal option to consider.

3. Strong form of holography suggested by strong form of general coordinate invariance strongly suggests that super-conformal symmetry is broken gauge invariance in the sense that the clasical super-conformal charges for a sub-algebra of the symplectic algebra with conformal weights vanishing modulo some integer n vanish. The proposal is that n corresponds to the effective Planck constant as $h_{eff}/h = n$. The standard conformal symmetries for spinors modes at string world sheets is always unbroken gauge symmetry.

The conserved charges associated with holomorphies

Generalized holomorphy not only solves explicitly the equations of motion [L160] but, as found quite recently, also gives corresponding conserved Noether currents and charges.

1. Generalized holomorphy algebra generalizes the Super-Virasoro algebra and the Super-Kac-Moody algebra related to the conformal invariance of the string model. The corresponding Noether charges are conserved. Modified Dirac action allows to construct the supercharges having interpretation as WCW gamma matrices. This suggests an answer to a longstanding question related to the isometries of the "world of the classical worlds" (WCW).
2. Either the generalized holomorphies or the symplectic symmetries of $H = M^4 \times CP_2$ or both together define WCW isometries and corresponding super algebra. It would seem that symplectic symmetries induced from H are not necessarily needed and might correspond to symplectic symmetries of WCW. One would obtain a close similarity with the string model, except that one has half-algebra for which conformal weights are proportional to non-negative integers and gauge conditions only apply to an isomorphic subalgebra. These are labeled by positive integers and one obtains a hierarchy.
3. By their light-likeness, the light cone boundary and orbits of partonic 2-surfaces allow an infinite-dimensional isometry group. This is possible only in dimension four. Its transformations are generalized conformal transformations of 2-sphere (partonic 2-surface) depending on light-like radial coordinate such that the radial scaling compensates for the usual conformal scaling of the metric. The WCW isometries would thus correspond to the isometries of the parton orbit and of the boundary of the light cone! These two representations could provide alternative representations for the charges if the strong form of holography holds true and would realize a strong form of holography. Perhaps these realizations deserve to be called inertial and gravitational charges.

   Can these transformations leave the action invariant? For the light-cone boundary, this looks obvious if the light-cone is sliced by a surface parallel to the light-cone boundary. Note however that the tip of this surface might produce problems. A slicing defined by the Hamilton-Jacobi structure would be naturally associated with partonic orbits.
4. What about Poincare symmetries? They would act on the center of mass coordinates of causal diamonds (CDs) as found already earlier [L169]. CDs form the "spine" of WCW, which can be regarded as fiber space with fiber for a given CD containing as a fiber the space-time surfaces inside it.

The super-symmetric counterparts of holomorphic charges for the modified Dirac action and bilinear in fermionic oscillator operators associated with the second quantization of free spinor fields in H, define gamma matrices of WCW. Their anticommutators define the Kähler metric of WCW. There is no need to calculate either the action defining the classical Kähler action defining the Kähler function or its derivatives with respect to WCW complex coordinates and their conjugates. What is important is that this makes it possible to speak about WCW metric also for number theoretical discretization of WCW with space-time surfaces replaced with their number theoretic discretizations.

Could generalized holomorphy allow to sharpen the existing views?

This picture is rather speculative, allows several variants, and is not proven. There is now however a rather convincing ansatz for the general form of preferred extremals. This proposal relies on the realization of holography as generalized 4-D holomorphy. Could it help to make the picture more precise?

1. Explicit solution of field equations in terms of the generalized holomorphy is now known. The solution ansatz is independent of action as long it is general coordinate invariance depending only on the induced geometric structures. Space-time surfaces would be minimal surfaces apart from lower-dimensional singular surfaces at which the field equations involve the entire action. Only the singularities, classical charges and positions of topological interaction vertices depend on the choice of the action [L160]. Kähler action plus volume term is the choice of action forced by twistor lift making the choice of H unique.
2. The universality has a very intriguing implication. One can assign to any action of this kind conserved Noether currents and their fermionic counterparts (also super counterparts). One would have a huge algebra of conserved currents characterizing the space-time geometry. The corresponding charges can be made conserved by suitably modifying the form of holomorphic functions of the ansatz and therefore the time derivatives $\partial_t h^k$ at the 3-D end of space-time surface at the boundary CD. This need not be the case for all deformations of partonic orbits. In any case, the 3-D holographic data seem to be dual as the strong form of holography suggests. The discussion of the symplectic symmetries leads to the conclusion that they give rise to conserved charges at the partonic 3-surfaces obeying Chern-Simons-Kähler dynamics, which is non-deterministic.
3. Hamilton-Jacobi structures emerge naturally as generalized conformal structures of space-time surfaces and M^4 [L164]. This inspires a proposal for a generalization of modular invariance and of moduli spaces as subspaces of Teichmüller spaces.
4. One can assign to holomorphy conserved Noether charges. The conservation reduces to the algebraic conditions satisfied for the same reason as field equations, i.e. the conservation conditions involving contractions of complex tensors of type (1,1) with tensors of type (2,0) and (0,2). The charges have the same form as Noether charges but it is not completely clear whether the action remains invariant under these transformations. This point is non-trivial since Noether theorem says that invariance of the action implies the existence of conserved charges but not vice versa. Could TGD represent a situation in which the equivalence between symmetries of action and conservation laws fails?

   Also string models have conformal symmetries but in this case 2-D area form suffers conformal scaling. Also the fact that holomorphic ansatz is satisfied for such a large class of actions apart from singularities suggests that the action is not invariant.
5. The action should define Kähler function for WCW identified as the space of Bohr orbits. WCW Kähler metric is defined in terms of the second derivatives of the Kähler action of type (1,1) with respect to complex coordinates of WCW. Does the invariance of the action under holomorphies imply a trivial Kähler metric and constant Kähler function?

   Here one must be very cautious since by holography the variations of the space-time surface are induced by those of 3-surface defining holographic data so that the entire space-time surface is modified and the action can change. The presence of singularities, analogous to poles and cuts of an analytic function and representing particles, suggests that the action represents the interactions of particles and must change. Therefore the action might not be invariant under holomorphies. The parameters characterizing the singularities should affect the value of the action just as the positions of these singularities in 2-D electrostatistics affect the Coulomb energy.

   Generalized conformal charges and supercharges define a generalization of Super Virasoro algebra of string models. Also Kac-Moody algebra assignable to the isometries of $\delta M^4_+ \times CP_2$ and light H generalizes trivially.
6. An absolutely essential point is that generalized holomorphisms are *not* symmetries of Kähler function since otherwise Kähler metric involving second derivatives of type (1,1) with respect to complex coordinates of WCW is non-trivial if defined by these symmetry generators as differential operators. If Kähler function is equal to Kähler action, as it seems, Kähler action cannot be invariant under generalized holomorphies.

   Noether's theorem states that the invariance of the action under a symmetry implies the conservation of corresponding charge but does *not* claim that the existence of conserved Noether currents implies invariance of the action. Since Noether currents are conserved now, one would have a concrete example about the situation in which the inverse of Noether's theorem does not hold true. In a string model based on area action, conformal transformations of complex string coordinates give rise to conserved Noether currents as one easily checks. The area element defined by the induced metric suffers a conformal scaling so that the action is not invariant in this case.

Challenging the existing picture of WCW geometry

These findings make it possible to challenge and perhaps sharpen the existing speculations concerning the metric and isometries of WCW.

I have considered the possibility that also the symplectomorphisms of $\delta M^4 + \times CP_2$ could define WCW isometries. This actually the original proposal. One can imagine two options.

1. The continuation of symplectic transformations to transformations of the space-time surface from the boundary of light-cone or from the orbits partonic 2-surfaces should give rise to conserved Noether currents but it is not at all obvious whether this is the case.
2. One can assign conserved charges to the time evolution of the 3-D boundary data defining the holographic data: the time coordinate for the evolution would correspond to the light-like coordinate of light-cone boundary or partonic orbit. This option I have not considered hitherto. It turns out that this option works!

The conclusion would be that generalized holomorphies give rise to conserved charges for 4-D time evolution and symplectic transformations give rise to conserved charged for 3-D time evolution associated with the holographic data.

About extremals of Chern-Simons-Kähler action

Let us look first the general nature of the solutions to the extremization of Chern-Simons-Kähler action.

1. The light-likeness of the partonic orbits requires Chern-Simons action, which is equivalent to the topological action $J \wedge J$, which is total divergence and is a symplectic in variant. The field equations at the boundary cannot involve induced metric so that only induced symplectic structure remains. The 3-D holographic data at partonic orbits would extremize Cherns-Simons-Kähler action. Note that at the ends of the space-time surface about boundaries of CD one cannot pose any dynamics.
2. If the induced Kähler form has only the CP_2 part, the variation of Chern-Simons-Kähler form would give equations satisfied if the CP_2 projection is at most 2-dimensional and Chern-Simons action would vanish and imply that instanton number vanishes.
3. If the action is the sum of M^4 and CP_2 parts, the field equations in M^4 and CP_2 degrees of freedom would give the same result. If the induced Kähler form is identified as the sum of the M^4 and CP_2 parts, the equations also allow solutions for which the induced M^4 and CP_2 Kähler forms sum up to zero. This phase would involve a map identifying M^4 and CP_2 projections and force induce Kähler forms to be identical. This would force magnetic charge in M^4 and the question is whether the line connecting the tips of the CD makes non-trivial homology possible. The homology charges and the 2-D ends of the partonic orbit cancel each other so that partonic surfaces can have monopole charge.

   The conditions at the partonic orbits do not pose conditions on the interior and should allow generalized holomorphy. The following considerations show that besides homology charges as Kähler magnetic fluxes also Hamiltonian fluxes are conserved in Chern-Simons-Kähler dynamics.

Can one assign conserved charges with symplectic transformations or partonic orbits and 3-surfaces at light-cone boundary?

The geometric picture is that symplectic symmetries are Hamiltonian flows along the light-like partonic orbits generated by the projection A_t of the Kähler gauge potential in the direction of the light-like time coordinate. The physical picture is that the partonic 2-surface is a Kähler charged particle that couples to the Hamilton $H = A_t$. The Hamiltonians H_A are conserved in this time evolution and give rise to conserved Noether currents. The corresponding conserved charge is integral over the 2-surface defined by the area form defined by the induced Kähler form.

Let's examine the change of the Chern-Simons-Kähler action in a deformation that corresponds, for example, to the CP_2 symplectic transformation generated by Hamilton H_A. M^4 symplectic transformations can be treated in the same way:here however M^4 Kähler form would be involved, assumed to accompany Hamilton-Jacobi structure as a dynamically generated structure.

1. Instanton density for the induced Kähler form reduces to a total divergence and gives Chern-Simons-Kähler action, which is TGD analog of topological action. This action should change in infinitesimal symplectic transformations by a total divergence, which should vanish for extremals and give rise to a conserved current. The integral of the divergence gives a vanishing charge difference between the ends of the partonic orbit. If the symplectic transformations define symmetries, it should be possible to assign to each Hamiltonian H_A a conserved charge. The corresponding quantal charge would be associated with the modified Dirac action.

2. The conserved charge would be an integral over X^2. The surface element is not given by the metric but by the symplectic structure, so that it is preserved in symplectic transformations. The 2-surface of the time evolution should correspond to the Hamiltonian time transformation generated by the projection $A_\alpha = A_k \partial_\alpha s^k$ of the Kähler gauge potential A_k to the direction of light-like time coordinate $x^\alpha \equiv t$.

3. The effect of the generator $j_A^k = J^{kl}\partial_l H_A$ on the Kähler potential A_l is given by $j_A^k \partial_k A_l$. This can be written as $\partial_k A_l = J_{kl} + \partial_l A_k$. The first term gives the desired total divergence $\partial_\alpha(\epsilon^{\alpha\beta\gamma} J_{\beta\gamma} H_A)$.

   The second term is proportional to the term $\partial_\alpha H_A - \{A_\alpha, H\}$. Suppose that the induced Kähler form is transversal to the light-like time coordinate t, i.e. the induced Kähler form does not have components of form $J_{t\mu}$. In this kind of situation the only possible choice for α corresponds to the time coordinate t. In this situation one can perform the replacement $\partial_\alpha H_A - \{A_\alpha, H\} \rightarrow dH_A/dt - \{A_t, H\}$ This corresponds to a Hamiltonian time evolution generated by the projection A_t acting as a Hamiltonian. If this is really a Hamiltonian time evolution, one has $dH_A/dt - \{A, H\} = 0$. Because the Poisson bracket represents a commutator, the Hamiltonian time evolution equation is analogous to the vanishing of a covariant derivative of H_A along light-like curves: $\partial_t H_A + [A, H_A] = 0$. The physical interpretation is that the partonic surface develops like a particle with a Kähler charge. As a consequence the change of the action reduces to a total divergence.

   An explicit expression for the conserved current $J_A^\alpha = H_A \epsilon^{\alpha\beta\gamma} J_{\beta\gamma}$ can be derived from the vanishing of the total divergence. Symplectic transformations on X^2 generate an infinite-dimensional symplectic algebra. The charge is given by the Hamiltonian flux $Q_A = \int H_A J_{\beta\gamma} dx^\alpha \wedge dx^\beta$.

4. If the projection of the partonic path CP_2 or M^4 is 2-D, then the light-like geodesic line corresponds to the path of the parton surface. If A_l can be chosen parallel to the surface, its projection in the direction of time disappears and one has $A_t = 0$. In the more general case, X^2 could, for example, rotate in CP_2. In this case A_t is nonvanishing. If J is transversal (no Kähler electric field), charge conservation is obtained.

Do the above observations apply at the boundary of the light-cone?

1. Now the 3-surface is space-like and Chern-Simons-Kähler action makes sense. It is not necessary but emerges from the "instanton density" for the Kähler form. The symplectic transformations of $\delta M^4_+ \times CP_2$ are the symmetries. The most time evolution associated with the radial light-like coordinate would be from the tip of the light-cone boundary to the boundary of CD. Conserved charges as homological invariants defining symplectic algebra would be associated with the 2-D slices of 3-surfaces. For closed 3-surfaces the total charges from the sheets of 3-space as covering of δM^4_+ must sum up to zero.

2. Interestingly, the original proposal [K29] for the isometries of WCW was that the Hamiltonian fluxes assignable to M^4 and CP_2 degrees of freedom at light-like boundary act define the charges associated with the WCW isometries as symplectic transformations so that a strong

form of holography would have been be realized and space-time surface would have been effectively 2-dimensional. The recent view is that these symmetries pose conditions only on the 3-D holographic data. The holographic charges would correspond to additional isometries of WCW and would be well-defined for the 3-surfaces at the light-cone boundary.

To sum up, one can imagine many options but the following picture is perhaps the simplest one and is supported by physical intuition and mathematical facts. The isometry algebra of $\delta M^4_+ \times CP_2$ consists of generalized conformal and KM algebras at 3-surfaces in $\delta M^4_+ \times CP_2$ and symplectic algebras at the light cone boundary and 3-D light-like partonic orbits. The latter symmetries give constraints on the 3-D holographic data. It is still unclear whether one can assign generalized conformal and Kac-Moody charges to Chern-Simons-Kähler action. The isomorphic subalgebras labelled by a positive integer and their commutators with the entire algebra would annihilate the physical states. The isomorphic subalgebras labelled by a positive integer and their commutators with the entire algebra would annihilate the physical states. These two representations would generalize the notions of inertial and gravitational mass and their equivalence would generalize the Equivalence Principle.

Objection against the idea about theoretician friendly Mother Nature

One of the key ideas behind the TGD view of dark matter is that Nature is theoretician friendly [L161]. When the coupling strength proportional to $\hbar_{eff}$ becomes so large that perturbation series ceases to converge, a phase transition increasing the value of h_{eff} takes place so that the perturbation series converges.

One can however argue that this argument is quantum field-theoretic and does not apply in TGD since holography changes the very concept of perturbation theory. There is no path integral to worry about. Path integral is indeed such a fundamental concept that one expects it to have some approximate counterpart also in the TGD Universe. Bohr orbits are not completely deterministic: could the sum over the Bohr orbits however translate to an approximate description as a path integral at the QFT limit? The dynamics of light-like partonic orbits is indeed non-deterministic and could give rise to an analog of path integral as a finite sum.

1. The dynamics implied by Chern-Simons-Kähler action assignable to the partonic 3-surface with light-one coordinate in the role of time, is very topological in that the partonic orbits is light-like 3-surface and has 2-D CP_2 and M^4 projections unless the induced M^4 and CP_2 Kähler forms sum up to zero. The light-likeness of the projection is a very loose condition and and the sum over partonic orbits as possible representation of holographic data analogous to initial values (light-likeness!) is therefore analogous to the sum over all paths appearing as a representation of Schrödinger equation in wave mechanics.

   One would have an analog of 1-D QFT. This means that the infinities of quantum field theories are absent but for a large enough coupling strength $g^2/4\pi\hbar$ the perturbation series fails to converge. The increase of h_{eff} would resolve the problem. For instance, Dirac equation in atomic physics makes unphysical predictions when the value of nuclear charge is larger than $Z \sim 137$.

2. I have also considered a discrete variant of this picture motivated by the fact that the presence of the volume term in the action implies that the M^4 projection of the CP_2 type extremal is a light-like geodesic line. The light-like orbits would consist of pieces of light-like geodesics implying that the average velocity would be smaller than c: this could be seen as a correlate for massivation.

   The points at which the direction of segment changes would correspond to points at which energy and momentum transfer between the partonic orbit and environment takes place. This kind of quantum number transfer might occur at least for the fermionic lines as boundaries of string world sheets. They could be described quantum mechanically as interactions with classical fields in the same way as the creation of fermion pairs as a fundamental vertex [L160]. The same universal 2-vertex would be in question.

   At these points the minimal surface property would fail and the trace of the second fundamental form would not vanish but would have a delta function-like singularity. The CP_2 part of the second fundamental form has quantum numbers of Higgs so that there would be an analogy with the standard description of massivation by the Higgs mechanism. Higgs would be only where the vertices are.

3. What is intriguing, that the light-likeness of the projection of the CP_2 type extremals in M^4 leads to Virasoro conditions assignable to M^4 coordinates and this eventually led to the idea of conformal symmetries as isometries as WCW. In the case of the partonic orbits, the light-like curve would be in $M^4 \times CP_2$ but it would not be surprising if the generalization of the Virasoro conditions would emerge also now.

   One can write M^4 and CP_2 coordinates for the light-like curve as Fourier expansion in powers of $exp(it)$, where t is the light-like coordinate. This gives $h^k = \sum h^k_n exp(int)$. If the CP_2 projection of the orbits of the partonic 2-surface is geodesic circle, CP_2 metric s_{kl} is constant, the light-likeness condition $h_{kl}\partial_t h^k \partial_t h^l = 0$ gives $Re[h_{kl} \sum_m h^k_{n-m}\overline{h}^l_m] = 0$. This does not give Virasoro conditions.

   The condition $d/dt(h_{kl}\partial_t h^k \partial_t h^l = 0) = 0$ however gives the standard Virasoro condition in quantization condition stating that the operator counterparts of quantities $L_n = Re[h_{kl} \sum_m (n-m)h^k_{n-m}\overline{h}^l_m]$ annihilate the physical states. What is interesting is that the latter condition also allows time-like (and even space-like) geodesics.

   Could massivation mean a failure of light-likeness? For piecewise light-like geodesics the light-likeness condition would be true only inside the segments. By taking Fourier transform one expects to obtain Virasoro conditions with a cutoff analogous to the momentum cutoff in condensed matter physics for crystals.

4. In TGD the Virasoro, Kac-Moody algebras and symplectic algebras are replaced by half-algebras and the gauge conditions are satisfied for conformal weights which are n-multiples of fundamentals with with n larger than some minimal value. This would dramatically reduce the effects of the non-determinism and could make the sum over all paths allowed by the light-likeness manifestly finite and reduce it to a sum with a finite number of terms. This cutoff in degrees of freedom would correspond to a genuinely physical cutoff due to the finite measurement resolution coded to the number theoretical anatomy of the space-time surfaces. This cutoff is analogous to momentum cutoff and could at the space-time picture correspond to finite minimum length for the light-like segments of the orbit of the partoic 2-surface.

Boundary conditions at partonic orbits and holography

TGD reduces coupling constant evolution to a number theoretical evolution of the coupling parameters of the action identified as Kähler function for WCW. An interesting question is how the 3-D holographic data at the partonic orbits relates to the corresponding 3-D data at the ends of space-time surfaces at the boundary of CD, and how it relates to coupling constant evolution.

1. The twistor lift of TGD strongly favours 6-D Kähler action, which dimensionally reduces to Kähler action plus volume term plus topological $\int J \wedge J$ term reducing to Chern Simons-Kähler action. The coefficients of these terms are proposed to be expressible in terms of number theoretical invariants characterizing the algebraic extensions of rationals and polynomials determining the space-time surfaces by $M^8 - H$ duality.

   Number theoretical coupling constant evolution would be discrete. Each extension of rationals would give rise to its own coupling parameters involving also the ramified primes characterizing the polynomials involved and identified as p-adic length scales.

2. The time evolution of the partonic orbit would be non-deterministic but subject to the light-likeness constraint and boundary conditions guaranteeing conservation laws. The natural expectation is that the boundary/interface conditions for a given action cannot be satisfied for all partonic orbits (and other singularities). The deformation of the partonic orbit requiring that boundary conditions are satisfied, does not affect X^3 but the time derivatives $\partial_t h^k$ at X^3 are affected since the form of the holomorphic functions defining the space-time surface would change. The interpretation would be in terms of duality of the holographic data associated with the partonic orbits *resp.* X^3.

   There can of course exist deformations, which require the change of the coupling parameters of the action to satisfy the boundary conditions. One can consider an analog of renormalization group equations in which the deformation corresponds to a modification of the coupling parameters of the action, most plausibly determined by the twistor lift. Coupling parameters would label different regions of WCW and the space-time surfaces possible for two different sets of coupling parameters would define interfaces between these regions.

In order to build a more detailed view one must fix the details related to the action whose value defines the WCW Kähler function.

1. If Kähler action is identified as Kähler action, the identification is unique. There is however the possibility that the imaginary exponent of the instanton term or the contribution from the Euclidean region is not included in the definition of Kähler function. For instance instanton term could be interpreted as a phase of quantum state and would not contribute.

2. Both Minkowskian and Euclidean regions are involved and the Euclidean signature poses problems. The definition of the determinant as $\sqrt{-g_4}$ is natural in Minkowskian regions but gives an imaginary contribution in Euclidean regions. $\sqrt{|g_4|}$ is real in both regions. $i\sqrt{g_4}$ is real in Minkowskian regions but imaginary in the Euclidean regions.

   There is also a problem related to the instanton term, which does not depend on the metric determinant at all. In QFT context the instanton term is imaginary and this is important for instance in QCD in the definition of CP breaking vacuum functional. Should one include only the 4-D or possibly only Minkowskian contribution to the Kähler function imaginary coefficient for the instanton/Euclidian term would be possible?

3. Boundary conditions guaranteeing the conservation laws at the partonic orbits must be satisfied. Consider the $\sqrt{|g_4|}$ case. Charge transfer between Euclidean and Minkowskian regions. If the C-S-K term is real, also the charge transfer between partonic orbit and 4-D regions is possible. The boundary conditions at the partonic orbit fix it to a high degree and also affect the time derivatives $\partial_t h^k$ at X^3. This option looks physically rather attractive because classical conserved charges would be real.

   If the C-S-K term is imaginary it behaves like a free particle since charge exchange with Minkowskian and Euclidean regions is not possible. A possible interpretation of the possible M^4 contribution to momentum could be in terms of decay width. The symplectic charges do not however involve momentum. The imaginary contribution to momentum could therefore come only from the Euclidean region.

4. If the Euclidean contribution is imaginary, it seems that it cannot be included in the Kähler function. Since in M^8 picture the momenta of virtual fermions are in general complex, one could consider the possibility that Euclidean contribution to the momentum is imaginary and allows an interpretation as a decay width.

The TGD counterparts of the gauge conditions of string models

The string model picture forces to ask whether the symplectic algebras and the generalized conformal and Kac-Moody algebras could act as gauge symmetries.

1. In string model picture conformal invariance would suggest that the generators of the generalized conformal and KM symmetries act as gauge transformations annihilate the physical states. In the TGD framework, this does not however make sense physically. This also suggests that the components of the metric defined by supergenerators of generalized conformal and Kac Moody transformations vanish. If so, the symplectomorphisms $\delta M^4_+ \times CP_2$ localized with respect to the light-like radial coordinate acting as isometries would be needed. The half-algebras of both symplectic and conformal generators are labelled by a non-negative integer defining an analog of conformal weight so there is a fractal hierarchy of isomorphic subalgebras in both cases.

2. TGD forces to ask whether only subalgebras of both conformal and Kac-Moody half algebras, isomorphic to the full algebras, act as gauge algebras. This applies also to the symplectic case. Here it is essential that only the half algebra with non-negative multiples of the fundamental conformal weights is allowed. For the subalgebra annihilating the states the conformal weights would be fixed integer multiples of those for the full algebra. The gauge property would be true for all algebras involved. The remaining symmetries would

be genuine dynamical symmetries of the reduced WCW and this would reflect the number theoretically realized finite measurement resolution. The reduction of degrees of freedom would also be analogous to the basic property of hyperfinite factors assumed to play a key role in thee definition of finite measurement resolution.

3. For strong holography, the orbits of partonic 2-surfaces and boundaries of the spacetime surface at δM^4_+ would be dual in the information theoretic sense. Either would be enough to determine the space-time surface.

5.3.3 Interior Dynamics For Fermions, The Role Of Vacuum Extremals, And Dark Matter

The key role of CP_2-type and M^4-type vacuum extremals has been rather obvious from the beginning but the detailed understanding has been lacking. Both kinds of extremals are invariant under symplectic transformations of $\delta M^4 \times CP_2$, which inspires the idea that they give rise to isometries of WCW . The deformations CP_2-type extremals correspond to lines of generalized Feynman diagrams. M^4 type vacuum extremals in turn are excellent candidates for the building bricks of many-sheeted space-time giving rise to GRT space-time as approximation. For M^4 type vacuum extremals CP_2 projection is (at most 2-D) Lagrangian manifold so that the induced Kähler form vanishes and the action is fourth-order in small deformations. This implies the breakdown of the path integral approach and of canonical quantization, which led to the notion of WCW .

If the action in Minkowskian regions contains also string area, the situation changes dramatically since strings dominate the dynamics in excellent approximation and string theory should give an excellent description of the situation: this of course conforms with the dominance of gravitation.

String tension would be proportional to $1/\hbar G$ and this raises a grave classical counter argument. In string model massless particles are regarded as strings, which have contracted to a point in excellent approximation and cannot have length longer than Planck length. How this can be consistent with the formation of gravitationally bound states is however not understood since the required non-perturbative formulation of string model required by the large valued of the coupling parameter GMm is not known.

In TGD framework strings would connect even objects with macroscopic distance and would obviously serve as correlates for the formation of bound states in quantum level description. The classical energy of string connecting say the two wormhole contacts defining elementary particle is gigantic for the ordinary value of $\hbar$ so that something goes wrong.

I have however proposed [K101, K83, K84] that gravitons - at least those mediating interaction between dark matter have large value of Planck constant. I talk about gravitational Planck constant and one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where $v_0/c < 1$ (v_0 has dimensions of velocity). This makes possible perturbative approach to quantum gravity in the case of bound states having mass larger than Planck mass so that the parameter GMm analogous to coupling constant is very large. The velocity parameter v_0/c becomes the dimensionless coupling parameter. This reduces the string tension so that for string world sheets connecting macroscopic objects one would have $T \propto v_0/G^2Mm$. For $v_0 = GMm/\hbar$, which remains below unity for Mm/m_{Pl}^2 one would have $h_{gr}/h = 1$. Hence action remains small and its imaginary exponent does not fluctuate wildly to make the bound state forming part of gravitational interaction short ranged. This is expected to hold true for ordinary matter in elementary particle scales. The objects with size scale of large neutron (100 μm in the density of water) - probably not an accident - would have mass above Planck mass so that dark gravitons and also life would emerge as massive enough gravitational bound states are formed. $h_{gr} = h_{eff}$ hypothesis is indeed central in TGD based view about living matter.

If one assumes that for non-standard values of Planck constant only n-multiples of super-conformal algebra in interior annihilate the physical states, interior conformal gauge degrees of freedom become partly dynamical. The identification of dark matter as macroscopic quantum phases labeled by $h_{eff}/h = n$ conforms with this.

The emergence of dark matter corresponds to the emergence of interior dynamics via breaking of super-conformal symmetry. The induced spinor fields in the interior of flux tubes obeying Kähler Dirac action should be highly relevant for the understanding of dark matter. The assumption that dark particles have essentially same masses as ordinary particles suggests that dark fermions correspond to induced spinor fields at both string world sheets and in the space-time interior: the spinor fields in the interior would be responsible for the long range correlations characterizing $h_{eff}/h = n$. Magnetic flux tubes carrying dark matter are key entities in TGD inspired quantum biology. Massless extremals represent second class of M^4 type non-vacuum extremals.

This view forces once again to ask whether space-time SUSY is present in TGD and how it is realized. With a motivation coming from the observation that the mass scales of particles and sparticles most naturally have the same p-adic mass scale as particles in TGD Universe I have proposed that sparticles might be dark in TGD sense. The above argument leads to ask whether the dark variants of particles correspond to states in which one has ordinary fermion at string world sheet and 4-D fermion in the space-time interior so that dark matter in TGD sense would almost by definition correspond to sparticles!

5.3.4 Classical Number Fields And Associativity And Commutativity As Fundamental Law Of Physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Embedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [A73]) are involved [K107] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the embedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K107]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of Kähler-Dirac action the identification of space-time surface as a hyper-quaternionic sub-manifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a hyper-quaternionic sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K127, K116].

How to achieve associativity in the fermionic sector?

In the fermionic sector an additional complication emerges. The associativity of the tangent- or normal space of the space-time surface need not be enough to guarantee the associativity at the level of Kähler-Dirac or Dirac equation. The reason is the presence of spinor connection. A possible cure could be the vanishing of the components of spinor connection for two conjugates of quaternionic coordinates combined with holomorphy of the modes.

1. The induced spinor connection involves sigma matrices in CP_2 degrees of freedom, which for the octonionic representation of gamma matrices are proportional to octonion units in

Minkowski degrees of freedom. This corresponds to a reduction of tangent space group $SO(1,7)$ to G_2. Therefore octonionic Dirac equation identifying Dirac spinors as complexified octonions can lead to non-associativity even when space-time surface is associative or co-associative.

2. The simplest manner to overcome these problems is to assume that spinors are localized at 2-D string world sheets with 1-D CP_2 projection and thus possible only in Minkowskian regions. Induced gauge fields would vanish. String world sheets would be minimal surfaces in $M^4 \times D^1 \subset M^4 \times CP_2$ and the theory would simplify enormously. String area would give rise to an additional term in the action assigned to the Minkowskian space-time regions and for vacuum extremals one would have only strings in the first approximation, which conforms with the success of string models and with the intuitive view that vacuum extremals of Kähler action are basic building bricks of many-sheeted space-time. Note that string world sheets would be also symplectic covariants.

   Without further conditions gauge potentials would be non-vanishing but one can hope that one can gauge transform them away in associative manner. If not, one can also consider the possibility that CP_2 projection is geodesic circle S^1: symplectic invariance is considerably reduces for this option since symplectic transformations must reduce to rotations in S^1.

3. The fist heavy objection is that action would contain Newton's constant G as a fundamental dynamical parameter: this is a standard recipe for building a non-renormalizable theory. The very idea of TGD indeed is that there is only single dimensionless parameter analogous to critical temperature. One can of coure argue that the dimensionless parameter is $\hbar G/R^2$, R CP_2 "radius".

   Second heavy objection is that the Euclidian variant of string action exponentially damps out all string world sheets with area larger than $\hbar G$. Note also that the classical energy of Minkowskian string would be gigantic unless the length of string is of order Planck length. For Minkowskian signature the exponent is oscillatory and one can argue that wild oscillations have the same effect.

   The hierarchy of Planck constants would allow the replacement $\hbar \rightarrow \hbar_{eff}$ but this is not enough. The area of typical string world sheet would scale as h_{eff} and the size of CD and gravitational Compton lengths of gravitationally bound objects would scale as $\sqrt{h_{eff}}$ rather than $\hbar_{eff} = GMm/v_0$, which one wants. The only way out of problem is to assume $T \propto (\hbar/h_{eff})^2 \times (1/h_{bar}G)$. This is however un-natural for genuine area action. Hence it seems that the visit of the basic assumption of superstring theory to TGD remains very short.

Is super-symmetrized Kähler-Dirac action enough?

Could one do without string area in the action and use only K-D action, which is in any case forced by the super-conformal symmetry? This option I have indeed considered hitherto. K-D Dirac equation indeed tends to reduce to a lower-dimensional one: for massless extremals the K-D operator is effectively 1-dimensional. For cosmic strings this reduction does not however take place. In any case, this leads to ask whether in some cases the solutions of Kähler-Dirac equation are localized at lower-dimensional surfaces of space-time surface.

1. The proposal has indeed been that string world sheets carry vanishing W and possibly even Z fields: in this manner the electromagnetic charge of spinor mode could be well-defined. The vanishing conditions force in the generic case 2-dimensionality.

   Besides this the canonical momentum currents for Kähler action defining 4 embedding space vector fields must define an integrable distribution of two planes to give string world sheet. The four canonical momentum currents $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ identified as embedding 1-forms can have only two linearly independent components parallel to the string world sheet. Also the Frobenius conditions stating that the two 1-forms are proportional to gradients of two embedding space coordinates Φ_i defining also coordinates at string world sheet, must be satisfied. These conditions are rather strong and are expected to select some discrete set of string world sheets.

2. To construct preferred extremal one should fix the partonic 2-surfaces, their light-like orbits defining boundaries of Euclidian and Minkowskian space-time regions, and string world sheets.

At string world sheets the boundary condition would be that the normal components of canonical momentum currents for Kähler action vanish. This picture brings in mind strong form of holography and this suggests that might make sense and also solution of Einstein equations with point like sources.

3. The localization of spinor modes at 2-D surfaces would would follow from the well-definedness of em charge and one could have situation is which the localization does not occur. For instance, covariantly constant right-handed neutrinos spinor modes at cosmic strings are completely de-localized and one can wonder whether one could give up the localization inside wormhole contacts.

4. String tension is dynamical and physical intuition suggests that induced metric at string world sheet is replaced by the anti-commutator of the K-D gamma matrices and by conformal invariance only the conformal equivalence class of this metric would matter and it could be even equivalent with the induced metric. A possible interpretation is that the energy density of Kähler action has a singularity localized at the string world sheet.

   Another interpretation that I proposed for years ago but gave up is that in spirit with the TGD analog of AdS/CFT duality the Noether charges for Kähler action can be reduced to integrals over string world sheet having interpretation as area in effective metric. In the case of magnetic flux tubes carrying monopole fluxes and containing a string connecting partonic 2-surfaces at its ends this interpretation would be very natural, and string tension would characterize the density of Kähler magnetic energy. String model with dynamical string tension would certainly be a good approximation and string tension would depend on scale of CD.

5. There is also an objection. For M^4 type vacuum extremals one would not obtain any non-vacuum string world sheets carrying fermions but the successes of string model strongly suggest that string world sheets are there. String world sheets would represent a deformation of the vacuum extremal and far from string world sheets one would have vacuum extremal in an excellent approximation. Situation would be analogous to that in general relativity with point particles.

6. The hierarchy of conformal symmetry breakings for K-D action should make string tension proportional to $1/h_{eff}^2$ with $h_{eff} = h_{gr}$ giving correct gravitational Compton length $\Lambda_{gr} = GM/v_0$ defining the minimal size of CD associated with the system. Why the effective string tension of string world sheet should behave like $(\hbar/\hbar_{eff})^2$?

   The first point to notice is that the effective metric $G^{\alpha\beta}$ defined as $h^{kl}\Pi_k^\alpha\Pi_l^\beta$, where the canonical momentum current $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ has dimension $1/L^2$ as required. Kähler action density must be dimensionless and since the induced Kähler form is dimensionless the canonical momentum currents are proportional to $1/\alpha_K$.

   Should one assume that α_K is fundamental coupling strength fixed by quantum criticality to $\alpha_K = 1/137$? Or should one regard g_K^2 as fundamental parameter so that one would have $1/\alpha_K = \hbar_{eff}/4\pi g_K^2$ having spectrum coming as integer multiples (recall the analogy with inverse of critical temperature)?

   The latter option is the in spirit with the original idea stating that the increase of h_{eff} reduces the values of the gauge coupling strengths proportional to α_K so that perturbation series converges (Universe is theoretician friendly). The non-perturbative states would be critical states. The non-determinism of Kähler action implying that the 3-surfaces at the boundaries of CD can be connected by large number of space-time sheets forming n conformal equivalence classes. The latter option would give $G^{\alpha\beta} \propto h_{eff}^2$ and $det(G) \propto 1/h_{eff}^2$ as required.

7. It must be emphasized that the string tension has interpretation in terms of gravitational coupling on only at the GRT limit of TGD involving the replacement of many-sheeted space-time with single sheeted one. It can have also interpretation as hadronic string tension or effective string tension associated with magnetic flux tubes and telling the density of Kähler magnetic energy per unit length.

   Superstring models would describe only the perturbative Planck scale dynamics for emission and absorption of $h_{eff}/h = 1$ on mass shell gravitons whereas the quantum description of bound states would require $h_{eff}/n > 1$ when the masses. Also the effective gravitational constant associated with the strings would differ from G.

The natural condition is that the size scale of string world sheet associated with the flux tube mediating gravitational binding is $G(M+m)/v_0$, By expressing string tension in the form $1/T = n^2\hbar G_1$, $n = h_{eff}/h$, this condition gives $\hbar G_1 = \hbar^2/M_{red}^2$, $M_{red} = Mm/(M+m)$. The effective Planck length defined by the effective Newton's constant G_1 analogous to that appearing in string tension is just the Compton length associated with the reduced mass of the system and string tension equals to $T = [v_0/G(M+m)]^2$ apart from a numerical constant ($2G(M+m)$ is Schwartschild radius for the entire system). Hence the macroscopic stringy description of gravitation in terms of string differs dramatically from the perturbative one. Note that one can also understand why in the Bohr orbit model of Nottale [E1] for the planetary system and in its TGD version [K101] v_0 must be by a factor 1/5 smaller for outer planets rather than inner planets.

Are 4-D spinor modes consistent with associativity?

The condition that octonionic spinors are equivalent with ordinary spinors looks rather natural but in the case of Kähler-Dirac action the non-associativity could leak in. One could of course give up the condition that octonionic and ordinary K-D equation are equivalent in 4-D case. If so, one could see K-D action as related to non-commutative and maybe even non-associative fermion dynamics. Suppose that one does not.

1. K-D action vanishes by K-D equation. Could this save from non-associativity? If the spinors are localized to string world sheets, one obtains just the standard stringy construction of conformal modes of spinor field. The induce spinor connection would have only the holomorphic component A_z. Spinor mode would depend only on z but K-D gamma matrix Γ^z would annihilate the spinor mode so that K-D equation would be satisfied. There are good hopes that the octonionic variant of K-D equation is equivalent with that based on ordinary gamma matrices since quaternionic coordinated reduces to complex coordinate, octonionic quaternionic gamma matrices reduce to complex gamma matrices, sigma matrices are effectively absent by holomorphy.
2. One can consider also 4-D situation (maybe inside wormhole contacts). Could some form of quaternion holomorphy [A99] [K116] allow to realize the K-D equation just as in the case of super string models by replacing complex coordinate and its conjugate with quaternion and its 3 conjugates. Only two quaternion conjugates would appear in the spinor mode and the corresponding quaternionic gamma matrices would annihilate the spinor mode. It is essential that in a suitable gauge the spinor connection has non-vanishing components only for two quaternion conjugate coordinates. As a special case one would have a situation in which only one quaternion coordinate appears in the solution. Depending on the character of quaternionion holomorphy the modes would be labelled by one or two integers identifiable as conformal weights.

   Even if these octonionic 4-D modes exists (as one expects in the case of cosmic strings), it is far from clear whether the description in terms of them is equivalent with the description using K-D equation based ordinary gamma matrices. The algebraic structure however raises hopes about this. The quaternion coordinate can be represented as sum of two complex coordinates as $q = z_1 + Jz_2$ and the dependence on two quaternion conjugates corresponds to the dependence on two complex coordinates z_1, z_2. The condition that two quaternion complexified gammas annihilate the spinors is equivalent with the corresponding condition for Dirac equation formulated using 2 complex coordinates. This for wormhole contacts. The possible generalization of this condition to Minkowskian regions would be in terms Hamilton-Jacobi structure.

   Note that for cosmic strings of form $X^2 \times Y^2 \subset M^4 \times CP_2$ the associativity condition for S^2 sigma matrix and without assuming localization demands that the commutator of Y^2 imaginary units is proportional to the imaginary unit assignable to X^2 which however depends on point of X^2. This condition seems to imply correlation between Y^2 and S^2 which does not look physical.

To summarize, the minimal and mathematically most optimistic conclusion is that Kähler-Dirac action is indeed enough to understand gravitational binding without giving up the associativity of the fermionic dynamics. Conformal spinor dynamics would be associative if the spinor modes are localized at string world sheets with vanishing W (and maybe also Z) fields guaranteeing well-definedness of em charge and carrying canonical momentum currents parallel to them. It is not quite clear whether string world sheets are present also inside wormhole contacts: for CP_2 type vacuum extremals the Dirac equation would give only right-handed neutrino as a solution (could they give rise to $N = 2$ SUSY?).

The construction of preferred extremals would realize strong form of holography. By conformal symmetry the effective metric at string world sheet could be conformally equivalent with the induced metric at string world sheets.

Dynamical string tension would be proportional to $\hbar/h_{eff}^2$ due to the proportionality $\alpha_K \propto 1/h_{eff}$ and predict correctly the size scales of gravitationally bound states for $\hbar_{gr} = \hbar_{eff} = GMm/v_0$. Gravitational constant would be a prediction of the theory and be expressible in terms of α_K and R^2 and $\hbar_{eff}$ ($G \propto R^2/g_K^2$).

In fact, all bound states - elementary particles as pairs of wormhole contacts, hadronic strings, nuclei [K74], molecules, etc. - are described in the same manner quantum mechanically. This is of course nothing new since magnetic flux tubes associated with the strings provide a universal model for interactions in TGD Universe. This also conforms with the TGD counterpart of AdS/CFT duality.

5.4 About some unclear issues of TGD

TGD has been in the middle of palace revolution during last two years and it is almost impossible to keep the chapters of the books updated. Adelic vision and twistor lift of TGD are the newest developments and there are still many details to be understood and errors to be corrected. The description of fermions in TGD framework has contained some unclear issues. Hence the motivation for the following brief comments.

There questions about the adelic vision about symmetries. Do the cognitive representations implying number theoretic disretization of the space-time surface lead to the breaking of the basic symmetries and are preferred embedding space coordinates actually necessary?

In the fermionic sector there are many questions deserving clarification. How quantum classical correspondence (QCC) is realized for fermions? How is SH realized for fermions and how does it lead to the reduction of dimension $D = 4$ to $D = 2$ (apart from number theoretical discretization)? Can scattering amplitudes be really formulated by using only the data at the boundaries of string sheets and what does this mean from the point of view of the modified Dirac equation? Are the spinors at light-like boundaries limiting values or sources? A long-standing issue concerns the fermionic anti-commutation relations: what motivated this article was the solution of this problem. There is also the general problem about whether statistical entanglement is "real".

5.4.1 Adelic vision and symmetries

In the adelic TGD SH is weakened: also the points of the space-time surface having embedding space coordinates in an extension of rationals (cognitive representation) are needed so that data are not precisely 2-D. I have believed hitherto that one must use preferred coordinates for the embedding space H - a subset of these coordinates would define space-time coordinates. These coordinates are determined apart from isometries. Does the number theoretic discretization imply loss of general coordinate invariance and also other symmetries?

The reduction of symmetry groups to their subgroups (not only algebraic since powers of e define finite-dimensional extension of p-adic numbers since e^p is ordinary p-adic number) is genuine loss of symmetry and reflects finite cognitive resolution. The physics itself has the symmetries of real physics.

The assumption about preferred embedding space coordinates is actually not necessary. Different choices of H-coordinates means only different and non-equivalent cognitive representations. Spherical and linear coordinates in finite accuracy do not provide equivalent representations.

5.4.2 Quantum-classical correspondence for fermions

Quantum-classical correspondence (QCC) for fermions is rather well-understood but deserves to be mentioned also here.

QCC for fermions means that the space-time surface as preferred extremal should depend on fermionic quantum numbers. This is indeed the case if one requires QCC in the sense that the fermionic representations of Noether charges in the Cartan algebras of symmetry algebras are equal to those to the classical Noether charges for preferred extremals.

Second aspect of QCC becomes visible in the representation of fermionic states as point like particles moving along the light-like curves at the light-like orbits of the partonic 2-surfaces (curve at the orbit can be locally only light-like or space-like). The number of fermions and antifermions dictates the number of string world sheets carrying the data needed to fix the preferred extremal by SH. The complexity of the space-time surface increases as the number of fermions increases.

5.4.3 Strong form of holography for fermions

It seems that scattering amplitudes can be formulated by assigning fermions with the boundaries of strings defining the lines of twistor diagrams [K46, L55]. This information theoretic dimensional reduction from $D = 4$ to $D = 2$ for the scattering amplitudes can be partially understood in terms of strong form of holography (SH): one can construct the theory by using the data at string worlds sheets and/or partonic 2-surfaces at the ends of the space-time surface at the opposite boundaries of causal diamond (CD).

4-D modified Dirac action would appear at fundamental level as supersymmetry demands but would be reduced for preferred extremals to its 2-D stringy variant serving as effective action. Also the value of the 4-D action determining the space-time dynamics would reduce to effective stringy action containing area term, 2-D Kähler action, and topological Kähler magnetic flux term. This reduction would be due to the huge gauge symmetries of preferred extremals. Sub-algebra of super-symplectic algebra with conformal weigths coming as n-multiples of those for the entire algebra and the commutators of this algebra with the entire algebra would annihilate the physical states, and thecorresponding classical Noether charges would vanish.

One still has the question why not the data at the entire string world sheets is not needed to construct scattering amplitudes. Scattering amplitudes of course need not code for the entire physics. QCC is indeed motivated by the fact that quantum experiments are always interpreted in terms of classical physics, which in TGD framework reduces to that for space-time surface.

5.4.4 The relationship between spinors in space-time interior and at boundaries between Euclidian and Minkoskian regions

Space-time surface decomposes to interiors of Minkowskian and Euclidian regions. At light-like 3-surfaces at which the four-metric changes, the 4-metric is degenerate. These metrically singular 3-surfaces - partonic orbits- carry the boundaries of string world sheets identified as carriers of fermionic quantum numbers. The boundaries define fermion lines in the twistor lift of TGD [K46, L55]. The relationship between fermions at the partonic orbits and interior of the space-time surface has however remained somewhat enigmatic.

So: What is the precise relationship between induced spinors Ψ_B at light-like partonic 3-surfaces and Ψ_I in the interior of Minkowskian and Euclidian regions? Same question can be made for the spinors Ψ_B at the boundaries of string world sheets and Ψ_I in interior of the string world sheets. There are two options to consider:

- Option I: Ψ_B is the limiting value of Ψ_I .
- Option II: Ψ_B serves as a source of Ψ_I .

For the Option I it is difficult to understand the preferred role of Ψ_B.

I have considered Option II already years ago but have not been able to decide.

1. That scattering amplitudes could be formulated only in terms of sources only, would fit nicely with SH, twistorial amplitude construction, and also with the idea that scattering amplitudes in gauge theories can be formulated in terms of sources of boson fields assignable to vertices and propagators. Now the sources would become fermionic.

2. One can take gauge theory as a guideline. One adds to free Dirac equation source term $\gamma^k A_k \Psi$. Therefore the natural boundary term in the action would be of the form (forgetting overall scale factor)

$$S_B = \overline{\Psi}_I \Gamma^\alpha (C-S) A_\alpha \Psi_B + c.c \quad .$$

Here the modified gamma matrix is $\Gamma^\alpha(C-S)$ (contravariant form is natural for light-like 3-surfaces) is most naturally defined by the boundary part of the action - naturally Chern-Simons term for Kähler action. A denotes the Kähler gauge potential.

3. The variation with respect to Ψ_B gives

$$G^\alpha (C-S) A_\alpha \Psi_I = 0$$

at the boundary so that the C-S term and interaction term vanish. This does not however imply vanishing of the source term! This condition can be seen as a boundary condition.

The same argument applies also to string world sheets.

5.4.5 About second quantization of the induced spinor fields

The anti-commutation relations for the induced spinors have been a long-standing issue and during years I have considered several options. The solution of the problem looks however stupifuingly simple. The conserved fermion currents are accompanied by super-currents obtained by replacing Ψ with a mode of the induced spinor field to get $\overline{u}_n \Gamma^\alpha \Psi$ or $\overline{\Psi} \Gamma^\alpha u_n$ with the conjugate of the mode. One obtains infinite number of conserved super currents. One can also replace both Ψ and $\overline{\Psi}$ in this manner to get purely bosonic conserved currents $\overline{u}_m \Gamma^\alpha u_n$ to which one can assign a conserved bosonic charges Q_{mn}.

I noticed this years ago but did not realize that these bosonic charges define naturally anti-commutators of fermionic creation and annihilation operators! The ordinary anti-commutators of quantum field theory follow as a special case! By a suitable unitary transformation of the spinor basis one can diagonalize the hermitian matrix defined by Q_{mn} and by performing suitable scalings one can transform anti-commutation relations to the standard form. An interesting question is whether the diagonalization is needed, and whether the deviation of the diagonal elements from unity could have some meaning and possibly relate to the hierarchy $h_{eff} = n \times h$ of Planck constants - probably not.

5.4.6 Is statistical entanglement "real" entanglement?

The question about the "reality" of statistical entanglement has bothered me for years. This entanglement is maximal and it cannot be reduced by measurement so that one can argue that it is not "real". Quite recently I learned that there has been a longstanding debate about the statistical entanglement and that the issue still remains unresolved.

The idea that all electrons of the Universe are maximally entangled looks crazy. TGD provides several variants for solutions of this problem. It could be that only the fermionic oscillator operators at partonic 2-surfaces associated with the space-time surface (or its connected component) inside given CD anti-commute and the fermions are thus indistinguishable. The extremist option is that the fermionic oscillator operators belonging to a network of partonic 2-surfaces connected by string world sheets anti-commute: only the oscillator operators assignable to the same scattering diagram would anti-commute.

What about QCC in the case of entanglement. ER-EPR correspondence introduced by Maldacena and Susskind for 4 years ago proposes that blackholes (maybe even elementary particles) are connected by wormholes. In TGD the analogous statement emerged for more than decade ago - magnetic flux tubes take the role of wormholes in TGD. Magnetic flux tubes were assumed to be accompanied by string world sheets. I did not consider the question whether string world sheets are always accompanied by flux tubes.

What could be the criterion for entanglement to be "real"? "Reality" of entanglement demands some space-time correlate. Could the presence of the flux tubes make the entanglement "real"? If statistical entanglement is accompanied by string connections without magnetic flux tubes, it would not be "real": only the presence of flux tubes would make it "real". Or is the presence of strings enough to make the statistical entanglement "real". In both cases the fermions associated with disjoint space-time surfaces or with disjoint CDs would not be indistinguishable. This looks rather sensible.

The space-time correlate for the reduction of entanglement would be the splitting of a flux tube and fermionic strings inside it. The fermionic strings associated with flux tubes carrying monopole flux are closed and the return flux comes back along parallel space-time sheet. Also fermionic string has similar structure. Reconnection of this flux tube with shape of very long flattened square splitting it to two pieces would be the correlate for the state function reduction reducing the entanglement with other fermions and would indeed decouple the fermion from the network.

5.5 About The Notion Of Four-Momentum In TGD Framework

The starting point of TGD was the energy problem of General Relativity [K121]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K121] ? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of zero energy ontology (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K76] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K116]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian [A25] [B29, B22, B23] variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [B15]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

5.5.1 Scale Dependent Notion Of Four-Momentum In Zero Energy Ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K102], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of zero energy ontology (ZEO) [K8, K124], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K124]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K127].

5.5.2 Are The Classical And Quantal Four-Momenta Identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

5.5.3 What Equivalence Principle (EP) Means In Quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

1. Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

2. QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

   Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

   Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

   The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

3. Does EP at quantum level reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

   Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K76] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors is extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep. It is of course possible that the two realizations of EP are equivalent and the natural conjecture is that this is the case.

For QCC option the GRT inspired interpretation of Equivalence Principle at space-time level remains to be understood. Is it needed at all? The condition that the energy momentum tensor of Kähler action has a vanishing divergence leads in General Relativity to Einstein equations with cosmological term. In TGD framework preferred extremals satisfying the analogs of Einstein's equations with several cosmological constant like parameters can be considered.

Should one give up this idea, which indeed might be wrong? Could the divergence of of energy momentum tensor vanish only asymptotically as was the original proposal? Or should one try to generalize the interpretation? QCC states that quantum physics has classical correlate at space-time level and implies EP. Could also quantum classical correspondence itself have a correlate at space-time level. If so, space-time surface would able to represent abstractions as statements about statements about.... as the many-sheeted structure and the vision about TGD physics as analog of Turing machine able to mimic any other Turing machine suggest.

5.5.4 TGD-GRT Correspondence And Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.
3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.
4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore. It has turned out that this line of approach is too adhoc to be taken seriously.

5.5.5 How Translations Are Represented At The Level Of WCW ?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW ? ZEO provides a possible answer to this question.

Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub- WCW : s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - - > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

The action of translations at space-time sheets

The action of embedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred extremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

5.5.6 Yangian And Four-Momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [B15]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K116], where also references to the work of pioneers can be found.

Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K116]. Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant andconcrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B22]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A11] and Virasoro algebras [A20] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.
2. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.
3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.
2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.
3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

5.6 Generalization Of Ads/CFT Duality To TGD Framework

AdS/CFT duality has provided a powerful approach in the attempts to understand the non-perturbative aspects of super-string theories. The duality states that conformal field theory in n-dimensional Minkowski space M^n identifiable as a boundary of $n+1$-dimensional space AdS_{n+1} is dual to a string theory in $AdS_{n+1} \times S^{9-n}$.

As a mathematical discovery the duality is extremely interesting but it seems that it need not have much to do with physics. From TGD point of view the reason is obvious: the notion of conformal invariance is quite too limited. In TGD framework conformal invariance is extended to a super-symplectic symmetry in $\delta M^4_\pm \times CP_2$, whose Lie-algebra has the structure of conformal algebra. Also ordinary super-conformal symmetries associated with string world sheets are present as well as generalization of 2-D conformal symmetries to their analogs at light-cone boundary and light-like orbits of partonic 2-surfaces. In this framework AdS/CFT duality is expected to be modified and this seems to be the case.

The matrix elements of Kähler metric of WCW can be expressed in two ways. As contractions of the derivatives $\partial_K \partial_{\overline{L}} K$ of the Kähler function of WCW with isometry generators or as anticommutators of WCW gamma matrices identified as supersymplectic Noether super charges assignable to fermioni strings connecting partonic 2-surfaces. Kähler function is identified as Kähler action for the Euclidian space-time regions with 4-D CP_2 projection. Kähler action defines the Kähler-Dirac gamma matrices appearing in K-D action as contractions of canonical momentum currents with embedding space gamma matrices. Bosonic and fermionic degrees of freedom are therefore dual in a well-defined sense.

This observation suggests various generalizations. There is super-symmetry between Kähler action and Kähler-Dirac action. The problem is that induced spinor fields are localized at 2-D string world sheets. Strong form of holography implying effective 2-dimensionality suggests the solution to the paradox. The paradox disappears if the Kähler action is expressible as string area for the effective metric defined by the anti-commutators of K-D gamma matrices at string world sheet. This expression allows to understand how strings connecting partonic 2-surfaces give rise to the formation of gravitationally bound states. Bound states of macroscopic size are however possible only if one allows hierarchy of Planck constants. This representation of Kähler action can be seen as one aspect of the analog of AdS/CFT duality in TGD framework.

One can imagine also other realizations. For instance, Dirac determinant for the spinors associated with string world sheets should reduce to the exponent of Kähler action.

5.6.1 Does The Exponent Of Chern-Simons Action Reduce To The Exponent Of The Area Of Minimal Surfaces?

As I scanned of hep-th I found an interesting article (see `http://tinyurl.com/ycpkrg4f`) by Giordano, Peschanski, and Seki [B42] based on AdS/CFT correspondence. What is studied is the high energy behavior of the gluon-gluon and quark-quark scattering amplitudes of $\mathcal{N} = 4$ SUSY.

1. The proposal made earlier by Aldaya and Maldacena (see `http://tinyurl.com/ybnk6kbs`) [B13] is that gluon-gluon scattering amplitudes are proportional to the imaginary exponent of the area of a minimal surface in AdS_5 whose boundary is identified as *momentum space*. The boundary of the minimal surface would be polygon with light-like edges: this polygon and its dual are familiar from twistor approach.
2. Giordano, Peschanski, and Seki claim that quark-quark scattering amplitude for heavy quarks corresponds to the exponent of the area for a minimal surface in the *Euclidian* version of AdS_5 which is hyperbolic space (space with a constant negative curvature): it is interpreted as a counterpart of WCW rather than momentum space and amplitudes are obtained by analytic continuation. For instance, a universal Regge behavior is obtained. For general amplitudes the exponent of the area alone is not enough since it does not depend on gluon quantum numbers and vertex operators at the edges of the boundary polygon are needed.

In the following my intention is to consider the formulation of this conjecture in quantum TGD framework. I hasten to inform that I am not a specialist in AdS/CFT and can make only general comments inspired by analogies with TGD and the generalization of AdS/CFT duality to TGD framework based on the localization of induced spinors at string world sheets, supersymmetry between bosonic and fermionic degrees of freedom at the level of WCW , and the notion of effective metric at string world sheets.

5.6.2 Does Kähler Action Reduce To The Sum Of Areas Of Minimal Surfaces In Effective Metric?

Minimal surface conjectures are highly interesting from TGD point of view. The weak form of electric magnetic duality implies the reduction of Kähler action to 3-D Chern-Simons terms. Effective 2-dimensionality implied by the strong form of General Coordinate Invariance suggests a further reduction of Chern-Simons terms to 2-D terms and the areas of string world sheet and of partonic 2-surface are the only non-topological options that one can imagine. Skeptic could of course argue that the exponent of the minimal surface area results as a characterizer of the quantum state rather than vacuum functional. In the following I end up with the proposal that the Kähler action should reduce to the sum of string world sheet areas in the effective metric defines by the anticommutators of Kähler-Dirac gamma matrices at string world sheets.

Let us look this conjecture in more detail.

1. In zero energy ontology twistor approach is very natural since all physical states are bound states of massless particles. Also virtual particles are composites of massless states. The possibility to have both signs of energy makes possible space-like momenta for wormhole contacts. Mass shell conditions at internal lines imply extremely strong constraints on the virtual momenta and both UV and IR finiteness are expected to hold true.
2. The weak form of electric magnetic duality [K127] implies that the exponent of Kähler action reduces to the exponent of Chern-Simons term for 3-D space-like surfaces at the ends of space-time surface inside CD and for light-like 3-surfaces. The coefficient of this term is complex since the contribution of Minkowskian regions of the space-time surface is imaginary ($\sqrt{g_4}$ is imaginary) and that of Euclidian regions (generalized Feynman diagrams) real. The Chern-Simons term from Minkowskian regions is like Morse function and that from Euclidian regions defines Kähler function and stationary phase approximation makes sense. The two contributions are different since the space-like 3-surfaces contributing to Kähler function and Morse function are different.

3. Electric magnetic duality [K127] leads also to the conclusion that wormhole throats carrying elementary particle quantum numbers are Kähler magnetic monopoles. This forces to identify elementary particles as string like objects with ends having opposite monopole charges. Also more complex configurations are possible.

   It is not quite clear what the scale of the stringyness is. The natural first guess inspired by quantum classical correspondence is that it corresponds to the p-adic length scale of the particle characterizing its Compton length. Second possibility is that it corresponds to electroweak scale. For leptons stringyness in Compton length scale might not have any fatal implications since the second end of string contains only neutrinos neutralizing the weak isospin of the state. This kind of monopole pairs could appear even in condensed matter scales: in particular if the proposed hierarchy of Planck constants [K43] is realized.

4. Strong form of General Coordinate Invariance requires effective 2-dimensionality. In given UV and IR resolutions either partonic 2-surfaces or string world sheets form a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). The string world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids intersect partonic 2-surfaces at discrete points carrying fermionic quantum numbers.

   This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces whose area by quantum classical correspondence depends on the quantum numbers of the external particles. String tension in turn should depend on gauge couplings -perhaps only Kähler coupling strength- and geometric parameters like the size scale of CD and the p-adic length scale of the particle.

5. One can of course ask whether the metric defining the string area is induced metric or possibly the metric defined by the anti-commutators of Kähler-Dirac gamma matrices. The recent view does not actually leave any other alternative. The analog of AdS/CFT duality together with supersymmetry demands that Kähler action is proportional to the sum of the areas of string world sheets in this effective metric. Whether the vanishing of induced W fields (and possibly also Z^0) making possible well-defined em charge for the spinor nodes is realized by the condition that the string world sheet is a miniml surface in the effective metric remains an open question.

   The assumption that ordinary minimal surfaces are in question is not consistent with the TGD view about the formation of gravitational bound states and if string tension is $1/\hbar G$ as in string models, only bound states with size of order Planck length are possible. This strongly favors effective metric giving string tension proportional to $1/h_{eff}^2$. How $1/h_{eff}^2$ proportionality might be understood is discussed in [?] in terms electric-magnetic duality.

6. One can of course still consider also the option that ordinary minimal surfaces are in question. Are the minimal surfaces in question minimal surfaces of the embedding space $M^4 \times CP_2$ or of the space-time surface X^4? All possible 2-surfaces at the boundary of CD must be allowed so that they cannot correspond to minimal surfaces in $M^4 \times CP_2$ unless one assumes that they emerge in stationary phase approximation only. The boundary conditions at the ends of CD could however be such that *any* partonic 2-surface correspond to a minimal surfaces in X^4. Same applies to string world sheets. One might even hope that these conditions combined with the weak form of electric magnetic duality fixes completely the boundary conditions at wormhole throats and space-like ends of space-time surface.

   The trace of the second fundamental form orthogonal to the string world sheet/partonic 2-surface as sub-manifold of space-time surface would vanish: this is nothing but a generalization of the geodesic motion obtained by replacing word line with a 2-D surface. It does not imply the vanishing of the trace of the second fundamental form in $M^4 \times CP_2$ having interpretation as a generalization of particle acceleration [K121]. Effective 2-dimensionality would be realized if Chern-Simons terms reduce to a sum of the areas of these minimal surfaces.

These arguments suggest that scattering amplitudes are proportional to the product of exponents of 2-dimensional actions which can be either imaginary or real. Imaginary exponent would be proportional to the total area of string world sheets and the imaginary unit would come naturally from $\sqrt{g_2}$, where g_2 is effective metric most naturally. Teal exponent proportional to the total area of partonic 2-surfaces. The coefficient of these areas would not in general be same.

The reduction of the Kähler action from Minkowskian regions to Chern-Simons terms means that Chern-Simons terms reduce to actions assignable to string world sheets. The equality of the Minkowskian and Euclidian Chern-Simons terms is suggestive but not necessarily true since there could be also other Chern-Simons contributions than those assignable to wormhole throats and the ends of space-time. The equality would imply that the total area of string world sheets equals to the total area of partonic 2-surfaces suggesting strongly a duality meaning that either Euclidian or Minkowskian regions carry the needed information.

5.6.3 Surface Area As Geometric Representation Of Entanglement Entropy?

I encountered a link to a talk by James Sully and having the title "Geometry of Compression" (see http://tinyurl.com/ycuu8xcr). I must admit that I understood very little about the talk. My not so educated guess is however that information is compressed: UV or IR cutoff eliminating entanglement in short length scales and describing its presence in terms of density matrix - that is thermodynamically - is another manner to say it. The TGD inspired proposal for the interpretation of the inclusions of hyper-finite factors of type II_1 (HFFs) [K126] is in spirit with this.

The space-time counterpart for the compression would be in TGD framework discretization. Discretizations using rational points (or points in algebraic extensions of rationals) make sense also p-adically and thus satisfy number theoretic universality. Discretization would be defined in terms of intersection (rational or in algebraic extension of rationals) of real and p-adic surfaces. At the level of "world of classical worlds" the discretization would correspond to - say - surfaces defined in terms of polynomials, whose coefficients are rational or in some algebraic extension of rationals. Pinary UV and IR cutoffs are involved too. The notion of p-adic manifold allows to interpret the p-adic variants of space-time surfaces as cognitive representations of real space-time surfaces.

Finite measurement resolution does not allow state function reduction reducing entanglement totally. In TGD framework also negentropic entanglement stable under Negentropy Maximization Principle (NMP) is possible [K70]. For HFFs the projection into single ray of Hilbert space is indeed impossible: the reduction takes always to infinite-D sub-space.

The visit to the URL was however not in vain. There was a link to an article (see http://tinyurl.com/y9h3qtr8) [B63] discussing the geometrization of entanglement entropy inspired by the AdS/CFT hypothesis.

Quantum classical correspondence is basic guiding principle of TGD and suggests that entanglement entropy should indeed have space-time correlate, which would be the analog of Hawking-Bekenstein entropy.

Generalization of AdS/CFT to TGD context

AdS/CFT generalizes to TGD context in non-trivial manner. There are two alternative interpretations, which both could make sense. These interpretations are not mutually exclusive. The first interpretation makes sense at the level of "world of classical worlds" (WCW) with symplectic algebra and extended conformal algebra associated with $\delta M^4_\pm$ replacing ordinary conformal and Kac-Moody algebras. Second interpretation at the level of space-time surface with the extended conformal algebras of the light-likes orbits of partonic 2-surfaces replacing the conformal algebra of boundary of AdS^n.

1. First interpretation

For the first interpretation 2-D conformal invariance is generalised to 4-D conformal invariance relying crucially on the 4-dimensionality of space-time surfaces and Minkowski space.

1. One has an extension of the conformal invariance provided by the symplectic transformations of $\delta CD \times CP_2$ for which Lie algebra has the structure of conformal algebra with radial light-like coordinate of δM^4_+ replacing complex coordinate z.

2. One could see the counterpart of AdS_n as embedding space $H = M^4 \times CP_2$ completely unique by twistorial considerations and from the condition that standard model symmetries are

obtained and its causal diamonds defined as sub-sets $CD \times CP_2$, where CD is an intersection of future and past directed light-cones. I will use the shorthand CD for $CD \times CP_2$. Strings in $AdS_5 \times S^5$ are replaced with space-time surfaces inside 8-D CD.

3. For this interpretation 8-D CD replaces the 10-D space-time $AdS_5 \times S^5$. 7-D light-like boundaries of CD correspond to the boundary of say AdS_5, which is 4-D Minkowski space so that zero energy ontology (ZEO) allows rather natural formulation of the generalization of AdS/CFT correspondence since the positive and negative energy parts of zero energy states are localized at the boundaries of CD.

Second interpretation

For the second interpretation relies on the observation that string world sheets as carriers of induced spinor fields emerge in TGD framework from the condition that electromagnetic charge is well-defined for the modes of induced spinor field.

1. One could see the 4-D space-time surfaces X^4 as counterparts of AdS_4. The boundary of AdS_4 is replaced in this picture with 3-surfaces at the ends of space-time surface at opposite boundaries of CD and by strong form of holography the union of partonic 2-surfaces defining the intersections of the 3-D boundaries between Euclidian and Minkowskian regions of space-time surface with the boundaries of CD. Strong form of holography in TGD is very much like ordinary holography.

2. Note that one has a dimensional hierarchy: the ends of the boundaries of string world sheets at boundaries of CD as point-like partices, boundaries as fermion number carrying lines, string world sheets, light-like orbits of partonic 2-surfaces, 4-surfaces, embedding space $M^4 \times CP_2$. Clearly the situation is more complex than for AdS/CFT correspondence.

3. One can restrict the consideration to 3-D sub-manifolds X^3 at either boundary of causal diamond (CD): the ends of space-time surface. In fact, the position of the other boundary is not well-defined since one has superposition of CDs with only one boundary fixed to be piece of light-cone boundary. The delocalization of the other boundary is essential for the understanding of the arrow of time. The state function reductions at fixed boundary leave positive energy part (say) of the zero energy state at that boundary invariant (in positive energy ontology entire state would remain unchanged) but affect the states associated with opposite boundaries forming a superposition which also changes between reduction: this is analog for unitary time evolution. The average for the distance between tips of CDs in the superposition increases and gives rise to the flow of time.

4. One wants an expression for the entanglement entropy between X^3 and its partner. Bekenstein area law allows to guess the general expression for the entanglement entropy: for the proposal discussed in the article the entropy would be the area of the boundary of X^3 divided by gravitational constant: $S = A/4G$. In TGD framework gravitational constant might be replaced by the square of CP_2 radius apart from numerical constant. How gravitational constant emerges in TGD framework is not completely understood although one can deduce for it an estimate using dimensional analyses. In any case, gravitational constant is a parameter which characterizes GRT limit of TGD in which many-sheeted space-time is in long scales replaced with a piece of Minkowski space such that the classical gravitational fields and gauge potentials for sheets are summed. The physics behind this relies on the generalization of linear superposition of fields: the effects of different space-time sheets particle touching them sum up rather than fields.

5. The counterpart for the boundary of X^3 appearing in the proposal for the geometrization of the entanglement entropy naturally corresponds to partonic 2-surface or a collection of them if strong form of holography holds true.

   There is however also another candidate to be considered! Partonic 2-surfaces are basic objects, and one expects that the entanglement between fundamental fermions associated with distinct partonic 2-surfaces has string world sheets as space-time correlates. Could the area of the string world sheet in the effective metric defined by the anti-commutators of K-D gamma matrices at string world sheet provide a measure for entanglement entropy? If this conjecture is correct: the entanglement entropy would be proportional to Kähler action. Also negative values are possible for Kähler action in Minkowskian regions but in TGD framework number theoretic entanglement entropy having also negative values emerges naturally.

Which of these guesses is correct, if any? Or are they equivalent?

With what kind of systems 3-surfaces can entangle?

With what system X^3 is entangled/can entangle? There are several options to consider and they could correspond to the two TGD variants for the AdS/CFT correspondence.

1. X^3 could correspond to a wormhole contact with Euclidian signature of induced metric. The entanglement would be between it and the exterior region with Minkowskian signature of the induced metric.

2. X^3 could correspond to single sheet of space-time surface connected by wormhole contacts to a larger space-time sheet defining its environment. More precisely, X^3 and its complement would be obtained by throwing away the wormhole contacts with Euclidian signature of induce metric. Entanglement would be between these regions. In the generalization of the formula

$$S = \frac{A}{4\hbar G}$$

area A would be replaced by the total area of partonic 2-surfaces and G perhaps with CP_2 length scale squared.

3. In ZEO the entanglement could also correspond to time-like entanglement between the 3-D ends of the space-time surface at opposite light-like boundaries of CD. M-matrix, which can be seen as the analog of thermal S-matrix, decomposes to a product of hermitian square root of density matrix and unitary S-matrix and this hermitian matrix could also define p-adic thermodynamics. Note that in ZEO quantum theory can be regarded as square root of thermodynamics.

Minimal surface property is not favored in TGD framework

Minimal surface property for the 3-surfaces X^3 at the ends of space-time surface looks at first glance strange but a proper generalization of this condition makes sense if one assumes strong form of holography. Strong form of holography realizes General Coordinate Invariance (GCI) in strong sense meaning that light-like parton orbits and space-like 3-surfaces at the ends of space-time surfaces are equivalent physically. As a consequence, partonic 2-surfaces and their 4-D tangent space data must code for the quantum dynamics.

The mathematical realization is in terms of conformal symmetries accompanying the symplectic symmetries of $\delta M^4_\pm \times CP_2$ and conformal transformations of the light-like partonic orbit [K127]. The generalizations of ordinary conformal algebras correspond to conformal algebra, Kac-Moody algebra at the light-like parton orbits and to symplectic transformations $\delta M^4 \times CP_2$ acting as isometries of WCW and having conformal structure with respect to the light-like radial coordinate plus conformal transformations of δM^4_+, which is metrically 2-dimensional and allows extended conformal symmetries.

1. If the conformal realization of the strong form of holography works, conformal transformations act at quantum level as gauge symmetries in the sense that generators with no-vanishing conformal weight are zero or generate zero norm states. Conformal degeneracy can be eliminated by fixing the gauge somehow. Classical conformal gauge conditions analogous to Virasoro and Kac-Moody conditions satisfied by the 3-surfaces at the ends of CD are natural in this respect. Similar conditions would hold true for the light-like partonic orbits at which the signature of the induced metric changes.

2. What is also completely new is the hierarchy of conformal symmetry breakings associated with the hierarchy of Planck constants $h_{eff}/h = n$ [K43]. The deformations of the 3-surfaces which correspond to non-vanishing conformal weight in algebra or any sub-algebra with conformal weights vanishing modulo n give rise to vanishing classical charges and thus do not affect the value of the Kähler action [K127].

   The inclusion hierarchies of conformal sub-algebras are assumed to correspond to those for hyper-finite factors. There is obviously a precise analogy with quantal conformal invariance conditions for Virasoro algebra and Kac-Moody algebra. There is also hierarchy of inclusions which corresponds to hierarchy of measurement resolutions. An attractive interpretation is that singular conformal transformations relate to each other the states for broken conformal symmetry. Infinitesimal transformations for symmetry broken phase would carry fractional conformal weights coming as multiples of $1/n$.

3. Conformal gauge conditions need not reduce to minimal surface conditions holding true for all variations.

4. Note that Kähler action reduces to Chern-Simons term at the ends of CD if weak form of electric magnetic duality holds true. The conformal charges at the ends of CD cannot however reduce to Chern-Simons charges by this condition since only the charges associated with CP_2 degrees of freedom would be non-trivial.

The way out of the problem is provided by the generalization of AdS/CFT conjecture. String area is estimated in the effective metric provided by the anti-commutator of K-D gamma matrices at string world sheet.

5.6.4 Related Ideas

p-Adic mass calculations led to the introduction of the p-adic variant of Bekenstein-Hawkin law in which Planck length is replaced by p-adic length scale. This generalization is in spirit with the idea that string world sheet area is estimated in effective rather than induced metric.

p-Adic variant of Bekenstein-Hawking law

When the 3-surface corresponds to elementary particle, a direct connection with p-adic thermodynamics suggests itself and allows to answer the questions above. p-Adic thermodynamics could be interpreted as a description of the entanglement with environment. In ZEO the entanglement could also correspond to time-like entanglement between the 3-D ends of the space-time surface at opposite light-like boundaries of CD. M-matrix, which can be seen as the analog of thermal S-matrix, decomposes to a product of hermitian square root of density matrix and unitary S-matrix and this hermitian matrix could also define p-adic thermodynamics.

1. p-Adic thermodynamics [K76] would not be for energy but for mass squared (or scaling generator L_0) would describe the entanglement of the particle with environment defined by the larger space-time sheet. Conformal weights would comes as positive powers of integers (p_0^L would replace $exp(-H/T)$ to guarantee the number theoretical existence and convergence of the Boltzmann weight: note that conformal invariance that is integer spectrum of L_0 is also essential).

2. The interactions with environment would excite very massive CP_2 mass scale excitations (mass scale is about 10^{-4} times Planck mass) of the particle and give it thermal mass squared identifiable as the observed mass squared. The Boltzmann weights would be extremely small having p-adic norm about $1/p^n$, p the p-adic prime: $M_{127} = 2^{127} - 1$ for electron.

3. One of the first ideas inspired by p-adic vision was that p-adic entropy could be seen as a p-adic counterpart of Bekenstein-Hawking entropy [K80]. $S = (R^2/\hbar^2) \times M^2$ holds true identically apart from numerical constant. Note that one could interpret $R^2M/\hbar$ as the counterpart of Schwartschild radius. Note that this radius is proportional to $1/\sqrt{p}$ so that the area A would correspond to the area defined by Compton length. This is in accordance with the third option.

What is the space-time correlate for negentropic entanglement?

The new element brought in by TGD framework is that number theoretic entanglement entropy is negative for negentropic entanglement assignable to unitary entanglement (in the sense that entanglement matrix is proportional to a unitary matrix) and NMP states that this negentropy increases [K70]. Since entropy is essentially number of energy degenerate states, a good guess is that the number $n = h_{eff}/h$ of space-time sheets associated with h_{eff} defines the negentropy. An attractive space-time correlate for the negentropic entanglement is braiding. Braiding defines unitary S-matrix between the states at the ends of braid and this entanglement is negentropic. This entanglement gives also rise to topological quantum computation.

5.6.5 The Importance Of Being Light-Like

The singular geometric objects associated with the space-time surface have become increasingly important in TGD framework. In particular, the recent progress has made clear that these objects might be crucial for the understanding of quantum TGD. The singular objects are associated not only with the induced metric but also with the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices appearing in the Kähler-Dirac equation and determined by the Kähler action.

The singular objects associated with the induced metric

Consider first the singular objects associated with the induced metric.

1. At light-like 3-surfaces defined by wormhole throats the signature of the induced metric changes from Euclidian to Minkowskian so that 4-metric is degenerate. These surfaces are carriers of elementary particle quantum numbers and the 4-D induced metric degenerates locally to 3-D one at these surfaces.

2. Braid strands at partonic orbits - fermion lines identified as boundaries of string world sheets in the more recent terminology - are most naturally light-like curves: this correspond to the boundary condition for open strings. One can assign fermion number to the braid strands. Braid strands allow an identification as curves along which the Euclidian signature of the string world sheet in Euclidian region transforms to Minkowskian one. Number theoretic interpretation would be as a transformation of complex regions to hyper-complex regions meaning that imaginary unit i satisfying $i^2 = -1$ becomes hyper-complex unit e satisfying $e^2 = 1$. The complex coordinates $(z, \overline{z})$ become hyper-complex coordinates $(u = t + ex, v = t - ex)$ giving the standard light-like coordinates when one puts $e = 1$.

The singular objects associated with the effective metric

There are also singular objects assignable to the effective metric. According to the simple arguments already developed, string world sheets and possibly also partonic 2-surfaces are singular objects with respect to the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices rather than induced gamma matrices. Therefore the effective metric might be more than a mere formal structure. The following is of course mere speculation and should be taken as such.

1. For instance, quaternionicity of the space-time surface *might* allow an elegant formulation in terms of the effective metric avoiding the problems due to the Minkowski signature. This is achieved if the effective metric has Euclidian signature $\epsilon \times (1,1,1,1)$, $\epsilon = \pm 1$ or a complex counterpart of the Minkowskian signature $\epsilon(1,1,-1,-1)$.

2. String word sheets and perhaps also partonic 2-surfaces might be be understood as singularities of the effective metric. What happens that the effective metric with Euclidian signature $\epsilon \times (1,1,1,1)$ transforms to the signature $\epsilon(1,1,-1,-1)$ (say) at string world sheet so that one would have the degenerate signature $\epsilon \times (1,1,0,0)$ at the string world sheet.

   What is amazing is that this works also number theoretically. It came as a total surprise to me that the notion of hyper-quaternions as a closed algebraic structure indeed exists. The hyper-quaternionic units would be given by $(1, I, iJ, iK)$, where i is a commuting imaginary unit satisfying $i^2 = -1$. Hyper-quaternionic numbers defined as combinations of these units with real coefficients do form a closed algebraic structure which however fails to be a number field just like hyper-complex numbers do. Note that the hyper-quaternions obtained with real coefficients from the basis $(1, iI, iJ, iK)$ fail to form an algebra since the product is not hyper-quaternion in this sense but belongs to the algebra of complexified quaternions. The same problem is encountered in the case of hyper-octonions defined in this manner. This has been a stone in my shoe since I feel strong disrelish towards Wick rotation as a trick for moving between different signatures.

3. Could also partonic 2-surfaces correspond to this kind of singular 2-surfaces? In principle, 2-D surfaces of 4-D space intersect at discrete points just as string world sheets and partonic 2-surfaces do so that this might make sense. By complex structure the situation is algebraically equivalent to the analog of plane with non-flat metric allowing all possible signatures (ϵ_1, ϵ_2) in various regions. At light-like curve either ϵ_1 or ϵ_2 changes sign and light-like curves for these two kinds of changes can intersect as one can easily verify by drawing what happens. At the intersection point the metric is completely degenerate and simply vanishes.

4. Replacing real 2-dimensionality with complex 2-dimensionality, one obtains by the universality of algebraic dimension the same result for partonic 2-surfaces and string world sheets. The braid ends at partonic 2-surfaces representing the intersection points of 2-surfaces of this kind would have completely degenerate effective metric so that the Kähler-Dirac gamma matrices would vanish implying that energy momentum tensor vanishes as does also the induced Kähler field.

5. The effective metric suffers a local conformal scaling in the critical deformations identified in the proposed manner. Since ordinary conformal group acts on Minkowski space and leaves the boundary of light-cone invariant, one has two conformal groups. It is not however clear whether the M^4 conformal transformations can act as symmetries in TGD, where the presence of the induced metric in Kähler action breaks M^4 conformal symmetry. As found, also in TGD framework the Kac-Moody currents assigned to the braid strands generate Yangian: this is expected to be true also for the Kac-Moody counterparts of the conformal algebra associated with quantum criticality. On the other hand, in twistor program one encounters also two conformal groups and the space in which the second conformal group acts remains somewhat mysterious object. The Lie algebras for the two conformal groups generate the conformal Yangian and the integrands of the scattering amplitudes are Yangian invariants. Twistor approach should apply in TGD if zero energy ontology is right. Does this mean a deep connection?

   What is also intriguing that twistor approach in principle works in strict mathematical sense only at signatures $\epsilon \times (1, 1, -1 - 1)$ and the scattering amplitudes in Minkowski signature are obtained by analytic continuation. Could the effective metric give rise to the desired signature? Note that the notion of massless particle does not make sense in the signature $\epsilon \times (1, 1, 1, 1)$.

These arguments provide genuine a support for the notion of quaternionicity and suggest a connection with the twistor approach.

5.7 Could One Define Dynamical Homotopy Groups In WCW?

Agostino Prastaro - working as professor at the University of Rome - has done highly interesting work with partial differential equations, also those assignable to geometric variational principles such as Kähler action in TGD [A31, A32]. I do not understand the mathematical details but the key idea is a simple and elegant generalization of Thom's cobordism theory, and it is difficult to avoid the idea that the application of Prastaro's idea might provide insights about the preferred extremals, whose identification is now on rather firm basis [K125].

One could also consider a definition of what one might call dynamical homotopy groups as a genuine characteristics of WCW topology. The first prediction is that the values of conserved classical Noether charges correspond to disjoint components of WCW. Could this mean that the natural topology in the parameter space of Noether charges zero modes of WCW metric) is p-adic? An analogous conjecture was made on basis of spin glass analogy long time ago. Second surprise is that the only the six lowest dynamical homotopy groups of WCW would be non-trivial. The finite number of these groups dictate by the dimension of embedding space suggests also an interpretation as analogs of homology groups.

In the following the notion of cobordism is briefly discussed and the idea of Prastaro about assigning cobordism with partial differential equations is discussed.

5.7.1 About Cobordism As A Concept

To get some background consider first the notion of cobordism (`http://tinyurl.com/y7wdhtmv`).

1. Thom's cobordism theory [A86] is inspired by the question "When an n-manifold can be represented as a boundary of $n+1$-manifold". One can also pose additional conditions such as continuity, smoothness, orientability, one can add bundles structures and require that they are induced to n-manifold from that of $n+1$-manifold. One can also consider sub-manifolds of some higher-dimensional manifold.

   One can also fix n-manifold M and ask "What is the set of n-manifolds N with the property that there exists $n+1$-manifold W having union of $M \cup N$ as its boundary". One can also allow M to have boundary and pose the same question by allowing also the boundary of connecting $n+1$-manifold W contain also the orbits of boundaries of M and N.

   The cobordism class of M can be defined as the set of manifolds N cobordant with M - that is connectable in this manner. They have same cobordism class since cobordism is equivalence relation. The classes form also a group with respect to disjoint union. Cobordism is much rougher equivalence relation than diffeomorphy or homeomorphy since topology changes are possible. For instance, every 3-D closed un-oriented manifold is a boundary of a 4-manifold! Same is true for orientable cobordisms. Cobordism defines a category: objects are (say closed) manifolds and morphisms are cobordisms.

2. The basic result of Morse, Thom, and Milnor is that cobordism as topology changes can be engineered from elementary cobordisms. One take manifold $M \times I$ and imbeds to its other n-dimensional end the manifold $S^p \times D^q$, $n = p + q$, removes its interior and glues back $D^{p+1} \times S^{q-1}$ along its boundary to the boundary of the resulting hole. This gives n-manifold with different topology, call it N. The outcome is a cobordism connecting M and N unless there are some obstructions.

   There is a connection with Morse theory (`http://tinyurl.com/ych4chg9`) in which cobordism can be seen as a mapping of W to a unit interval such that the inverse images define a slicing of W and the inverse images at ends correspond to M and N.

3. One can generalize the abstract cobordism to that for n-sub-manifolds of a given embedding space. This generalization is natural in TGD framework. This might give less trivial results since not all connecting manifolds are imbeddable into a given embedding space. If connecting 4-manifolds connecting 3-manifolds with Euclidian signature (of induced metric) are assumed to have a Minkowskian signature, one obtains additional conditions, which might be too strong (the classical result of Geroch [A88] implies that non-trivial cobordism implies closed time loops - impossible in TGD).

   From TGD point of view this is too strong a condition and in TGD framework space-time surfaces with both Euclidian and Minkowskian signature of the induced metric are allowed. Also cobordisms singular as 4-surfaces are analogous to 3-vertices of Feynman diagrams are allowed.

5.7.2 Prastaro's Generalization Of Cobordism Concept To The Level Of Partial Differential Equations

I am not enough mathematician in technical sense of the word to develop overall view about what Prastaro has done and I have caught only the basic idea. I have tried to understand the articles

[A31, A32] with title "Geometry of PDE's. I/II: Variational PDE's and integral bordism groups" (`http://tinyurl.com/yb9wey8c` and `http://tinyurl.com/y9x55qmk`), which seem to correspond to my needs. The key idea is to generalize the cobordism concept also to partial differential equations with cobordism replaced with the time evolution defined by partial differential equation. In particular, to geometric variational principles defining as their extremals the counterparts of cobordisms.

Quite generally, and especially so in the case of the conservation of Noether charges give rise to strong selection rules since two n-surfaces with different classical charges cannot be connected by extremals of the variational principle. Note however that the values of the conserved charges depend on the normal derivatives of the embedding space coordinates at the n-dimensional ends of cobordism. If one poses additional conditions fixing these normal derivatives, the selection rules become even stronger. In TGD framework Bohr orbit property central for the notion of WCW geometry and holography allows to hope that conserved charges depend on 3-surfaces only.

What is so beautiful in this approach that it promises to generalize the notion of cobordism and perhaps also the notions of homotopy/homology groups so that they would apply to partial differential equations quite generally, and especially so in the case of geometric variational principles giving rise to $n+1$-surfaces connecting n-surfaces characterizing the initial and final states classically. TGD with $n = 3$ seems to be an ideal applications for these ideas.

Prastaro also proposes a generalization of cobordism theory to super-manifolds and quantum super-manifolds. The generalization in the case of quantum theory utilizing path integral does not not pose conditions on classical connecting field configurations. In TGD framework these generalizations are not needed since fermion number is geometrized in terms of embedding space gamma matrices and super(-symplectic) symmetry is realized differently.

5.7.3 Why Prastaro's Idea Resonates So Strongly With TGD

Before continuing I want to make clear why Prastaro's idea resonates so strongly with TGD.

1. One of the first ideas as I started to develop TGD was that there might be selection rules analogous to those of quantum theory telling which 3-surfaces can be connected by a space-time surface. At that time I still believed in path integral formalism assuming that two 3-surfaces at different time slices with different values of Minkowski time can be connected by any space-time surface for which embedding space coordinates have first derivatives.

   I soon learned about Thom's theory but was greatly disappointed since no selection rules were involved in the category of abstract 3-manifolds. I thought that possible selection rules should result from the imbeddability of the connecting four-manifold to $H = M^4 \times CP_2$ but my gut feeling was that these rules are more or less trivial since so many connecting 4-manifolds exist and some of them are very probably imbeddable.

   One possible source of selection rules could have been the condition that the induced metric has Minkowskian signature - one could justify it in terms of classical causality. This restricts strongly topology change in general relativity (`http://tinyurl.com/y6vuopgj`). Geroch's classical result [A88] states that non-trivial smooth Lorentz cobordism between compact 3-surfaces implies the existence of closed time loop - not possible in TGD framework. Second non-encouraging result is that scalar field propagating in trouser topology leads to an occurrence of infinite energy burst (`http://tinyurl.com/ybbuwyfj`).

   In the recent formulation of TGD however also Euclidian signature of the induced metric is allowed. For space-time counterparts of 3-particle vertices three space-time surfaces are glued along their smooth 3-D ends whereas space-time surface fails to be everywhere smooth manifold. This picture fits nicely with the idea that one can engineer space-time surfaces by gluing them together along their ends.

2. At that time (before 1980) the discovery of the geometry of the "World of Classical Worlds" (WCW) as a possible solution to the failures of canonical quantization and path integral formalism was still at distance of ten years in future. Around 1985 I discovered the notion of WCW. I made some unsuccessful trials to construct its geometry, and around 1990 finally realized that 4-D general coordinate invariance is needed although basic objects are 3-D surfaces.

This is realized if classical physics is an exact part of quantum theory - not only something resulting in a stationary phase approximation. Classical variational principle should assign to a 3-surface a physically unique space-time surface - the analog of Bohr orbit - and the action for this surface would define Kähler function defining the Kähler geometry of WCW using standard formula.

This led to a notion of preferred extremal: absolute minimum of Kähler action was the first guess and might indeed make sense in the space-time regions with Euclidian signature of induced metric but not in Minkowskian regions, which give to the vacuum functional and exponential of Minkowskian Kähler action multiplied by imaginary unit coming from $\sqrt{g}$ - just as in quantum field theories. Euclidian regions give the analog of the free energy exponential of thermodynamics and transform path integral to mathematically well-defined functional integral.

3. After having discovered the notion of preferred extremal, I should have also realized that an interesting generalization of cobordism theory might make sense after all, and could even give rise to the classical counterparts of the selection rules! For instance, conservation of isometry charges defines equivalence classes of 3-surfaces endowed with tangent space data. Bohr orbit property could fix the tangent space data (normal derivatives of embedding space coordinates) so that conserved classical charges would characterize 3-surfaces alone and thus cobordism equivalence classes and become analogous to topological invariants. This would be in spirit with the attribute "Topological" in TGD!

5.7.4 What Preferred Extremals Are?

The topology of WCW has remained mystery hitherto - partly due to my very limited technical skills and partly by the lack of any real physical idea. The fact, that p-adic topology seems to be natural at least as an effective topology for the maxima of Kähler function of WCW gave a hint but this was not enough.

I hope that the above summary has made clear why the idea about dynamical cobordism and even dynamical homotopy theory is so attractive in TGD framework. One could even hope that dynamics determines not only Kähler geometry but also the topology of WCW to some extend at least! To get some idea what might be involved one must however first tell about the recent situation concerning the notion of preferred extremal.

1. The recent formulation for the notion of preferred extremal relies on strong form of General Coordinate Invariance (SGCI). SGCI states that two kinds of 3-surfaces can identified as fundamental objects. Either the light-light 3-D orbits of partonic 2-surfaces defining boundaries between Minkowskian and Euclidian space-time regions or the space-like 3-D ends of space-time surfaces at boundaries of CD. Since both choices are equally good, partonic 2-surfaces and their tangent space-data at the ends of space-time should be the most economic choice.

   This eventually led to the realization that partonic 2-surfaces and string world sheets should be enough for the formulation of quantum TGD. Classical fields in the interior of space-time surface would be needed only in quantum measurement theory, which demands classical physics in order to interpret the experiments.

2. The outcome is strong form of holography (SH) stating that quantum physics should be coded by string world sheets and partonic 2-surfaces inside given causal diamond (CD). SH is very much analogous to the AdS/CFT correspondence but is much simpler: the simplicity is made possible by much larger group of conformal symmetries.

   If these 2-surfaces satisfy some consistency conditions one can continue them to 4-D space-time surface inside CD such that string world sheets are surfaces inside them satisfying the condition that charged (possibly all) weak gauge potentials identified as components of the induced spinor connection vanish at the string world sheets and also that energy momentum currents flow along these surfaces. String world sheets carry second quantized free induced spinor fields and fermionic oscillator operator basis is used to construct WCW gamma matrices.

3. The 3-surfaces at the ends of WCW must satisfy strong conditions to guarantee effective 2-dimensionality. Quantum criticality suggests the identification of these conditions. All Noether charges assignable to a sub-algebra of super-symplectic algebra isomorphic to it and having conformal weights which are n-multiples of those of entire algebra vanish/annihilate quantum states. One has infinite fractal hierarchy of broken super-conformal symmetries with the property that the sub-algebra is isomorphic with the entire algebra. This like a ball at the top of ball at the top of ....

   The speculative vision is that super-symplectic subalgebra with weights coming as n-ples of those for the entire algebra acts as an analog of conformal gauge symmetries on light-like orbits of partonic 2-surfaces, and gives rise to a pure gauge degeneracy whereas other elements of super-symplectic algebra act as dynamical symmetries. The hierarchy of quantum criticalities defines hierarchies of symmetry breakings characterized by hierarchies of sub-algebras for which one n_{i+1} is divisible by n_i. The proposal is that conformal gauge invariance means that the analogs of Bohr orbits are determined only apart from conformal gauge transformations forming to n_i conformal equivalence classes so that effectively one has n_i discrete degrees of freedom assignable to light-like partonic orbits.

4. In this framework manifolds M and N would correspond the 3-surfaces at the boundaries of CD and containing a collection strings carrying induced spinor fields. The connecting 4-surface W would contain string world sheets and the light-like orbits of partonic 2-surfaces as simultaneous boundaries for Minkowskian and Euclidian regions.

Propagator line has several meanings depending on whether one considers particles as strings, as single fermion states localizable at the ends of strings, or as Euclidian space-time regions or their light-like boundaries with singular induced metric having vanishing determinant. Vertices appear as generalizations of the stringy vertices and as generalization of the vertices of Feynman diagrams in which the incoming 4-surfaces meet along their ends.

1. Propagator line has several meanings depending on whether one considers particles as strings, as single fermion states localizable at the ends of strings, or as Euclidian space-time regions or their light-like boundaries with degenerate induced metric with vanishing determinant. Vertices appear as generalizations of the stringy vertices and as generalization of the vertices of Feynman diagrams in which the incoming 4-surfaces meet along their ends.

   (a) The lines of generalized Feynman graphs defined in topological sense are identified as slightly deformed pieces of CP_2 defining wormhole contacts connecting two Minkowskian regions and having wormhole throats identified as light-like parton orbits as boundaries. Since there is a magnetic monopole flux through the wormhole contacts they must appear as pairs (also larger number is possible) in order that magnetic field lines can close. Elementary particles correspond to pairs of wormhole contacts. At both space-time sheets the throats are connected by magnetic flux tubes carrying monopole flux so that a closed flux tube results having a shape of an extremely flattened square and having wormhole contacts at its ends. It is a matter of taste, whether to call the light-like wormhole throats or their interiors as lines of the generalized Feynman/twistor diagrams.

   The light-like orbits of partonic 2-surfaces bring strongly in mind the light-like 3-surfaces along which radiation fields can be restricted - kind of shockwaves at which the signature of the induced space-time metric changes its signature.

   (b) String world sheets as orbits of strings are also in an essential role and could be seen as particle like objets. String world sheets could as kind of singular solutions of field equations analogous to characteristics of hyperbolic differential equations. The isometry currents of Kähler action flow along string world sheets and field equations restricted to them are satisfied. As if one would have 2-dimensional solution. $\sqrt{g_4}$ would of course vanishes for genuinely 2-D solution but this one can argue that this is not a problem since $\sqrt{g_4}$ can be eliminated from field equations. String world sheets could serve as 2-D a analoga for a solution of hyperbolic field equations defining expanding wave front localized at 3-D light-like surface.

(c) Propagation in the third sense of word is assignable to the ends of string world sheets at the light-like orbits of partonic 2-surfaces and possibly carrying fermion number. One could say that in TGD one has both fundamental fermions serving as building bricks of elementary particles and strings characterizing interactions between particles. Fermion lines are massless in 8-D sense. By strong form of holography this quantum description has 4-D description space-time description as a classical dual.

2. The topological description of interaction vertices brings in the most important deviation from the standard picture behind cobordism: space-time surfaces are not smooth in TGD framework. One allows topological analogs of 3-vertices of Feynman diagrams realized by connecting three 4-surfaces along their smooth 3-D ends. 3-vertex is also an analog (actually much more!) for the replication in biology. This vertex is *not* the analog of stringy trouser vertex for which space-time surface is continuous whereas 3-surface at the vertex is singular (also trouser vertex could appear in TGD).

   The analog of trouser vertex for string world sheets means splitting of string and fermionic field modes decompose into superposition of modes propagating along the two branches. For instance, the propagation of photon along two paths could correspond to its geometric decay at trouser vertex not identifiable as "decay" to two separate particles.

   For the analog of 3-vertex of Feynman diagram the 3-surface at the vertex is non-singular but space-time surface is singular. The gluing along ends corresponds to genuine 3-particle vertex.

   The view about solution of PDEs generalizes dramatically but the general idea about cobordism might make sense also in the generalized context.

5.7.5 Could Dynamical Homotopy/Homology Groups Characterize WCW Topology?

The challenge is to at least formulate (with my technical background one cannot dream of much more) the analog of cobordism theory in this framework. One can actually hope even the analog of homotopy/homology theory.

1. To a given 3-surface one can assign its cobordism class as the set of 3-surfaces at the opposite boundary of CD connected by a preferred extremal. The 3-surfaces in the same cobordism class are characterized by same conserved classical Noether charges, which become analogs of topological invariants.

   One can also consider generalization of cobordisms as analogs to homotopies by allowing return from the opposite boundary of CD. This would give rise to first homotopy groupoid. One can even go back and forth several times. These dynamical cobordisms allow to divide 3-surfaces at given boundary of CD in equivalence classes characterized among other things by same values of conserved charges. One can also return to the original 3-surface. This could give rise to the analog of the first homotopy group Π_1.

2. If one takes the homotopy interpretation literally one must conclude that the 3-surfaces with different conserved Noether charges cannot be connected by any path in WCW - they belong to disjoint components of the WCW! The zeroth dynamical homotopy group Π_0 of WCW would be non-trivial and its elements would be labelled by the conserved Noether charges defining topological invariants!

   The values of the classical Noether charges would label disjoint components of WCW. The topology for the space of these parameters would be totally disconnected - no two points cannot be connected by a continuous path. p-Adic topologies are indeed totally disconnected. Could it be that p-adic topology is natural for the conserved classical Noether charges and the sectors of WCW are characetrized by p-adic number fields and their algebraic extensions?

   Long time ago I noticed that the 4-D spin glass degeneracy induced by the huge vacuum degeneracy of Kähler action implies analogy between the space of maxima of Kähler function and the energy landscape of spin glass systems [K80]. Ultrametricity (`http://tinyurl.com/y6vswdoh`) is the basic property of the topology of the spin glass energy landscape. p-Adic topology is ultrametric and the proposal was that the effective topology for the space of maxima could be p-adic.

3. Isometry charges are the most important Noether charges. These Noether charges are very probably not the only conserved charges. Also the generators in the complement of the gauge sub-algebra of symplectic algebra acting as gauge conformal symmetries could be conserved. All these conserved Noether charges would define a parameter space with a natural p-adic topology.

   Since integration is problematic p-adically, one can ask whether only discrete quantum superpositions of 3-surfaces with different classical charges are allowed or whether one should even assume fixed values for the total classical Noether charges appearing in the scattering amplitudes.

   I have proposed this kind of approach for the zero modes of WCW geometry not contributing to the Kähler metric except as parameters. The integration for zero modes is also problematic because there is no metric, which would define the integration measure. Since classical charges do not correspond to quantum fluctuating degrees of freedom they should correspond to zero modes. Hence these arguments are equivalent.

The above argument led to the identification of the analogs of the homotopy group Π_0 and led to the idea about homotopy groupoid/group Π_1. The elements of Π_1 would correspond to space-time surfaces, which run arbitrary number of times fourth and back and return to the initial 3-surface at the boundary of CD. If the two preferred extremals connecting same pair of 3-surfaces can be deformed to each other, one can say that they are equivalent as dynamical homotopies (or cobordisms). What could be the allowed deformations? Are they cobordisms of cobordisms? What this could mean? Could they define the analog of homotopy groupoid Π_2 as foliations of preferred extremals connecting the same 3-surfaces?

1. The number theoretic vision about generalized Feynman diagrams suggests a possible approach. Number theoretic ideas combined with the generalization of twistor approach [K125, K116] led to the vision that generalized Feynman graphs can be identified as sequences or webs of algebraic operations in the co-algebra defined by the Yangian assignable to super-symplectic algebra [A25] [B29, B22, B23] and acting as symmetries of TGD. Generalized Feynman graphs would represent algebraic computations. Computations can be done in very many different ways and each of them corresponds to a generalized Feynman diagram. These computations transform give same final collection of "numbers" when the initial collection of "numbers" is given. Does this mean that the corresponding scattering amplitudes must be identical?

   If so, a huge generalization of the duality symmetry of the hadronic string models would suggest itself. All computations can be reduced to minimal computations. Accordingly, generalized Feynman diagrams can be reduced to trees by eliminating loops by moving the ends of the loops to same point and snipping the resulting tadpole out! The snipped of tadpole would give a mere multiplicative factor to the amplitude contributing nothing to the scattering rate - just like vacuum bubbles contribute nothing in the case of ordinary Feynman diagrams.

2. How this symmetry could be realized? Could one just assume that only the minimal generalized Feynman diagrams contribute? - not a very attractive option. Or could one hope that only tree diagrams are allowed by the classical dynamics: this was roughly the original vision? The huge vacuum degeneracy of Kähler action implying non-determinism does not encourage this option. The most attractive and most predictive realization conforming with the idea about generalized Feynman diagrammatics as arithmetics would be that all the diagrams differing by these moves give the same result. An analogous symmetry has been discovered for twistor diagrams.

3. Suppose one takes seriously the snipping of a tadpole away from diagram as a move, which does not affect the scattering amplitude. Could this move correspond to an allowed elementary cobordism of preferred extremal? If so, scattering amplitudes would have purely

topological meaning as representations of the elements of cobordism classes! TGD would indeed be what it was proposed to be but in much deeper sense than I thought originally. This could also conform with the interpretation of classical charges as topological invariants, realize adelic physics at the level of WCW, and conform with the idea about TGD as almost topological QFT and perhaps generalizing it to topological QFT in generalized sense.

4. One can imagine several interpretations for the snipping operation at space-time level. TGD allows a huge classical vacuum degeneracy: all space-time surfaces having Lagrangian manifold of CP_2 as their CP_2 projection are vacuum extremals of Kähler action. Also all CP_2 type extremals having 1-D light-like curve as M^4 projection are vacuum extremals but have non-vanishing Kähler action. This would not matter if one does not have superpositions since multiplicative factors are eliminated in scattering amplitudes. Could the tadpoles correspond to CP_2 type vacuum extremals at space-time level?

   There is also an alternative interpretation. In ZEO causal diamonds (CD) form a hierarchy and one can imagine that the sub-CDs of given CD correspond to quantum fluctuations. Could tadpoles be assigned to sub-CDs of CD be considered+

5. In this manner one could perhaps define elements of homotopy groupoid Π_2 as foliations preferred extremals with same ends - these would be 5-D surfaces. If one has two such 5-D foliations with the same 4-D ends, one can form the reverse of the other and form a closed surface. This would be analogous to a map of S^2 to WCW. If the two 5-D foliations cannot be transformed to each other, one would have something, which might be regarded as a non-trivial element of dynamical homotopy group Π_2.

One can ask whether one could define also the analogs of higher homology or homotopy grouppoids and groupoids Π_3 up to Π_5 - the upper bound $n = 5 = 8 - 3$ comes from the fact that foliations of foliations.. can have maximum dimension $D = 8$ and from the dimension of $D = 3$ of basic objects.

1. One could form a foliation of the foliations of preferred extremals as the element of the homotopy groupoid Π_3. Could allowed moves reduce to the snipping operation for generalized Feynman diagrams but performed along direction characterized by a new foliation parameter.

2. The topology of the zero mode sector of WCW parameterized by fixed values of conserved Noether charges as element of Π_0 could be characterized by dynamical homotopy groups Π_n, $n = 1, ..., 5$ - at least partially. These degrees of freedom could correspond to quantum fluctuating degrees of freedom. The Kähler structure of WCW and finite-D analogy suggests that all odd dynamical homotopy groups vanish so that Π_0, Π_2 and Π_4 would be the only non-trivial dynamical homotopy groups. The vanishing of Π_1 would imply that there is only single minimal generalized Feynman diagram contributing to the scattering amplitude. This also true if Feynman diagrams correspond to arithmetic operations.

3. Whether one should call these groups homotopy groups or homology groups is not obvious. The construction means that the foliations of foliations of ... can be seen as images of spheres suggesting "homotopy". The number of these groups is determined by the dimension of embedding space, which suggests "homology".

4. Clearly, the surfaces defining the dynamical homotopy groups/groupoids would be analogs of branes of M-theory but would be obtained constructing paths of paths of paths... by starting from preferred extremals. The construction of so called n-groups (`http://tinyurl.com/yckcjcln`) brings strongly in mind this construction.

5.7.6 Appendix: About Field Equations Of TGD In Jet Bundle Formulation

Prastaro utilizes jet bundle (`http://tinyurl.com/yb2575bm`) formulation of partial differential equations (PDEs). This notion allows a very terse formulation of general PDEs as compared to the old-fashioned but much more concrete formulation that I have used. The formulation is rather formula rich and reader might lose easily his/her patience since one must do hard work to learn which formulas follow trivially from the basic definitions.

I will describe this formulation in TGD framework briefly but without explicit field equations, which can be found at [K15]. To my view a representation by using a concrete example is always more reader friendly than the general formulas derived in some reference. I explain my view about the general ideas behind jet bundle formulation with minimal number amount of formulas. The reader can find explicit formulas from the Wikipedia link above.

The basic goal is to have a geometric description of PDE. In TGD framework the geometric picture is of course present from beginning: field patterns as 4-surfaces in field space - somewhat formal geometric objects - are replaced with genuine 4-surfaces in $M^4 \times CP_2$.

Field equations as conservation laws, Frobenius integrability conditions, and a connection with quaternion analyticity

The following represents qualitative picture of field equations of TGD trying to emphasize the physical aspects. Also the possibility that Frobenius integrability conditions are satisfied and correspond to quaternion analyticity is discussed.

1. Kähler action is Maxwell action for induced Kähler form and metric expressible in terms of embedding space coordinates and their gradients. Field equations reduce to those for embedding space coordinates defining the primary dynamical variables. By GCI only four of them are independent dynamical variables analogous to classical fields.

2. The solution of field equations can be interpreted as a section in fiber bundle. In TGD the fiber bundle is just the Cartesian product $X^4 \times CD \times CP_2$ of space-time surface X^4 and causal diamond $CD \times CP_2$. CD is the intersection of future and past directed light-cones having two light-like boundaries, which are cone-like pieces of light-boundary $\delta M^4_\pm \times CP_2$. Space-time surface serves as base space and $CD \times CP_2$ as fiber. Bundle projection Π is the projection to the factor X^4. Section corresponds to the map $x \rightarrow h^k(x)$ giving embedding space coordinates as functions of space-time coordinates. Bundle structure is now trivial and rather formal.

   By GCI one could also take suitably chosen 4 coordinates of $CD \times CP_2$ as space-time coordinates, and identify $CD \times CP_2$ as the fiber bundle. The choice of the base space depends on the character of space-time surface. For instance CD, CP_2 or $M^2 \times S^2$ (S^2 a geodesic sphere of CP_2), could define the base space. The bundle projection would be projection from $CD \times CP_2$ to the base space. Now the fiber bundle structure can be non-trivial and make sense only in some space-time region with same base space.

3. The field equations derived from Kähler action must be satisfied. Even more: one must have a *preferred* extremal of Kähler action. One poses boundary conditions at the 3-D ends of space-time surfaces and at the light-like boundaries of $CD \times CP_2$.

   One can fix the values of conserved Noether charges at the ends of CD (total charges are same) and require that the Noether charges associated with a sub-algebra of super-symplectic algebra isomorphic to it and having conformal weights coming as n-ples of those for the entire algebra, vanish. This would realize the effective 2-dimensionality required by SH. One must pose boundary conditions also at the light-like partonic orbits. So called weak form of electric-magnetic duality is at least part of these boundary conditions.

   It seems that one must restrict the conformal weights of the entire algebra to be non-negative $r \geq 0$ and those of subalgebra to be positive: $mn > 0$. The condition that also the commutators of sub-algebra generators with those of the entire algebra give rise to vanishing Noether charges implies that all algebra generators with conformal weight $m \geq n$ vanish so the dynamical algebra becomes effectively finite-dimensional. This condition generalizes to the action of super-symplectic algebra generators to physical states.

   M^4 time coordinate cannot have vanishing time derivative dm^0/dt so that four-momentum is non-vanishing for non-vacuum extremals. For CP_2 coordinates time derivatives ds^k/dt can vanish and for space-like Minkowski coordinates dm^i/dt can be assumed to be non-vanishing if M^4 projection is 4-dimensional. For CP_2 coordinates $ds^k/dt = 0$ implies the vanishing of electric parts of induced gauge fields. The non-vacuum extremals with the largest conformal gauge symmetry (very small n) would correspond to cosmic string solutions for which induced gauge fields have only magnetic parts. As n increases, also electric parts are generated. Situation becomes increasingly dynamical as conformal gauge symmetry is reduced and dynamical conformal symmetry increases.

4. The field equations involve besides embedding space coordinates h^k also their partial derivatives up to second order. Induced Kähler form and metric involve first partial derivatives $\partial_\alpha h^k$ and second fundamental form appearing in field equations involves second order partial derivatives $\partial_\alpha \partial_\beta h^k$.

   Field equations are hydrodynamical, in other worlds represent conservation laws for the Noether currents associated with the isometries of $M^4 \times CP_2$. By GCI there are only 4 independent dynamical variables so that the conservation of $m \leq 4$ isometry currents is enough if chosen to be independent. The dimension m of the tangent space spanned by the conserved currents can be smaller than 4. For vacuum extremals one has $m = 0$ and for massless extremals (MEs) $m = 1$! The conservation of these currents can be also interpreted as an existence of $m \leq 4$ closed 3-forms defined by the duals of these currents.

5. The hydrodynamical picture suggests that in some situations it might be possible to assign to the conserved currents flow lines of currents even globally. They would define $m \leq 4$ global coordinates for some subset of conserved currents (4+8 for four-momentum and color quantum numbers). Without additional conditions the individual flow lines are well-defined but do not organize to a coherent hydrodynamic flow but are more like orbits of randomly moving gas particles. To achieve global flow the flow lines must satisfy the condition $d\phi^A/dx^\mu = k^A_B J^B_\mu$ or $d\phi^A = k^A_B J^B$ so that one can special of 3-D family of flow lines parallel to $k^A_B J^B$ at each point - I have considered this kind of possibly in [K15] at detail but the treatment is not so general as in the recent case.

   Frobenius integrability conditions (`http://tinyurl.com/yc6apam2`) follow from the condition $d^2\phi^A = 0 = dk^A_B \wedge J^B + k^A_B dJ^B = 0$ and implies that dJ^B is in the ideal of exterior algebra generated by the J^A appearing in $k^A_B J^B$. If Frobenius conditions are satisfied, the field equations can define coordinates for which the coordinate lines are along the basis elements for a sub-space of at most 4-D space defined by conserved currents. Of course, the possibility that for preferred extremals there exists $m \leq 4$ conserved currents satisfying integrability conditions is only a conjecture.

   It is quite possible to have $m < 4$. For instance for vacuum extremals the currents vanish identically For MEs various currents are parallel and light-like so that only single light-like coordinate can be defined globally as flow lines. For cosmic strings (cartesian products of minimal surfaces X^2 in M^4 and geodesic spheres S^2 in CP_2 4 independent currents exist). This is expected to be true also for the deformations of cosmic strings defining magnetic flux tubes.

6. Cauchy-Riemann conditions in 2-D situation represent a special case of Frobenius conditions. Now the gradients of real and imaginary parts of complex function $w = w(z) = u + iv$ define two conserved currents by Laplace equations. In TGD isometry currents would be gradients apart from scalar function multipliers and one would have generalization of C-R conditions. In citeallbprefextremals,twistorstory I have considered the possibility that the generalization of Cauchy-Riemann-Fuerter conditions [A99, A75] (`http://tinyurl.com/yb8l34b5`) could define quaternion analyticity - having many non-equivalent variants - as a defining property of preferred extremals. The integrability conditions for the isometry currents would be the natural physical formulation of CRF conditions. Different variants of CRF conditions would correspond to varying number of independent conserved isometry currents.

7. The problem caused by GCI is that there is infinite number of coordinate choices. How to pick a physically preferred coordinate system? One possible manner to do this is to use coordinates for the projection of space-time surface to some preferred sub-space of embedding - geodesic manifold is an excellent choice. Only $M^1 \times X^3$ geodesic manifolds are not possible but these correspond to vacuum extremals.

One could also consider a philosophical principle behind integrability. The variational principle itself could give rise to at least some preferred space-time coordinates in the same manner as TGD based quantum physics would realize finite measurement resolution in terms of inclusions of HFFs in terms of hierarchy of quantum criticalities and fermionic strings connecting partonic 2-surfaces. Frobenius integrability of the isometry currents would define some preferred coordinates. Their number need not be the maximal four however.

For instance, for massless extremals only light-like coordinate corresponding to the light-like momentum is obtained. To this one can however assign another local light-like coordinate uniquely to obtain integrable distribution of planes M^2. The solution ansatz however defines directly an integrable choice of two pairs of coordinates at embedding space level usable also as space-time coordinates - light-like local direction defining local plane M^2 and polarization direction defining a local plane E^2. These choices define integrable distributions of orthogonal planes and local hypercomplex and complex coordinates. Pair of analogs of C-R equations is the outcome. I have called these coordinates Hamilton-Jacobi coordinates for M^4.

8. This picture allows to consider a generalization of the notion of solution of field equation to that of integral manifold (`http://tinyurl.com/yajn7cuz`. If the number of independent isometry currents is smaller than 4 (possibly locally) and the integrability conditions hold true, lower-dimensional sub-manifolds of space-time surface define integral manifolds as kind of lower-dimensional effective solutions. Genuinely lower-dimensional solutions would of course have vanishing $\sqrt{g_4}$ and vanishing Kähler action.

   String world sheets can be regarded as 2-D integral surfaces. Charged (possibly all) weak boson gauge fields vanish at them since otherwise the electromagnetic charge for spinors would not be well-defined. These conditions force string world sheets to be 2-D in the generic case. In special case 4-D space-time region as a whole can satisfy these conditions. Well-definedness of Kähler-Dirac equation [K127, K96] demands that the isometry currents of Kähler action flow along these string world sheets so that one has integral manifold. The integrability conditions would allow $2 < m \leq n$ integrable flows outside the string world sheets, and at string world sheets one or two isometry currents would vanish so that the flows would give rise 2-D independent sub-flow.

9. The method of characteristics (`http://tinyurl.com/y9dcdayt`) is used to solve hyperbolic partial differential equations by reducing them to ordinary differential equations. The (say 4-D) surface representing the solution in the field space has a foliation using 1-D characteristics. The method is especially simple for linear equations but can work also in the non-linear case. For instance, the expansion of wave front can be described in terms of characteristics representing light rays. It can happen that two characteristics intersect and a singularity results. This gives rise to physical phenomena like caustics and shock waves.

   In TGD framework the flow lines for a given isometry current in the case of an integrable flow would be analogous to characteristics, and one could also have purely geometric counterparts of shockwaves and caustics. The light-like orbits of partonic 2-surface at which the signature of the induced metric changes from Minkowskian to Euclidian might be seen as an example about the analog of wave front in induced geometry. These surfaces serve as carriers of fermion lines in generalized Feynman diagrams. Could one see the particle vertices at which the 4-D space-time surfaces intersect along their ends as analogs of intersections of characteristics - kind of caustics? At these 3-surfaces the isometry currents should be continuous although the space-time surface has "edge".

10. The analogy with ordinary analyticity suggests that it might be possible to interpret string world sheets and partonic 2-surfaces appearing in strong form of holography (SH) as co-dimension 2 surfaces analogous to poles of analytic function in complex plane. Light-like 3-surfaces might be seen as analogs of cuts. The coding of analytic function by its singularities could be seen as analog of SH.

Jet bundle formalism

Jet bundle formalism (`http://tinyurl.com/yb2575bm`) is a modern manner to formulate PDEs in a coordinate independent manner emphasizing the local algebraic character of field equations. In

TGD framework GCI of course guarantees this automatically. Beside this integrability conditions formulated in terms of Cartan's contact forms are needed.

1. The basic idea is to take the partial derivatives of embedding space coordinates as functions of space-time coordinates as independent variables. This increases the number of independent variables. Their number depends on the degree of the jet defined and for partial differential equation of order r, for n dependent variables, and for N independent variables the number of new degrees of freedom is determined by r, n, and N - just by counting the total number of various partial derivatives from $k = 0$ to r. For $r = 1$ (first order PDE) it is $N \times (1 + n)$.

2. Jet at given space-time point is defined as a Taylor polynomial of the embedding space coordinates as functions of space-time coordinates and is characterized by the partial derivatives at various points treated as independent coordinates analogous to embedding space coordinate. Jet degree r is characterized by the degree of the Taylor polynomial. One can sum and multiply jets just like Taylor polynomials. Jet bundle assigns to the fiber bundle associated with the solutions of PDE corresponding jet bundle with fiber at each point consisting of jets for the independent variables ($CD \times CP_2$ coordinates) as functions of the dependent variables (space-time coordinates).

3. The field equations from the variation of Kähler action are second order partial differential equations and in terms of jet coefficients they reduce to local algebraic equations plus integrability conditions. Since TGD is very non-linear one obtains polynomial equations at each point - one for each embedding space coordinate. Their number reduces to four by GCI. The minimum degree of jet bundle is $r = 2$ if one wants algebraic equations since field equations are second order PDEs.

4. The local algebraic conditions are not enough. One must have also conditions stating that the new independent variables associated with partial derivatives of various order reduces to appropriate multiple partial derivatives of embedding space coordinates. These conditions can be formulated in terms of Cartan's contact forms, whose vanishing states these conditions. For instance, if dh^k is replaced by independent variable u^k, the condition $dh^k - u^k = 0$ is true for the solution surfaces.

5. In TGD framework there are good motivations to break the non-orthodoxy and use 1-jets so that algebraic equations replaced by first order PDEs plus conditions requiring vanishing of contact forms. These equations state the conservation of isometry currents implying that the 3-forms defined by the duals of isometry currents are closed. As found, this formulation reveals in TGD framework the hydrodynamic picture and suggests conditions making the system integrable in Frobenius sense.

5.8 Twistor lift of TGD and WCW geometry

In the following a view about WCW geometry forced by twistor lift of TGD [K116, L31, L55, L72] is summarized. Twistor lift brings to the action a volume term but without breaking conformal invariance and without introducing cosmological constant as a fundamental dimensional dynamical coupling. The proposed construction of the gamma matrices of WCW giving rise to Kähler metric as anti-commutators is now in terms of the Noether super charges associated with the super-symplectic algebra. This I dare to regard as a very important step of progress.

5.8.1 Possible weak points of the earlier vision

To make progress it is wise to try to identify the possible weak points of the earlier vision.

1. The huge vacuum degeneracy of Kähler action [K57] defining the Kähler function of WCW Kähler metric is analogous to gauge degeneracy of Maxwell action and coded by symplectic transformations of CP_2. It implies that the degeneracy of the metric increases as one approaches vacuum extremals and is maximal for the space-time surfaces representing canonical embeddings of Minkowski space: Kähler action vanishes up to fourth order in deformation.

The original interpretation was in terms of 4-D spin glass degeneracy assumed to be induced by quantum degeneracy.

One could however argue that classical non-determinism of Kähler action is not acceptable and that a small term removing the vacuum degeneracy is needed to make the situation mathematically acceptable. There is an obvious candidate: a volume term having an interpretation in terms of cosmological constant. This term however seems to mean the presence of length scale as a fundamental constant and is in conflict with the basic lesson learned from gauge theories teaching that only dimensionless couplings can be allowed.

2. The construction of WCW Kähler metric relies on the hypothesis that the basic result from the construction of loop space geometries [A44] generalizes: the Kähler metric should be essentially unique from the condition that the isometry group is maximal - this guarantees the existence of Riemann connection. For $D = 3$ this condition is expected to be even stronger than for $D = 1$.

   The hypothesis is that in zero energy ontology (ZEO) the symplectic group acting at the light-like boundaries of causal diamond (CD) (one has CD= $cd \times CP_2$, where cd is the intersection of future and past directed light-cones) acts as the isometries of the Kähler metric.

   It would be enough to identify complexified WCW gamma matrices and define WCW metric in terms of their anti-commutators. The natural proposal is that gamma matrices are expressible as linear combinations of fermionic oscillator operators for second quantized induced spinor fields at space-time surface. One could even ask whether fermionic super charges and conserved fermionic Noether charges are involved with the construction.

   The explicit construction of gamma matrices [K127, K96] has however been based on somewhat ad hoc formulas, and what I call effective 2-dimensionality argued to follow from quantum criticality is somewhat questionable as exact notion.

5.8.2 Twistor lift of TGD and ZEO

Twistor lift of TGD and ZEO meant a revolution in the view about WCW geometry and spinor structure.

1. The basic idea is to replace 4-D Kähler action with dimensionally reduced 6-D Kähler for the analog of twistor space of space-time surface. The induction procedure for the spinors would be generalized so that it applies to twistor structure [L70]. The twistor structure of the embedding space is identified as the product of twistor spaces $M^4 \times S^2$ of M^4 and $SU(3)/U(1) \times U(1)$ of CP_2. In momentum degrees of freedom the twistor space of M^4 would be the usual CP_3.

   Remarkably, M^4 and CP_2 are the only spaces allowing twistor space with Kähler structure [A63]. In the case of M^4 the Kähler structure is a generalization of that for E^4. TGD would be unique from the existence of twistor lift. This predicts CP breaking at fundamental level possibly responsible for CP breaking and matter-antimatter asymmetry.

2. One would still have Kähler coupling strength α_K as the only single dimensionless coupling strength, whose spectrum is dictated by quantum criticality meaning that it is analogous to critical temperature. All coupling constant like parameters would be determined by quantum criticality. Cosmological constant would not be fundamental constant and this makes itself visible also in the concrete expressions for conserved Noether currents. The breaking of the scale invariance removing vacuum degeneracy of 4-D Kähler action would be analogous to spontaneous symmetry breaking and would remove vacuum degeneracy and classical non-determinism.

   The volume term would emerge from dimensional reduction required to give for the 6-surface the structure of S^2 bundle having space-time surfaces as base space. Cosmological constant would be determined by dynamics and depend on p-adic length scale depending in the average on length scale of space-time sheet proportional to the cosmic time sense like $1/a^2$, a cosmic time. This would solve the problem of large cosmological constant and predict extremely small cosmological constant in cosmic scales in the recent cosmology. This suggests that in long length scales one still has spin glass degeneracy realized in terms of many-sheeted space-time.

3. In ZEO 3-surface correspond to a union of 3-surfaces at the ends of space-time surfaces at boundaries of CD. There are many characterizations of quantum criticality.

   (a) Preferred extremal property and quantum criticality would mean that one has simultaneously an extremal of both 4-D Kähler action and volume term except at singular 2-surfaces identified as string world sheets and their boundaries. In accordance with the universality of quantum critical dynamics, one would have outside singularities local dynamics without dependence on Kähler coupling strength. The interpretation would be as geometric generalization of massless fields also characterizing criticality.

   (b) Another characterization of preferred extremal is as a space-time surfaces using sub-algebra S_m of symplectic algebra S for which generators have conformal weights coming as m-tuples of those for the full symplectic algebra. Both S_m and $[S, S_m]$ would have vanishing Noether charges. For the induced spinor fields analogous condition would hold true. Effectively the infinite number of radial conformal weights of the symplectic algebra associated with the light-like radial coordinate of $\delta M^4_\pm$ would reduce to a finite number.

   (c) A further characterization would be in terms of $M^8 - H$ duality [L48]. Preferred extremals in H would be images of of space-time surfaces in M^8 under $M^8 - H$ duality. The latter would correspond to roots of octonionic polynomials with coefficients in an extension of rationals. Therefore space-time surfaces in H satisfying field equations plus preferred extremal conditions would correspond to surfaces described by algebraic equations in M^8. Algebraic dynamics would be dual to differential dynamics.

   (d) In adelic physics [L54, L53] the hierarchy of Planck constants $h_{eff}/h_0 = n$ with n having an interpretation as dimensions of Galois group of extension of rationals would define further correlate of quantum criticality. The scaled up Compton lengths proportional to h_{eff} would characterize the long range fluctuations associated with quantum criticality.

5.8.3 The revised view about WCW metric and spinor structure

In this framework one can take a fresh approach to the construction of the spinor structure and Kähler metric of WCW. The basic vision is rather conservative. Rather than inducing ad hoc formulas for WCW gamma matrices one tries to identify Noether the elements super-algebra as Noether charges containing also the gamma matrices as Noether super charges.

1. The simplest guess is that the algebra generated by fermionic Noether charges Q^A for symplectic transformations $h^k \rightarrow h^k + \epsilon j^{Ak}$ assumed to induce isometries of WCW and Noether supercharges Q_n and their conjugates for the shifts $\Psi \rightarrow \Psi + \epsilon u_n$, where u_n is a solution of the modified Dirac equation, and ϵ is Grassmann number are enough to generate algebra containing the gamma matrix algebra.

2. The commutators $\Gamma^A_n = [Q^A, Q_n]$ are super-charges labelled by (A, n). One would like to identify them as gamma matrices of WCW. The problem is that they are labelled by (A, n) whereas isometry generators are labelled by A only just as symplectic Noether charges. Do all supercharges Γ^A_n except Γ^A_0 corresponding to $u_0 = constant$ annihilate the physical states so that one would have 1-1 correspondence? This would be analogous to what happens quite generally in super-conformal algebras.

3. The anti-commutators of Γ^A_0 would give the components of the Kähler metric. The allowance of singular surfaces having 2-D string world sheets as singularities would give to the metric also stringy component besides 3-D component and possible 0-D components at the ends of string. Metric 2-D property would not be exact as assumed originally.

This construction can be blamed for the lack of explicitness. The general tendency in the development of TGD has been replacement of explicit but somewhat ad hoc formulas with principles. Maybe this reflects to my own ageing and increasing laziness but my own view is that principles are what matter and get abstracted only very slowly. The less formulas, the better!

5.9 Does 4-D action generate lower-dimensional terms dynamically?

The original proposal was that the action defining the preferred extremals is 4-D Kähler action. Later it became obvious that there must be also 2-D string world sheet term present and probably also 1-D term associated with string boundaries at partonic 2-surfaces. The question has been whether these lower-D terms in the action are primary of generated dynamically. By super-conformal symmetry the same question applies to the fermionic part of the action. The recent formulation based on the twistor lift of TGD contains also volume term but the question remains the same.

During years several motivations for the proposal that preferred extremals of action principle including also volume term for twistor lift of Kähler action are minimal surfaces which are singular at 2-D string world sheets and perhaps also at their boundaries.

In particular, quantum criticality would be realized as a minimal surface property realized by holomorphy in suitably generalized sense [L79, L70]. The reason is that the holomorphic solutions of minimal surface equations involve no coupling parameters as the universality of the dynamics at quantum criticality demands.

Minimal surface equation would be true apart from possible singular surfaces having dimension $D = 2, 1, 0$. $D = 2$ corresponds to string world sheets and partonic 2-surfaces. If there are 0-D singularities they would be associated with the ends of orbits of partonic 2-surfaces at boundaries of causal diamond (CD). Minimal surfaces are solutions of non-linear variant of massless d'Alembertian having as effective sources the singular surfaces at which d'Alembertian equation fails. The analogy with gauge theories is highly suggestive: singular surfaces would act as sources of massless field.

Strings world sheets seem to be necessary. The basic question is whether the singular surfaces are postulated from the beginning and there is action associated with them or whether they emerge dynamical from 4-D action. One can consider two extreme options.

Option I: There is an explicit assignment of action to the singular surfaces from the beginning. A transfer of Noether charges between space-time interior and string world sheets is possible. This kind of transfer process can take place also between string world sheets and their light-like boundaries and happens if the normal derivatives of embedding space coordinates are discontinuous at the singular surface.

Option II: No separate action is assigned with the singular surfaces. There could be a transfer of Noether charges between 4-D Kähler and volume degrees of freedom at the singular surfaces causing the failure of minimal surface property in 4-D sense. But could singular surfaces carry Noether currents as 2-D delta function like densities?

This is possible if the discontinuity of the normal derivatives generates a 2-D singular term to the action. Conservation laws require that at string world sheets energy momentum tensor should degenerate to a 2-D tensor parallel to and concentrated at string world sheet. Only 4-D action would be needed - this was actually the original proposal. Strings and particles would be essentially edges of space-time - this is not possible in GRT. Same could happen also at its boundaries giving rise to point like particles. Super-conformal symmetry would make this possible also in the fermionic sector.

For both options the singular surfaces would provide a concrete topological picture about the scattering process at the level of single space-time surface and telling what happens to the initial state. The question is whether Option I actually reduces to Option II. If the 2-D term is generated to 4-D action dynamically, there is no need to postulate primary 2-D action.

5.9.1 Can Option II generate separate 2-D action dynamically?

The following argument shows that Option II with 4-D primary action can generate dynamically 2-D term into the action so that no primary action need to be assigned with string world sheets.

Dimensional hierarchy of surfaces and strong form of holography

String world sheets having light-like boundaries at the light-like orbits of partonic 2-surfaces are certainly needed to realize strong form of holography [K127]. Partonic 2-surfaces emerge automatically as the ends of the orbits of wormhole contacts.

1. There could (but neet not) be a separate terms in the primary action corresponding to string world sheets and their boundaries. This hierarchy bringing in mind branes would correspond to the hierarchy of classical number fields formed by reals, complex numbers, quaternions (space-time surface), and octonions (embedding space in M^8-side of M^8 duality). The tangent - or normal spaces of these surfaces would inherit real, complex, and quaternionic structures as induced structure. The number theoretic interpretation would allow to see these surfaces as images of those surfaces in M^8 mapped to H by $M^8 - H$ duality. Therefore it would be natural to assign action to these surfaces.

2. This makes in principle possible the transfer of classical and quantum charges between space-time interior and string world sheets and between from string world sheets to their light-like boundaries. TGD variant of twistor Grassmannian approach [L55, L72] relies on the assumption that the boundaries of string world sheets at partonic orbits carry quantum numbers. Quantum criticality realized in terms of minimal surface property realized holomorphically is central for TGD and one can ask whether it could play a role in the definition of S-matrix and identification of particles as geometric objects.

3. For preferred extremals string world sheets (partonic 2-surfaces) would be complex (co-complex) manifolds in octonionic sense. Minimal surface equations would hold true outside string world sheets. Conservation of various charges would require that the divergences of canonical momentum currents at string world sheet would be equal to the discontinuities of the normal components of the canonical momentum currents in interior. These discontinuities would correspond to discontinuities of normal derivatives of embedding space coordinates and are acceptable. Similar conditions would hold true at the light-like boundaries of string world sheets at light-like boundaries of parton orbits. String world sheets would not be minimal surfaces and minimal surface property for space-time surface would fail at these surfaces.

   Quantum criticality for string world sheets would also correspond to minimal surface property. If this is realized in terms of holomorphy, the field equations for Kähler and volume parts at string world sheets would be satisfied separately and the discontinuities of normal components for the canonical momentum currents in the interior would vanish at string world sheets.

4. The idea about asymptotic states as free particles would suggest that normal components of canonical momentum currents are continuous near the boundaries of CD as boundary conditions at least. The same must be true at the light-like boundaries of string world sheets. Minimal surface property would reduce to the property of being light-like geodesics at light-like partonic 2-surface. If this is not assumed, the orbit is space-like. Even if these conditions are realized, one can imagine the possibility that at string world sheets 4-D minimal surface equation fails and there is transfer of charges between Kähler and volume degrees of freedom (Option II) and therefore breaking of quantum criticality.

   If the exchange of Noether charges vanishes everywhere at string world sheets and boundaries, one could argue that they represent independent degrees of freedom and that TGD reduces to string model. The proposed equation for coupling constant evolution however contains a coefficients depending on the total action so that this would not be the case.

5. Assigning action to the lower-D objects requires additional coupling parameters. One should be able to express these parameters in terms of the parameters appearing in 4-D action (α_K and cosmological constant). For string sheets the action containing cosmological term is 4-D and Kähler action for $X^2 \times S^2$, where S^2 is non-dynamical twistor sphere is a good guess. Kähler action gets contributions from X^2 and S^2. If the 2-D action is generated dynamically as a singular term of 4-D action its coupling parameters are those of 4-D action.

6. There is a temptation to interpret this picture as a realization of strong form of holography (SH) in the sense that one can deduce the space-time surfaces by using data at string world sheets and partonic 2-surfaces and their light-like orbits. The vanishing of normal components of canonical momentum currents would fix the boundary conditions.

If double holography $D = 4 \to D = 2 \to D = 1$ were satisfied it might be even possible to reduce the construction of S-matrix to the proposed variant of twistor Grassmann approach. This need not be the case: p-adic mass calculations rely on p-adic thermodynamics for the excitions of massless particles having CP_2 mass scale and it would seem that the double holography can makes sense for massless states only.

In M^8-picture [L48] the information about space-time surface is coded by a polynomial defined at real line having coefficients in an extension of rationals. This real line for octonions corresponds to the time axis in the rest system rather than light-like orbit as light-like boundary of string world sheet.

Stringy quantum criticality?

The original intuition [L79] was that there are canonical momentum currents between Kähler and volume degrees of freedom at singular surfaces but no transfer of canonical momenta between interior and string world sheets nor string world sheets and their boundaries. Also string world sheets would be minimal surfaces as also the intuition from string models suggests. Could also the stringy quantum criticality be realized?

1. Some embedding space coordinates h^k must have discontinuous partial derivatives in directions normal to the string world sheet so that 3-surface has 1-D edge along fermionic string connecting light-like curves at partonic 2-surfaces in both Minkowskian and Euclidian regions. A closed highly flattened rectangle with long and short edges would be associated with closed monopole flux tube in the case of wormhole contact pairs assigned with elementary particles. 3-surfaces would be "edgy" entities and space-time surfaces would have 2-D and 1-D edges. In condensed matter physics these edges would be regarded as defects.

2. Quantum criticality demands that the dynamics of string world sheets and of interior effectively decouple. Same must take place for the dynamics of string world sheets and their boundaries. Decoupling allows also string world sheets to be minimal surfaces as analogs of complex surfaces whereas string world sheet boundaries would be light-like (their deformations are always space-like) so that one obtains both particles and string like objects.

3. By field equations the sums for the divergences of stringy canonical momentum currents and the corresponding singular parts of these currents in the interior must vanish. By quantum criticality in interior the divergencespf Kähler and volume terms vanish separately. Same must happen for the sums in case of string world sheets and their boundaries. The discontinuity of normal derivatives implies that the contribution from the normal directions to the divergence reduces to the sum of discontinuities in two normal directions multiplied by 2-D delta function. Thid contribution is in the general case equal to the divergence of corresponding stringy canonical momentum current but must vanish if one has quantum criticality also at string world sheets and their boundaries.

   The separate continuity of Kähler and volume parts of canonical momentum currents would guarantee this but very probably implies the continuity of the induced metric and Kähler form and therefore of normal derivatives so that there would be no singularity. However, the condition that total canonical momentum currents are continuous makes sense, and indeed implies a transfer of various conserved charges between Kähler action and volume degrees of freedom at string world sheets and their boundaries in normal directions as was conjectured in [L79].

4. What about the situation in fermionic degrees of freedom? The action for string world sheet X^2 would be essentially of Kähler action for $X^2 \times S^2$, where S^2 is twistor sphere. Since the modified gamma matrices appearing in the modified Dirac equation are determined in terms of canonical momentum densities assignable to the modified Dirac action, there could be similar transfer of charges involved with the fermionic sector and the divergences of Noether charges and super-charges assignable to the volume action are non-vanishing at the singular surfaces. The above mechanism would force decoupling between interior spinors and string world sheets spinors also for the modified Dirac equation since modified gamma matrices are determined by the bosonic action.

Remark: There is a delicacy involved with the definition of modified gamma matrices, which for volume term are proportional to the induced gamma matrices (projections of the embedding space gamma matrices to space-time surface). Modified gamma matrices are proportional to the contractions $T^{\alpha}_{k}\Gamma^k$ of canonical momentum densities $T^{\alpha_k} = \partial L/\partial(\partial_\alpha h^k)$ with embedding space gamma matrices Γ^k. To get dimension correctly in the case of volume action one must divide away the factor $\Lambda/8\pi G$. Therefore fermionic super-symplectic currents do not involve this factor as required.

It remains an open question whether the string quantum criticality is realized everywhere or only near the ends of string world sheets near boundaries of CD.

String world sheet singularities as infinitely sharp edges and dynamical generation of string world sheet action

The condition that the singularities are 2-D string world sheets forces 1-D edges of 3-surfaces to be infinitely sharp.

Consider an edge at 3-surface. The divergence from the discontinuity contains contributions from two normal coordinates proportional to a delta function for the normal coordinate and coming from the discontinuity. The discontinuity must be however localized to the string rather than 2-surface. There must be present also a delta function for the second normal coordinate. Hence the value of also discontinuity must be infinite. One would have infinitely sharp edge. A concrete example is provided by function $y = |x|^\alpha$ $\alpha < 1$. This kind of situation is encountered in Thom's catastrope theory for the projection of the catastrophe: in this case one has $\alpha = 1/2$. This argument generalizes to 3-D case but visualization is possible only as a motion of infinitely sharp edge of 3-surface.

Kähler form and metric are second degree monomials of partial derivatives so that an attractive assumption is that $g_{\alpha\beta}$, $J_{\alpha\beta}$ and therefore also the components of volume and Kähler energy momentum tensor are continuous. This would allow $\partial_{n_i} h^k$ to become infinite and change sign at the discontinuity as the idea about infinitely sharp edge suggests. This would reduce the continuity conditions for canonical momentum currents to rather simple form

$$T^{n_i n_j}\Delta\partial_{n_j} h^k = 0 \quad . \tag{5.9.1}$$

which in turn would give

$$T^{n_i n_j} = 0 \tag{5.9.2}$$

stating that canonical momentum is conserved but transferred between Kähler and volume degrees of freedom. One would have a condition for a continuous quantity conforming with the intuitive view about boundary conditions due to conservation laws. The condition would state that energy momentum tensor reduces to that for string world sheet at the singularity so that the system becomes effectively 2-D. I have already earlier proposed this condition.

The reduction of 4-D locally to effectively 2-D system raises the question whether any separate action is needed for string world sheets (and their boundaries)? The generated 2-D action would be similar to the proposed 2-D action. By super-conformal symmetry similar generation of 2-D action would take place also in the fermionic degrees of freedom. I have proposed also this option already earlier. This would mean that Option II is enough.

The following gives a more explicit analysis of the singularities. The vanishing on the discontinuity for the sum of normal derivative gives terms with varying degree of divergence. Denote by n_i resp. t_i the coordinate indices in the normal resp. tangent space. Suppose that some derivative $\partial_{n_i} h^k$ become infinite at string. One can introduce degree n_D of divergence for a quantity appearing as part of canonical momentum current as the degree of the highest monomial consisting of the diverging derivatives $\partial_{n_i} h^k$ appearing in quantity in question. For the leading term in continuity conditions for canonical momentum currents of total action one should have $n_D = 2$ to give the required 2-D delta function singularity.

- $\partial_{n_i} h^k$ has $n_D \leq 1$. If it is also discontinuous - say changes sign - one has $n_D = 2$ for $\Delta \partial_{n_i} h^k$ in direction n_i.

- One has $n_D(g_{t_i t_j}) = 0$, $n_D(g_{t_i n_j}) = 1$, $n_D(g_{n_i n_i}) = 2$ and $n_D(g_{n_i n_j}) = 1$ or 2 for $i \neq j$. One has $n_D(g) = 4$ ($g = det(g_{\alpha\beta})$). For contravariant metric one gas $n_D(g^{t_i t_j}) = 0$ and $n_D(g^{n_i j}) = n_D(g^{n_i n_j}) = -2$ as is easy to see from the formula for $g^{\alpha\beta}$ in terms of cofactors.

- Both Kähler and volume terms in canonical momentum current are proportional to $\sqrt{g}$ with $n_D(\sqrt{g}) = 2$ having leading term proportional to 2-determinant $\sqrt{det(g_{n_i n_j})}$. In Kähler action the leading term comes from tangent space part J_{ij} and has $n_D = -1$ coming from the partial derivative. The remaining parts involving $J_{t_i n_j}$ or $J_{n_i n_j}$ have $n_D < 0$.

- Consider the behavior of the contribution of volume term to the canonical momentum currents. For $g^{n_i t_j} \partial_{t_j} h^k \sqrt{g}$ one has $n_D = 0$ so that this term is finite. For $g^{n_i n_j} \partial_{n_j} h^k \sqrt{g}$ one has $n_D \leq 1$ and this term can be infinite as also its discontinuity coming solely from the change of sign for $\partial_{n_j} h^k$. If $\partial_{n_j} h^k$ is infinite and changes sign, one can have $n_D = 2$ as required by 2-D delta function singularity.

  The continuity condition for the canonical momentum current would state the vanishing of $n_D = 2$ discontinuity but would not imply separate vanishing of discontinuity for Kähler and volume parts of canonical momentum currents - this in accordance with the idea about canonical momentum transfer. If the sign of partial derivative only changes the coefficient of the partial derivative must vanish so that the condition reduces to the condition $T^{n_i n_j} = 0$ already given for the components of the total energy momentum tensor, which would be continuous by the above assumption.

5.9.2 Kähler calibrations: an idea before its time?

While updating book introductions I was surprised to find that I had talked about so called calibrations of sub-manifolds as something potentially important for TGD and later forgotten the whole idea! A closer examination however demonstrated that I had ended up with the analog of this notion completely independently later as the idea that preferred extremals are minimal surfaces apart form 2-D singular surfaces, where there would be exchange of Noether charges between Kähler and volume degrees of freedom.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

   Calibrated k-manifolds are minimal surfaces in their homology class, in other words their volume is minimal. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces and representing fundamental fermions play a key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality outside singular surfaces would be realized as decoupling of the two parts of the action. May be all preferred extremals be regarded as calibrated in generalized sense.

   If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

   It is interesting to recall that the original proposal for the preferred extremals as absolute minima of Kähler action has transformed during years to a proposal that they are absolute minima of volume action within given homology class and having fixed ends at the boundaries of CD.

4. The experience with CP_2 would suggest that the Kähler structure of M^4 defining the counterpart of form ϕ is unique. There is however infinite number of different closed self-dual Kähler forms of M^4 defining what I have called Hamilton-Jacobi structures. These forms can have subgroups of Poincare group as symmetries. For instance, magnetic flux tubes correspond to given cylindrically symmetry Kähler form. The problem disappears as one realizes that Kähler structures characterize families of preferred extremals rather than M^4 itself.

If the notion of calibration indeed generalizes, one ends up with the same outcome - preferred extremals as minimal surfaces with 2-D string world sheets and partonic 2-surfaces as singularities - from many different directions.

1. Quantum criticality requires that dynamics does not depend on coupling parameters so that extremals must be separately extremals of both volume term and Kähler action and therefore minimal surfaces for which these degrees of freedom decouple except at singular 2-surfaces, where the necessary transfer of Noether charges between two degrees of freedom takes place at these. One ends up with string picture but strings alone are of course not enough. For instance, the dynamical string tension is determined by the dynamics for the twistor lift.

2. Almost topological QFT picture implies the same outcome: topological QFT property fails only at the string world sheets.

3. Discrete coupling constant evolution, vanishing of loop corrections, and number theoretical condition that scattering amplitudes make sense also in p-adic number fields, requires a representation of scattering amplitudes as sum over resonances realized in terms of string world sheets.

4. In the standard QFT picture about scattering incoming states are solutions of free massless field equations and interaction regions the fields have currents as sources. This picture is realized by the twistor lift of TGD in which the volume action corresponds to geodesic length and Kähler action to Maxwell action and coupling corresponds to a transfer of Noether charges between volume and Kähler degrees of freedom. Massless modes are represented by minimal surfaces arriving inside causal diamond (CD) and minimal surface property fails in the scattering region consisting of string world sheets.

5. Twistor lift forces M^4 to have generalize Kähler form and this in turn strongly suggests a generalization of the notion of calibration. At physics side the implication is the understanding of CP breaking and matter anti-matter asymmetry.

6. $M^8 - H$ duality requires that the dynamics of space-time surfaces in H is equivalent with the algebraic dynamics in M^8. The effective reduction to almost topological dynamics implied

by the minimal surface property implies this. String world sheets (partonic 2-surfaces) in H would be images of complex (co-complex sub-manifolds) of $X^4 \subset M^8$ in H. This should allows to understand why the partial derivatives of embedding space coordinates can be discontinuous at these edges/folds but there is no flow between interior and singular surface implying that string world sheets are minimal surfaces (so that one has conformal invariance).

The analogy with foams in 3-D space deserves to be noticed.

1. Foams can be modelled as 2-D minimal surfaces with edges meeting at vertices. TGD space-time could be seen as a dynamically generated foam in 4-D many-sheeted space-time consisting of 2-D minimal surfaces such that also the 4-D complement is a minimal surface. The counterparts for vertices would be light-like curves at light like orbits of partonic 2-surfaces from which several string world sheets can emanate.

2. Can one imagine something more analogous to the usual 3-D foam? Could the light-like orbits of partonic 2-surfaces define an analog of ordinary foam? Could also partonic 2-surfaces have edges consisting of 2-D minimal surfaces joined along edges representing strings connecting fermions inside partonic 2-surface?

   For years ago I proposed what I called as symplectic QFT (SQFT) as an analog of conformal QFT and as part of quantum TGD [K22]. SQFT would have symplectic transformations as symmetries, and provide a description for the symplectic dynamics of partonic 2-surfaces. SQFT involves an analog of triangulation at partonic 2-surfaces and Kähler magnetic fluxes associated with them serve as observables. The problem was how to fix this kind of network. Partonic foam could serve as a concrete physical realization for the symplectic network and have fundamental fermions at vertices. The edges at partonic 2-surfaces would be space-like geodesics. The outcome would be a calibration involving objects of all dimensions $0 \leq D \leq 4$ - a physical analog of homology theory.

5.10 Could metaplectic group have some role in TGD framework?

Metaplectic group appears as a covering group of linear symplectic group $Sp(2n, F)$ for any number field and its representations cam be regarded as analog of spinor representations of the rotation group. Since infinite-D symplectic group of $\delta M^4_+ \times CP_2$, where δM^4_+ is light-cone boundary, appears as an excellent candidate for the isometries of "world of classical worlds" in zero energy ontology (ZEO), one can ask whether and how the notion of metaplectic group generalizes to TGD framework [K57, ?, K96, K75, K28, K27].

The condition for the existence of metaplectic structure is same as those for the spinor structure and not met in the case of CP_2. One however expects that also in the case of metaplectic structure the modified metaplectic structure exists is one couples spinors to an odd integer multiple of Kähler gauge potential. For triality 1 representation assignable to quarks one has $n = 1$. The fact that the center of $SU(3)$ is Z_3 suggests that metaplectic group for CP_2 is 3- or 6-fold covering of symplectic group instead of 2-fold covering.

Besides the ordinary representations of $SL(2, C)$ also the possibly existing analogs of metaplectic representations of $SL(2, C) = Sp(2, C)$ acting on wave functions at hyperbolic space H_3 at $a^2 = t^2 - r^2$ hyperbolololoid of M^4_+ are cosmologically interesting since the many-sheeted space-time in number theoretic vision allows quantum coherence in even cosmological scales and there are indications for periodic redshift suggests tessellations of H_3 analogous to lattices in E^3 and defined by discrete subgroup of $Sl(2, C)$.

5.10.1 Heisenberg group, symplectic group, and metaplectic group

The following gives a brief summary of basics related to Heisenberg group, symplectic group, and metaplectic group.

Heisenberg group

1. The matrix representation of the simplest Heisenberg group http://tinyurl.com/y2fomegs is given by matrices

$$\begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \tag{5.10.1}$$

   A 3-D Lie group is in question. The multiplication for group elements (a_1, b_1, c_1) and (a_2, b_2, c_2) is given by $(a_1, b_1, c_1) \circ (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2 - a_1 b_2)$. The coefficients (a,b,c) can be belong to any ring sin the inverse can be expressed using only product and sum as $(-a, -b, ab - c)$. In particular, discrete variants of Heisenberg group such as those associated with extensions of rationals, exist. For odd primes one can define Heisenberg group modulo p as group of order p^3 in finite field F_p.

   $2n + 1$-D Heisenberg group consists of upper triangular with unit matrix at diagonal.

2. Continuous Heisenberg group is a nilpotent Lie group of dimension $d = 3$. Nilpotency means that it is Lie algebra elements are nilpotent. The Lie algebra is generated by upper-diagonal matrices and the commuation relations for the Lie algebra basis are $[X, Y] = Z$, $[X, Z] = 0$, $[Y, Z] = 0$. The coordinate $X = q$ and differential operator $Y = p = \hbar \partial_q$, $Z = i\hbar 1$satisfying $[p, q] = i\hbar Id$, define a concrete representation of the Lie algebra of the simplest 3-D Heisenberg group in the space of functions $f(q)$. By introducing n pairs of coordinates commuting to unit matrix one obtain $2n + 1$-D Heisenberg group.

Symplectic group

Symplectic group acts as automorphisms of Heisenberg group. Symplectic group leaves acts in function algebra of function $H(p, q)$ leaving invariant Poisson bracket $\{H_1, H_2\} = \partial_q H_1 \partial_p H_2 - \partial_q H_2 \partial_p H_1$. The Poisson bracket $\{p, q\} = 1$ giving the element of $J_{p,q} = 1$ symplectic form remaining invariant under symplectic transformations. Exponentiation of any Hamiltonian $H(p, q)$ acting as Hamiltonian generates symplectic flows. Symplectic group is infinite-D.

3-D linear symplectic group $Sp(2, F)$ is obtained as a special case. In continuous case Hamiltoniansare linear functions of p and q so that the action by Poisson bracket is linear. General linear symplectic group$Sp(2n, F)$ acts in $2n$-D space spanned spanned by the analogs of (q_p, p_i). When symplectic form is accompanied by complex structure and Kähler form symplectic isometries define a finite-D subgroup of symplectic group. For instance, in case of CP_2 symplectic isometries define group $SU(3)$.

Metaplectic group

Metaplectic group $Mp_m(2n, F)$ (see http://tinyurl.com/y5mpswy8 and http://tinyurl.com/y4kjys3e) is an m-fold covering of the linear symplectic group $Sp(2n, F)$. Metaplectic group like also linear symplectic group metaplectic grop is defined for all number fields, in particular p-adic number fields and even adeles. All representations of the metapelectic group are infinite-D (non-compactness is not the only reason: even finite-D non-unitary matrix representations fail to exist).

$Sp(2, R)$ co-incides with a covering group the special linear group $Sl(2, R)$ acting as real Möbius transformations in upper half-plane. Metaplectic group does not allow finite-D matrix representations and all representations are infinite-dimensional. Metaplectic group can be regarded as m-fold cover of symplectic group and in Weil representation the cover can be chosen to be 2-fold cover.

The elements for the metaplectic group $M_2(2,R)$ as 2-fold covering of $Sp(2,R)$ have representation as pairs (g,ϵ) with g a Möbius transformation represented by matrix $(a,b;c,d)$ with unit determinant acting as $z \rightarrow (az+b)/(cz+d)$ and with $\epsilon(z)^2 = cz+d$. The product of group elements is given by $(g_1, epsilon_1)(g_2,\epsilon_2) = (g_1g_2,\epsilon)$, $\epsilon(z) = \epsilon_1(g_2(z))\epsilon_2(z)$. The entities transforming in this manner are not functions but analogous to spinors and one can speak of symplectic spinors.

3. One can generalize the notion of symplectic structure to that of metaplectic structure. The topological conditions (the second Stiefel-Withney class vanishes) for the existence of metaplectic structure for given symplectic manifold are same as for the spinor structure.

   Interestingly, in the case of CP_2 this condition is not satisfied and the problem is circumvented by coupling CP_2 spinors to an odd multiple of Kähler gauge potential giving rise to Kähler form: this is essential for obtain electroweak couplings correctly for the induced spinor structure at space-time surface. Since Kähler form relates so closely to symplectic structure, it is reasonable to expect that also in case of CP_2(CP_{2n}) symplectic spinors exist.

   The center of isometry group $SU(3)$ of CP_2 is Z_3 acting trivial on CP_2 coordinates. The action is analogous to that of Möbius transformations being induced by linear action oof $SU(3)$ on projective coordinates (z_1,z_2,z_3) and by the projective map such as $(z_1,z_2,z_3) \rightarrow (z_1/z_3, z_2/z_3, 1)$ in given coordinate patch defined by a choice of two complex coordinates (z_i,z_j) now $(z_1/z_3, z_2/z_3)$. Do symplectic spinors spinors transform like CP_2 spinors under metaplectic action of $SU(3)$?

   CP_2 spinors with unit coupling to Kähler gauge potential allow triality $t = \pm 1$ partial impossible without the coupling making possible spinor structure and presumabley also metaplectic struture. Does this mean that in the case of CP_2 the metaplectic group must be identified as 3-fold or possibly 6-fold covering of symplectic group. The holomy group is electroweak $U(2)$ and acts like $SU(2) \times U(1)$. Does holonomy group acts as double covering of $SO(3)$ and as 3-fold covering of $U(1)$ giving 6-fold covering of tangent space group $SO(4)$?

5.10.2 Symplectic group in TGD

In TGD the symplectic transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is light-cone boundary, and generated by Hamiltonian algebra, are central and act in the "world of classical worlds" (WCW) [K57, **?**, K96, K75, K28, K27].

1. WCW is formed by pairs of 3-surfaces with members at opposite boundaries of causal diamond $CD = cd \times CP_2$ of embedding space $H = M^4 \times CP_2$. cd is causal diamond of M^4 defined as intersection of future and past directed light-cones. The members of the pair are connected by preferred extremal of action defined by twistor lift of TGD: it is sum of Kähler action and volume term. Preferred extremal is analogous to Bohr orbit.

2. The obvious question is whether also infinite-D symplectic group of $\delta M^4_+ \times CP_2$ allows metaplectic variant. Second question is how symplectic spinors relate to ordinary spinors. Are ordinary spinors of H symplectic spinors as one might expect?

3. In TGD the spinors of "world of classical worlds" (WCW) [K57, **?**, K96] should have interpretation as symplectic spinors. Spinors of WCW are fermionic Fock states created by quark oscillator operators replacing theta parameters in super-coordinates and in super-spinors of super variant of embedding space H. Their local composites appear as monomials with vanishing quark number in hermitian super-coordinates of super-variant of H and in super-quark-spinors of super-H containing only monomials with odd quark number. These super-fields differ from those of standard SUSY since monomials of theta parameters are replaced with monomials of quark oscillator operators and Majorana spinors are not in question.

   Infinite-D metaplectic group $\delta M^4_+ \times CP_2$ should act on WCW spinor fields and the action should be induced from action in H.

5.10.3 Kac-Moody type approach to representations of symplectic/metaplectic group

Representations of the symplectic/metaplectic group. Kac-Moody type approach is strongly suggested physically. Kac-Moody group has Lie-algebra which is central extension of the Lie-algebra of local gauge transformation. Kac-Moody algebra elements are labelled by elements with conformal weight $n \in Z$ but also the variant $n \geq 0$ ("half-algebra" exists as sub-algebra is clear from the commutation relations.

1. Let r denote the radial light-like coordinate of light-cone boundary $\delta M^4_+ \times CP_2$. $\delta M^4_+ = S^2 \times R^+$ is metrically 2-sphere S^2 and this implies extension of usual conformal invariance for S^2 to conformal invariance localized with respect to r and explains why 4-D Minkowski space is physically unique.

   Radially local conformal transformations $z \to f(r,z)$of light-cone boundary with scaling $r \to |df(r,z,zbar)/dz|^{-1} \times r$ in light-cone radial coordinate r compensating for the conformal scaling factor $|df(r,z,zbar)/dz|^2$ as isometries of light-cone boundary as also color rotation local with respect to r. One has radially local $S = SO(3) \times SU(3)$ as isometries of light-cone boundary. This would serve as the TGD variant of color gauge symmetry.

2. Effective localization of the symplectic algebra of $S^2 \times CP_2$ with respect to the radial light-like coordinate r. Denote the radial conformal weight h.

   Option 1: Radial waves of form r^h, $h = -1/2+iy$ (something to do with zeros of zeta) behave like plane waves with wave vector y for in inner product defined by integration measure dr. Orthogonal plane-wave basis effectively.

   Restriction to causal diamond CD defined as intersection of future and past directed light-cones implies $r \leq r_{max}$ defining the size of CD and periodic boundary conditions for a discrete basis r^h. If $h = -1/2 + iy$ corresponds to a zero of zeta, the size of CD determined by r_{max} is quantized. For instance, $sin(yln(r_{max})) = 0$ would imply $ln(r_{max}) = n \times \pi/y$. Also $cos(yln(r_{max})) = 0$ can be considered.

   Option 2: One can include the real part of h to the integration measure of inner product defined as $d\mu = dr/r$. This is dimensionless and very natural by scaling invariance. For this choice one has $h = iy$ and the connection with Riemann zeta is not anymore natural. $r_{max} = exp(n \times \pi/y)$ would give periodic boundary conditions.

   For $y = k\pi$ one would have $r_{max} = exp(1/k)$, k integer. This conforms with the adelic picture since the infinite-D extension of rationals generated by $e^{1/k}$ induces finite-D extension of p-adic numbers since e^p is ordinary p-adic number.

   $y = k\pi/log(p)$ gives $r_{max} = p^{n/k}$ and one can construct finite-D extensions of rationals allowing roots of p.

3. Super-symplectic algebra is assumed to have fractal structure . There is a hierarchy of isomorphic super-sympletcic sub-algebras SSA_n, $n = 1, 2...$, for which conformal weights n-multiples of the weights for the entire algebra.

   Option 1: One would have also conformal weights $n(-1/2 + iy)$ for these radial waves however inner product using dmu= dr as integration measures does reduce to inner product for plane waves but to $\int r^{-n+1} exp(in(y_1 - y_2))du$, $u = log(r/r_0)$. This leads out from the original state space. The modification of the integration measure to $d\mu = r^(n-1)dr$ does not seem plausible.

   Option 2: Identify the conformal weight as $h = iy$ and include the real part -1/2 to the dimensionless integration measure $d\mu = dr/r$. This allows fractal hierarchy $h = niy$. This seems to be the only elegant option so that the connection with Riemann zeta seems artificial

This picture leads to some conjectures and questions.

1. Sub-algebra SSA_n and its commutator with entire algebra SSA represented trivially for physical states. Also classical Noether charges vanish: this gives strong conditions on preferred

extremals and makes them analogs of Bohr orbits: only preferred pairs of 3-surfaces at opposite boundaries of CD are connected by preferred extremal. Hierarchy of state spaces is the outcome.

This would be generalization of Super Virasoro conditions for which only the entire algebra would act trivially apart from the scaling generator L_0.

2. Could the hierarchies of extensions of rationals with dimensions $n_1|n_2|...$ (| is for "divides") correspond to hierarchies of inclusions of hyper-finite factors.

3. Could the hierarchies of SSA_n with $n_1|n_2|...$ correspond to hierarchies of extensions of extensions of.... of rationals with dimensions $n_1|n_2|....$

$\delta M^4_+ \times CP_2$ is metrically $S^2 \times CP_2$ and this leads to some questions.

1. Could one have Kac-Moody type representation of the symplectic algebra of $S^2 \times CP_2$, which is radially local and involves central extension? This is physically suggestive.

2. Symplectic isometries of $S^2 \times CP_2$ local with respect to r would define a sub-representation.

   Hamiltonians products of $\delta M^4_+ \times CP_2$ Hamiltonians for δM^4_+ and CP_2 labelled by angular momentum j and by the 2 Casimirs of triality $t = 0$ color representations.

   Isometry algebras $SO(3)$ and $SU(3)$ are sub-algebras of symplectic algebra determined by Hamiltonians at light-cone boundary in given representation to themselves. There are no higher-D sub-algebras so that one cannot consider hierarchy analogous to the hierarchy of sub-algebras labelled by radial conformal weights as n-multiples of weights of the entire algebra.

This in turn leads to a series of questions concerning what happens if one takes gauge symmetry and Kac-Moody symmetry as its analog as a physical guideline.

1. The metaplectic group of $SL(2,R)$ has only infinite-D representations but no matrix representations. Can this be true also for the metaplectic representation of infinite-D for $SO(3) \times SU(3)$ which is compact and allow finite-D unitary ordinary representations. $SO(3)$ must be lifted to $SU(2)$ and this is natural for quark spinors. SU(3) allows only triality $t = 0$ partial waves.

   Since $SU(3)$ has Z_3 as center one expects that the notion of metaplectic representation in this case generalizes so that one has 3-fold covering of function space instead of 2-fold one. Quark spinors indeed allow CP_2 partial waves which are in $t = 1$ representations. As already noticed CP_2 allows does not allow metaplectic structure in standard sense but the coupling to the Kähler gauge potential probably makes this possible since the condition for the existence of generalized metaplectic structure is same as for the existence of modified spinor structure.

2. Should one treat all S^2 Hamiltonians with $l > 1$ as gauge degrees of freedom? A possible interpretation would be in terms of finite measurement resolution and analog of Kac-Moody symmetry acting very much like gauge symmetry representing the finite measurement resolution. Symplectic group would effectively reduce to $SO(3) \times SU(3)$. If so, one would have $SO(3) \times SU(3)$ gauge theory with $l = 1$ states and spin 1/2 states with color as particles.

3. Only quark triplets and singlets of fermions and color octets of gluons are observed. Without any additional conditions TGD predicts infinite number of spinor harmonics. For CP_2 spinor harmonics there is a correlation for the color quantum numbers and electroweak quantum numbers of spinor harmonic. In QCD the color representation of quark does not however depend on electroweak quantum numbers. Also the masses of spinor harmonics depend on electroweak quantum numbers and are typically very large.

   Remark: One could of course ask whether quarks could move in different color partial waves but having $t = 1$. This however seems rather implausible.

   The proposal is that Kac-Moody type generators can be used to build massless states with have correct correlation between color represented as angular momentum like quantum number and electroweak quantum numbers. Could the experimental absence of higher color partial waves be due to the fact the gauge nature of higher excitations of symplectic algebra making higher color partial waves of quarks and leptons gauge degrees of freedom?

4. What about $l = 1$ states assignable to $SO(3)$? Twistor lift of TGD predicts that also M^4 has analog of Kähler form and induced $U(1)$ gauge field analogous to induced Kähler form. The physical effects are weak and would be responsible for CP breaking and matter antimatter asymmetry. Could the $l = 1$ triplet correspond to this $U(1)$ gauge boson somewhat like $SU(3)$ octet corresponds to gluon (gluon is identified as pair of quark and antiquark at different positions)?

5. How does this relate to the analog of metaplectic group for $SO(3) \times SU(3)$? What about the central extension of $SO(3) \times SU(3)$ assignable to spinor representations with weight $n = 1/2$. If one adds to the Hamilton associated with rotation generator L_z around z-axis in $SO(3)$ and to hyper-charge generator Y of $SU(3)$ a constant, one obtains what looks like central extension at the level of Poisson brackets since right hand side of brackets receives an additive constant. In $SU(3)$ degrees of freedom one can have only $t = 0$ color partial waves for scalars but for spinors one obtains the $t = 1$ waves and can say that color partial waves possess and anomalous hyper-charge Y.

   The spectra of L_z and Y are shifted but Killing vector fields are not affected. The couplings of isometry generators are changed since there is coupling proportional to Hamiltonian. This does not seem to have have interpretation as a mere gauge transformation since it makes $t = 1$ color partial waves possible for quarks.

5.10.4 Relationship to modular functions

The metaplectic representations involve in basic form $Sp(2n, F)$, F any number field.

1. $n = 1$ is physically special: one has $Sp(2, C) = SL(2, C)$, which is double covering of Lorentz group. The so called modular representations giving rise to basic functions appearing in number theory are related to the representations of $SL(2, C)$ with the condition that SL(2,Z) or its discrete subgroup (there are infinite number of them) is represented either trivially or mere projective factor. In the representation realizing SL(2,C) as Möbius transformations $z \to (Az + B)/Cz + D)$ or upper half-plane one has $f(z) \to (Cz + D)^k f(z)$ when (A,B;C,D) represents element of SL(2,Z) or its subgroup G. k is integer or half integer. One has modular invariance apart from the projective factor.

   Although these nodularity conditions apply only to a discrete subgroup of $SL(2, R)$ they they imply projective invariance of the analytic functions involved so that projectively their support of the function reduces to $G\backslash H$, H upper complex plane analogous to unit cell. Could this kind of conditions correspond to the proposed analogs of Kac-Moody type gauge conditions proposed for symplectic symmetries of $\delta M^4_\pm \times CP_2$?

2. $SO(3, 1)$ acts as isometries of the hyperbolic space H_3 identifiable as the hyperboloid H_3 as $a^2 = t^2 - r^2 = constant$ surface of future light-cone M^4_+: a defines in TGD Lorentz invariant cosmic time and is natural embedding space coordinate in ZEO. Since $SL(2, Z)$ has infinite number if discrete subgroups, one has infinite number of tessellations of H_3 analogous to lattices in 3-D Euclidian space.

   In TGD quantum coherence is possible in even cosmological scales since TGD predicts hierarchy of effective values of Planck constants. Could one have quantum coherent structures represented as tessellations of the hyperboloid? The prediction would be quantization of redshift as reflection of quantization of distances from given point of tessellations to other points. Evidence for this kind of quantization has been observed.

3. Finite measurement resolution suggests consideration of tessellations as discretization of H_3 and assignable to extensions of rationals and also to subgroups of $SL(2, Z)$. This would mean discretized wave functions in the tessellation. This would be like wave function for particle in discrete lattice in E^3. On the other hand, modular functions with projective modular invariance would be analogs for wave functions of particles periodic symmetry implied by lattice but represented projectively.

Could one decompose the representation to products of modular forms as projective representations in coset space $SL(2,C)/\Gamma$, Γ a discrete subgroup of $SL(2,C)$ and of representations of discrete subgroup corresponding to finite measurement resolution. This would be like representation of wave functions as products of discrete lattice wave function and wave functions in the space of momenta modulo lattice momenta: Fermi sphere would be replaced by the coset space $SU(2)/G$.

4. The projective factor $\epsilon^2(Z) = (Cz + D)^k$ is essential for the projective representation of $Sp(2,C)$. Is it possible to generalize this factor acting on upper complex plane to the case of H_3? If subgroup $SO(3)$ is represented projectively, then one can use for H_3 coordinates (r,θ,ϕ), such that r as radius of sphere S^2 remains invariant under r and $SO(3)$ actsob the complex coordinate of S^2 transforming linearly under $SO(1)$ as $z \to (Az + B)/(Cz + D)$ so that the projective factor can be identified. These representations would be analogous to modular representations: the discrete subgroup of $SL(2,C)$ would be replaced with $SU(2)$.

   It would seem that it must be replaced with $SU(2)$ as subgroup. Could one generalize the notion of modular form invariant under discrete subgroup of $SL(2,C)$ so that the discrete subgroup would become discrete subgroup of $SO(3)$ ($SU(2)$).

   Platonic solids are lattices at S^2 and their isometries and finite subgroups $D(2n)$ appear in McKay correspondence relating discrete subgroups of $SU(2)$ and ADE Lie groups. Finite measurement resolution as dual interpretation. What about infinite discrete subgroups. Does invariance mean projective SU(2) invariance (the case when $n = 0$)

Chapter 6

Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occarm's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Käction in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent embedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic 2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At embedding space level the modes of embedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at embedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

6.1 Introduction

Occam's razor argument is one the standard general purpose arguments used in debunking: the debunked theory is claimed to be hopelessly complicated. This argument is more refined that mere "You are a crackpot!" but is highly subjective and often the arguments pro or con are not given. Combined with the claim that the theory does not predict anything Occam's razor is very powerful argument unless the audience includes people who have bothered to study the debunked theory.

Let us take a closer look on this argument and compare TGD superstring models and seriously ask which of these theories is simple.

In superstring models one has strings as basic dynamical objects. They live in target space M^{10}, which in some mysterious way (something "non-perturbative" it is) spontaneously compactifies to $M^4 \times C$, C is Calabi-Yau space. The number of them is something like 10^{500} or probably infinite: depends on the counting criterion. And this estimate leaves their metric open. This leads to landscape and multiverse catastrophe: theory cannot predict anything. As a matter fact $M^4 \times C$:s must be allowed to deform still in Kaluza-Klein paradigm in which space-time has Calabi-Yau as small additional dimensions. An alternative way to obtain space-time is as 3-brane. One obtains also higher-D objects. Again by some "nonperturbative" mechanisms. One does not even know what space-time is! Situation looks to me a totally hopeless mess. Reader can conclude whether to regard this as simple and elegant.

I will consider TGD at three levels. At the level of "world of classical worlds" (WCW), at space-time level, and at the level of embedding space $H = M^4 \times CP_2$. I hope that I can convince the reader about the simplicity of the approach. The simplicity is actually quite shocking and certainly an embarrassing experience for the unhappy super string theorists meandering around in the landscape and multiverse. Behind this simplicity are however principles - something, which colleagues usually regard as unpractical philosophizing: "shut-up-and-calculate!"!

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

6.2 Simplicity at various levels

6.2.1 WCW level: a generalization of Einstein's geometrization program to entire quantum physics

I hope that the reader would read the following arguments keeping in mind the question "Is TGD really hopelessly complicated mess of pieces picked up randomly from theoretical physics?" as one debunker who told that he does not have time to read TGD formulated it.

1. Einstein's geometrization program for gravitation has been extremely successful but has failed for other classical fields, which do not have natural geometrization in the case of abstract four-manifolds with metric. One should understand standard model quantum numbers and also family replication for fermions.

   However, if space-time can be regarded surface in $H = M^4 \times CP_2$ also the classical fields find a natural geometrization as induced fields obtained basically by projecting. Also spinor structure can be induced and one avoids the problems due the fact that generic space-time as abstract 4-manifold does not allow spinor structure. The dynamics of space-time surfaces incredibly simple: only 4 field-like variables corresponding to *four* embedding space coordinates and induced that of classical geometric fields. Nowadays one would speak of emergence. The complexity emerges from the topology of space-time surfaces giving rise to many-sheeted space-time.

2. Even this view about geometrization is generalized in TGD. Einstein's geometrization program is applied to the entire quantum physics in terms of the geometry of WCW consisting of 3-D surfaces of H. More precisely, in zero energy ontology (ZEO) it consists of pairs of 3-surfaces at opposite boundaries of causal diamond (CD) connected by a preferred extremals of a variational principle to be discussed.

   Quantum states of the Universe would correspond to the modes of formally classical WCW spinor field satisfying the analog of Dirac equation. No quantization: just the construction of WCW geometry and spinor structure. The only genuinely quantal element of quantum theory would be state function reduction and in ZEO its description leads to a quantum theory of consciousness.

To me this sounds not only simple but shockingly simple.

WCW geometry

Consider first the generalization of Einstein's program of at the level of WCW geometry [K96, K57, K29].

1. Since complex conjugation must be geometrized, WCW must allow a geometric representation of imaginary unit as an antisymmetric tensor, which is essentially square root of the negative

of the metric tensor and thus allow Kähler structure coded by Kähler function. One must have 4-D general coordinate invariance (GCI) but basic objects are 3-D surfaces. Therefore the definition of Kähler function must assign to 3-surface a unique 4-surface.

Kähler function should have physical meaning and the natural assumption is that it is Kähler action plus possibly also volume term (twistor lift implies it). Space-time surface would be a preferred extremal of this action. The interpretation is also as an analog of Bohr orbit so that Bohr orbitology would correspond exact rather than only approximate part of quantum theory in TGD framework. One could speak also of quantum classical correspondence.

2. The action principle involves coupling parameters analogous to thermodynamical parameters. Their value spectrum is fixed by the conditions that TGD is quantum critical. For instance Kähler couplings strength is analogous to critical temperature. Different values correspond to different phases. Coupling constant evolution correspond to phase transitions between these phases and loops vanish as in free field theory for $\mathcal{N} = 4$ SYM.

3. The infinite-dimensionality of WCW is a crucial element of simplicity. Already in the case of loop spaces the geometry is essentially unique: loop space is analogous to a symmetric space points of the loop space being geometrically equivalent. For loop spaces Riemann connection exists only of the metric has maximal isometries defined by Kac-Moody algebra.

   The generalization to 3-D case is compelling. In TGD Kac-Moody algebra is replaced by super-symplectic algebra, which is much larger but has same basic structure (conformal weights of two kinds) and a fractal hierarchy of isomorphic sub-algebas with conformal weights coming as multiples of those for the entire algebra is crucial. Physics is unique because of its mathematical existence. WCW decompose to a union of sectors, which are infinite-D variants of symmetric spaces labelled by zero modes whose differentials do not appear in the line element of WCW.

All this sounds to me shockingly simple.

WCW spinor structure

One must construct also spinor structure for WCW [K127, K96].

1. The modes of WCW spinor fields would correspond to the solutions of WCW Dirac equation and would define the quantum states of the Universe. WCW spinors (assignable to given 3-surface) would correspond to fermionic Fock states created by fermionic creation operators. In ZEO 3-surfaces are pairs of 3-surfaces assignable to the opposite boundaries of WCW connected by preferred extremal.

   The fermionic states are superpositions of pairs of fermion states with opposite net quantum numbers at the opposite ends of space-time surface at boundaries of CD. The entanglement coefficients define the analogs of S-matrix elements. The analog of Dirac equation is analog for super-Virasoro conditions in string models but assignable to the infinite-D supersymplectic algebra of WCW defining its isometries.

2. The construction of the geometry of WCW requires that the anticommuting gamma matrices of WCW are expressible in terms of fermionic oscillator operators assignable to the induced spinor fields at space-time surface. Fermionic anti-commutativity at space-time level is not assumed but is forced by the anticommutativity of gamma matrices to metric. Fermi statistics is geometrized.

3. The gamma matrices of WCW in the coordinates assignable to isometry generators can be regarded as generators of superconformal symmetries. They correspond to classical charges assignable to the preferred extremals and to fermionic generators. The fermionic isometry generators are fermionic bilinears and super-generators are obtained from them by replacing the second second quantized spinor field with its mode. Quantum classical correspondence between fermionic dynamics and classical dynamics (SH) requires that the eigenvalues of the fermionic Cartan charges are equal to corresponding bosonic Noether charges.

4. The outcome is that quantum TGD reduces to a theory of formally *classical* spinor fields at the level of WCW and by infinite symmetries the construction of quantum states reduces to the construction of representations of super-symplectic algebra which generalizes to Yangian algebra as twistorial picture suggests. In ZEO everything would reduce to group theory, even the construction of scattering amplitudes! In ZEO the construction of zero energy states and thus scattering amplitudes would reduce to that for the representations of Yangian variant of super-symplectic algebra [A25] [B29, B22, B23].

5. One can go to the extreme and wonder whether the scattering amplitudes as entanglement coefficients for Yangian zero energy states are just constant scalars for given values of zero modes as group invariant for isometries. This would leave only integration over zero modes and if number theoretical universality is assumed this integral reduces to sum over points with algebraic coordinates in the preferred coordinates made possible by the symmetric space property. Certainly this is one of the lines of research to be followed in future.

   Personally I find it hard to imagine anything simpler!

6.2.2 Space-time level: many-sheeted space-time and emergence of classical fields and GRT space-time

At space-time level one must consider dynamics of space-time surface and spinorial dynamics.

Dynamics of space-time surfaces

Consider first simplicity at space-time level.

1. Space-time is identified as 4-D surface in certain embedding space required to have symmetries of special relativity - Poincare invariance. This resolves the energy problem and many other problems of GRT [K129].

   This allows also to see TGD as generalization of string models obtained by replacing strings with 3-surfaces and 2-D string world sheets with 4-D space-time surfaces. Small space-time surfaces are particles, large space-time surfaces the background space-time in which these particles “live”. There are only 4 dynamical field like variables for 8-D $M^4 \times CP_2$ since GCI eliminates 4 embedding space coordinates (they can be taken as space-tme coordinates). This should be compared with the myriads of classical fields for 10-D Einstein’s theory coupled to matter fields (do not forget landscape and multiverse!)

2. Classical fields are induced at the level of single space-time sheet from their geometric counterparts in embedding space. A more fashionable way to say the same is that they emerge. Classical gravitational field correspond to the induced metric, electroweak gauge potentials to induced spinor connection of CP_2 and color gauge potentials to projections of Killing vector fields for CP_2.

3. In TGD the space-time of GRT is replaced by many-sheeted space-time constructed from basic building bricks, which are preferred extremals of Kähler action + volume term. This action emerges in twistor lift of TGD existing only for $H = M^4 \times CP_2$: TGD is completely unique since only M^4 and CP_2 allows twistor space with Kähler structure. This also predicts Planck length as radius of twistor sphere associated with M^4. Cosmological constant appears as the coefficient of the volume term and obeys p-adic length scale evolution predicting automatically correct order of magnitude in the scale of recent cosmos. Besides this one has CP_2 size which is of same order of magnitude as GUT scale, and Kähler coupling strength. By quantum criticality the various parameters are quantized.

   Quantum criticality is basic dynamical principle [K57, K46] and discretizes coupling constant evolution: only coupling constants corresponding to quantum criticality are realized and discretized coupling constant evolution corresponds to phase transitions between these values of coupling constants. All radiative corrections vanish so that only tree diagram contribute.

4. Preferred extremals realize strong form of holography (SH) implied by strong form of GCI (SGCI) emerging naturally in TGD framework. That GCI implies SH meaning an enormous simplification at the conceptual level.

   One has two choices for fundamental 3-D objects. They could be light-like boundaries between regions of Minkowskian and Euclidian signatures of the induced metric or they could be pairs of space-time 3-surfaces at the ends of space-time surface at opposite boundaries of causal diamond (CD) (CDs for a scale hierarchy). Both options should be correct so that the intersections of these 3-surfaces consisting of partonic 2-surfaces at which light-like partonic orbits and space-like 3-surfaces intersect should carry the data making possible holography. Also data about normal space of partonic 2-surface is involved.

   SH generalizes AdS/CFT correspondence by replacing holography with what is very much like the familiar holography. String world, sheets, which are minimal surfaces carrying fermion fields and partonic 2-surfaces intersecting string world sheets at discrete points determine by SH the entire 4-D dynamics. The boundaries of string world sheets are world lines with fermion number coupling to classical Kähler force. In the interior Kähler force vanishes so that one has "dynamics of avoidance" [L27] required also by number theoretic universality satisfied if the coupling constants do not appear in the field equations at all: they are however seen in the boundary values stating vanishing of the classical super-symplectic charges (Noether's theorem) so that one obtains dependence of coupling constants via boundary conditions and coupling constant evolutions makes it manifest also classically. Hence the preferred extremals from which the space-time surfaces are engineered are extremely simple objects.

5. In twistor formulation the assumption that the inverse of Kähler coupling strength has zeros of Riemann zeta [L23] as the spectrum of its quantum critical values gives excellent prediction for the coupling constant of U(1) coupling constant of electroweak interactions. Complexity means that extremals are extremals of both Kähler action and volume term: minimal surfaces extremals of Kähler action. This would be part of preferred extremal property.

   Why α_K should be complex? If α_K is real, both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions decouple completely. This is not physically attractive. If α_K is complex there is coupling between the two regions and the simplest assumption is that there is no Chern-Simons term in the action and one has just continuity conditions for canonical momentum current and hits super counterpart. Note the analogy with the possibility of blackhole evaporation. The presence of momentum exchange is also natural since it gives classical space-time correlates for interactions as momentum exchange.

   The conditions state that sub-algebra of super-symplectic algebra isomorphic to itself and its commutator with the entire algebra annihilate the physical states (classical Noether charges vanish). The condition could follow from minimal surface extremality or provide additional conditions reducing the degrees of freedom. In any case, 3-surfaces would be almost 2-D objects.

6. GRT space-time emerges from many-sheeted space-time as one replaces the sheets of many-sheeted space-time (4-D M^4 projection) to single slightly curved region of M^4 defining GRT space-time. Since test particle regarded as 3-surface touching the space-time sheets of many-sheeted spacetime, test particle experiences the sum of forces associated with the classical fields at the space-time sheets. Hence the classical fields of GRT space-time are sums of these fields. Disjoint union for space-time sheets maps to the sum of the induced fields. This gives standard model and GRT as long range scale limit of TGD.

How to build TGD space-time from legos?

TGD predicts shocking simplicity of both quantal and classical dynamics at space-time level. Could one imagine a construction of more complex geometric objects from basic building bricks - space-time legos?

Let us list the basic ideas.

1. Physical objects correspond to space-time surfaces of finite size - we see directly the non-trivial topology of space-time in everyday length scales.

2. There is also a fractal scale hierarchy: 3-surfaces are topologically summed to larger surfaces by connecting them with wormhole contact, which can be also carry monopole magnetic flux in which one obtains particles as pairs of these: these contacts are stable and are ideal for nailing together pieces of the structure stably.

3. In long length scales in which space-time surface tend to have 4-D M^4 projection this gives rise to what I have called many-sheeted spacetime. Sheets are deformations of canonically imbedded M^4 extremely near to each other (the maximal distance is determined by CP_2 size scale about 10^4 Planck lengths. The sheets touch each other at topological sum contacts, which can be also identified as building bricks of elementary particles if they carry monopole flux and are thus stable. In $D = 2$ it is easy to visualize this hierarchy.

What could be the simplest surfaces of this kind - the legos?

1. Assume twistor lift [K46, L31] so that action contain volume term besides Kähler action: preferred extremals can be seen as non-linear massless fields coupling to self-gravitation. They also simultaneously extremals of Kähler action. Also hydrodynamical interpretation makes sense in the sense that field equations are conservation laws. What is remarkable is that the solutions have no dependence on coupling parameters: this is crucial for realizing number theoretical universality. Boundary conditions however bring in the dependence on the values of coupling parameters having discrete spectrum by quantum criticality.

2. The simplest solutions corresponds to Lagrangian sub-manifolds of CP_2: induced Kähler form vanishes identically and one has just minimal surfaces. The energy density defined by scale dependent cosmological constant is small in cosmological scales - so that only a template of physical system is in question. In shorter scales the situation changes if the cosmological constant is proportional the inverse of p-adic prime.

   The simplest minimal surfaces are constructed from pieces of geodesic manifolds for which not only the trace of second fundamental form but the form itself vanishes. Geodesic sub-manifolds correspond to points, pieces of lines, planes, and 3-D volumes in E^3. In CP_2 one has points, circles, geodesic spheres, and CP_2 itself.

3. CP_2 type extremals defining a model for wormhole contacts, which can be used to glue basic building bricks at different scales together stably: stability follows from magnetic monopole flux going through the throat so that it cannot be split like homologically trivial contact. Elementary particles are identified as pairs of wormhole contacts and would allow to nail the legos together to from stable structures.

Amazingly, what emerges is the elementary geometry. My apologies for those who hated school geometry.

1. Geodesic minimal surfaces with vanishing induced gauge fields

Consider first *static* objects with 1-D CP_2 projection having thus *vanishing* induced gauge fields. These objects are of form $M^1 \times X^3$, $X^3 \subset E^3 \times CP_2$. M^1 corresponds to time-like or possible light-like geodesic (for CP_2 type extremals). I will consider mostly Minkowskian space-time regions in the following.

1. Quite generally, the simplest legos consist of 3-D geodesic sub-manifolds of $E^3 \times CP_2$. For E^3 their dimensions are $D = 1, 2, 3$ and for CP_2, $D = 0, 1, 2$. CP_2 allows both homologically non-trivial resp. trivial geodesic sphere S^2_I resp. S^2_{II}. The geodesic sub-manifolds cen be products $G_3 = G_{D_1} \times G_{D_2}$, $D_2 = 3 - D_1$ of geodesic manifolds G_{D_1}, $D_1 = 1, 2, 3$ for E^3 and G_{D_2}, $D_2 = 0, 1, 2$ for CP_2.

2. It is also possible to have twisted geodesic sub-manifolds G_3 having geodesic circle S^1 as CP_2 projection corresponding to the geodesic lines of $S^1 \subset CP_2$, whose projections to E^3 and CP_2 are geodesic line and geodesic circle respectively. The geodesic is characterized by S^1 wave vector. One can have this kind of geodesic lines even in $M^1 \times E^3 \times S^1$ so that the solution is characterized also by frequency and is not static in CP_2 degrees of freedom anymore.

These parameters define a four-D wave vector characterizing the warping of the space-time surface: the space-time surface remains flat but is warped. This effect distinguishes TGD from GRT. For instance, warping in time direction reduces the effective light-velocity in the sense that the time used to travel from A to B increases. One cannot exclude the possibility that the observed freezing of light in condensed matter could have this warping as space-time correlate in TGD framework.

For instance, one can start from 3-D minimal surfaces $X^2 \times D$ as local structures (thin layer in E^3). One can perform twisting by replacing D with twisted closed geodesics in $D \times S^1$: this gives valued map from D to S^1 (subset CP_2) representing geodesic line of $D \times S^1$. This geodesic sub-manifold is trivially a minimal surface and defines a two-sheeted cover of $X^2 \times D$. Wormhole contact pairs (elementary particles) between the sheets can be used to stabilize this structure.

3. Structures of form $D^2 \times S^1$, where D^2 is polygon, are perhaps the simplest building bricks for more complex structures. There are continuity conditions at vertices and edges at which polygons D^2_i meet and one could think of assigning magnetic flux tubes with edes in the spirit of homology: edges as magnetic flux tubes, faces as 2-D geodesic sub-manifolds and interiors as 3-D geodesic sub-manifolds.

   Platonic solids as 2-D surfaces can be build are one example of this and are abundant in biology and molecular physics. An attractive idea is that molecular physics utilizes this kind of simple basic structures. Various lattices appearing in condensed matter physics represent more complex structures but could also have geodesic minimal 3-surfaces as building bricks. In cosmology the honeycomb structures having large voids as basic building bricks could serve as cosmic legos.

4. This lego construction very probably generalizes to cosmology, where Euclidian 3-space is replaced with 3-D hyperbolic space $SO(3,1)/SO(3)$. Also now one has pieces of lines, planes and 3-D volumes associated with an arbitrarily chosen point of hyperbolic space. Hyperbolic space allows infinite number of tessellations serving as analogs of 3-D lattices and the characteristic feature is quantization of redshift along line of sight for which empirical evidence is found.

5. The structures as such are still too simple to represent condensed matter systems. These basic building bricks can glued together by wormhole contact pairs defining elementary particles so that matter emerges as stabilizer of the geometry: they are the nails allowing to fix planks together, one might say.

2. Geodesic minimal surfaces with non-vanishing gauge fields

What about minimal surfaces and geodesic sub-manifolds carrying non-vanishing gauge fields - in particular em field (Kähler form identifiable as U(1) gauge field for weak hypercharge vanishes and thus also its contribution to em field)? Now one must use 2-D geodesic spheres of CP_2 combined with 1-D geodesic lines of E^2. Actually both homologically non-trivial resp. trivial geodesic spheres S^2_I resp. S^2_{II} can be used so that also non-vanishing Kähler forms are obtained.

The basic legos are now $D \times S^2_i$, $i = I, II$ and they can be combined with the basic legos constructed above. These legos correspond to two kinds of magnetic flux tubes in the ideal infinitely thin limit. There are good reasons to expected that these infinitely thin flux tubes can be thickened by deforming them in E^3 directions orthogonal to D. These structures could be used as basic building bricks assignable to the edges of the tensor networks in TGD.

3. Static minimal surfaces, which are not geodesic sub-manifolds

One can consider also more complex static basic building bricks by allowing bricks which are not anymore geodesic sub-manifolds. The simplest static minimal surfaces are form $M^1 \times X^2 \times S^1$, $S^1 \subset CP_2$ a geodesic line and X^2 minimal surface in E^3.

Could these structures represent higher level of self-organization emerging in living systems? Could the flexible network formed by living cells correspond to a structure involving more general minimal surfaces - also non-static ones - as basic building bricks? The Wikipedia article about minimal surfaces in E^3 suggests the role of minimal surface for instance in bio-chemistry (see `http://tinyurl.com/zqlv322`).

The surfaces with constant positive curvature do not allow embedding as minimal surfaces in E^3. Corals provide an example of surface consisting of pieces of 2-D hyperbolic space H^2 immersed in E^3 (see `http://tinyurl.com/ho9uvcc`. Minimal surfaces have negative curvature as also H^2 but minimal surface immersions of H^2 do not exist. Note that pieces of H^2 have natural embedding to E^3 realized as light-one proper time constant surface but this is not a solution to the problem.

Does this mean that the proposal fails?

1. One can build approximately spherical surfaces from pieces of planes. Platonic solids represents the basic example. This picture conforms with the notion of monadic manifold having as a spine a discrete set of points with coordinates in algebraic extension of rationals (preferred coordinates allowed by symmetries are in question). This seems to be the realistic option.

2. The boundaries of wormhole throats at which the signature of the induced metric changes can have arbitrarily large M^4 projection and they take the role of blackhole horizon. All physical systems have such horizon and the approximately boundaries assignable to physical objects could be horizons of this kind. In TGD one has minimal surface in $E^3 \times S^1$ rather than E^3. If 3-surface have no space-like boundaries they must be multi-sheeted and the sheets co-incide at some 2-D surface analogous to boundary. Could this 3-surface give rise to an approximately spherical boundary.

3. Could one lift the immersions of H^2 and S^2 to E^3 to minimal surfaces in $E^3 \times S^1$? The constancy of scalar curvature, which is for the immersions in question quadratic in the second fundamental form would pose one additional condition to non-linear Laplace equations expressing the minimal surface property. The analyticity of the minimal surface should make possible to check whether the hypothesis can make sense. Simple calculations lead to conditions, which very probably do not allow solution.

4. Dynamical minimal surfaces: how space-time manages to engineer itself?

At even higher level of self-organization emerge dynamical minimal surfaces. Here string world sheets as minimal surfaces represent basic example about a building block of type $X^2 \times S^2_i$. As a matter fact, S^2 can be replaced with complex sub-manifold of CP_2.

One can also ask about how to perform this building process. Also massless extremals (MEs) representing TGD view about topologically quantized classical radiation fields are minimal surfaces but now the induced Kähler form is non-vanishing. MEs can be also Lagrangian surfaces and seem to play fundamental role in morphogenesis and morphostasis as a generalization of Chladni mechanism [K115, L31]. One might say that they represent the tools to assign material and magnetic flux tube structures at the nodal surfaces of MEs. MEs are the tools of space-time engineering. Here many-sheetedness is essential for having the TGD counterparts of standing waves.

Spherically symmetry metric as minimal surface

Physical intuition and the experience with the vacuum extremals as models for GRT space-times suggests that Kähler charge is not important in the case of astrophysical objects like stars so that it might be possible to model them as minimal surfaces, which in the simplest situation have spherically symmetric metric analogous to Schwartschild solution. The vanishing of the induced Kähler form does not of course exclude the presence of electromagnetic fields. It must be of course emphasized that the assumption that single-sheeted space-time surface can model GRT-QFT limit based on many-sheeted space-time could be un-realistic.

At 90's I studied the embeddings of Schwartschild-Nordström solution as vacuum extremals of Kähler action and found that the solution is necessarily electromagnetically charged [K121]. This property is unavoidable. The embedding in coordinates (t, r, θ, ϕ) for X^4, (m^0, r, θ, ϕ) for M^4 and (Θ, Φ) for the trivial geodesic sphere S^2_{II} of CP_2 was not stationary as the first guess might be. m^0 relates to Schwartschild time and radial coordinate r by a shift $m^0 = \Lambda t + h(r)$. Without this shift the perihelion shift would be negligibly small.

One has $(cos(\Theta) = f(r), \Phi = \omega t + k(r))$. Also the dependence of Φ is not the first possibility to come in mind. The shifts $h(r)$ and $k(r)$ are such that the non-diagonal contribution g_{tr} to the induced metric vanishes. The question is whether one obtains spherically symmetric metric as a minimal surface.

5. General form of minimal surface equations

Consider first the minimal surface equations generally.

1. The field equations are analogous to massless wave equations for scalar fields defined by CP_2 coordinates having gravitational self coupling and also covariant derivative coupling due to the non-flatness of CP_2. One might therefore expect that the Newtonian gravitation based on Laplace equation in empty space-time regions follows as an approximation. Therefore also something analogous to Schwartschild metric is to be expected. Note that also massless extremals (MEs) are obtained as minimal surfaces so that also the topologically quantized counterparts of em and gravitational radiation emerge.

2. The general field equations can be written as vanishing of the covariant divergence for canonical momentum current $T^{k\alpha}$

$$\begin{aligned} D_\alpha(T^{k\alpha}\sqrt{g}) &= \partial_\alpha\left[T^{k\alpha}\sqrt{g}\right] + \left\{ \begin{matrix} k \\ \alpha\ m \end{matrix} \right\} T^{m\alpha}\sqrt{g} = 0 \ , \\ T^{k\alpha} &= g^{\alpha\beta}\partial_\beta h^k \ , \\ \left\{ \begin{matrix} k \\ \alpha\ m \end{matrix} \right\} &= \left\{ \begin{matrix} k \\ l\ m \end{matrix} \right\} \partial_\alpha h^l \ . \end{aligned} \tag{6.2.1}$$

D_α is covariant derivative taking into account that gradient $\partial_\alpha h^k$ is embedding space vector.

3. For isometry currents $j^{A,k}$ (Killing vector fields)

$$T^{A,\alpha} = T^{\alpha k} h_{kl} j^{A,l} \tag{6.2.2}$$

the covariant divergence simplifies to ordinary divergence

$$\partial_\alpha\left[T^{A,\alpha}\sqrt{g}\right] = 0 \ . \tag{6.2.3}$$

This allows to simplify the equations considerably.

6. Spherically symmetric stationary minimal surface

Consider now the spherically symmetric stationary metric representable as minimal surface.

1. In the following we consider only the region exterior to the surface defining the TGD counterpart of Schwartschild horizon and the possible horizon at which the signature of the induced metric. The first possibility is $g_{tt} = 0$ at horizon. If g_{rr} remains non-vanishing, the signature changes to Euclidian. If also $g_{rr} = 0$, both g_{tt} and g_{rr} can change sign so that one has a smooth variant of Schwartschild horizon.

   Second possibility is $g_{rr} = 0$ at radius r_E in the region below Schwartschild radius. At r_E the determinant of 4-metric would vanish and the signature of the induced metric would change to Euclidian.

2. The reduction to the conservation of isometry currents can be used for isometry current corresponding to the rotation $\Phi \rightarrow \Phi + \epsilon$ and time translation $m^0 \rightarrow m^0 + \epsilon$.

3. With the experience coming from the embedding of Reissner-Nordström metric the ansatz is exactly the same and can be written as

$$m^0 = \Lambda t + h(r) \ , \quad \Phi = \omega t + k(r) \ , \quad u \equiv cos(\Theta) = u(r) \ , \tag{6.2.4}$$

4. The condition $g_{tr} = 0$ gives

$$\Lambda \partial_r h = R^2 \omega sin^2(\Theta) \partial_r k = 0 \ . \tag{6.2.5}$$

   This allows to integrate $h(r)$ in terms of $k(r)$.

5. The interesting components of the induced metric are

$$g_{tt} = \Lambda^2 - R^2\omega^2 sin^2(\Theta) \ , \quad g_{rr} = -1 - R^2(\partial_r\Theta)^2 + \Lambda^2(\partial_r h)^2 \ . \tag{6.2.6}$$

6. The field equations reduce to conservation laws for various isometry currents. Consider energy current and the current related to the $SO(3) \subset SU(3)$ rotation acting on Φ as shift (call this current isospin current). The stationary character of the induced metric implies that the field equations reduce to the conservation of the radial current for energy current and isospin current. These two equations fix the solution together with diagonality condition. One obtains the following equations

$$\partial_r(\partial_r h \times g^{rr}\sqrt{g}) = 0 \ , \quad \partial_r(sin^2(\Theta)\partial_r k \times g^{rr}\sqrt{g}) = 0) \ . \tag{6.2.7}$$

   These two equations can be satisfied simultaneously only if one has

$$\partial_r h \times g^{rr} r^2 \sqrt{g_2} = A sin^2(\Theta)\partial_r k \times g^{rr} r^2 \sqrt{g_2} + B \ , \quad g_2 \equiv -g_{tt}g_{rr} \ . \tag{6.2.8}$$

   Note the presence of constant B.

   Second implication is

$$g^{rr}\partial_r h\sqrt{g_2} = \frac{C}{r^2} \ , \quad g^{rr} sin^2(\Theta)\partial_r k\sqrt{g_2} = \frac{D}{r^2} \ , \quad C = AD + B \ . \tag{6.2.9}$$

   By substituting the expressions for the metric one has

$$\partial_r h = \sqrt{-\frac{g_{rr}}{g_{tt}}} \times \frac{C}{r^2} \ , \quad sin^2(\Theta)\partial_r k = \sqrt{-\frac{g_{rr}}{g_{tt}}} \times \frac{D}{r^2} \ . \tag{6.2.10}$$

7. It is natural to look what one obtains in the approximation that the metric is flat expected to make sense at large distances. Putting $g_{tt} = -g_{rr} = 1$, one obtains

$$\partial_r h \simeq \frac{C}{r^2} \ , \quad sin^2(\Theta)\partial_r k \simeq \frac{D}{r^2} \quad . \tag{6.2.11}$$

The time component of the induced metric is given by

$$g_{tt} = \Lambda^2 - R^2\omega^2 sin^2(\Theta) \simeq \Lambda^2 - \frac{D}{r^2\partial_r k} \quad . \tag{6.2.12}$$

This gives $1/r$ gravitational potential of a mass point if one has $\partial_r k \simeq E/r$ giving for $\Lambda = 1$

$$g_{tt} = 1 - \frac{r_S}{r} \ , \quad r_S = 2GM = \frac{D}{E} \ . \tag{6.2.13}$$

with the identification $r_S = 2GM = D/E$ inspired by the behavior of the Scwartschild metric. It seems that one can take $\Lambda = 1$ without a loss of generality.

8. Using $g_{tr} = 0$ condition this gives for h the approximate expression

$$\partial_r h \simeq \frac{D}{r^2} \ , \quad D = \frac{R^2\omega^2}{\Lambda} \ . \tag{6.2.14}$$

so that the field equations are consistent with the $1/r$ behavior of gravitational potential. The solution carries necessarily a non-vanishing Abelian electroweak gauge field.

9. The asymptotic behaviors of k and h would be

$$k \simeq k_0 log(\frac{r}{r_0}) \ , \quad h \simeq h_0 - \frac{C}{r} \ . \tag{6.2.15}$$

7. Two horizons and layered structure as basic prediction

A very interesting question is whether $g_{tt} = 0$ defines Schwartschild type horizon at which the roles of the coordinates t and r change or whether one obtains horizon at which the signature of the induced metric becomes Euclidian. The most natural option turns out to be Schwartschild like horizon at which the roles of time and radial coordinate are changed and second inner horizon at which g_{rr} changes sign again so that the induced metric has Euclidian signature below this inner horizon.

1. Unless one has $g_{tt}g_{rr} = C \neq 0$ ($C = -1$ holds true in Scwhartschild-Nordström metric) the surface $g_{tt} = 0$ - if it exists - defines a light-like 3-surface identifiable as horizon at which the signature of the induced metric changes. The conditions $g_{tt} = 0$ gives

$$\Lambda^2 - R^2\omega^2(1 - u^2) = 0 \ . \tag{6.2.16}$$

giving

$$0 < sin^2(\Theta) = 1 - u^2 \quad = \quad \frac{\Lambda^2}{R^2\omega^2} < 1 \ . \tag{6.2.17}$$

For $\Lambda = 1$ this condition implies that ω is a frequency of order of the inverse of CP_2 radius R. Note that $g_{tt} = 0$ need mean change of the metric signature to Euclidian if the analog of Schwarschild horizon is in question.

2. $g_{tt} = 0$ surface is light-like surface if g_{rr} has non-vanishing and finite value at it. g_{rr} could diverges at this surface guaranteeing $g_{tt}g_{rr} > 0$. The quantities $\partial_r h$ and $sin^2(\Theta)\partial_r k$ are proportional to $\sqrt{g_{rr}/g_{tt}}$, which diverges for $g_{tt} = 0$ unless also g_{rr} vanishes so that also these derivatives would diverge. The behavior of g_{rr} at this surface is

$$g_{rr} = -1 - R^2 \frac{(\partial_r u)^2}{1-u^2} + \Lambda^2(\partial_r h)^2 \quad , \quad u \equiv cos(\Theta) \ . \tag{6.2.18}$$

There are several options to consider.

   (a) Option I: The divergence of $(\partial_r h)^2$ as cause for the divergence of g_{rr} is out of question. If this quantity increases for small values of r, g_{rr} can change sign for with finite value of $\partial_r h$ and $u^2 < 1$ at some larger radius r_S analogous to Schwartschild radius. Since it is impossible to have two time-like directions also the sign of g_{tt} must change so that one would have the analog of Schwartschild horizon at this radius - call it r_S: $r_S = 2GM$ need not hold true. The condition $g_{tt} = 0$ at this radius fixes the value of $sin^2(\Theta)$ at this radius

$$sin^2(\Theta_S) \quad = \quad \frac{\Lambda^2}{R^2\omega^2} \ . \tag{6.2.19}$$

   If g_{rr} has finite value and is continuous, the metric has Euclidian signature in interior. If g_{rr} is discontinuous and changes sign as in the case of Schwartschild metric, one has counterpart of Scwartschild horizon without infinities. This option will be called Option I.

   (b) Second possibility giving rise to would be that u becomes equal 1. This is not consistent with $sin^2(\Theta_S) = 0$.

   (c) Option II: Voth g_{tt} and g_{rr} change their sign and vanish at r_S. This however requires both radial and time-like direction become null directions locally. Space-time surface would become locally metrically 2-dimensional at the horizon. This would conform with the idea of strong form of holography (SH) but it is not possible to have two different light-like directions simultaneously unless these directions are actually same. Mathematically it is certainly possible to have surfaces for which the dimension is locally reduced from the maximal one but it is difficult to visualize what this kind of metric reduction of local space-time dimension could mean. This option will be considered in what follows.

   To sum up, g_{rr} changes sign at horizon. For Option I g_{rr} is finite and dis-continuous. For Option II g_{rr} vanishes and is continuous. Whether g_{rr} vanishes at horizon or not, remains open.

3. For Schwartschild-Nordström metric g_{rr} becomes infinite and changes sign at horizon. The change of the roles of g_{tt} and g_{rr} could for Option II take place smoothly so that both could become zero and change their sign at r_S. This would keep $\partial_r h$ and $sin^2(\Theta)\partial_r k$ finite. One would have the analog of the interior of Schwartschild metric.

   What happens at the smaller radii? The obvious constraint is that $sin^2(\Theta)$ remains below unity. If g_{rr}/g_{tt} remains bounded, the condition for $sin^2(\Theta)\partial_k$ however suggests that $sin^2(\Theta) = 1$ is eventually achieved. This is the case also for the embedding of Schwartschild metric. Could this horizon correspond to a surface at which the signature of the metric changes? g_{rr} should becomes zero in order to obtain light-like surface. g_{rr} contains indeed a term proportional to $1/sin^2(\Theta)$ which diverges at $u = 1$ so that g_{rr} must change sign for second time already above the radius for $sin^2(\Theta) = 1$ if h and k behaves smoothly enough. At this radius - call it r_E - g_{tt} would be finite and the signature would become Euclidian below this radius.

   One would therefore have two special radii r_S and r_E and a layer between these radii. $r_S = 2GM$ need not hold true but is expected to give a reasonable order of magnitude estimate.

Is there any empirical evidence for the existence of two horizons? There is evidence that the formation of the recently found LIGO blackhole (discussed from TGD view point in [L35]) is not fully consistent with the GRT based model (see `http://tinyurl.com/zbbz58w`). There are some indications that LIGO blackhole has a boundary layer such that the gravitational radiation is reflected forth and back between the inner and outer boundaries of the layer. In the proposed model the upper boundary would not be totally reflecting so that gravitational radiation leaks out and gave rise to echoes at times .1 sec, .2 sec, and .3 sec. It is perhaps worth of noticied that time scale .1 sec corresponds to the secondary p-adic time scale of electron (characterized by Mersenne prime $M_{127} = 2^{127} - 1$). If the minimal surface solution indeed has two horizons and a layer like structure between them, one might at least see the trouble of killing the idea that it could give rise to repeated reflections of gravitational radiation.

The proposed model (see `http://tinyurl.com/zbbz58w`) assumes that the inner horizon is Schwarstchild horizon. TGD would however suggests that the outer horizon is the TGD counterpart of Schwartschild horizon. It could have different radius since it would not be a singularity of g_{rr} (g_{tt}/g_{rr} would be finite at r_S which need not be $r_S = 2GM$ now). At r_S the tangent space of the space-time surface would become effectively 2-dimensional for $g_{rr} = 0$: the interpretation in terms of strong holography (SH) has been already mentioned.

The condition that the normal components of the canonical momentum currents for Kähler action and volume term are finite implies that $g^{nn}\sqrt{g_4}$ is finite at both sides of the horizon. Also the weak form of electric magnetic duality for Kähler form requires this. This condition can be satisfied if g_{tt} and g_{nn} approach to zero in the same manner at both sides of the horizon. Hence it seems that strong form of holography in the horizon is forced by finiteness.

One should understand why it takes rather long time $T = .1$ seconds for radiation to travel forth and back the distance $L = r_S - r_E$ between the horizons. The maximal signal velocity is reduced for the light-like geodesics of the space-time surface but the reduction should be rather large for $L \sim 20$ km (say). The effective light-velocity is measured by the coordinate time $\Delta t = \Delta m^0 + h(r_S) - h(r_E)$ needed to travel the distance from r_E to r_S. The Minkowski time Δm^0_{-+} would be the from null geodesic property and $m^0 = t + h(r)$

$$\Delta m^0_{-+} = \Delta t - h(r_S) + h(r_E) \quad , \quad \Delta t = \int_{r_E}^{r_S} \sqrt{\frac{g_{rr}}{g_{tt}}} dr \equiv \int_{r_E}^{r_S} \frac{dr}{c\#} \quad . \tag{6.2.20}$$

Note that $c\#$ approaches zero at horizon if g_{rr} is non-vanishing at horizon.

The time needed to travel forth and back does not depend on h and would be given by

$$\Delta m^0 = 2\Delta t = 2\int_{r_E}^{r_S} \frac{dr}{c\#} \quad . \tag{6.2.21}$$

This time cannot be shorter than the minimal time $(r_s - r_E)/c$ along light-like geodesic of M^4 since light-like geodesics at space-time surface are in general time-like curves in M^4. Since .1 sec corresponds to about 3×10^4 km, the average value of $c_\#$ should be for $L = 20$ km (just a rough guess) of order $c\# \sim 2^{-11}c$ in the interval $[r_E, r_S]$. As noticed, $T = .1$ sec is also the secondary p-adic time assignable to electron labelled by the Mersenne prime M_{127}. Since g_{rr} vanishes at r_E one has $c_\# \to \infty$. $c_\#$ is finite at r_S.

There is an intriguing connection with the notion of gravitational Planck constant. The formula for gravitational Planck constant given by $h_{gr} = GMm/v_0$ characterizing the magnetic bodies topologically for mass m topologically condensed at gravitational magnetic flux tube emanating from large mass M [K101, K83, **?**, K84]. The interpretation of the velocity parameter v_0 has remained open. Could v_0 correspond to the average value of $c_\#$? For the 4 inner planets one has $v_0 \simeq 2^{-11}$ so that the order of magnitude is same as for the estimate for $c_\#$.

Remark: More than year after after writing the above text I learned about additional evidence for blackhole echoes. Sabine Hossenfelder (see `http://tinyurl.com/ybd9gswm`) tells about the new evidence reported by Niayesh Afshordi, Professor of astrophysics at Perimeter Institute in the article "*Echoes from the Abyss: A highly spinning black hole remnant for the binary neutron star merger GW170817*" (see `https://arxiv.org/abs/1803.10454`). Now the earlier 2.5 sigma evidence has grown into 4.2 sigma evidence. 5 sigma is regarded as a criterion for discovery.

What about TGD inspired cosmology?

Before the discovery of the twistor lift TGD inspired cosmology has been based on the assumption that vacuum extremals provide a good estimate for the solutions of Einstein's equations at GRT limit of TGD [K121, K102] . One can find embeddings of Robertson-Walker type metrics as vacuum extremals and the general finding is that the cosmological with super-critical and critical mass density have finite duration after which the mass density becomes infinite: this period of course ends before this. The interpretation would be in terms of the emergence of new space-time sheet at which matter represented by smaller space-time sheets suffers topological condensation. The only parameter characterizing critical cosmologies is their duration. Critical (over-critical) cosmologies having $SO3\times E^3$ ($SO(4)$) as isometry group is the duration and the CP_2 projection at homologically trivial geodesic sphere S^2: the condition that the contribution from S^2 to g_{rr} component transforms hyperbolic 3-metric to that of E^3 or S^3 metric fixes these cosmologies almost completely. Sub-critical cosmologies have one-dimensional CP_2 projection.

Do Robertson-Walker cosmologies have minimal surface representatives? Recall that minimal surface equations read as

$$\begin{aligned}
D_\alpha(g^{\alpha\beta}\partial_\beta h^k\sqrt{g}) &= \partial_\alpha\left[g^{\alpha\beta}\partial_\beta h^k\sqrt{g}\right]+\left\{\begin{matrix}k\\ \alpha\ m\end{matrix}\right\}g^{\alpha\beta}\partial_\beta h^m\sqrt{g}=0\ ,\\
\left\{\begin{matrix}k\\ \alpha\ m\end{matrix}\right\} &= \left\{\begin{matrix}k\\ l\ m\end{matrix}\right\}\partial_\alpha h^l\ .
\end{aligned} \tag{6.2.22}$$

Sub-critical minimal surface cosmologies would correspond to $X^4\subset M^4\times S^1$. The natural coordinates are Robertson-Walker coordinates, which co-incide with light-cone coordinates ($a=\sqrt{(m^0)^2-r_M^2}, r=r_M/a,\theta,\phi$) for light-cone M^4_+. They are related to spherical Minkowski coordinates (m^0,r_M,θ,ϕ) by ($m^0=a\sqrt{1+r^2}, r_M=ar$). $\beta=r_M/m_0=r/\sqrt{1+r^2}$ corresponds to the velocity along the line from origin (0,0) to (m^0,r_M). r corresponds to the Lorentz factor $\gamma\beta=\beta/\sqrt{1-\beta^2}$. The metric of M^4_+ is given by the diagonal form $[g_{aa}=1, g_{rr}=a^2/(1+r^2), g_{\theta\theta}=a^2r^2, g_{\phi\phi}=a^2r^2sin^2(\theta)]$. One can use the coordinates of M^4_+ also for X^4.

The ansatz for the minimal surface reads is $\Phi=f(a)$. For $f(a)=constant$ one obtains just the flat M^4_+. In non-trivial case one has $g_{aa}=1-R^2(df/da)^2$. The g^{aa} component of the metric becomes now $g^{aa}=1/(1-R^2(df/da)^2)$. Metric determinant is scaled by $\sqrt{g_{aa}}=1\rightarrow\sqrt{1-R^2(df/da)^2}$. Otherwise the field equations are same as for M^4_+. Little calculation shows that they are not satisfied unless one as $g_{aa}=1$.

Also the minimal surface embeddings of critical and over-critical cosmologies are impossible. The reason is that the criticality alone fixes these cosmologies almost uniquely and this is too much for allowing minimal surface property.

Thus one can have only the trivial cosmology M^4_+ carrying dark energy density as a minimal surface solution! This obviously raises several questions.

1. Could $\Lambda=0$ case for which action reduces to Kähler action provide vacuum extremals provide single-sheeted model for Robertson-Walker cosmologies for the GRT limit of TGD for which many-sheeted space-time surface is replaced with a slightly curved region of M^4? Could $\Lambda=0$ correspond to a genuine phase present in TGD as formal generalization of the view of mathematicians about reals as $p=\infty$ p-adic number suggest. p-Adic length scale would be strictly infinite implying that $\Lambda\propto 1/p$ vanishes.

2. Second possibility is that TGD is quantum critical in strong sense. Not only 3-space but the entire space-time surface is flat and thus M^4_+. Only the local gravitational fields created by topologically condensed space-time surfaces would make it curved but would not cause smooth expansion. The expansion would take as quantum phase transitions reducing the value of $\Lambda\propto 1/p$ as p-adic prime p increases. p-Adic length scale hypothesis suggests that the preferred primes are near but below powers of 2 $p\simeq 2^k$ for some integers k. This led for years ago to a model for Expanding Earth [L65].

3. This picture would explain why individual astrophysical objects have not been observed to expand smoothly (except possibly in these phase transitions) but participate cosmic expansion only in the sense that the distance to other objects increase. The smaller space-time sheets glued to a given space-time sheet preserving their size would emanate from the tip of M^4_+ for given sheet.

4. RW cosmology should emerge in the idealization that the jerk-wise expansion by quantum phase transitions and reducing the value of Λ (by scalings of 2 by p-adic length scale hypothesis) can be approximated by a smooth cosmological expansion.

One should understand why Robertson-Walker cosmology is such a good approximation to this picture. Consider first cosmic redshift.

1. The cosmic recession velocity is defined from the redshift by Doppler formula.

$$z = \frac{1+\beta}{1-\beta} - 1 \simeq \beta = \frac{v}{c} \quad . \tag{6.2.23}$$

In TGD framework this should correspond to the velocity defined in terms of the coordinate r of the object.

Hubble law tells that the recession velocity is proportional to the proper distance D from the source. One has

$$v = HD \quad , \quad H = (\frac{da/dt}{a}) = \frac{1}{\sqrt{g_{aa}}a} \quad . \tag{6.2.24}$$

This brings in the dependence on the Robertson-Walker metric.

For M^4_+ one has $a = t$ and one would have $g_{aa} = 1$ and $H = 1/a$. The experimental fact is however that the value of H is larger for non-empty RW cosmologies having $g_{aa} < 1$. How to overcome this problem?

2. To understand this one must first understand the interpretation of gravitational redshift. In TGD framework the gravitational redshift is property of observer rather than source. The point is that the tangent space of the 3-surface assignable to the observer is related by a Lorent boost to that associated with the source. This implies that the four-momentum of radiation from the source is boosted by this same boost. Redshift would mean that the Lorentz boost reduces the momentum from the real one. Therefore redshift would be consistent with momentum conservation implied by Poincare symmetry.

g_{aa} for which a corresponds to the value of cosmic time for the observer should characterize the boost of observer relative to the source. The natural guess is that the boost is characterized by the value of g_{tt} in sufficiently large rest system assignable to observer with t is taken to be M^4 coordinate m^0. The value of g_{tt} fluctuates do to the presence of local gravitational fields. At the GRT limit g_{aa} would correspond to the average value of g_{tt}.

3. There is evidence that H is not same in short and long scales. This could be understood if the radiation arrives along different space-time sheets in these two situations.

4. If this picture is correct GRT description of cosmology is effective description taking into account the effect of local gravitation to the redshift, which without it would be just the M^4_+ redshift.

Einstein's equations for RW cosmology [K121, K102] should approximately code for the cosmic time dependence of mass density at given slightly deformed piece of M^4_+ representing particular sub-cosmology expanding in jerkwise manner.

1. Many-sheeted space-time implies a hierarchy of cosmologies in different p-adic length scales and with cosmological constant $\Lambda \propto 1/p$ so that vacuum energy density is smaller in long scale cosmologies and behaves on the average as $1/a^2$ where a characterizes the scale of the cosmology. In zero energy ontology given scale corresponds to causal diamond (CD) with size characterized by a defining the size scale for the distance between the tips of CD.

2. For the comoving volume with constant value of coordinate radius r the radius of the volume increases as a. The vacuum energy would increase as a^3 for comoving volume. This is in sharp conflict with the fact that the mass decreases as $1/a$ for radiation dominated cosmology, is constant for matter dominated cosmology, and is proportional to a for string dominated cosmology.

   The physical resolution of the problem is rather obvious. Space-time sheets representing topologically condensed matter have finite size. They do not expand except possibly in jerkwise manner but in this process Λ is reduced - in average manner like $1/a^2$.

   If the sheets are smaller than the cosmological space-time sheet in the scale considered and do not lose energy by radiation they represent matter dominated cosmology emanating from the vertex of M^4_+. The mass of the co-moving volume remains constant.

   If they are radiation dominated and in thermal equilibrium they lose energy by radiation and the energy of volume behaves like $1/a$.

   Cosmic strings and magnetic flux tubes have size larger than that the space-time sheet representing the cosmology. The string as linear structure has energy proportional to a for fixed value of Λ as in string dominated cosmology. The reduction of Λ decreasing on the average like $1/a^2$ implies that the contribution of given string is reduced like $1/a$ on the average as in radiation dominated cosmology.

3. GRT limit would code for these behaviours of mass density and pressure identified as scalars in GRT cosmology in terms of Einstein's equations. The time dependence of g_{aa} would code for the density of the topologically condensed matter and its pressure and for dark energy at given level of hierarchy. The vanishing of covariant divergence for energy momentum tensor would be a remnant of Poincare invariance and give Einstein's equations with cosmological term.

4. Why GRT limit would involve only the RW cosmologies allowing embedding as vacuum extremals of Kähler action? Can one demand continuity in the sense that TGD cosmology at $p \to \infty$ limit corresponds to GRT cosmology with cosmological solutions identifiable as vacuum extremals? If this is assumed the earlier results are obtained. In particular, one obtains the critical cosmology with 2-D CP_2 projection assumed to provide a GRT model for quantum phase transitions changing the value of Λ.

If this picture is correct, TGD inspired cosmology at the level of many-sheeted space-time would be extremely simple. The new element would be many-sheetedness which would lead to more complex description provided by GRT limit. This limit would however lose the information about many-sheetedness and lead to anomalies such as two Hubble constants.

Induced spinor structure

The notion of induced spinor field deserves a more detailed discussion. Consider first induced spinor structures [K127].

1. Induced spinor field are spinors of $M^4 \times CP_2$ for which modes are characterized by chirality (quark or lepton like) and em charge and weak isospin.

2. Induced spinor spinor structure involves the projection of gamma matrices defining induced gamma matrices. This gives rise to superconformal symmetry if the action contains only volume term.

   When Kähler action is present, superconformal symmetry requires that the modified gamma matrices are contractions of canonical momentum currents with embedding space gamma matrices. Modified gammas appear in the modified Dirac equation and action, whose solution at string world sheets trivializes by super-conformal invariance to same procedure as in the case of string models.

3. Induced spinor fields correspond to two chiralities carrying quark number and lepton number. Quark chirality does not carry color as spin-like quantum number but it corresponds to a color partial wave in CP_2 degrees of freedom: color is analogous to angular momentum. This reduces to spinor harmonics of CP_2 describing the ground states of the representations of super-symplectic algebra.

   The harmonics do not satisfy correct correlation between color and electroweak quantum numbers although the triality t=0 for leptonic waves and t=1 for quark waves. There are two ways to solve the problem.

   (a) Super-symplectic generators applied to the ground state to get vanishing ground states weight instead of the tachyonic one carry color and would give for the physical states correct correlation: leptons/quarks correspond to the same triality zero(one partial wave irrespective of charge state. This option is assumed in p-adic mass calculations [K65].

   (b) Since in TGD elementary particles correspond to pairs of wormhole contacts with weak isospin vanishing for the entire pair, one must have pair of left and right-handed neutrinos at the second wormhole throat. It is possible that the anomalous color quantum numbers for the entire state vanish and one obtains the experimental correlation between color and weak quantum numbers. This option is less plausible since the cancellation of anomalous color is not local as assume in p-adic mass calculations.

The understanding of the details of the fermionic and actually also geometric dynamics has taken a long time. Super-conformal symmetry assigning to the geometric action of an object with given dimension an analog of Dirac action allows however to fix the dynamics uniquely and there is indeed dimensional hierarchy resembling brane hierarchy.

1. The basic observation was following. The condition that the spinor modes have well-defined em charge implies that they are localized to 2-D string world sheets with vanishing W boson gauge fields which would mix different charge states. At string boundaries classical induced W boson gauge potentials guarantee this. Super-conformal symmetry requires that this 2-surface gives rise to 2-D action which is area term plus topological term defined by the flux of Kähler form.

2. The most plausible assumption is that induced spinor fields have also interior component but that the contribution from these 2-surfaces gives additional delta function like contribution: this would be analogous to the situation for branes. Fermionic action would be accompanied by an area term by supersymmetry fixing modified Dirac action completely once the bosonic actions for geometric object is known. This is nothing but super-conformal symmetry.

   One would actually have the analog of brane-hierarchy consisting of surfaces with dimension D= 4,3,2,1 carrying induced spinor fields which can be regarded as independent dynamical variables and characterized by geometric action which is D-dimensional analog of the action for Kähler charged point particle. This fermionic hierarchy would accompany the hierarchy of geometric objects with these dimensions and the modified Dirac action would be uniquely determined by the corresponding geometric action principle (Kähler charged point like particle, string world sheet with area term plus Kähler flux, light-like 3-surface with Chern-Simons term, 4-D space-time surface with Kähler action).

3. This hierarchy of dynamics is consistent with SH only if the dynamics for higher dimensional objects is induced from that for lower dimensional objects - string world sheets or maybe even their boundaries orbits of point like fermions. Number theoretic vision [K125] suggests that this induction relies algebraic continuation for preferred extremals. Note that quaternion analyticity [K46] means that quaternion analytic function is determined by its values at 1-D curves.

4. Quantum-classical correspondences (QCI) requires that the classical Noether charges are equal to the eigenvalues of the fermionic charges for surfaces of dimension $D = 0, 1, 2, 3$ at the ends of the CDs. These charges would not be separately conserved. Charges could flow between objects of dimension $D+1$ and D - from interior to boundary and vice versa. Four-momenta and also other charges would be complex as in twistor approach: could complex values relate somehow to the finite life-time of the state?

   If quantum theory is square root of thermodynamics as zero energy ontology suggests, the idea that particle state would carry information also about its life-time or the time scale of CD to which is associated could make sense. For complex values of α_K there would be also flow of canonical and super-canonical momentum currents between Euclidian and Minkowskian regions crucial for understand gravitational interaction as momentum exchange at embedding space level.

5. What could be the physical interpretation of the bosonic and fermionic charges associated with objects of given dimension? Condensed matter physicists assign routinely physical states to objects of various dimensions: is this assignment much more than a practical approximation or could condensed matter physics already be probing many-sheeted physics?

SUSY and TGD

From this one ends up to the possibility of identifying the counterpart of SUSY in TGD framework [K100].

1. In TGD the generalization of much larger super-conformal symmetry emerges from the super-symplectic symmetries of WCW. The mathematically questionable notion of super-space is not needed: only the realization of super-algebra in terms of WCW gamma matrices defining super-symplectic generators is necessary to construct quantum states. As a matter of fact, also in QFT approach one could use only the Clifford algebra structure for super-multiplets. No Majorana condition on fermions is needed as for $\mathcal{N} = 1$ space-time SUSY and one avoids problems with fermion number non-conservation.

2. In TGD the construction of sparticles means quite concretely adding fermions to the state. In QFT it corresponds to transformation of states of integer and half-odd integer spin to each other. This difference comes from the fact that in TGD particles are replaced with point like particles.

3. The analog of $\mathcal{N} = 2$ space-time SUSY could be generated by covariantly constant right handed neutrino and antineutrino. Quite generally the mixing of fermionic chiralities implied by the mixing of M^4 and CP_2 gamma matrices implies SUSY breaking at the level of particle masses (particles are massless in 8-D sense). This breaking is purely geometrical unlike the analog of Higgs mechanism proposed in standard SUSY.

There are several options to consider.

1. The analog of brane hierarchy is realized also in TGD. Geometric action has parts assignable to 4-surface, 3-D light like regions between Minkowskian and Euclidian regions, 2-D string world sheets, and their 1-D boundaries. They are fixed uniquely. Also their fermionic counterparts - analogs of Dirac action - are fixed by super-conformal symmetry. Elementary particles reduce so composites consisting of point-like fermions at boundaries of wormhole throats of a pair of wormhole contacts.

   This forces to consider 3 kinds of SUSYs! The SUSYs associated with string world sheets and space-time interiors would certainly be broken since there is a mixing between M^4 chiralities in the modified Dirac action. The mass scale of the broken SUSY would correspond to the length scale of these geometric objects and one might argue that the decoupling between the degrees of freedom considered occurs at high energies and explains why no evidence for SUSY has been observed at LHC. Also the fact that the addition of massive fermions at these dimensions can be interpreted differently. 3-D light-like 3-surfaces could be however an exception.

2. For 3-D light-like surfaces the modified Dirac action associated with the Chern-Simons term does not mix M^4 chiralities (signature of massivation) at all since modified gamma matrices have only CP_2 part in this case. All fermions can have well-defined chirality. Even more: the modified gamma matrices have no M^4 part in this case so that these modes carry no four-momentum - only electroweak quantum numbers and spin. Obviously, the excitation of these fermionic modes would be an ideal manner to create spartners of ordinary particles consting of fermion at the fermion lines. SUSY would be present if the spin of these excitations couples - to various interactions and would be exact in absence of coupling to interior spinor fields.

   What would be these excitations? Chern-Simons action and its fermionic counterpart are non-vanishing only if the CP_2 projection is 3-D so that one can use CP_2 coordinates. This strongly suggests that the modified Dirac equation demands that the spinor modes are co-variantly constant and correspond to covariantly constant right-handed neutrino providing only spin.

   If the spin of the right-handed neutrino adds to the spin of the particle and the net spin couples to dynamics, $\mathcal{N}=2$ SUSY is in question. One would have just action with unbroken SUSY at QFT limit? But why also right-handed neutrino spin would couple to dynamics if only CP_2 gamma matrices appear in Chern-Simons-Dirac action? It would seem that it is independent degree of freedom having no electroweak and color nor even gravitational couplings by its covariant constancy. I have ended up with just the same SUSY-or-no-SUSY that I have had earlier.

3. Can the geometric action for light-like 3-surfaces contain Chern-Simons term?

   (a) Since the volume term vanishes identically in this case, one could indeed argue that also the counterpart of Kähler action is excluded. Moreover, for so called massless extremals of Kähler action reduces to Chern-Simons terms in Minkowskian regions and this could happen quite generally: TGD with only Kähler action would be almost topological QFT as I have proposed. Volume term however changes the situation via the cosmological constant. Kähler-Dirac action in the interior does not reduce to its Chern-Simons analog at light-like 3-surface.

   (b) The problem is that the Chern-Simons term at the two sides of the light-like 3-surface differs by factor $\sqrt{-1}$ coming from the ratio of $\sqrt{g_4}$ factors which themselves approach to zero: oOne would have the analog of dipole layer. This strongly suggests that one should not include Chern-Simons term at all.

   Suppose however that Chern-Simons terms are present at the two sides and α_K is real so that nothing goes through the horizon forming the analog of dipole layer. Both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions would decouple completely but currents would flow to the analog of dipole layer. This is not physically attractive.

   The canonical momentum current and its super counterpart would give fermionic source term $\Gamma^n\Psi_{int,\pm}$ in the modified Dirac equation defined by Chern-Simons term at given side $\pm$: $\pm$ refers to Minkowskian/Euclidian part of the interior. The source term is proportional to $\Gamma^n\Psi_{int,\pm}$ and Γ^n is in principle mixture of M^4 and CP_2 gamma matrices and therefore induces mixing of M^4 chiralities and therefore also 3-D SUSY breaking. It must be however emphasized that Γ^n is singular and one must be consider the limit carefully also in the case that one has only continuity conditions. The limit is not completely understood.

   (c) If α_K is complex there is coupling between the two regions and the simplest assumption has been that there is no Chern-Simons term as action and one has just continuity conditions for canonical momentum current and hits super counterpart.

   The cautious conclusion is that 3-D Chern-Simons term and its fermionic counterpart are absent.

4. What about the addition of fermions at string world sheets and interior of space-time surface ($D=2$ and $D=4$). For instance, in the case of hadrons $D=2$ excitations could correspond

to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. Let us consider the interior ($D = 4$). For instance, inn the case of hadrons $D = 2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. The smallness of cosmological constant implies that the contribution to the four-momentum from interior should be rather small so that an interpretation in terms of broken SUSY might make sense. There would be mass $m \sim .03$ eV per volume with size defined by the Compton scale $\hbar/m$. Note however that cosmological constant has spectrum coming as inverse powers of prime so that also higher mass scales are possible.

This interpretation might allow to understand the failure to find SUSY at LHC. Sparticles could be obtained by adding interior right-handed neutrinos and antineutrinos to the particle state. They could be also associated with the magnetic body of the particle. Since they do not have color and weak interactions, SUSY is not badly broken. If the mass difference between particle and sparticle is of order $m = .03$ eV characterizing dark energy density ρ_{vac}, particle and sparticle could not be distinguished in higher energy physics at LHC since it probes much shorter scales and sees only the particle. I have already earlier proposed a variant of this mechanism but without SUSY breaking.

To discover SUSY one should do very low energy physics in the energy range $m \sim .03$ eV having same order of magnitude as thermal energy $kT = 2.6 \times 10^{-2}$ eV at room temperature 25 °C. One should be able to demonstrate experimentally the existence of sparticle with mass differing by about $m \sim .03$ eV from the mass of the particle (one cannot exclude higher mass scales since Λ is expected to have spectrum). An interesting question is whether the sparticles associated with standard fermions could give rise to Bose-Einstein condensates whose existence in the length scale of large neutron is strongly suggested by TGD view about living matter.

6.2.3 Embedding space level

In GRT the description of gravitation involve only space-time and gravitational force is eliminated. In TGD also embedding space level is involved with the description [K46].

1. The incoming and outgoing states of particle reaction are labelled by the quantum numbers associated with the isometries of the embedding space and by the contributions of super-symplectic generators and isometry generators to the quantum numbers. This follows from the fact that the ground states of super-symplectic representations correspond to the modes of embedding space spinors fields. These quantum numbers appear in the S-matrix of QFT limit too. In particular, color quantum numbers as angular momentum like quantum numbers at fundamental level are transformed to spin-like quantum numbers at QFT limit.

2. In GRT the applications rely on Post-Newtonian approximation (PNA). This means that the notion of gravitational force is brought to the theory although it has been eliminated from the basic GRT. This is not simple. One could argue that there is genuine physics behind this PNA and TGD suggests what this physics is.

   At the level of space-time surfaces particles move along geodesic lines and in TGD minimal surface equation states the generalization of the geodesic line property for 3-D particles. At the embedding space level gravitational interaction involves exchanges of four-momentum and in principle of color quantum numbers too. Indeed, there is an exchange of classical charges through the light-like 3-surfaces defining the boundaries of Euclidian regions defining Euclidian regions as “lines” of generalized scattering diagrams. This however requires that Kähler coupling strength is allowed to be complex (say correspond to zero of Riemann Zeta). Hence in TGD also Newtonian view would be correct and needed.

6.3 Some questions about TGD

In Face Book I was made a question about general aspects of TGD. It was impossible to answer the question with few lines and I decided to write a blog posting, which then gave rise to this section.

This text talks from different perspective about same topics as the article *Can one apply Occam's razor as a general purpose debunking argument to TGD?* [L29] trying o emphasize the simplicity of the basic principles of TGD and of the resulting theory.

6.3.1 In what aspects TGD extends other theory/theories of physics?

I will replace "extends" with "modifies" since TGD also simplifies in many respects. I shall restrict the considerations to the ontological level which to my view is the really important level.

1. Space-time level is where TGD started from. Space-time as an abstract 4-geometry is replaced as space-time as 4-surface in $M^4 \times CP_2$. In GRT space-time is small deformation of Minkowski space.

   In TGD both Relativity Principle (RP) of Special Relativity (SRT) and General Coordinate Invariance (GCI) and Equivalence Principle (EP) of General Relativity hold true. In GRT RP is given up and leads to the loss of conservation laws since Noether theorem cannot be applied anymore: this is what led to the idea about space-time as surface in H. Strong form of holography (SH) is a further principle reducing to strong form of GCI (SGCI).

2. TGD as a physical theory extends to a theory of consciousness and cognition. Observer as something external to the Universe becomes part of physical system - the notion of self - and quantum measurement theory which is the black sheet of quantum theory extends to a theory of consciousness and also of cognition relying of p-adic physics as correlate for cognition. Also quantum biology becomes part of fundamental physics and consciousness and life are seen as basic elements of physical existence rather than something limited to brain.

   One important aspect is a new view about time: experienced time and geometric time are not one and same thing anymore although closely related. ZEO explains how the experienced flow and its direction emerges. The prediction is that both arrows of time are possible and that this plays central role in living matter.

3. p-Adic physics is a new element and an excellent candidate for a correlate of cognition. For instance, imagination could be understood in terms of non-determinism of p-adic partial differential equations for p-adic variants of space-time surfaces. p-Adic physics and fusion of real and various p-adic physics to adelic physics provides fusion of physics of matter with that of cognition in TGD inspired theory of cognition. This means a dramatic extension of ordinary physics. Number Theoretical Universality states that in certain sense various p-adic physics and real physics can be seen as extensions of physics based on algebraic extensions of rationals (and also those generated by roots of e inducing finite-D extensions of p-adics).

4. Zero energy ontology (ZEO) in which so called causal diamonds (CDs, analogs Penrose diagrams) can be seen as being forced by very simple condition: the volume action forced by twistor lift of TGD must be finite. CD would represent the perceptive field defined by finite volume of embedding space $H = M^4 \times CP_2$.

   ZEO implies that conservation laws formulated only in the scale of given CD do not anymore fix select just single solution of field equations as in classical theory. Theories are strictly speaking impossible to test in the old classical ontology. In ZEO testing is possible be sequence of state function reductions giving information about zero energy states.

   In principle transition between any two zero energy states - analogous to events specified by the initial and final states of event - is in principle possible but Negentropy Maximization Principle (NMP) as basic variational principle of state function reduction and of consciousness restricts the possibilities by forcing generation of negentropy: the notion of negentropy requires p-adic physics.

   Zero energy states are quantum superpositions of classical time evolutions for 3-surfaces and classical physics becomes exact part of quantum physics: in QFTs this is only the outcome of stationary phase approximation. Path integral is replaced with well-defined functional integral- not over all possible space-time surface but pairs of 3-surfaces at the ends of space-time at opposite boundaries of CD.

ZEO leads to a theory of consciousness as quantum measurement theory in which observer ceases to be outsider to the physical world. One also gets rid of the basic problem caused by the conflict of the non-determinism of state function reduction with the determinism of the unitary evolution. This is obviously an extension of ordinary physics.

5. Hierarchy of Planck constants represents also an extension of quantum mechanics at QFT limit. At fundamental level one actually has the standard value of h but at QFT limit one has effective Planck constant $h_{eff}/h = n$, $n = 1, 2, ...$ This generalizes quantum theory. This scaling of h has a simple topological interpretation: space-time surface becomes n-fold covering of itself and the action becomes n-multiple of the original which can be interpreted as $h_{eff}/h = n$.

   The most important applications are to biology, where quantum coherence could be understood in terms of a large value of h_{eff}/h. The large n phases resembles the large N limit of gauge theories with gauge couplings behaving as $\alpha \propto 1/N$ used as a kind of mathematical trick. Also gravitation is involved: h_{eff} is associated with the flux tubes mediating various interactions (being analogs to wormholes in ER-EPR correspondence). In particular, one can speak about h_{gr}, which Nottale introduced originally and $h_{eff} = h_{gr}$ plays key role in quantum biology according to TGD.

6.3.2 In what sense TGD is simplification/extension of existing theory?

1. Classical level: Space-time as 4-surface of H means a huge reduction in degrees of freedom. There are only 4 field like variables - suitably chosen 4 coordinates of $H = M^4 \times CP_2$. All classical gauge fields and gravitational field are fixed by the surface dynamics. There are no primary gauge fields or gravitational fields nor any other fields in TGD Universe and they appear only at the QFT limit [K15, K9, L31].

   GRT limit would mean that many-sheeted space-time is replaced by single slightly curved region of M^4. The test particle - small particle like 3-surface - touching the sheets simultaneously experience sum of gravitational forces and gauge forces. It is natural to assume that this superposition corresponds at QFT limit to the sum for the deviations of induced metrics of space-time sheets from flat metric and sum of induce gauge potentials. These would define the fields in standard model + GRT. At fundamental level effects rather than fields would superpose. This is absolutely essential for the possibility of reducing huge number field like degrees of freedom. One can obviously speak of emergence of various fields.

   A further simplification is that only preferred extremals for which data coding for them are reduced by SH to 2-D string like world sheets and partonic 2-surfaces are allowed. TGD is almost like string model but space-time surfaces are necessary for understanding the fact that experiments must be analyzed using classical 4-D physics. Things are extremely simple at the level of single space-time sheet.

   Complexity emerges from many-sheetedness. From these simple basic building bricks - minimal surface extremals of Kähler action (not the extremal property with respect to Kähler action and volume term strongly suggested by the number theoretical vision plus analogs of Super Virasoro conditions in initial data) - one can engineer space-time surfaces with arbitrarily complex topology - in all length scales. An extension of existing space-time concept emerges. Extremely simple locally, extremely complex globally with topological information added to the Maxwellian notion of fields (topological field quantization allowing to talk about field identify of system/field body/magnetic body.

   Another new element is the possibility of space-time regions with Euclidian signature of the induced metric. These regions correspond to 4-D “lines” of general scattering diagrams. Scattering diagrams has interpretation in terms of space-time geometry and topology.

2. The construction of quantum TGD using canonical quantization or path integral formalism failed completely for Kähler action by its huge vacuum degeneracy. The presence of volume term still suffers from complete failure of perturbation theory and extreme non-linearity. This led to the notion of world of classical worlds (WCW) - roughly the space of 3-surfaces.

Essentially pairs of 3-surfaces at the boundaries of given CD connected by preferred extremals of action realizing SH and SGCI.

The key principle is geometrization of the entire quantum theory, not only of classical fields geometrized by space-time as surface vision. This requires geometrization of hermitian conjugation and representation of imaginary unit geometrically. Kähler geometry for WCW [K57, K29, K96] makes this possible and is fixed once Kähler function defining Kähler metric is known. Kähler action for a preferred extremal of Kähler action defining space-time surface as an analog of Bohr orbit was the first guess but twistor lift forced to add volume term having interpretation in terms of cosmological constant.

Already the geometrization of loop spaces demonstrated that the geometry - if it exists - must have maximal symmetries (isometries). There are excellent reasons to expect that this is true also in $D = 3$. Physics would be unique from its mathematical existence!

3. WCW has also spinor structure [K127, K96]. WCW spinors correspond to fermionic Fock states using oscillator operators assignable to the induced spinor fields - free spinor fiels. WCW gamma matrices are linear combinations of these oscillator operators and Fermi statistics reduces to spinor geometry.

4. There is **no quantization** in TGD framework at the level of WCW [K28, K46]. The construction of quantum states and S-matrix reduces to group theory by the huge symmetries of WCW. Therefore zero energy states of Universe (or CD) correspond formally to **classical** WCW spinor fields satisfying WCW Dirac equation analogous to Super Virasoro conditions and defining representations for the Yangian generalization of the isometries of WCW (so called super-symplectic group assignable to $\delta M^4_+ \times CP_2$. In ZEO stated are analogous to pairs of initial and final states and the entanglement coefficients between positive and negative energy parts of zero energy states expected to be fixed by Yangian symmetry define scattering matrix and have purely group theoretic interpretation. If this is true, entire dynamics would reduce to group theory in ZEO.

6.3.3 What is the hypothetical applicability of the extension - in energies, sizes, masses etc?

TGD is a unified theory and is meant to apply in all scales. Usually the unifications rely on reductionistic philosophy and try to reduce physics to Planck scale. Also super string models tried this and failed: what happens at long length scales was completely unpredictable (landscape catastrophe).

Many-sheeted space-time however forces to adopt fractal view. Universe would be analogous to Mandelbrot fractal down to CP_2 scale. This predicts scaled variants of say hadron physics and electroweak physics. p-Adic length scale hypothesis and hierarchy of phases of matter with $h_{eff}/h = n$ interpreted as dark matter gives a quantitative realization of this view.

1. p-Adic physics shows itself also at the level of real physics [K76]. One ends up to the vision that particle mass squared has thermal origin: the p-adic variant of particle mass square is given as thermal mass squared given by p-adic thermodynamics mappable to real mass squared by what I call canonical identification. p-Adic length scale hypothesis states that preferred p-adic primes characterizing elementary particles correspond to primes near to power of 2: $p \simeq 2^k$. p-Adic length scale is proportional to $p^{1/2}$.

   This hypothesis is testable and it turns out that one can predict particle mass rather accurately. This is highly non-trivial since the sensitivity to the integer k is exponential. So called Mersenne primes turn out to be especially favoured. This part of theory was originally inspired by the regularities of particle mass spectrum. I have developed arguments for why the crucial p-adic length scale hypothesis - actually its generalization - should hold true. A possible interpretation is that particles provide cognitive representations of themselves by p-adic thermodynamics.

2. p-Adic length scale hypothesis leads also to consider the idea that particles could appear as different p-adically scaled up variants. For instance, ordinary hadrons to which one can

assign Mersenne prime $M_{107} = 2^{107} - 1$ could have fractally scaled variants. M_{89} and $M_{G,107}$ (Gaussian prime) would be two examples and there are indications at LHC for these scaled up variants of hadron physics [K71, K72]. These fractal copies of hadron physics and also of electroweak physics would correspond to extension of standard model.

3. Dark matter hierarchy predicts zoomed up copies of various particles. The simplest assumption is that masses are not changed in the zooming up. One can however consider that binding energy scale scales non-trivially. The dark phases would emerge are quantum criticality and give rise to the associated long range correlations (quantum lengths are typically scaled up by $h_{eff}/h = n$).

6.3.4 What is the leading correction/contribution to physical effects due to TGD onto particles, interactions, gravitation, cosmology?

1. Concerning particles I already mentioned the key predictions.

   (a) The existence of scaled variants of various particles and entire branches of physics. The fundamental quantum numbers are just standard model quantum numbers code by CP_2 geometry.

   (b) Particle families have topological description meaning that space-time topology would be an essential element of particle physics [K26]. The genus of partonic 2-surfaces (number of handles attached to sphere) is $g = 0, 1, 2, ...$ and would give rise to family replication. $g < 2$ partonic 2-surfaces have always global conformal symmetry Z_2 and this suggests that they give rise to elementary particles identifiable as bound states of g handles. For $g > 2$ this symmetry is absent in the generic case which suggests that they can be regarded as many-handle states with mass continuum rather than elementary particles. 2-D anyonic systems could represent an example of this.

   (c) A hierarchy of dynamical symmetries as remnants of super-symplectic symmetry however suggests itself [K28, K96]. The super-symplectic algebra possess infinite hierarchy of isomorphic sub-algebras with conformal weights being n-multiples of for those for the full algebra (fractal structure again possess also by ordinary conformal algebras). The hypothesis is that sub-algebra specified by n and its commutator with full algebra annihilate physical states and that corresponding classical Noether charges vanish. This would imply that super-symplectic algebra reduces to finite-D Kac-Moody algebra acting as dynamical symmetries. The connection with ADE hierarchy of Kac-Moody algebras suggests itself. This would predict new physics. Condensed matter physics comes in mind.

   (d) Number theoretic vision suggests that Galois groups for the algebraic extensions of rationals act as dynamical symmetry groups. They would act on algebraic discretizations of 3-surfaces and space-time surfaces necessary to realize number theoretical universality. This would be completely new physics.

2. Interactions would be mediated at QFT limit by standard model gauge fields and gravitons. QFT limit however loses all information about many-sheetedness and there would be anomalies reflecting this information loss. In many-sheeted space-time light can propagate along several paths and the time taken to travel along light-like geodesic from A to B depends on space-time sheet since the sheet is curved and warped. Neutrinos and gamma rays from SN1987A arriving at different times would represent a possible example of this. It is quite possible that the outer boundaries of even macroscopic objects correspond to boundaries between Euclidian and Minkowskian regions at the space-time sheet of the object.

   The failure of QFTs to describe bound states of say hydrogen atom could be second example: many-sheetedness and identification of bound states as single connected surface formed by proton and electron would be essential and taken into account in wave mechanical description but not in QFT description.

3. Concerning gravitation the basic outcome is that by number theoretical vision all preferred extremals are extremals of both Kähler action and volume term. This is true for all known

extremals what happens if one introduces the analog of Kähler form in M^4 is an open question) [L31].

Minimal surfaces carrying no Kähler field would be the basic model for gravitating system. Minimal surface equation are non-linear generalization of d'Alembert equation with gravitational self-coupling to induce gravitational metric. In static case one has analog for the Laplace equation of Newtonian gravity. One obtains analog of gravitational radiation as "massless extremals" and also the analog of spherically symmetric stationary metric.

Blackholes would be modified. Besides Schwartschild horizon which would differ from its GRT version there would be horizon where signature changes. This would give rise to a layer structure at the surface of blackhole [L31].

4. Concerning cosmology the hypothesis has been that RW cosmologies at QFT limit can be modelled as vacuum extremals of Kä hler action. This is admittedly ad hoc assumption inspired by the idea that one has infinitely long p-adic length scale so that cosmological constant behaving like $1/p$ as function of p-adic length scale assignable with volume term in action vanishes and leaves only Kähler action [K84]. This would predict that cosmology with critical is specified by a single parameter - its duration as also over-critical cosmology [K102]. Only sub-critical cosmologies have infinite duration.

   One can look at the situation also at the fundamental level. The addition of volume term implies that the only RW cosmology realizable as minimal surface is future light-cone of M^4. Empty cosmology which predicts non-trivial slightly too small redshift just due to the fact that linear Minkowski time is replaced with light-cone proper time constant for the hyperboloids of M^4_+. Locally these space-time surfaces are however deformed by the addition of topologically condensed 3-surfaces representing matter. This gives rise to additional gravitational redshift and the net cosmological redshift. This also explains why astrophysical objects do not participate in cosmic expansion but only comove. They would have finite size and almost Minkowski metric.

   The gravitational redshift would be basically a kinematical effect. The energy and momentum of photons arriving from source would be conserved but the tangent space of observer would be Lorentz-boosted with respect to source and this would course redshift.

   The very early cosmology could be seen as gas of arbitrarily long cosmic strings in H (or M^4) with 2-D M^4 projection [K102, K68]. Horizon would be infinite and TGD suggests strongly that large values of h_{eff}/h makes possible long range quantum correlations. The phase transition leading to generation of space-time sheets with 4-D M^4 projection would generate many-sheeted space-time giving rise to GRT space-time at QFT limit. This phase transition would be the counterpart of the inflationary period and radiation would be generated in the decay of cosmic string energy to particles.

Part II

GENERAL THEORY

Chapter 7

Construction of Quantum Theory: Symmetries

7.1 Introduction

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the configuration space - "world of the classical worlds" (WCW) - identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

7.1.1 Physics As Infinite-Dimensional Kähler Geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the WCW forms what I have used to call super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD.

4. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

5. Kähler-Dirac equation gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the Kähler-Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

   A second breakthrough came from the realization that the well-definedness of em charge forces in the generic situation localization of the modes to 2- surfaces at which induced W fields and also Z^0 fields above weak scale vanish.

6. The effective 2-dimensionality of the space-like 3-surfaces realizing quantum holography can be formulated as a symmetry stating that the replacement of wormhole throat by any light-like 3-surfaces parallel to it in the slicing of the space-time sheet induces only a gauge transformation of WCW Kähler function adding to it a real part of a holomorphic function of complex coordinate of WCW depending also on zero modes. This means that the Kähler metric of WCW remains invariant. It is also postulated that measurement interaction added to the Kähler-Dirac action induces similar gauge symmetry.

7. The study of the Kähler-Dirac equation leads to a detailed identification of super charges of the super-conformal algebras relevant for TGD [K127]: these results represent the most recent layer in the development of ideas about supersymmetry in TGD Universe. Whereas many considerations related to supersymmetry represented earlier rely on general arguments, the results deriving from the Kähler-Dirac equation are rather concrete and clarify the crucial role of the right-handed neutrino in TGD based realization of super-conformal symmetries. $\mathcal{N} = 1$ SUSY- now almost excluded at LHC - is not possible in TGD because it requires Majorana spinors. Also $\mathcal{N} = 2$ variant of the standard space-time SUSY seems to be excluded in TGD Universe. Fermionic oscillator operators for the induced spinor fields restricted to 2-D surfaces however generate large $\mathcal{N}$ SUSY and super-conformal algebra and the modes of right-handed neutrino its 4-D version.

7.1.2 P-Adic Physics As Physics Of Cognition

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization, which involves no ad hoc elements and is inherent to the physics of TGD.

The original idea was that the notion of number theoretic braid could pose strong number theoretic conditions on physics just as p-adic thermodynamics poses on elementary particle mass spectrum. A practically oriented physicist would argue that general braids must be allowed if one wants to calculate something and that number theoretic braids represent only the intersection between the real and various p-adic physics. He could also insist that at the level of WCW various sectors must be realized in a more abstract way - say as hierarchies of polynomials with coefficients belonging to various extensions or rationals so that one can speak about surfaces common to real and various p-adic sectors. In this view the fusion of various physics would be analogous to the completion of rationals to various number fields.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic embedding spaces are glued together along rational embedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of WCW spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K105]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The idea is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and embedding space points become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

7.1.3 Hierarchy Of Planck Constants And Dark Matter Hierarchy

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzles leads to a profound generalization of quantum TGD through a generalization of the notion of embedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the embedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups.

The original intuition was the space-time would be n-sheeted for $h_{eff} = n$. Quantum criticality expected on basis of the vacuum degeneracy of Kähler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have n conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by n. This defines infinite number of inclusion hierarchies $.. \subset C(n_1) \subset C(n_3)...$ such that n_{i+1} divides n_i. These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

This topic will not be discussed in this chapter since it is discussed in earlier chapter [?].

7.1.4 Number Theoretical Symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely ways objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by WCW spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the Kähler-Dirac gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4, one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4. One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H. so that TGD could be seen as a generalized number theory.

This idea will not be discussed in this chapter since it has better place in the book about physics as generalized number theory [K81].

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L17].

7.2 Symmetries

The most general expectation is that WCW can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$.

Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and H and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what "preferred" means.

7.2.1 General Coordinate Invariance And Generalized Quantum Gravitational Holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional configuration space of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremals are absolute minima of Kähler action.

This picture turned out to be too simple.

1. I have already described the recent view about light-like 3-surfaces as generalized Feynman diagrams and space-time surfaces as preferred extremals of Kähler action and will not repeat what has been said.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of WCW to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3_l as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet must define the preferred 3-surface X^3_l.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and symplectic symmetries allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

7.2.2 Light Like 3-D Causal Determinants And Effective2-Dimensionality

The light like 3-surfaces X^3_l of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry (see **Fig. 7.1**) related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

1. Field-particle duality is realized. Light-like 3-surfaces X^3_l -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X^3_l)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

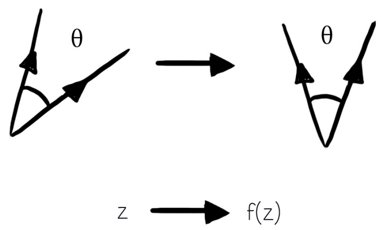

Figure 7.1: Conformal symmetry preserves angles in complex plane

2. One could also say that light-like 3-surfaces X_l^3 and the space-like 3-surfaces X^3 in the intersections of $X^4(X_l^3) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

3. Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This construction in turn led to the realization that WCW for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M_\pm^4 \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M_\pm^4 \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X_l^3 \times \delta M_\pm^4 \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M_\pm^4$ implies that the data at either X^3 or X_l^3 should be enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light-likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD: s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. What was regarded originally as a victory was that it simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M_+^4 \times CP_2$ reducing now to 2-dimensional integrals. One can of course criticize so strong form of effective 2-dimensionality as unphysical. As often happens, the later progress led to the comeback of the formulation involving 3-surfaces! The stringy picture implied by the solutions of Kähler-Dirac action led to the 3-D picture with effective 2-dimensionality realized in terms of super conformal symmetries.

7.2.3 Magic Properties Of Light Cone Boundary And Isometries OfWCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM_+^4, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are paramet3rized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

   Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of WCW .

2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations wiykd correspond to zero modes having zero norm in the Kähler metric of WCW . This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

3. The groups G and H, and thus WCW itself, should inherit the complex structure of the light cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

7.2.4 Symplectic Transformations Of $\Delta M^4_+ \times CP_2$ As Isometries Of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

7.2.5 Does The Symmetric Space Property Correspond To Coset Construction For Super Virasoro Algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{7.2.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

1. WCW geometry allows two super-conformal symmetries. The first one corresponds to super-symplectic transformations acting at the level of embedding space. The second one corresponds to super Kac-Moody symmetry acting as deformations of light-like 3-surfaces respecting their light-likeness.

2. It took considerable amount of trials and errors to realize that both symplectic and Kac-Moody algebras are needed to generate the entire isometry algebra g. h is sub-algebra of this extended algebra. In general case the elements of both algebras are non-vanishing at the prefered partonic 2-surfaces considered.

3. Strong form of holography implies that transformations located to the interior of space-like 3-surface and light-like partonic orbit define zero modes and act like gauge symmetries. The physically non-trivial transformations correspond to transformations acting non-trivially at space-like 3-surfaces. g corresponds to the algebra generated by these transformations. For preferred p3-surface - identified as (say) maximum of Kähler function - h corresponds to the elements of this algebra reducing to infinitesimal diffeomorphisms.

4. Coset representation has five tensor factors as required by p-adic mass calculations and they correspond to color algebra, to two factors from electroweak $U(2)$, to one factor from transversal M^4 translations and one factor from symplectic algebra (note that also Hamiltonians which are products of δM^4_+ and CP_2 Hamiltonians are possible.

5. The realization of WCW sectors with fixed values of zero modes as symmetric spaces G/H (analogous to $CP_2 = SU(3)/U(2)$) suggests that one can assign super-Virasoro algebras with G and H as a generalized coset representation for g and h so that the differences of the generators of two super Virasoro algebras annihilate the physical states for coset representations. This obviously generalizes Goddard-Olive-Kent construction [A61]. It however does not imply Equivalence Principle as believed for a long time.

7.2.6 Symplectic And Kac-Moody Algebras As Basic Building Bricks

Concerning the interpretation of the relationship between symplectic and Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with precise interpretation of strong form of holography.

The basic building bricks are symplectic algebra of δCD (this includes CP_2 besides light-cone boundary) and Kac-Moody algebra assignable to the isometries of δCD [K29]. It seems however that the longheld view about the role of Kac-Moody algebra must be modified. Also the earlier realization of super-Hamiltonians and Hamiltonians seems too naïve.

1. I have been accustomed to think that Kac-Moody algebra could be regarded as a sub-algebra of symplectic algebra. p-Adic mass calculations however requires five tensor factors for the coset representation of Super Virasoro algebra naturally assigned to the coset structure G/H of a sector of WCW with fixed zero modes. Therefore Kac-Moody algebra cannot be regarded as a sub-algebra of symplectic algebra giving only single tensor factor and thus inconsistent with interpretation of p-adic mass calculations.

2. The localization of Kac-Moody algebra generators with respect to the internal coordinates of light-like 3-surface taking the role of complex coordinate z in conformal field theory is also questionable: the most economical option relies on localization with respect to light-like radial coordinate of light-cone boundary as in the case of symplectic algebra. Kac-Moody algebra cannot be however sub-algebra of the symplectic algebra assigned with covariantly constant right-handed neutrino in the earlier approach.

3. Right-handed covariantly constant neutrino as a generator of super symmetries plays a key role in the earlier construction of symplectic super-Hamiltonians. What raises doubts is that other spinor modes - both those of right-handed neutrino and electro-weakly charged spinor modes - are absent. All spinor modes should be present and thus provide direct mapping from WCW geometry to WCW spinor fields in accordance with super-symmetry and the general idea that WCW geometry is coded by WCW spinor fields.

   Hence it seems that Kac-Moody algebra must be assigned with the modes of the induced spinor field which carry electroweak quantum numbers. If would be natural that the modes of right-handed neutrino having no weak and color interactions would generate the huge symplectic algebra of symmetries and that the modes of fermions with electroweak charges generate much smaller Kac-Moody algebra.

4. The dynamics of Kähler action and Kähler-Dirac action action are invisible in the earlier construction. This suggests that the definition of WCW Hamiltonians is too simplistic. The proposal is that the conserved super charges derivable as Noether charges and identifiable as super-Hamiltonians define WCW metric and Hamiltonians as their anti-commutators. Spinor modes would become labels of Hamiltonians and WCW geometry relates directly to the dynamics of elementary particles.

5. Note that light-cone boundary $\delta M^4_+ = S^2 \times R_+$ allows infinite-dimensional group of isometries consisting of conformal transformation of the sphere S^2 with conformal scaling compensated by an S^2 local scaling or the light-like radial coordinate of R_+. These isometries contain as a subgroup symplectic isometries and could act as gauge symmetries of the theory.

Gauge symmetry property means that the Kähler metric of the WCW is same for all choices of preferred X^3. Kähler function would however differ by a real part of a holomorphic function of WCW coordinates for different choices of preferred X^3.

Strong form of holography (or strong form of GCI) implies that one can take either space-like or light-like 3-surfaces as basic objects and consider the action the super-symplectic algebra also for the light-like 3-surfaces. This is possible by just parallelly translating the light-like boundary of CD so that one obtains slicing of CD by these light-like 3-surfaces. The equality of four-momenta associated with the two super-conformal representations might allow interpretation in terms of equivalence of gravitational and inertial four-momenta.

7.2.7 Comparison Of TGD And Stringy Views About Super-Conformal Symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

1. In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K29]. This kind of representation

applies also in Kac-Moody sector since the local transversal isometries localized in X_l^3 and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

2. A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K27].

3. The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an $1/2$ disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

4. If Kähler action defines the Kähler-Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

5. What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D \ mod\ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator

$S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned} \{G_m, G_n^\dagger\} &= 2L_{m+n} + \frac{c}{3}(m^2 - \frac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} &= 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} &= 0 \ . \end{aligned} \tag{7.2.2}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n < 0$ and $G_n^\dagger$, $n > 0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

1. Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K29]. Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

2. The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K127].

   One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

3. The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M_\pm^4 \times CP_2$. Clearly, wormhole throats X_l^3 would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

4. An alternative identification for TGD parts of conformal fields is inspired by M^8-H duality. Conformal fields would be fields in WCW . The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

7.3 WCW As A Union Of Homogenous Or Symmetric Spaces

The physical interpretation and detailed mathematical understanding of super-conformal symmetries has developed rather slowly and has involved several side tracks. In the following I try to summarize the basic picture with minimal amount of formulas with the understanding that the statement "Noether charge associated with geometrically realized Kac-Moody symmetry" is enough for the reader to write down the needed formula explicitly. Formula oriented reader might deny the value of the approach giving weight to principles. My personal experience is that piles of formulas too often hide the lack of real understanding.

In the following the vision about WCW as union of coset spaces is discussed in more detail.

7.3.1 Basic Vision

The basic view about coset space construction for WCW has not changed.

1. The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A8] (`http://tinyurl.com/y7u2t8jo`) means at Lie algebra level the decomposition $g=h\oplus t$ to sub-Lie-algebra h and its complement t such that $[h,t]\subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t,t]\subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A21] (`http://tinyurl.com/ycouv7uh`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

   An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t,t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

2. The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW . The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW . In particular, Hamiltonians must be represented in completely

new manner. The key idea is to construct WCW Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and embedding space coordinates are treated purely classically.

3. It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

   CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r = 0$ is left invariant by $U(2)$ but for $r > 0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW .

   How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

4. The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h + t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

   For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

   For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

7.3.2 Equivalence Principle And WCW

7.3.3 Equivakence Principle At Quantum And Classical Level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K121].

   A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore: this idea is however not promising.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

7.3.4 Criticism Of The Earlier Construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

1. Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

   One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that embedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

2. The fact that the dynamics of Kähler action and Kähler-Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K127] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the Kähler-Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

7.3.5 Is WCW Homogenous Or Symmetric Space?

A key question is whether WCW can be symmetric space [A21] (`http://tinyurl.com/y8ojglkb`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h,t] \subset t$, $[t,t] \subset h$. The latter condition is the difficult one.

1. δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

2. For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$

and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

3. In the case of symplectic algebra one can construct the embedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level embedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

4. A more promising involution is the inversion $r_M \to 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

   $r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by $/r_M$-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in pointwise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

5. One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW .

   For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t, t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

7.3.6 Symplectic And Kac-Moody Algebras As Basic Building Bricks

7.3.7 WCW As A Union Of Symmetric Spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) = \cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

If G is symplectic group of $\delta M^4_\pm \times CP_2$ then H is its subgroup, and one can wonder whether this is really consistent with the identification of H as Kac-Moody algebra assignable to light-like

3-surfaces. This raises the possibility that SKM acts as pure gauge symmetries and has nothing to do with the coset decomposition.

The improved understanding of solutions of the Kähler-Dirac equation [K127] also leads to the realization that the direct sum of super-symplectic algebra and isometry algebra is more natural spectrum generating algebra. For super-symplectic algebra super-generators are represented in terms of contractions of covariantly constant right-handed neutrino mode with second quantized spinor field. For isometry sub-algebra super generators have representation in terms of contractions of modes of induced spinor field localized at string world sheets is a more natural identification of the fundamental conformal algebra and gives five tensor factors as required by p-adic mass calculations.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the WCW). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the WCW . Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality) are zero modes and WCW allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would in turn correspond to the symplectic isometries reducing to diffeomorphisms at the 3-surfaces or possibly at partonic 2-surfaces only.

H could but not not need to corresponds to the Kac-Moody symmetries respecting light-likeness of X^3_l and acting in X^3_l but trivially at the partonic 2-surface X^2. The action of course extends also to the interior of space-like 3-surface X^3 at the boundary of CD. This coset structure was originally suggested via coset construction for super Virasoro algebras of super-symplectic and super Kac-Moody algebras.

WCW isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the space of 3-surfaces in $\delta M^4_+ \times CP_2$. WCW is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all on topologies of X^3 plus possibly other “zero modes”. Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

7.3.8 Isometries Of WCW Geometry As SymplecticTransformations Of $\Delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of WCW as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) = S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \tag{7.3.1}$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

1. The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

2. CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of WCW gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

7.3.9 SUSY Algebra Defined By The Anti-Commutation Relations Of Fermionic Oscillator Operators And WCW Local Clifford Algebra Elements As Chiral Super-Fields

Whether TGD allows space-time supersymmetry has been a long-standing question. Majorana spinors appear in $N = 1$ super-symmetric QFTs- in particular minimally super-symmetric standard model (MSSM). Majorana-Weyl spinors appear in M-theory and super string models. An undesirable consequence is chiral anomaly in the case that the numbers of left and right handed spinors are not same. For $D = 11$ and $D = 10$ these anomalies cancel which led to the breakthrough of string models and later to M-theory. The probable reason for considering these dimensions is that standard model does not predict right-handed neutrino (although neutrino mass suggests that right handed neutrino exists) so that the numbers of left and right handed Weyl-spinors are not the same.

In TGD framework the situation is different. Covariantly constant right-handed neutrino spinor acts as a super-symmetry in CP_2. One might think that right-handed neutrino in a well-defined sense disappears from the spectrum as a zero mode so that the number of right and left handed chiralities in $M^4 \times CP_2$ would not be same. For light-like 3-surfaces covariantly constant right-handed neutrino does not however solve the counterpart of Dirac equation for a non-vanishing four-momentum and color quantum numbers of the physical state. Therefore it does not disappear from the spectrum anymore and one expects the same number of right and left handed chiralities.

In TGD framework the separate conservation of baryon and lepton numbers excludes Majorana spinors and also the the Minkowski signature of $M^4 \times CP_2$ makes them impossible. The conclusion that TGD does not allow super-symmetry is however wrong. For $\mathcal{N} = 2N$ Weyl spinors are indeed possible and if the number of right and left handed Weyl spinors is same super-symmetry is possible. In 8-D context right and left-handed fermions correspond to quarks and leptons and since color in TGD framework corresponds to CP_2 partial waves rather than spin like quantum number, also the numbers of quark and lepton-like spinors are same.

The physical picture suggest a new kind of approach to super-symmetry in the sense that the anti-commutations of fermionic oscillator operators associated with the modes of the induced spinor fields define a structure analogous to SUSY algebra. This means that $\mathcal{N} = 2N$ SUSY with large N is in question allowing spins higher than two and also large fermion numbers. Recall that $\mathcal{N} \leq 32$ is implied by the absence of spins higher than two and the number of real spinor components is $N = 32$ also in TGD. The situation clearly differs from that encountered in super-string models and SUSYs and the large value of N allows to expect very powerful constraints on dynamics irrespective of the fact that SUSY is broken. Right handed neutrino modes define a sub-algebra for which the SUSY is only slightly broken by the absence of weak interactions and one could also consider a theory containing a large number of $\mathcal{N} = 2$ super-multiplets corresponding to the addition of right-handed neutrinos and antineutrinos at the wormhole throat.

Masslessness condition is essential for super-symmetry and at the fundamental level it could be formulated in terms of Kähler-Dirac gamma matrices using octonionic representation and assuming that they span local quaternionic sub-algebra at each point of the space-time sheet. SUSY algebra has standard interpretation with respect to spin and isospin indices only at the partonic 2-surfaces so that the basic algebra should be formulated at these surfaces. Effective 2-dimensionality would require that partonic 2-surfaces can be taken to be ends of any light-like 3-surface Y_l^3 in the slicing of the region surrounding a given wormhole throat.

Super-algebra associated with the Kähler-Dirac gamma matrices

Anti-commutation relations for fermionic oscillator operators associated with the induced spinor fields are naturally formulated in terms of the Kähler-Dirac gamma matrices. Super-conformal symmetry suggests that the anti-commutation relations for the fermionic oscillator operators at light-like 3-surfaces or at their ends are most naturally formulated as anti-commutation relations for SUSY algebra. The resulting anti-commutation relations would fix the quantum TGD.

$$\begin{aligned} \{a_{n\alpha}^{\dagger}, a_{n\beta}\} &= D_{mn} D_{\alpha\beta} \ , \\ D &= (p^{\mu} + \sum_a Q_a^{\mu})\hat{\sigma}^{\mu} \ . \end{aligned} \tag{7.3.2}$$

Here p^{μ} and Q_a^{μ} are space-time projections of momentum and color charges in Cartan algebra. Their action is purely algebraic. The anti-commutations are nothing but a generalization of the ordinary equal-time anti-commutation relations for fermionic oscillator operators to a manifestly covariant form. The matrix $D_{m,n}$ is expected to reduce to a diagonal form with a proper normalization of the oscillator operators. The experience with extended SUSY algebra suggest that the anti-commutators could contain additional central term proportional to $\delta_{\alpha\beta}$.

One can consider basically two different options concerning the definition of the super-algebra.

1. If the super-algebra is defined at the 3-D ends of the intersection of X^4 with the boundaries of CD, the modified gamma matrices appearing in the operator D appearing in the anti-commutator are associated with Kähler action. If the generalized masslessness condition $D^2 = 0$ holds true -as suggested already earlier- one can hope that no explicit breaking of super-symmetry takes place and elegant description of massive states as effectively massless states making also possible generalization of twistor is possible. One must however notice that also massive representatives of SUSY exist.

2. SUSY algebra could be also defined at 2-D ends of light-like 3-surfaces.

According to considerations of [K127] these options are equivalent for a large class of space-time sheets. If the effective 3-dimensionality realized in the sense that the effective metric defined by the Kähler-Dirac gamma matrices is degenerate, propagation takes place along 3-D light-like 3-surfaces. This condition definitely fails for string like objects.

One can realize the local Clifford algebra also by introducing theta parameters in the standard manner and the expressing a collection of local Clifford algebra element with varying values of fermion numbers (function of CD and CP_2 coordinates) as a chiral super-field. The definition of a chiral super field requires the introduction of super-covariant derivatives. Standard form for the anti-commutators of super-covariant derivatives D_α make sense only if they do not affect the Kähler-Dirac gamma matrices. This is achieved if p_k acts on the position of the tip of CD (rather than internal coordinates of the space-time sheet). Q_a in turn must act on CP_2 coordinates of the tip.

Super-fields associated with WCW Clifford algebra

WCW local Clifford algebra elements possess definite fermion numbers and it is not physically sensible to super-pose local Clifford algebra elements with different fermion numbers. The extremely elegant formulation of super-symmetric theories in terms of super-fields encourages to ask whether the local Clifford algebra elements could allow expansion in terms of complex theta parameters assigned to various fermionic oscillator operator in order to obtain formal superposition of elements with different fermion numbers. One can also ask whether the notion of chiral super field might make sense.

The obvious question is whether it makes sense to assign super-fields with the Kähler-Dirac gamma matrices.

1. Kähler-Dirac gamma matrices are not covariantly constant but this is not a problem since the action of momentum generators and color generators on space-time coordinates is purely algebraic.

2. One can define the notion of chiral super-field also at the fundamental level. Chiral super-field would be continuation of the local Clifford algebra of associated with CD to a local Clifford algebra element associated with the union of CDs. This would allow elegant description of cm degrees of freedom, which are the most interesting as far as QFT limit is considered.

3. Kähler function of WCW as a function of complex coordinates could be extended to a chiral super-field defined in quantum fluctuation degrees of freedom. It would depend on zero modes too. Does also the latter dependence allow super-space continuation? Coefficients of powers of theta would correspond to fermionic oscillator operators. Does this function define the propagators of various states associated with light-like 3-surface? WCW complex coordinates would correspond to the modes of induced spinor field so that super-symmetry would be realized very concretely.

7.3.10 Identification Of Kac-Moody Symmetries

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating WCW degrees of freedom contributing to the metric. The recent vision looks like follows.

1. The recent interpretation is that these symmetries are due to the non-determinism of Kähler action and transform to each other preferred extremals with same space-like surfaces as their ends at the boundaries of causal diamond. These space-time surfaces have same Kähler action and possess same conserved quantities.

2. The sub-algebra of conformal symmetries acts as gauge transformations of these infinite set of degenerate preferred extremals and there is finite number n of gauge equivalence classes. n corresponds to the effective (or real depending on interpretation) value of Planck constant $h_{eff} = n \times h$. The further conjecture is that the sub-algebra of conformal algebra for which conformal weights are integers divisible by n act as genuine gauge symmetries. If Kähler action reduces to a sum of 3-D Chern-Simons terms for preferred extremals, it is enough to consider the action on light-like 3-surfaces. For gauge part of algebra the algebra acts trivially at space-like 3-surfaces.

3. A good guess is that the Kac-Moody type algebra corresponds to the sub-algebra of symplectic isometries of $\delta M^4_\pm \times CP_2$ acting on light-like 3-surfaces and having continuation to the interior.

A stronger assumption is that isometries are in question. For CP_2 nothing would change but light-cone boundary $\delta M^4_\pm = S^2 \times R_+$ has conformal transformations of S^2 as isometries. The conformal scaling is compensated by S^2-local scaling of the light like radial coordinate of R_+.

4. This super-conformal algebra realized in terms of spinor modes and second quantized induced spinor fields would define the Super Kac-Moody algebra. The generators of this Kac-Moody type algebra have continuation from the light-like boundaries to deformations of preferred extremals and at least the generators of sub-algebra act trivially at space-like 3-surfaces.

The following is an attempt to achieve a more detailed identification of the Kac-Moody algebra is considered.

Identification of Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{7.3.3}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{7.3.4}$$

Ansatz as an X^3-local conformal transformation of embedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the embedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{7.3.5}$$

This gives

$$\begin{aligned} c_A(x) & \left[D_k j^A_l + D_l j^A_k\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l \\ = & \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \end{aligned} \tag{7.3.6}$$

If an X^3-local variant of a conformal transformation of the embedding space is in question, the first term is proportional to the metric since one has

$$D_k j^A_l + D_l j^A_k = 2h_{kl} \ . \tag{7.3.7}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the embedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{7.3.8}$$

A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \ . \tag{7.3.9}$$

The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r)d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \ . \tag{7.3.10}$$

so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i \quad = \quad \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \ . \tag{7.3.11}$$

The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj} \partial_i \xi^k + g_{ki} \partial_j \xi^k \quad = \quad \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \ . \tag{7.3.12}$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^{μ} are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] = c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \ , \tag{7.3.13}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k \partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl} D - M^{kl} \ . \end{array} \tag{7.3.14}$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak} h_{kl} \partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labeled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3)\times E^3$ respecting the radial conformal weights would reduce to $SO(2)\times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

Hamiltonians

The action of these transformations on Kähler action is well-defined and one can deduce the conserved quantities having identification as WCW Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_\pm \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

The action of Kac-Moody algebra on spinors and fermionic representations of Kac-Moody algebra

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. The basic goal is to deduce the fermionic Noether charge associated with the bosonic Kac-Moody symmetry and this can be done by a standard recipe. The first contribution to the charge comes from the transformation of Kähler-Dirac gamma matrices appearing in the Kähler-Dirac action associated with fermions. Second contribution comes from spinor rotation.

2. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors.

How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-symplectic algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

About the interpretation of super Kac-Moody symmetries

Also the light like 3-surfaces X^3_l of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to WCW metric. In this case the symmetries correspond to the isometries of the embedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X^3_l so that Kac-Moody type symmetry results.

Also the condition $\sqrt{g_3} = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction.

If is enough to localize only the H-isometries with respect to X^3_l, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation need not be equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them.

The following arguments support one possible identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X^3_l-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the WCW supports the view that action of the generators of X^3_l-local color transformations on WCW spinor fields represents local color transformations. If the action of X^3_l-local $SU(3)$ transformations on WCW spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N = 4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only right-handed neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of WCW gamma matrices forming a subalgebra of the full super-algebra.

$N = 2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N = 2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N = 2$ super-conformal representations are modular invariant, and that $N = 2$ algebra defines so called chiral ring defining a topological quantum field theory [A59], lend a further support for the belief that $N = 2$ super-conformal algebra acts in super-symplectic degrees of freedom.

The values of c and conformal weights for $N=2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ l=0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ m=-l,-l+2,....,l-2,l \ . \end{aligned} \quad (7.3.15)$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k=1$ one would have $q=0,1/3,-1/3$, which brings in mind anyons. $\Delta_{l=0,m=0}=0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [A59].

For Ramond representation $L_0-c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta=c/24$ giving the condition $k=2\left[l(l+2)-m^2\right]$ (note that k must be even and that $(k,l,m)=(4,1,1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac}=\pm c/12=\pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N=4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on WCW Hamiltonians expressible in terms of Hamiltonians of $X^3_l\times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h=1$ whereas T and G have conformal weights $h=2$ and $h=3/2$.

The experience with $N=4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h=1/2$ and their super-partners with $h=0$ and realized as fermion-anti-fermion bilinears. Since G and Ψ are labeled by 2×4 spinor indices, super-partners would correspond to $2\times(3+1)=8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

7.3.11 Coset Space Structure For WCW As A SymmetricSpace

The key ingredient in the theory of symmetric spaces is that the Lie-algebra of G has the following decomposition

$$\begin{aligned} &g=h+t \ , \\ &[h,h]\subset h \ , \quad [h,t]\subset t \ , \quad [t,t]\subset h \ . \end{aligned}$$

In present case this has highly nontrivial consequences. The commutator of *any* two infinitesimal generators generating nontrivial deformation of 3-surface belongs to h and thus vanishing norm in the WCW metric at the point which is left invariant by H. In fact, this same condition follows from Ricci flatness requirement and guarantees also that G acts as isometries of WCW . This generalization is supported by the properties of the unitary representations of Lorentz group at the light cone boundary and by number theoretical considerations.

The algebras suggesting themselves as candidates are symplectic algebra of $\delta M^\pm\times CP_2$ and Kac-Moody algebra mapping light-like 3-surfaces to light-like 3-surfaces to be discussed in the next section.

The identification of the precise form of the coset space structure is however somewhat delicate.

1. The essential point is that both symplectic and Kac-Moody algebras allow representation in terms of X^3_l-local Hamiltonians. The general expression for the Hamilton of Kac-Moody algebra is

$$H = \sum \Phi_A(x) H^A \ . \tag{7.3.16}$$

Here H^A are Hamiltonians of $SO(3) \times SU(3)$ acting in $\delta X_l^3 \times CP_2$. For symplectic algebra any Hamiltonian is allowed. If x corresponds to any point of X_l^3, one must assume a slicing of the causal diamond CD by translates of $\delta M_\pm^4$.

2. For symplectic generators the dependence of form on r^Δ on light-like coordinate of $\delta X_l^3 \times CP_2$ is allowed. Δ is complex parameter whose modulus squared is interpreted as conformal weight. Δ is identified as analogous quantum number labeling the modes of induced spinor field.

3. One can wonder whether the choices of the $r_M = constant$ sphere S^2 is the only choice. The Hamiltonin-Jacobi coordinate for $X^4_{X_l^3}$ suggest an alternative choice as E^2 in the decomposition of $M^4 = M^2(x) \times E^2(x)$ required by number theoretical compactification and present for known extremals of Kähler action with Minkowskian signature of induced metric. In this case $SO(3)$ would be replaced with $SO(2)$. It however seems that the radial light-like coordinate u of $X^4(X_l^3)$ would remain the same since any other curve along light-like boundary would be space-like.

4. The vector fields for representing Kac-Moody algebra must vanish at the partonic 2-surface $X^2 \subset \delta M_\pm^4 \times CP_2$. The corresponding vector field must vanish at each point of X^2:

$$j^k = \sum \Phi_A(x) J^{kl} H_l^A = 0 \ . \tag{7.3.17}$$

   This means that the vector field corresponds to $SO(2) \times U(2)$ defining the isotropy group of the point of $S^2 \times CP_2$.

   This expression could be deduced from the idea that the surfaces X^2 are analogous to origin of CP_2 at which $U(2)$ vector fields vanish. WCW at X^2 could be also regarded as the analog of the origin of local $S^2 \times CP_2$. This interpretation is in accordance with the original idea which however was given up in the lack of proper realization. The same picture can be deduced from braiding in which case the Kac-Moody algebra corresponds to local $SO(2) \times U(2)$ for each point of the braid at X^2. The condition that Kac-Moody generators with positive conformal weight annihilate physical states could be interpreted by stating effective 2-dimensionality in the sense that the deformations of X_l^3 preserving its light-likeness do not affect the physics. Note however that Kac-Moody type Virasoro generators do not annihilate physical states.

5. Kac-Moody algebra generator must leave induced Kähler form invariant at X^2. This is of course trivial since the action leaves each point invariant. The conditions of Cartan decomposition are satisfied. The commutators of the Kac-Moody vector fields with symplectic generators are non-vanishing since the action of symplectic generator on Kac-Moody generator restricted to X^2 gives a non-vanishing result belonging to the symplectic algebra. Also the commutators of Kac-Moody generators are Kac-Moody generators.

7.3.12 The Relationship Between Super-Symplectic And SuperKac-Moody Algebras, Equivalence Principle, And Justification Of P-Adic Thermodynamics

The relationship between super-symplectic algebra (SS) acting at light-cone boundary and Super Kac-Moody algebra (SKM) assumed to act on light-like 3-surfaces and by continuation of the action also to the space-like 3-surfaces at the boundaries of CD has remained somewhat enigmatic due to the lack of physical insights.

Corresponding to the coset decomposition G/H of WCW there is also the sub-algebra SD of SS acting as diffeomorphisms of given 3-surface. This algebra acts as gauge algebra. It seems that SKM and SD cannot be the same algebra.

The construction of WCW gamma matrices and study of the solutions of Kähler-Dirac equation support strongly the conclusion that the construction of physical states involves the direct sum of two algebras SS and SI. The super-generators of SS are realized using only covariantly constant mode for the right-handed neutrino. The isometry sub-algebra SI is realized using all spinor modes. The direct sum $SS \oplus SI$ has the 5 tensor factors required by p-adic mass calculations. SI is Kac-Moody algebra and could be a natural identification for SKM. This forces to give up the construction of coset representation for the Super-Virasoro algebras.

This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP and to wait for an answer for rather long time. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra - in obvious conflict with the basic wisdom that this generator should annihilate physical states - remained lacking.

One cannot still exclude the possibility that these three problems could have a common solution in terms of an appropriate coset representation. Quantum variant of EP cannot not follow from the coset representation for SS and SD. The coset representation of SS and $SI = SKM$ could however make sense and would be realized in the tensor product for the representations of SS and SI and would have the five tensor factors. Physical states would correspond to those for the direct sum $SS \oplus SI$. Since $SS \oplus SI$ acts as a spectrum generating algebra rather than gauge algebras, the condition that L_0 annihilates the physical states is not necessary. The coset representation would differ from the representation for $SS \oplus SI$ only that the states would be annihilated by the differences of the SV generators rather than their sums.

New vision about the relationship between various algebras

Consider now the new vision about the relationship between SSV, its sub-algebra acting as diffeomorphisms of 3-surface and $SKMV$.

1. The isometries G of sub- WCW associted with given CD are symplectic transformations of $\delta CD \times CP_2$ [K29] (note that I have used the attribute "canonical" instead of "symplectic" in some contexts) reducing to diffeomorphisms at partonic 2-surfaces or at the entire 3-surfaces at the boundaries of CD. H acts a symplectic subgroup acting as diffeomorphisms of X^3 or partonic 2-surfaces. It should annihilate physical states so that SD associated with $H \subset G$ is not interesting as far as coset representations are considered.

   Only the sub-algebra SI associated with symplectic isometries can provide coset representation. The representation space would be generated by the action of $SS \oplus SI$ in terms of fermionic oscillator operators and WCW isometry algebra. The same representation space allows also the representation of sums of super generators so that one has two options. $SS \oplus SI$ and $SS - SI$.

2. Consider first the $SS \oplus SI$ option. In this case the number of tensor factors in Super-Virasoro algebra is five as required by the p-adic mass calculations. L_n annihilated physical states but there is no need for L_0 to annihilate them since symplectic algebra is not gauge algebra.

3. Consider next the $SS - SI$ obtain, the coset representation. A generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of Super-Virasor algebra elements for SS and SI would annihilate physical states. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$. For coset representation one could even require that the difference of the scaling generators L_0 annihilates the physical states.

   The problem is however that the Super Virasoro algebra generators do note reduce to the sums of generators assignable to SS and SI so that one does not obtain the five tensor factors.

The coset representation motivated the proposal was that identical action of the Dirac operators assignable to G and H in coset representation could provide the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to G are equal to the gravitational four-momentum and color quantum numbers assignable to H. One can argue that since super-symplectic transformations correspond to the isometries of the "world of classical worlds", the assignment of the attribute "inertial" to them is natural.

This interpretation is not feasible if H corresponds acts as diffeomorphisms: the four-momentum associated with SD most naturally vanishes since it represents diffeomorphisms. If H corresponds to SI, one has the problem with the number of tensor factors. Therefore $SS \oplus SI$ seems to be the only working option.

A more feasible realization of EP quantum level is as Quantum Classical Correspondence (QCC) stating that the conserved four-momentum associated with Kähler action equals to an eigenvalue of the conserved Kähler-Dirac four-momentum having natural interpretation as gravitational four-momentum due the fact that well-defined em charge for spinor modes forces them in the generic case to string world sheets. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

1. The hope was that for SS/SI coset representations the p-adic thermal expectation values of the SS and SI conformal weights would be non-vanishing and identical and mass squared could be identified equivalently either as the expectation value of SI or SS scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SS - SI$.

2. There seems consistency with p-adic mass calculations for hadrons [K78] since the non-perturbative SS contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SS is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SS - SI$ duality. The perturbative quark contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SI whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SS. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [K71] remains intact in this framework.

3. The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N = 5$ and during years I have proposed several explanations for this number. This excldes the coset representation SS/SI. $SS \oplus SI$ however survives. It indeed seems that holonomic contributions related to spinor modes other than covariantly constant right-handed neutrino- that is electro-weak and spin contributions- must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew} \times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2) \subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

1. In positive energy ontology Lorentz invariance requires the interpretation of mass squared as thermal expectation value of the conformal weight assignable to vibrational degrees of freedom. In Zero Energy Ontology (ZEO) quantum theory can be formally regarded as a square root of thermodynamics and it is possible to speak about thermal expectation value of mass squared without losing Lorentz invariance since the zero energy state corresponds to a square root of density matrix expressible as product of hermitian and unitary matrices.

This implies that one can speak about thermal expectation value of mass squared rather than conformal weight. This might have some non-trivial experimental consequences since the energies of states with the same free momentum contributing to the thermal expectation value are different.

2. The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. The identification of the spinor partial waves is in terms of ground states of super-conformal representations.

3. In the case of M^4 degrees of freedom it is strictly speaking not possible to talk about momentum eigen states since translations take parton out of δH_+. This would suggests that 4-momentum must be assigned with the tip of the light-cone containing the particle but this is not consistent with zero energy ontology. Hence it seems that one must restrict the translations of X_l^3 to time like translations in the direction of geometric future at $\delta M^4_+ \times CP_2$. The decomposition of the partonic 3-surface X_l^3 to regions $X_{l,i}^3$ carrying non-vanishing induced Kähler form and the possibility to assign $M^2(x) \subset M^4$ to the tangent space of $X^4(X_l^3)$ at points of X_l^3 suggests that the points of number theoretic braid to which oscillator operators can be assigned can carry four-momentum in the plane defined by $M^2(x)$. One could assume that the four-momenta assigned with points in given region X_i^3 are collinear but even this restriction is not necessary.

4. The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \tag{7.3.18}$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \tag{7.3.19}$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

How it is possible to have negative conformal weights for ground states?

p-Adic mass calculations require negative conformal weights for ground states [K65]. The only elegant solution of the problems caused by this requirement seems to be p-adic: the conformal weights are positive in the real sense but as p-adic numbers their dominating part is negative integer (in the real sense), which can be compensated by the conformal weights of Super Virasoro generators.

1. If $\pm\lambda_i^2$ as such corresponds to a ground state conformal weight and if λ_i is real the ground state conformal weight positive in the real sense. In complex case (instanton term) the most natural formula is $h = \pm|\lambda|^2$.

2. The first option is based on the understanding of conformal excitations in terms of CP breaking instanton term added to the modified Dirac operator. In this case the conformal weights are identified as $h = n-|\lambda_k|^2$ and the minus sign comes from the Euclidian signature of the effective metric for the Kähler-Dirac operator. Ground state conformal weight would be non-vanishing for non-zero modes of $D(X_l^3)$. Massless bosons produce difficulties unless one has $h = |\lambda_i(1) - \lambda_i(2)|^2$, where $i = 1, 2$ refers to the two wormhole throats. In this case the difference can vanish and its non-vanishing would be due to the symmetric breaking. This scenario is assumed in p-adic mass calculations. Fermions are predicted to be always massive since zero modes of $D(X^2)$ represent super gauge degrees of freedom.

3. In the context of p-adic thermodynamics a loop hole opens allowing λ_i to be real. In spirit of rational physics suppose that one has in natural units $h = \lambda_i^2 = xp^2 - n$, where x is integer. This number is positive and large in the real sense. In p-adic sense the dominating part of this number is $-n$ and can be compensated by the net conformal weight n of Super Virasoro generators acting on the ground state. xp^2 represents the small Higgs contribution to the mass squared proportional to $(xp^2)_R \simeq x/p^2$ ($_R$ refers to canonical identification). By the basic features of the canonical identification $p > x \simeq p$ should hold true for gauge bosons for which Higgs contribution dominates. For fermions x should be small since p-adic mass calculations are consistent with the vanishing of Higgs contribution to the fermion mass. This would lead to the earlier conclusion that xp^2 and hence B_K is large for bosons and small for fermions and that the size of fermionic (bosonic) wormhole throat is large (small). This kind of picture is consistent with the p-adic modular arithmetics and suggests by the cutoff for conformal weights implied by the fact that both the number of fermionic oscillator operators and the number of points of number theoretic braid are finite. This solution is however tricky and does not conform with number theoretical universality.

7.4 Are Both Symplectic And Conformal Field Theories Needed?

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light-cone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [K28] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

7.4.1 Symplectic QFT At Sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [K83]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctuation $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is

convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \quad . \tag{7.4.1}$$

Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl}f(A(s_1,s_2,s))Idd\mu_s$ so that one has

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl}f(A(s_1,s_2,s))d\mu_s \quad . \tag{7.4.2}$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n=1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

7.4.2 Symplectic QFT With Spontaneous Breaking Of Rotational And Reflection Symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized embedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized embedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{7.4.3}$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\begin{aligned}\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle &= c_{kl}^r \int f(\Delta A(s_1, s_2, s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s \\ &= \end{aligned} \tag{7.4.4}$$

$$c_{kl}^r c_{rm} \int f(\Delta A(s_1, s_2, s)) f(\Delta A(s, s_3, t)) d\mu_s d\mu_t \ . \tag{7.4.5}$$

Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl} \int f(\Delta A(s_1, s_2, s)) d\mu_s \tag{7.4.6}$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

7.4.3 Generalization To Quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the "world of classical worlds".

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a way that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n)+N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naïve first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of WCW . Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [K107].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time.

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and WCW degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone

boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of WCW Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules can be treated separately. This separation indeed happens since conformal degrees of freedom correspond to quantum fluctuations contributing to the WCW metric and affecting the induced metric whereas symplectic invariants correspond to non-quantum fluctuating zero modes defining the part of quantum state not affected by quantum fluctuations parameterized by the symplectic group of $\delta M^4_\pm \times CP_2$. Also the dream about symplectic fusion rules have been realized. An explicit construction of symplectic fusion algebras is represented in [K22].

Chapter 8

Zero Energy Ontology

8.1 Introduction

Zero energy ontology (ZEO) has become gradually one of the corner stones of quantum TGD. This motivates the collection of material related to ZEO in a single chapter providing an overall view about the development of ideas. The sections are independent and reflect different views about ZEO.

8.1.1 ZEO in its recent form (2021)

The following gives a brief summary of the most recent view (2021) of ZEO [L91].

1. The notion of a causal diamond (CD) (see **Fig. ??**) is a central concept. Its little cousin "cd" can be identified as a union of two half-cones of M^4 glued together along their bottoms (3-D balls). The half-cones are mirror images of each other. CD=$cd \times CP_2$ is the Cartesian product of cd with CP_2 and obtained by replacing the points of cd with CP_2. The notion of CD emerges naturally in the number theoretic vision of TGD (adelic physics [L53])via the $M^8 - H$ duality [L85, L109, L110].

2. In ZEO, quantum states are not 3-dimensional if the determinism does not fail as it actually does, but superpositions of 4-dimensional deterministic time evolutions connecting ordinary 3-dimensional states. For the strongest form of holography implied by general coordinate invariance (GCI), the time evolutions are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution.

   The failure of determinism probably implies that a given 3-surface at the passive boundary of CD (PB) corresponds to a finite number of 4-D minimal surfaces and that the minimal surface can be regarded as an analog of soap film spanned by a frame having fixed parts at the boundaries of CD and dynamically generated parts in the interior of CD. The frame can be identified as a topological analog of a Feynman diagram.

3. Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced by a new superposition. The classical determinism of individual time evolution is not violated. This solves the basic paradox of quantum measurement theory. There are two kinds of SFRs: BSFRs (counterparts of ordinary SFRs) changing the arrow of time (AT) and SSFRs (analogs of "weak" measurements) preserving the arrow of time that give rise to an analog of the Zeno effect (`https://cutt.ly/yl7oIUy`) [L91]. The findings of Minev *et al* [L77] provide strong support for ZEO [L77].

8.1.2 Topics of the chapter

The goal is to provide some conceptual background for the attempts to identify scattering amplitudes in the framework provided by ZEO.

In the section "*Zero Energy Ontology in its original form*" the basic ideas and implications of ZEO are described. I will represent motivations for ZEO in TGD framework, compare ZEO with the ordinary positive energy ontology, and try to make clear the implications of ZEO for quantum measurement theory since they relate also directly to the notion of conscious observer as it is understood in TGD inspired theory of consciousness.

In the section "*Some comments related to Zero Energy Ontology*" the basic ideas and notions of ZEO in its original form are critically discussed and also TGD based theory of consciousness is formulated in this framework.

In the section "*Still about quantum measurement theory in ZEO*" quantum measurement theory in ZEO framework is considered with a particular emphasis on number theoretic universality brought in by number theoretic aspects of TGD [L54, L53].

In the section "*Some questions concerning zero energy ontology*" the most recent(2021) view about ZEO is developed by making making objections against the the earlier view [L54, L53]. This view provides a formulation of ZEO at the level of "world of classical worlds" (WCW) and provides surprisingly insights to quantum TGD and TGD inspired theory of consciousness.

For a fixed 3-surface at PB, quantum non-determinism corresponds to a discrete classical non-determinism due to the failure of a strict determinism. The classical orbits of the 3-surface form a tree and zero energy states are superpositions of paths of this tree. Interestingly, this notion of non-determinism is equivalent with the notion of association sequence that I introduced in the early developmental phases of TGD inspired theory of consciousness.

8.2 Zero Energy Ontology in its original form

Zero energy ontology has changed profoundly the views about the construction of S-matrix and forced to introduce the separate notions of M-matrix and U-matrix. M-matrix generalizes the notion of S-matrix as used in particle physics. The unitary U-matrix is something new having a natural place in TGD inspired theory of consciousness. Therefore it it best to begin the discussion with a brief summary of zero energy ontology.

8.2.1 Motivations For Zero Energy Ontology

Zero energy ontology was first forced by the finding that the embeddings of Robertson-Walker cosmologies to $M^4 \times CP_2$ are vacuum extremals. The interpretation is that positive and negative energy parts of states compensate each other so that all quantum states have vanishing net quantum numbers. One can however assign to state quantum numbers as those of the positive energy part of the state. At space-time level zero energy state can be visualized as having positive energy part in geometric past and negative energy part in geometric future. In time scales shorter than the temporal distance between states positive energy ontology works. In longer time scales the state is analogous to a quantum fluctuation.

Zero energy ontology gives rise to a profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

8.2.2 Zero Energy Ontology

Zero energy ontology (ZEO) is one of the cornerstones of TGD and has become part of TGD during last six years. Zero energy states are identified as superpositions of pairs of positive and negative energy states assigned with the future and past boundaries of causal diamonds (CDs) and correspond in ordinary ontology to physical events with positive and negative energy parts of the state identified as counterparts for the initial and final states of the event. Effective 2-dimensionality allows a further reduction to the level of partonic 2-surfaces: also their 4-D tangent space data matter. Symmetry considerations lead to a beautiful view about generalizations S-matrix to U-matrix in terms o forthogonal M-matrices which in turn are expressible as products of orthogonal basis of hermitian square roots of density matrices and unitary S-matrix [K75]. One can say that quantum theory is "complex" square root of thermodynamics.

Therefore one should try to find tests for ZEO.

The hierarchy of CDs

The basic assumption is that the sizes of CDs come as integer multiples of CP_2 scale R and for prime multiples of R correspond to secondary p-adic length scales $L_{p,2} = L_{p,1}\sqrt{p}$, $L_{p_1} = R\sqrt{p}$, where R denotes CP_2 scale. For electron with $p = M_{127} = 2^{127} - 1$ one has $T_{p_2} = .1$ seconds and defines a fundamental bio-rhythm. This time scale should have preferred role in physics. More generally the secondary p-adic time scales assignable to elementary particles should define time scales relevant to macroscopic physics. The corresponding size scale can be assigned to the magnetic body of the elementary particle. Also it should be possible to assign to quark mass scales special biological time scales as has been indeed done [K10]. h predictions could be tested.

Generalization of standard conservation laws in ZEO

ZEO together with sub-manifold geometry provides a new view about conservation laws and resolves the problem posed by the fact that gravitational interactions do not seem to respect energy conservation in cosmological time scales. Conservation laws holds true only in the scale associated with given CD, not universally (this would allow only single infinitely large CD).

Superconducting coherent states involve quantum superposition of states with different numbers of Cooper pairs and therefore break the super-selection rule associated with fermion number in ordinary ontology. In ZEO they could be understood without giving up the superselection rule associated with fermion number.

Experimental tests should try to prove that quantum number conservation is a length scale dependent notion. For instance, creation of matter from vacuum is possible in ZEO, and one might hope that its occurrenece could be in some scale for CDs aritficially.

Breaking of second law in standard form

In standard physics second law states that all systems are entropic but a system can reduce its entropy by feeding its entropy to the environment. Negentropic entanglement carries genuine information and life can be seen as islands of negentropy in the sea of entropy. This forces to generalized second law. The proposed generalization (see `http://tinyurl.com/ybg8qypx`) [L13] [K70] can be characterized as maximally pessimistic.

The generation of negentropic entanglement is assumed to be accompanied by generation of compensating entropic entanglement. The modified form of second law is suggested by the mechanism of directed attention based on negentropic entanglement assignable to magnetic flux tube connecting selfandtarget. Negentropic entanglement prevails during the attention but disappears after state function reduction giving rise to entropy at the level of ensemble. Second law would hold true above time scale assignab le to the duration of negentropic entanglement.

There are also other reasons to reconsider second law. The breaking of second law in standard form since the arrow of geometric time can change locally. Living systems are indeed accompanied by syntropic effects as realized by Italian quantum physicist Fantappie [J7, J9]. These effects could be understood as entropic effects but with a reversed arrow of geometric time. The mechanism would be based on negative energy signals. Phase conjugate laser waves are known to obey second law in reversed direction of geometric time. Cooling effects due to the absorption of negative energy signals inducing the breaking of the standard form of the second law are predicted to be possible. One can also imagine a spontaneous excitation of atoms generating radiation in the return to ground state in a situation when there is a target able to receive negative energy signals emitted in spontaneous excitation.

Standard form of second law assumes that quantum coherence is absent in the scales in which it is applied. Both the hierarchy of Planck constants and negentropic entanglement however make possible macroscopic quantum coherence characterized by the scale involved and the natural guess is that the time scale associated with causal diamond in question defines the scale above which one can expect second law to hold. There is evidence for the breaking of second law in time scale of.1 seconds [D2].

Negative energy signals

Zero energy ontology allows to assign to zero energy states an arrow of time naturally since one can require that states have well defined single particle quantum numbers at either upper or lower boundary of CD. Also the spontaneous change of the arrow of geometric time is possible. The simplest possible description for U-process is that U-matrix relates to each other these two kinds of states and state function reductions can occur at upper and lower boundaries of CD meaning reduction to single particle states with well defined quantum numbers. The precise correlates for the generation of geometric arrow of time are not completely understood.

Negative energy signals to geometric past would serve as counterparts for time reversed states in the case of radiation and phase conjugate laser waves are natural counterparts for them. The signal property requires a dissipative process proceeding in preferred time direction and this kind of process has been assigned to sub-CDs and should proceed as state function reduction sequence in preferred direction of time determined by the quantum arrow of time for the zero energy state. This process would be essential for the experience of flow of time in preferred direction and for generation of arrow of geometric time as explain in previous chapter and also in [K8]. For phase conjugate laser beams the reversed time direction for dissipation is observed.

Negative energy signals make possible remote metabolism as sucking of energy from remote energy source provided resonance conditions for transitions are satisfied. The counterpart of population inverted laser could serve as ideal source and the negative energy signal could serve as a control switch inducing phase transition like process taking the excited atom like systems to ground state (induce emission). This process should occur in living matter. Anomalous excitation of atomic state by absorbing energy by remote metabolism and subsequent generation of radiation could also serve as a signature. It could also lead to cooling effects breaking second law.

Negative energy signals would also make possible realization of intentional action by initiating the activity already in geometric past. This would be very desirable in rapidly changing circumstances. The time anomalies of Libet for active aspect of consciousness could be interpreted in terms of time mirror mechanism [J2] and further experiments in longer time scales might be perhaps carried out.

Negative energy signals could be also essential for the mechanism of long term memory. They would induce a breakdown for a system analogous to population reversed laser via induced emission meaning generation of strong positive energy signal [K93].

Definition of energy in zero energy ontology

The approach relying on the two super conformal structures of quantum TGD gives hopes of defining the notion of energy for positive and negative energy parts of the state.

1. CD allows translational invariance only in its interior and since partonic two surfaces are located to the boundary of CD, one can argue that translations assigned to them lead out from CD. One can however argue that if it is enough to assign eigenstates of four-momentum to partons and require that only the total four-momentum generators acts on the physical state by shifting CD. Since total four-momentum vanishes for CD this would mean that wave function in cm degrees of CD is just constant plane wave. Super-conformal invariance would indeed allow to assign momentum eigenstates to the super-conformal representations.

2. A more stringent condition would be that four-momentum generators act as translation like operators on partons themselves. Since light-like 3-surfaces assignable to incoming and outgoing legs of the generalized Feynman diagrams are the basic objects, one can hope of having enough translational invariance to define the notion of energy. If translations are restricted to time-like translations acting in the direction of the future (past) then one has local translation invariance of dynamics for classical field equations inside $\delta M^4_\pm$ as a kind of semigroup. Also the M^4 translations leading to interior of X^4 from the light-like 2-surfaces surfaces act

as translations. Classically these restrictions correspond to non-tachyonic momenta defining the allowed directions of translations realizable as particle motions. These two kinds of translations can be assigned to super-symplectic conformal symmetries at $\delta M^4_\pm \times CP_2$ and and super Super-Kac-Moody type conformal symmetries acting as super-symplectic isometries. Super-symplectic algebra is realized in terms of second quantized spinor fields and covariantly constant modes of right-handed neutrino. Symplectic group has as sub-group symplectic isometries and the Super-Kac-Moody algebra associated with this group and represented in terms of spinor modes localized to string world sheets plays also a key role in TGD.

Finite M^4 translations to the interior of CD do not respect the shape of the partonic 2-surface. Local M^4 translations vanishing at the boundary of CD however act as Kac-Moody symmetries of the light-like 3-surfaces and reduce physically to gauge transformations: hence one could allow also the deformations of the partonic 2-surface in the interior of the light-like 3-surface. This corresponds to the effective metric 2-dimensionality stating that all information both about the geometry of WCW and quantum physics is carried by the partonic 2-surfaces X^2 resulting as intersections of the light-like 3-surfaces X^3_l and space-like 3-D surfaces X^3 at the boundaries of CD and the distribution of 4-D tangent planes of X^2.

3. The condition selecting preferred extremals of Kähler action is induced by a global selection of $M^2 \subset M^4$ as a plane belonging to the tangent space of X^4 at all its points [K28] and interpreted as a plane of nonphysical polarizations so that direct connection with number theory and gauge symmetries emerges. The M^4 translations of X^4 as a whole in general respect the form of this condition in the interior. Furthermore, if M^4 translations are restricted to M^2, also the condition itself - rather than only its general form - is respected. This observation, the earlier experience with p-adic mass calculations, and also the treatment of quarks and gluons in QCD encourage to consider the possibility that translational invariance should be restricted to M^2 translations so that mass squared, longitudinal momentum and transversal mass squared would be well defined quantum numbers. This would be enough to realize zero energy ontology. Encouragingly, M^2 appears also in the generalization of the causal diamond to a book-like structure forced by the realization of the hierarchy of Planck constant at the level of the embedding space.

4. That the cm degrees of freedom for CD would be gauge like degrees of freedom sounds strange. The paradoxical feeling disappears as one realizes that this is not the case for sub-CDs, which indeed can have non-trivial correlation functions with either upper or lower tip of the CD playing a role analogous to that of an argument of n-point function in QFT description. One can also say that largest CD in the hierarchy defines infrared cutoff.

Objection against zero energy ontology

In Zero Energy Ontology (ZEO), the basic geometric structure is causal diamond (CD), which is a subset of $M^4 \times CP_2$ identified as an intersection of future and past directed light cones of M^4 with points replaced with CP_2. Poincare symmetries are isometries of $M^4 \times CP_2$ but CD itself breaks Poincare symmetry.

Whether Poincare transformations can act as global symmetries in the "world of classical worlds" (WCW), the space of space-time surfaces - preferred extremals - connecting 3-surfaces at opposite boundaries of CD, is not quite clear since CD itself breaks Poincare symmetry. One can even argue that ZEO is not consistent with Poincare invariance. By holography one can either talk about WCW as pairs of 3-surfaces or about space of preferred extremals connecting the members of the pair.

First some background.

1. Poincare transformations act symmetries of space-time surfaces representing extremals of the classical variational principle involved, and one can hope that this is true also for preferred extremals. Preferred extremal property is conjectured to be realized as minimal surface property realize in terms of appropriately generalized holomorphy property meaning that field equations are separately satisfied for Kähler action and volume action apart from 2-D string world sheets and their boundaries to which one can assign the analog of Kähler action.

2. String world sheets and their light-like boundaries carry elementary particle quantum numbers identified as conserved Noether charges assigned with second quantized induced spinors solving modified Dirac equation determined by the action principle determining preferred extremals. Quantum classical correspondence (QCC) states that classical charges are equal to the eigenvalues of Poincare generators in Cartan algebra of Poincare algebra. This would hold quite generally.

3. The ground states of super-symplectic and super-Kac-Moody representations correspond to spinor harmonics with well-defined Poincare quantum numbers. Excited states are obtained using generators of symplectic algebra and have well-defined four-momenta identifiable also as classical momenta.

4. In ZEO one assigns opposite total quantum numbers to the boundaries of CD: this codes for the conservation laws. The action of Poincare transformations can be non-trivial at second (active) boundary of CD only and one has two kinds of realizations of Poincare algebra leaving either boundary invariant. Since Poincare symmetries extend to Kac-Moody symmetries analogous to local gauge symmetries it should be possible to achieve trivial action at passive boundary of CD so that the Cartan algebra of symmetries acts non-trivially only at the active boundary of CD. Physical intuition suggests that Poincare transformations on the entire CD treating it as a rigid body correspond to trivial center of mass quantum numbers.

How do the Poincare transformations act on 3-surfaces at the active boundary of CD?

1. Zero energy states are superpositions of 4-D preferred extremals connecting 3-D surfaces at boundaries of CD, the ends of space-time. One should be able to construct the analogs of plane waves as superpositions of space-time surfaces obtained by translating the active boundary of CD and 3-surfaces at it so that the size of CD increases or decreases. The translate of a preferred extremal is a preferred extremal associated with the new pair of 3-surfaces and has size and thus also shape different from those of the original. Classical theory becomes obviously an essential part of quantum theory.

2. Four-momentum eigenstate is an analog of plane wave which is superposition of the translates of a preferred extremal. In practice it is enough to have wave packets so that in given resolution one has a cutoff for the size of translations in various directions. As noticed, QCC requires that the eigenvalues of Cartan algebra generators such as momentum components are equal to the classical charges.

8.2.3 The Anatomy Of Quantum Jump In Zero Energy Ontology (ZEO)

Zero energy ontology (ZEO) emerged around 2005 and has had profound consequences for the understanding of quantum TGD. The basic implication is that state function reductions occur at the opposite light-like boundaries of causal diamonds (CDs) forming a hierarchy, and produce zero energy states with opposite arrows of time. Also concerning the identification of quantum jump as moment of consciousness ZEO encourages rather far reaching conclusions. In ZEO the only difference between motor action and sensory representations is that the arrows of embedding space time (CDs) are opposite for them. Furthermore, sensory perception followed by motor action corresponds to a basic structure in the sequence of state function reductions and it seems that these processes occur fractally for CDs of various size scales.

1. State function reduction can be performed to either boundary of CD but not both simultaneously. State function reduction at either boundary is equivalent to state preparation giving rise to a state with well defined quantum numbers (particle numbers, charges, four-momentum, etc...) at this boundary of CD. At the other boundary single particle quantum numbers are not well defined although total conserved quantum numbers at boundaries are opposite by the zero energy property for every pair of positive and negative energy states in the superposition. State pairs with different total energy, fermion number, etc.. for other boundary are possible: for instance, the coherent states of super-conductor for which fermion number is ill defined are possible in zero energy ontology and do not break the super-selection rules.

2. The basic objects coding for physics are U-matrix, M-matrices and S-matrix. M-matrices correspond to hermitian square roots of density matrices multiplied by a universal S-matrix which depends on the scale n of CD in very simple manner: $S(n) = S^n$ giving thus a unitary representation for scalings. The explicit construction of a unitary U-matrix in terms of M-matrices is carried out in [K75]: U-matrix elements are essentially inner products of M-matrices associated with CDs with various size scales. One can say that quantum theory is formally a square root of thermodynamics. The thermodynamics in question would however relate more naturally to NMP rather than second law, which at ensemble level and for ordinary entanglement can be seen as a consequence of NMP.

   The non-triviality of M-matrix requires that for given state reduced at say the "lower" boundary of CD there is entire distribution of states at "upper boundary" (given initial state can lead to a continuum of final states). Even more, all size scales of CDs are possible since the position of only the "lower" boundary of CD is localized in quantum jump whereas the location of upper boundary of CD can vary so that one has distribution over CDs with different size scales and over their Lorentz boots and translates.

3. The quantum arrow of time follows from the asymmetry between positive and negative energy parts of the state: the other is prepared and the other corresponds to the superposition of the final states resulting when interactions are turned on: also quantum superposition over CDs of different sizes with second boundary belonging to the same fixed $\delta M^4_{\pm}$ is possible. What is remarkable that the arrow of time at embedding space level (at least) changes direction as quantum jump occurs to opposite boundary.

   It is however possible to have sequences of quantum jumps occurring at the same boundary: these periods are counterparts for repeated state function reductions, which do not change the state at all in standard quantum measurement theory. During these periods the superposition of opposite boundaries of CDs and states at them change, and the average distance between the tips of CDs tends to increase, hence the flow of subjective time and its arrow.

   NMP dictates when the first quantum jumps to the opposite boundary of CD takes place. The sequence of state function reduction at the same boundary defines self as a conscious entity and the increase of the average distance between the tips of CD defines the life-time of self.

   This brings strongly in mind the old proposal of Fantappie [J7] that in living matter the arrow of time is not fixed and that entropy and its diametric opposite syntropy apply to the two arrows of the embedding space time. The arrow of subjective time assignable to second law would hold true but the increase of syntropy would be basically a reflection of second law since only the arrow of the geometric time at embedding space level has changed direction. The arrow of geometric at space-time level which conscious observer experiences directly could be always the same if quantum classical correspondence holds true in the sense that the arrow of time for zero energy states corresponds to arrow of time for preferred extremals. The failure of strict non-determinism making possible phenomena analogous to multi-furcations makes this possible.

4. This picture differs radically from the standard view and if quantum jump represents a fundamental algorithm, this variation of the arrow of geometric time should manifest itself in the functioning of brain and living organisms. The basic building brick in the functioning of brain is the formation of sensory representation followed by motor action/volition realized as the first reduction at the opposite boundary.

   These processes look very much like temporal mirror images of each other such as the state function reductions to opposite boundaries of CD look like. The fundamental process could correspond to a sequences of these two kinds of state function reductions at opposite boundaries of CDs and maybe independently for CDs of different size scales in a "many-particle" state defined by a union of CDs.

How the formation of cognitive and sensory representations could relate to quantum jump?

1. The earlier view was based on the idea that p-adic space-time sheets can transform to real ones and vice versa in quantum jump and these process correspond to a realization of intention as

action and formation of though. This view is mathematically awkward and has been replaced with the adelic vision in which all systems have both sensory (real space-time sheets) and cognitive (p-adic space-time sheets) space-time correlates. The real and p-adic number fields form a book like structure - adele- with an algebraic extension of rationals as its back. Same applieds at the level of embedding space, space-time surfaces, and WCW. In this framedwork holography makes it possible to understand real and p-adic space-time surfaces as continuations of string world sheets and partonic 2-surfaces to space-time surfaces, either real or p-adic. The string world sheets themselves are in the intersection of reality and various p-adicities in the sense that the parameters characterizing them belong to an extension of rational numbers.

2. Self having the mental image about intention can be be seen as the agent transforming intention to action. By NMP negentropy is typically generated in this transition tending to increase the value of Planck constant $h_{eff} = n \times h$ and thus reducing quantum criticality and occurring therefore spontaneously. Negentropy Maximization Principle eventually forces the occurrence of volitional action - self experiences the urge to perform the action so strong that cannot resist. Subself representing the mental image about intention tries to prevent it as long as possible because it means death: all living systems try to stay at the existing level of criticality and avoid the fatal final state function reduction by practicing homeostasis and using metabolic energy. Weak form of NMP states that self has freedom to decide whether it performs the reduction producing maximal entanglement negentropy. It can also perform ordinary quantum jump reducing entanglement entropy to zero and destroying entanglement. The outcome is isolation from the external world. The motivation for the weak form of NMP is that we do not live in the best possible world and have free will to choose between Good and Evil. Strong form of NMP would produce always mazimal negentropy gain and would mean best possible world.ur in various length scales in fractal manner.

8.2.4 Conscious Entities And Arrow Of Time In TGD Universe

"Fractality from your blog" posed an interesting question about possible asymmetry between boundaries of causal diamond CD. The answer to the question led to recall once again the incomplete understanding of details about how the arrow of time emerges in zero energy ontology (ZEO).

The basic vision is following.

1. CDs form a fractal scale hierarchy. Zero energy states possess a wave function in moduli degrees of freedom characterizing sizes of CDs as well telling what Lorentz boost leaving boundary invariant are allowed for them. Boosts form by number theoretic constraints a discrete subgroup of Lorentz group defining analogs of lattices generated by boosts instead of translations.

2. The arrow of subjective time maps to that of geometric time somehow. The origin of arrow comes from the fact that state function reductions can occur to either boundary of given CD and reduction creates time-asymmetric state since second boundary of CD is in a quantum superposition of different sizes and there is a superposition of many-particle states with different particles numbers and quantum number distributions. It is possible that each state function reduction leaving the passive boundary intact, involves localization in the moduli space of CDs with second boundary fixed.

3. Subjective existence corresponds to a sequence *of moments of consciousness*: state function reductions at opposite boundaries of CDs. State function reduction localizes either boundary but the second boundary is in a quantum superposition of several locations and size scales for CD. This predicts that the arrow of time is not constant. In fact, there is considerable evidence for the variation of the arrow of time in living systems and Fantappie [J7] introduced long time ago the notion of syntropy to describe his view about the situation.

4. The first very naïve proposal was that state function reductions occur *alternately* to the two boundaries of CD. This assumption would be indeed natural if one considered single fixed

CD rather than superposition CDs with different size and state function reduction localizing their either boundary: restriction to single CD was what I indeed did first.

5. This assumption leads to the question about why do we do not observe this alternation of the arrow of time all the time in our personal experience. Some people actually claim to have actually experienced a temporary change of the arrow of time: I belong to them and I can tell that the experience is frightening. But why do we experience the arrow of time as stable in the standard state of consciousness?

One possible way to solve the problem - perhaps the simplest one - is that state function reduction to the same boundary of CD can occur many times repeatedly. This solution is so absolutely trivial that I could perhaps use this triviality to defend myself for not realizing it immediately!

I made this totally trivial observation only after I had realized that also in this process the wave function in the moduli space of CDs change in these reductions. Zeno effect in ordinary measurement theory relies on the possibility of repeated state function reductions. In the ordinary quantum measurement theory repeated state function reductions do not affect the state in this kind of sequence but in ZEO the wave function in the moduli space labelling different CDs with the same boundary could change in each quantum jump. It would be natural that this sequence of quantum jumps give rise to the experience about flow of time? This option would allow the size scale of CD associated with human consciousness be rather short, say.1 seconds. It would allow to understand why we do not observe continual change of arrow of time.

Maybe living systems are working hardly to keep the personal arrow of time un-changed - living creatures try to prevent kettle from boiling by staring at it intensely. Maybe it would be extremely difficult to live against the collective arrow of time.

An objection against this picture as compared to the original one assuming alternate reductions to the opposite boundaries of CD is that is that one can understand state preparation as state function reduction to the opposite boundary. This interpetation makes sense almost as such also in the new picture if the average time period for which the reductions occur to a given boundary is shorter in elementary particles scales than in macroscopic scales characteristic for human consciousness. The approximate reversibility in elementary particle scales can be understood as summing up of the two arrows of time to no arrow at all.

This picture allows also to identify self as a continuous entity as the sequence of state function reductions occurring at the same boundary of CD. The average increase of the temporal distance between the tips of cD defines the life-time of self. The number of reductions would give a measure for the subjectively experienced of life-time of self.

In elementary particle time scales reversibility is a good approximation and this suggests that in elementary particle scales the number of state function reductions at the same boundary of CD is small so that the effects due to the change of the arrow of time cancel on the average.

NMP would eventually force "death" of self since the state function reduction at opposite boundary would generate more negentropy. "Death" of self would mean birth of self asssociated with the opposite boundary of CD. The age of self identified as the proper time distance between the tips would increase in statistical sense even when its arrow can change. The act of volition would have a natural identification as the first state function reduction at the opposite boundary of CD.

This picture raises a series of questions. Do our wake-up periods correspond to sequences of state function reductions for self and are sleeping periods wake-up periods of the self at the opposite boundary of CD? The arrow of geometric time should change at some space-time sheet associated with the self hierarchy. How could one demonstrate this? Are the memories of the "other" self predictions of future from our point of view? Do we sleep in order to get information from future, to remember what the future will be?

How the hierarchy of Planck constants defining a hierarchy of quantum criticalities does relate to this picture? The ageing of self having has as a correlate the increase of the size scale of CD. Could this increase be due to the increase of h_{eff} expected to occur spontaneously since it corresponds to a reduction of criticality and therefore to the appearance of new physical degrees of freedom as symplectic gauge degrees of freedom transform to physical ones in gauge symmetry breaking. This is not the case. The time evolution must be analogous to shift in time rather than scaling. This of course corresponds to the QFT view about time evolution.

In the first state function reduction to the opposite boundary of CD however scaling of CD is possible and would correspond to the scaling of CD represented by exponent of infinitesimal scaling operator as in conformal field theories. The emergence of bew physical degrees of freedom suggest increasing perceptive and cognitive capabilities. The increase of h_{eff} could be seen as evolution as also the associated increase of resources of negentropic entanglement suggests. The total increase of h_{eff} measured by the ratio $h_{eff}(final)/h_{eff}(initial)$ could be seen as a measure for the progress per single life period of self.

8.2.5 Copenhagen interpretation dead: long live ZEO based quantum measurement theory!

I encountered a very interesting ScienceDaily article "*Physicists can predict the jumps of Schrödinger's cat (and finally save it)*" (see `http://tinyurl.com/y5lpe2eo`). The experimental findings described in the article are extremely interesting from the point of view provide by TGD inspired quantum measurement theory relying on Zero Energy Ontology (ZEO) and provides a test for it.

In standard quantum measurement theory (Copenhagen interpretation) of Bohr quantum jump is random in the sense that it occurs with predictable probabilities to an eigenstate of the measured observables. Their occurrence cannot be predicted and even less prevented - except by monitoring - Zeno effect.

The findings of Minev *et al* are described in the article "*To catch and reverse a quantum jump mid-flight*" [L77] (see `https://arxiv.org/abs/1803.00545`). The outcome of quantum jump is indeed unpredictable but the time of occurrence is to high degree predictable: there is a detectable warning signal, period of "flight" from the initial to the final state!

A curious feature is that the external signal responsible for the quantum jump can be stopped during the "flight" from the initial to final state. As if the quantum jump is analogous to a domino effect. It is also claimed that the jump can be reversed during flight period by a control signal: if jump has already occurred then one might argue that the control signal induces quantum jump in opposite direction when applied at time which is roughly the mid-time of "flight".

If the findings by Minev *et al* are replicable, one is forced to give up the basic assumption of the standard quantum measurement theory stating that state function reductions occur completely randomly and instantaneously. State function reduction (SR) looks like a continuous, deterministic process. Bohr's theory would be dead also officially and one must finally go back to the blackboard and start serious thinking about fundamentals. It took 92 years - almost a century! State function reduction (SR) is definitely more complex phenomenon than predicted by Bohr.

What is most intriguing that SR looks smooth, deterministic classical time evolution although the outcome is not predictable. People loving hidden variables might be happy but better to think about this more precisely before jumping to any conclusions. Authors apply so called quantum trajectory theory to describe the findings [B47] and report that the model is able to predict the parameters of the parameterization with one per cent accuracy.

Zero energy ontology (ZEO) based view about quantum measurement and the relationship between geometric and subjective time explains why state function reduction looks like a deterministic process. Unfortunately, what ZEO is, is not completely clear [L81]. This allows to consider two options.

1. Both options imply that one can apparently anticipate quantum jump. This could be however an illusion: the observed classical time evolution could occur *after* the quantum jump in opposite direction of time. The fact that the absence of the signal inducing quantum jump does not affect the occurrence of quantum jump suggests that the "flight" period indeed represents the classical evolution after the quantum jump in the reversed direction of time so that the absence of the external signal would not anymore affect the situation. Generalized Zeno effect is essential element ZEO based quantum measurement theory so that SR might be prevented. Perhaps a more plausible interpretation is that the control signal induces the reversal of the quantum jump already occurred. A careful analysis to distinguish between subjective and geometric time and arrows of time for the observer and atom would be needed.

2. The more conventional option nearer to the interpretation of experimenters is that the observered time evolution occurs *before* the quantum jump in standard direction. The period

before quantum jump consists of a sequence of "small" state function reductions - "weak" measurements. $M^8 - H$ duality suggests a concrete assignment of the moments of time to them [L81] and there would be also the last moment of this kind. After these things proceed to "big" state function reduction in analogy with domino effect. It is not however obvious why the classical time evolution should appear to converge to the final outcome deterministically.

First ZEO based based view about the findings

What about TGD and zero energy ontology (ZEO) based quantum measurement theory [K75]? Could it explain the revolutionary findings?

1. The new element is that quantum states are not time= constant snapshots for time evolution but superpositions of entire deterministic time evolutions at the level of space-time surfaces and at the level of induced spinor fields. SR replaces super position of classical time evolutions with a new one. This like selecting and starting new deterministic computer program. Non-determinism is in these choices [L56].

2. The notion of causal diamond (CD) identified as an intersection of future and past directed light-cones of M^4 with points replaced with CP_2 is crucial. The notion of CD is strongly suggested by the gigantic symmetries of CD essential for the construction of quantum TGD. CD could be seen as embedding space correlate for the perceptive field of a conscious entity - self. The upper boundary of CD - to be called active boundary A represents the boundary for space-time region from which self can receive classical signals and is therefore natural. The lower boundary, to be called passive boundary B, brings in mind cosmic expansion and follows as a prediction from $M^8 - H$ duality.

3. There are two kinds of state function reductions in ZEO.

   (a) In "small" SRs (SSRs) the states change at active boundary of causal diamond (CD) (call it A) but remain unchanged at passive boundary (call it P): generalized Zeno effect occurs at the passive boundary and "weak measurements" (see `http://tinyurl.com/zt36hpb`) at A. The observables measured commute with those determining the states at P as their eigenstates. In particular, the location of A is measured localizing it and corresponds to the measurement of time as distance between the tips of CD.

   "Big" SRs (BSRs) reverse the arrow of time of zero energy states and the roles of A and P. BSR is preceded by a sequence of SSRs - "weak" or almost classical measurements. In TGD inspired theory of consciousness [L56, L81] [K62] this sequence defines the life cycle of a conscious entity - self.

What is of crucial importance that BSR creates the illusion that it is an outcome of a continuous process: this realizes quantum classical correspondence (QCC). Standard observer assumes standard arrow of time and the space-time surfaces in the final time reversed state seem to lead to the 3-surface serving as a correlate for the final state! As if BSR would be outcome of a smooth deterministic process, which it is not! There is actually a superposition of these 3-surfaces at A after BSR but in the resolution used this is not detected. Putting it more precisely:

1. The time reversal of time evolution is in good approximation obtained by time reflection symmetry T but not quite since T is slightly broken. This is extremely small effect.

2. Before BSR one has a distribution of 3-surfaces X^3 defining the ends of space-time surfaces X^4 at A: 3-surfaces X^3 corresponds to different outcomes of BSR and can differ dramatically. Observer is not conscious of this. This is like a situation of Schrödinger cat before measurement: it is impossible to be conscious about the superposition of dead and alive cat.

   After BSR one has quantum superposition of space-time surfaces directed to geometric past. Near the end of space-time at A they look like leading to a unique classical counterpart of final state of state function reduction. As if the state function reduction were a smooth, continuous, deterministic process. BSR guarantees this but BSR is not a smooth evolution.

The experimental findings could be understood by applying this general picture.

1. One can assign to the evolution from initial state G of atom at P to final state E at A a sequence of small reductions, weak measurements and also superposition of classical time evolutions approximated by single evolution in given measurement resolution. The state E is superposition of various measurement outcomes and each of them corresponds to a superposition of space-time surfaces identical in the measurement resolution used.

2. Then occurs the BSR: atom jumps from state E to state D. This selects from the superposition of space-time surfaces/time only the evolutions apparently leading to D. Or more precisely: the superposition of reversed time evolutions starting from D at A and very similar near A but deviating farther from it. The illusion about continuous, smooth, deterministic time evolution from G to D is created!

3. Also the possibility to anticipate the reduction would be an illusion due to the different arrows of time for observer and the observed system after BSR. The time reversed time evolution actually starts from the final state. The warning signal (absence of photon emission would be natural consequence of the reduction but in reversed arrow of time. The illusion would be due to the identification of arrows of time of observer and the atom that made state function reduction. This conforms with the observation that one can drop away the periodic signal inducing the quantum jumps during the "flight" period identified as the deterministic process representing the quantum jump.

   The lesson would be that one must always check whether the arrow of time for the target of attention is same as my own. Not a good idea to be on the wrong lane (means death also in ZEO based consciousness theory).

   It is also claimed that on can prevent the quantum jump using a signal during the "flight" period. Generalized Zeno effect is basic element of TGD but the signal forcing the state to remain in P would be present before the quantum jump. This would suggest that the control signal induced quantum jump in opposite direction. To really understand the situation a careful analysis of the relationships between subjective and geometric times of observer and between geometric time of observer and atomic system after and before the quantum jump would be needed.

Also Libet's findings about active aspects of consciousness [J2] can be interpreted in ZEO along the same lines. The observation that the neural activity begins before conscious decision can be understood by saying that the act of free will as a big state function reduction changed the arrow of time for an appropriate subsystem of the system studied. Tte time reversed classical evolutions from the outcome of the volitional action were interpreted erratically as a time evolution leading to the conscious decision. A less precise manner to say this is that conscious decision (big state function reduction) sent a classical signal to geometric past with opposite arrow of time initiating neural activity. Libet's finding led physicalistic neuroscientists to conclude that free will is an illusion. The actual illusions were physicalism and the belief that arrow of time is always the same.

To sum up, ZEO is fantastic magician. Maybe this magic is necessary for the mental health of observer: a world without this illusion would be like nightmare where one cannot trust anything.

Second ZEO based based view about the findings inspired by M^8-H duality

I have learned to take experimental findings very seriously and I am ready to aks whether the above described option the only possibility allowed by ZEO or can one think other alternatives? It would be nice to answer "No" but one can consider variants of ZEO [L81] inspired by so called M^8-H duality [L48, L96].

The sequence of "small" state function reductions (SSRs) should have the last one. Is the "big" state function reduction (BSR) forced by some condition? One idea is that the life cycle of self corresponds to a measurement of all observables assignable to the active boundary A of CD and commuting with those defining the unaffected states at passive boundary P are measured (time as a location of A belongs to these observables measured in each SSR).

I have discussed in [L81] possible modifications of ZEO inspired by so called M^8-H duality [L48, L96]. One motivation is that time flow as shifting M^4 time $t = constant$ hyper-plane can be argued to be more natural than that for light-cone boundary. Light-cone boundaries are however favored by its huge symmetries essential for the definition of the geometry of "world of classical worlds" (WCW). M^8-H duality forces passive light-cone boundary P and the identification of A as boundary of region where sensory signals can arrive to self is natural.

M^8-H duality allows to consider variants the original ZEO.

1. M^8-H duality

Let us first briefly summarize what M^8-H duality [L48] is.

1. M^8-H duality is one of the key ideas of TGD, and states that one can regard space-times as surfaces in either complexified octonionic M^8 or in $M^4\times CP_2$. The dynamics M^8 is purely algebraic and requires that either tangent or normal space of space-time surface is associative (quaternionic).

2. The algebraic equations for space-time surfaces in M^8 state the vanishing of either the real or imaginary part (defined in quaternionic sense) for octonion valued polynomial $P(o)$ with real coefficients. Besides 4-D roots one obtains as universal exceptional roots 6-spheres at boundary of the light-cone of M^8 with radii given by the roots r_n of the polynomial in question. They correspond to the balls $t=r_n$ (t is octonionic real coordinate) inside Minkowski light-cone with each point have as fiber a 3-sphere S^3 with radius contracting to zero at the boundary of the light-cone of M^4. These 6-spheres are clearly analogous to branes connected by 4-D space-time surfaces.

3. The intersections of space-time surfaces with 6-spheres would be 2-D and I have interpreted them as partonic 2-surfaces identifiable as topological particle reaction vertices - partonic 2-surfaces - at which incoming and outgoing light-like 3-surfaces meet along their ends. These light-like 3-surfaces - partonic orbits - would represent the boundaries between space-time regions with Euclidian and Minkowskian signatures of the induced metric. Partonic 2-surfaces would be analogs of the vertices of Feynman diagrams. The boundaries of string world sheets predicted as singularities of minimal surfaces defining space-time surfaces would be along the partonic orbits and give rise to QFT type description using cognitive representations and analogs of twistor diagrams consisting of lines.

2. M^8-H duality and consciousness

One can ask whether M^8-H duality and this braney picture has implications for ZEO based theory of consciousness. Certain aspects of M^8-H duality indeed challenge the recent view about consciousness based on ZEO (zero energy ontology) and ZEO itself.

1. The moments $t=r_n$ defining the 6-branes correspond classically to special moments for which phase transition like phenomena occur. Could $t=r_n$ have a special role in consciousness theory?

   (a) For some SSRs the increase of the size of CD reveals new $t=r_n$ plane inside CD. One can argue that these SSRS define very special events in the life of self. This would not modify the original ZEO considerably but could give a classical signature for how many ver special moments of consciousness have occurred: the number of the roots of P would be a measure for the lifetime of self and there would be the largest root after which BSR would occur.

   (b) Second possibility is more radical. One could one think of replacing CD with single truncated future- or past-directed light-cone containing the 6-D universal roots of P up to some r_n defining the upper boundary of the truncated cone? Could $t=r_n$ define a sequence of moments of consciousness? To me it looks more natural to assume that they are associated with very special moments of consciousness.

2. For both options SSRs increase the number of roots r_n inside CD/truncated light-one gradually and thus its size? When all roots of $P(o)$ would have been measured - meaning that the largest value r_{max} of r_n is reached -, BSR would be unavoidable.

BSR could replace $P(o)$ with $P_1(r_1 - o)$: r_1 must be real and one should have $r_1 > r_{max}$. The new CD/truncated light-cone would be in opposite direction and time evolution would be reversed. Note that the new CD could have much smaller size size if it contains only the smallest root r_0. One important modification of ZEO becomes indeed possible. The size of CD after BSR could be much smaller than before it. This would mean that the re-incarnated self would have "childhood" rather than beginning its life at the age of previous self - kind of fresh start wiping the slate clean.

One can consider also a less radical BSR preserving the arrow of time and replacing the polynomial with a new one, say a polynomial having higher degree (certainly in statistical sense so that algebraic complexity would increase).

3. Is a more conservative view possible?

Could this picture allow to build a more conservative picture more akin to that proposed by experimenters?

1. The interpretation of the detected time evolution as that *before* the quantum jump would conform with the interpretation of experimentalists that a kind of domino effect is involved and also with the observation that stopping the signal causing the quantum jumps does not anymore affect the situation.

2. It is however unclear how to understand why the evolution looks like leading to the outcome unless the sequence of r_n:s defines a sequence of steps gradually taking the system near the final state.

3. What about preventing the BSR by external signal and even reversing the quantum jump? This would require an external perturbation of the octonionic polynomial increasing the value of the largest root r_{max} or even the degree of the polynomial and bringing in additional significant moments of life. Is it possible to speak about external perturbations of the coefficients of polynomials assumed to be rational numbers? The perturbations would come from a higher level in the hierarchy of selves (experimentalist), and one can imagine them in the framework of many-sheeted space-time.

To sum up, to my opinion (which could change) the first option looks more plausible. The introduction of moments $t = r_n$ as special moments in the life of self looks highly attractive and also the possibility of wiping the slate clear.

8.2.6 Could ZEO allow over-unity effects and quantum error correction mechanism?

The notion of free energy used by free energy researches is different from the standard notion of free energy appearing in thermodynamics. Free energy researchers typically claim over-unity effects. In strong form it would mean non-conservation of energy and academic community concludes that since energy is conserved, these claims are crackpot non-sense. In weaker form the claims would mean transformation of heat to work with efficiency larger than the upper bound predicted by Carnot law.

Second law kills hopes about perpetual motion machines based on this kind of over-unity effects. Second law however assumes fixed arrow of time. There exists however strong empirical evidence for the possibility that the arrow of time can change - phase conjugate waves are the key example. This led Fantappie [J7] to propose the notion of syntropy as entropy in reversed time direction. Second law in reverse time direction could also allow an error correction mechanism in quantum computation: Nature itself would do it. Phase conjugate waves are indeed known to perform error correction.

Quantum TGD relies on zero energy ontology (ZEO). ZEO allows both arrows of time and a temporary change of the arrow of time could make possible to break the standard laws of thermodynamics at least temporarily and in short enough scales. ZEO indeed plays a key role in TGD inspired quantum biology and quantum theory of consciousness.

In the following I will consider the issue of energy conservation in ZEO. Classically energy is well-defined and conserved in TGD Universe. But what about energy conservation in quantum sense? ZEO involves delocalization of states in time and this could allow energy conservation only in some resolution determined by the scale of the increment of time in given state function reduction inducing a shift of the active boundary of CD farther from the passive one.

Energy is conserved classically in TGD but what about conservation in quantum sense in ZEO?

ZEO guarantees classical conservation laws. What about the situation at quantum level? Could the energy associated with the positive energy part of zero energy state increase in quantum transitions and lead to over-unity effects? In principle, conservation laws do not prevent this quantally.

1. Recall that zero energy states [K75] [L56] are identified as superpositions of pairs (a, b) formed from states a and b having opposite total quantum numbers and being assigned with the opposite boundaries of causal diamond (CD). The states at the passive boundary B of CD are not affected whereas the states at the active boundary A are affected by a sequence of unitary time evolutions also shifting A farther away from B (in statistical sense at least).

   Each unitary evolution induces a de-localization of A in its moduli space and "small" SFR induces its localization (including time localization meaning time measurement). This sequence would approximately conserve the energies of the states in the superposition. This in the approximation that their energies are large in the energy scale $\Delta E = \hbar_{eff}/Deltat$ defined by the time increment Δt in single unitary time evolution. Large value of h_{eff} makes the conservation worse for a given Δt. Unitarity together with the approximate energy conservation implies that the average energy is approximately conserved.

2. Negative energy signals sent from A to its geometric past and received at B in remote metabolism would correspond to "big" SFR. If the notion of remote metabolism giving effectively rise to over-unitary effect is to make sense, the approximate energy conservation should fail in "big" SFRs in quantal sense. For this to be the case, the first unitary evolution of B followed by "small" SFR energy conservation should be a bad approximation. This does not however seem plausible if one assumes energy conservation for the next state function reductions. What could be so special in the first state function reduction?

3. Why the energy conservation made approximate by the finite size of CD and finite duration of unitary evolution, should fail badly in some situations? According to the number theoretic vision [L53], "small" SFRs preserve the extension of rationals defining the adele and therefore also $\hbar_{eff}/\hbar_0 = n$ identifiable as the dimension of the extension. $\hbar_{eff}/\hbar_0 = n$ can however change $n_{old} \rightarrow n_{new}$ in "big" SFRs forced to occur when "small" SFRs preserving n_{old} are not anymore possible. If a large increase of h_{eff} occurs in the "big" SFR, the $\Delta E = \hbar_{eff}/Deltat$ increases if Δt is still of the same order of magnitude. The approximate energy conservation could fail badly enough to make possible remote metabolism.

4. In the subsequent SFRs energy conservation should however hold true in good approximation. The values of Δt should be large in the subsequent "small" SFRs, and Δt should scale as $\Delta t \propto n$ to guarantee that ΔE remains the same. As a quantum scale Δt analogous to Compton length is indeed proportional to n. In the first reduction one must have of $n = n_{old}$ but in the subsequent reductions one must have $n = n_{new}$ to guarantee energy conservation in the same approximation as before.

   To sum up: in the first "small" SFR one should have $\Delta E \propto n_{new}$ and $\Delta t \propto n_{new}$. Can one really deduce this from the basic TGD?

5. ZEO suggests that evolution [L62] means a continual increase of the size of CD so that arbitrarily small CD could eventually grow to even cosmic size (whether this occurs always or whether zero energy state can become pure vacuum at both boundaries of CD remains an open problem). CD with a cosmic size should however have huge energy. This would not only require non-conservation of energy in quantal sense but also its increase in statistical sense at least. Why should the energy increase?

The increase would relate directly to the basic defining property of ZEO. Preferred time direction means that the transfer of quantum numbers, in positive energy with fixed sign, can take place only from the active boundary of CD to the passive boundary in "big" SFR. This allows interpretation as remote metabolism implying increase of the magnitude of energy.

Criticism

Quite recently (towards end of 2019) I found a more precice formulation for the intuitive notion of remote metabolism, which strongly suggests that energy is conserved in ZEO. There is a decomposition to system and the energy energy source: call them A and B. Intuitively, A receives energy from B by sending negative energy to B. What does this really mean?

1. A "big" state function reduction reversing arrow of time takes place: this would correspond to sending negative energy signal to past. The energy of A+B in the final time reversed state at new passive boundary of CD would be shared in new manner such that one can say that A has received from B the metabolic energy.

2. Energy would be conserved. I have also considered the interpretation that the total energy of the system associated with CD increases [K75] [L93]: since CD itself breaks Poincare invariance, it seems that one cannot exclude this. However, the Poincare invariance is realized at the level of moduli space for the positions of the either boundary of CD, and one can assume energy conservation. Even the wave functions at the boundary of CD can be taken to be in the representations of Lorentz group acting as its isometries. Plane waves correspond to wave functions in the moduli space for the boundary of CD keeping second boundary fixed.

3. To make this more precise one must define metabolic energy more precisely by introducing the hierarchy of Planck constants and the fact that the increase of h_{eff} of sub-system keeping other parameters constant increases it energy. Second law means that A tends to loose energy due to the decrease of h_{eff} for its sub-systems. This is true also for the time-reversed state but in opposite direction of geometric time so that with respect to standard direction of time the energy increases. This would provide extremely general purely thermodynamical mechanism of remote metabolism.

Could Nature provide an error correction mechanism for quantum computation?

Error correction has turned out to be major problem in the attempts to construct quantum computers. It is believed to be necessary because quantum entanglement is extremely fragile for the standard value of Planck constant. In TGD the situation changes. Large values of h_{eff} increasing the time scale of entanglement are possible and reversed time evolutions in quantum sense imply second law in reversed time direction meaning spontaneous reduction of entropy in the standard time direction. Nature itself would provide the needed error correction mechanism perhaps applied routinely in living systems (for instance, to correct mutations of DNA and transcription and translation errors).

To sum up, this picture is extremely interesting from the point of view of future technologies. One can even challenge the cherished law of energy conservation at quantum level (classically it remains exact in TGD Universe). Could one consider the possibility that the energy of system could be increased by the evolution by "big" state function reductions increasing the value of h_{eff}? Could one at least temporarily reduce entropy by inducing time evolutions in opposite time direction? TGD strongly suggests that these mechanisms are at work in biology. Maybe energy and iquantum nformation technologists could learn something from living matter?

8.2.7 Ballistic resonance and zero energy ontology

The popular article "*Scientists have discovered a new physical paradox*" (`https://cutt.ly/lp6VL4l`) tells about the work of Vitaly A. Kuzkin *et al* published in Phys Rev E (`https://cutt.ly/Jp8XHYY`) as an article [D9] with title "*Ballistic resonance and thermalization in the Fermi-Pasta-Ulam-Tsingou chain at finite temperature*" . The article describes very interesting experimental findings, which could provide a direct application of zero energy ontology (ZEO) based theory of self-organization.

The findings and their explanation provided by experimenters

Researchers from the Peter the Great St. Petersburg Polytechnic University (SPbPU) have discovered a new physical effect: the amplitude of mechanical vibrations can grow without external influence in which system converts its thermal energy to mechanical energy. The phenomenon is known as ballistic resonance. The description of the phenomenon involves also an abnormally high heat conductivity - one speaks of ballistic heat conductivity.

The electromagnetic analogy is very high electric conductivity: the work of Bandyopadhyay related to effects of oscillating voltage on currents flowing along microtubules demonstrates ballistic conductivity possibly reflecting underlying super-conductivity [L20].

This behavior seems to be in conflict with second law of thermodynamics telling that the vibrations should be attenuated. The researchers propose also a theoretical explanation of this paradox (`https://cutt.ly/Cp6V52j`) based on a model assuming ballistic heat conduction. One can of course wonder whether the notion of ballistic heat conduction is consistent with second law in its standard form.

Fermi-Past-Ulam-Tsingou problem (`https://cutt.ly/cp8CxtA`) was a finding about a theoretical model of a vibrating string with a non-linear dynamics. The expectation was that the situation develops ergodic so that energy is evenly divided between the modes of the string. It however turned out that the behavior was essentially periodic. The model explaining the behavior relies on solitons assignable to Korteveg-de-Vries equation. This phenomenon is different from the ballistic resonance observed in the experiments. In Korteveg de-Vries equation there is no dissipative term and the unexpected phenomenon is that wave pattern preserves it shape. Dissipation without energy feed would attenuate the wave.

ZEO based model for the findings

TGD suggests that a genuine explanation requires a profound change in the thinking about time- in particular the relationship between geometric time and experienced time must be updated. I call the new conceptual framework zero energy ontology ZEO) [L91]. The identification of these two times in standard ontology is in conflict with simple empirical facts, and leads to a paradox related to state function reduction (SFR) taking place in quantum measurement. The non-determinism of SFR is in conflict with the determinism of Schrödinger equation.

1. According to ZEO in ordinary state function reduction (SFR) the arrow of time subsystem changes: this solves the basic paradox of quantum measurement theory. The experiments of Minev *et al* [L77] give impressive experimental support for the notion in atomic scales, and sow that SFR looks completely classical deterministic smooth time evolution for the observer with opposite arrow of time. This is just what TGD predicts. Macroscopic quantum jump can occur in all scales but ZEO takes care that the world looks classical! The endless debate about the scale in which quantum world becomes classical would be solely due to complete misunderstanding of the notion of time.

2. Non-standard arrow of time forces a generalization of thermodynamics. For time reversed system generalized second law applies in reverse direction of time. Dissipation with reversed arrow of time extracts energy from environment, in particular thermal energy from internal thermal environment. The energy feed necessary for self-organization reduces to dissipation in reversed arrow of time.

   This explains why self-organization is possible [L86]. Standard form of the second flow would imply that also energy flows between systems go to zero: this would mean thermodynamical equilibrium everywhere - heat death. This has led to desperate theoretical proposals such as life as gigantic thermodynamical fluctuation. The recent empirical understanding suggests that this giant fluctuation would have occurred in the scale of the entire Universe and continue forever!

3. Macroscopic quantum coherence is however a necessary prerequisite for macroscopic effects. TGD predicts hierarchy of phases of ordinary matter residing at magnetic body (MB) of the system with value of effective Planck constant $h_{eff} = nh_0$ ($h = 6h_0$) of h_{eff} behaving like dark matter and controlling ordinary matter. The larger the value of h_{eff}, the longer the

scale of quantum coherence scale at MB. MB acts as master for ordinary matter in the role of slave and induces coherent behaviour. This gives rise to self-organization.

This picture could explain the observations of self induced resonance using thermal energy. A subsystem or its MB in time reversed mode would extract the thermal energy. There are many other applications. The phenomenon of stochastic resonance in which system extracts energy from external noise could have explanation along these lines. Stochastic resonance plays an important role in sensory perception by making possible amplification of weak signal in large background. There is evidence for it even in astrophysical scales. In biology metabolic energy could be extracted from metabolites and maybe also from thermal energy by time reversed dissipation by some subsystems related to metabolism.

TGD picture does not exclude the possibility of delicate models mimicking this behavior in the framework of thermodynamics. The basic challenge in this kind of effective model is to describe the presence time reversed dissipation inducing self-organization and the presence of dark matter at magnetic body phenomenologically. Energy feed as parameter gives rise to states far from thermodynamical equilibria.

For instance, the thermodynamics of ion distributions inside and outside cell is far far from thermodynamical equilibrium and and non-equilibrium thermodynamics has been developed for the modelling of this kind of systems utilizing the notions of ionic pumps and channels. The phenomenological description introduces chemical potentials as parameters to describe the non-equilibrium situation in the framework ordinary thermodynamics. Chemical potentials would model the neglected presence of $h_{eff} \geq h$ phases of dark matter at magnetic body of the system.

8.2.8 $M^8 - H$ duality and consciousness

$M^8 - H$ duality is one of the key ideas of TGD and one can ask whether it has implications for TGD inspired theory of consciousness and it indeed forces to challenge the recent ZEO based view about consciousness [L56] .

Objections against ZEO based theory of consciousness

Consider first objections against ZEO based view about consciousness.

1. ZEO (zero energy ontology) based view about conscious entity can be regarded as a sequence of "small" state function reductions (SSRs) identifiable as analogs of so called weak measurements at the active boundary of causal diamond (CD) receding reduction by reduction farther away from the passive boundary, which is unchanged as also the members of state pairs at it. One can say that weak measurements commute with the observables, whose eigenstates the states at passive boundary are. This asymmetry assigns arrow of time to the self having CD as embedding space correlate. "Big" state function reductions (BSRs) would change the roles of boundaries of CD and the arrow of time. The interpretation is as death and re-incarnation of the conscious entity with opposite arrow of time.

   The question is whether quantum classical correspondence (QCC) could allow to say something about the time intervals between subsequent values of temporal distance between weak state function reductions.

2. The questionable aspect of this view is that $t_M = constant$ sections look intuitively more natural as seats of quantum states than light-cone boundaries forming part of CD boundaries. The boundaries of CD are however favoured by the huge symplectic symmetries assignable to the boundary of M^4 light-cone with points replaced with CP_2 at level of H. These symmetries are crucial or the existence of the geometry of WCW ("world of classical worlds").

3. Second objection is that the size of CD increases steadily: this nice from the point of view of cosmology but the idea that CD as correlate for a conscious entity increases from CP_2 size to cosmological scales looks rather weird. For instance, the average energy of the state assignable to either boundary of CD would increase. Since zero energy state is a superposition of states with different energies classical conservation law for energy does not prevent this [L93]: essentially quantal effect due to the fact that the zero energy states are not exact eigenstates

of energy could be in question. In BSRs the energy would gradually increase. Admittedly this looks strange and one must be keen for finding more conventional options.

4. Third objection is that re-incarnated self would not have any "childhood" since CD would increase all the time.

One can ask whether $M^8 - H$ duality and this braney picture has implications for ZEO based theory of consciousness. Certain aspects of $M^8 - H$ duality indeed challenge the recent view about consciousness based on ZEO (zero energy ontology) and ZEO itself.

1. The moments $t = r_n$ defining the 6-branes correspond classically to special moments for which phase transition like phenomena occur. Could $t = r_n$ have a special role in consciousness theory?

   (a) For some SSRs the increase of the size of CD reveals new $t = r_n$ plane inside CD. One can argue that these SSRS define very special events in the life of self. This would not modify the original ZEO considerably but could give a classical signature for how many ver special moments of consciousness have occurred: the number of the roots of P would be a measure for the lifetime of self and there would be the largest root after which BSR would occur.

   (b) Second possibility is more radical. One could one think of replacing CD with single truncated future- or past-directed light-cone containing the 6-D universal roots of P up to some r_n defining the upper boundary of the truncated cone? Could $t = r_n$ define a sequence of moments of consciousness? To me it looks more natural to assume that they are associated with very special moments of consciousness.

2. For both options SSRs increase the number of roots r_n inside CD/truncated light-one gradually and thus its size? When all roots of $P(o)$ would have been measured - meaning that the largest value r_{max} of r_n is reached -, BSR would be unavoidable.

   BSR could replace $P(o)$ with $P_1(r_1 - o)$: r_1 must be real and one should have $r_1 > r_{max}$. The new CD/truncated light-cone would be in opposite direction and time evolution would be reversed. Note that the new CD could have much smaller size size if it contains only the smallest root r_0. One important modification of ZEO becomes indeed possible. The size of CD after BSR could be much smaller than before it. This would mean that the re-incarnated self would have "childhood" rather than beginning its life at the age of previous self - kind of fresh start wiping the slate clean.

   One can consider also a less radical BSR preserving the arrow of time and replacing the polynomial with a new one, say a polynomial having higher degree (certainly in statistical sense so that algebraic complexity would increase).

Could one give up the notion of CD?

A possible alternative view could be that one the boundaries of CD are replaced by a pair of two $t = r_N$ snapshots $t = r_0$ and $t = r_N$. Or at least that these surfaces somehow serve as correlates for mental images. The theory might allow reformulation also in this case, and I have actually used this formulation in popular lectures since it is easier to understand by laymen.

1. Single truncated light-cone, whose size would increase in each SSR would be present now since the spheres correspond to balls of radius r_n at times r_n. If $r_0 = 0$, which is the case for $P(o) \propto o$, the tip of the light-cone boundary is one root. One cannot avoid association with big bang cosmology. For $P(0) \neq r_0$ the first conscious moment of the cosmology corresponds to $t = r_0$. One can wonder whether the emergence of consciousness in various scales could be described in terms of the varying value of the smallest root r_0 of $P(o)$.

   If one allows BSR:s this picture differs from the earlier one in that CDs are replaced with alternation of light-cones with opposite directions and their intersections would define CD.

2. For this option the preferred values of t for SSRs would naturally correspond to the roots of the polynomial defining $X^4 \subset M^8$. Moments of consciousness as state function reductions would be due to collisions of 4-D space-time surfaces X^4 with 6-D branes! They would replace the sequence of scaled CD sizes. CD could be replaced with light-one and with the increasing sequence $(r_0, ...r_n)$ of roots defining the ticks of clock and having positive and negative energy states at the boundaries r_0 and r_n.

3. What could be the interpretation for BSRs representing death of a conscious entity in the new variant of ZEO? Why the arrow of time would change? Could it be because there are no further roots of $P(o)$? The number of roots of $P(o)$ would give the number of small state function reductions?

   What would happen to $P(o)$ in BSR? The vision about algebraic evolution as increase of the dimension for the extension of rationals would suggest that the degree of $P(o)$ increases as also the number of roots if all complex roots are allowed. Could the evolution continue in the same direction or would it start to shift the part of boundary corresponding to the lowest root in opposite direction of time. Now one would have more roots and more algebraic complexity so that evolutionary step would occur.

   In the time reversal one would have naturally $t_{max} \geq r_{n_{max}}$ for the new polynomial $P(t-t_{max})$ having $r_{n_{max}}$ as its smallest root. The light-cone in M^8 with tip at $t = t_{max}$ would be in opposite direction now and also the slices $t - t_{max} = r'_n$ would increase in opposite direction! One would have two light-cones with opposite directions and the $t = r_n$ sections would replace boundaries of CDs. The reborn conscious entity would start from the lowest root so that also it would experience childhood.

This option could solve the argued problems of the previous scenario and give concrete connection with the classical physics in accordance with QCC. On the other hand, a minimal modification of original scenario combined with $M^8 - H$ duality with moments $t = r_n$ as special moments in the life of conscious entity allows also to solve these problems if the active boundary of CD is interpreted as boundary beyond which classical signals cannot contribute to perceptions.

What could be the minimal modification of ZEO based view about consciousness?

What would be the minimal modification of the earlier picture? Could one *assume* that CDs serve as embedding space correlates for the perceptive field?

1. Zero energy states would be defined as before that is in terms of 3-surfaces at boundaries of CD: this would allow a realization of huge symmetries of WCW and the active boundary A of CD would define the boundary of the region from which self can receive classical information about environment. The passive boundary P of CD would define the boundary of the region providing classical information about the state of self. Also now BSR would mean death and reincarnation with an opposite arrow of time. Now however CD would shrink in BSR before starting to grow in opposite time direction. Conscious entity would have "childhood".

2. If the geometry of CD were fixed, the size scale of the $t = r_n$ balls of M^4 would first increase and then start to decrease and contract to a point eventually at the tip of CD. One must however remember that the size of $t = r_n$ planes increases all the time as also the size of CD in the sequences of SSRs. Moments $t = r_n$ could represent special moments in the life of conscious entity taking place in SSRs in which $t = r_n$ hyperplane emerges inside CD with increased size. The recent surprising findings challenging the Bohrian view about quantum jumps [L77] can be understood in this picture [L77].

3. $t = r_n$ planes could also serve as correlates for memories. As CD increases at active boundary new events as $t = r_n$ planes would take place and give rise to memories. The states at $t = r_n$ planes are analogous to seats of boundary conditions in strong holography and the states at these planes might change in state function reductions - this would conform with the observations that our memories are not absolute.

To sum up, the original view about ZEO seems to be essentially correct. The introduction of moments $t = r_n$ as special moments in the life of self looks highly attractive as also the possibility of wiping the slate clear by reduction of the size of CD in BSR.

8.3 Some comments related to Zero Energy Ontology (ZEO)

Zero energy ontology (ZEO) lies behind TGD based quantum measurement theory in turn giving rise to a theory of consciousness by making observed part of system as a conscious entity - self [L56]. ZEO solves the basic paradox of quantum measurement theory forcing to give up ontology altogether in the Copenhagen interpretation. ZEO has become a key aspect of the entire TGD based physics.

The basic prediction of ZEO is that ordinary ("big") state function reductions (BSFRs) involve change of the arrow of time. There is a lot of support for this prediction. The recent highly counterintuitive findings of Minev *et al*l provided support for the time reversal in atomic systems [L77] [L77]. Fantappie [J7] proposed decades ago time reversal in living systems and introduced syntropy as time reversed entropy. In living matter the generation of more complex molecules from their building bricks can be seen as decay in time reversed direction. Phase conjugate laser beams are known to obey time reversed second law.

Also Libet's findings [J2] related to the active aspects of conscious experience find a nice explanation in terms of the time reversal. The latest application is to the understanding of the mysterious looking findings about earthquakes and volcanic eruptions suggesting that macroscopic quantum jumps involving time reversal are in question [L80]. This suggest that experimental verification of the time reversal and occurrence of macroscopic quantum jumps is possible by studying causal anomalies. For these reasons is important to try to develop the details of the view about ZEO as precise as possible.

In the sequel I will consider more precise mathematical formulation and physical interpretation of ZEO. ZEO forms also the cornerstone of TGD inspired theory of consciousness and quantum biology and I will consider also some related aspects of ZEO such as the notions of free will and intentionality, the notions of memory and precognition as its time reversal, intuitive in contrast to formal reasoning, and remote metabolism as a universal thermodynamical mechanism of metabolism in ZEO based thermodynamics.

8.3.1 General view about ZEO

The details of ZEO - in particular the technical details related to the conservation laws BSFR and SSFR - are from well-understood and the following is an attempt to fix these details by using analogy with cosmology.

Rough view about ZEO

Consider first what ZEO roughly means.

1. The realization of ZEO [L94, L56, L67, L90] involves besides the notions of "small" (SSFR) and "big" state function reduction (BSFR) also the notion of causal diamond (CD). CD defines perceptive field of conscious entity as a 8-D region $cd \times CP_2$, where cd is the 4-D causal diamond of M^4 defined as the intersection of future and past directed light-cones.

2. At the classical level the basic entity is space-time surface connecting 3-surfaces at the opposite boundaries of CD. The space-time surfaces inside sub-CD continue outside and there is a hierarchy of CDs with largest CD beyond which space-time surfaces do not continue. This defines a space-time correlate for the hierarchy of selves.

   Space-time surfaces are preferred extremals of the basic action principle defined by the twistor lift of TGD [L72]. Minimal surfaces with 2-D string world sheets as singularities would be in question. They connect 3-surfaces at the boundaries of CD and are analogous to Bohr orbits so that not any pair is possible and the conditions characterizing preferred extremal property might even imply 1-1 correspondence between these 3-surfaces.

3. Zero energy states are superpositions of preferred extremals. One can also understand zero energy states as superpositions of deterministic programs - quantum programs, functions in the sense of quantum biology, or quantum behaviors. ZEO allows to solve the basic paradox of quantum measurement theory since the non-determinism of quantum jump between zero energy states corresponds to the causality of free will and is not in conflict with the classical

determinism realizing the causality of field equations. Experienced time and geometric time are not same but there is a strong correlation between them.

4. In SSFRs the active boundary of CD shifts to future - at least in statistical sense. This is preceded by a unitary time evolution generating superposition of CDs with different sizes but having fixed passive boundary and same superposition of 3-surfaces at it. SSFR involves time-localization to single CD with fixed temporal distance between its tips. Essentially time measurement is in question.

5. In BSFR the arrow of time changes and one can say that state function reduction measuring set of observables takes place at the active boundary of CD, which becomes a passive boundary at which state does not change during subsequent SSFRs in which CD increases in opposite direction with the former passive boundary becoming an active boundary. The change of the arrow of time in BSFR creates the illusion that instantaneous quantum jump corresponds to a smooth and deterministic time evolution leading to the final state [L77] [L77].

The mathematical and physical details of the picture are not completely nailed down, and the best manner to proceed is to return to basic questions again and again and to challenge the details of the existing picture. In the following I will do my best to invent nasty arguments against ZEO.

ZEO and conservation laws

The geometry of CD breaks Poincare invariance. Lorentz invariance with respect to the either tip of CD is exact symmetry and is extremely attractive in the construction of members of state pairs in ZEO. Classically Poincare invariance is exact and one can deduce expressions for conserved quantities for both bosonic and fermionic sector: the latter have interpretation as operators, whose eigenvalues in Cartan algebra are by quantum classical correspondence (QCC) identified as classical values of conserved quantities.

ZEO involves the somewhat questionable assumption that one can assign well-defined Poincare quantum numbers to both boundaries and that these quantum numbers are opposite: this motivates the term ZEO.

1. M^8-H duality [L85] allows to assign to CDs with either boundary fixed a moduli space, which corresponds to Poincare group. The proposal is that Poincare invariance is realized at this level and that the values of conserved charges in Cartan algebra correspond to the Poincare quantum numbers labelling these wave functions. The wave functions at the boundaries of CD could be arranged in representations of Lorentz group acting as exact symmetry of the boundary.

2. There is further little nuisance involved. Only time translations, which correspond to a non-negative time value as distance from the fixed boundary of CD are possible. One would obtain momentum eigenstates restricted to a future or past light-cone. This is of course what happens in TGD based cosmology. Maybe one must just accept this as a physical fact forcing to give up mathematical idealization.

   Formally one would replace the plane wave basis with a basis multiplied by characteristic function for future or past light-cone equal to 1 inside the light-cone and vanishing elsewhere. This basis is closed with respect to summation. This would mean that the states are not anymore exact eigenstates of momentum globally but superposition of Lorentz boosts of the basic momentum obtained by Fourier expanding the characteristic function of future/past light-cone.

   But what about CD which is intersection of future and past directed light-cones? Can one really assign to both boundaries wave functions defined in entire future (or past) directed light-cone? It seems that this is the case. Zero energy state would be entangled state as a superposition of products of boosted momentum eigenstates with opposite momenta representing the characteristic function of CD.

   The usual idea about unitary time evolution for Schrödinger amplitude would be given up inside CD, and replaced by a sequence of unitary time evolutions producing de-localization of the active boundary of CD and followed by a localization.

3. There is still a problem. A complete de-localization for the boundaries of CD is not consistent with the intuitive idea that CD has definite size scale. In wave mechanics the plane waves are only idealizations and in the real world one replaces plane waves with wave packets. Gaussian wave packets have the nice feature that they remain Gaussian in Fourier transformation.

   If one has Gaussian wave packet for the temporal distance between the tips of CD concentrated on certain value of time, the Fourier transform for this is Gaussian wave packet concentrated around certain relative energy, which is two times the energy assignable to say passive boundary of CD. Instead of sharp value of time as distance between the tips of CD one would have Gaussian distribution for its value. This is consistent with Lorentz invariance since zero energy states allow superposition over states with varying momenta assignable to say active boundary. The wave function would be essentially Gaussian in energy in the rest system and one can consider also wave functions in Lorentz group leaving the passive boundary of CD invariant.

SSFRs in ZEO

In the proposed picture the sequence of SSFRs could mean gradual widening of the Gaussian wave packet for the value of measured time as the temporal distance between the tips of CD by discrete steps.

The basic condition is that the states at passive boundary of CD identified as superpositions of 3-surfaces remain unaffected during the sequences of SSFRs increasing the size of CD. This corresponds to generalized Zeno effect and in consciousness theory thr unchanging part of zero energy state corresponds to unchanging part of self, one might call it soul. One can imagine two options.

Option I: CD increases statistically in SSFRs but classical energy is conserved for space-time surfaces connecting its boundaries. Energy density would decreases as CD increases. This does not seem too bad actually: it would be analogous to matter dominated cosmology.

Not only superpositions of 3-surfaces at passive boundary of CD would be conserved but also their 4-D tangent spaces would be unaffected: this is unnecessarily strong a condition for generalized Zeno effect.

Option II: CD increases but classical energies decrease. This looks more plausible- if not the only - option and is strongly favoured by the analogy of CD with expanding cosmology. It also conforms with uncertainty principle. The process would be essentially quantum analog of cooling or analog for what happens for particle in a box expanding adiabatially. The classical energies of the space-time surfaces in zero energy state would thus decrease as CD increases.

Also this option allows the states as superpositions of 3-surfaces to at passive boundary of CD to remain unffected in expansion of CD. The classical energies can however decrease because the space-time surfaces - tangent spaces of space-time surfaces at passive boundary - can change so that also energies can change.

This option is completely analogous to quantum adiabatic change in which the coefficients in the superposition of energy eigenstates are unaffected but energies change.

Option II looks more natural and will be considered in more detail.

1. The constraint that SSFRs as quantum measurements are for observables, which commute with observables, whose eigenstate the state at the passive boundary is, poses very strong constraints on what happens SSFR. Furthermore, preferred extremal is analog of Bohr orbit and cannot cannot be arbitrary pair of 3-surfaces. Therefore, when the CD changes, the preferred extremal also changes as a whole meaning also that also energy changes. These conditions could force adiabatic picture and the analog of Uncertainty Principle for classical energies as function of CD size.

2. The sequence of SSFRs could be also analogous analogous to what happens for a particle in box as the size of the box increases adiabatically: adiabaticity would actually be a hypothesis about what happens in the steps consisting of unitary evolution and SSFR. In adiabatic approximation the coefficients in the superposition of the energy eigenstates do not change at all: only the energies would change.

3. In thermodynamics this kind of process would correspond to a cooling, which could serve as a natural quantum correlate for the cooling in cosmology. In accordance with the idea that quantum TGD in ZEO corresponds to a complex square root of thermodynamics, one could interpret zero energy state as complex square root of thermal partition function for cosmology assignable to CD. The hierarchy of CDs would define Russian doll cosmology.

4. A further manner to understand this is in terms of Uncertainty Principle. As the size scale of CD given by temporal distance between its dips increases, the classical energy decreases. Intuitively the reduction of the classical energy is easy to understand. Increasing CD and keeping the 3-surface as such at passive boundary reduces time gradients at the passive boundary and space-time surface becomes more flat. Energy density is proportional to time gradients of coordinates and its therefore reduced. This argument is also used in inflation theories.

5. Change is the prerequisite of conscious experience and there would be indeed change also at the passive boundary of CD contributing to conscious experience. But in some sense this contribution - the "soul" - should *not* be changing! "Adiabaticity" would translate this idea to the language of physics.

What happens to CD in long run? There are two options.

1. The original assumption was that the location of formerly passive boundary is not changed. This would mean that the size of CD would increase steadily and the outcome would be eventually cosmology: this sounds counter-intuitive. Classically energy and other Poincare charges are conserved for single preferred extremal could fail in BSFRs due to the fact that zero energy states cannot be energy eigenstates.

2. The alternative view suggested strongly M^8-H duality [L48] is that the size of CD is reduced in BSFR so that the new active boundary can be rather near to the new passive boundary. One could say that the reincarnated self experiences childhood. In this case the size of CD can remain finite and its location in M^8 more or less fixed. One can say that the self associated with the CD is in a kind of Karma's cycle living its life again and again. Since the extension of rationals can change in BSFR and since the number of extensions larger than given extension is infinitely larger than those smaller than it, the dimension of extension identifiable in terms of effective Planck constant increases. Since $n=h_{eff}/h_0$ serves as a kind of IQ, one can say that the system becomes more intelligent.

   Also the temperature assignable to CD remains finite. In cosmological scales it could correspond to the analog of the temperature assignable to CMB. TGD based view about stars as blackhole like entities [L78] leads to the identification of the Hagedorn temperature assignable to the volume filling flux tube giving rise to star with the Hawking temperature of dark radiation at gravitational flux tubes. Even CMB temperature could be assigned with dark photons at gravitational flux tubes. The asymptotic temperature for CD before BSFR could correspond to this temperature.

   One expects that the center of mass coordinates of cm do not appreciably change during the quantum evolution. The hierarchy of CDs would imply that the Universe decomposes effectively to sub-Universes behaving to some degree independently. The view about Karma's cycles provides a more precise formulation of the pre-ZEO idea that systems are artists building themselves as 4-D sculptures. In particular, this applies to mental images in TGD based view about brain. The assumption that stars correspond to repeatedly re-incarnating conscious entities allows to solve several time anomalies in cosmology [L78] so that there would be a direct connection between cosmology and theory of consciousness.

There could be a relationship between quantal flow of geometric time by SSFRs and p-adic variant of time coordinates giving a reason why for p-adicity.

1. TGD predicts geometric time as a real variant and p-adic variants in extensions of various p-adics induced by given extension of rationals (adelic space-time and adelic geometric time). Real and p-adic times share discrete points in the extension of rationals considered: roots of

octonionic polynomials defining space-time surfaces as roots for their “real” and “imaginary” parts in quaternionic sense [L81]. The roots of the real polynomial with rational coefficients giving octonionic polynomial as its continuation define space moments of M^4 linear time assignable to special SSFRs. p-Adic time associated with the p-adic balls assignable the points are not well-ordered. One cannot tell about two moments of time which is earlier and which later.

2. This could relate to the corresponding lack of well ordering related to “clock time” associated with self at given level of evolutionary hierarchy defined by the extension of rationals. The increase of “clock time” as a distance between tips of CD for a sequence of small state function reductions (weak measurements) occurs only in statistical sense and “clock time” can also decrease. The moments of time correspond to roots of the real polynomial define “special moments in the life of self”, one might say.

   At the limit of infinite-D extension the roots of the polynomial define algebraic numbers forming a dense set in the set of reals. Cognitive representation becomes dense set. These “special moments” need not however become dense.

3. One can raise an interesting question inspired by self inspection. As one types text, it often happen that the letters of the word become in wrong order, change places, and even jump from a word to another one. The experienced order of letters assignable to a sequence of SSFRs is not the same as the order of letters representing the order for the moments of geometric time. When one is tired, the phenomenon is enhanced.

   Neuroscientists can certainly propose an explanation for this. But could this be at deeper level quantum effect based on the above mechanism and have a description in terms of p-adicity assignable to prime p defining a ramified prime for the extension of rationals involved? When one is tired the metabolic resources have petered out and the IQs $n = h_{eff}/h_0$ defined by dimensions of extensions of rationals for the distribution of extensions tend to reduce, cognitive resolution for time becomes lower and mistakes of this kind become worse.

There is a further technical detail involved. For SSFRs the temporal distance between active boundary and passive boundary increases at least in statistical sense. It seems that one must define the inner product in S-matrix elements for the unitary step preceding SSFR using the previous state basis as sub-basis of the new state basis in the case that CD increases. In adiabatic approximation the S-matrix elements would be overlaps for the states with different size of CD and analogous to matrix elements between states of particle in boxes with the same fixed end but different moving end.

BSFRs in ZEO

Details of BSFR are not completely fixed. One can consider two options. Both options must satisfy the condition that the states at passive boundary of CD identified as superpositions of 3-surfaces remain invariant during the sequence of SSFRs. The tangent space-to the space-time surfaces need not however remain invariant. Therefore the classical energies of space-time surfaces can change since the energy densities are proportional to time derivatives of embedding space coordinates.

1. The size of CD increases steadily as was the original proposal and is thus not reduce in BSFRs. The problem with the steady increase seems to be that the size of CD becomes infinite eventually and the state evolves to what looks like cosmology. If the energy assignable with zero energy state is conserved, the energy density of matter inside CD increasing without limit becomes arbitrarily small. Is this a catastrophe?

   For TGD inspired cosmology this is the case at the limit of big bang in the sense that the energy density goes like $1/a^2$ (cosmic string dominance) and energy in a co-moving volume vanishes like a, where a is light-cone proper time. One can think that CD defines only perceptive field and that space-time surfaces continue also outside CD up to the maximal size of CD in the hierarchy of selves involved. The zero energy state would have finite energy energy but density of energy would go to zero at the boundary of CD. The perceptive field of conscious entity would increase steadily in size.

As found, energy need not be conserved in the subsequence SSFRs because Gaussian wave packets of CDs around given size are required so that eigenstates of energy are not in question and the reduction of the width of Gaussian in the sequence of SSFRs implies reduction of average energy. Only the superpositions of 3-surfaces at the passive boundary of CD would be conserved.

Even the conservation of energy combined with the increase of CD need not be a catastrophe. In matter dominated cosmology the conservation of mass takes place with respect to cosmological time which corresponds to the proper time measured as temporal distance from the passive tip of CD. This cosmological mass is not energy but closely relates to it. What looks of course counter-intuitive is that every self would evolve to a cosmology.

2. The size of CD could be also reduced in BFSR [L81]. $M^8 - H$ duality and existence of "braney" solutions encourages to take this option serious. The 6-D brane like entities correspond to $t = constant$ sections for linear M^4 time t. They would represent special moments in the life of self. The exceptional 6-D roots of octonionic polynomials as branes would emerge to the perceptive field conscious entity at these moment. Discontinuity of classical space-time evolution as SSFR. Every time-reversed re-incarnation of self would have have "childhood" and experience increase of CD from some minimal size to maximal size.

   Since the size of CD can be reduced, it could happen that the CD remains stuck below certain maximal size for ever. The associated mental images would continue living in the geometric past of bigger CD associated with self. The sub-CDs in past would represent memories of self. Cosmos in 4-D sense would be full of life. The interpretation of CD as perceptive field allows this. CD could also increase and become even a cosmology! This picture looks attractive from the view point of consciousness.

3. One can however invent an objection against ZEO, one might even speak about paradox.

   (a) Suppose that in biological death I indeed re-incarnate with opposite arrow of time and continue to live towards geometric past. Suppose also that I re-incarnate as more advanced human being - at least in statistical sense. Human beings have parents. But how can I have parents in the former geometric future, if my parents how have already died live in the former geometric past?

   (b) The only solution of the paradox seems to be that the magnetic body (MB) - the boss - does not disappear in the death of biological body (BB). The MBs of my parents continue their existence and in my biological death means their separation in stanard time direction and meeting in the new time direction. They meet, fall in love, and give rise to my birth but all this in opposite time direction.

   This would provide an answer to a long-standing question about whether MBs are preserved in biological death or not. My view has been that biological death is more or less that MB loses interest in my BB and directs attention to something more interesting. One could however argue that also MB is generated in birth and genes code also for it so that it would die. If directing attention corresponds to BSFR MB would continue to exist after biological death. This particular reincarnation - CD - would be like vortex in the flow of time.

   (c) Can one find any support for this crazy looking proposal? TGD Universe is fractal and lower levels in the length scale hierarchies are slaves. In particular, bio-chemical level serves as the slave of MB expected to obey kind of shadow dynamics. If the proposed topological dynamics of MBs solving the above paradox has a miniature representation at the level of DNA, one could take the proposal with some seriousness.

   In meiosis (`http://tinyurl.com/n5eqkdn`) germ cells, whose chromosomes are coctails of paternal and maternal chromoses (PCs and MCs), are formed. In fertilization (`http://tinyurl.com/ngzwhcq`) - in some sense a (time?) reversal of meiosis - pairs of PCs and MCs are formed. The fusion of paternal and maternal germ cells could be indeed seen in topological sense as a time reversal of replication. The replication of soma cells involves mitosis (`http://tinyurl.com/p35lkwr`) forming pairs of chromosomes of PCs and MCs.

Could the chromosomal dynamics be a miniature version of the proposed dynamics at the level of MB even at the level of organisms? If so, mitosis at the level of MB would correspond to a loose pairing of paternal and maternal MBs - formation of a relationship. Our personal MBs as analogs of germ cells would be coctails of MBs of PCs and MCs formed by reconnection process.

What about replication? In the case of asexual reproduction (`http://tinyurl.com/y8odomtf`) one could speak about replication at the level of MB of the entire organism. Also cell - and DNA replication would represent examples of asexual reproduction and in meiosis sexual reproduction of also DNA would take place.

When does BSFR occur? I have imagined several options, which need not exclude each other.

1. Could BSFR occur, when there are no observables at the active boundary commuting with those diagonalized at passive boundary. Measurement of observable at means generation of eigenstate in the extension of rationals and it typically occurs that the resulting state is outside the extension. Could BSFR occur when there are no observables in the extension of rationals in question.

2. $M^8 - H$ duality predicts universal special solutions besides 4-D space-time surfaces. These 6-D analogs of branes correspond to n moments of linear M^4 time, where n is the polynomial whose octonionic continuation defines space-time surfaces in M^4 as roots of its real or imaginary part in quaternionic sense. At these branes 4-D space-time surfaces are glued together along their ends- space-time looks is analogous to piecewise continuous curve in time direction - and they would correspond to "special moments in the life of self" [L81]. When all these moments as special roots of the octonionic polynomial are experienced, BSFR would be the only possibility. The polynomial with rationals coefficients defining the octonionic polynomial defines the extension of rationals used so that this option could be consistent with the first option.

3. Is BSFR is forced to occur because there are no preferred extremals connecting the pairs of 3-surfaces exists anymore. Could it happen that the state becomes increasingly classical during the sequence of SSFRs and thus becoming more and more local in WCW (the "world of classical worlds", which is essentially the space of 3-surfaces at either boundary of CD). The unchanging part of the zero energy state associated with the time-reversed state as outcome of BSFR at the new passive boundary would be maximally classical. This might relate to the fact that the world looks so classical. Also the fact BSFRs themselves look classical smooth time evolution ending to the outcome of BSFR, creates the illusion of classicality [L77].

8.3.2 ZEO, life, and consciousness

The most important implications of ZEO relate to consciousness and quantum biology. One can understand act of free will and motor action in terms of BSFR. BSFR corresponds to motor action and its time-reversal. SSFRs correspond to sensory perception in either direction of time [L69]. Model for memory is one prediction and predicts precognition as time reversal of memory [K93] [L97]. Also the relationship between generation of insight and mechanical logic deductions can be understood. In biology ZEO leads to remote metabolism as a universal purely thermodynamical mechanism of metabolism. One can also understand zero energy states as superpositions of deterministic programs - quantum programs, functions in the sense of quantum biology, or quantum behaviors.

Act of free will, intentionality, and ZEO

Act of free will would correspond to BSFR that is quantum jump leading to final state with opposite arrow of time. Final state is a superposition of deterministic time evolution connecting the 3-surfaces in the superpositions defining initial and the final states. In this picture state function reduction leads to final state inducing time reversed time evolution so that classically the causal order is changed. What in standard picture - say neural activities - causes the outcome, is caused by the outcome. Could it be that mere volitional act with sharp enough intention is needed? The correct deterministic time evolution is dictated by intention as consequence rather than cause!

Here I cannot avoid the temptation to tell about my own strange experiences. At this age one must remember to take the pills every morning. I have the habit of filling my pill dispenser every Monday morning. I do not bother to count the pills one by one. I just take randomly a bunch of them hoping that their number is correct. And it is! Quite too often! Similar thing happens in market when I pay with coins: I do not count the coins but just take a handful of them. The sum of the coins is correct quite too often! Could a mere sharp intention dictate the outcome. Could one learn gradually this kind of sharp intentions.

Could this be crucial for various skills like playing tennis or computer game, where one simply cannot react rapidly by computing the outcome since time does not allow it? Could this explain also mathematical/physical/.. intuition as skill to solve problems by making quantum jump directly to the solution of the problem.

Precognition and ZEO

It seems that neuroscientists are beginning to take remote mental interactions such as precognition, telepathy, and psychokinesis seriously. The popular article entitled "*Scientists Discover That The Heart & Brain Respond To Future Events – Before They Happen*" (see `http://preview.tinyurl.com/y494hw5u`) describes changing views of neuroscientists towards precognition.

In ZEO precognitions are naturally time-reversed memories. Classical signals giving rise to sensory experience arrive from geometry future in the standard frame. During sleep state precognition should be possible if sleep corresponds to time-reversed state for the self.

In the associative and computational models of brain our ability to predict the future is taken to be an extrapolation based on memories and experience of earlier life. This looks very reasonable but when one asks how these memories are represented, problems begin to appear. In TGD framework ZEO predicts that memories correspond to mental images in geometric past, in the simplest case, when the original event took place. This solves a huge problem of standard since memory storage becomes brain in 4-D sense rather than in 3-D sense [K93].

ZEO however implies that also time reversed memories are possible. If sleep state correspond to time reversed self about which we do not have direct memories, memories with reversed arrow of time would be possible in this state. Precognition becomes possible if these memories can be communicated to the wake-up state with the ordinary arrow of time. In dreams some parts of brain are awake and they could make possible this communication. The communicated information could be also conscious to some selves above or below us in the hierarchy. Dreams can indeed predict what happens during the next day. The classical book "*An Experiment with Time* " (see `http://tinyurl.com/jtqysty`) of J. W. Dunne tells about precognitive dreams that he experienced.

Intuitive and formal logical reasoning in ZEO

The basic vision is that adelic space-time geometry provides correlates for sensory experience and cognition/imagination. Fermionic degrees of freedom would represent quantal Boolean mind. In ZEO given deterministic time evolution for 3-surface and induced spinor fields would give rise to sensory and cognitive time evolution and to Boolean evolution having interpretation as analog of logical deduction leading from premises to conclusions.

1. The basis of fermionic Fock states can be regarded as Boolean algebra. Superpositions and thus entanglement of fermionic qubits are however possible and one can speak about quantum Boolean logic. In standard view concepts are formally regarded as sets containing the instances of concept as elements. Quantum concepts could be superposition of quantum states representing the instances so that quantum abstraction would be much more complex notion than ordinary abstraction. Non-classical Boolean states would be superpositions of statements identifiable as abstractions. Schrödinger cat would be seen abstraction. "Dead" and "alive" would represent instances of this abstraction.

2. Zero energy states are superpositions of initial and final fermion states and there is also a superposition over 3-surfaces, and could be interpreted as representations for implications.

The sum $\sum_n S_{mn}|n\rangle$, where S denotes unitary S-matrix, represents a superposition over all transitions $|m\rangle \rightarrow |n\rangle$ allowed by laws of physics. These transitions could be interpreted as logical implications.

One could argue that by diagonalizing S-matrix one obtains only diagonal transitions and the situation is rather trivial: just logical identities. The point is however that in number theoretical physics the diagonalization of S would in general lead outside the extension of rationals determining the adele and is therefore not possible. Same number theoretical mechanism would also stabilize negentropic entanglement and could force BSFR. Only state big state function reduction extending the extension of rationals can reduce this kind of entanglement.

3. Probably every mathematician has pondered the mystery of mathematical insight. How for instance mathematical insight is generated? What eureka experience is basically? Insight would correspond naturally to a big state function reduction leading to a new state reversing the arrow of time.

   Truth can be deduced in given system of axioms also mechanically - at least in principle. How does insight relate to a logical deduction leading to a theorem? The final state of quantum jump is superposition of classical time evolutions leading from the final state to geometric past. With respect to standard arrow of time it is superposition of logical deductions leading from various initial states- initial assumptions - to the final state - to the outcome of the deduction. Superposition of states at boundary of CD could be seen as an abstraction. Deterministic time evolutions would represent the mechanical deductions.

   Note however that in the time reversed state arbitrary long time evolution in opposite time direction is in principle possible and would correspond to an arbitrary long ordinary deduction or computation [L36]. After that a return to the original arrow of time would take place and provide the solution. The formal deduction leading to the outcome would be indeed forced by the outcome rather than vice versa?

Metabolism in ZEO

ZEO has also deep implications for biology. As already explained, ZEO allows to understand what behaviors, biological functions are at fundamental level.

Why metabolism is needed can be understood in TGD view about dark matter as phases of ordinary matter labelled by the value of effective Planck constant $h_{eff} = n \times h_0$, where n has also interpretation as dimension of extension of rationals giving rise to the extension of adeles [L53, L54]. n serves as a kind of IQ labelling different evolutionary levels and is bound to increase in statistical sense. Not only biology but also self-organization involving also energy feed could be understand in terms of the hierarchy of Planck constant.

In ZEO remote metabolism suggests itself as a completely universal purely thermodynamical mechanism of metabolism. Usually system loses its energy by dissipation. If the arrow of time is non-standard, systems seems to receive energy from environment. Note that the duration of time spent in time reversed state does not matter! What matters is the increment of time between states with same arrow of time! Sleep state could be seen also as a way to collect metabolic energy. BSFR can be seen as an act of free will - motor action and sucking of metabolic energy from "environment" would be very natural.

The interpretation for the return to the original time direction by second BSFR would be as beginning of sensory perceptions in standard arrow of time as sequences of SSFRs. During this period subsystem would be dissipating energy to environment.

8.3.3 Under what conditions does BSFR take place and what happens in it?

In the following the question under what conditions "Big" state function reduction (BSFR) takes place and what happens in it.

Two kinds of state function reductions

The discussion however requires the basic ideas of ZEO as background.

1. "Small" state function reductions (SSFRs)

Small state function reductions (SSFRs) are counterparts of so called "weak measurements", which are rather near to classical measurements in the sense that nothing drastic happens.

1. The passive boundary of CD does not shift but changes in size because active boundary shifts and this induces change of size. For state pairs defining zero energy states the members at passive boundary do not change and the coefficients of possibly time-entangled state defined as their superposition do not change. The members of state pairs at active boundary change and this change is induced by unitary time evolution between too SSFRs. This time evolution could be regarded as a generalization of adiabatic time evolution.

2. In statistical sense the active boundary shifts towards future and the size of CD increases. The temporal distance between the tips defines clock time in one-one correspondence with SSFRs. Note that the unitary evolution forms a superposition of CDs with different sizes and SSFR means localization to single CD size.

3. The moment "Now" of self would naturally correspond to the M^4 hyper-plane dividing CD into two pieces of identical size. The radius of this 3-ball would be $r = T/2$, where T is the temporal distance between the tips of CD. At this hyperplane expansion of 3-ball with light-velocity would transform to contraction.

4. The mental images of self would correspond sub-CDs and also they would shifts towards geometric future in the sequence SSFRs. They would form a kind of log file about the life history of self such that geometric time order would be opposite to subjective time order. Self could remember these experiences by sending signals to geometric future reflecting back in time direction - seeing in time direction would be in question.

   What is in sharp conflict with natural expectation is that the memories would be stored in geometric future and part of them would become un-changing permanent part for the time reversed re-incarnation of self- kind of Karma.

   Note however that self might have also mental images represented as sub-CDs in geometric past.

$M^8 - H$-duality suggests space-time picture about the "log files".

1. 4-D space-time surfaces in complexified M^8 having interpretation as complexified octonions are 4-D roots for octonion valued polynomial obtained as an algebraic continuation of a real polynomial with rational or even algebraic coefficients. $M^8 - H$ correspondence maps thee surfaces to minimal surfaces with 2-D singularities in H [L85, L81].

2. Besides this one obtains for any polynomial also special solutions as analogs of branes in M-theory. They have topology of 6-D ball and their projection to M^4 is $t = r_n$ hyperplane intersecting CD and with topology of 3-ball. r_n is a root of P and thus an algebraic number. I have called $t = r_n$ "very special moments in the life of self". Generalized vertices for particle reactions would correspond to partonic 2-surfaces localized at these 6-surfaces. At these surfaces incoming and outgoing partonic orbits would be glued together along their ends. The roots define positions of external particles at the boundaries of CD.

3. In SSFRs these balls at the active half of CD would shift towards future and new roots would emerge. These roots wold define a geometric representation of the memories of CD as "log file" increasing in size. If there are sub-CDs associate with them, one would have mental images shifting towards future.

2. "Big" state function reductions (BSFRs)

"Big" state function reductions (BSFRs) correspond to ordinary state function reductions (SFRs) in ZEO. In BSFR the roles of active and passive boundaries of CD are changed and the arrow of geometric time changes since the formerly passive boundary starts to shift to opposite time direction. State function reduction not commuting with the observables defining states at passive boundary as their eigenstates would takes place and the state at passive boundary would be changed. It would be however fixed by quantum dynamics. The findings of Minev *et al* provide support for the change of the arrow of time in ordinary SFR [L77].

The passive boundary can be shifted towards future so that the size of CD would decrease. One can say that the re-incarnate would be experience childhood. Note that also part of the "log file" about often personal experiences of self towards end of its life defining the permanent part of self-hood of the re-incarnate would disappear. The interpretation in terms of Karma is suggestive.

Remark: During a discussion with Marko Manninen, Marko noticed that people who have had near death experience often report that they experienced their entire life like a film during these moments. Could the "log file" representing stored mental images give rise to this experience at the moment of death?

What happens in biological death from TGD perspective?

What happens in biological death can be taken as a guideline in attempts to understand what happens in BSFR.

1. Death certainly occurs if there is no metabolic energy feed to the system. Metabolic energy feed is guaranteed by nutrition using basic molecules as metabolites. Since the increase of h_{eff} quite generally requires energy if other parameters are kept constant and since the reduction of h_{eff} can take spontaneously, the metabolic energy is needed to keep the distribution of values of h_{eff} stationary or even increase it - at least during the growth of organism and perhaps also during the mature age when it would go to increase of h_{eff} at MB.

   If the size of CD for at least MB correlates with the maximum value of h_{eff} or its average, the size of CD cannot grow and can be even reduced if the metabolic energy feed is too low. The starving organism withers and its mental abilities are reduced. This could correspond to the reduction of maximum/average value of h_{eff} and also size of CD.

   One can argue that if the organism loses metabolic energy feed or is not able to utilize the metabolic energy death and therefore also BSFR must take place.

2. In ZEO self-organization reduces to the second law in reversed direction of geometric time at the level of MB inducing effective change of arrow of time at the level of biological body [L86]. The necessary energy feed correspond to dissipation of energy in opposite time direction. In biological matter energy feed means its extraction from the metabolites fed to the system. One could say that system sends negative energy to the systems able to receive it. A more precise statement is that time reversed subs-system dissipates and metabolites receive the energy but in reversed time direction.

   In living matter sub-systems with non-standard arrow of time are necessary since their dissipation is needed to extract metabolic energy. The highest level dissipates in standard time direction and there must be a transfer of energy between different levels. This hierarchy of levels with opposite arrows of geometric time would be realized at the level of MB.

Death as a re-incarnation with opposite arrow of time

These observations suggest that one should consider the reincarnation with opposite arrow of time with wisdom coming from the death of biological systems.

1. We know what happens in death and birth in biological systems. What happens in biological death should have analogy at general level. In particular, in death the decay of the system to components should occur. Also the opposite of this process with reversed arrow of time should take place and lead at molecular level to the replication of DNA and RNA and build-up of basic biomolecules and at the cell level to cell replications and development of organs. How these processes could correspond to each other?

2. The perceived time corresponds to the hyperplane $t = T/2$ dividing CD to parts of same size. Here T is the distance between the tips of CD and therefore to maximal diameter of temporal slice of cd, which is 3-ball. The part of CD above it shifts towards future in SSFRs. In BSFR parts of the boundary of space-time surfaces at the active boundary of CD become

unchanging permanent parts of the re-incarnate - kind of log file about the previous life. One can say that the law of Karma is realized.

If CD decreases in size in BSFR the former active boundary keeps its position but its size as distance between its tips is scaled down: $T \to T_1 \leq T$. The re-incarnate would start from childhood at $T - T_1/2$ and would get partially rid of the permanent part of unchanging self-hood corresponding to interval $[T - T_1/2, T/2]$ so that the permanent part of reincarnate would correspond to $[T - T_1/2, T]$. Reincarnate would start almost from scratch, so to say. The part between $T - T_1/2$ and T would be preserved as analog of what was called BIOS in personal computers.

3. At the moment of birth CD possibly would thus decrease in size and the former passive boundary now in the range $[T - T_1/2, T - T_1]$ and lower tip of new CD at $T - T_1$ would become active and the seat of sensory experience. Arrow of time would change. Where the analog of biological decay is located? The region of CD in the range $[T/2, T - T_1/2]$ disappearing from "log file" is the natural candidate. This region is also the place, where the events related to birth in opposite time direction should take place.

4. The decay of the organism should therefore correspond to the development and birth of re-incarnated organism at the level of MB (it must be also remembered that genuine time reversal takes place at the level of MB and induces only effective time reversal at the level of ordinary bio-matter). The decay of organism dissipates energy in standard time direction: this energy could used by the re-incarnate as metabolic energy. How long lasting biochemical processes have effective time reversals depends on the quantum coherence scale determined by the size scale of corresponding CD.

Could the re-incarnations with opposite arrow of time be seen in bio-chemistry?

The possible occurrence of effective time reversals at the level of bio-chemistry could be perhaps tested experimentally.

1. Could the replication of DNA and RNA and build-up of various bio-molecules be effective time-reversals for their decays. Could the same apply to the replication of cells and generation of organs. Replication of DNA is self-organization process in which second DNA strand serves as a template for a new one. The decay of DNA should therefore involve two DNA strands such that the second DNA strand serves as a template for the effectively time reversed replication. The double strand structure indeed makes possible for the other strand to decay first. Cell replication should use another cell as replicate and same would happen in the cell decay.

2. An interesting mental exercise is to imagine the time reversals of various basic processes like transcription and translation. In the time reversal of translation of mRNA to amino-acid sequence the amino-acid sequence and mRNA would return to ribosome machinery, and amino-acid and tRNA codon associated with tRNA would return to form tRNA. mRNA strand would shift one step backwards and the process would repeat itself and finally mRNA strand would return to open DNA strand. In the time reversal of transcription of DNA to mRNA, mRNA strand would return to open part of DNA strand, decay to RNA codons and eventually DNA strand would close. It should be easy to check whether these processes really occur in the decay process.

3. The formation of stem cells involves de-differentiation. Could it mean time reversal of the entire process leading to a differentiated cell? Also this idea could be tested.

In biology pairs of various structures often occur. Could they correspond in some sense to effective time reversals of each other whereas at the level of magnetic body one would have genuine time reversals

1. Could the opposite inherent chiralities of MBs of DNA strands correspond to opposite arrows of time at the level of MB of DNA realizing dark genetic code [L28]? Could this be seen as a kind of explanation for the double strand structure of DNA. Could the passivity of DNA

strand with respect to transcription correspond to opposite arrow of time at the level of MB? Could the passive strand become active in time reversal?

2. Even brain has this kind of pairing. Right brain hemisphere is passive in the sense that it does not seem to contribute to wake-up intelligence (presumably identified as analytic intelligence). Could either hemisphere serve as a template in the development of brain or could this happen only at the level of MB of brain? Could different time arrows at the level of MB be used to understand the strange passive character of right brain and could one one understand the holism of right brain *viz.* analytic reductionism of left brain as reflection of the fact that dissipation as decay corresponds to time reversal for self-organization generating structures at the level of MB.

What about ordinary re-incarnation?

A couple of comments relating to the notion of re-incarnation in standard sense are in order.

1. Eastern philosophies talk about the possibility of liberation from Karma's cycle. Can one imagine something like this? The above picture would suggest that in this kind of process the reduction of the size of CD does not occur at all and therefore there would be no decay process equivalent to the growth of time reversed organism. This would serve as an empirical signature for the liberation - if possible at all. CD would continue to increase in size or perhaps keep its size. It would seem that a new kind of non-biological source of metabolic energy would be needed.

2. Reincarnation is a basic notion in Eastern philosophies. In ordinary reincarnation person has memories about life of a person, who lived earlier. There is evidence for this. This cannot be understood in terms of time reversed re-incarnation.

   Recall that there would be a hierarchy of selves and corresponding CDs within CDs. It has remained an open question whether CDs could also overlap? Could re-incarnation in ordinary sense be explained in terms of this kind of overlap?

   Suppose that one has two overlapping CDs: CD_1 and CD_2 and that CD_2 extends farther to the future of CD_1. The sub-CDs of CD_1 shift to future as the active part of CD_1 shifts to future and increases in size giving rise to a kind of log file defining the personal memories of CD_1. In this kind of situation the mental images of CD_1 can enter to CD_2 and become mental images of CD_2. This would be sharing of mental images but in different sense as compared to the fusion of mental images by entanglement, which could also require intersection of sub-CDs of mental images.

   Could one imagine that the cosmos is full of selves serving as counterparts of memes wandering around and finding for selves hosting them by providing metabolic energy? Note that ZEO means that CD center of mass degrees of freedom do not carry any conserved quantum numbers so that the motion of these lonely CDs would not be restricted by conservation laws!

3. This picture suggests that CD:s form a conscious fractal atlas consisting of charts with various resolutions analogous to the atlas defining a covering of manifold by open sets. The earlier proposal was that in biological death MB redirects its attention to a new system. This picture would be modified: the MB of of CD_1 would still attend the time-reversed system and experience time-reversed life. Some sub-CDs of CD_1 would however belong to a new CD in its geometric future - CD_2. This conforms with the intuitive expectation that space-time surfaces continue outside CD and only the perceptive field of conscious entity is restricted to CD.

4. Mental images should correspond to sub-selves and therefore sub-CDs of CD. Contrary to what I have proposed earlier, it seems that after images cannot correspond to BSFR type re-incarnations of mental images nor re-incarnations in standard sense.

   Mental images would shift towards the future together with active part of CD and form a kind of log file. Could after images be memories of previous mental images involving a signal time reflect from the the mental image in log file and creating the after image as a sensory memory of the earlier visual mental image? Or could one understand after images in terms of propagation of dark photon signals along closed magnetic loops giving rise to periodically occurring mental images.

In [L113] I discussed how the evolution of self by BSFRs could correspond to a transition to chaos as iteration of the polynomial defining the space-time surface. The proposed picture was that the evolution by SSFRs corresponds to iteration of a polynomial P assignable to the active boundary of CD. This would predict a continual increase of the degree of the polynomial involved. This is however only one possibility to interpret the evolution of self as iteration leading to chaos.

1. One could argue that the polynomial $P_{nk} = P_n \circ .... \circ P_n$ associated with the active boundary remains the same during SSFRs as long as possible. This because the increase of degree from nk to $n(k+1)$ in $P_{nk} \rightarrow P_{nk} \circ P_n$ increases h_{eff} by factor $(k+1)/k$ so that the metabolic feed needed to preserve the value of h_{eff} increases.

   Rather, when all roots of the polynomials P assignable to the active boundary of CD are revealed in the gradual increase of CD preserving P_{nk}, the transition $P_{nk} \rightarrow P_{nk} \circ P_n$ could occur provided the metabolic resources allow this. Otherwise BSFR occurs and self dies and re-incarnates. The idea that BSFR occurs when metabolic resources are not available is very natural for this option.

2. Could $P_{nk} \rightarrow P_{nk} \circ P_n$occur only in BSFRs so that the degree n of P would be preserved during single life cycle of self - that n can increase only in BSFRs was indeed the original guess.

While preparing this contribution I learned about a highly interesting claim (`https://tinyurl.com/yap8ss4p`) made by the research group led by Harold Katcher. The claim is that the epigenetic age (there are several measures for it such as methylation level of DNA) of rats has been reduced up to 50 percent. The theory goes that epigenetic age of molecules would be controllable by hormonal signalling globally.

BSFR would mean death of conscious entity and its reincarnation with opposite arrow of time. The system would rejuvenate in the transition starting a new life in opposite time direction from childhood so to say - rejuvenation would be in question. Doing this twice would lead to life with original arrow of time but starting in rejuvenated state. The claim of the group suggests that living matter could do this systematically using hormonal control.

Tukdam and TGD

This piece of text was inspired by a document (`https://rb.gy/abt8za`) about a strange phenomenon known as Tukdam. What happens is that in Tukdam the person is physically dead but is believe to be in a continued meditation. There is no EEG, the heart does not beat, and there is no normal metabolism. However, the decomposition processes do not start. The condition can last up to a couple of weeks. Similar longer-lasting ones have been reported: a yogi can be buried underground for months in an oxygen-free state and then wake up.

This challenges neuroscience's view of the brain as the seat of consciousness. According to reports there could be awareness and a sensory experience consisting of different light sensations. The Tibetan Book of the Dead describes these experiences. Near-death experiences have many similar features [L130].

In the body in Tukdam, the area of the heart is reported to feel warmer to the touch than the rest of the body, but the thermometer does not detect this difference. This would indicate that the body receives metabolic energy at the cellular level from some other source than in the normal metabolism, and that living matter can detect what measuring devices based on the recent knowledge provided by modern physics cannot detect. Where could this energy come from? If one wants to answer this, one must also ask what happens in death and what is consciousness and what is life.

1. Dark energy and matter are the two basic puzzles of recent day physics. In the TGD approach, I have identified dark matter as a phase of ordinary matter, for which the effective Planck constant h_{eff} is much larger than normally.

In particular, the gravitational Planck constant $h_{eff} = h_{gr}$ assignable to gravitational flux tubes can be very large and makes quantum coherence possible even on astrophysical scales. Large Planck constants would be associated with the dark matter magnetic body, which would be the TGD counterpart to the magnetic field of Maxwell's theory, but would differ from it in many respects. As a quantum coherent unit, this magnetic body would control the ordinary biological body and induce its coherence. The classical energy of a magnetic body, consisting of volume energy and magnetic energy, would be dark energy.

2. In the TGD Universe dominated by zero energy ontology, consciousness is a universal phenomenon and present on all scales, from elementary particles to the level of the cosmos. Even galaxies, stars and planets would be conscious beings. Also life and death would be universal phenomena. Likewise, the biological decomposition process associated with death would correspond to the universal decomposition process, which would essentially correspond to the decomposition of magnetic monopole flux tubes (magnetic catabolism), which would induce the catabolism of the breakdown of biomolecules. Its time-reversed version would be magnetic anabolism and induce the building of bio-structures such as molecules.

3. The fundamental metabolic processes would be essentially magnetic anabolism and catabolism induced by "big" state function reductions (BSFRs) changing the arrow of time and inducing the biological anabolism and catabolism. Death would mean reincarnation with the opposite arrow of time.

In Tukdam, the biological body would be dead, but the magnetic body would still be alive and prevent the biological decay from starting. The disintegration of the magnetic body would start in Tukdam much later than normally, and initiate the disintegration of the biological body. The content of the conscious experience in Tukdam, light sensations and deep peace, would come from the magnetic body. The dead biological body would not provide contribution from sensory input, motor activity, and cognition.

By a strange accident, just before seeing the document about Tukdam, I wrote an article [L159, L165] about a seemingly completely unrelated topic, solar flares related to the reversal of the direction of the sun's magnetic field in the solar cycle, which has a period of 11+11 years.

The reversal of the Sun's magnetic field would correspond to magnetic catabolism as the breakdown of long monopole flux tubes into very short parts. It would be followed by magnetic anabolism as their re-fusion into long flux tubes. The solar cycle would correspond to the sleep-wake cycle, or more precisely: a series of lives in different directions of time. Death would only be a change of time's arrow, nothing final.

The model unexpectedly leads to a biological analogy and to understanding what might happen to the magnetic body in biological death.

8.3.4 Conditions on the periods with reversed arrow of time

In zero energy ontology (ZEO) falling asleep (death at"my" level of self the hierarchy) corresponds to ordinary - or "big" - state function reduction (BSFR) and also means a reincarnation with opposite arrow of time. We would be therefore conscious during sleep and wake-up would correspond to falling sleep of that other, time reversed self.

When I fall asleep, I wake-up later tomorrow morning for instance, not yesterday morning. It is interesting to see what kind of conditions this implies and whether it is possible to satisfy this easily and even more interesting is to see whether a time travel to the geometric past - maybe the Golden Youth - could be possible.

The following assumptions are made about what happens in BSFR.

1. Causal diamond (CD) is a correlate for self. CD is obtained by gluing together two identical half-cones along their bottoms. Moment "Now" corresponds to the largest hyperplane $T_{now} = T$ (origin of time coordinate is at either (call it "lower") tip of CD) .

2. During the sequence of SSFRs defining self, the 3-surfaces at the passive boundary of self are fixed although their 4-D tangent space changes and corresponds to the unchanging part of selfhood - soul one might say. The opposite active boundary of CD and 3-surfaces at it

change and shift towards geometric future.This gives rise to wake-up consciousness involving sensory input and thoughts, emotions etc. induced by it. Each SSFR is preceded by the analog of unitary time evolution.

3. BSFR means a death of self (subself) and its reincarnation with an opposite arrow of time. One can equally well speak about the analog of falling in sleep and waking up after that for some level of hierarchy of selves. The self born in the death of the self with an opposite arrow of time self has no direct memories about the state. Self can however have memories about dreams in which part of say brain is awake. These memories store information about what self experienced during the sleep.

   In BSFR the active boundary of the CD becomes passive and is frozen. The size of CD is scaled down so that CD becomes small: this implies that the reincarnated self has a childhood and much of the memories - often not pleasant - stored near the active boundary as subselves living forth and back as conscious entities disappear. The surviving memories of self become "silent wisdom" of the reincarnated self.

4. If CD belongs to a larger CD, call it CD_{super} representing a larger unit of consciousness, the sub-CDs must shift to the same direction as the active boundary of CD_{super}. Otherwise the sub-CDs would drop from the flow of consciousness. This is analogous to co-movement of matter in cosmology.

   Note that the mental images of self correspond to sub-CDs around T_{now} and shift towards geometric future as CD increases and new mental images emerges at T_{now} plane: by M^8-H correspondence these special moments in the life of self correspond to roots of the polynomial defining space-time surface and reside are the upper half-cone of the CD. As CD increases, new roots pop up inside the upper half-cone near the T_{now} hyper-plane for some particular SSFRs. Completely counterintuitively, the mental images about past experiences are therefore in the geometric future of T_{now} hyperplane!

The proposed picture must be consistent with everyday experience. Call the two periods of self sleep wake-up and sleep label the two different BSFRs by "sleep" and "wake-up".

1. In each SSFR CD size increases - at least in statistical sense this implies that T grows. Each SSFR corresponds to a scaling for the CD shifting its active boundary towards the geometric future. During its life cycle CD experiences scaling Λ:

$$T_{now} \to T_{now,sleep_1} = \Lambda(SSFR)T_{now} \ \ , \ \ \Lambda(SSFR) > 1 \ \ .$$

2. When the system falls in sleep the size of CD is scaled down so that also the value of T_{now} is scaled down by $\Lambda_{BSFR} < 1$:

$$T_{now,sleep_2} = (1-\Lambda(BSFR))2T_{now,sleep_1} = (1-\Lambda(BSFR))\Lambda(SSFR)2T_{now} \ \ , \ \ \Lambda(BSFR) < 1 \ \ .$$

   After that the CD begins to increase in size by small scalings in SSFRs to opposite time direction and T_{now} begins to decrease from its value $T_{now,sleep}$ begins to decrease.

3. If CD belongs to a bigger CD - call it super-CD - representing a larger unit of consciousness with a longer life cycle, one can argue that the CD must shift to the same direction as the larger CD increases. Otherwise the CD would drop from the flow of consciousness defined by super-CD. This is analogous to co-movement of matter in cosmology. Therefore a given life cycle corresponds also a shift ΔT of sub-CDs towards the growth direction of super-CD takes place and one has for the time coordinate $T_{super,now}$ of the super-CD. Therefore one must perform shift$T \to T + \Delta T$ for $T_{now,sleep_1}$ and $T_{now,sleep_2}$ to take into account the drifting. This gives for the moments "Now" before ad after the shrinking of CD in BSFR (falling asleep):

$$T_{super,now,sleep_1} = T_0 + T_{now,sleep_1} + \Delta T \ \ ,$$

$$T_{super,now,sleep_2} = T_0 + (1-\Lambda(BSFR))2T_{now,sleep_1} + \Delta T \ \ .$$

4. Similar formula holds true for the moment of wake-up. In the previous formula T_{now} is replaced with $T_{now,sleep_2}$ and one has

$$T_{super,now,wakeup_1} = T_0 + \Lambda^{1)}(SSFR)T_{now,sleep_2} + \Delta T^{1)} \ ,$$

$$T_{super,now,wakeup_2} = T_0 + (1-\Lambda^{1)}(BSFR))\Lambda^{1)}(SSFR)2T_{now,sleep_2} + \Delta T^{1)} \ .$$

The parameter T_0 depends on the choice of the origin of time for super-CD but is irrelevant.

One can deduce a consistency condition for the parameters of the model.

1. During the sleep period the time coordinate $T_{super,now}$ for moment "Now" in the coordinates of larger CD changes in the following manner:

$$T_{super,now,sleep} = T_0 + T_{now,sleep_1} \rightarrow T_{super,now,wakeup}$$
$$= T_0 + \Lambda^{1)}(BSFR)T_{super,now,sleep_2} + \Delta T^{1)} \ .$$

T_0 is an irrelevant parameter associated with super-CD. Note that there is breaking of time reversal symmetry since self associated with CD_{super} has fixed arrow of time unlike CD. Hence ΔT has at least in a statistical sense the same sign irrespective of the arrow of time of self.

2. This picture should be consistent with what we observe. When the tired average self fall a sleep at the evening, it wakes wake-up at the morning and is full of energy. Quite generally, wake-up occurs after time $\Delta T(sleep)$ meaning that the value of time T_{super} has increased by

$$T_{super,now,wakeup} = T_{super,now}(sleep_1) + \Delta T(sleep) \ .$$

These two expressions for the value of $T_{super,now}(wakeup)$ must be consistent and this gives a conditions on the parameters involved:

$$(1-\Lambda^{1)}(BSFR))\Lambda^{1)}(SSFR)2T_{now,sleep_1} + \Delta T^{1)}$$
$$= T_{now,sleep_1} + \Delta T + \Delta T(sleep) \ .$$

$\Delta T(sleep)$ is given by

$$\Delta T(sleep) = [(1-\Lambda^{1)}(BSFR))\Lambda^{1)}(SSFR)2-1]T_{now,sleep_1} + \Delta T^{1)} - \Delta T \ .$$

Intuitively it seems clear that for a given arrow of time it is not possible to wake-up before one falls asleep, and the condition $\Delta T(sleep) > 0$ for the standard arrow of time gives a constraint on the parameters. One cannot however exclude the possibility of time travel without dying or falling asleep first of the duration of time travel is much longer than that of wave-up period: $\Delta T^{1)} - \Delta T$.

A special solution corresponds to $\Delta T(sleep) = \Delta T^{1)} - \Delta T$ and $(1-\Lambda^{1)}(BSFR))2\Lambda^{1)}(SSFR) = 1$ giving $T_{now,sleep_2} = T_{now}$.

8.4 Still about quantum measurement theory in ZEO

The relation between zero energy ontology (ZEO) based quantum measurement theory and adelic vision could be much clearer. The following considerations suggest a more precise picture about cognitive representations and formulation of quantum measurement theory for them.

In the sequel ZEO based theory of consciousness [L56, L91] as quantum measurement theory is discussed first by starting with a criticism of physicalism and after that introducing ZEO based view about consciousness as quantum measurement theory as a solution to the problems of physicalism.

After this the relation between zero energy ontology (ZEO) based quantum measurement theory and adelic vision [L54, L53] is discussed. The considerations suggest a more precise picture about cognitive representations and formulation of quantum measurement theory for them. One can generalize classical cognitive representations as number theoretical discretizations of space-time surfaces in the extension of rationals considered to their quantum counterparts as wave functions in the Galois group of the extension and introduce also fermions as spinors in the group algebra of Galois group. The strongest option is purely number theoretical representations of spinors as spinors in this group algebra. Presumably however M^8 spinors are required and have interpretation in terms of octonion structure.

An attractive vision is that number theoretical quantum measurements reduce to measurement cascades involving a sequence of state function reductions reducing the entanglement between wave functions in sub-Galois group H and group G/H and ends up to a prime Galois group for group algebra has prime dimension and represents Hilbert space prime not decomposable to tensor product.

Also time measurement is considered from the number theoretic perspective assuming M^8-H duality [L81]. Clock readings are realized as roots of the rational polynomial determining the space-time surface in M^8. Time measurement would involve a localization to a definite extension of rationals, whose dimension n must be proportional to the temporal distance T between the tips of causal diamond (CD) to guarantee fixed time and energy resolution.

8.4.1 ZEO based theory of consciousness as quantum measurement theory

Consider first zero energy ontology (ZEO) based quantum measurement theory as a theory of consciousness.

Criticism of physicalism

It is good to start with a criticism of physicalism.

1. In physicalism consciousness would reduce to a physical property, like energy, momentum or charge and one would have the hard problem. There would be absolutely no idea why for instance sensory qualia emerge and how they correspond to sensory input. For instance, the assignment of sensory qualia to brain regions leads to a mystery: auditory, visual, etc. areas look exactly the same. How they can give rise to so different qualia?

   Remark: The answer to the question is that this is not possible. I n TGD framework macroscopic quantum coherence and ZEO allow to assume that sensory qualia are seated at sensory organs [L47].

2. This is not the only problem: free will is not possible and we must stop talking about ethics and moral as we have indeed done in modern free market economy, which threatens to destroy our civilization.

3. The third problem of physicalism and also idealism is that conscious experience is about something: it carries information about something, external world, my body, even about my thoughts. It is associated with a pair of systems- me and the rest of the world - rather than single system as consciousness as a physical property implies. This “aboutness”, kills the physicalistis view and actually idealism and under reasonable assumptions also dualism. Standard ontologies of consciousness fail.

Physicalistic approach has also problems with quantum measurement theory. The basic problems are basically due to the fact that observer as a conscious entity remains an outsider: observations affect the measured system but theory cannot say anything about observer as subjective entity. In ZEO the situation is different [L91] (`http://tinyurl.com/wd7sszo`) .

1. Quantum jump defines the basic building brick of conscious experience. It is something between two different quantum worlds, not in the world as a physical property of quantum system. Consciousness is a moment of re-creation. This a solves the hard problem and problem of free will.

2. Also the paradox of state function reduction can be solved if one can understand the problems related to the notion of time. There are two times: experienced time and geometric time, or the clock time. They are very different. Experienced time irreversible and has preferred moment "Now". Geometric time reversible and without preferred "Now". For some reason these times have been however identified.

ZEO based quantum measurement theory

In ZEO physical states as time= constant snapshots are replaced by pairs of "initial" and "final" states A and B or - by holography - with superpositions of deterministic time evolutions from A to B with respect to geometric time - note the analogy with computer program in computer science, behavior pattern in neuroscience, and function in biology.

1. In "small" state function reductions (SFRs) -"weak" measurements - the superposition of time evolutions from A to B is replaced with a new one such that states A at passive end - "initial state" - are not changed. Classical determinism is respected although one has quantum jump and generalization of quantum measurement theory. Two times - two causalities. The temporal distance T between A and B increases in statistical sense and this gives the correspondence between experienced time as sequence of state function reductions and geometric time is identified as T. These measurements changing B correspond to "weak" measurements analogous to classical measurements and to sensory input. A represents permanent part of selfness, "soul" one might say.

2. In "big" (ordinary) state function reductions (BSFRs) the roles of "initial" and "final" states change and the arrow of geometric time changes. Self dies and reincarnates with an opposite arrow of geometric time.

3. In more precise view the pairs of time=constant snapshots are replaced with what I call causal diamonds (CDs). The assumption that the size of CD is preserved In BSFR as assumed originally leads to some paradoxical looking implications. For instance, the size of CDs assignable to our sub-selves identifiable as mental images would increase without bound. $M^8 - H$ duality suggests strongly that the sizes of CDs can decrease in BSFR: the formerly active boundary would be frozen but the temporal distance of formerly passive boundary would be reduced so that the size of CD would decrease. One could say that self has childhood and starts from scratch with all sins of previous life forgiven.

This picture about state function reduction finds considerable empirical support.

1. The paradoxical experimental findings of Minev *et al* in atomic systems challenging standard quantum measurement theory give strong support for the reversal of the arrow of time in BSFR [L77] [L77] (`http://tinyurl.com/yj9prkho`).

2. Also Libet's finding that experience of free will [J2] seems to be preceded - caused - by neural activity, can be understood. It is not anymore support for the claim that free will is an illusion. State function reduction changing time order happens, and free will causes neural activity in the geometric past.

3. There is are lot of support for the new view about time from biology. For instance, self-organization - not only biological - could be understood as involving time reversal meaning that the time reversed reduction of order implied by generalization of second law looks from standard observer's viewpoint like increase of order. Self-assembly and generation of structures in long scales would involve increase of time order. Evolution is second aspect of self-organization and reduces to the unavoidable increase of h_{eff} as dimension for extension of rationals. Also the need for energy feed - metabolic energy feed in living matter - can be understood because the increase of h_{eff} keeping other parameters constant, increases energy scale. Dark matter would be visible everywhere in sharp contrast with standard prejudices.

4. There is support even from cosmology and astrophysics, where TGD predicts quantum jumps in macroscopic scales. For instance, stars older than Universe can be understood in more detailed picture about ZEO [L78, L80] (`http://tinyurl.com/tf38xnx`).

One can of course criticize the view about the role of clock time as the distance T between the tips of CD as over-simplified [L91].

1. The state function reductions preceding SSFRs are preceded by unitary processes U. What one can say about "time evolution" U. First of all, U is assumed to produce a zero energy state de-localized in the space of CDs - in particular with respect to the distance T between the tips of CD.

   The simplest guess is that in SSFR a complete localization in T - measurement of T - and other moduli of CD (say boost with respect to the lower tip of CD) occurs. Can one reduce the localization in T to a SSFR reducing quantum entanglement or is time measurement something different? What entanglement of CD sizes with different values of T with the measurement apparatus could mean? What the presence of a measurement apparatus for time T - the clock at fundamental level, could mean mathematically? Later also the question whether one could reduce this measurement to pure number theory emerges?

2. The notion of completely localized state is over-idealization and also mathematically poorly defined. Gaussian wave packet over classical states with well-defined classical conserved energy (by Poincare invariance) with respect to T localized around some value T_0 is a more realistic notion and time measurement would mean localization to a wave packet around T_0.

   In [L91] the proposal that the time evolution of self could be seen an analog of cooling process analogous to cosmic cooling is considered. This would correspond to an adiabatic time evolution happening for a particle in box whose size increases slowly. In this process the coefficients in a superposition of states with given classical energy remain unaffected but the classical energies of the states themselves decrease. This would conform with Uncertainty Principle stating that the classical energies scale as $1/T$.

A more detailed view about quantum measurement in ZEO

Consider next in more detail what state function as quantum measurement means in TGD.

1. In standard quantum measurement theory quantum measurements are often thought to be performed by humans only. In TGD one assumes that state function reduction as analog of quantum measurement is universal and can take place for any pair of mutually entangled systems unentangled from its complement.

2. Density matrix for the entangled pair of systems is the fundamental observable. This applies to both BSFRs and SSFRs at active boundary of CD, which correspond to "weak" measurements commuting with the observables diagonalized at the passive boundary of CD and thus leaving the states at it invariant.

3. Quantum measurement involves typically measurement of several observables. This is realized as a measurement cascade. First the quantum measurement of density matrix occurs for some pair formed sub-system S_1 and its complement S_2 forming together system S. After the same occurs for S_1 and S_2. Observables correspond to density matrices in this cascade. One proceeds as along as new decompositions are found. If the final state belongs to a sub-space with prime dimension the cascade stops since there is no further decomposition to tensor product.

4. The density matrix for subsystem in general case decomposes to a sum of projectors t sub-spaces and the state function reduction takes to one of them. The outcome of the measurement can be sub-space rather than ray.

   Number theoretic vision suggests also a second possibility. The SSFR would take place only if the eigenvalue of density matrix having probability interpretation associated with the subspace or ray is in the extension of rationals associated with the matrix elements of the density matrix and space-time surfaces considered (defining the cognitive representation). If one assumes frequency interpretation of probability theory, this probability must be rationals. Entanglement can be number theoretically stable. This would that one can have stable entanglement.

It is natural to assume that BSFR can can increase the extension of rationals associated with the eigenvalues of density matrix in the extension of the extension associated with its matrix elements.

5. Stable entanglement could be crucial for quantum computation as also the possibility of large values of h_{eff} and of time reversal. One can also assign to entanglement with coefficients in an extension of rationals p-adic variant of entanglement entropy by replacing logarithms of probabilities with the logarithms of their p-adic norms. These p-adic entanglement negentropies can be positive so that the entanglement carries information. This negentropy is different from the real negative entropy due to the loss of precise knowledge about entangled states. Quite generally, the sum of p-adic negentropies can be larger than real entropy. This would explain the paradoxical looking fact that highly evolved biological systems are highly entropic [?] [L21]. england

8.4.2 The relationship between adelic physics and ZEO based quantum theory

The challenge is to formulate quantum measurement theory taking into account the constraints from adelic physics [L54, L53]. One can consider the possibility is that the quantum physics could reduces at the level of cognitive representations to purely number theoretic physics. This would mean huge simplification. I have considered quantum theory at the level of cognitive representations from the point of view of number theory in [L88] and from the perspective of scattering amplitudes in [L87].

Two kinds of cognitive representations

One can consider two kinds of cognitive representations. The cognitive representations considered hitherto correspond to number theoretical discretization of space-time surface determined by an extension of rationals, they are "classical". The bosonic wave functions in Galois group of extension acting on cognitive representations and their fermionic counterparts based on fermionic dynamics in the group algebra of Galois group and its normal subgroups (Galois groups too) would define quantal cognitive representations.

1. There are cognitive representations both at the classical level in terms number theoretical discretizations of space-time surfaces defined by the extension of rationals and at the quantum level based on spinorial wave functions in Galois group of the representation. Also the spinorial wave functions in factor sub-groups and normal subgroups of Galois group are involved.

2. One can assign preferred primes p_{pref} to the classical space-time dynamics as ramified primes p_{ram} of the extension. For these the polynomial defining extension has double root in $O(p) = 0$ approximation. This would be the realization of quantum criticality for cognition: criticality is typically in potential models a situation in which two or more extrema of the potential function co-incide - catastrophe theory of Thom is classical example.

3. At the level of state (spinorial) space wave functions in Galois group acting on cognitive representations are natural candidate for a bosonic state space. Quantum states would be wave functions in Galois group G with normal subgroup H acting as a Galois group of lower-D extension.

   G/H is group itself and one can express wave functions in G as superpositions of products wave functions in G/H and H. The wave functions in G/H and H define naturally a tensor product and an attractive idea is that state function reduction can be regarded as measurement in G/H or equivalently in H. When H has prime order further reduction is not possible since Hilbert spaces with prime dimension are primes of tensor product.

   A natural candidate for preferred primes p_{pref} is as orders of smallest possible normal subgroups of Galois group, kind of primitive generating Galois groups.

Remark: One must consider also the possibility that quark and possibly also leptonic degrees of freedom are present as additional spinor indices. The fact that M^8 has octonionic structure could require also M^8 spinor structure.

4. In TGD dark matter is identified as $h_{eff} = n \times h_0$ phases of ordinary manner. n is identified as the order of Galois group of Galois extensions and thus of the extension itself. For ordinary value of Planck constant empirical inputs suggests the identification $h = 6h_0$ [L32, L64].

   Quite interestingly, one has $6 = 2 \times 3$ so that there is factorization to 2-D and 3-D subspaces assignable to massless particles, and massive gauge bosons. This indeed suggests that number theoretical vision could allows to represent all many-particle states in terms of wave functions (spinor fields) in the group algebra of Galois group.

5. How to construct cognitive representations for fermions? A natural generalization of the bosonic dynamics in n-D group algebra of Galois group is introduction of spinor structure in terms of 2^k-dimensional spinors in the group algebra. For $k = n$ both chiralities are present and for $k = n-1$ only second chirality. In fact, one could pose even more chirality conditions giving $2^{n/2}$-D ($[n+1]/2$-D) spinors for even (odd) n. Indeed, the recent view about SUSY in TGD framework suggests that only quarks - second embedding space chirality - appear as fundamental fermions and that leptons are local composites of 3 quarks - spartners of quarks in well-defined sense [L92] (`http://tinyurl.com/y4pdb2xz`).

   The simplest option is that at the level of cognitive representations the fermionic oscillator operator algebra corresponds to the oscillator operator algebra creating fermions states having at most $k = n$, $k = n-1$,...$n/2$ ($[n+1]/2$) fermions assignable to these spinors in finite measurement resolution. Entire quantum dynamics at the level of cognitive representations would reduce to the dynamics of fermions in the group algebra of Galois group and its Galois sub-groups.

6. There is also question about the Galois groups of the extensions of various p-adic number fields Q_p induced by the extension of rationals with dimension n. For p-adic numbers in approximation the extension reduces to a finite field $G(p,k)$, $k \leq n$, and one has k-dimensional extension. Galois group G_p is smaller than the Galois group G for rationals. G_p would act naturally in the p-adic counterparts of cognitive representations and the representations of G would reduce to direct sums of representations of G_p. Note that the distinction between sensory and cognitive (real and p-adic) would emerge only at the quantum level.

   For $p < n+1$ the fact that one has $x^{p-1} = 1$ for $G(p)$ implies that the irreducible polynomial P defining the extension Q reduces to a polynomial with degree $nmodp-1 \leq p-1$. Information is lost for $p < n+1$. For $p \geq n+1$ situation is different but also in this case the reduction occurs for ramified primes since polynomial P as in this case multiple roots. This would be the counterpart of quantum criticality at the level of cognitive representations.

7. Could the primes appearing as factors of n be preferred p-adic primes? Since these primes as p-adic primes mean a loss of information, they are distinguished but hardly preferred in p-adic evolution. Ramified primes larger than n are more plausible candidates and can be assigned even with polynomials of order 2. The preferred p-adic primes assignable to elementary particles are indeed large: electron would correspond to $M_{127} = 2^{127} - 1 \sim 10^{38}$ [K65].

Quantum measurement theory for cognitive representations

What can one say about quantum measurement theory for cognitive representations? The basic questions concern the tensor products. How many tensor factorizations there are and can one pose some conditions on them? Assume that fermionic Fock states for second quantized spinor fields in n-D group algebra are enough for quantum physics at the level of cognitive representations.

1. Tensor product decomposition for n-D group algebra corresponds to the factorization $n = k \times l$. All factorizations of n define a possible quantum measurement situation and state function reduction can take place in bosonic sector to k or equivalently l-dimensional space. These factorizations would be highly unique since they correspond to pairs of Galois group G and its Galois subgroup H. They are defined modulo discrete automorphism of G. It is

not clear whether the choice of this automorphism has physical content: one might consider a discrete variant of gauge invariance.

For the fermionic oscillator algebra analogous statement holds true. Now the decompositions are induced by $n = k \times l$ decompositions.

2. State function reduction cascades would correspond to sequences of Galois subgroups $G \supset G_1 \supset ....G_k$ such that G_k corresponds to either trivial group of group with prime order. In this case the final state would be reached by a factorization in which the density matrix for G_k does not allow eigenvalues in the extension considered. This extension could be G, G_1 or perhaps rationals (frequency interpretation for probabilities).

$M^8 - H$ duality and measurement cascade

$M^8 - H$ duality [L81] suggests much more concrete picture about the measurement cascade.

1. $M^8 - H$ duality predicts that the roots r_n of a rational polynomial defining the space-time surfaces at the level of M^8 correspond "very special moments in the life of self" $t = r_n$ for the M^4 linear time in the rest system of CD, and that once these moments have been experienced, BSFR can take place. This is possible but not the only possible interpretation.

2. $M^8 - H$ duality and the view about evolution as analog of genetic evolution in which genes are conserved suggests that the polynomials can be regarded as functional composites of simple polynomials $P = P_{n_1} \circ P_{n_2} \circ ...P_{n_k}$ satisfying $P_{n_r} = 0$ (n_i refers to the degree of the polynomial). P possesses the roots of P_i and the corresponding Galois groups as normal subgroups as the counterpart for the conservation of genes in evolution.

   One can distinguish also primitive polynomials as those defining extensions which do not decompose further. Galois groups with prime number of elements corresponds to such extensions. Note that the same extension can appear at several levels in hierarchy and would correspond to a realization of extension at different hierarchy level defining a kind of abstraction level.

3. Intuitively the measurement cascade should correspond to a cascade proceeding to shorter time and length scales by increasing the resolution and also to a process in which abstraction is gradually concretized.

   Could the measurement cascade for a state localized to a given extension of rationals start with the measurement of the root set $X_1 = \{r_{1,1}\}$ of P_{n_1} corresponding to the lowest time resolution. After than P_2 and the root set $X_2 = \{r_{2,i}\}$ would be measured meaning a refined of time resolution replacing $r_{1,i}$ with as subset of X_2 around it.

   Here one must be however very cautious: one could also consider a hierarchy of CDs with decreasing size scales as the counterpart of the measurement cascade. I do not understand well enough the scale hierarchy to answer the question whether these two views might relate.

Measurement of time number theoretically

Could the measurement of clock time T as (average) distance between the tips of CD [L91] be understood as number theoretical measurement?

1. What about the measurement of time as the distance T between tips of CD or more generally as the center of mass value T_0 of T in the case that one has Gaussian wave packets localized around varying T_0? How could one realize the measurement apparatus - the clock - in terms of entanglement?

   Suppose that the superposition over CDs with different values of T corresponds at the level of space-time surfaces in M^8 to that for space-time surfaces determined by polynomials P_n with varying degrees and rational coefficients. The measurement fixing the extension and Galois group would not fix P_n since there is a large number of polynomials with rational coefficients but same Galois group. The measurement fixing the extension leads to a partial (at least) localization in T or T_0 but this is not expected to be enough.

2. A stronger localization in the state function reduction measuring n would require that T or T_0 correlates with the degree n. How could this be achieved in a natural manner? Intuitively the requirement of some fixed time resolution based on the preferred moments $t = r_n$ interpreted as clock readings has fixed resolution as the average time lapse $\Delta T = \langle \Delta T_{i,i+1} = r_{i+1} - r_i \rangle$ would require $n \propto T$ or $n \propto T_0$. How could this be achieved concretely? Could one specify the zero energy states by giving the time resolution as ΔT and being equivalent to energy resolution. This would also dictate the resolution of the cognitive representation as the set of space-time points in the extension.

8.5 Some questions concerning zero energy ontology

Zero energy ontology (ZEO) [L91] gives rise to quantum measurement theory, which naturally extends to a theory of consciousness. In this article also consciousness aspect is central and my sincere hope is that it would not expel those physicist readers for whom consciousness still remains an unscientific notion.

Zero energy ontology (ZEO) briefly

ZEO provides a new ontology solving the key problem of the standard quantum measurement theory and quantum theory itself. It must be emphasized that ZEO is not a new interpretation created to put under the rug the logical paradox due to the conflict between non-determinism of state function reduction (SFR) and the determinism of unitary time evolution. Also the problem about the scale in which quantum world becomes classical disappers: the Universe is quantal im all scales and ZEO view about quantum jump makes the Universe to look like classical.

1. At the level of space-time dynamics, the notion of preferred extremal (PE) as a space-time surface is central: PE is an extremal of an action principle, which by general coordinate invariance must be highly unique once its intersection with either boundary of causal CD $= cd \times CP_2$ (cd is the intersection of future and past directed light-cones of M^4) is given. In the ideal situation this implies holography. Space-time surface is an analog of Bohr orbit and classical theory is an exact part of quantum theory.

   There is probably a finite and discrete non-determinism analogous to that associated with soap films spanned by a frame: space-time is indeed a minimal surface as also soap films, and the 3-surfaces at its ends at boundaries of CD are part of the frame. Besides space-time surface is an extermal for Kähler action analogous to Maxwell action. The challenge is to interpret this finite non-determinism.

2. Quantum states, which I call zero energy states, can be interpreted as pairs of analogs of ordinary 3-D quantum states with positive energy. The members of the pair are at the opposite boundaries of CD. The convenient convention used also in quantum field theories (QFTs) is that the conserved quantum numbers at opposite boundaries sum up to zero classically: this brings in nothing new. At quantum level, 4-momenta are conserved only at the limit when CD has infinite size whereas classically the conservation holds true for all CD sizes: this reflects the Uncertainty Principle [L141]. Also in QFTs exact momentum conservation is obtained only at the limit of infinite quantization volume.

   At the space-time level, zero energy states can be regarded also as superpositions of deterministic time evolutions: this is central for the interpretation.

3. SFRs are quantum jumps between zero energy states. SFR does not affect any deterministic time evolution but only replaces their superposition with a new one. This solves the paradox that was one of the key motivations for ZEO.

4. Zeno effect strongly suggests that there are 2 kinds of quantum measurements assignable to SFRs. For "weak measurements", "small" SFRs (SSFRs), the component of zero energy state at the either boundary of CD, to be called passive boundary (PB), is unaffected. Also the PB is unaffected apart from scaling. At the active boundary (AP) state changes and AP is scaled up (at least in statistical sense) and due to the scaling shifts to the geometric future.

The unitary time evolution preceding each SSFR corresponds to a scaling of CD (or rather, its M^4 projection cd) rather than time translation as its counterpart in string models. In A unitary evolution B between two SSFRs a superposition of CDs with varying sizes is formed and SFR localizes CD to a fixed size, which means the measurement of geometric time identifiable as the distance between the tips of CD. This geometric time correlates with the subjective time defined by the sequences of SSFRs. Subjective and geometric times are not identical as in standard ontology but only correlated.

5. "Big" SFRs (BSFRs) are the counterparts of ordinary quantum measurements. In the BSFR the roles of AB and PB of CD change so that the arrow of time changes since CD increases in the opposite direction of time (at least in statistical sense). For an observer with an opposite arrow of time, BSFR looks like an average deterministic time evolution leading to the final state of BSFR as observed experimentally by Minev *et al* [L77] [L77]. This illusion makes BSFR look classical in all scales although the TGD based dynamics is quantal in all scales due to the hierarchy of Planck constants predicted by TGD.

   The possibility of time reversal forces a generalization of thermodynamics to allow both arrows of time: this kind of generalization was proposed long ago by Fantappie [J7] with motivation coming from biology. Quite generally, self-organization processes seem to violate the arrow of time. External energy feed explains this partially but BSFR would be an important additional element of self-organization [L86, L135], especially so in living matter.

   The assignment of "free will" to BSFR allows us to understand how free will can be consistent with the classical non-determinism of physics which would be exact.

ZEO based quantum measurement theory and therefore also physics naturally extends to a theory of consciousness, and one cannot avoid using this word, which is still a cursed word in the physicalistic camp.

Problems related to the mathematical realization of ZEO

There are several open questions related to ZEO and TGD inspired theory of consciousness and the existing view involves several working hypothesis which should be reduced to deeper principles or shown to be wrong.

At least the following questions related to physical interpretation of ZEO are still waiting for a detailed answer.

1. Preferred extremal (PE) property of space-time surfaces is central for quantum TGD [L119]. It follows from holography forced by general coordinate invariance (GCI), which however need not be ideal. How uniquely does the PE property of the space-time surface fix the space-time surface inside a given CD? The simplest situation is that the data at the end of the space-time surface at either boundary of the CD,fixes it completely. Space-time surface would be an analog of Bohr orbit.

   Full determinism would imply that WCW for CD effectively reduces to the space of 3-surfaces assignable to either end of CD. The dynamics of SSFRs would reduces to that in fermionic degrees of freedom assignable to Boolean cognition since WCW degrees of freedom assignable to sensory perception would be fixed.

   However, the dynamics of soap films spanned by frames suggests that this is not the case. The 3-D ends of the space-time surface define a frame and also dynamically generated portions of frame are allowed by the variational principle defined by the sum of a volume term and Kähler action as an analog of Maxwell action. The coefficient of the volume term has an interpretation in terms of a length scale dependent cosmological constant Λ.

   Outside the frame space-time surface would be at least for a very large portion of extremals an analog of complex surface and therefore a minimal surface [L146] and also an extremal of Kähler action. At the frames only the equations for the entire action (sum of volume term and Kähler action) would be satisfied. The divergences of the conserved isometry currents for the volume term and Kähler action would have delta function type singularities but they would cancel each other. The portions of the frame could be analogous to singularities of analytic functions such as cuts and poles.

2. Number theoretic universality [L53, L54] in turn suggests that the inherent non-determinism of p-adic differential equations [K79] [L91] proposed to be a correlate of imagination could also relate to this non-determinism. How do the non-determinism of space-time surface, p-adic non-determinism, and non-determinism of the state function reduction relate to each other: could they be even one and the same thing?

ZEO based quantum measurement theory defines a theory of consciousness. How unique is the interpretation of zero energy ontology (ZEO) [L91]? Here 3 options suggest themselves corresponding to "western" and "eastern" world views and their hybrid.

1. For the western option, the space-time surface continues outside any CD as external world, in particular sub-CD and sub-CD is a correlate for the perceptive field of self.

2. For the eastern option, space-time ends at the boundary of any CD and sub-CD is not a correlate for the perceptive field of self and there is no constraint from the external world at boundaries of CD.

3. For the hybrid of these two options, conscious entity corresponds to a hierarchy of CD for which the highest level corresponds to CD for which space-time does not continue outside the CD. The highest level represents a God-like entity.

Problems related to ZEO based theory of consciousness

The new picture about sub-CDs at WCW level raises questions related to the TGD inspired theory of consciousness. This view involves several ad hoc assumptions related to the notions such as attention, mental image, memory, volition and intentions. Do these assumptions follow from more general assumptions or can some of them be simply wrong?

1. CD is a correlate for the perceptive field of self. Sub-CDs of CD define perceptive fields of subselves identified as mental images. What is the precise definition of sub-CD? Can one say that a sub-CD is created when a mental image is created. How does this happen? What determines the position and size of the sub-CD?

   The sub-CD is defined by the restriction of zero energy state to sub-CDs so that sub-CDs are induced by CD. This condition is analogous to boundary condition in classical physics and freezes WCW degrees of freedom of sub-CD at the passive boundary (PB) but the failure of determinism leaves discrete degrees of freedom at the active boundary (AB) so that the dynamics of SSFRs is restricted to these sub-WCW degrees of freedom and fermionic degrees of freedom.

2. Where sub-CDs and subselves are located? The natural location for a minimal sub-CD and mental images is around 3-surface at which the classical non-determinism fails: the frames of the soap film in soap film analogy. One can develop a rather detailed picture about frames [L146] based on number theoretic vision realized in terms of M^8-H duality [L109, L110, L128].

3. How sub-selves (sub-CDs) are created? Can they disappear? The notion of attention as generation of sub-CD achieved by a location of WCW ("world of classical worlds") spinor field at spacetime surfaces having their intersection with the PB of CD in a fixed set of 3-surfaces defining the sub-WCW is highly suggestive. This also affects the WCW spinor field of CD.

   The attention can be directed in several ways. Redirection of attention means a movement of the region defining the content of mental images in the interior of a CD. Entanglement and classical communications would be naturally associated with attention defined in this manner. If minimal subselves are associated with the frames as loci of classical non-determinism, the set of targets of attention is discrete and finite.

   This view about attention makes it possible to see also memory, anticipation, and intentions as special cases of attention.

4. The time evolution of CD itself would correspond to a scaling of CD (rather than translation), which by the failure of strict determinism brings in new discrete degrees of freedom related to the new frames becoming into the daylight as space-time surfaces increase. In the new picture, the sub-WCW property poses strong restrictions to the earlier picture about the development of sub-CD. The idea about silent wisdom as mental images preserved from the previous life after BSFR is not lost but is considerably modified.

   In this picture, the small failure of classical determinism would be an absolutely essential element in that it makes possible a non-trivial theory of consciousness at the level of CD and at space-time level. Otherwise would have only fermionic degrees of freedom forgiven sub-CD. What is intriguing is that everything would be finite. SFRs would involve choices between finitely many alternatives and in this respect the theory would be analogous to the computationalistic approach: in fact, preferred extremals are analogous to computer programs.

8.5.1 Some background

In the sequel, some understanding of the basic ideas and notions of TGD proper [L119] is needed. Also ZEO as the target of critical discussion is briefly summarized.

TGD view briefly

Very concisely, TGD emerges as fusion of special and general relativities and has Poincare invariance of special relativity and General Coordinate Invariance (GCI) and Equivalence Principle (EP) as basic principles. Also the interpretation as a generalization of string models is possible: point-like particles are replaced by 3-surfaces instead of strings and world lines become space-time surfaces.

The notion of induction makes it possible to eliminate classical boson fields as primary dynamical variables and reduce them to the sub-manifold geometry of the space-time surface. For the simplest option, free second quantized quark fields of the embedding space $H = M^4 \times CP_2$ induced to the space-time surface remain as fundamental fermion fields and quarks serve as basic building bricks of both bosons and fermions as elementary particles [L92, L129].

Some understanding of notions such as the "world of classical worlds" (WCW) [K96], preferred extremal (PE) [K9], and various variants of holography [L109, L110] implied by general coordinate invariance (GCI) in TGD framework is assumed. Inclusions of hyperfinite factors of type II_1 (HFFs) [K126, K44] are central elements of quantum TGD proper.

Adelic physics [L54, L53] replacing real number based with number theoretical universal physics based on the hierarchy of adeles defined by extensions of rationals (EQs) and $M^8 - H$ duality (see Appendix 8.5.6) allowing number theoretic and geometric views about physics dual to each other is also assumed as the background.

Hierarchy of Planck constants $h_{eff} = n \times h_0$, with n identified as dimension of EQ, is the basic implication of adelic physics and central for quantum TGD. The phases labelled by h_{eff} behave like dark matter [K33, K34, K35, K36]. This hierarchy serves as a correlate for quantum criticality in arbitrarily long length scales.

Cognitive representations identified as points of space-time surface for which preferred coordinates of embedding space are in an extension of rationals are also central for the construction of the theory using $M^8 - H$ duality [L109, L110].Galois group of EQ becomes number theoretical symmetry and is central in the description of quantum variants of cognitive representations [L16, L121].

Zero energy ontology (ZEO) [L91] is a key notion of quantum measurement theory. The basic prediction is that time reversal occurs in the ordinary state function reduction (SFR). This has profound implications for the interpretation of the quantum measurement theory [L77].

TGD inspired theory of consciousness can be seen as an extension of quantum measurement theory and relies on Negentropy Maximization Principle (NMP) as a basic dynamical principle [K70] [L135] implying second law for ordinary entanglement entropy.

$M^8 - H$ duality as it is towards the end of 2021

The view of $M^8 - H$ duality (see Appendix 8.5.6) has changed considerably towards the end 2021 [L141] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L141] of the M^8-H duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L141] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L109, L110]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$. This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L81]. This notion does not make sense at the level M^8 anymore.

The modified $M^8 - H$ duality forces us to modify the original interpretation [L141]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L128] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L141]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

ZEO

The TGD based view of consciousness relies on ZEO solving the basic paradox of quantum measurement theory. First, a brief summary of the recent view of ZEO [L91] is required. Some aspects of this view will be challenged in the sequel for sub-CDs.

1. The notion of a causal diamond (CD) (see **Fig. ??**) is a central concept. Its little cousin "cd" can be identified as a union of two half-cones of M^4 glued together along their bottoms (3-D balls). The half-cones are mirror images of each other. CD=$cd \times CP_2$ is the Cartesian product of cd with CP_2 and obtained by replacing the points of cd with CP_2. The notion of CD emerges naturally in the number theoretic vision of TGD (adelic physics [L53])via the $M^8 - H$ duality [L85, L109, L110].

2. In the ZEO, quantum states are not 3-dimensional if the classical determinism does not fail as it actually does, but superpositions of 4-dimensional deterministic time evolutions connecting ordinary 3-dimensional states. By holography forced by general coordinate invariance, time evolutions are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution.

Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced by a new superposition. The classical determinism of individual time evolution is not violated. This solves the basic paradox of quantum measurement theory. There are two kinds of SFRs: BSFRs (counterparts of ordinary SFRs) changing the arrow of time (AT) and SSFRs (analogs of "weak" measurements) preserving the arrow of time that give rise to an analog of the Zeno effect (`https://cutt.ly/yl7oIUy`) [L91]. The findings of Minev *et al* [L77] provide strong support for ZEO [L77].

To avoid confusion, one may emphasize some aspects of ZEO.

1. ZEO does not mean that the physical states identified in standard quantum theory as 3-D time= constant snapshots - and assigned in ZEO to the opposite boundaries of a causal diamond (CD) - would have zero energy. Rather, these 3-D states have the same conserved quantities, such as energy. Conservation laws allow us to adopt the convention that the values of conserved quantities are opposite for these states so that their sum vanishes.

   This is not new: in quantum field theories (QFTs), one speaks, instead of incoming and outgoing particles, external particles arriving from the geometric past and future and having opposite signs of energy. That conserved quantities vanish in the *4-D* sense, expresses only the content of conservation laws. A weaker form of this condition [L134] states that the total conserved Poincare charges are opposite only at the limit of infinitely large CD. CD would be an analog of quantization volume in QFTs, whose finiteness implies a small conservation of momentum.

2. ZEO implies *two* times: subjective time as a sequence of quantum jumps and geometric time as a space-time coordinate: for instance, the proper time of the observer. Since subjective time does not correspond to a real continuum, these times are not identifiable but are strongly correlated. This correlation has led to their identification although they are different.

8.5.2 How uniquely PE property fixes the space-time surface?

How uniquely the PE property fixes the space-time surface if its 3-D intersections with the boundaries of CD are given? This is the key question in this section.

Various variants of holography

General coordinate invariance (GCI) forces holography in the TGD framework. One can however consider several variants of holography [L109, L110, L135].

1. Holography in the standard sense would fix the space-time surface from the data of its intersection with either boundary of CD or the data associated with the light-like 3-surfaces at which the signature of the induced metric changes.

2. Strong form of holography (SH) states that 2-D data at the intersections of the light-like 3-surfaces and boundary of CD are enough to determine the space-time surface.

3. The strongest form of holography inspired by M^8-H duality [L109, L110, L134] states that space-time region is determined by a rational value coefficients of a real polynomial extended to an octonionic polynomials, whose "root" is the space-time surface in M^8. The n roots of a real polynomial would determine a 4-D region in M^8 and its image in $H=M^4\times CP_2$ would be interpreted as space-time surface.

4. There is a variant of holography, which gives up the full determinism of classical field equations and gives rise to what look like classical topological analogs of Feynman diagrams.

   (a) Consider first the particle level at the level of H. Particle lines generalized to 4-D orbits of 3-D surfaces representing particles. Particles as 4-D orbits of 3-surfaces contain light-line 3- D orbits of partonic 2-surfaces.

(b) Partons as building bricks of particles in the information theoretic sense, and correspond to partonic 2-surfaces at which the orbits of partonic 2-surfaces meet. Their orbits are 3-D light-like surfaces at which the signature of the induced metric of the space-time surface changes.

The partonic 2-D surfaces defining topological vertices belong to the 3-D sections of space-time surface with a constant value of M^4 time coordinate t to which one can map the 6-D brane-like entities of M^8 predicted by $M^8 - H$ duality [?]

This picture suggests that, besides the data at the boundaries of CD, also the data at the partonic 2-surfaces in the interior of CD are needed. This failure of classical determinism brings in the failure of the strongest form of holography. There would be a large number of PEs connecting the 3-surfaces at the ends of CD and they would correspond to the analogs of Feynman diagrams.

Zero energy state as a scattering amplitude would be a superposition over these diagrams. This superposition would not be however pre-determined as in the path integral but the zero energy state would define the superposition of paths in question.

Is the failure of classical determinism possible?

The possibility of classical non-determinism is suggested by the interpretation of space-time surfaces as generalized Feynman diagrams. These Feynman diagram entities would not however define an analog of path integral in TGD framework. Classical non-determinis would be a space-time correlate for the non-nondetermism at quantum level.

In this framework partonic 2-surfaces or equivalently the 3-D sections of the space-time surfaces with constant value of M^4 time would act as 3-surfaces at which the deterministic time evolution as a minimal surface would fail.

Another option is that light-like 3-surfaces containing the partonic 2-surfaces at very special moments of M^4 time define frames. These special values $t = t_n$ of M^4 time would be associated with 6-D branes predicted by M^8 picture as universal special solutions and their images in H would define "very special moments in the life of self" defined by the sequences of SSFRs defining the self.

1. The first hint comes from the dynamics of soap films. Soap films are minimal surfaces. The soap films spanned by 1-D frames consist of minimal surfaces glued together at the frames and this dynamics is non-deterministic in the sense that it allows several soap film configurations due to the different branchings at frames. At frames the minimal surface equations fail.

2. In TGD framework space-time surfaces as PEs are both minimal surfaces and extremals of Kähler action. In this case the 3-surfaces associated with "very special moments of time" $t = t_n$ could define an analog of a dynamically generated frame defining a 4-D soap film. The 3-surfaces at the ends of the CD would be fixed frames like those for soap films.

   This realizes quantum criticality in the sense that the field equations outside frame do not involve the parameters of the action which sum of volume term and Kähler action. The interpretation as a non-linear analog of massless free field theory outside the frame conforms with the basic spirit of quantum field theory. These solutions of field equations rely on a a generalization of holomorphy to 4-D situation so that field equations reduce to purely algebraic conditions involving only the first derivatives of embedding space coordinates. The analogy is defined by the solution of 2-D Laplacian equation in terms of real or imaginary part of an analytic function.

   Field equations consist of two terms, which are divergences for the conserved currents (4-momentum currents plus color currents) defined by the induced metric in the case of volume term. In the interior of the space-time surface these divergences vanish separately for the volume term and Kähler action but not at the frame.

3. The field equations must hold true also at the 3-D frame but this need not be true for both volume term and Kähler action separately. The coupling parameters of the theory make themselves visible only via the frame. For the volume action the divergences of the conserved

currents are orthogonal to the space-time surface. For K "ahler action, the divergences of the conserved currents contain to terms. The first term is proportional to the energy momentum tensor of Kähler action and orthogonal to the space-time surface.

Second term is not orthogonal to the space-time surface. For twistor lift the Kähler also has an M^4 part with a similar decomposition.

The sums of the parts of divergences orthogonal to the space-time surface and parallel to it must sum up to zero separately. This gives 8 conditions altogether so that the number of field equations is doubled at the frame.

4. Could it happen that the divergences of these two isometry currents are singular and proportional to 3-D delta function but that their sum vanishes and conservation laws are respected? The part of the frame in the space-time interior would be dynamically generated whereas the part of the frame at the ends of CD would be fixed.

5. The restriction to 3-D frames is not the most general option. The delta function singularities could be located also at 2-D partonic 2-surfaces, at light-like 3-surfaces at which the induced metric changes its signature, and at string world sheets which connect these light-like 3-surfaces and have 1-D light-like boundaries at them. The light-like 3-D surfaces would be analogs of the cuts for analytic functions. Partonic 2-surfaces at the ends of light-like 3-surfaces could be analogs for the ends of the cuts. String world sheets could serve as analogs of poles.

6. The non-determinism associated with the soap films and with frames suggests that there is a large number of 4-D "soap films with a given frame", which is fixed at the boundaries of CD but not in the interior of CD.

8.5.3 Questions related to the theory of consciousness

At the level of TGD inspired theory of consciousness theory, causal diamond (CD) defines a correlate of self or of its perceptive field. CD has sub-CDs which correspond to subselves experienced by self as mental images [L91, L135].

Concerning the evolution of self, the basic notions of "small" state function reduction (SSFR) as an analog of "weak measurement" and "big" SFR (BSFR) as an analog of ordinary SFR.

1. The first deviation from the standard ontology is that BSFR changes the arrow of time defined by the selection of PB of CD at which 3-D part of zero energy states remains unchaged during SSFRs.

2. The second deviation is that either boundary of CD and states at it remain unaffected in SSFRs whose sequence defines self as a conscious entity. This is the TGD counterpart for the Zeno effect of ordinary quantum theory in which repeated measurements of the same observable leave the state unaffected.

The details of the evolution of self are not fully understood and the proposed general view can be criticized.

1. How the constraint that sub-CD serves as a correlate for a classical perceptive field can be taken into account?

2. What is the precise definition of mental images as subselves? Are they at some special positions inside space-time surface?

3. What are the precise definitions of memories and conscious memory recall? The same question applies to the notions of intention, anticipation and attention.

4. Can the mental images be destroyed or do they only experience BSFR and continue to live with an opposite arrow of time and become unconscious to self? If a mental image can completely disappear, what could be the physical mechanism leading to its disappearance?

5. One can challenge the detailed picture of the notion of time evolution by SSFRs. The assumption about the drift of mental images towards future in the second half-cone of CD is ad hoc. Should it be replaced with a deeper assumption. Could one simply assume that they are stationary.

Three ontological options

The basic problem of ZEO is whether the causal diamond (CD) represents a perceptive field in the sense that the space-time surface continues outside the CD or whether CD is an independent entity in the sense that space-time surfaces do not continue outside CD. Conservation laws do not exclude either option.

ZEO allows 3 ontological options which might be called easter, western, and intermediate views.

Option I: Space-time surfaces are restricted inside CDs. Quantum universe is a collection of CDs containing space-time surfaces, which have ends at the boundaries of CD.

In this framework, space-time in cosmological scales is an idealization and could be perhaps explained in terms of the correlations between CDs. CDs do not form a fractal atlas of something unless one says that the atlas *is* the territory. CD is an independent entity rather than a perceptive field of sub-self.

One can argue that for sub-CDs this picture is problematic since it seems that one loses totally the notion of objective reality as something existing outside CD. There are no sensory perceptions. Could the overlaps with other CDs create the experience about the existence of the external world?

Cosmology would be a mental construct and correspond to a very large CD. One would have a multiverse but only at the level of conscious experience. Option I is consistent with the eastern view that only subjective experience exists but not with the western view.

Option II: Space-time surface continues always outside all CDs and CDs can be interpreted always as perceptive fields. Option II conforms with the westerm option and implies that cosmology is something real.

Option III: Self is a hierarchy of CDs such that for sub-CDs the space-time surfaces continue outside the CD but for the largest CD this would not be the case. Sub-CDs would represent perceptive fields but the largest CD would be a God-like entity experiencing itself as the entire cosmos.

Meditators report altered states of consciousness in which the separation to self and external world ceases and the mind is empty. Also the experience of timelessness is mentioned. Could these states correspond to experiences without mental images (sub-CDs) created by SFRs at this highest level?

Option III is roughly consistent with both western and eastern views about consciousness. If one requires the notion of the external world as objective reality and accepts the proposed explanation of altered states of consciousness, option III remains the only possible option.

A general picture about the dynamics of sub-CDs

The ZEO based view of quantum measurement theory and the theory of consciousness inspired by it have not been precisely formulated for sub-CDs. In particular, the question of how sub-CDs as mental images are created, has remained unanswered.

The following proposal provides such a formulation and is consistent with Options I and III.

1. CDs form a fractal atlas of conscious maps but the map would be the territory since in general the space-time surfaces need not continue outside the CD. There would be no external particles as 4-D lines for generalized Feynman diagrams outside CD.

2. Sub-CDs correspond to mental images of CD as a conscious entity. From the point of view of consciousness theory, there are only experiencers (CDs) which can have experiences as mental images (have sub-CDs), be mental images of experiencers (be sub-CDs) and share mental images (intersecting CDs with common sub-CDs).

3. Consistency conditions for the quantum dynamics of CDs and sub-CDs and for the overlapping CDs give rise to correlations between the regions of the map. The shared regions are geometrically analogs for the intersections of the intersections of a covering of a manifold by open sets.

4. For sub-CD the interpretation of sub-CD as a perceptive field would be natural.

The first question is what does one really mean with sub-CD at the level of space-time surfaces.

1. Do the space-time surfaces of sub-CD continue outside sub-CD as space-time surfaces of CD? Does this imply that the quantum dynamics of sub-CDs in ZEO is completely dictated by that of CD? This is certainly not the case. Fermionic zero energy states associated with the sub-CD are possible and are analogous to quantum fluctuations. Note that in the TGD framework all elementary particles can be constructed from fundamental fermions (quarks).

2. If the PE (PE) property fixes completely the space-time surface, its intersections with the boundary of CD, this seems to be the case. If the classical dynamics is not completely deterministic, as suggested by the analogy with minimal surfaces spanned by frames, the situation changes.

   Sub-CD defines a subsystem of CD with boundary conditions at the boundary of CD which do not completely fix the quantum dynamics of sub-CD. Quantum states as WCW spinor fields inside sub-CD could change in SFRs of sub-CD.

   The tensor product of sub-CD with CD would not be ordinary tensor product but much more restricted one and Connes tensor product, related to inclusions of HFFs, would be a possible identification. A sub-system would be like an included hyper-finite factor of type II_1 (HFF).

Suppose that the classical dynamics is indeed non-deterministic and sub-CDs are defined in the proposed manner. How the view about WCW spinor fields changes as one restricts the consideration to sub-WCW.

1. The failure of the classical determinism forces to replace each 3-surface at PB with a discrete tree-like structure consisting of all PEs connecting it to AB. Sub-WCW as the space of PEs is larger than the space of 3-surfaces X^3 at PB. Zero energy states are defined in this sub-WCW and assign to a given X^3 a wave function in this discrete set allowing interpretation as wave function in a set of paths of the tree.

   One cannot avoid the association with cognitive representations of adelic physics involving the number theoretic degrees of freedom characterized by Galois group of the extension of rationals associated with the polynomial defining the space-time region [L42, L121].

2. The activation of sub-WCW would mean an SFR selecting in WCW of CD such sub-WCW for which the space-time surfaces are such that their ends at sub-CD are fixed. This would correspond to SFR creating a sub-CD and corresponding mental image. This would answer the long standing question whether and how mental images can appear as if from scratch. This SFR would also represent a third kind of SFR having interpretation as a partial localization in WCW associated with CD. This also suggest that mental images could disappear suddenly. This "activation" could be seen as a directed attention.

3. WCW degrees of freedom at the boundaries of sub-CD are fixed. Also sub-WCW spinor fields make sense. One can allow the tensor product of Fock spaces of many-fermion states associated with the boundaries of CD. One would have a QFT like picture with sub-WCW degrees of freedom fixed at boundaries of sub-CD.

4. The tensor product of fermionic state spaces at the boundaries of sub-WCW makes sense and one can define zero energy states in the same manner as proposed hitherto. The only difference is that WCW degrees of freedom are frozen at the boundaries of sub-CD. At the level of conscious experience this means that the subself experiences the external world as fixed. This would be by definition the meaning of being subself.

The fermionic Fock state basis has an interpretation as a Boolean algebra so that fermionic zero energy states have an interpretation as Boolean statements of form $A \rightarrow B$. This would mean that consciousness of the subself would be Boolean, cognitive consciousness, thinking. This conforms with the Eastern view that ordinary consciousness is essentially thinking and that the higher level of consciousness as that associated with the highest level of the CD hierarchy of self is pure consciousness. Thinking assignable to the fermionic degrees of freedom would be seen as an endless generation of illusions. "Reality" in this interpretation would correspond to WCW degrees of freedom.

What restrictions must one pose on the quantum dynamics of CDs in the case of sub-CDs? Does the subjective evolution of sub-CD states by SSFRs and BSFRs make sense for sub-CDs?

1. The increase of the size of sub-CD makes sense and the proposed subjective evolution by scalings and SSFRs makes sense. The time evolution is also now induced by the increase of the perceptive field of a subself defined by the WCW associated with increasing sub-CD bringing in new 4-surfaces due to the classical non-determinism.

2. What about the interaction between CD and sub-CDs. Does this time evolution respect the condition that the space-time surfaces meet the fixed 3-surfaces at boundaries of sub-CD or is it possible that the SSFRs of CD destroy the subself by delocalization so that sub-CD as a mental images must be regenerated by localization in WCW.

3. Also the interaction between overlapping CDs and the sharing of mental images can be understood in this framework.

8.5.4 Comparison of the revised view of self with the earlier one

The revised view about TGD inspired theory of consciousness relies on the definition of subself at the level of WCW unlike the older view. In the following the new view is compared with the old view.

The view about SSFRs

Earlier picture

The earlier view about SSFRs was inspired by the M^8 picture.

1. The dynamics was assumed to involve both scaling of CD with respect to either tip of CD. The lower half-cone was only scaled whereas the upper half-cone was also shifted as required by the stationarity of the passive boundary. Dynamics at PB was passive in the sense that only a portion of the space-time surface became visible making also new states visible at it (Zeno effect) in the sequence of SSFRs . The idea about scaling leads to a rather concrete proposal for the S-matrix characterizing the scalings of CD.

2. The surfaces inside CD (or sub-CD) were assumed to be mirror symmetric with respect to the middle plane of CD. This assumption does not conform with the assumption that these surfaces define a perceptive field in the sense that they are parts of large space-times and continue outside CD.

The old view had several ad hoc features.

1. The creation of mental images was implicitly assumed without specifying what this could mean mathematically. These mental images were assumed to be created in the upper half-cone just above the $t = T$ mid-plane of CD and shift to the geometric future with the upper half-cone of CD. The asymmetry between upper and half-cone could be seen as reflecting geometrically the future-past asymmetry but was ad hoc.

2. One can criticize the assumption that the memories about the events of the subjective past are located in the geometric future with respect to the mid-plane of CD.

3. Whether mental images can disappear or only die and reincarnate by BSFR, was not specified.

New picture

In the new picture the situation is the following.

1. Also in the new picture, the time evolution by SSFRs would be a sequence of scalings of CD. The assumption about reflection symmetry of space-time surfaces is given up since it is inconsistent with the identification of sub-CD as a perceptive field. Also now the time evolution is passive in the sense that only a new portion of the space-time surface extending outside sub-CD is revealed at each step.

2. As in the previous picture, new discrete WCW degrees of freedom appear during the sequence of SSFRs and complexity increases. For both options only fermionic degrees of freedom remain if full determinism is assumed and if QCC is required also at the level of SFRs.

3. In the new view both directed attention, memory, and intention correspond to a generation of sub-CD by a localization in WCW fixing a subset of 3-surfaces at the PB of CD. Redirecting of attention would allow apparent movement of the sub-CD in the interior of CD and as a special case shifting the mental images in the time direction assumed in the earlier picture.

4. In the new view the loci of mental images are naturally associated with the loci of classical non-determinism that is 3-surfaces at the 4-D minimal surface branches.

5. M^8-H duality suggests that the branchings occur at H image points of the M^8 cognitive representation defined by the quark momenta which are algebraic integers for the extension of rationals defined by the polynomial defining $X^4\subset M^8$. The non-determinism at $X^4\subset H$ point set would correspond to non-determinism assignable to a bound state of quarks at corresponding point of M^8.

   Note that physical states correspond to total quark momenta which are rational integers, one can speak of Galois confinement meaning that physical states are Galois singlets. This gives an infinite hierarchy of bound states formed by a universal, purely number theoretical mechanism. All bound states could be formed in this manner.

   The non-determinism at $X^4\subset H$ point which corresponds to a subset of points as images of quark momenta composing the bound state would correspond to non-determinism assignable to a bound state of quarks at corresponding point of M^8. There would be a hierarchy of CDs within CDs and hierarchy of mental images corresponding to the hierarchy of bound states.

   The bound state momenta are mapped to $X^4\subset H$ by M^8-H duality already described. In particular, the positions of quarks contained in 6-branes X^6 with a constant energy $E=E_n$ are mapped to a sequence of points at the boundary of cd of the system by M^8-duality and it can be said to represent the positions of these quarks. These point sets define sequences of "very special moments in the life of self".

   The targets of attention would therefore form a discrete set assignable to bound states of quarks and antiquarks. Note however that each 3-surface X^3 in the superposition defining the WCW spinor field at the PB of CD has its own discrete set loci of non-determinism. BSFRs can change the superposition of these 3-surfaces. The selection between branches is possible in BSFR but not in SSFRs.

6. An attractive idea motivated by ZEP is that volitional action could be interpreted in the new view as an SFR selecting one path at the node of a tree characterizing the non-determinism. Single deterministic time evolution analogous to a computer program would be selected rather than modifying the deterministic time evolution as in standard ontology. In the M^8 picture, the very special moments $t=r_n$ in the life of self correspond to the roots of a real polynomial. What happens when all roots have been experienced? Does NMP force the BSFR to occur since nothing new can be learned?

Comparison of the views about BSFR

Those aspects of BSFR in which old and new views differ are of special interest.

Earlier view

The fact that the notion of sub-CD and mental image were not properly formulated led to several ad hoc assumptions.

1. The possible failure of a strict determinism was realized. The failure of strict determinism was assigned to "very special moments in the life of self" associated with the images $E = E_n$ planes of $M^4 \subset M^8$ at which the partonic vertices as loci of non-determinism were assigned.

2. The mental images of previous life near the AB of CD were assumed to be inherited as "silent wisdom". Their contents was from the early period of life with opposite arrow of time and one can of course ask whether they were really "wisdom".

3. There were also assumptions about the change of the size scale of CD in BSFR. The idea that the reduction of the size scale guarantees that re-incarnate has childhood was considered. This assumption also prevents unlimited increase of the size scale of sub-CD.

New view
The new view makes it possible to develop a more detailed picture of what happens in BSFR.

1. The WCW localization at the AB of CD selects one of the branches of the space-time surface beginning at the PB. This selection of the branch happens to each 3-surface in the superposition of 3-surfaces at the PB defined by the WCW spinor field before BSFR.

2. The future directed tree becomes a past directed tree beginning from one particular branch at the AB. The initial and final space-time surface share a common space-time surface connecting the roots of the old and new trees. This is essential for having a non-trivial transition amplitude for BSFR at WCW level.

In the earlier view, the mental images interpreted as memory mental images and located near the boundary of CD were assumed to be inherited as "silent wisdom" by the time-reversed reincarnate. What happens now?

The notion of "silent wisdom" as inherited information still makes sense.

1. The new space-time surfaces originate from 3-surface which was selected by WCW localization in BSFR. Therefore the new space-time surfaces carry classical information about previous life.

2. The space-time surfaces originating from the new root are near to the space-time surface connecting the old and new roots. The WCW spinor field before and after BSFR musthave a strong overlap in order to make the transition amplitude large. This implies that information about previous life is transferred to the new life.

3. The nearness property could imply that they are easily re-created as perceptions by directed attention so that they would indeed be "silent" wisdom. These mental images are from the later part of the life cycle rather than from the early life as in the earlier picture. If aging means getting wisdom, then silent wisdom would be in question.

Does the notion of "silent wisdom" as mental images make sense?

1. Mental images - this includes both sensory and memory mental images and intentions) are naturally assignable to the loci of classical non-determinism at the images of the planes $E = E_n$ of the branched space-time surfaces associated with the new root ("very special moments in the life of self").

   For the special space-time surface connecting the roots of old and new space-time surface, the surfaces $E = E_n$ in M^8 would not change and the mental images would carry information about previous life. Could one talk about potentially conscious "silent wisdom".

2. What happens to the mental images of self in BSFR? Can they be preserved or do they disappear or do they reincarmate by BSFR? The idea about preservation makes sense only for space-time surfaces connecting the roots.

3. What can happen to the size scale of CD in BSFR? The extreme option that CD decreases in size by shift of the formerly PB such that the time evolutions are fully determinimistic in the superposition of 3-surfaces. There would be no inherited silent wisdom and the self would start from scratch, live a chilhood. Otherwise these loci would define candidate for inherited silent wistom.

In the earlier picture the mental images corresponding to sub-CD could not disappear although it could die by BSFR and reincarnate with a reversed arrow of time. Can the mental image disappear now? Creation of mental image require metabolic energy feed: this explains 7 ± 2rule for the number of simultaneous mental images. Could this happen when attention is redirected? Therefore one could argue that mental image must totally disappear when the attention is redirected.

On the other hand, time reversed mental image apparently feeds energy to the environment in the original arrow of time, i.e. apparently dissipates. Could this dissipation be interpreted as an energy feed for its time reversal.

Note that the total disappearance of the mental image means delocalization at the level of WCW and seems possible. The new view clearly challenges the idea about the Karma's cycle of self. This cycle appears in many applications of BSFR.

8.5.5 Conclusions

Also the article *Some comments related to Zero Energy Ontology (ZEO)*" [L91] written for few years ago challenged the basic assumptions of ZEO. One tends to forget the unpleasant questions but now it was clear that it is better to face the fear that there might be something badly wrong. ZEO however survived and several ad hoc assumptions were eliminated.

Progress at the level of basic TGD

The basic goal is to improve the understanding about quantum-classical correspondence. The dynamics of soap films serves as an intuitive starting point.

1. In TGD frame 3-surfaces at the boundaries of CD define the analog of frame for a 4-D soap film as a minimal surface outside frame. This minimal surface would be an analog of a holomorphic minimal surface and simultaneous exremal of Kähler action except at the frame where one would have delta function singularities analogous to sources for massless d'Alembert equation.

2. There is also a dynamically generated part of the frame since the action contains also Kähler action. The dynamically generated parts of the frame would mean a failure of mimimal surface property at frame and also the failure of complete determinism localized at these frames.

3. At the frame only the equations for the entire action containing both volume term and Kähler term would be satisfied. This guarantees conservation laws and gives very strong constraints to what can happen at frames.

   The frame portions with various dimensions are analogous to the singularities of analytic functions at which the analyticity fails: cuts and poles are replaced with 3-, 2-, and 1-D singularities acting effectively as sources for volume term or equvavelently Kähler term. The sum of volume and Kähler singularities vanish by field equations. This gives rise to the interaction between volume and Kähler term at the loci of non-determinism.

4. H-picture suggests that the frames as singularities correspond to 1-D core for the deformations of CP_2 type extremals with light-like geodesic as M^4 projection, at partonic 2-surfaces and string world sheets, and at 3-D $t = t_n$ balls of CD as "very special moments in the life of self" which integrate to an analog of catastrophe. T

   Deformations of Euclidean CP_2 type extremals, the light-like 3-surfaces as partonic orbits at which the signature of the induced metric changes, string world sheets, and partonic 2-surfaces at $r = t_n$ balls taking the role of vertices give rise to an analog of Feynman (or twistor -) diagram. The external particles arriving the vertex correspond to different roots of the polynomial in M^8 picture co-inciding at the vertex.

The proposed picture at the level of $H = M^4 \times CP_2$ has dual at the level of (complexified) M^8 identifiable as complexified octonions. The parts of frame correspond to loci at which the space-time as a covering space with sheet defined by the roots of a polynomial becomes degenerate, i.e. touch each other.

Concerning the physical interpretation, a crucial step of progress was the interpretation of M^8 as analog of momentum space allowing to interpret M^8-H duality as an analog of momentum-position duality and of complementarity principle of wave mechanics [L141]. This forced to modify M^8-H duality in M^4 degrees of freedom to satify the constraints posed by UP.

There is a nice analogy with the catastrophe theory of Thom [A86, A50]. The catastrophe graph for cusp catastrophe serves as an intuitive guide line. embedding space coordinates serve as behaviour variables and space-time coordinates as control variables. One obtains a decomposition of space-time surface to regions of various dimension characterized by the degeneracy of the root.

Progress in the understanding of TGD inspired theory of consciousness

The improved view about ZEO makes it possible to define the basic notions like self, sub-self, BSFR and SSFR at the level of WCW. Also the WCW correlates for various aspects of consciousness like attention, volition, memory, memory recall, anticipation are proposed. Attention is the basic process: attention creates sub-CD and subself by a localization in WCW and projects WCW spinor field to a subset of WCW. This process is completely analogous to position measurement at the level of H. At the level of M^8 it is analogous to momentum measurement.

One can distinguish between the Boolean aspects of cognition assignable to WCW spinors as fermionic Fock states (WCW spinor field restricted to given 3-surface). Fermionic consciousness is present even in absence of non-determinism. The non-determinism makes possible sensory perceptions and spatial consciousness.

A precise definition of sub-CD as a correlate of perceptive field at WCW level implies that the space-time surfaces associated with sub-CDs continue outside it. This gives powerful boundary conditions on the dynamics. For the largest CD in the hierarchy of CDs of a given self, this constraint is absent, and it is a God-like entity in ZEO. This leads to a connection between the western and eastern views about consciousness.

A connection with the minimal surface dynamics emerges [L146]. The sub-CDs to which mental image as subselves are assigned would be naturally associated with portions of dynamically generated frames as loci of non-determinism. If one identifies partonic 2-surfaces as vertices, one can interpret the collection of possible space-time surfaces for a fixed 3-surface at PB as a tree. All paths along the tree are possible time-evolutions of subself. The dynamics of consciousness for fixed 3-surface at PB becomes discrete and provides discrete correlate for a volitional action as selection of a path or a subset of paths in the tree. The reduction of dynamics of mental imagines to discrete dynamics would mean a huge simplification and conforms with the discreteness of cognitive representations.

Challenges

There are many challenges to be faced. The discrete dynamics of sub-self consciousness certainly correlates with the notion of cognitive representation based on adelic physics [L54, L53] and implying a discretization at both space-time level and WCW level. The Galois group for the extension of rationals acting on the roots of the polynomial plays a key role in this dynamics [L121, L128].

One teaser question remains. Localization requires energy quite generally and this conforms with the fact that mental images demand metabolic energy feed. It is possible to redirect attention and it remains unclear whether the mental image disappears totally or suffers BSFR.

This relates directly to the question whether consciousness continues after the physical death. If mental images (and corresponding sub-CDs) can disappear, the same can happen to us since we are mental images of some higher level self. If this cannot happen, BSFR means death and reincarnation with an opposite arrow of time in a completely universal sense. For instance, sleep period could correspond to a kind of death at some level of the personal self hierarchy generalizing the Id-ego-superego hierarchy of Freud. This would explain why we have no memories of the sleep period.

8.5.6 Appendix: M^8- and H views about classical non-determinism and particle reactions

M^8 picture and $M^8 - H$ duality

In M^8 picture, space-time surfaces correspond to real projections of 4-D complex "roots" of octonionic polynomials obtained from real polynomials with rational coefficients by algebraic continuation, i.e. by replacing real coordinate by complexified octonion coordinate [L49, L50, L51] [L109, L110]. The interested reader finds a rather detailed summary of M^8-H duality in Appendix 8.5.6.

$M^8 - H$ duality maps the point of $M^4 \times E^4$ to a point of $M^4 \times CP_2$ such that the point of $M^4 \subset M^4 \times E^4$ is mapped to some point of $M^4 \subset M^4 \times CP_2$. $M^8 - H$ duality is not a local map. Rather, the normal space of a $x \in X^4 \subset M^8$ goes to a point of CP_2 characterizing its quaternionic normal space.

1. To be a 4-D "root" in the complex sense means that the real part of a complexified octonionic polynomial determining the space-time surfaces vanishes. The number theoretic content of this condition is that the normal space of the space-time surface is quaternionic and therefore associative. The second option would be that the tangent space is associative but this gives only M^4 as a solution.

2. At a given point there are n roots and some of them can coincide in some regions of the space-time surface. These regions correspond to the branchings of the space-time surface at which particle-like entities identified as space-time surfaces meet and interact.

   The quaternionic normal plane at this intersection is not unique so that several CP_2 points of $X^4 \subset H$ correspond to a single point of $X^4 \subset M^8$. The extreme situation is encountered in a point-like singularity when the normal plane at a given point of M^4 is a sub-manifold of CP_2.

   The interpretation is as particle vertices. The intuitive expectation is that they correspond to partonic 2-surfaces and perhaps also string world sheets. These surfaces are mapped to those in $M^4 \times CP_2$ by $M^8 - H$ correspondence.

3. Also 6-D brane like entities are predicted as universal "roots" they correspond to 6-spheres in M^8 with M^4 projection which is a 3-ball with constant value $E = E_n$ of energy as counterpart of the Minkowski time coordinate such that E_n is the root of the real polynomial defining the octonionic polynomial. The momenta $(E_n, p = 0)$ are mapped to points $t_n = (\hbar_{eff}/E_n, 0)$ and define "very special moments of time in the life of self".

   The points with $p \neq 0$, in particular the points corresponding to quark momentum, however correspond to $t < t_n$ at the boundary of cd with size $L(p) = \hbar_{ef}/\sqrt{E_n^2 - p^2}$. To these moments the failure of classical determinism giving rise to one particular kind of quantum non-determinism is concentrated. Note that points of double hyperboloid of M^4 with opposite energies are mapped to opposite boundaries of cd.

4. The intersections of 4-D "roots" with 6-D brane-like entities are 2-D and it might be possible to interpret them as analogs of either partonic 2-surfaces or string world sheets at which several roots become degenerate of octonionic polynomial co-incide. Outside the singularity, the roots do not coincide and define separate space-time sheets and it is natural to interpret them as external particles of a particle reaction.

5. At the light-like orbits of partonic 2-surfaces the induced metric for the H-image of the space-time surface becomes degenerate since its signature changes. Could one say that the Minkowskian and Euclidean roots coincide at the partonic orbits?

   One can also wonder what the M^8 interpretation of wormhole contacts having two throats could be. Do the two throats correspond to two coincing roots at the level of M^8 having different normal spaces and mapped to separate 2-surfaces in H?

Catastrophe theoretic analogy

Consider the analogy with the catastrophe theory of Thom [A86] in more detail.

1. Catastrophe map is the graph of solutions for the vanishing of the gradient of a potential function as a function of control parameters. One considers only real roots as function of variable control parameters and the number of real roots varies as a function of parameters and one obtains lower-dimensional regions at which the number of roots to catastrophe polynomial changes as roots become degenerate [A86, A50]. Cusp catastrophe serves as the school example.

2. In the recent case, space-time surfaces correspond to roots of complexified octonionic polynomials and the coefficients of the polynomial appear as control parameters. Also complex roots are allowed and real 4-D space-time surface is obtained as a real projection and mapped to H by M^8-H duality and conjectured to correspond to a preferred extremal of an action determined by the twistor lift of TGD.

3. The basic motivations for this assumption are quantum criticality requiring preferred extremal property, which requires at the level of H the independence of the dynamics on coupling parameters of the twistor lift of Kähler action outside the loci of non-determinism demanded by M^8 level.

Connection between singularities and preferred extremals of various types

The above picture suggests the characterization of the space-time surfaces in terms of their singularities as surfaces of M^8.

At the level of H one can consider 4 kinds of very simple preferred extremals, which give rise to prototype singularities.

1. Einsteinian spacetime $X^4 \subset M^8$ with a 4-D M^4 projection and a unique normal space as a point of CP_2. $X^4 = M^4$ defines a prototype.

2. Cosmic string extremal $X^2 \times Y^2$ with Y^2 a complex surface in CP_2 and defining a set of normal spaces assignable to a point of X^2. $M^2 \times S^2$, S^2 a geodesic sphere defines a proto type. S^2 can be either homological trivial or non-trivial.

3. $X^3 \times S^1 \subset M^4 \times CP_2$, where S^1 is a geodesic circle of CP_2, is a candidate for a preferred extremal and singular surface. Both $M^3 \times S^1$ and $E^3 \times S^1$ are minimal surfaces and vacuum extremals of Kähler action.

   For the Euclidean signature, X^3 could be space-like and define a 3-ball compactifying to S^3 as a sub-manifold of the S^6 brane. The very special moments t_n would be singular in the sense that the normal space at a given point of $X^3 \subset M^4 \subset M^8$ would not be unique and would give rise S^1 singularity.

4. CP_2 type extremal with light-like geodesic as $M^4 \subset H$ projection and corresponding to a light-like geodesic in M^8 with normal spaces forming a 3-D surface in CP_2. Also $M^1 \times Y^3 \subset M^4 \times CP_2$ can be considered but is probably not a preferred extremal.

The intuitive picture is that these 4 types of preferred extremals correspond to singularities of the normal space of $X^4 \subset M^8$ of dimension $d = 0, 1, 2, 4$ and codimension $d_c = 4 - d$.

Analogy with knot theory

In knot theory a knot in 3-D space is projected to 2-plane where one obtains a diagram containing crossings. Knot invariants can be constructed in terms of this diagram. A knot theory inspired intuition is that space-time surfaces near to these special cases are projected to these special surfaces to get the toy model.

1. Canonically embedded $M^4 \subset M^8$ (or $M^4 \subset M^4 \times CP_2$) is an analog of the plane to which the knot is projected. One can project the space-time regions with 4-D $M4$ projection to M^4. In particular, those with a Minkowskian signature of the induced metric.

2. The M^4 projection of CP_2 type extremal is 1-D light-like geodesic. One must project the deformations of CP_2 type extermals to CP_2 type extremal at the level of H. At the level of H, CP_2 type extremal could correspond to a light-like geodesic of M^8 such that each point of the geodesic is singular point such that the union of quaternionic normal spaces defines a 3-D quaternionic surface in CP_2.

   A puncture in E^3 as an infinitesimal hole serves as an analogy. At the puncture, one can say that all normal spaces labelled by points of S^2 are realized.

   At the given point of the light-like geodesic, the quaternionic normal space of point is not unique but a 3-D union of normal spaces and defines a 3-D subset CP_2.

3. For the $X^2 \times Y^2 \subset M^4 \times CP_2$ type cosmic string extremals and their small deformations, one must project to $M^2 \times S^2 \subset CP_2$. For a point of X^2 the normal spaces define $Y^2 \subset CP_2$ so that the singularity is milder.

   For $X^3 \times S^1 \subset M^4 \times CP_2$ the normal spaces at a point of X^3 would define $S^1 \subset CP_2$. If X^3 is Euclidean, these 3-D singularities could correspond to the $t = t_n$ planes associated with the branes. The small deformations of these surfaces would project to $M^3 \times S^1$. This picture would integrate all 3 kinds of singularities and various types of preferred extremals to a single unified picture.

A toy model for the singularities

The following toy model for the singularities in the case of CP_2 type extremals generalizes also to other singularities.

1. A rather general class of CP_2 type extremals can be represented as a map $M^4 \to CP_2$ given by

$$m^k = p^k f(r) \ ,$$

   where p^k is light-like momentum and r is radial $U(2)$ invariant CP_2 coordinate labelling 3-spheres of CP_2 such that $r = \infty$ gives homologically non-trivial geodesic 2-sphere instead of 3-sphere.

   If $f(r)$ approaches constant value for $r \to \infty$, one can say that M^4 time stops at this limit, and one obtains a homologically non-trivial geodesic sphere instead of 3-D surface identifiable as an intersection with 6-D brane. Various external particles of the vertex would correspond to $m^k = p_k f_i(r)$ such that their values at $r = \infty$ co-incide.

   It is not possible to obtainh omologically trivial 2-sphere in this manner.

2. Outside the vertex, the CP_2 type space-time sheets have distinct light-like geodesics as M^4 projections and they can be continued to distinct regions of M^4 in the toy model.

   The analog of the knot diagram would be a set of M^4:s with different constant values of CP_2 coordinates. The CP_2 type extremals would be glued along light-like geodesics to various M^4s.

   The CP_2 points of M^4:s meeting at the same geodesic sphere must belong to the same geodesic sphere S^2. The S^2:s associated with different vertices are different. Note that any two geodesic spheres must have common points.

3. In the toy model for the string world sheets $X^2 \times Y^2$ would be projected to a piece of $M^2 \times S^2$ connecting two partonic vertices with the same S^2. S^2:s would be at the ends of the string, whose orbit is a piece of M^2.

   $B^3 \times S^1$ could be interpreted as a subset of 6-D brane with B^3 identified as the $t = t_n$ cross section of M^4 light-cone.

This picture would suggest that the singularities could be indeed located to $t = t_n$ planes and integrated together to form a rough analog of catastrophe map.

Some examples of minimal surfaces with 1-D CP_2 projection

This subsection is not directly relevant to the basic topic and is added to give ideas about the possible role of volume term.

The original proposal was that preferred extremals are extremals of Kähler action but the twistor lift introduced the volume term as an additional term. This removed the huge vacuum degeneracy of Kähler action meaning that any 4-surface for which CP_2 projection was so called Lagrange manifold with the property that induced Kähler form vanishes, was a solution of field equations. For these surface induced Kähler potential is pure gauge.

The addition of the volume term removes this degeneracy and only minimal surfaces of this kind are possible as extremals. It is however not clear whether they are preferred extremals (are they analogs of complex surfaces?).

These solutions have not been studied previously [K9]. Space-time surfaces representing a warped embedding of M^4 with a flat metric represent the simplest example.

1. Denoting the angle coordinate of the geodesic sphere S^1 byΦ and the metric of S^1 by $ds^2 = -R^2 d\Phi^2$ the ansatz reads in linear Minkowski coordinates as $\Phi = k \cdot m$, where k is analog of four-momentum. The induced metric is flat and the second fundamental form vanishes by the linearity of Φ in m so that the field equations are satisfied.

   Boundary conditions require the vanishing of the normal components of momentum currents and give $(\eta^{\alpha\beta} - R^2 p^\alpha p^\beta) n_\beta = 0$ .This condition cannot be satisfied so that these solutions should have infinite size, which looks unphysical.

   The presence of the volume term in the action implies that the induced metric appears in the boundary conditions and this represents a problem quite generally. The only way to overcome the problem is that there are no boundaries. The many-sheetedness indeed makes this possible.

   The warped extremals could represent a reasonable approximation of the space-time surface in the regions which are almost empty.

2. The light velocity defined in terms of time taken to get from the M^4 position A to B, is reduced to $c_1 = \sqrt{1 - |k \cdot k|}$. If k is light-like this does not happen.

   Although the analog of gravitational force is vanishing in warped metric, the deviation the flat metric from M^4 metric given by $|k \cdot k|$ in flat case could it be interpreted as gravitational potential and the gravitational potential energy of test mass would be given by by $E_{gr} = -m|k \cdot k|$.

   Could Nature provide a kind of cognitive representation or toy model of a gravitational field as a piecewise constant function in terms of CDs with which warped vacuum extremals would be associated? The representation would contain length scale dependent Λ as second parameter assigning momentum 4-momentum proportional to Λ p^k to the CD. The volume energy would include its gravitational potential energy represented in terms of warping?

   For warped solutions the space-time light cone - to be distinguished from its embedding space counterpart - would be defined by $c_1^2 t^2 - r^2 = 0$ and space-time CD would be modified accordingly.

Only single extremal - canonically embedded M^4 - remains from the spectrum of cosmological vacuum extremals for Kähler action having 1-D CP_2 projection and defined by $\Phi = f(a)$, where f is an arbitrary function of light-cone proper time coordinate $a = \sqrt{t^2 - r_M^2}$.

At QFT-GRT limit, the many-sheeted space-time is approximated with Einsteinian cosmology with the deviation of the induced metric from M^4 metric defined by the sum of the corresponding deviations for the sheets. Since the value of Λ becomes large in short p-adic length scales, a cosmology resembling GRT type cosmology could emerge and Einstein's equations would be a remnant of Poincare symmetry.

The induced metric for the solutions has very little to do with the metric appearing at the Einsteininian limit. The models of cosmology as space-time surfaces based on Kähler action with vanishing Λ could however make sense in very long scales for which Λ approaches zero.

For string dominated cosmology, the comoving mass is proportional to a [K102, K9, K68]. One has a silent whisper amplified to a Big bang in GRT sense. Also critical cosmology [K9] as an analog of inflationary cosmology for which curvature scalar as dimensional quantity vanishes can be regarded as a silent whisper amplified to a Big Bang and also it becomes Euclidean for a critical value $a = a_0$ of cosmic time.

8.6 What could 2-D minimal surfaces teach about TGD?

In the quantum TGD based on zero energy ontology (ZEO) space-time surfaces within causal diamonds (CDs) are fundamental objects [L91, L138]. $M^8 - H$ duality plays a central role: the earlier views can be found in [L49, L50, L51] and the recent view in [L109, L110, L134] differing in some aspects from the earlier view. $M^8 - H$ duality means that one can interpret the space-time surfaces in two ways: either as an algebraic surfaces in complexified M^8 or as minimal surfaces in $H = M^4 \times CP_2$ [L138]. $M^8 - H$ duality maps these surfaces to each other.

The twistor lift of TGD is another key element [L55, L72]. It replaces space-time surfaces with their 6-D twistor spaces represented as 6-D surfaces in the product of twistor spaces assignable to M^4 and CP_2 and having an induced twistor structure. This implies dimensional reduction of a 6-D Kähler action to a sum of a 4-D Kähler action and volume term having interpretation in terms of cosmological constant Λ. Kähler structure exists only for the twistor spaces of M^4 and CP_2 [A63] so that the theory is unique.

Each extension of rationals (EQ) corresponds to a different value $\Lambda > 0$. For $\Lambda = 0$, the finite-D extension of rationals determined by real polynomials would be replaced with real analytic functions or subset of them.

Whether $\Lambda = 0$ can be accepted physically, will be one of the key topics of this article. At the level of adelic theory of cognition [L54, L52] this question boils down to the question whether cognition is always finite and related to finite-D extensions of rationals of whether also infinite-D extensions and transcendence can be allowed.

8.6.1 Basic notions

$M^8 - H$ duality and twistor lift of TGD are the basic notions relevant for what follows and its is appropriate to discuss them briefly.

Space-time surfaces at the level of M^8

The recent view of $M^8 - H$ duality [L109, L110, L134] deserves a brief summary.

At M^8 level, space-time surfaces can be regarded as algebraic 4-surfaces in complexified M^8 having interpretation as complexified octonions. The dynamical principle states that the normal space of the space-time surface at each point is associative and therefore quaternionic. The space-time surfaces are determined by the condition that the real part of an octonionic polynomial obtained as an algebraic continuation of a real polynomial with rational coefficients vanishes.

This gives a complex surface which is minimal surface from which one takes a real part by projecting to real part of complexified M^8: it is not clear whether it is minimal surface of M^8. Minimal surface property is the geometric analog of a massless d'Alembert equation [L27, L83].

Also real analytic functions can be considered [L109, L110] but this leads to infinite-D extensions of rationals in the adelization requiring that also the p-adic counterparts of the space-time surfaces exist. Whether this phase which would correspond to $\Lambda = 0$, can be accepted physically, will be one of the key topics in the sequel.

The conditions defining the space-time surfaces are exactly solvable and the conjecture is that these surfaces are minimal surfaces by their holomorphy (the induced metric of the space-time surface does not however play any role and its role is taken by the complexification number theoretic octonion norm which is real valued for the real projections) [L109, L110, L134].

Space-time surfaces at the level of $H = M^4 \times CP_2$

At the level of $H = M^4 \times CP_2$, space-time surfaces are preferred extremals (PEs) of a 6-D Kähler action fixed by the twistor lift of TGD [L72]. The existence of the twistor lift makes TGD unique since only the twistor spaces of $T(M^4)$ and $T(CP_2)$ have the needed Kähler structure [A63]. The 6-D twistor space $T(X^4)$ of the space-time surface X^4 is represented as a 6-surface X^6 in $T(M^4) \times T(CP_2)$. $T(X^4)$ has S^2 as fiber and X^4 as base. The twistor structure of $T(X^4)$ is induced from the product of twistor structures of $T(M^4)$ and $T(CP_2)$. The S^2 bundle structure of X^6 requires dimensional reduction and dimensionally reduced 6-D Kähler action consists of a volume term having an interpretation in terms of length scale dependent cosmological constant Λ and 4-D Kähler action.

Physically "preferred" means holography: to a given 3-surface at the either boundary of CD one can assign a unique space-time surface as an analog of Bohr orbit. This assumption is very probably too strong: the number of Bohr orbits is finite and the dynamically determined frames of the space-time surface would characterize the non-determinism [L138]. "Preferred" has several mathematical meanings, which are conjectured to be equivalent.

One of those meanings is that space-time surfaces simultaneous extremals of both volume term and Kähler action and field equations reduce almost everywhere to the analogs of the conditions satisfied by complex surfaces of complex manifolds. Note that the field equations express local conservation laws for the isometries of $H = M^4 \times CP_2$ and are in this sense hydrodynamic.

The field equations for preferred extremals do not depend on coupling parameters. This expresses quantum criticality and reduces the number of solutions dramatically as required by the fact that at the level the field equations are algebraic rather than differential equations.

Space-time surfaces are therefore minimal surfaces everywhere except at singularities, which are lower-dimensional surfaces. At singularities they are satisfied only for the entire action. The divergences of the isometry currents for the volume term and Kähler action would have delta function singularities, which must cancel each other to guarantee conservation laws.

The singular surfaces can be wormhole throats as boundaries of CP_2 type extremals at which the signature of the induced metric changes, partonic 2-surfaces acting as analogs of vertices at which light-like partonic orbits representing the lines of generalized Feynman (or twistor) diagram meet, and string world sheets having light-like boundaries at partonic orbits.

Also 3-D singularities are predicted and could be associated to time= constant hyperplanes of M^4, which in M^8 picture are associated with the roots of the polynomials determining space-time region: I have christened these roots "very special moments in the life of self" [L81]. The roots define 6-spheres as universal special solutions and they intersect future light-cone along $t = r_n$ hyper-plane. It is possible to glue different solutions together along these planes so that they can serve as loci of classical non-determinism.

The singular surfaces are analogous to the frames of soap films [L138]: part of them are fixed and at the boundaries of CD and part of them are dynamically generated. Classical conservation laws for the isometry currents expressing field equations pose strong conditions on what can happen in vertices.

$M^8 - H$ correspondence for the singularities

By $M^8 - H$ correspondence, the singular surfaces of $X^4 \subset H$ correspond to the singularities of the pre-image at the level of M^8. For the singularities $X^4 \subset M^8$ the quaternionic normal space of X^4 is not unique at points of a $d < 4$ dimensional surface but is replaced with a union of quaternionic normal spaces labelled by the points of sub-manifold of CP_2 for which the dimension is $d_c = 4 - d$. At the level of H, the singular points blow-up to d_c-dimensional surfaces. What happens for the normal space at a puncture of 3-space serves as a good analog.

In particular, the deformation of a CP_2 type extremal as a singularity corresponds to an image of a 1-D singularity with $(d = 1, d_c = 3)$ and $d_c = 3$-dimensional blow up. The properties of CP_2 type extremals suggest the 1-D curve is light-like curve for mere Kähler action and light-like geodesic for the Kähler action plus volume term.

These situations correspond to $\Lambda = 0$ and $\Lambda > 0$, where Λ is length scale dependent cosmological constant as coefficient of the volume term of action.

Membrane like structures as particularly interesting singularities

Membrane-like structures appear in all length scales from soap bubbles to large cosmic voids and it would be nice if they were fundamental objects in the TGD Universe. The Fermi bubble in the galactic center is an especially interesting membrane-like structure also from the TGD point of view as also the membrane- like structure presumably defining the analog of horizon for the TGD counterpart of a blackhole. Cell membrane is an example of a biological structure of this kind. I have however failed to identify candidates for the membrane-like structures.

An especially interesting singularity would be a static 3-D singularity $M^1 \times X^2$ with a geodesic circle $S^1 \subset CP_2$ as a local blow-up.

1. The simplest guess is a bubble-like structure as a product $M^1 \times S^2 \times S^1 \subset M^4 \times CP_2$. The problem is that a soap bubble is not a minimal surface: a pressure difference between interior and exterior of the bubble is required so that the trace of the second fundamental form is constant. Quite generally, closed 2-D surfaces cannot be minimal surfaces in a flat 3-space since the vanishing curvature of the minimal surface forces the local saddle structure.

2. A correlation between M^4 and CP_2 degrees of freedom is required. In order to obtain a minimal surface, one must achieve a situation in which the S^2 part of the second fundamental form contains a contribution from a geodesic circle $S^1 \subset CP_2$ so that its trace vanishes. A simple example would correspond to a soap bubble-like minimal surface with M^4 projection $M^1 \times X^2$, which has having geodesic circle S^1 as a local CP_2 projection, which depends on the point of $M^1 \times X^2$.

3. The simplest candidate for the minimal surface $M^1 \times S^2 \subset M^4$. One could assign a geodesic circle $S^1 \subset CP_2$ to each point of S^2 in such a way that the orientation of $S^1 \subset CP_2$ depends on the point of S^2.

4. A natural simplifying assumption is that one has $S^1 \subset S^2_1 \subset CP_2$, where S^2_1 is a geodesic sphere of CP_2 which can be either homologically trivial or non-trivial. One would have a map $S^2 \rightarrow S^2_1$ such that the image point of point of S^2 defines the position of the North pole of S^2_1 defining the corresponding geodesic circle as the equatorial circle.

   The maps $S^2 \rightarrow S^2_1$ are characterized by a winding number. The map could also depend on the time coordinate for M^1 so that the circle S^1 associated with a given point of S^1 would rotate in S^2_1. North pole of S^2_1 defining the corresponding geodesic circle as an equatorial circle. These maps are characterized by a winding number. The map could also depend on the time coordinate for M^1 so that the circle S^1 associated with a given point of S^1 would rotate in S^2_1.

   The minimal surface property might be realized for maximally symmetric maps. Isometric identification using map with winding number $n = \pm 1$ is certainly the simplest imaginable possibility.

Large voids of size scale or order 10^8 light years forming honeycomb like structures are rather mysterious objects, or rather non-objects. The GRT based proposal is that the formation of gravitational bound states leads to these kinds of structures in general relativity but I do now know how convincing these arguments really are.

One should answer two questions: what are these voids and why do they form these lattice-like structures?

One explanation of large voids is based on the TGD based view about space-time as a 4-surface in $H = M^4 \times CP_2$.

1. Space-time surfaces have M^4 projection, which is 4-D for what I call Einsteinian space-times. At this limit general relativity is expected to be a good approximation for the field theory limit of TGD.

   However, the M^4 projection can be also 3-D , 2-D or 1-D. In these cases one has what looks like a membrane, string, or point-like particle. All these options are realized. The simplest membranes would look like $M^1 \times S^2 \times S^1$, S^1 a geodesic circle of CP_2, which depends on a point of $M^1 \times S^2$ defining the M^4 projection. Only this assumption allows us to have a minimal surface. Varying S^1 creates the analog of pressure difference making soap films possible. I discovered this quite recently although the existence of membrane like entities was almost obvious from the beginning.

Small perturbations tend to thicken the dimension of M^4 projection to 4 but the deformed objects are in an excellent approximation still 3-D, 2-D or 1-D.

2. Large voids could be really voids in a good idealization! Even 4-D space-time would be absent! The void would be the true vacuum. It should be noticed that matter as smaller objects, say cosmic strings thickened to flux tubes, would in turn have galaxies as tangles, which in turn would have stars as tangles. The TGD counterparts of blackholes would be dense flux tube spaghettis filling the entire volume.

3. What is remarkable that membranes are everywhere: large voids, blackhole horizons, Fermi bubbles, cell membranes, soap bubbles, bubbles in water, shock wave fronts, etc....

What could then give rise to the lattice like structures formed from voids? Here TGD suggests a rather obvious solution.

1. The lattices could correspond to tessellations of the 3-D hyperbolic space H^3 for which cosmic time coordinate identified as light-cone proper time is constant. H^3 allows an infinite number of tessellations whereas Euclidean 3-space allows a relatively small number of lattices.

   There is even empirical evidence for these tessellations. Along the same line of sight there are several sources of light and the redshifts are quantized. One speaks of God's fingers [E3] [K102]. This is what any tessellation of cosmic voids would predict: cosmic redshift would define effective distance. Of course also tessellations in smaller scales can be considered.

2. Also ordinary atomic lattices could involve this kind of tessellations with atomic nuclei at the centers of the unit cells as voids. The space between nucleus and atom would literally be empty, even 4-D space-time would be absent!

3. Also the TGD inspired model for genetic code [L133] involves a particular tessellation of H^3 realized at the magnetic body (MB) of a biological system and realizing genetic code. This leads to the conjecture that genetic code is universal and does not characterize only living matter. It would be induced to the space-time surface in the sense that part of tessellation would define a tessellation at the space-time surface. At the level of dark matter at MB, 1-D DNA could also have 2-D and even 3-D analogs, even in ordinary living matter!

8.6.2 Key questions

The basic question to be discussed in the following is what the general ideas about 2-D minimal surfaces can teach about minimal surfaces in M^8 and H, and more generally, about quantum TGD.

Uncertainty Principle and $M^8 - H$ duality

The interpretation of M^8 as analog of momentum space [L109, L110] meant a breakthrough in the understanding of $M^8 - H$ duality but created also a problem. How can one guarantee that $M^8 - H$ duality is consistent with Uncertainty Principle (UP)? The surfaces to which one can assign well defined momentum in M^8 should correspond to the analogs of plane waves in H and geometrically to periodic surfaces.

The fact that at the level of M^8 the surfaces are algebraic surfaces defined by polynomials with rational coefficients poses therefore a problem. Periodicity requires trigonometric functions. The introduction of real anlytic functions with rational Taylor coeffients would force the introduction of infinite-D extensions of rationals and make this possible. This is however in conflict with the idea about the finiteness of cognition forming the basic principle of adelic physics [L54, L53].

Is the category of polymials enough?

Is it possible to have periodic minimal surfaces at the level of H or at the level of both M^8 and H without leaving the category polynomials?

1. Could the non-local character of the $M^8 - H$ duality in CP_2 degrees freedom miraculously give rise to periodic functions at the level of H? Or should one perhaps modify $M^8 - H$ duality itself to achieve this [L134].

2. Periodic frames assignable to light-like curves in M^8 as light-like curves would allow to achieve periodicity in the same manner as for helicoid but this requires the extension of the category of real polynomials to real analytic functions in M^8. One could even give up the assumption about a Taylor expansion with rational coefficients and assume that the coefficients belong to some possibly transcendental extension of rationals. This option would make sense in $\Lambda = 0$ phase.

3. Or could geometry come in rescue of algebra? Could one construct periodic surfaces both at the level of M^8 and H purely geometrically by gluing minimal surfaces together to form repeating patterns as is done for 2-D minimal surfaces? This option could work in $\Lambda > 0$ phases: smoothness at the junctions would be given up but local conservation laws would hold true for the entire action rather than for volume term and Kähler action separately.

If transcendental extensions are allowed, they would naturally contain some maximal root $e^{1/n}$ and its powers. The induced extension of p-adics is finite-D since e^p is an ordinary p-adic number. Logarithms of $log(k)$, $1 \leq k \leq p$, and their powers are needed to define p-adic logarithm for given p. The outcome is an infinite-D extension. Also π and its powers are expected to belong to the minimal transcendental extension.

It came as a surprise to me that is not known whether e and π are algebraically independent over rationals, that is whether a polynomial equation $P(x,y) = 0$ with rational coefficients is true for $(x,y) = (\pi, e)$ (`https://cutt.ly/xmyL23W`.) This would imply that π belongs to the extension defined by the polynomial $P(y,e)$ in an extension of rationals by e. Same would be true in the corresponding finite-D extensions of p-adic numbers. The algebraic independence of π and e would have rather dramatic implications for the TGD view about cognition. That π and e are algebraically independent follows from a more general conjecture by Schanuel and `https://cutt.ly/ImyL1YJ`).

Is also $\Lambda > 0$ phase physically acceptable?

Can one allow also $\Lambda = 0$ phase for the action. In this case the action reduces to mere Kähler action defined by M^4 and CP_2 Kähler forms analogous to self-dual covariantly constant $U(1)$ gauge fields? Could one see $\Lambda = 0$ phase as an analog of Higgs=0 phase?

In this phase the category of rational functions would expand to a category of real analytic functions and infinite extensions of rationals containing transcendental numbers would be unavoidable and allow light-like curves as frames instead of piecewise light-like geodesics.

One could argue that since the evolution of mathematical consciousness has led to the notion transcendentals and transcendental functions, they must be realized also at the level of space-time surfaces.

One can invent objections against the $\Lambda = 0$ phase for which Kähler action has only CP_2 part and serving at the same time as arguments for the necessity of M^4 part.

1. For a mere CP_2 Kähler action, the CP_2 type extremals representing building bricks of elementary particles become vacuum extremals and are lost from the spectrum. However, also the M^4 part of Kähler action predicted by the twistor lift gives rise to Chern-Simons (C-S) term assignable to the light-like 3-surface X^3_L as the orbit of partonic 2-surface and one can assign a momentum to X^3_L. The boundary conditions guaranteeing momentum conservation make possible momentum exchange between interior and X^3_L.

2. CP_2 Kähler action has a huge vacuum degeneracy since space-time surfaces with 2-D Lagrangian manifold as a CP_2 projection are vacuum extremals. $\Lambda > 0$ eliminates most of these extremals. Also the M^4 part of Kähler action, which vanishes for canonically imbedded M^4, implies that most vacuum extremals of CP_2 Kähler action cease to be extremals even for $\Lambda = 0$.

While writing the first version of this article I had not realized that what the correct form for the Kähler property in M^4 case is.

1. Suppose for definiteness the simplest option that the M^4 Kähler form are associated with the decomposition $M^4 = M^2 \times E^2$. A more general decomposition corresponds to Hamilton-Jacobi structure in which the distributions for $M^2(x)$ and $E^2(x)$ orthogonal to each other are integrable and define slicings of M^4 [L141].

2. The naive guess was that $J^2 = -g$ condition must be satisfied. This implies that the M^2 part of Kähler form of $M^4 = M^2 \times E^2$ decomposition has an electric part, which is imaginary so that the energy density is of form $-E^2 + B^2$ ($= 0$ for M^4). For instance, solutions of $M^2 \times Y^2$, where Y^2 is any Lagrangian manifold of CP_2 would have negative energy for $\Lambda = 0$. Even worse, Kähler gauge potential would be imaginary and the modified Dirac equation would be non-hermitian.

3. The problem disappears by noticing that the M^2 by its signature has hypercomplex rather than complex structure, which means that the counterpart of the imaginary unit satisfies $e^2 = 1$ rather than $i^2 = -1$. This allows a real Kähler electric field and the situation is the same as in Maxwell's theory.

8.6.3 About 2-D minimal surfaces

A brief summary about 2-D minimal surfaces and questions raised by them in TGD framework is in order. One can classify minimal surfaces to those without frame and with frame.

Some examples of 2-D minimal surfaces

The following examples about minimal surfaces are collected from the general Wikipedia article about minimal surface `https://cutt.ly/Hn673ry`) and various other Wikipedia articles. This article gives also references to articles (for instance the article "The classical theory of minimal surfaces" of Meeks and Perez [A98]) and textbooks discussing minimal surfaces , see for instance [A85]. Also links to online sources are given. "Touching Soap Films - An introduction to minimal surfaces" `https://cutt.ly/dmwMnJ7`) serves as a general introduction to minimal surfaces). There is also a gallery of periodic minimal surfaces (`https://cutt.ly/RmwMQ49`), which is of special interest from the TGD point of view.

1. Minimal surfaces without frame

In E^3 frameless minimal surfaces have an infinite size and are often glued from pieces, which asymptotically approach a flat plane.

Catenoid (`https://cutt.ly/in675Z6`) is obtained by a rotation of a catenoid, which is the form of the chain spanned between poles of equal height in the gravitational field of Earth. Catenoid has two planes as asymptotics and is obtained from torus by adding two punctures. Costa's minimal surface (`https://cutt.ly/in65wyP`) is obtained from torus by adding a single puncture and its second end looks like a catenoid.

Frameless minimal surfaces in E^3 allow also lattice-like structures. Schwarz minimal surface (`https://cutt.ly/dn65rJm`) is an example about minimal giving rise to 3-D lattice like structure. These surfaces have minimal genus $g = 3$.

In compact spaces closed minimal surfaces are possible and some quite surprising results hold true, see the popular article "*Math Duo Maps the Infinite Terrain of Minimal Surfaces*" (`http://tinyurl.com/yyetb7c7`). These surfaces have area proportional to volume of the embedding space and the explanation is that these surfaces fill the volume densely [A65, A76].

2. Minimal surfaces with lattice like structure

There exists also minimal surfaces with lattice-like structure.

1. Riemann described a one parameter of minimal surfaces with a 1-D lattice structure consisting of shelfs connected by catenoids (`https://cutt.ly/Pn65y3f`).

2. Scherk surfaces (`https://cutt.ly/3n65oeB`) are singly or doubly periodic. Scwartz surfaces (`https://cutt.ly/un65pCK`) are triply periodic structures defining 3-D lattices and have minimal genus $g = 3$. This kind of surfaces have been used to model condensed matter lattices. These surfaces have also hyperbolic counterparts.

3. Minimal surfaces spanned by frames

Minimal surfaces with frames allow to models soap films and are obtained as a solution of the Plateau's problem (`https://cutt.ly/7n65fgT`).

1. Helicoid (https://cutt.ly/Wn65jgT) represents a basic example of a simply periodic framed surface. Also helicoid involves transcendental functions. A portion of helicoid is locally isometric to catenoid.

2. Arbitrary curves can serve as frames with some mild restrictions. The minimal surface need not be unique. A given 2-D minimal surface is obtained in topological sense from a compact manifold by adding a puncture to represent boundaries defined by frames or the boundaries at infinity.

Some comments on 2-D minimal surfaces in relation to TGD

The study of the general properties of 2-D minimal surfaces from the TGD perspective suggest a generalization to the TGD framework and also makes possible a wider perspective about TGD itself.

1. Frameless minimal surfaces in TGD framework

Frameless minimal surfaces in E^3 have infinite sizes since they are locally saddle like. In TGD framework, the most interesting space-time surface are expected to be framed. Despite this frameless minimal surfaces are of interest.

1. In the TGD framework the minimal surfaces could extend to infinity in time-direction and remain finite in spatial directions. The asymptotically flat 2-plane could in TGD correspond to the simplest extremals of action: M^4 and "massless extremals" (MEs); surfaces $X^2 \times Y^2$ with X^2 a string world sheet and Y^2 complex manifold of CP_2; and CP_2 type extremals with 1-D light-like curve as CP_2 projection.

   Conservation laws do not allow M^4 even in principle unless the total angular momentum and color charges vanish. Various singularities could deform flat M^4 in close analogy with point and line charges.

2. In curved compact spaces also closed minimal surfaces are possible [A65, A76] (http://tinyurl.com/yyetb7c7). One can wonder whether CP_2 as a curved space might allow a volume-filling closed 2-D or 3-D minimal surfaces besides complex surfaces and minimal Lagrangian manifolds [L83]. For $\Lambda > 0$, only complex surfaces defined by polynomials in M^8 appear in PEs. It is difficult to see how this kind of exotic structure could define a physically interesting partonic 2-surface although formally one could consider a product of string world sheet and this kind of 2-surface.

2. Minimal surfaces with lattice structure

2-D minimal surfaces in E^3 allow lattice-like structures with dimensions 1, 2 and even 3. They are are interesting also in TGD framework.

1. Scwartz surface (https://cutt.ly/un65pCK), call it S, allows in the TGD framework a variant of form $M^1 \times S \times S^1$, where S^1 is a geodesic sphere. Same applies to all 2-D minimal surfaces allowing a lattice structure and could be in a central role in condensed matter physics according to TGD. Also hyperbolic variants of a lattice like structure expected to relate to the tessellations of hyperbolic 3-space can be considered and could play important role at the level of magnetic bodies (MBs) as indeed suggested [L133].

2. If $\Lambda = 0$ phase is physically acceptable, it would make possible light-like curves as frames and also lattice-like minimal surfaces with periodicity forced by that of the light-like curve assignable to to CP_2 type extremal as M^8 pre-image.

   Note that $\Lambda = 0$ phase relates to $\Lambda > 0$ phase by the breaking of conformal symmetry transforming light-like curves to light-like geodesics. The interpretation of $\Lambda = 0$ phase in terms of the emergence of continuous string world sheet degrees of freedom is attractive.

   Another interpretation would be based on the hierarchy of Jones inclusions of hyper-finite factors of type II_1 (HFFs). $\Lambda > 0$ phase would define the reduced configuration space ("world of calassical worlds" (WCW)) in finite measurement resolution defined by the included HFF representing measurement resolution and $\Lambda = 0$ phase as the factor without this reduction. The approximation of real analytic functions by polynomials of a given degree would define the inclusion. This sequence of approximations would be realized as genuine physical systems ,rather than only approximate descriptions of them.

3. For $\Lambda > 0$ allowing only polynomial function, periodic smooth minimal surfaces in M^8. The construction of Schwartz surface suggests how one can circumvent this difficulty.

   Schwartz surface defines a 3-D lattice obtained by gluing together analogs of unit cells. If a region of a minimal surface intersects orthogonally a plane, the gluing of this surface together with its mirror image gives rise to a larger minimal surface and one can construct an entire lattice-like system in this way. These surfaces are not smooth at the junctions.

   In the TGD framework, one would construct lattice in time direction and the gluing would occur at edges defined by 3-D $t = r_n$ planes ("very special moments in the life of self" [L81]). Local conservation laws as limits of field equations are enough and derivatives can be discontinuous at $t = r_n$ planes. The expected non-uniqueness of the gluing procedure would mean a partial failure of the strict classical determinism having a crucial role in the understanding of cognition in ZEO. This is discussed in [L138].

   M^8-picture suggests a very concrete geometric recipe for constructing minimal surfaces periodic in time direction and this would make it possible to realize UP for $M^8 - H$ duality.

The general vision would be that $\Lambda > 0$ phases the periodic minimal surfaces can be constructed as piecewise smooth lattice-like structures in the category of real polynomials by using the gluing procedure whereas in $\Lambda = 0$ phase they correspond to smooth surfaces in the category of real analytic functions.

3. Minimal surfaces spanned by frames

Minimal surfaces spanned by frames are of special interest from TGD point of view.

1. In the TGD framework. Minimal surfaces are spanned by fixed frames at the boundary of CD and by dynamically generated frames in the interior of CD. The dynamically generated frames break strict determinism, which means that space-time surfaces as analogs of Bohr orbits becomes non-unique [L138] and holography (for its various forms see [L109, L110]) forced by the General Coordinate Invariance is not completely unique.

2. CP_2 type extremal in H would correspond to 1-D singularity in M^8 analogous to a frame assigned 2-D minimal surfaces. The physical picture suggests that this curve is a light-like curve for the Kähler action ($\Lambda = 0$) and a light-like geodesic for action involving also volume term ($\Lambda > 0$). In the first case the periodicity of the light-like curve could give rise to periodic minimal surfaces as generalization of helicoid. In the second case discretized variants could replace these curves.

3. For the minimal surfaces discussed above, polynomials are not enough for their construction and the examples involve transcendental functions like trigonometric, exponential and logarithmic functions in their definition.

   The same is expected to be true also in TGD. Should one leave the category of polynomials and allow all real analytic functions with rational Taylor coefficients? Or should one assume also the $\Lambda = 0$ phase making possible real analytic functions?

   As far as cognitive representations are involved, this would mean that cognition becomes infinite since the extensions of p-adic become infinite. Could $\Lambda = 0$ phase be associated with an expansion of consciousness, kind of enlightment, and relate to mathematical consciousness?

8.6.4 Periodic minimal surfaces with periodicity in time direction

There are several motivations for the periodic minimal surfaces.

Consistency of M^8-H duality with Uncertainty Principle

Consistency of M^8-H duality with UP is one motivation.

1. M^8 is interpreted as an analog of momentum space. M^8-H correspondence must be consistent with UP. If M^8-H correspondence in M^4 degrees of freedom involves inversion of form $m^k \rightarrow \hbar_{eff} m^k/m^2$. [L109, L110, L134]. This solves the problem only partially. M^8-H correspondence should realize also the idea about plane wave as space-time counterpart of point in momentum space.

   The first guess [L134] would be that the $X^4 \subset CD \subset M^8$ is mapped to a union of translates of images of CD by inverse of P^k, where is the total momentum assignable to CD. What I saw as a problem, was that this gives a lattice-like many-particle state rather than a single particle state as a counterpart of a plane wave.

   If the momentum is space-like, this is indeed the case. Therefore I proposed that the image is a quantum superposition of translates rather than their union and represents an analog of plane wave. I failed to realize that this is not the case for time-like momentum since periodicity in time direction does not mean lattice as many-particle state.

   A geometric correspondence for time-like momenta is possible after all! The problem is a concrete realization of this correspondence and here the geometric construction gluing together the analogs of unit cells to form a periodic structure in time direction suggests itself.

2. Quite concretely, one could take part of $X^4 \subset CD \subset M^8$ defining particle and construct a periodic surface with a period determined by the total time-like momentum assignable to this part of X^4. X^4 has a slicing by planes $e=e_n$ [L81] assignable to 6-branes with topology of S^6 defining universal special solutions of algebraic equations. Here e_n is a root of the real polynomial defining X^4.

   One could take a piece $[e_1,...,e_k]$ of $X^4 \subset CD$ and glue it to its time reversal in M^8 to get a basic unit cell and fuse these unit cells together to obtain a periodic structure.

   The differences $e_i - e_j$, which for M^8 correspond to energy differences, are mapped by inversion to time differences $t_i - t_j$ in H. The order of magnitude for the p-adic length scale assignable to CD in question is the same as for the largest difference for the roots as conjectured on basis of the conjecture that the p-adic length scale correspond to a ramified prime of the extension dividing $|t_i - t_j|^2$ for some pair (i,j). The p-adic prime for CD need not however be a ramified prime and one can develop an argument for how it emerges [L138].

3. Rather remarkably, one can glue together portions $[t_1,..t_r]$ and the mirror image of $[t_k,t_r]$, for any k. All possible sequences of this kind are possible! This suggests an analogy to logical reasoning: $[t_n,t_{n+1}]$ would represent a basic step $t_n \rightarrow t_{n+1}$ in the reasoning and one could combine these steps. Could this process serve as the geometric correlate for logical thought or as engineeering at the level of fundamenta interactions?

   The physicalists refusing to accept non-determinism at the fundamental level fail to realize that our technology relies on a fusion of deterministic processes and is therefore not consistent with strict determinism. Also computer programs consist of deterministic pieces.

4. There is still one open question. Does the construction of the time lattices occur only at the level of H or both at the level of M^8 and H? One can argue that the realization of the analog of inverse Fourier transform forces the construction at both sides.

Bohr orbitology for particles in terms of minimal surfaces

In TGD, space-time surfaces correspond to analogs of Bohr orbits. One should also have classical space-time analogs for ordinary bound states as Bohr orbits for particles. Atoms represent the basic example. In TGD Universe, Bohr model should be much more than mere semiclassical model. Also the geodesic orbits of particles in gravitational fields should have minimal surface analogs.

The Bohr orbits should be representable as parts of minimal surfaces identifiable as deformed CP_2 type extremals. There are two options to consider corresponding to $\Lambda = 0$ phase and to $\Lambda > 0$ phases.

1. $\Lambda = 0$ *phase*

$\Lambda = 0$ phase corresponds to a long length scale limit but general consideratons encourage its inclusion as a genuine phase. Its relation to $\Lambda > 0$ phases would be like the relation of real numbers to extensions of rationals and transcendental functions to polynomials.

1. For $\Lambda = 0$, CP_2 type extremals are vacuum extremals and correspond to 1-D singularities, which are light-like curves in M^8 blown up to orbits of wormhole contacts in H.

   Light-like curve as an M^4 projection of Bohr orbit of this kind can give rise to "zitterbewegung" as a helical motion with average cm velocity $v < c$. The proposal for the TGD based geometric description of Higgs mechanism realizes this zitterbewegung of CP_2 type extremals for Kähler action. This makes it possible to assign to any particle orbit - be it Bohr orbit in an atom or a geodesic path in a gravitational field, an average of a light-like curve.

2. Light-likeness gives rise to Virasoro conditions emerging in the bosonic string theories. This served as a stimulus leading to the assignment of extended Kac-Moody symmetries to the light-like partonic orbits X^3 . The isometries of H define the extended Kac-Moody group. The generators of the Kac-Moody algebra depend on the complex coordinate z of the partonic 2-surface and on the light-like radial coordinate of X^3. Super-symplectic symmetries assigned to the light-like $\delta M^4_\pm \times CP_2$ and identified as isometries of WCW have an analogous structure [K96] [L119].

   The light-like orbits of the partonic 2-surfaces in H are connected by string world sheets. The interpretation could be that in $\Lambda = 0$ phase strings emerge as additional degrees of freedom.

3. For CP_2 part of Kähler action $\Lambda = 0$ CP_2 type extremals are vacua (this need not be the case for the deformations). The C-S term for CP_2 Kähler action carries no momentum and cannot contribute to momentum and cannot realize momentum conservation for deformed CP_2 type extremals.

   However, the C-S term for the M^4 part of Kähler action defines the partonic orbits as dynamical entities. If the projection of the deformation of CP_2 type extremal at the wormhole throat has M^4 projection with dimension $D = 3$, M^4 C-S term gives rise to non-vanishing momentum currents and the smooth light-orbit is consistent with the momentum conservation if boundary conditions are realized. What is remarkable that M^4 C-S term also gives rise to small CP breaking, whose origin is not understood in the standard model. The tiny C-S breaking term would be paramount for the existence of elementary particles!

The implications of this picture are rather profound. It could be possible to assign to any physical system rather detailed view about the minimal surfaces involved both at the level of H and M^8.

Could tachyonic states appear as parts of non-tachyonic states somewhat like tachyonic virtual particles appear in Feynman graphs?

1. The possibly existing periodic minimal surfaces with tachyonic total momenta would have an interpretation as lattice-like many-particle states. This excludes them as unphysical. In fact, one cannot construct tachyonic periodic minimal surfaces in the proposed way since the planes $t = t_n$ have time-like normal.

2. M^8 picture allows to interpret tachyonicity as a trick. In the M^8 picture the choice of $M^4 \subset M^8$ is in principle free. The mass squared of the particle depends on this choice since M^4 momentum is a projection of M^8 momentum to $M^4 \subset M^8$. For eigenstates of M^4 mass, one can rotate $M^4 \subset M^8$ in such a way that the mass squared vanishes. For a superposition of states with different mass squared possible in ZEO this is not possible but one can choose M^4 so that mass squared is minimized. This gives rise to p-adic thermodynamics as a description for the mixing with heavier states.

One could understand the tachyonic ground state as an effective description for the choice of M^4 in this manner.

2. $\Lambda > 0$ phase

For $\Lambda > 0$ only light-like geodesics are possible and this forces a modification of the above picture by replacing light-like curves with piece-wise light-like geodesics.

1. A discrete variant of zitterbewegung consisting of pieces of light-like geodesics is suggestive. The dynamics in stringy degrees of freedom would be almost frozen and completely dictated by the ends of the string. Discretized version of smooth dynamics would be in question. This kind of phenomenological model for hadronic strings has been proposed.
2. The change of the direction of the partonic orbit takes place in a vertex. In M^8 picture it is associated with a partonic 2-surface associated with a $t = r_n$ hyperplane at which several CP_2 type extremals meet at the level of H. These reactions could be seen as ordinary particle reactions.
3. Another way to change the direction would be based on the interaction of parton with the interior degrees of freedom so that conservation laws are not lost. The interaction between the 3-D orbit of wormhole throat and interior is defined by the condition that normal components of the isometry currents of the total Kähler action are equal to the divergences of C-S currents the partonic orbit. For the M^4 part of C-S action only momentum currents are non-vanishing whereas for CP_2 only color currents are non-vanishing.

   At the turning points the normal current of the entire Kähler action - and the divergence of the isometry current for C-S part CP_2 type extremal must become non-vanishing and divergent but cancel each other. Local conservation laws hold true and one can speak of a momentum exchange between interior and wormhole throat. This picture applies also to color currents.

3. A connection with Higgs mechanism

The fact that zitterbewegung makes the particle effectively massive in long enough scales, suggests an analogy with the massivation by the Higgs mechanism.

1. The interactions between partonic orbits and the interior of the space-time surface are analogous to the interactions of particles with a Higgs field leading to the massivation as the Higgs field develops a vacuum expectation value.
2. M^4 Kähler form represents a constant self-dual Abelian gauge field. Although this field is not a scalar field, it is analogous to the vacuum expectation value of the Higgs field as far as its effects are considered.

4. A connection twistor diagrams and generalization of cognitive representations

Also a connection with twistor diagrams is suggestive. The light-like geodesic lines appearing as 1-D singularities in M^8 would correspond to light-like differences of the time-like momenta assignable to vertices. In H they are assignable with partonic 2-surfaces identifiable as boundaries of 3-D blow ups of 1-D singularities in M^8. In M^8, the graphs containing time-like momenta connected by singular lines would define analogs of twistor diagrams. Also at the level of H the lines connecting partonic 2-surfaces would be light-like as also the distances between them since the inversion map preserves light-likeness of the tangent curves.

This would pose additional conditions on cognitive representations.

1. The original proposal [**?**]as that cognitive representation consists of points of X^4 for which M^8 coordinates belong to the EQ associated with the polynomial considered. The expectation was that one has a generic situation so that this set is automatically finite.

   The explicit solution of the polynomial equations however led to a surprising finding was that the number of these points was a dense set for the space-time surfaces satisfying co-associativity conditions [L109, L110]. The second surprise was that co-associativity (associativity of normal space) is the only possible option.

2. The additional conditions guaranteeing that the cognitive representation consists of a finite number of objects, generalize it from a discrete set of points to a union of singularities with co-dimension $d_c = 4 - d$, $d = 1, 2, 3$.

   The vertices would be connected by $d = 1$ light-like singularities and belong to 2-D partonic 2-surfaces as $d = 2$ singularities at $t = r_n$ surfaces in turn defining $d = 3$ singularities. Also 2-D string world sheets having $d = 1$ singularities as boundaries would be included.

3. This would also generalize twistor diagrams as a frame holographically coding for the space-time surface as an analog of Bohr orbit. At the M^8 level, the definition of the parts of this structure would involve only parameters with values in EQ (say the end points of a light-like geodesic defining it).

Periodic self-organization patterns, minimal surfaces, and time crystals

Periodic self-organization patterns which die and are reborn appear in biology. Even after images, which die and reincarnate, form this kind of periodic pattern. Presumably these patterns would relate to the magnetic body (MB), which carries dark matter in the TGD sense and controls the biological body (BB) consisting of ordinary matter. The periodic patterns of MB represented as minimal surface would induce corresponding biological patterns.

The notion of time crystal [B37] (`https://cutt.ly/2n65x0k`) as a temporal analog of ordinary crystals in the sense that there is temporal periodicity, was proposed by Frank Wilczeck in 2012. Experimental realization was demonstrated in 2016-2017 [D6] but not in the way theorized by Wilczek. Soon also a no-go theorem against the original form of the time crystal emerged [B64] and motivated generalizations of the Wilzeck's proposal.

Temporal lattice-like structures defined by minimal surfaces would be obvious candidates for the space-time correlates of time crystals.

1. One must first specify what one means with time crystals. If the time crystal is a system in thermo-dynamic equilibrium, the basic thermodynamics denies periodic thermal equilibrium. A thermodynamical non-equilibrium state must be in question and for the experimentally realized time crystals periodic energy feed is necessary.

   Electrons constrained on a ring in an external magnetic field with fractional flux posed to an energy feed form a time crystal in the sense that due to the repulsive Coulomb interaction electrons form a crystal-like structure which rotates. This example serves as an illustration of what time crystal is.

2. Breaking of a discrete time translation symmetry of the energy feed takes place and the period of the time crystal is a multiple of the period of the energy feed. The periodic energy feed guarantees that the system never reaches thermal equilibrium. According to the Wikipedia article, there is no energy associated with the oscillation of the system. In rotating coordinates the state becomes time-independent as is clear from the example. What comes to mind is a dynamical generation of Galilean invariance applied to an angle variable instead of linear spatial coordinate.

3. Also the existence of isolated time crystals has been proposed assuming unusual long range interactions but have not been realized in laboratory.

Time crystals are highly interesting from the TGD perspective.

1. The periodic minimal surfaces constructed by gluing together unit cells would be time crystals in geometric sense (no thermodynamics) and would provide geometric correlates for plane waves as momentum eigenstates and for periodic self-organization patterns induced by the periodic minimal surfaces realized at the level of the magnetic body. It is difficult to avoid the idea that geometric analogs of time crystals are in question.

2. The hierarchy of effective Planck constants $h_{eff} = nh_0$ is realized at the level of MB. To preserve the values of h_{eff} energy feed is needed since h_{eff} tends to be reduced spontaneously. Therefore energy feed would be necessary for this kind of time crystals. In living systems, the energy feed has an interpretation as a metabolic energy feed.

The breaking of the discrete time translation symmetry could mean that the period at MB becomes a multiple of the period of the energy feed. The periodic minimal surfaces related to ordinary matter and dark matter interact and this requires con-measurability of the periods to achieve resonance.

3. Zero energy ontology (ZEO) predicts that ordinary ("big") state function reduction (BSFR) involves time reversal [L91, L138]. The experiments of Minev *et al* [L77] [L77] give impressive experimental support for the notion in atomic scales, and that SFR looks completely classical deterministic smooth time evolution for the observer with opposite arrow of time. Macroscopic quantum jump can occur in all scales but ZEO together with h_{eff} hierarchy takes care that the world looks classical! The endless debate about the scale in which quantum world becomes classical would be solely due to complete misunderstanding of the notion of time.

4. Time reversed dissipation looks like self-organization from the point of view of the external observer. A sub-system with non-standard arrow of time apparently extracts energy from the environment [L86]. Could this mechanism make possible systems in which periodic oscillations take place almost without external energy feed?

Could periodic minimal surfaces provide a model for this kind of system?

1. Suppose that one has a basic unit consisting of the piece $[t_1, .., t_k]$ and its time reversal glued together. One can form a sequence of these units.

   Could the members of these pairs be in states, which are time reversals of each other? The first unit would be in a self-organizing phase and the second unit in a dissipative phase. During the self-organizing period the system would extract part of the dissipated energy from the environment. This kind of state would be "breathing" [L177].

   There is certainly a loss of energy from the system so that a metabolic energy feed is required but it could be small. Could living systems be systems of this kind?

2. One can consider also more general non-periodic minimal surfaces constructed from basic building bricks fitting together like legos or pieces of a puzzle. These minimal surfaces could serve as models for thinking and language and behaviors consisting of fixed temporal patterns.

8.7 Double slit experiment in time domain from the TGD perspective

The temporal analog of the double slit experiment carried out by a research team led by Riccardo Sapienza [D5] has gained a lot of attention. The experiment is a generalization of the regular double slit experiment to the time domain (`rebrand.ly/tmbz3gu`).

I am grateful to Tuomas Sorakivi for turning my attention to a Youtube video "Light Can Interfere in Time as well as Space" by Ben Milles (`rebrand.ly/zz3jq2a`). The discussions in our Zoom group about the experiment inspired this article and led to an idea that the testing of the basic predictions of zero energy ontology might be possible.

There are also popular articles about the experiment in Live Science (`rebrand.ly/o0ojyya`) and in Quantum Insider (`rebrand.ly/y3k24gl`).

The following is the abstract of the research article [D5]:

"Double-slit experiments—where a wave is transmitted through a thin double aperture in space—have confirmed the wave–particle duality of quantum objects, such as single photons, electrons, neutrons, atoms and large molecules. Yet, the temporal counterpart of Young's double-slit experiment—a wave interacting with a double temporal modulation of an interface—remains elusive. Here we report such a time-domain version of the classic Young's double-slit experiment: a beam of light twice gated in time produces an interference in the frequency spectrum.

The 'time slits', narrow enough to produce diffraction at optical frequencies, are generated from the optical excitation of a thin film of indium tin oxide near its epsilon-near-zero point. The separation between time slits determines the period of oscillations in the frequency spectrum, whereas the decay of fringe visibility in frequency reveals the shape of the time slits.

Surprisingly, many more oscillations are visible than expected from existing theory, implying a rise time that approaches an optical cycle. This result enables the further exploration of time-varying physics, towards the spectral synthesis of waves and applications such as signal processing and neuromorphic computation."

The temporal analog of double slit experiments are expected to have many technological applications:

"The observation of temporal Young's double-slit diffraction paves the way for the optical realizations of time-varying metamaterials, promising enhanced wave functionalities such as non-reciprocity, new forms of gain, time reversal and optical Floquet topology. The visibility of oscillations can be used to measure the phase coherence of the wave interacting with it, similar to wave–matter interferometers. Double-slit time diffraction could be extended to other wave domains, for example, matter waves, optomechanics and acoustics, electronics and spintronics, with applications for pulse shaping, signal processing and neuromorphic computation."

Double slit experiment in spatial domain

One can look at the description of the situation in Maxwell's classical theory first. One has two slits at the first screen characterized by their distance and their widths. They parameterize the diffraction pattern at the second screen. The beam can be assumed to be monochromatic and normal to the screen.

Formally one can say that the transparency of the screen containing the slits is varying. It vanishes outside the slits and is maximal at the slits. The transparency of the screen depends on position. The modelling based on Maxwell's equations assumes incoming beam of light, say plane wave in definite direction and presence of the first screen described in terms of transparency. One can solve Maxwell's equations and predict the diffraction pattern.

One can study the spatial interference pattern at the second screen or equivalently, its Fourier transform with respect to a coordinate parallel to the screen and in the direction vertical to the parallel slits. One obtains a diffraction pattern also in the space of wave vectors (photon momenta in the quantum situation). The dominating peak is associated with the wave vector associated with the incoming beam.

In the diffraction, the normal component of the wave vector is conserved but the tangential component can change. This gives rise to dispersion with respect to the tangential component of the wave vector. At single photon level one can say that the slit scatters the incoming photons to various directions.

Hitherto everything has been purely classical. What makes the phenomenon so remarkable is that the double slit diffraction patterns appear also when single photons are used. This takes place also for other particles. The classical view of a particle would predict two peaks behind the slits.

8.7.1 Double slit experiment in time domain

What happens in the temporal double slit experiment is very similar to what happens in the regular case. One cannot directly measure the temporal patterns of the reflected wave. One can however measure the intensity as a function of frequency characterized by the Fourier transform of the reflected electromagnetic field. If one is also able to determine the phases of the Fourier components theoretically, one could also estimate the fields at the level of space-time, even those before the first pump pulse.

The frequency distribution is qualitatively similar to that for the wave vector distribution in the regular double slit experiment. Slit becomes pulse, slit width becomes the duration of pulse, and the distance between slits becomes the interval between pulses.

The effect is stronger than expected for short pulse durations approaching the optical cycle $T = 1/f$ where f is the frequency of the incoming laser beams. The proposal is that what matters is the time scale τ for the change from transparency to reflectivity lasting as the duration of the pulse. One has $\tau = 2.3$ fs is roughly one half of the period $T = 1/f = 4.4$ fs of the period of incoming laser beam. The duration between pulses is .8 ps.

During period τ the dispersion of the chromatic beam to frequencies around the peak frequency would take place and could be understood mathematically as following from the time dependence of the refractive index (or reflection coefficient).

The frequency dispersion occurs already for a single pulse and gives rise to a single maximum as in the case of the ordinary single slit experiment. For two pulses a diffraction pattern in the frequency domain resembling that of the double slit experiment is observed. The shorter the duration of the pulse is and the shorter the time interval between slits is, the more pronounced the diffraction pattern is.

Maxwell's equations in the presence of the screen with time dependent refractive index describes the situation classically in the first approximation. Time dependent refractive index characterizes the screen. Refractive index does not change during the period between pump pulses but changes during the period τ to a constant value preserved during the pump pulse and after that returns to its original value. The interpretation would be that pump pulse feeds to the system energy needed to keep the value of the refractive index constant allowing reflection. One can say that during the pulse the system is not closed.

In the Maxwellian view, the reflected fields propagate with a light velocity. This gives rise to classical causality. The temporal field pattern of the reflected wave deducible by the inverse Fourier transform from its frequency pattern should vanish before the first pulse. But is this really the case? Time reversal is mentioned in the abstract of the article: is there evidence that this is actually not the case?

8.7.2 TGD view of the double-slit experiment in temporal domain

Consider next the TGD [L141] based quantum view of the situation.

Basic incredients

There are several new ingredients involved.

1. TGD leads to a new view of space-time [L170]. Point-like particles are replaced by 3-surfaces in $H = M^4 \times CP_2$ and their orbits determine space-time regions. The regions of space-time surface obey almost deterministic holography, being analogous to Bohr orbits.

2. Holography forces zero energy ontology (ZEO) [L91, L68, L149, L138] replacing the standard ontology of quantum theory and solving the basic paradox of the quantum measurement theory. ZEO also leads to a new view of what occurs in state function reductions (SFRs).

3. Number theoretic view of TGD predicts a hierarchy of effective Planck constants $h_{eff} = nh_0$ [K33, K34, K35, K36] characterizing phases of ordinary matter behaving in many respects like dark matter. They allow long length scale quantum coherence. In the recent case, laser beams could have $h_{eff} > h$.

4. Momentum position duality is generalized to $M^8 - H$ duality [L109, L110, L168], which states that the M^8 picture providing algebraic and number theoretic view of physics is complementary to the geometric view of physics.

This framework could allow totally new insights of the double slit experiment in the time domain.

ZEO briefly

I have explained ZEO so many times that I will give only a very brief sketch.

1. In ZEO, quantum states are superpositions of classical time evolutions obeying almost, but not quite(!) exact holography [L169, L170, L161]. These time evolutions correspond to space-time surfaces analogous to Bohr orbits. These space-time surfaces are within a causal diamond (CD) and zero energy states are pairs of 3-D states as analogs of ordinary 3-D quantum states at the opposite boundaries of the CD identifiable as superpositions of 3-surfaces.

2. There are two kinds of state function reductions (SFRs) [K130] [L91, L138, L68]. "Small" SFRs (SSFRs) preserve the arrow of time, and their sequence is the counterpart for repeated

quantum measurements which do not affect the system at all (Zeno effect). SSFRs do not affect the passive boundary nor 3-D states at it. SSFRs affect the states at the active boundary of CD and in a statistical sense shift it farther away from the passive boundary. This shifting corresponds to the increase of the geometric time identified as the distance between the tips of CD. Geometric time therefore correlates with the subjective time identified as the sequence of SSFRs.

3. In "big" SFRs (BSFRs), which are counterparts of ordinary SFRs the arrow of time changes and the roles of active and passive boundaries of CD are changed. SSFRs correspond to a sequence of measurements of fixed observables. The states at the passive boundary are eigenstates for a subset of these observables and are not affected in SSFRs. When the set of measured observables changes by external perturbation, BSFR must occur and change the arrow of time temporarily. Quantum tunnelling would correspond to a temporary change of the arrow of time caused by two BSFRs.

Could ZEO make it possible to say something interesting about the time slit experiments?

1. In Maxwellian view the reflected fields propagate with light velocity. This gives rise to classical causality. In the Maxwellian picture, the temporal field pattern of the reflected wave, possibly deducible by inverse Fourier transform from its Fourier transform, should vanish before the first pulse. Is this really the case? Can this be tested? Time reversal is mentioned in the abstract of the article, is there evidence for the occurrence of a temporary time reversal.

2. Could the pump pulse induce BSFR and a temporary time reversal? The end of the pulse would induce a second BSFR and bring back the original arrow of time. Since SFRs replace the superposition of classical space-time surfaces (classical time evolutions and therefore field patterns) with a new one, the pair of BSFRs would change the classical field patterns also in the geometric past.

   Could the classical field patterns be deduced from the frequency spectrum of reflected waves? If it is possible to estimate the phases of the Fourier components theoretically, it is in principle to estimate the classical spatiotemporal field patterns in the geometric past.

   If they violate classical causality, i.e. are non-vanishing before the first pump pulse, one can conclude that the experiment provides a strong support for ZEO. Quite generally, one might someday be able to measure what happens to the geometric past in BSFR pairs.

Can one determine the classical fields in the geometric by measuring the Fourier transform?

In principle, there are good hopes that in the TGD Universe it might be possible to deduce from the frequency spectrum of the reflected light the spatiotemporal behavior of the reflected wave in the geometric past and therefore test ZEO. There are several reasons for why this should be the case.

1. Classical fields are geometrized in TGD. For a single space-time surface the classical fields are determined completely by the surface itself as induced fields. This simplifies dramatically the mathematical picture.

2. Topological quantization means that Maxwellian classical fields decompose to topological field quanta, which or at least their M^4 projections have a finite size. One obtains a variety of field quanta. Massless extremals (MEs) are highly analogous to laser beams and represent precisely targeted propagation in which the pulses consisting of Fourier components with only a single direction of wave vector in the direction of ME preserve their shape.

   The pulse beam, incoming laser beam and reflected light beam(s) could be modelled in terms of MEs. The reflected beam could correspond to a ME representing a superposition of analogs of parallel plane waves with different frequencies. Monochromatic ME becomes multichromatic ME in the reflection. One could say that the frequency dispersion represents new physics predicted by TGD.

3. Holography makes it possible to deduce a given space-time surface from a 3-D holographic data, basically a set of 3-surfaces. There is no need to know the 4-D tangent spaces at the 3-surfaces.

   There is however a small violation of determinism for the holography, which in fact forces ZEO. At the space-time level this corresponds to the fact that space-time surfaces as minimal surfaces are not completely fixed by holographic data, which is analogous to frames spanning soap films.

4. Complexified M^8 is an analog of 8-D momentum space and a given 4-surface in M^8 is determined by roots of a polynomials, which correspond to mass shells $H^3 \subset M^4 \subset M^8 = M^4 \times E^4$. The 4-surface $Y^4 \subset M^8$ must go through 3-surfaces at these mass shells defining the holographic data.

   The condition that the normal space of 4-surface $Y^4 \subset M^8$ going through these mass shells is associative allows us to realize M^8 holography almost uniquely.

5. One can also apply M^8-H duality, which is analogous to momentum position duality of wave mechanics. M^8-H duality maps $Y^4 \subset M^8$ to a space-time surface $X^4 \subset H = M^4 \times CP_2$ and induces holography at the level of H. For space-time surfaces, holography means a generalization of holomorphy from 2-D to a 4-D situation.

   In particular, mass shells are mapped by the inversion mapping M^4 momentum value at mass shell to a light-cone proper time $a = constant$ hyperboloid in $M^4 \subset H$. The momentum components are in general algebraic integers and therefore complex. Therefore one must take the real part of the image by inversion. One has $p^k \rightarrow Re[h_{eff}p^k/p_lp^l]$ [L168].

   This works when mass squared $p^lkp_l = m^2$ is nonvanishing. Photons are however massless. In this case the inversion is ill-defined. In the massless case polynomials determine energy shells in M^4. In this case, the associative holography is based on energy shells. The inversion must map light-cones boundaries to light-cone boundaries. Complex algebraic integer valued energy E is mapped to $t = Re[h_{eff}/E]$.

6. 3-D holographic data at the mass shells in H^3, and in the special case at light-cone boundaries for massless photons, codes for the frequency spectrum and this in principle determines the classical fields in X^4 by M^8-H duality.

   Associative holography and M^8-H duality would make it possible to calculate the classical fields in the geometric past before the first pump pulse and test ZEO by comparing them with those predicted by the classical causality.

7. Number theoretical view brings in also the notion of finite measurement resolution, necessary for number theoretical universality needed when p-adic physics is introduced as a correlate for cognition. This simplifies further the situation.

   Finite measurement resolution requires a number theoretical discretization of mass shells, which is unique. Allowed 4-momenta components, which are algebraic integers for an algebraic extension determined by the polynomial P whose roots define mass shells in turn determining Y^4. The momentum unit corresponds to the size scale of the causal diamond (CD).

   The physical states expressible as bound states of fundamental fermions and antifermions are Galois singlets for which momentum components are ordinary integers.

   An interesting possibility is that, in a finite measurement resolution determined by the polynomial P, all space-time surfaces in the superposition defining a given zero energy state have the same number theoretic discretization. Finite measurement resolution would make the quantum states effectively classical apart from the non-determinism associated with the holography!

In principle, this picture would allow us to determine the classical fields in the geometric past from the frequency spectrum and to check whether they are consistent with the classical causality. The appearance of the reflected wave is already before the first pulse could be regarded as an empirical support, if not a proof for the ZEO!

Of course, in reality the space-time of standard model is determined as an approximation by replacing the space-time visualizable as topologically extremely complex many-sheeted structure with its M^4 projection such that the sums of the induced fields at various sheets define standard model gauge fields and gravitational field as the sum of the deviations of the induce metric from the Minkowski metric.

Chapter 9

New result about causal diamonds from the TGD view point of view

9.1 Introduction

This work is a response to two very interesting articles related to the notion of causal diamond (CD).

9.1.1 The metric for the space of causal diamonds

Dainis Zeps sent article [B20] (`https://rb.gy/gcfvus`) related to the space of causal diamonds (cd) of M^4 (CD is the Cartesian product $cd \times CP_2$ of cd, defined as intersection of future and past directed light-cones, and of CP_2).

Although it remains unclear whether the proposal for the identification of the moduli space is realistic in the TGD framework, where also a simpler realization of the moduli space can be considered, the article led to a clarification of many aspects related to ZEO and the space of CDs as the backbone of the "world of classical worlds" (WCW).

9.1.2 The existence of a hyperbolic generator of conformal group defining a flow, which maps causal diamond to itself

Gary Ehlenberger sent a link to a very interesting article [B56] (`https://rb.gy/z7o7wj`) related to causal diamonds (cd) of 4-D Minkowski space.

The article is very interesting from the TGD point view since causal diamond (CD) of $H = M^4 \times CP_2$ is the key object in zero energy ontology defining the ontology of TGD. CD is defined as the Cartesian product $CD = cd \times CP_2$ of the causal diamond $cd \subset M^4$, identified as an intersection of future and past directed light cones, and of CP_2.

The basic findings of [B56] are as follows.

1. The generator of K of special conformal transformation in time direction, scaling generator D and generator P_0 of time translation generate an $SL(2,R)$ algebra. A suitable combination of these 3 generators defines a generator of what generates hyperbolic time translations, call it S. Its exponentiation generates an analog of time evolution in conformal quantum mechanics. What is remarkable is that this flow takes the CD to itself so that the time range of time evolution can be said to correspond to the distance between the tips of the CD.

   The flow lines of the exponentiated Killing vector would have an interpretation as the world lines of accelerated observables with constant acceleration determined by the initial value of the radial M^4 coordinate.

   The hyperbolic time evolution is time-like only in the interior of the second light-cone associated with CD. Only a single arrow of time is allowed by time-likeness. It is time-like also outside the light-cones.

2. The time evolution of a static CD observer is equivalent to a time evolution of conformal QM defined by a hyperbolic Hamiltonian. The flow states for a fixed value of radial coordinate r correspond to a time evolution from the value of hyperbolic time $\tau = -\infty$ to $\tau = \infty$. This evolution corresponds to a finite time interval with respect to the ordinary Minkowski time t.

   One obtains a family of states corresponding to time evolutions for various values of t. These states are so-called coherent states, which are not orthogonal and their inner product for values $t = t_1$ and $t = t_2$ defines a correlation function of conformal QM.

   The behaviour of 2-points functions of conformal QM in CD is equivalent to that for thermal QFT in M^4. One can say that static CD observers falling freely along the flow lines of the Killing vector observe the vacuum state of the inertial M^4 system as a thermal state characterized by Unruh temperature $T = \hbar/2\pi L$ (`https://rb.gy/qxyp8q`), where L is the size of CD .

9.1.3 Could these findings have significance for TGD

An obvious question is whether these findings could have significance for TGD, in particular for the zero energy ontology (ZEO) [L91, L149, L141] [K130], which replaces the standard ontology as a foundation of quantum theory and in this way solves the basic paradox of quantum measurement theory.

Zero energy ontology (ZEO) and CDs

Causal diamond (CD) defines the analog of quantization volume in the embedding space $H = M^4xCP_2$ inside which the zero energy states are superpositions of space-time surfaces, which connect the light-like boundaries of CD. In the TGD inspired theory of consciousness, CD corresponds to a 4-D perceptive field of self.

The allowed space-time surfaces are preferred extremals (PEs) analogous to Bohr orbits and satisfy almost completely deterministic holography forced by the general coordinate invariance. Quantum states can be also regarded as superpositions of pairs of 3-D states assignable the opposite boundaries of CD and constructed using second quantized fermions of H satisfying free Dirac equation or its modification in the case that also M^4 allows the analog of Kähler structure suggested by the twistor lift of TGD. At the limit of large CDs also the Poincare quantum numbers are opposite at the opposite boundaries of CD.

The TGD based proposal is that the time evolution at the level of "world of classical worlds" (WCW), which consists of these preferred extremals for given CD, is defined by a conformal scaling leaving the passive boundary of causal diamond and states at it invariant but affect the active boundary and 3-D states at it. The gradual increase of CD would correspond to the growth of geometric time correlating with the flow of subjective time defined by "small" state function reductions (SSFRs) as analogs of weak measurements and this time evolution replaces the trivial time evolution of Zeno effect in TGD and gives rise to self as a conscious entity. In "big" SFRs (BSFRs) the roles of the active and passive boundaries change and the arrow of time changes.

Under certain conditions the evolution by scaling can be approximated by a time translation. The scalings commute with the Lorentz transformations mapping the passive boundary of CD to itself. This time evolution allows us to understand spin-glass type systems [L139]).

From the point of view of ordinary time evolution defined by energy, the difference is that the relaxation processes obey fractal power law rather than exponential decay. This is a characteristic feature of the N-point functions of conformal field theories (CFTs) as opposed to those of massive quantum field theories (QFTs). The conformal invariance of 2-D conformal theories generalizes since the light-cone boundary $\delta M^4_\pm \times CP_2$ and light-like 3-surfaces in general allow extended conformal invariance. $\delta M^4_+ \times CP_2$ allows also supersymplectic invariance for light-like radial coordinate, which takes the role of complex coordinate.

One can say that in the TGD framework the 2-D conformal field theories generalize to dimension $D = 4$. In particular, the boundaries of space-time surfaces and the 3-D surfaces at the boundaries of CDs define 3-D holographic data for the generalized conformal field theory.

Poincare group acts in WCW rather than at the level of space-time and M^4

In the TGD framework, the great conceptual leap is made possible by the moduli space of CDs is that 4-D Poincare and conformal groups would not act at the level of space-time or of CD but at the level moduli space of CDs forming the backbone of WCW.

In TGD, Poincare invariance need not be a symmetry of the action at the space-time level as has become clear already earlier. Indeed, the twistor lift of TGD suggests that M^4 has a Kähler form contributing to Kähler action of H so that Poincare and Lorentz symmetries would be broken at the level of M^4.

This picture would fit nicely with the breaking of Lorentz invariance implied by the momentum and polarization vectors assignable to massless particles and also with the view of hadrons based on quarks characterized by a place carrying 2-D longitudinal momenta. This kind of flexibility gives a strong support for the WCW approach.

The quantum numbers of the zero energy states assignable to a given CD must correlate or be equal with the quantum numbers, such as four-momentum and angular momentum assignable to the wave function in the moduli space of CD.

1. The value of mass squared for CD should correspond to that for the generator of conformal scalings realized in super symplectic and super-Kac-Moody type degrees of freedom in the interior of CD.

2. One can assign to the modes of the second quantized spinor field of H 4-momenta and the total momentum associated with these at either half-cone of CD would naturally correspond to the momentum assignable to CD.

3. Classical charges are conserved for action and also these should be identical with those assignable to CD and with the wave function in the moduli space of CDs. Here interesting questions relate to the violation of translational invariance and Lorentz invariance induced by the M^4 Kähler form.

4. In case of TGD, CD means actually $cd \times CP_2$: also the total quantum numbers assignable to CP_2 should be the same for CD and the wave function in the space of CDs.

Problem and questions

In the TGD framework, the existence of a hyperbolic generator S mapping cd to itself and the properties of the corresponding flow raise some questions.

1. The exponentiation of a hyperbolic generator defines a diffeormorphism of CD, which would deform the space-time surface in the interior of the CD. One would have a problem since this deformation is not expected respect the preferred extremal property.

   In TGD, hyperbolic generator S, just like supersymplectic generators, can act only at the boundaries of CD and affect the data at them. These data define boundary conditions for holography, which extends this action to an action on the space-time surface inside the CD. If the hyperbolic flow is modified at all at the boundaries of CD, this induces a modification of the preferred extremal at the level of the space-time surface.

2. The flow induced by S corresponds to a flow that maps the states for a fixed value of radial coordinate r ($r = constant$ means that system is "at rest") from time $\tau = -\infty$ to $\tau = \infty$ and this corresponds to relativistic at the level of CD accelerate motion with acceleration $a = 1/L$, L the size of CD. The initial and final points of motion correspond to point $(t_\pm(r), r)$ at the boundaries of CD. Somewhat disappointgly, the flow does not affect the points at the boundaries at all.

3. What is however possible is that the time evolution as an exponentiation of the hyperbolic generator labelled by t affects the quantum states at the boundaries of CD for each value value of t. One can say that time evolution is represented as unitary time evolution at the boundary of CD.

   This would relate the states at the points of boundary with different values of light-like radial coordinate. This correlation could be interpreted in terms of radial conformal invariance and could be manifested in terms of conformal correlation functions. This is just what one expects since the light-like radial coordinate for the light-cone boundary is analogous to the complex coordinate for ordinary conformal invariance.

Hyperbolic time evolution would define unitary action on zero energy states and could define "small" SFRs (SSFRs), which does not scale CD so that the geometric time defined by the distance between the tips of CD is not changed. In TGD inspired theory of consciousness this kind of SSFRs would correspond to "timeless" conscious experience. Could the sequences of these time evolutions followed by SSFRs correspond to meditative states, which are reported to be timeless? SSFRs involving scalings of CD would in turn correspond to ordinary ordinary conscious experience involving the sensation of time flow.

These findings force to reconsider the basic assumptions of ZEO.

1. The assumption has been that only SSFRs scaling the size of CD possible? Could one think that only the SSFRs corresponding to the hyperbolic time evolution are possible. The size of CD remains unaffected in the sequence of SSFRs.

   Could the scalings of CD correspond to SSFRs or could they precede BSFRs in which the arrow of time changes? Note that the scalings would affect the WCW spinor fields representing zero energy states at the level of moduli space of CDs whereas hyperbolic time evolutions would affect only the WCW spinor field in "internal" CD degrees of freedom.

2. One can assign to the 2-point functions at the boundary of the CD a temperature given by $T = \hbar/L$, where L is the size of the CD. Could this temperature and thus the size of CD correlate with the physical temperature of the environment? In fact, I have already ended up with the view that L codes for the analog of cosmic temperature gradually reduces during the sequence of SFRs.

9.2 Two possible identifications for the space of CDs and its metric

The space of causal diamonds (CDs) forms the backbone of the "world of classical worlds" (WCW). Each CD interpreted as a geometric correlate for the perceptive field corresponds to a sub-WCW. Zero energy states are realized as superpositions of space-time surfaces inside CD and pairs of fermionic Fock states at boundaries of CD. The space of CDs would define cm degrees of freedom in WCW.

A possible interpretation [B50] (`https://rb.gy/m1dvwq`) of CDs assigned with, say geodesic lines, is that the size of the CD (temporal distance between its tips) defines the duration for a "moment of consciousness". This interpretation makes no distinction between subjective and geometric time. This distinction is however very real as many examples from everyday life demonstrate.

The increase of the size of CD in "small" state function reduction (SSFR) inducing a small scaling of CD would correspond to the increment of geometric time identified as the distance between the tips of CD. The basic TGD based proposal is that this corresponds to the duration of experienced moment of subjective time in SSFR. The duration of the moment of consciousness between two SSFRs would thus correspond to an increase of the size of the CD rather than to its size.

One can consider two very different identifications of the moduli space of CDS.

9.2.1 TGD in inspired option for the moduli space of CDs

Consider first the TGD inspired identification for the moduli space of CDs.

1. It is easy to see that the space of CDs is 8-D. The position of either tip or of center of the CD gives M^4. The Lorentz transformations $SO(3,1)$ leaving either tip of CD invariant and the group D of scalings leaving the cm of CD invariant give new CDs. The isotropy group of the CD is $SO(3)$. This gives 8-D space.

2. The first guess is that the space of CDs is the coset space $D \rtimes P/SO(3)$. The division by $SO(3)$ is due to the fact that a given CD corresponds to a unique rest system and is invariant under rotations. Translations of M^4, that is M^4, code for the position of the cm or tip of CD, and D corresponds to the size of CD. Locally one would have an 8-D product $D \rtimes M^4 \times H^3$, H^3 is 3-D hyperbolic space analogous to mass shell. There might be some delicate effects changing the topology.

   The naive guess that the signature of the metric is (1,7), is wrong. For the Lorentz group $SO(1,3)$ the signature is $(3,3)$ and for H^3 as coset space $H^3 = SO(1,3)/SO(3)$ the signature is $(3,0)$ rather than $(0,3)$ as one might expect. This has no physical implications. If D has space-like signature, $D \rtimes M^4 \times H^3$ has signature $(0,1) + (1,3) + (3,0) = (4,4)$. This is what one obtains also for the $SO(2,4)/SO(1,3) \times SO(1,1)$ option so that options could be equivalent.

9.2.2 The identification of the moduli space of CDs inspired by conformal invariance

An identification of the moduli space of CDs inspired by conformal invariance is considered in [B20] (`https://rb.gy/gcfvus`). The 5-D $AdS_5 = SO(2,4)/SO(1,3)$ appears in AdS/CFT correspondence and is associated with $4 \rightarrow 5$ holography whereas the space-time surfaces inside CD realize $3 \rightarrow 4$ holography. This might have served as a motivation for the proposal: maybe the idea has been that one cold generalize holography by introducing 8-D space $SO(2,4)/SO(1,3) \times SO(1,1)$ as analog of AdS and its "boundary" as a 7-D space.

The proposal for the space of CDs

The proposal of the article is that the space of CDs is given by $SO(2,4)/SO(1,3) \times SO(1,1)$ 8-D space.

1. $SO(2,4)$ is the 15-D conformal group of M^4 including Poincare transformations, scalings and also the 4-D group of special conformal transformations. $SO(2,4)$ acts linearly in 2+4-D Minkowski space $M(2,4)$. The action of special conformal transformation is however singular and this might mean difficulties. CD is defined by a 2-D sphere of M^4 defining the maximal ball of CD and thus CD.

   The space of CDs would be 8-D have signature (4,4) also now. This can be seen by noticing that boosts correspond to time-like directions and rotations to space-like directions so that the signature of $SO(2,4)$ is $(8,7)$. $SO(1,3)$ has signature $(3,3)$ and $SO(1,1)$ has signature $(1,0)$. Signature $(4,4)$ is indeed the outcome. Maybe the two candidates for the space of CDs are identical.

2. Why $SO(2,4)$ would be a correct choice for the group considered. The hyperbolic time evolution as an exponential of special conformal transformation in the time direction defined by CD maps the lower boundary of CD to the upper boundary. Special conformal transformation can be composed from an inversion with respect to the middle point of the CD followed by an ordinary translation followed by inversion. Inversion $x^\mu \rightarrow x^\mu/x^2$ performs a local scaling of the point of the point $(\pm t, r = T \mp t)$ of the CD boundary to $(\pm t, r = T \mp t)/(-T^2 \pm 2Tt)$ and scales the CD boundary. This scaled boundary is translated and preserves its shape. The inversion scales this CD down. Therefore special conformal transformations can leave the shape of CD invariant but induce Lorentz transformation, scaling, and translation.

3. The intuitive expectation is that the subgroup dividing $SO(2,4)$ should leave CD invariant. If so, $SO(1,1)$ should map the CD to itself. A possible identification is as the group of special conformal transformations in the time direction defined by CD. If $SO(1,3)$ corresponds to the usual Lorentz group, it does not leave the CD invariant unlike $SO(3)$.

   This suggests a different interpretation. Perhaps the division means that $SO(1,3) \times SO(1,1)$ consists of Lorentz boosts and hyperbolic special conformal transformations leaving the center point of CD invariant. Only $SO(3) \times SO(1,1)$ would leave the CD unaffected.

4. The basic distinction from the TGD inspired proposal is that there is no holography involved with CD. Rather, it seems that one starts from the idea that one has $7 \rightarrow 8$ holography is associated with the 8-D $SO(2,4)/SO(1,3) \times SO(1,1)$. 7-D subset of CDs would provide the holographic data. One can, of course, consider this kind of holography in WCW cm degrees of freedom.

   For this option, the identification $SO(3,1) \times SO(1,1)$ could have the same role as $SO(3)$ in the case of the Poincare group or Lorentz group. This would give for CD Lorentz degrees of freedom and assignable with the $SO(1,1)$ mapping CD to itself.

How representations of the Poincare group could be realized in $SO(2,4)/O(1,3) \times SO(1,1)$?

The representations of the Poincare group should be realized in the moduli space of CDs. How this is achieved is not obvious for the conformal group option.

1. It would seem that instead of finite-D possibly non-unitary representations of the rotation group $SO(3)$ as in the case of P, non-unitary finite-D representations of $SO(1,3) \times SO(1,1)$ characterize what might be called generalized spin degrees of freedom? The situation would resemble that in Poincare invariant QFTs for which one has non-unitary finite-D representations of $SO(3,1)$ given by say spinors. It is not clear whether the induction gives rise to a direct sum of state spaces associated with mass shells H^3 as the physical intuition would suggest.

2. What seems like a paradox is that the Poincare group allows 4-D translations as Cartan algebra whereas $SO(2,4)$ has 3-D Cartan algebra. Constant mass squared for Poincare irreps however means that only 3 momentum generators are dynamical. $SO(2,4)$ Lie algebra contains 3-D Cartan algebra, which consists of commuting rotation generators of $SO(4)$ and $SO(2)$ having a discrete spectrum. Momenta are not expected to have a similar discrete spectrum without special assumptions. Intriguingly, the number theoretical vision of TGD, involving the notion of Galois confinement TGD, predicts that the spectrum of physical stats is integer valued in suitable units.

3. For the TGD inspired option the moduli space is identified as $D \rtimes P/SO(3)$ and the situation is much simpler and differs from the construction of the representations of the Poincare group only by the presence of the scaling group with defines the analogs of unitary time evolutions preceding SSFRs. The scaling group would reduce by a symmetry breaking forced by the number theoretical picture to a discrete subgroup of scalings giving a discrete mass squared spectrum.

4. $SO(2,4)$ acts as symmetries of massless theories to which ordinary twistor approach applies so that the proposed picture is attractive in the framework of massless QFTs in M^4. Masslessness in M^4 sense conforms with the invariance of the state space of massless particles under scalings.

   In the TGD framework, the masslessness holds true in the 8-D sense and also the twistor space of CP_2 enters the game so that masslessness in the M^4 sense is not absolutely necessary and the physical expectation is that the mass square spectrum is integer valued using as the momentum unit defined by CD size scale. Note however that the ground states of super symplectic representations are massless in this sense.

5. The space of CDs would be non-compact and would have 4 time-like directions. Holography analogous to AdS/CFT correspondence, with AdS interpreted as space-time, does not look plausible.

9.2.3 Questions about the dynamics in the space of CDs

CDs define a decomposition of WCW to sub-WCWs. The intuitive picture is that one has a network of CDs acting as analogs of interaction volumes and that "particle lines" connect the CDs to each other. One would approach standard ontology as these networks analogous to Feynman diagrams increase. In standard ontology one can speak not only about events, but something that exists. This would be like a transition from Eastern to Western world view.

Should one try to describe these particle lines by modifying the space of CDs and by introducing interactions between CDs? Does it make sense to assume that overlapping CDs interact in the sense that the space-time surfaces belonging to two different CDs touch? If CDs are interpreted as perceptive fields, this does not look like an attractive idea. The emergence of larger Feynman diagram-like structures would only mean emergence of larger CDs containing sub-CDs. Of course, understanding the interaction between CD and sub-CDs remains a challenge.

For the simplest option based one has the space of CDs, in which CDs are like particles with internal degrees of freedom. How can one construct transition amplitudes in the space of CDs?

1. If one can assign representations of the Poincare group to CDs, they would be analogous to particles characterized by momenta and angular momenta having also conformal weight as a quantum number associated with scalings. They would also have internal dynamics, which have been the main target of attention hitherto.

2. Transitions are expected to occur between CDs with different positions, sizes and different rest systems (direction of time line defined by the tips). In the recent picture, state function reductions are assumed to correspond to scalings only.

   One would expect that the transition amplitude between quantum states for the moduli space propagation between two different CDs has a kinematic part, which one might hope to reduce to symmetry considerations just as for the propagation of particles in Minkowski space.

3. There is also an inner product of zero energy states related to CDs. The basis of the zero energy states characterized by holography are expected to differ by the action of an element $g(CD_1, CD_2)$ of the group $SO(4,2)$ or of $D \times P$ transforming the CDs to each other.

   The transition amplitude should be proportional to the overlap of these states and therefore to the matrix element of $g(CD_1, CD_2)$ between the zero energy states associated with CD_1 and CD_2.

4. Physical intuition suggests that the transition amplitudes are small for "large" transformations $g(CD_1, CD_2)$ and that in good approximation small translations, Lorentz boosts, and scalings are preferred. In the approximation that translations and Lorentz boosts affecting the center point of CD are trivial, only scalings and hyperbolic evolutions remain under consideration.

9.3 TGD inspired questions and ideas related to the interpretation of the hyperbolic flow

The interpretation of the findings related to hyperbolic time evolution in the TGD framework inspire several questions and ideas.

9.3.1 The flow lines of time-like special conformal transformation as a motion with a constant acceleration

The exponentiated hyperbolic generator S corresponds to a relativistic motion in M^4 with a constant acceleration a, which is essentially the inverse of the size L of CD: $a = hbar/L$ for $c = 1$, where L is the size of CD identified as distance between its tips. Could this constant acceleration correspond to a representation for an acceleration of the system defined by CD in an external gravitational field, which is constant in the first approximation?

Note that this acceleration is huge when compared with that assignable to macroscopic systems. Gravitational acceleration g at the surface of Earth corresponds to a thermal energy of order 10^{-21} eV. The size of the CD would be in this case of order 10^{15} m.

It has been already noticed that in the TGD framework S can affect only the holographic data at the boundaries of CD. The action of S exponentiates to a unitary hyperbolic time evolution on quantum states at the boundaries of CD. The two-point functions of the conformal QM are thermal with temperature determined by the scale L of CD so that the acceleration $a = 1/L$ can be said to make itself visible via Unruh effect (`https://rb.gy/qxyp8q`).

9.3.2 Can one assign thermodynamics to CD?

One can assign to the CD a temperature. I have earlier proposed that the expansion of CD by sequences of SSFRs could be interpreted as a stepwise cosmic expansion with temperature decreasing like $\hbar/L$. This would conform with the finding that the astrophysical objects themselves do not seem to participate in the expansion.

1. In QFT in M^4, constant acceleration corresponds to the so-called Unruh temperature proportional to $\hbar \times a$. For the ordinary value of $\hbar$, this temperature is extremely small for the accelerations encountered in macroscopic quantum systems. An accelerated system sees the vacuum of an inertial system as a black body at Unruh temperature.

   Note that the blackhole temperature is analogous to Unruh temperature and proportional to $\hbar GM/R^2 \propto \hbar/GM$ (this temperature is extremely small for astrophysical blackhole-like entities).

2. The conformal 2-point functions of a CFT inside a CD with "energy"/Hamiltonian associated with infinitesimal special conformal transition, behave like those for ordinary CFT at finite temperature $T_{CD} = hbar/L$, L the size of CD. T_{CD} is analogous to Unruh temperature but much larger.

   For a massive particle with Compton length L , the CD temperature would be of order $T_{CD} = m = hbar/L$. This would be more like ordinary temperature for a system of mass m moving in a volume defined by Compton length. I have proposed that CDs are characterized by this temperature and the expansion of CD in the sequence of SSFRs leads to the reduction of this temperature analogous to that taking place in cosmology.

What could be the interpretation of T_{CD}?

1. For the cell scale the CD temperature T_{CD} would be of order of the energy of visible light. Could T_{CD} relate to bio-photons? Could CD temperatures correspond to those in ordinary thermodynamics and could they be interpreted as a kind of sensory/cognitive representations for the real temperatures in terms of the internal physics of CD?

2. In p-Adic thermodynamics [K65] [L158], energy is replaced with mass squared interpreted as proportional to conformal weight as in string models. The p-adic analog of the temperature is inverse of an integer and corresponds to a dimensional p-adic temperature $T_p = \hbar log(p)/L_p$, where L_p is the p-adic length scale proportional to $\sqrt{p}$. p-Adic length scale corresponds to the length scale defined by the Compton length of the particle. p-Adic length scale hypothesis states that preferred p-adic primes are near to powers of 2, or possibly also other small primes and that these primes correspond to fixed points of discrete p-adic coupling constant evolution. Number theoretic vision of TGD suggests a concrete mechanism implying this [L170].

   A possible interpretation is that p-adic massivation and p-adic temperature characterizes the density matrix for the particle entanglement with the environment [L158]. p-Adic temperature is assigned with a scaling which changes the size of the CD and could be perhaps associated with "big" CDs.

   T_{CD} would naturally correspond to the p-adic temperature $T_p = \hbar log(p)/L_p$.

3. $T_{CD} = \hbar/L$ generalizes to $T_{CD} = \hbar_{eff}/L$. For gravitational Planck constant $\hbar_gr/\hbar = GMm/beta_0$, $\beta_0 < 1$, where M is some large mass, say Earth mass, one has $T \propto (GMm/beta_0)m$.

   If the size scale of CD is expected to scale lik h_{eff}, one obtains a zoomed version of the system and temperature is not changed. This applies also to the temperature $T_{CD,gr} = \hbar_g r/L$ if L scales like $hbar_{gr}$. Note that for $\hbar_{gr}(Earth)$ assignal to dark particles at gravitational flux tubes, gravitational Compton L is about .45 cm and does not depend on mass m (Equivalence Principle). For electrons and protons $T_{CD,gr}$ would be unrealistically high.

9.3.3 Could the dynamics of CD define a sensory map of the exterior of CD?

The time evolution by a special conformation maps the CD into itself. This is a surprising result. The infinitesimal generator vanishes at the light-like boundaries and the generator is time-like for either half-cone of the cd.

In TGD, CD is identified both as the analog of quantization volume and the perceptive field of self. This raises some questions.

1. The special conformal transformation in time direction consists of an inversion I with respect to the center point of the CD followed by an ordinary time translation followed by the same inversion I.

   Ordinary time translation in the exterior is mapped to the special conformal transformation inside the cd. Does this imply some kind of cognitive or sensory map of the exterior world to the interior world? Note that the same can be done also for the other special conformal transformations. Is this something that the monads of Leibniz might be doing?

2. Inversion is also involved with the M^8-H duality and gives a semiclassical realization of Uncertainty Principle [L109, L110]. The mass shells $H^3 \subset M^4 \subset M^8$ with mass m are mapped by inversion $m \rightarrow a = \hbar_{eff}/m$ to cosmic time $a = constant$ hyperboloids of $M^4 \subset H$.

   M^8-H duality, has an interpretation as a generalization of momentum-position duality. This duality is natural since quantum TGD is essentially the generalization of wave mechanics obtained by replacing point-like particles with 4-D surfaces obeying holography. Could one interpret the special conformal transformations in terms of M^8-H duality. If so, special conformal transformations could be seen as mirror images of translations of in momentum space.

   Twistor approach [B25] to scattering amplitudes involves a duality between two Minkowski spaces in which dual conformal groups act. M^8-H duality suggests a natural identification of these two Minkowski spaces as space-time and momentum space and could correspond to this duality.

3. In zero energy ontology (ZEO) [L91, L149, L141] [K130], the time flow corresponds to the increase of CD (at least in statistical sense) taking place in scalings of CD? For a sequence of time-like special conformal transformations realized as unitary hyperbolic time evolutions the experienced flow of time should therefore cease. Could these sequences serve as correlates for "timeless" meditative states of consciousness?

9.3.4 Could astrophysical SSFRs correspond to special conformal transformations leaving CD size unaffected?

There are also questions related to the TGD inspired cosmology, which can be quantum coherent in arbitrarily long length scales. This would explain the constancy of the temperature of CMB.

In the TGD inspired cosmology [K102, K68], the cosmic expansion does not take place for astrophysical objects (stars and planets at least) as a smooth process but as rapid "jerks" in which flux tubes as thickened cosmic strings suffer phase transitions increasing their thickness and reducing their string tension [L71, L78, K114]. This is also an empirical fact and not understood in general relativity.

On the other hand, the emergence of complex life forms in the Cambrian Explosion is a biological mystery. TGD explains it in terms of this kind of rapid expansion of the radius of Earth by factor 2 bursting highly developed life forms from underground oceans to the surface of Earth [L65, L145, L131, L163].

Should one modify the views of ZEO [L91, L149, L141] [K130]? Could one think that at quantum level the TGD analog of unitary time evolution could correspond to a sequence of "small" state function reductions (SSFRs) followed by hyperbolic time evolutions, which do not scale up the CD? This even in astrophysical scales. In this view, only "big" SFRs (BSFRs) changing the arrow of time and occurring at the level of magnetic body (MB) would correspond to the scalings of CD and a pair of BSFR would bring back the original arrow of time. This would be the case if CD size corresponds to that for a *real* physical system rather than for the perceptive field of a conscious entity as assumed hitherto.

9.3.5 About the quantum realization of hyperbolic time evolution in TGD framework

What one can say in the TGD framework about the quantum realization of the exponentiated S using holography?

1. The situation can be understood by noticing that in the lowest approximation the motion is given by $r = r_0 + gt^2/2$ so that a parabola intersecting the light-cone boundary is obtained in a finite time. S is well-defined at the light-like boundaries of CD.

2. Holography inside CD does not allow assignment of a flow inside CD to the representation of the exponentiation of S represented as an operator. S is not parallel to the light-like boundaries, in which case the action of the superposition of 3-surfaces would be non-trivial. Therefore only the action in fermionic degrees of freedom can be non-trivial.

3. The action of S on the quantum state at the boundary makes sense and is analogous to that of a Hamiltonian in Hilbert space at time= constant hyper-surface. Zero energy state is the sum of over pairs of 3-D states located at passive and active boundaries of CD and by holography the action of exponentiation of S at the passive boundary and determines the action of S at the active boundary. There is no flow inside the CD. Holography defines the state at the opposite boundary.

   One obtains a one parameter family of exponentiations of S analogous as unitary time evolution operators $U(\tau)$. The value of the parameter τ in the exponential runs from $-\infty$ to $+\infty$ and corresponds to a finite range for the ordinary time coordinate t. This is indeed analogous to 1-D conformal QM since the angular and radial degrees of freedom are effectively absent. The light-like radial coordinates take the role of complex variable z in conformal field theories.

9.4 Motion of CDs in the moduli space and kinematic qualia

The proposal that CD serves as a correlate for the perceptive field of self implies that the motions of CDs in the moduli space of CDs gives rise to the kinematic qualia assignable to various kinds of motions.

The first thing to notice is that the transformations, which leave the position of CD that is the center point of CD invariant, are in a special role.

1. These transformations consist of Lorentz group $SO(3,1)$, scalings, plus possibly also the subgroup $SL(2,R)$ of special conformal transformations leaving the center point of CD invariant. This group would characterize the internal degrees of freedom analogous to the orientations of a rigid body.

2. Besides this there are transformations of $SO(2,4)$ affecting the center point of the CD. The moduli space $SO(4,2)/SO(1,3) \times SO(1,1)$ could correspond to these transformations in the space of generalized positions of CD.

3. $SO(1,3) \times SO(1,1)$ would leave the center point of the CD invariant and code for various shapes of the CD with one point fixed. This space would consist of various shapes of CD representing the states of motion of the CD. For instance, if a CD moves in a particular direction with some velocity. The CD would have a shape characterized by a corresponding Lorentz transformation. This group includes also scalings leaving center point invariant.

9.4.1 The motion at quantum level

One can consider the situation both at quantum and classical level.

1. At the quantum level the system would be characterized by a wave function in moduli space and small state function reduction (SSFRs) would correspond to steps in the motion.

   The analogues of both position and momentum measurements are possible and these could by $M^8 - H$ duality correspond to position measurements in the space of CDs assignable to M^8 and H. This duality could correspond to the duality of twistor Grasmannian amplitude realized in terms of ordinary and momentum twistors [L55].

   If a localization takes place in the degrees of freedom considered as commuting degrees of freedom at each step, one obtains a discrete motion in these degrees of freedom.

2. The M^4 position of CD would define naturally Cartan algebra and SSFRs involving a position measurement in E^3 would define the discrete motion in M^4.

3. This discrete motion in H would give rise to kinematic qualia such as experience of motion with constant velocity and acceleration. The localization in the space of CDs with respect to scaling would fix the size of CD and therefore geometric time as a correlate of subjective time identified as a distance between the tips of CD. The localization with respect to the time coordinate of CD is impossible and does not allow to identify Minkowski time as an observable.

   Scalings increasing the size of CD in a statistical sense would correspond to the growth of geometric time as distance between the tips of CD correlating with the subjective time identifiable as a sequence of SSFRs.

4. D and boost and rotation generators (call them K_z and J_z) of the Lorentz group leaving the center point of the CD invariant can serve as observables for states localized in the moduli space.

   What about momentum eigenstates in the moduli space? D does not commute with momentum so that for momentum eigenstates D is not well-defined. K_z and J_z would be analogous to helicity since their identification depends on the position of CD or momentum of momentum eigenstate. The measurement of D implies delocalization with respect to the size of CD. The dual measurement of the size of CD, call it L, means a measurement of geometric time.

   For a given size L of CD one can construct momentum eigenstates as analogues of plane waves so that the measurement of L seems to commute with the measurement of momentum. Geometric time as a gradual increase of the size of CD would flow only during the measurements involving measurement of L.

Can the scaling operator D correspond to the "stringy" scaling operator L_0?

There is a problem related to the identification of conformal weights as eigenvalues of the scaling operator L_0 as mass squared eigenvalues. In string models, the scaling operator L_0 acting in complex coordinate z of the string world sheet is proportional to the mass squared operator and commutes with it. L_0 commutes with the mass square operator but the scaling operator D does not. What could be the counterpart of L_0 in TGD?

One can consider several candidates for L_0 in TGD. Light-like 3-surfaces appear at the level of both embedding space and space-time surface so that there are two basic types of candidates. These two kinds of scaling generators could relate to the notions of inertial and gravitational masses and therefore to the Equivalence Principle.

Consider first the embedding space level.

1. The light-like radial coordinate r for the light-cone boundary would provide the first guess for L_0 as scaling $L_0 = r\partial_r$. L_0 however scales the size of the light-like boundary and therefore CD so that this interpretation can be challenged.

   On the other hand, the basic assumption is that the states at the passive boundary of CD are unaffected under SSFRs so that scaling should not affect the momenta. Note that the center point of CD must be shifted in time direction under the action of CD so that one has a combination of D and P_0.

2. The most stringy candidate for L_0 would be as the scaling generator $L_0 = zd/dz$ for the conformal transformations of S^2 of light-cone boundary representable as $S^2 \times R_+$. Remarkably, L_0 does not affect the size of the CD.

   The generators of globally defined conformal transformations of S^2 are representable as Möbius transformations representing $SL(2,C)$. One can even compensate for the conformal scaling factor associated with these transformations by a suitable radial scaling of r depending on z so that they act as isometries.

Light-like 3-surfaces appear also at the level of space-time surfaces. Both the light-like 3-surfaces defining boundaries of space-time surfaces [L155], and the boundaries between Euclidean wormhole contacts and Minkowskian space-time regions, identified as deformed CP_2 type extremals, allow by their metric 3-dimensionality extended conformal invariance.

1. One assign light-like coordinate r with the light-like 3-surface and the scaling generator $L_0 = r\partial_r$ is highly analogous to L_0.

2. Also now one has the representations $X^2 \times R_+$ and complex coordinate for X^2 defines counterpart of L_0.

3. Could L_0 assignable to X^2, represent gravitational mass squared? Could holography assign momenta to the light-like 3-surfaces identifiable as gravitational momenta and by Equivalence Principle be identical with inertial momenta assignable to the boundaries of CDs. Gravitational-inertial dictotomy would reflect the space-time embedding space dichotomy.

4. Classical TGD is an exact part of quantum TGD and the conserved classical momenta assignable to the space-time surfaces are also natural candidates for gravitational momenta. The inertial momenta would be assignable to the CDs.

9.4.2 The motion at the classical level

At the classical level would have motions in moduli space consisting of small pieces, which are geodesic lines in $D \rtimes P$ or $SO(4,2)$.

1. Causality would mean that only discretized world lines consisting of pieces which are time-like or light-like geodesic lines are considered (recall that the signature of the moduli space is $(4,4)$.

2. A small transformation of the group $D \rtimes P$ or of $SO(2,4)$, which is representable as an action of a Lie algebra generator, would describe a basic step in the motion. In the quantum case, one must have Cartan algebra and one can argue that all classical motions consist of small translations or scaling rotation, and boost commuting with it.

   The measurements of the size and position of CD are dual measurements and the measurement of the size of CD would give rise to the experience flow of time correlating with the flow of the geometric time.

3. For $D \rtimes P$ small translations along geodesics of M^4 would be basic building bricks of translational motion and the direction of small translation would change step by step in curvilinear motion.

   For $SO(2,4)$ also special conformal transformations affecting the center point of the CD could be considered classically. They do not however commute with translations so that at least quantum mechanically they would be excluded. The 4 special conformal transformations could however serve as a dual set of observables instead of translations.

4. A Lorentz boost of CD would code for the velocity associated with a given step of translational motion and would be determined by the direction and size of the step. It would be coded by the shape of the CD characterized by the direction of the time-like axis connecting its tips. The shape of the CD would change during the motion.

5. Rotational motion around the center point would leave center point and CD itself invariant but affect the zero energy state of CD. More general rotational motion would correspond to steps consisting of translations.

9.5 Questions related to ZEO

ZEO involves several questions which are not completely understood. Do SSFRs correspond to repeated measurements for a set O of commuting observables? Does BSFR occur when a new set of observables not commuting with the set O are measured? What exactly happens in SSFR?

9.5.1 Questions related to SSFRS

SSFRs as a generalization of Zeno effect and weak measurements

Consider once again the question related to the identification of SSFRs. SSFRs are identified as the TGD counterpart for weak measurements, generalizing the notion of repeated measurements giving rise to the Zeno effect.

1. The most straightforward generalization of the Zeno effect is that in the kinematic degrees of freedom for CDs the sequence of SSFRs corresponds to a sequence of measurements of commuting observables. BSFR would take place always when the set of measured observables changes to a new one, not commuting with the original set.

2. D, K_z and J_z leave the center point of CD, identified as position of CD, invariant. D does not commute with momenta. Should one just accept that momenta and $\{D, K_z, J_z\}$ are two sets of mutually commuting observables and that the change of this set induces BSFR.

   The size of CD and therefore the value of the geometric time would change in the sequence of measurements of D, K_z and J_z but not in the sequence of momentum measurements one would have superposition over different sizes of CD and time would be ill-defined as also Uncertainty Principle requires. This would conform with the original view.

What really happens in SSFRs?

I have written a lot of what might happen in SSFRs and BSFRs but I must admit that the situation is still unclear and the proposals depend on what one takes as starting point assumptions, which can be overidealizations.

On the more general level, the sequence of SSFRs would correspond to dispersion in the moduli space of CDs and if SSFRs correspond to the measurement of same commuting observables identified as generators of $SO(2,4)$ or $D \rtimes P$ or their duals as generalized position in the moduli space, rather simple picture emerges of what can happen.

BSFR would take place when the new set of observables not commuting with the original set emerges. What are the conditions forcing this? If one assumes that sleep is induced by BSFR, it becomes clear that this does not happen at will but when metabolic energy resources are depleted and the system must rest. The dissipation of the time reversed system looks like self-organization and the system heals during sleep. Also homeostasis would rely on BSFRs in various scales making it possible to stay near quantum criticality.

But what exactly happens in SSFR? It seems clear that the states at the passive boundary are not changed. But what happens to the passive boundary?

1. Do the contents of sensory experience assigned with the sequence of SSFRs localize

   Option a: to the active boundary of the CD or

   Option b: to the 3-ball at which the half-cones of the CD meet.

2. What happens to the passive boundary itself in SSFR? The scaling occurs for the entire CD but there are two basic options.

   Option 1: The scaling leaves the *center point* of the CD invariant. Passive boundary is shifted towards past just like active boundary towards future.

   If the sensory experience is assigned to the active boundary (Option a)), option 1) is consistent with what happens when we wake up. The time has been flowing during sleep but we have not been aware of this. The arrow time would be determined solely by the change of the state at the active boundary.

If the sensory experience is assigned with the 3-ball (option b)) at the center of CD (Option b)), time does not flow in the sequences of SSFRs.

Option 2: The scaling leaves invariant the tip of CD associated with the passive boundary so that it is not shifted at all but is scaled. This option is consistent with both option a) and b) for the localization of the experience of time flow. However, waking-up from sleep would take at the time when we fell asleep: this does not make sense.

The model for sleep favours option a)+1) for which CDs would define ever expanding sub-cosmologies changing the arrow of time repeatedly. Any conscious entity would eventually evolve to a cosmology, a kind of God-like conscious entity.

3. One can also consider other empirical inputs. There are stars and even galaxies older than the Universe. Their existence is consistent with option a)+1).

   CDs form a scaling hierarchy. CDs in the distant geometric past assignable to stars and galaxies are much smaller than the cosmological CD. The scaling cosmological CD inducing the time flow takes place much faster than the scaling of the much smaller astrophysical CDs. Cosmological time runs much faster and astrophysical CDs remain in the distant geometric past.

4. A third test is based on after images, which appear repeatedly. They correspond to sub-CDs of a CD. Could the after images correspond to life cycles of the *same* sub-CD as I have proposed? This is the case if the sub-CDs are comoving in the scalings of the CD shift. This looks rather natural.

9.5.2 More questions and objections related to ZEO and consciousness

The best way to make progress is to make questions and objections against the existing view, which is often far from clear. In the following I raise some questions of this kind.

What could BSFR mean biologically?

1. In have considered the possibility that BSFR could mean as biologically birth in opposite time direction. This however leads to rather complex speculations.

   The most natural assumption is that it means what it says, the emergence of a new CD [L172] as a perceptive field of a conscious entity. This does not require that biological death would be a birth in the opposite time direction although this cannot be excluded. This means one counter argument less.

2. I have considered the idea that in BSFR the size of a CD could decrease dramatically so that the reincarnated CD would be much smaller than before BSFR. This would make possible what one might call childhood. The idea is that the painful memories from the end of the lifecycle could be deleted. This model however requires rather detailed assumptions about how the memories of life cycle are stored at the active boundary of CD. The oldest memories would reside near the tip of CD and newest nearest to the intersection of the half-cones of the CD.

   Is this picture consistent with the view about SFR as a localization in the space of CDs? Since the number of CDs larger than given CD is much larger than those with size smaller than it, one can argue that the size of CD increases in statistical sense without limit in SFRs. If one can assume that death involves localization in the space-like degrees in the space of CDs (E^3 position and size of CD), the reduction of CD size looks rather implausible. If the preceding SSFR involved also this kind of localization then the CD after BSFR would in statistical sense be larger than it was before BSFR.

3. Can CDs interact? For instance, can a CD catch the sub-CD defining a mental image of the CD with which it overlaps? This is not the case: it is not possible to catch the spotlight of consciousness.

CD serves as a correlate for the perceptive field of self. Self is also an active causal agent. This aspect must relate to the zero energy states defined as superpositions of space-time surfaces inside a CD.

1. CD defines a perceptive field, a kind of spotlight of consciousness, which makes it possible to sensorily perceive the space-time surface, which continues outside CD although one can also imagine a situation in which this is not the case. Saying that mental image co-moves means that the spotlight moves.

2. Self has also causal powers. SSFRs change the state at the active boundary of CD. This induces changes inside the future light-cone in turn define perturbations of CDs of the geometric future possibly inducing BSFRs.

   Since the superposition of 3-surfaces at the active boundary of CD changes in SSFR, SSFRs have an effect on the geometric future. This is of course the case: our acts of free will affect the world around us but conform with causality.

   Almost deterministic holography for space-time surfaces and zero energy states dramatically reduces the freedom of free will due to state function reductions . The delocalization in WCW taking place in the space of CDS during the analogues of unitary time evolutions preceding SSFR improves the situation.

   One can also imagine a situation in which nothing changes at the boundaries of CD: self is completely passive: this is of course true at the passive boundary and can be true also at the active boundary in special situations. The classical time evolution for preferred extremals is not fully deterministic. Space-time surface is analogous to a 4-D soap film with frames and the case of 2-D soap films suggests that a finite non-determinism is assignable to the frames. This kind of SSFRs would not affect the space-time surface around CD at all. Pure cognition or meditative states might correspond to this kind of SSFRs.

The notion of ego is central in Eastern philosophies. How could one understand this notion in the ZEO based theory of consciousness?

1. Ego means that mental images want to survive. Self survival instinct is an analogous notion although it refers to the biological body. The quantum state at the passive boundary of the CD defines a good candidate for ego since it is indeed preserved during the sequence of SSFRs during which the set of measured observables is preserved.

2. BSFRs means death of self or subself as a sub-CD. Also the external physical perturbations arriving at the passive or active boundary can affect the quantum state at it and can induce BSFR. The self assignable to CD is exposed to perturbations, which might induce BSFR. A simple example of this kind of perturbation would be a blow in the head inducing a loss of consciousness.

   Ego preservation could mean that self does its best to make the periods of time with an opposite arrow of time as short as possible. This is not in conflict with the fact that the durations of sleep and awake states are roughly the same if a given arrow of time means that the time fraction spent in a state with this arrow of time dominates over that in a state with an opposite arrow of time.

   At the magnetic bodies carrying dark matter as phases with large h_{eff}, the interactions perturbing the boundaries of CD are expected to be rather weak. One has something analogous to a quantum computer isolated from the external world.

3. This suggests a more quantitative definition of the period with a fixed arrow of time. One expects that consciousness with a given arrow of time can have gaps. There is indeed empirical evidence suggesting that our flow of consciousness has gaps. Perhaps the wake-up-sleep ratio of the periods with different arrows of time is what matters. For a given arrow of time, the system would be dominantly in wake-up state or in sleep state.

   At a given level of self-hierarchy there is some average time for a given arrow of time and it is expected to increase at the higher levels. Magnetic bodies carrying dark matter interact only weakly with lower levels of the hierarchy, in particular ordinary matter, would make possible long periods with a given arrow of time, in the first guess proportional to say h_{eff}.

4. What could biological death as a process at the level of ordinary biomatter mean? Is biological death determined by the situation at the lower hierarchy levels? On the other hand, dark

matter at MBs defines a control hierarchy and is gradually thermalized as suggested in [L177] so that the ability to perform biocontrol is reduced. Also the ability to gain metabolic energy is reduced and makes it difficult to preserve the arrow of time. Since the average value of h_{eff} is reduced, the system becomes more vulnerable to perturbations inducing a BSFR changing the arrow of time.

There are also questions related to metabolism.

1. A metabolic energy feed is needed to preserve the distribution for the values of h_{eff}. The energies of quantum states increase with h_{eff} and in the absence of a metabolic energy feed, the values of h_{eff} at MBs tend to decrease. The system becomes more vulnerable to perturbations and the BSFRs changing the arrow of time occur more often. The system becomes drowsy.

2. Sun serves as a fundamental source of metabolic energy but TGD leads to a proposal that also radiation from the core of Earth, which happens to be at the same wavelength range as solar radiation could have served and maybe still serve as a source of metabolic energy.

3. I have proposed remote metabolism as a mechanism in which the system contains a subsystem with an opposite arrow which emits energy, say dissipates, in opposite time direction and thus seems to gain metabolic energy if seen from the standard arrow of time.

   This is possible if there is a system able to receive the *effective* negative energy signals. For instance, a population reversed laser could serve as such a system. The second option is that the environment loses thermal energy so that the second law in its standard form would be violated. For instance, heat could be transferred from a system with a given temperature to a system with higher temperature. The dissipation for the time reversed system looks like self-organization. Sleep periods would in this picture mean gain of metabolic resources and healing.

4. Also life with the opposite arrow of time needs metabolic energy. We receive metabolic energy basically from the Sun. Could the Sun serve as a source of metabolic energy also for the time reversed systems? The answer is positive.

   To understand why, one must clarify what the change of the arrow of time means. Time reversed signals have positive energy and only the reversed time direction makes them look like negative energy signals. The sum of energies for the sub systems with opposite arrows of time is conserved apart from effects due to finite sizes of CDs (Uncertainty Principle). Also life with an opposite arrow of time can use solar energy as a metabolic energy source.

5. The biological death is assumed to be due to the loss of quantum coherence at the level of MBs inducing a loss of ordinary coherence in short scales implying bodily decay. What could the situation be in the next reincarnation with the same arrow of time? Does the next life with the same arrow of time end at roughly the same time so that the size of the CD would become rather stationary. There would not be much progress.

   Or could the MB be able to preserve the quantum coherence for a longer time in the next reincarnation? Since the quantum coherence of MB naturally explains the coherence of the ordinary biomatter, impossible to understand in the standard physics framework, there is no reason why MB could not achieve this feat in the next incarnation.

9.5.3 Is Negentropy Maximization Principle needed as an independent principle?

The proposal has been that Negentropy Maximization Principle (NMP) [K70] [L135, L43] serves as the basic variation principle of the dynamics of conscious experience. NMP says that the information related to the contents of consciousness increases for the whole system even though it can decrease for the subsystem. Mathematically, NMP is very similar to the second law although it states something completely opposite. Second law follows from statistical physics and is not an independent physical law. Is the situation the same with the NMP? Is NMP needed at all as a fundamental principle or does it follow from number theoretic physics?

The number theoretic evolution is such a powerful principle that one must ask whether NMP is needed as a separate principle or whether it is a consequence of number theoretical quantum physics, just like the second law follows from ordinary quantum theory.

Two additional aspects are involved. Evolution can in adelic physics [L53] be seen as an unavoidable increase in the algebraic complexity characterized by the dimension $n = h_{eff}/h_0$ of extension of rationals associated with the polynomial define space-time surface at the fundamental level by socalled $M^8 - H$ duality [L109, L110]. There is also the possibility to identify a quantum correlate for ethics in terms of quantum coherence: a good deed corresponds to a creation of quantum coherence and the evil deed to its destruction.

How do these two aspects relate to the NMP? Is NMP an independent dynamical principle or a consequence of number theoretic (adelic) quantum physics?

Consider in the sequel "big" state function reduction (BSFR) as the counterpart of the ordinary state function reduction. I'm not completely sure whether the following arguments can be also applied to SSFRs for which the arrow of time does not change.

One can consider two alternative formulations for NMP.

Option I

Option I is the simpler and physically more plausible option.

1. BSFR divides the quantum entangled system at the active boundary of CD into two parts, which are analogous to the measurement apparatus and the measured system. The selection of this partition is completely free and decided by the system. This choice corresponds to an act of free will. Depending on conditions to be discussed, the action of the measurement to this pair can be trivial in which case the entanglement is not reduced. The measurement can also reduce the entanglement partially or completely and the p-adic entanglement negentropy and entropy decreases or becomes zero.

2. If the partition into two parts is completely free and if the choice is such that NMP, or whatever the principle in question is, allows BSFR, the quantum coherence decreases. Number theoretic evolution suggests that the principle telling when BSFR can occur is number theoretic.

   There is a cascade of BSFRs since BSFRs are also possible for the emerging untangled subsystem and its complement. The cascade stops when the entanglement becomes stable.

3. What condition could determine whether the reduction of the entanglement takes place? What could make the entanglement stable against BSFR?

   Number theoretical vision suggests an answer. Physical intuition suggests that bound states represent a typical example of stable quantum entanglement. Bound states correspond to Galois confined states [L156, L132, L147, L148] for which the momenta of fermions are algebraic integers in an extension of rationals but total momentum has integer valued components. This mechanism for the formation of the bound states would be universal.

   A natural number theoretical proposal is that the entanglement is stable if the entanglement probabilities obtained by diagonalizing the density matrix characterizing the entanglement belong to an extension of rationala, which is larger than the extension, call it E, defined by the polynomial P defining the space-time surface. An even stronger condition, inspired by the fact that cognition is based on rational numbers, is that BSFR can take place only if they are rational.

   This kind of entanglement would be outside the number system used and one can argue that this forces the stability of the entanglement. A weaker statement is that the reduction is possible to a subspace of the state space for which the entanglement probabilities belong to E (or are rational).

4. This option could replace NMP as a criterion with a purely number theoretical principle. This does not however mean that NMP would not be preserved as a principle analogous to the second law and implied by the number theoretic evolution implied by the hierarchy of extensions of rationals.

Could free will as the ability to do evil or good deeds reduce to number theory that is to the choice of a partition, which leads to either increase or decrease of entanglement negentropy and therefore of quantum coherence?

The basic objection can be formulated as a question. How can the conscious entity know whether a given choice of partition leads to BSFR or not? Memory must be involved. Only by making this kind of choices, a system with a memory can learn the outcome of a given choice. How could the self learn, which deeds are good and which are evil? The answer is suggested by the biologically motivated view of survival instinct and origin of ego [L172] based on SSFRs as a generalization of Zeno effect.

1. Conscious entity has a self characterized by the set of observables measured in the sequence of SSFRs. BSFR as a reduction of entanglement occurs when a new set of observables not commuting with the original set are measured. In BSFR self "dies" (loses consciousness). Second BSFR means reincarnation with the original arrow of time.

2. The perturbations of the system at both boundaries of CD are expected to induce BSFRs and to occur continually. Therefore the arrow of time is fixed only in the sense that it dominates over the opposite arrow.

3. Self preserves its identity (in particular memories defining it) if the second BSFR leads to a set of observables, which does not differ too much from the original one. The notions of survival instinct and ego would reduce to an approximate Zeno effect.

4. This mechanism would allow the self to learn the distinction between good and evil and also what is dangerous and what is not. A BSFR inducing only a brief period of life with a reversed arrow of time could teach the system when the BSFR leads to a reduction of entanglement and loss of coherence.

   The harmless BSFRs could provide a mechanism of imagination making survival possible. Intelligent systems could do this experimentation at the level of a self representation of a system rather than in real life and the development of complex self representations would distinguish higher life forms from those at a lower evolutionary level.

Option II

Option II is stronger than Option I but looks rather complex. I have considered it already before. NMP would select a partition for which the negentropy gain is maximal in BSFR or at least, the decrease of the negentropy is minimal. One must however define what one means with negentropy gain.

Before considering whether this condition can be precise, it is good to list some objections.

1. Is the selection of this kind of optimal partition possible? How can the system know which partition is optimal without trying all alternatives? Doing this would reduce the situation to the first option.

2. Free will as ability do also evil deeds seems to be eliminated as a possibility to either increase or decrease entanglement negentropy and therefore quantum coherence by choosing the partition of the system so that it reduces negentropy.

3. If the BSFR cascade would lead to a total loss of quantum entanglement, the entanglement negentropy would always be zero and NMP would not say anything interesting. On the other hand, if the selection of the partition is optimal and the number theoretic criterion for the occurrence of the reduction holds true, it could imply that nothing happens for the entanglement. Again the NMP would be trivial.

4. What does one mean with the maximal negentropy gain?

What does one mean with a maximal negentropy gain?

Option II for NMP says that for a given partition BSFR occurs if the entanglement negentropy increases maximally. What does one mean with entanglement negentropy gain? This notion is also useful for Option I although it is not involved with the criterion.

1. Entanglement negentropy refers to the negentropy related to the *passive* edge of the CD (Zeno effect). Passive boundary involves negentropic entanglement because NMP does not allow a complete elimination of quantum entanglement (bound state entanglement is stable). The new passive boundary of CD emerging in the BSFR corresponds to the previously active boundary of CD.

2. For option I for which the concept of good/bad is meaningful, the number theoretical criterion could prevent BSFR and stop the BSFR cascade. There is however no guarantee that the total entanglement negentropy would increase in the entire BSFR cascade. This would make the term "NMP" obsolete unless NMP follows in a statistical sense from number theoretic evolution: this looks however plausible.

   The unavoidable increase of the number theoretical complexity would force the increase of p-adic entanglement negentropy and NMP as an analog of the second law would follow from the hierarchy of extensions of rationals.

9.6 Appendix: About the action of $SL(2,R)$ inside causal diamond

$SL(2,R)$ appearing as conformal symmetries of conformal field theories and mapping the upper half plane of the complex plane to itself by Möbius transformations. I have collected the basic facts about how SL(2,R) is realized for causal diamonds. These facts can be found also from [B56].

The Lie algebra of $SL(2,R)$ is spanned by the generator $D_0 r\partial_r + t\partial_t$ of scaling with respect to the center point of CD, by the generator $P_0 = \partial_t$ of time translation in the direction defined by the line connecting the tips of CD, and by the generator $K_0 = 2tr\partial_r + (t^2+r^2)\partial_t = 2tD_0 + (r^2-t^2)P_0$ of a special conformal transformation in time direction obtained as IP_0I, where I is the inversion with respect to the center point of CD. Conformal Killing vector is linear combination of form

$$\xi = aK_0 + bD_0 + cT_0 \ . \tag{9.6.1}$$

For $b = 0$ the center of point of the CD is at origin ($t = 0, r = 0$). The generators obey the Lie-algebra

$$[P_0, D_0] = P_0 \ , \quad [K_0, D_0] = -K_0 \ , \quad [P_0, K_0] = m2D_0 \ . \tag{9.6.2}$$

The time evolution of conformal QM is defined by a Hamiltonian which in its most general form is given as $G = i(uP_0 + vD_0 + wK_0)$. The conformal Hamiltonians G can be classified by the Casimir invariant

$$C = D_0^2 - \frac{1}{2}(K_0P_0 + P_0K_0) \ . \tag{9.6.3}$$

This corresponds to the invariance of the determinant of the matrix $(b, 2c; 2a, b$ given by $\Delta = b^2 - 4ac$.

1. For $\Delta < 0$ one has generators of elliptic transformations analogous to rotations in the Lorentz group.

$$R = \frac{1}{2}(\alpha P_0 + \frac{K_0}{\alpha}) \ . \tag{9.6.4}$$

is a representative of this class. One has $\alpha = L$, where L is the radius of CD defined by the maximal radial distance from the time axis of CD. In the sequel will use the notation α used also in [B56] The distance between the tips of the CD is $2L$. The radial conformal Killing vector is everywhere time-like.

2. Generators with $\Delta = 0$ generate parabolic transformations, null rotations. Also now the radial conformal Killing vector is time-like everywhere except for the light-cone emanating from $t = -b/2a$, $r = 0$

3. Generators with $\Delta > 0$ generate hyperbolic transformations analogous to Lorentz group perspective. Dilation D and the generator

$$S_0 = \frac{1}{2}(\alpha P_0 - \frac{K_0}{\alpha}) \ . \tag{9.6.5}$$

serves as a representative for this class. In this case the conformal Killing vector is null at the tips of the light-cone $(t = t_\pm, r = 0)$, $t_\pm 0 - b + \sqrt{\Delta}/2a$, time-like inside either light-cone or outside both light-cones, and space-like everywhere else.

One can assign to S_0 *resp.* R_0 time coordinates T *resp.* τ in such a way that one has

$$R_0 = \partial_T \ \ , \quad S_0 = \partial_\tau \ . \tag{9.6.6}$$

These time coordinates are related to Minkowski time t by

$$t = \alpha tan(T/2) = \alpha tanh(\tau/2) \ . \tag{9.6.7}$$

One finds that the range $(-\alpha, \alpha)$ for t corresponds to the range $(-\infty, \infty)$ for τ and to the range $(-\pi, \pi)$ for T.

S_0 has a representation as a differential operator

$$S_0 = \frac{1}{2\alpha}[(\alpha^2 - t^2 - r^2)\partial_t - 2tr\partial_r] \ . \tag{9.6.8}$$

S_0 maps CD identifiable as the region $|t| + vertr| < \alpha$ to itself.

One can identify so-called diamond coordinates via the formulas

$$t = \alpha\frac{sinh(\tau)}{cosh(x)+cosh(\tau)} \ \ , \quad r = \frac{sinh(x)}{cosh(x)+cosh(\tau)} \ . \tag{9.6.9}$$

The ratio of the equations gives $t/r = sinh(\tau)/sinh(x)$. For small values of τ and x this gives $(t \simeq \alpha\tau, r \simeq \alpha x$. From $t = \alpha tanh(\tau/2)$ one can solve $sinh(\tau)$ and use it to the expression of t/r to get

$$sinh(x) = 2\frac{t}{\alpha}/(1 - (\frac{t}{\alpha})^2) \ . \tag{9.6.10}$$

$sinh(x)$ is constant for the flow lines.

In diamond coordinates, the line element of Minkowski metric reads as

$$ds^2 = \frac{\alpha^2}{(cosh(x)+cosh(\tau))^2}(-d\tau^2 + dx^2 + sinh(x)^2\Omega^2) \ . \tag{9.6.11}$$

and the flow lines correspond to a particle at rest. Diamond coordinates are analogous to the natural coordinates of a freely falling system.

The integral curves of the Killing flows defined by S_0 are obtained as

$$t^2 - (r - \alpha\omega)^2 = \alpha^2(1 - \omega^2) \ \ , \quad \omega = \frac{1}{tanh(x)} \ . \tag{9.6.12}$$

ω is constant along these curves and varies in the range $[1, \infty)$. This equation should be equivalent with the equation already obtained. The integral curves correspond to a relativistic motion with constant acceleration given by $a(x) = sinh(x)/\alpha$ which is constant along each integral curve for which x indeed remains constant. For the line passing through origin one has $a = 0$.

Chapter 10

Number theoretic vision, Hyper-finite Factors and S-matrix

10.1 Introduction

Quantum criticality has been the key idea from beginning but its understanding has evolved rather slowly. Quantum criticality accompanies several hierarchies: hierarchy of p-adic length scales and hierarchy of space-time sheets glued to larger space-timer sheets; hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter; hierarchy of breakings of super-symplectic symmetry represented as gauge symmetry; hierarchy of causal diamonds (CDs); hierarchies of inclusions of hyperfinite factors of type II_1 (HFFs); hierarchies of extensions of extensions of of rationals emerging in M^8 picture about TGD; hierarchies of conscious entities with lower level represented as mental images.

10.1.1 Zero energy ontology and the interpretation of light-like 3-surfaces as generalized Feynman diagrams

Zero energy ontology (ZEO) is discussed in [K130] but its role in the construction of scattering amplitudes deserves a brief discussion also here.

1. ZEO is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X_l^3 connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

2. Either space-like 3-surfaces at the boundaries of CDs or light-like 3-surfaces connecting the boundaries of CDs can be seen as the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

3. String word sheets and partonic 2-surfaces however appear also in TGD as carriers of spinor modes: this follows from the condition that em charge is well defined for the modes. The condition follows also from number theoretic arguments and is assumed quite generally. This

has far reaching consequences for the understanding of gravitation in TGD framework and profound deviations from string models are predicted due to the hierarchy of Planck constants absolutely essential for the description of gravitational bound states in terms of strings connecting partonic 2-surfaces. Macroscopic quantum coherence in even astrophysical scales is predicted [?, K84].

10.1.2 About the identification of various TGD counterparts of S-matrix

The identification of the TGD counterpart of S-matrix is the key topic of this chapter. What this matrix actually means is far from obvious.

1. One can characterize zero energy state by a "square root" of density matrix which is product of hermitian matrix and unitary matrix: I have called this matrix M-matrix. The unitary matrix related to the M-matrix could relate closely to the S-matrix assigned with particle reactions.
2. One can assign the analog of unitary S-matrix to "small" state function reductions (SSFRs) defining the TGD counterparts of "weak" measurements. The states at the passive boundary PB are unaffected, which has interpretation as the TGD counterpart of Zeno effect. This S-matrix could relate to the evolution of self as a conscious entity and to its cognitive time evolution [L121, L142].
3. One can also assign an S-matrix like entity to "big" SFRs (BSFRs) in which the arrow of time changes. This S-matrix would be the counterpart of the ordinary S-matrix and should closely relate to the M-matrix.
4. I have also introduced the notion of U-matrix, which would be defined between zero energy states without fixing states at the passive boundary essential for fixing the arrow of time. This notion has remained somewhat misty and it seems that this notion is not needed since the matrices assigned SSFRs and BSFRs indeed are between zero energy states.

The TGD counterpart of S-matrix - call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD.

1. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several arguments suggesting that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.
2. In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics or at least formalism resembling thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

In the chapter "*Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory*" [L124] of this book, the idea that scattering amplitudes could allow a geometrization in terms of the Kähler metric of WCW is considered. The role of M^8-H duality in the construction of scattering amplitudes as M-matrix is discussed in chapter "*Breakthrough in understanding of M^8-H duality*" [K20] of this book. The idea would be that the descriptions at the level of M^8 and H provide momentum-space and space-time descriptions of the scattering amplitudes.

10.1.3 Topics of the chapter

The first sections provide conceptual background for the attempts to identify scattering amplitudes in TGD framework. The other chapters discuss more detailed attempts.

$M^8 - H$ duality, hierarchy of Planck constants, and p-adic length scale hypothess

$M^8 - H$ duality provides a cornerstone of TGD and one can consider the construction of scattering amplitudes both at M- and H-level. This motivates the discussion of in the section "*About relationship between $M^8 - H$ duality, hierarchy of Planck constants, and p-adic length scale hypothesis*".

The meaning of $M^8 - H$ duality in fermionic sector is considered in the section "*Fermionic variant of $M^8 - H$ duality*". The role of second quantized spinors in H is well-understood but in M^8 the situation is different. The basic guideline is that also fermionic dynamics at the level of M^8 should be algebraic and number theoretical.

1. Spinors should be octonionic. I have already earlier considered their possible physical interpretation [L18].

2. Dirac equation as linear partial differential equation should be replaced with a linear algebraic equation for octonionic spinors which are complexified octonions. The momentum space variant of the ordinary Dirac equation is an algebrac equation and the proposal is obvious: $P\Psi = 0$, where P is the octonionic continuation of the polynomial defining the space-time surface and multiplication is in octonionic sense. The conjugation in O_c is induced by the conjugation of the commuting imaginary unit i. The square of the Dirac equation is real if the space-time surface corresponds to a projection $O_c \rightarrow M^8 \rightarrow M^4$ with real time coordinate and imaginary spatial coordinates so that the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

   The masslessness condition restricts the solutions to light-like 3-surfaces $m_{kl}P^kP^l = 0$ in Minkowskian sector analogous to mass shells in momentum space - just as in the case of ordinary massless Dirac equation. $P(o)$ rather than octonionic coordinate o would define momentum. These mass shells should be mapped to light-like partonic orbits in H.

3. This picture leads to the earlier phenomenological picture about induced spinors in H. Twistor Grassmann approach suggests the localization of the induced spinor fields at light-like partonic orbits in H. If the induced spinor field allows a continuation from 3-D partonic orbits to the interior of X^4, it would serve as a counterpart of virtual particle in accordance with quantum field theoretical picture.

Hyper-finite factors and M-matrix

The notion of hyper-finite factor is expected to play central role in the mathematical description of finite measurement resolution, in the realization of the hierarchy of Planck constants [K43, **?**], the hierarchy quantum criticalities, and the hierarchy of gauge symmetry breakings for the super-symplectic algebra. This motivates the discussion of the basic results and ideas are about HFFs. The views about M-matrix as a characterizer of time-like entanglement and M-matrix as a functor are analyzed. The role of hyper-finite factors in the construction of M-matrix is considered. One section is devoted to the possibility that Connes tensor product could define fundamental vertices. A more detailed discussion can be found in the book [K54], in particular in chapter [K126].

I do not pretend of having handle about the huge technical complexities and can only recommend the works of von Neumann [A81, A96, A87, A69]. Tomita [A80]. [B45, B16, B46]. the work of Powers and Araki and Woods which served as starting point for the work of Connes [A28, A27]. The work of Jones [A56], and other leading figures in the field. What is may main contribution is fresh physical interpretation of this mathematics which also helps to make mathematical conjectures. The book of Connes [A28] available in web provides an excellent overall view about von Neumann algebras and non-commutative geometry.

The role of HFFs in the construction of M-matrix is considered in the section "*A vision about the role of HFFs in TGD*".

Number theoretic approch to the S-matrix associated with SFRs

Adelic physics, $M^8 - H$ duality, and ZEO to a proposal that the dynamics involved with "small" state function reductions (SSFRs) as counterparts of "weak" measurements could be basically number theoretical dynamics with SSFRs identified as SFR cascades leading to completely unentangled state in the space of wave functions in Galois group of extension of rationals identifiable as wave functions in the space of cognitive representations. This is discussed in the section "it The dynamics of SSFRs as quantum measurement cascades in the group algebra of Galois group" [L121].

As a side product a prime factorization of the order of Galois group is obtained. The result looks even more fascinating if the cognitive dynamics is a representation for the dynamics in real degrees of freedom in finite resolution characterized by the extension of rationals. If cognitive representations represent reality approximately, this indeed looks very natural and would provide an analog for adele formula expressing the norm of a rational as the inverse of the product of is p-adic norms. The results can be appplied to the TGD inspired model of genetic code.

The last section "it The relation between U-Matrix and M-matrices" includes some old and perhaps obsolete speculations about the admittedly misty U-matrix. The negative and positive energy parts of zero energy state can contain zero energy parts in shorter scales - quantum field theorist might talk about quantum fluctuations. One can have also U-matrix and M-matrix elements between this kind of states and even between zero energy states and a hierarchy suggests itself. Since fermions could be seen as correlates of Boolean cognition and zero energy states in fermion sectors as quantal Boolean statements, one can ask whether these matrices could define Boolean hierarchies: statements about statements about...

10.2 About M^8-H-duality, p-adic length scale hypothesis and dark matter hierarchy

M^8-H duality, p-adic length scale hypothesis and dark matter hierarchy as phases of ordinary matter with effective Planck constant $h_{eff}=nh_0$ are basic assumptions of TGD, which all reduce to number theoretic vision. In the sequel M^8-H duality, p-adic length scale hypothesis and dark matter hierarchy are discussed from number theoretic perspective.

Several new results emerge. Strong form of holography (SH) allows to weaken strong form of M^8-H duality mapping space-time surfaces $X^4\subset M^8$ to $H=M^4\times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4\subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained.

M^8 duality allows to relate p-adic length scales L_p to differences for the roots of the polynomial defining the extension defining "special moments in the life of self" assignable causal diamond (CD) central in zero energy ontology (ZEO). Hence p-adic length scale hypothesis emerges both from p-adic mass calculations and M^8-H duality. It is proposed that the size scale of CD correspond to the largest dark scale nL_p for the extension and that the sub-extensions of extensions could define hierarchy of sub-CDs. Skyrmions are an important notion if nuclear and hadron physics, M^8-H dyality suggests an interpretation of skyrmion number as winding number as that for a map defined by complex polynomial.

10.2.1 Some background

A summary of the basic notions and ideas involved is in order.

p-Adic length scale hypothesis

In p-adic mass calculations [K65] real mass squared is obtained by so called canonical identification from p-adic valued mass squared identified as analog of thermodynamical mass squared using p-adic generelization of thermodynamics assuming super-conformal invariance and Kac-Moody algebras assignable to isometries ad holonomies of $H=M^4\times CP_2$. This implies that the mass squared is essentially the expectation value of sum of scaling generators associated with various tensor factors of the representations for the direct sum of super-conformal algebras and if the number of factors is 5 one obtains rather predictive scenario since the p-adic temperature T_p must be inverse integer in order that the analogs of Boltzmann factors identified essentially as p^{L_0/T_p}.

The p-adic mass squared is of form $Xp+O(p^2)$ and mapped to $X/p+O(1/p^2)$. For the p-adic primes assignable to elementary particles ($M_{127}=2^{127}-1$ for electron) the higher order corrections are in general extremely small unless the coefficient of second order contribution is larger integer of order p so that calculations are practically exact.

Elementary particles seem to correspond to p-adic primes near powers 2^k. Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of period doubling in which prime $p=2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p=3$ (period tripling as approach to chaos). Many elementary particles and also hadron physics and electroweak physics seem to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2.

For given prime p also higher powers of p define p-adic length scales: for instance, for electron the secondary p-adic time scale is .1 seconds characterizing fundamental bio-rhythm. Quite generally, elementary particles would be accompanied by macroscopic length and time scales perhaps assignable to their magnetic bodies or causal diamonds (CDs) accompanying them.

This inspired p-adic length scale hypothesis stating the size scales of space-time surface correspond to primes near half-octaves of 2. The predictions of p-adic are exponentially sensitive to the value of k and their success gives strong support for p-adic length scale hypothesis. This hypothesis applied not only to elementary particle physics but also to biology and even astrophysics and cosmology. TGD Universe could be p-adic fractal.

Dark matter as phases of ordinary matter with $h_{eff}=nh_0$

The identification of dark matter as phases of ordinary matter with effective Planck constant $h_{eff}=nh_0$ is second key hypothesis of TGD. To be precise, these phases behave like dark matter and galactic dark matter could correspond to dark energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes.

There are good arguments in favor of the identification $h=6h_0$ [L32, L64]. "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining space-time surface as a covering. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

M^8-H duality

M^8-H duality ($H=M^4\times CP_2$) [L85] has taken a central role in TGD framework. M^8-H duality allows to identify space-time regions as "roots" of octonionic polynomials P in complexified M^8 - M^8_c - or as minimal surfaces in $H=M^4\times CP_2$ having 2-D singularities.

Remark:O_c,H_c,C_c,R_c will be used in the sequel for complexifications of octonions, quaternions, etc.. number fields using commuting imaginary unit i appearing naturally via the roots of real polynomials.

The precise form of M^8-H duality has however remained unclear. Two assumptions are involved.

1. Associativity stating that the tangent or normal space of at the point of the space-time space-time surface M^8 is associative - that is quaternionic. There are good reasons to believe that this is true for the polynomial ansatz everywhere but there is no rigorous proof.

2. The tangent space of the point of space-time surface at points mappable from M^8 to H must contain fixed $M^2\subset M^4\subset M^8$ or an integrable distribution of $M^2(x)$ so that the 2-surface of M^4 determined by it belongs to space-time surface.

The strongest, global form of M^8-H duality states that $M^2(x)$ is contained to tangent spaces of X^4 at all points x. Strong form of holography (SH) states allows also the option for which this holds true only for 2-D surfaces - string world sheets and partonic 2-surfaces - therefore mappable to H and that SH allows to determined $X^4\subset H$ from this data. In the following a realization of this weaker form of M^8-H duality is found. Note however that one cannot exclude the possibility that also associativity is true only at these surfaces for the polynomial ansatz.

Number theoretic origin of p-adic primes and dark matter

There are several questions to be answered. How to fuse real number based physics with various p-adic physics? How p-adic length scale hypothesis and dark matter hypothesis emerge from TGD?

The properties of p-adic number fields and the strange failure of complete non-determinism for p-adic differential equations led to the proposal that p-adic physics might serve as a correlate for cognition, imagination, and intention. This led to a development of number theoretic vision which I call adelic physics. A given adele corresponds to a fusion of reals and extensions of various p-adic number fields induced by a given extension of rationals.

The notion of space-time generalizes to a book like structure having real space-time surfaces and their p-adic counterparts as pages. The common points of pages defining is back correspond to points with coordinates in the extension of rationals considered. This discretization of space-time surface is in general finite and unique and is identified as what I call cognitive representation. The Galois group of extension becomes symmetry group in cognitive degrees of freedom. The ramified primes of extension are exceptionally interesting and are identified as preferred p-adic primes for the extension considered.

The basic challenge is to identify dark scale. There are some reasons to expect correlation between p-adic and dark scales which would mean that the dark scale would depend on ramified primes, which characterize roots of the polynomial defining the extensions and are thus not defined completely by extension alone. Same extension can be defined by many polynomials. The naïve guess is that the scale is proportional to the dimension n of extension serving as a measure for algebraic complexity (there are also other measures). p-Adic length scales L_p would be proportional nL_p, p ramified prime of extension? The motivation would be that quantum scales are typically proportional to Planck constant. It turns out that the identification of CD scale as dark scale is rather natural.

10.2.2 New results about $M^8 - H$ duality

In the sequel some new results about M^8-H duality are deduced. Strong form of holography (SH) allows to weaken the assumptions making possible $M^8 - H$ duality. It would be enough to map only certain complex 2-D sub-manifolds of quaternionic space-time surface in M^8 to H: SH would allow to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds would be determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and they form a discrete set.

Strong form of holography (SH)

Ordinary 3-D holography is forced by general coordinate invariance (GCI) and loosely states that the data at 3-D surfaces allows to determined space-time surface $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

This poses additional strong conditions on the space-time surface.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SC_n with radial conformal weights coming as n-multiples of those for the entire algebra SC and its commutator $[SC_n, SC]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations.

   This hierarchy of minimal surfaces would naturally corresponds to the hierarchy of extensions of rationals with n identifiable as dimension of the extension giving rise to effective Planck constant. At the level of Hilbert spaces the inclusion hierarchies for extensions could also correspond to the inclusion hierarchies of hyper-finite factors of type I_1 [K126] so that M^8-H duality would imply beautiful connections between key ideas of TGD.

2. Second conjecture is that the preferred extremals (PEs) are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor structure from the product of twistor spaces of M^4 and CP_2 to 6-D

S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [A63] so that TGD is unique.

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated either with

- light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian or
- the space-like 3-surfaces at the ends of CD to determine space-time surface as PE (in case that it exists).

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken strong form of $M^8 - H$ duality mapping space-time surfaces $X^4 \subset M^8$ to $H = M^4 \times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained.

Space-time as algebraic surface in M^8_c regarded complexified octonions

The octonionic polynomial giving rise to space-time surface as its "root" is obtained from ordinary real polynomial P with rational coefficients by algebraic continuation. The conjecture is that the identification in terms of roots of polynomials of even real analytic functions guarantees associativity and one can formulate this as rather convincing argument [?] Space-time surface X^4_c is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. These options correspond to complexified-quaternionic tangent- or normal spaces. For $P(x) = x^n + ..$ ordinary roots are algebraic integers. The real 4-D space-time surface is projection of this surface from M^8_c to M^8. One could drop the subscripts "$_c$" but in the sequel they will be kept.

M^4_c appears as a special solution for any polynomial P. M^4_c seems to be like a universal reference solution with which to compare other solutions.

One obtains also brane-like 6-surfaces as 6-spheres as universal solutions. They have M^4 projection, which is a piece of hyper-surface for which Minkowski time as time coordinate of CD corresponds to a root $t = r_n$ of P. For monic polynomials these time values are algebraic integers and Galois group permutes them.

One cannot exclude rational functions or even real analytic functions in the sense that Taylor coefficients are octonionically real (proportional to octonionic real unit). Number theoretical vision - adelic physics [?, ?] suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

How do the solutions assignable to the opposite boundaries of CD relate to each other?

CD has two boundaries. The polynomials associated with them could be different in the general formulation discussed in [L113, L121] but they could be also same. How are the solutions associated with opposite boundaries of CD glued together in a continuous manner?

1. The polynomials assignable to the opposite boundaries of CD are allowed to be polynomials of o *resp.* $(o - T)$: here T is the distance between the tips of CD.
2. CD brings in mind the realization of conformal invariance at sphere: the two hemispheres correspond to powers of z and $1/z$: the condition $z = \overline{1/z}$ at unit circle is essential and there

is no real conjugation. How the sphere is replaced with 8-D CD which is also complexified. The absence of conjugation looks natural also now: could CD contain a 3-surface analogous to the unit circle of sphere at which the analog of $z = \overline{1/z}$ holds true? If so, one has $P(o,z) = P(1/o,z)$ and the solutions representing roots fo $P(o,z)$ and $P(1/o,z)$ can be glued together.

Note that $1/o$ can be expressed as $\overline{o}/o\overline{o}$ when the Minkowskian norm squared $\overline{o}o$ is non-vanishing and one has polynomial equation also now. This condition is true outside the boundary of 8-D light-cone, in particular near the upper boundary of CD.

The counter part for the length squared of octonion in Minkowskian signature is light-one proper time coordinate $a^2 = t^2 - r^2$ for M^8_+. Replacing o which scaled dimensionless variable $o_1 = o/(T/2)$ the gluing take place along $a = T/2$ hyperboloid.

One has algebraic holomorphy with respect to o but also anti-holomorphy is possible. What could these two options correspond to? Could the space-time surfaces assignable to self and its time-reversal relate by octonionic conjugation $o \to \overline{o}$ relating two Fock vacuums annihilated by fermionic annihilation *resp.* creation operators?

In [L113, L121] the possibility that the sequence of SSFRs or BSFRs could involve iteration of the polynomial defining space-time surface - actually different polynomials were allowed for two boundaries. There are 3 options: each SSFR would involve the replacement $Q = P \circ .. \circ P \to P \circ Q$, the replacement occurs only when new "special moments in the life of self" defined by the roots of P as $t = r_n$ balls of cd, or the replacement can occur in BSFR when the metabolic resources do not allow to continue the iteration (the increase of h_{eff} during iteration increases the needed metabolic feed).

The iteration is compatible with the proposed picture. The assumption $P(0) = 0$ implies that iterates of P contain also the roots of P as roots - they are like conserved genes. Also the 8-D light-cone boundary remains invariant under iteration. Even more general function decompositions $P \to Q \to P$ are consistent with the proposed picture.

Brane-like solutions

One obtains also 6-D brane-like solutions to the equations.

1. In general the zero loci for imaginary or real part are 4-D but the 7-D light-cone δM^8_+ of M^8 with tip at the origin of coordinates is an exception [L49, L50, L51]. At δM^8_+ the octonionic coordinate o is light-like and one can write $o = re$, where 8-D time coordinate and radial coordinate are related by $t = r$ and one has $e = (1 + e_r)/\sqrt{2}$ such that one as $e^2 = e$.

   Polynomial $P(o)$ can be written at δM^8_+ as $P(o) = P(r)e$ and its roots correspond to 6-spheres S^6 represented as surfaces $t_M = t = r_N$, $r_M = \sqrt{r_N^2 - r_E^2} \leq r_N$, $r_E \leq r_N$, where the value of Minkowski time $t = r = r_N$ is a root of $P(r)$ and r_M denotes radial Minkowski coordinate. The points with distance r_M from origin of $t = r_N$ ball of M^4 has as fiber 3-sphere with radius $r = \sqrt{r_N^2 - r_E^2}$. At the boundary of S^3 contracts to a point.

2. These 6-spheres are analogous to 6-D branes in that the 4-D solutions would intersect them in the generic case along 2-D surfaces X^2. The boundaries $r_M = r_N$ of balls belong to the boundary of M^4 light-cone. In this case the intersection would be that of 4-D and 3-D surface, and empty in the generic case (it is however quite not clear whether topological notion of "genericity" applies to octonionic polynomials with very special symmetry properties).

3. The 6-spheres $t_M = r_N$ would be very special. At these 6-spheres the 4-D space-time surfaces X^4 as usual roots of $P(o)$ could meet. Brane picture suggests that the 4-D solutions connect the 6-D branes with different values of r_n.

   The basic assumption has been that particle vertices are 2-D partonic 2-surfaces and light-like 3-D surfaces - partonic orbits identified as boundaries between Minkowskian and Euclidian regions of space-time surface in the induced metric (at least at H level) - meet along their 2-D ends X^2 at these partonic 2-surfaces. This would generalize the vertices of ordinary Feynman diagrams. Obviously this would make the definition of the generalized vertices mathematically elegant and simple.

Note that this does not require that space-time surfaces X^4 meet along 3-D surfaces at S^6. The interpretation of the times t_n as moments of phase transition like phenomena is suggestive. ZEO based theory of consciousness suggests interpretation as moments for state function reductions analogous to weak measurements ad giving rise to the flow of experienced time.

4. One could perhaps interpret the free selection of 2-D partonic surfaces at the 6-D roots as initial data fixing the 4-D roots of polynomials. This would give precise content to strong form of holography (SH), which is one of the central ideas of TGD and strengthens the 3-D holography coded by ZEO alone in the sense that pairs of 3-surfaces at boundaries of CD define unique preferred extremals. The reduction to 2-D holography would be due to preferred extremal property realizing the huge symplectic symmetries and making M^8-H duality possible as also classical twistor lift.

   I have also considered the possibility that 2-D string world sheets in M^8 could correspond to intersections $X^4\cap S^6$? This is not possible since time coordinate t_M constant at the roots and varies at string world sheets.

   Note that the compexification of M^8 (or equivalently octonionic E^8) allows to consider also different variants for the signature of the 6-D roots and hyperbolic spaces would appear for $(\epsilon_1, \epsilon_i, .., \epsilon_8)$, $epsilon_i=\pm 1$ signatures. Their physical interpretation - if any - remains open at this moment.

5. The universal 6-D brane-like solutions S^6_c have also lower-D counterparts. The condition determining X^2 states that the C_c-valued "real" or "imaginary" for the non-vanishing Q_c-valued "real" or "imaginary" for P vanishes. This condition allows universal brane-like solution as a restriction of O_c to M^4_c (that is CD_c) and corresponds to the complexified time=constant hyperplanes defined by the roots $t=r_n$ of P defining "special moments in the life of self" assignable to CD. The condition for reality in R_c sense in turn gives roots of $t=r_n$ a hyper-surfaces in M^2_c.

Explicit realization of M^8-H duality

M^8-H duality allows to map space-time surfaces in M^8 to H so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D singularities in H satisfying an infinite number of additional conditions stating vanishing of Noether charges for super-symplectic algebra actings as isometries for the "world of classical worlds" (WCW). Twistor lift allows variants of this duality. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of extensions of rationals defining an evolutionary hierarchy. This forms the basis for the number theoretical vision about TGD.

M^8-H duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions.

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of M^8-H duality and means that tangent - or normal space is quaternionic.

2. The tangent space of space-time surface and thus space-time surface itself must contain a preferred $M^2_c\subset M^4_c$ or more generally, an integrable distribution of tangent spaces $M^2_c(x)$ and similar distribution of their complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c\subset X^4_c$ in R_c sense. $E^2_c(x)$ would correspond to partonic 2-surface.

One can imagine two realizations for this condition.

Option I: Global option states that the distributions $M^2_c(x)$ and $E^2_c(x)$ define slicing of X^4_c.

Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce space-time surfaces in H. This would be the minimal option.

That the selection between these options is not trivial is suggested by following.

1. For massless extremals (MEs, topological light rays) parameterized by light-like vector vector k defining $M^2 \subset M^2 \times E^2 \subset M^4$ at each point and by space-like polarization vector ϵ depending on single transversal coordinate of E^2 [K9].

2. CP_2 coordinates have an arbitrary dependence on both $u = k \cdot m$ and $w = \epsilon \cdot m$ and can be also multivalued functions of u and w. Single light-like vector k is enough to identify M^2. CP_2 type extremals having metric and Kähler form of CP_2 have light-like geodesic as M^4 projection defining M^2 and its complement E^2 in the normal space.

3. String like objects $X^2 \times Y^2 \subset M^4 \times CP_2$ are minimal surfaces and X^2 defines the distribution of $M^2(x) \subset M^4$. Y^2 ddefines the complement of this distribution.

   Option I is realized in all 3 cases. It is not clear whether M^2 can depend on position in the first 2 cases and also CP_2 point in the third case. It could be that only a discrete set of these string world sheets assignable to wormhole contacts representing massless particles is possible (**Option II**).

How these conditions would be realized?

1. The basic observation is that X^2c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M^2_c would be the simplest solution but also more general complex sub-manifolds $X^2_c \subset M^4_c$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

   These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. As found, there are also classes special global solutions for which the choice of M^2_c is global and does not depend on space-time point. The interpretation would be in terms of modes of classical massless fields characterized by polarization and momentum. If the identification of M^2_c is correct, these surfaces are however unstable against perturbations generating discrete string world sheets and orbits of partonic 2-surfaces having interpretation space-time counterparts of quanta. That fields are detected via their quanta was the revolutionary observation that led to quantum theory. Could quantum measurement induce the instability decomposing the field to quanta at the level of space-time topology?

3. One can generalize this condition so that it selects 1-D surface in X^2_c. By assuming that R_c-valued "real" or "imaginary" part of quaternionic part of P at this 2-surface vanishes. one obtains preferred M^1_c or E^1_c containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

   This option could be made possible by SH. SH states that preferred extremals are determined by data at 2-D surfaces of X^4. Even if the conditions defining X^2_c have only a discrete set of solutions, SH at the level of H could allow to deduce the preferred extremals from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. Associativity and existence of $M^2(x)$ would be required only at the 2-D surfaces.

4. I have proposed that physical string world sheets and partonic 2-surfaces appear as singularities and correspond to 2-D folds of space-time surfaces at which the dimension of the quaternionic tangent space degenerates from 4 to 2 [L83] [K9]. This interpretation is consistent with a book like structure with 2-pages. Also 1-D real and imaginary manifolds could be interpreted as folds or equivalently books with 2 pages.

   For the singular surfaces the dimension quaternionic tangent or normal space would reduce from 4 to 2 and it is not possible to assign CP_2 point to the tangent space. This does not of course preclude the singular surfaces and they could be analogous to poles of analytic function. Light-like orbits of partonic 2-surfaces would in turn correspond to cuts.

5. What could the normal space singularity mean at the level of H? Second fundamental form defining vector basis in normal space is expected to vanish. This would be the case for minimal surfaces.

   (a) String world sheets with Minkowskian signature (in M^4 actually) are expected to be minimal surfaces. In this case T matters and string world sheets could be mapped to H by M^8-H duality and SH would work for them.

   (b) The light-like orbits of partonic 2-surfaces with Euclidian signature in H would serve as analogs of cuts. N is expected to matter and partonic 2-surfaces should be minimal surfaces. Their branching of partonic 2-surfaces is thus possible and would make possible (note the analogy with the branching of soap films) for them to appear as 2-D vertices in H.

   The problem is to identify the pre-images of partonic 2-surfaces in M^8. The light-likeness of the orbits of partonic 2-surfaces (induced 4-metric changes its signature and degenerates to 3-D) should be important. Could light-likeness in this sense define the pre-images partonic orbits in M^8?

Remark: It must be emphasized that SH makes possible M^8-H correspondence assuming that also associativity conditions hold true only at partonic 2-surfaces and string world sheets. Thus one could give up the conjecture that the polynomial ansatz implies that tangent or normal spaces are associative. Proving that this is the case for the tangent/normal spaces of these 2-surfaces should be easier.

Does M^8-H duality relate hadron physics at high and low energies?

During the writing of this article I realized that M^8-H duality has very nice interpretation in terms of symmetries. For $H=M^4\times CP_2$ the isometries correspond to Poincare symmetries and color $SU(3)$ plus electroweak symmetries as holonomies of CP_2. For octonionic M^8 the subgroup $SU(3)\subset G_2$ is the sub-group of octonionic automorphisms leaving fixed octonionic imaginary unit invariant - this is essential for M^8-H duality. $SU(3)$ is also subgroup of $SO(6)\equiv SU(4)$ acting as rotation on $M^8=M^2\times E^6$. The subgroup of the holonomy group of $SO(4)$ for E^4 factor of $M^8=M^4\times E^4$ is $SU(2)\times U(1)$ and corresponds to electroweak symmetries. One can say that at the level of M^8 one has symmetry breaking from $SO(6)$ to $SU(3)$ and from $SO(4)=SU(2)\times SO(3)$ to $U(2)$.

This interpretation gives a justification for the earlier proposal that the descriptions provided by the old-fashioned low energy hadron physics assuming $SU(2)_L\times SU(2)_R$ and acting acting as covering group for isometries $SO(4)$ of E^4 and by high energy hadron physics relying on color group $SU(3)$ are dual to each other.

Skyrmions and M^8-H duality

I received a link (`https://tinyurl.com/ycathr3u`) to an article telling about research (`https://tinyurl.com/yddwhr2o`) carried out for skyrmions, which are very general condensed matter quasiparticles. They were found to replicate like DNA and cells. I realized that I have not clarified myself the possibility of skyrmions on TGD world and decided to clarify my thoughts.

1. What skyrmions are?

Consider first what skyrmions are.

1. Skyrmions are topological entities. One has some order parameter having values in some compact space S. This parameter is defined in say 3-ball such that the parameter is constant at the boundary meaning that one has effectively 3-sphere. If the 3rd homotopy group of S characterizing topology equivalence classes of maps from 3-sphere to S is non-trivial, you get soliton-llike entities, stable field configurations not deformable to trivial ones (constant value). Skyrmions can be assigned to space S which is coset space $SU(2)_L\times SU(2)_R/SU(2)_V$, essentially S^3 and are labelled by conserved integer-valued topological quantum number.

2. One can imagine variants of this. For instance, one can replace 3-ball with disk. $SO(3) = S^3$ with 2-sphere S^2. The example considered in the article corresponds to discretized situation in which one has magnetic dipoles/spins at points of say discretized disk such that spins have same direction about boundary circle. The distribution of directions of spin can give rise to skyrmion-like entity. Second option is distribution of molecules which do not have symmetry axis so that as rigid bodies the space of their orientations is discretized version of $SO(3)$. The field would be the orientation of a molecule of lattice and one has also now discrete analogs of skyrmions.

3. More generally, skyrmions emerge naturally in old-fashioned hadron physics, where $SU(2)_L \times SU(2)_R/SU(2)_V$ involves left-handed, right-handed and vectorial subgroups of $SO(4) = SU(2)_L \times SU(2)_R$. The realization would be in terms of 4-component field (π, σ), where π is charged pion with 3 components - axial vector - and σ which is scalar. The additional constraint $\pi \cdot \pi + \sigma^2 = constant$ defines 3-sphere so that one has field with values in S^3. There are models assigning this kind of skyrmion with nucleon, atomic nuclei, and also in the bag model of hadrons bag can be thought of as a hole inside skyrmion. These models seem to have something to do with reality so that a natural question is whether skyrmions might appear in TGD.

2. Skyrmion number as winding number

In TGD framework one can regard space-time as 4-surface in either octonionic M^8_c, c refers here to complexification by an imaginary unit i commuting with octonions, or in $M^4 \times CP_2$. For the solution surfaces M^8 has natural decomposition $M^8 = M^2 \times E^6$ and E^6 has $SO(6)$ as isometry group containing subgroup $SU(3)$ having automorphisms of octonions as subgroup leaving M^2 invariant. $SO(6) = SU(4)$ contains $SU(3)$ as subgroup, which has interpretation as isometries of CP_2 and counterpart of color gauge group. This supports $M^8 - H$ duality, whose most recent form is discussed in [L111].

The map $S^3 \to S^3$ defining skyrmion could be taken as a phenomenological consequence of $M^8 - H$ duality implying the old-fashioned description of hadrons involving broken $SO(4)$ symmetry (PCAC) and unbroken symmetry for diagonal group $SO(3)_V$ (CCV). The analog of $(\pi, sigma)$ field could correspond to a B-E condensate of pions $(\pi, sigma)$.

The obvious question is whether the map $S^3 \to S^3$ defining skyrmion could have a deeper interpretation in TGD framework. I failed to find any elegant formulation. One could however generalize and ask whether skyrmion like entities characterize by winding number are predicted by basic TGD.

1. In the models of nucleon and nuclei the interpretation of conserved topological skyrmion number is as baryon number. This number should correspond to the homotopy class of the map in question, essentially winding number. For polynomials of complex number degree corresponds to winding number. Could the degree $n = h_{eff}/h_0$ of polynomial P having interpretation as effective Planck constant and measure of complexity - kind of number theoretic IQ - be identifiable as skyrmion number? Could it be interpreted as baryon number too?

2. For leptons regarded as local 3 anti-quark composites in TGD based view about SUSY [L92] the same interpretation would make sense. It seems however that the winding number must have both signs. Degree is n is however non-negative.

   Here complexification of M^8 to M^8_c is essential. One an allow both holomorphic and anti-holomorphic continuations of real polynomials P (with rational coefficients) using complexification defined by commutative imaginary unit i in M^8_c so that one has polynomials $P(z)$ *resp.* $P(\overline{z})$ in turn algebraically continued to complexified octonionic polynomials $P(z, o)$ *resp.* $P(\overline{z}, o)$.

   Particles *resp.* antiparticles would correspond to the roots of octonionic polynomial $P(z, o)$ resp. $P(\overline{z}, o)$ meaning space-time geometrization of the particle-antiparticle dichotomy and would be conjugates of each other. This could give a nice physical interpretation to the somewhat mysterious complex roots of P.

3. More detailed formulation

To make this formulation more detailed on must ask how 4-D space-time surfaces correspond to 8-D “roots” for the “imaginary” (“real”) part of complexified octonionic polynomial as surfaces in M^8_c.

1. Equations state the simultaneous vanishing of the 4 components of complexified quaternion valued polynomial having degree n and with coefficients depending on the components of O_c, which are regarded as complex numbers $x + iy$, where i commutes with octonionic units. The coefficients of polynomials depend on complex coordinates associated with non-vanishing “real” (“imaginary”) part of the O_c valued polynomial.

2. To get perspective, one can compare the situation with that in catastrophe theory in which one considers roots for the gradient of potential function of behavior variables x^i. Potential function is polynomial having control variables as parameters. Now behavior variable correspond “imaginary” (“real”) part and control variables to “real” (“imaginary”) of octonionic polynomial.

   For a polynomial with real coefficients the solution divides to regions in which some roots are real and some roots are complex. In the case of cusp catastrophe one has cusp region with 3-D region of the parameter defined by behavior variable x and 2 control parameters with 3 real roots, the region in which one has one real root. The boundaries for the projection of 3-sheeted cusp to the plane defined by control variables correspond to degeneration of two complex roots to one real root.

   In the recent case it is not clear whether one cannot require the M^8_c coordinates for space-time surface to be real but to be in $M^8 = M^1 + iE^7$.

3. Allowing complex roots gives 8-D space-time surfaces. How to obtain real 4-D space-time surfaces?

   (a) One could project space-time surfaces to real M^8 to obtain 4-D real space-time surfaces. For M^8 this would mean projection to $M^1 + iE^7$ and in time direction the real part of root is accepted and is same for the root and its conjugate. For E^7 this would mean that imaginary part is accepted and means that conjugate roots correspond to different space-time surfaces and the notion of baryon number is realized at space-time level.

   (b) If one allows only real roots, the complex conjugation proposed to relate fermions and anti-fermions would be lost.

4. One can select for 4 complex M^8_c coordinates X^k of the surface and the remaining 4 coordinates Y^k can be formally solved as roots of n:th degree polynomial with dynamical coefficients depending on X^k and the remaining Y^k. This is expected to give rise to preferred extremals with varying dimension of M^4 and CP_2 projections.

5. It seems that all roots must be complex.

   (a) The holomorphy of the polynomials with respect to the complex M^8_c coordinates implies that the coefficients are complex in the generic point M^8_c. If so, all 4 roots are in general complex but do not appear as conjugate pairs. The naïve guess is that the maximal number of solutions would be n^4 for a given choice of M^8 coordinates solved as roots. An open question is whether one can select subset of roots and what happens at $t = r_n$ surfaces: could different solutions be glued together at them.

   (b) Just for completeness one can consider also the case that the dynamical coefficients are real - this is true in the E^8 sector and whether it has physical meaning is not clear. In this case the roots come as real roots and pairs formed by complex root and its conjugate. The solution surface can be divided into regions depending on the character of 4 roots. The n roots consist of complex root pairs and real roots. The members or complex root pairs are mapped to same point in E^8.

4. Could skyrmions in TGD sense replicate?

What about the observation that condensed matter skyrmions replicate? Could this have analog at fundamental level?

1. The assignment of conserved topological quantum number to the skyrmion is not consistent with replication unless the skyrmion numbers of outgoing states sum up to that of the initial state. If the system is open one can circumvent this objection. The replication would be like replication of DNA in which nucleotides of new DNA strands are brought to the system to form new strands.

2. It would be fascinating if all skyrmions would correspond to space-time surfaces at fundamental M^8 level. If so, skyrmion property also in magnetic sense could be induced by from a deeper geometric skyrmion property of the MB of the system. The openness of the system would be essential to guarantee conservation of baryon number. Here the fact that leptons and baryons have opposite baryon numbers helps in TGD framework. Note also ordinary DNA replication could correspond to replication of MB and thus of skyrmion sequences.

10.2.3 About p-adic length scale hypothesis and dark matter hierarchy

It is good to introduce first some background related to p-adic length scale hypothesis discussed in chapters of [K76] and dark matter hierarchy discussed in chapters [K55, K56], in particular in chatper [**?**].

General form of p-adic length scale hypothesis

The most general form of p-adic length scale hypothesis does not pose conditions on allowed p-adic primes and emerges from p-adic mass calculations [K26, K65, K78]. It has two forms corresponding to massive particles and massless particles.

1. For massive particles the preferred p-adic mass calculations based on p-adic thermodynamics predicts the p-adic mass squared m^2 to be proportional to p or its power- the real counterpart of m^2 is proportional to $1/p$ or its power. In the simplest case one has

$$m^2 = \frac{X}{p}\frac{\hbar}{L_0} \quad ,$$

where L_0 is apart from numerical constant the length R of CP_2 geodesic circle. X is a numerical constant not far from unity. $X \geq 1$ is small integer in good approximation. For instance for electron one has $x = 5$.

By Uncertainty Principle the Compton length of particle is characterizing the size of 3-surfaces assignable to particle are proportional to $\sqrt{p}$:

$$L_c(m) = \frac{\hbar}{m} = \sqrt{\frac{1}{X}}L_p \quad , \quad L_p = \sqrt{p}L_0 = \quad .$$

Here L_p is p-adic length scale and corresponds to minimal mass for given p-adic prime. p-Adic length scale would be would characterize the size of the 3-surface assignable to the particle and would correspond to Compton length.

2. For massless particles mass vanishes and the above picture is not possible unless there is very small mass coming from p-adic thermodynamics and determined by the size scale of CD - this is quite possible. The preferred time/spatial scales p-adic energy- equivalently 3-momentum are proportional to p-adic prime p or its power. The real energy is proportional to $1/p$. At the embedding space level the size of scale causal diamond (CD) [L91] would be proportional to p: $L = T = pL_0$, $L_0 = T_0$ for $c = 1$. The interpretation in terms of Uncertainty Principle is possible.

There would be therefore two levels: space-time level and embedding space level . At the space-time level the primary p-adic length scale would be proportional to $\sqrt{p}$ whereas the p-adic length scale at embedding space-time would correspond to secondary p-adic length scale proportional to p. The secondary p-adic length scales would assign to elementary new physics in macroscopic scales. For electron the size scale of CD would be about .1 seconds, the time scale associated with the fundamental bio-rhythm of about 10 Hz.

3. A third piece in the picture is adelic physics [L53, L54] inspiring the hypothesis that effective Planck constant h_{eff} given by $h_{eff}/h_0 = n$, $h = 6h_0$, labels the phases of ordinary matter identified as dark matter. n would correspond to the dimension of extension of rationals.

   The connection between preferred primes and the value of $n = h_{eff}/h_0$ is interesting. One proposal is that preferred primes p in p-adic length scale hypothesis determining the mass scale of particle correspond to so called ramified primes, which characterize the extensions. The p-adic variant of the polynomial defining space-time surfaces in M^8 picture would have vanishing discriminant in order $O(p)$. Since discriminant is proportional to the product of differences of different roots of the polynomial, two roots would be very near to each other p-adically. This would be mathematical correlate for criticality in p-adic sense.

   M^8-H duality [L85, L81] leads to the prediction that the roots r_n of polynomial defining the space-time region in M^8 correspond to preferred time values $t = t_n =\propto r_n$- I have called $t = t_n$ "special moments in the life of self". Since the squares for the differences for the roots are proportional to ramified primes, these time differences would code for ramified primes assignable to the space-time surface. There would be several p-adic time scales involved and they would be coded by $t_{ij} = r_i - r_j$, whose moduli squared are divided by so called ramified primes defining excellent candidates for preferred p-adic primes. p-Adic physics would make itself visible at the level of space-time surface in terms of "special moments in the life of self".

4. p-Adic length scales emerge naturally from M^8-H duality [L85, L81]. Ramified primes would in M^8 picture appear as factors of time differences associated with "special moments in the life of self" associated with CD [L81]. One has $|t_i - t_j| \propto \sqrt{p_{ij}}$, p_{ij} ramified prime. It is essential that square root of ramified prime appears here.

   This suggests strongly that p-adic length scale hypothesis is realized at the level of space-time surface and there are several p-adic length scales present coded to the time differences. Knowing of the polynomial would give information about p-adic physics involved. If dark scales correlate with p-adic length scales as proposed, the definition of dark scale should assume the dependence of ramified primes quite generally rather than as a result of number theoretic survival of fittest as one might also think.

   The factors $t_i - t_j$ are proportional - not only to the typically very large p-adic prime p_{max} charactering the system - but also smaller primes or their powers. Could the scales in question be of form $l_p = \sqrt{X}\sqrt{p_{max}}L_0$ rather than p-adic length scales $L_{p_{ram}}$ defined by various ramified primes. Here X would be integer consisting of small ramified primes.

   p-Adic mass calculations predict in an excellent approximation the mass of the particle is given by $m = (\sqrt{X}/\sqrt{p})m_0$, X small integer and $m_0 = 1/L_0$. Compton length would be given by $L_c(p) = \sqrt{p}/\sqrt{X})L_0$. The identification $l_p = L_c(p)$ would be attractive but is not possible unless one has $X = 1$. In this case one would be considering p-adic length scale L_p. the interpretation in terms of multi-p-adicity seems to be the realistic option.

About more detailed form of p-adic length scale hypothesis

More specific form of p-adic length scale hypothesis poses conditions on physically preferred p-adic primes. There are several guesses for preferred primes. They could be primes near to integer powers 2^k, where k could be positive integer, which could satisfy additional conditions such as being odd, prime or be associated with Mersenne prime or Gaussian Mersenne. One can consider also powers of other small primes such as $p = 2, 3, 5$. p-Adic length scale hypothesis in is basic form would generalize the notion of period doubling. For odd values of k one would indeed obtain period doubling, tripling, etc... suggesting strongly chaos theoretic origin.

1. p-Adic length scale hypothesis in its basic form

Consider first p-adic length scale hypothesis in its basic form.

1. In its basic form states that primes $p \simeq 2^k$ are preferred p-adic primes and correspond by p-adic mass calculations p-adic length scales $L_p \equiv L(k) \propto \sqrt{p} = 2^{k/2}$. Mersenne primes and primes associated with Gaussian Mersennes as especially favored primes and charged leptons ($k \in \{127, 113, 107\}$) and Higgs boson ($k = 89$) correspond to them. Also hadron physics ($k = 107$) and nuclear physics ($k = 113$) correspond to these scales. One can assign also to hadron physics Mersenne prime and the conjecture is that Mersennes and Gaussian Mersennes define scaled variants of hadron physics and electroweak physics. In the length scale between cell membrane thickness fo 10 nm and nuclear size about 2.5 μm there are as many as 4 Gaussian Mersennes corresponding to $k \in \{151, 157, 163, 167\}$.

   Mersenne primes correspond to prime values of k and I have proposed that k is prime for fundamental p-adic length scales quite generally. There are also however also other p-adic length scales - for instance, for quarks k need not be prime - and it has remained unclear what criterion could select the preferred exponents k. One can consider also the option that odd values of k defined fundamental p-adic length scales.

2. What makes p-adic length scale hypothesis powerful is that masses of say scaled up variant of hadron physics can be estimated by simple scaling arguments. It is convenient to use electron's p-adic length scale and calculate other p-adic length scales by scaling $L(k) = 2^{(k-127)/2}L(127)$.

Here one must make clear that there has been a confusion in the definitions, which was originally due to a calculational error.

1. I identified the p-adic length scale $L(151)$ mistakenly as $L(151) = 2^{(k-127)/2}L_e(127)$ by using instead of $L(127)$ electron Compton length $L_e \simeq L(127/\sqrt{5}$. The notation for these scales would be therefore $L_e(k)$ identified as $L_e(k) = 2^{(k-127)/2}L_e(127)$ and I have tried to use it systematically but failed to use the wrong notation in informal discussions.

2. This mistake might reflect highly non-trivial physics. It is scaled up variants of L_e which seem to appear in physics. For instance, $L_e(151) \simeq 10$ nm corresponds to basic scale in living matter. Why the biological important scales should correspond to scaled up Compton lengths for electron? Could dark electrons with scaled up Compton scales equal to $L_e(k)$ be important in these scales? And what about the real p-adic length scales relate to these scales by a scaling factor $\sqrt{5} \simeq 2.23$?

2. Possible modifications of the p-adic length scale hypothesis

One can consider also possible modifications of the p-adic length scale hypothesis. In an attempt to understand the scales associated with INW structures in terms of p-adic length scale hypothesis it occurred to me that the scales which do not correspond to Mersenne primes or Gaussian Mersennes might be generated somehow from the these scales.

1. Geometric mean $L = \sqrt{L(k_1)L(k_2)}$ would length scale which would correspond to L_p with $p \simeq 2^{(k_1+k_2)/2}$. This is of the required form only if $k = k_1 + k_2$ is even so that k_1 and k_2 are both even or odd. If one starts from Mersennes and Gaussian Mersennes the condition is satisfied. The value of $k = (k_1 + k_2)/2$ can be also even.

   Remark: The geometric mean $(127 + 107)/2 = 117$ of electronic and hadronic Mersennes corresponding to mass 16 MeV rather near to the mass of so called X boson [L39] (`https://tinyurl.com/ya3yuzeb`).

2. One can also consider the formula $L = (L(k_1)L(k_2)..L(k_n))^{1/n}$ but in this case the scale would correspond to prime $p \simeq 2^{k_1+...k_n)/n}$. Since $(k_1 + ..k_n)/n$ is integer only if $k_1 + ...k_n$ is proportional to n.

What about the allowed values of fundamental integers k? It seems that one must allow all odd integers.

1. If only prime values of k are allowed, one can obtain obtain for twin prime pair $(k-1,k+1)$ even integer k as geometric mean $\sqrt{k}$ if k is square. If prime k is not a member of this kind of pair, it is not possible to get integers $k-1$ and $k+1$. If only prime values of k are fundamental, one could assign to $k=89$ characterizing Higgs boson weak bosons $k=90$ possibly characterizing weak bosons. Therefore it seems that one must allow all odd integers with the additional condition already explained.

2. Just for fun one can check whether $k=161$ forced by the argument related to electroweak scale and h_{eff} corresponds to a geometric mean of two Gaussian Mersennes. One has $k(k_1,k_2)=(k_2+k_2)/2$ giving the list $k(151,157)=154)$, $k(151,163)=157$ Gaussian Mersenne itself, $k(151,167)=159$, $k(157,163)=160$, $k(157,167)=162$, $k(163,167)=165$. Unfortunately, $k=161$ does not belong to this set. If one allows all odd values of k as fundamental, the problem disappears.

One can also consider refinements of p-adic length scale hypothesis in its basic form.

1. One can consider also a generalization of p-adic length scale hypothesis to allow length scales coming as powers of small primes. The small primes $p=2,3,5$ assignable to Platonic solids would be especially interesting. $p=2,3,5$ and also Fermat primes and Mersenne primes are maximally near to powers of two and their powers would define secondary and higher p-adic length scales. In this sense the extension would not actually bring anything new.

   There is evidence for the occurrence of long p-adic time scales coming as powers of 3 [?, ?] (`http://tinyurl.com/ycesc5mq`) and [K79] (`https://tinyurl.com/y8camqlt`. Furthermore, prime 5 and Golden Mean are related closely to DNA helical structure. Portion of DNA with L(151) contains 10 DNA codons and is the minimal length containing an integer number of codons.

2. The presence of length scales associated with 1 nm and 2 nm thick structures encourage to consider the possibility of p-adic primes near integers $2^k3^l5^m$ defining generators of multiplicative ideals of integers. They do not satisfy the maximal nearness criterion anymore but would be near to integers representable as products of powers of primes maximally near to powers of two.

What could be the interpretation of the integer k appearing in $p\simeq 2^k$? Elementary particle quantum numbers would be associated with wormhole contacts with size scale of CP_2 whereas elementary particles correspond to p-adic size scale about Compton length. What could determine the size scale of wormhole contact? I have proposed that to p-adic length scale there is associated a scale characterizing wormhole contact and depending logarithmically on it and corresponds to $L_k=(1/2)log(p)L_0=(k/2)log(2)L_0$. The generalization of this hypothesis to the case of $p\simeq 2^k3^l5^m...$ be straightforward and be $L_{k,l,m}=(1/2)(klog(2)+llog(3)+mlog(5)+..)$.

Dark scales and scales of CDs and their relation to p-adic length scale hierarchy

There are two length scale hierarchies. p-Adic length scale hierarchy assignable to space-time surfaces and the dark hierarchy assignable to CDs. One should find an identification of dark scales and understand their relationship to p-adic length scales.

1. Identification of dark scales

The dimension n of the extension provides the roughest measure for its complexity via the formula $h_{eff}/h_0=n$. The basic - rather ad hoc - assumption has been that n as dimension of extension defines not only h_{eff} but also the size scale of CD via $L=nL_0$.

This assumption need not be true generally and already the attempt to understand gravitational constant [L112] as a prediction of TGD led to the proposal that gravitational Planck constant $h_{gr}=n_{gr}h_0=GMm/v_0$ [E1] could be coded by the data relating to a normal subgroup of Galois group appearing as a factor of n.

The most general option is that dark scale is coded by a data related to extension of its sub-extension and this data involves ramified primes. Ramified primes depend on the polynomial defining the extension and there is large number polynomials defining the same extension.

Therefore ramified ramifies code information also about polynomial and dynamics of space-time surface.

First some observations.

1. For Galois extension the order n has a natural decomposition to a product of orders n_i of its normal subgroups serving also as dimensions of corresponding extensions: $n = \prod_i n_i$. This implies a decomposition of the group algebra of Galois group to a tensor product of state spaces with dimensions n_i [L121].

2. Could one actually identify several dark scales as the proposed identifications of gravitational, electromagnetic, etc variants of h_{eff} suggest? The hierarchy of normal subgroups of Galois group of rationals corresponds to sub-groups with orders given by $N(i,1) = n_i n_{i-1}...n_{i-1}$ of n define orders for the normal subgroups of Galois group. For extensions of $k-1$:th extension of rationals one has $N(i,k) = n_i n_{i-1}...n_{i-k}$. The most general option is that these normal subgroups provide only the data allowing to associate dark scales to each of them. The spectrum of h_{eff} could correspond to the $\{N_{i,k}\}$ or at least the set $\{N_{i,1}\}$.

3. The extensions with prime dimension $n = p$ have no non-trivial normal subgroups and $n = p$ would hold for them. For these extensions the state space of group algebra is prime as Hilbert space and does not decompose to tensor product so that it would represent fundamental system. Could these extensions be of special interest physically? SSFRs would naturally involve state function reduction cascades proceeding downwards along hierarchy of normal subgroups and would represent cognitive measurements [L121].

The original guess was that dark scale $L_D = nL_p$, where n is the order n for the extensions and p is a ramified prime for the extension. A generalized form would allow $L_D = N(i,1)L_{p_k}$ for the sub-extension such that p_k is ramified prime for the sub-extension.

2. Can one identify the size scale of CD as dark scale?

It would be natural if the scale of CD would be determined by the extension of rationals. Or more generally, the scales of CD and hierarchy of sub-CDs associated with the extension would be determined by the inclusion hierarchy of extensions and thus correspond to the hierarchy of normal sub-groups of Galois group.

The simplest option would be $L_{CD} = L_D$ so that the size scales of sub-CD would correspond dark scales for sub-extension given by $L_{CD,i} = N(i,1)L_{p_k}$, p_k ramified prime of sub-extension.

1. The differences $|r_i - r_j|$ would correspond to differences for Minkowski time of CD. CD need not contain all values of hyperplanes $t = r_i$ and the evolution by SSFR would gradually bring in day-light all roots r_n of the polynomial P defining space-time surface as "very special moments in the life of self". If the size scale of CD is so large that also the largest value of $|r_i|$ is inside the upper or lower half of CD, the size scale of CD would correspond roughly to the largest p-adic length scale.

   CD contains sub-CDs and these could correspond to normal subgroups of Galois extension as extension of extension of ....

2. One can ask what happens when all special moments $t = r_n$ have been experienced? Does BSFR meaning death of conscious entity take place or is there some other option? In [L113] I considered a proposal for how chaos could emerge via iterations of P during the sequence of SSFRs.

   One could argue that when CD has reached by SSFRs following unitary evolutions a size for which all roots r_n have become visible, the evolution could continues by the replacement of P with $P \circ P$, and so on. This would give rise to iteration and space-time analog for the approach to chaos.

3. Eventually the evolution by SSFRs must stop. Biological arguments suggests that metabolic limitations cause the death of self since the metabolic energy feed is not enough to preserve the distribution of values of h_{eff} (energies increase with $h_{eff} \propto Nn$, for N:th iteration and h_{eff} is reduced spontaneously) [L122].

10.3 Fermionic variant of $M^8 - H$ duality

The topics of this section is $M^8 - H$ duality for fermions. Consider first the bosonic counterpart of $M^8 - H$ duality.

1. The octonionic polynomial giving rise to space-time surface X^4 as its "root" is obtained from ordinary real polynomial P with rational coefficients by algebraic continuation. The conjecture is that the identification in terms of roots of polynomials of even real analytic functions guarantees associativity and one can formulate this as rather convincing argument [L49, L50, L51]. Space-time surface X^4_c is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. These options correspond to complexified-quaternionic tangent- or normal spaces. For $P(x) = x^n + ..$ ordinary roots are algebraic integers. The real 4-D space-time surface is projection of this surface from M^8_c to M^8. One could drop the subscripts "$_c$" but in the sequel they will be kept.

   M^4_c appears as a special solution for any polynomial P. M^4_c seems to be like a universal reference solution with which to compare other solutions.

   One obtains also brane-like 6-surfaces as 6-spheres as universal solutions. They have M^4 projection, which is a piece of hyper-surface for which Minkowski time as time coordinate of CD corresponds to a root $t = r_n$ of P. For monic polynomials these time values are algebraic integers and Galois group permutes them.

2. One cannot exclude rational functions or even real analytic functions in the sense that Taylor coefficients are octonionically real (proportional to octonionic real unit). Number theoretical vision - adelic physics [L53], suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

   The generalization of the relationship between reals, extensions of p-adic number fields, and algebraic numbers in their intersection is suggestive. The "world of classical worlds" (WCW) would contain the space-time surfaces defined by polynomials with general real coefficients. Real WCW would be continuous space in real topology. The surfaces defined by rational or perhaps even algebraic coefficients for given extension would represent the intersection of real WCW with the p-adic variants of WCW labelled by the extension.

3. $M^8 - H$ duality requires additional condition realized as condition that also space-time surface itself contains 2-surfaces having commutative (complex) tangent or normal space. These surfaces can be 2-D also in metric sense that is light-like 3-D surfaces. The number of these surfaces is finite in generic case and they do not define a slicing of X^4 as was the first expectation. Strong form of holography (SH) makes it possible to map these surfaces and their tangent/normal spaces to 2-D surfaces $M4 \times CP_2$ and to serve as boundary values for the partial differential equations for variational principle defined by twistor lift. Space-time surfaces in H would be minimal surface apart from singularities.

Concerning $M^8 - H$ duality for fermions, there are strong guidelines: also fermionic dynamics should be algebraic and number theoretical.

1. Spinors should be octonionic. I have already earlier considered their possible physical interpretation. [L18].

2. Dirac equation as linear partial differential equation should be replaced with a linear algebraic equation for octonionic spinors which are complexified octonions. The momentum space variant of the ordinary Dirac equation is an algebraic equation and the proposal is obvious: $P\Psi = 0$, where P is the octonionic continuation of the polynomial defining the space-time surface and multiplication is in octonionic sense. The conjugation in O_c is induced by the conjugation of the commuting imaginary unit i. The square of the Dirac operator is real if the space-time surface corresponds to the projection $O_c \to M^8 \to M^4$ with real time coordinate and imaginary spatial coordinates so that the metric defined by the octonionic norm is real

and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

The masslessness condition restricts the solutions to light-like 3-surfaces $m_{kl}P^kP^l = 0$ in Minkowskian sector analogous to mass shells in momentum space - just as in the case of ordinary massless Dirac equation. $P(o)$ rather than octonionic coordinate o would define momentum. These mass shells should be mapped to light-like partonic orbits in H.

3. This picture leads to the earlier phenomenological picture about induced spinors in H. Twistor Grassmann approach suggests the localization of the induced spinor fields at light-like partonic orbits in H. If the induced spinor field allows a continuation from 3-D partonic orbits to the interior of X^4, it would serve as a counterpart of virtual particle in accordance with quantum field theoretical picture.

10.3.1 $M^8 - H$ duality for space-time surfaces

It is good to explain $M^8 - H$ duality for space-time surfaces before discussing it in fermionic sector.

Space-time as 4-surface in $M^8_c = O_c$

One can regard real space-time surface $X^4 \subset M^8$ as a M^8–-projection of $X^4_c \subset M^8_c = O_c$. M^4_c is identified as complexified quaternions H_c [L85, L111]. The dynamics is purely algebraic and therefore local an associativity is the basic dynamical principle.

1. The basic condition is associativity of $X^4 \subset M^8$ in the sense that either the tangent space or normal space is associative - that is quaternionic. This would be realized if X^4_c as a root for the quaternion-valued "real" or "imaginary part" for the O_c algebraic continuation of real analytic function $P(x)$ in octonionic sense. Number theoretical universality requires that the Taylor coefficients are rational numbers and that only polynomials are considered.

   The 4-surfaces with associative normal space could correspond to elementary particle like entities with Euclidian signature (CP_2 type extremals) and those with associative tangent space to their interaction regions with Minkowskian signature. These two kinds space-time surfaces could meet along these 6-branes suggesting that interaction vertices are located at these branes.

2. The conditions allow also exceptional solutions for any polynomial for which both "real" and "imaginary" parts of the octonionic polynomial vanish. Brane-like solutions correspond to 6-spheres S^6 having $t = r_n$ 3-ball B^3 of light-cone as M^4 projection: here r_n is a root of the real polynomial with rational coefficients and can be also complex - one reason for complexification by commuting imaginary unit i. For scattering amplitudes the topological vertices as 2-surfaces would be located at the intersections of X^4_c with 6-brane. Also Minkowski space M^4 is a universal solution appearing for any polynomial and would provide a universal reference space-time surface.

3. Polynomials with rational coefficients define EQs and these extensions form a hierarchy realized at the level of physics as evolutionary hierarchy. Given extension induces extensions of p-adic number fields and adeles and one obtains a hierarchy of adelic physics. The dimension n of extension allows interpretation in terms of effective Planck constant $h_{eff} = n \times h_0$. The phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ behave like dark matter and galactic dark matter could correspond to classical energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes. It is not completely clear whether number galactic dark matter must have $h_{eff} > h$. Dark energy in would correspond to the volume part of the energy of the flux tubes.

   There are good arguments in favor of the identification $h = 6h_0$ [L59]. "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining X^4 as a covering space locally. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

The ramified primes of extension in turn are identified as preferrred p-adic primes. The moduli for the time differences $|t_r - t_s|$ have identification as p-adic time scales assignable to ramified primes [L111]. For ramified primes the p-adic variants of polynomials have degenerate zeros in $O(p) = 0$ approximation having interpretation in terms of quantum criticality central in TGD inspired biology.

4. During the preparation of this article I made a trivial but overall important observation. Standard Minkowski signature emerges as a prediction if conjugation in O_c corresponds to the conjugation with respect to commuting imaginary unit i rather than octonionic imaginary units as though earlier. If the space-time surface corresponds to the projection $O_c \rightarrow M^8 \rightarrow M^4$ with real time coordinate and imaginary spatial coordinates the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

Realization of $M^8 - H$ duality

$M^8 - H$ duality allows to $X^4 \subset M^8$ to $X^4 \subset H$ so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D preferred 2-surfaces defining holography making possible M^8-H duality and possibly appearing as singularities in H. The dynamics of minimal surfaces, which are also extremals of Kähler action, reduces for known extremals to purely algebraic conditions analogous to holomorphy conditions in string models and thus involving only gradients of coordinates. This condition should hold generally and should induce the required huge reduction of degrees of freedom proposed to be realized also in terms of the vanishing of super-symplectic Noether charges already mentioned [K96].

Twistor lift allows several variants of this basic duality [L99]. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of EQs defining an evolutionary hierarchy. This forms the basics for the number theoretical vision about TGD.

As already noticed, $X^4 \subset M^8$ would satisfy an infinite number of additional conditions stating vanishing of Noether charges for a sub-algebra $SSA_n \subset SSA$ of super-symplectic algebra SSA actings as isometries of WCW.

M^8-H duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions [L85].

1. Associativity condition for tangent-/normal spaces is the first essential condition for the existence of M^8-H duality and means that tangent - or normal space is associative/quaternionic.

2. Each tangent space of X^4 at x must contain a preferred $M^2_c(x) \subset M^4_c$ such that $M^2_c(x)$ define an integrable distribution and therefore complexified string world sheet in M^4_c. This gives similar distribution for their orthogonal complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense. $E^2_c(x)$ would correspond to partonic 2-surface. This condition generalizes for X^4 with quaternionic normal space. A possible interpretation is as a space-time correlate for the selection of quantization axes for energy (rest system) and spin.

One can imagine two realizations for the additional condition.

Option I: Global option states that the distributions $M^2_c(x)$ and $E^2_c(x)$ define a slicing of X^4_c.

Option II: Only a discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce $X^4 \subset H$. This would be the minimal option.

It seems that only **Option II** can be realized.

1. The basic observation is that X^2_c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c-valued "real" or "imaginary" part in C_c sense for P vanishes. M^2_c would be the simplest solution but also more general complex sub-manifolds $X^2_c \subset M^4_c$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. SH makes possible $M^8 - H$ duality assuming that associativity conditions hold true only at 2-surfaces including partonic 2-surfaces or string world sheets or perhaps both. Thus one can give up the conjecture that the polynomial ansatz implies the additional condition globally.

   SH indeed states that PEs are determined by data at 2-D surfaces of X^4. Even if the conditions defining X^2_c have only a discrete set of solutions, SH at the level of H could allow to deduce the PEs from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. The existence of $M^2(x)$ would be required only at the 2-D surfaces.

3. There is however a delicacy involved: X^2 might be 2-D only metrically but not topologically! The 3-D light-like surfaces X^3_L indeed have metric dimension $D = 2$ since the induced 4-metric degenerates to 2-D metric at them. Therefore their pre-images in M^8 would be natural candidates for the singularities at which the dimension of the quaternionic tangent or normal space reduces to $D = 2$ [L83] [K9]. If this happens, SH would not be quite so strong as expected. The study of fermionic variant of $M^8 - H$-duality supports this conclusion.

One can generalize the condition selecting X^2_c so that it selects 1-D surface inside X^2_c. By assuming that R_c-valued "real" or "imaginary" part of complex part of P sense at this 2-surface vanishes. One obtains preferred M^1_c or E^1_c containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as a complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \to C_c \to H_c \to O_c$ realized as surfaces.

10.3.2 What about $M^8 - H$ duality in the fermionic sector?

During the preparation of this article I become aware of the fact that the realization M^8-H duality in the fermionic sector has remained poorly understood. This led to a considerable integration of the ideas about $M^8 - H$ duality also in the bosonic sector and the existing phenomenological picture follows now from $M^8 - H$ duality. There are powerful mathematical guidelines available.

Octonionic spinors

By supersymmetry, octonionicity should have also fermionic counterpart.

1. The interpretation of M^8_c as complexified octonions suggests that one should use complexified octonionic spinors in M^8_c. This is also suggested by SO(1,7) triality unique for dimension $d = 8$ and stating that the dimensions of vector representation, spinor representation and its conjugate are same and equal to $D = 8$. I have already earlier considered the possibility to interpret M^8 spinors as octonionic [L18]. Both octonionic gamma matrices and spinors have interpretation as octonions and gamma matrices satisfy the usual anti-commutation rules. The product for gamma matrices and gamma matrices and spinors is replaced with non-associative octonionic product.

2. Octonionic spinors allow only one M^8-chirality, which conforms with the assumption of TGD inspired SUSY that only quarks are fundamental fermions and leptons are their local composites [L92].

3. The decomposition of $X^2 \subset X^4 \subset M^8$ corresponding to $R \subset C \subset Q \subset O$ should have analog for the O_c spinors as a tensor product decomposition. The special feature of dimension $D = 8$ is that the dimensions of spinor spaces associated with these factors are indeed 1, 2, 4, and 8 and correspond to dimensions for the surfaces!

   One can define for octonionic spinors associative/co-associative sub-spaces as quaternionic/co-quaternionic spinors by posing chirality conditions. For $X^4 \subset M^8_c$ one could define the analogs of projection operators $P_\pm = (1 \pm \gamma_5)/2$ as projection operators to either factor of the spinor space as tensor product of spinor space associated with the tangent and normal

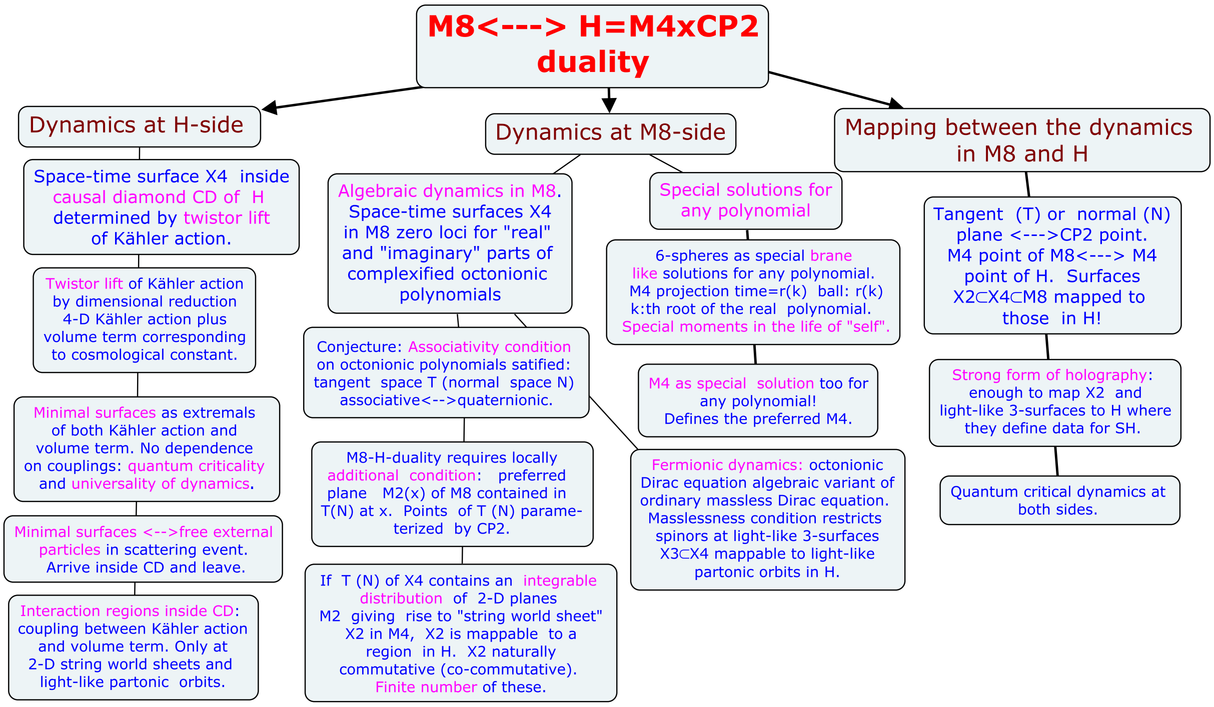

Figure 10.1: M^8-H duality.

spaces of X^4: the analog of γ_5 would correspond to tangent or normal space depending on whether tangent or normal space is associative. For the spinors with definite chirality there would be no entanglement between the tensor factors. The condition would generalize the chirality condition for massless M^4 spinors to a condition holding for the local M^4 appearing as tangent/normal space of X^4.

4. The chirality condition makes sense also for $X^2 \subset X^4$ identified as complex/co-complex surface of X^4. Now γ_5 is replaced with γ_3 and states that the spinor has well-defined spin in the direction of axis defined by the decomposition of X^2 tangent space to $M^1 \times E^1$ with M^1 defining real octonion axis and selecting rest frame. Interpretation in terms of quantum measurement theory is suggestive.

What about tangent space quantum numbers in M^8 picture. In H-picture they correspond to spin and electroweak quantum numbers. In M^8 picture the geometric tangent space group for a rest system is product $SU(2) \times SU(2)$ with possible modifications due to octonionicity reducing tangent space group to those respecting octonionic automorphisms.

What about the sigma matrices for the octonionic gamma matrices? The surprise is that the commutators of M^4 sigma matries and those of E^4 sigma matrices close to the sama $SO(3)$ algebra allowing interpretation as representation for quaternionic automorphisms. Lorentz boosts are represented trivially, which conforms with the fact that octonion structure fixes unique rest system. Analogous result holds in E^4 degrees of freedom. Besides this one has unit matrix assignable to the generalize spinor structure of CP_2 so that also electroweak $U(1)$ factor is obtained.

One can understand this result by noticing that octonionic spinors correspond to 2 copies of a tensor products of the spinor doublets associated with spin and weak isospin. One has $2\otimes 2 = 3\oplus 1$ so that one must have $1\oplus 3\oplus 1\oplus 3$. The octonionic spinors indeed decompose like $1+1+3+\overline{3}$ under $SU(3)$ representing automophisms of the octonions. $SO(3)$ could be interpreted as $SO(3)\subset SU(3)$. $SU(3)$ would be represented as tangent space rotations.

Dirac equation as partial differential equation must be replaced by an algebraic equation

Algebraization of dynamics should be also supersymmetric. The modified Dirac equation in H is linear partial differential equation and should correspond to a linear algebraic equation in M^8.

1. The key observation is that for the ordinary Dirac equation the momentum space variant of Dirac equation for momentum eigenstates is algebraic! Could the interpretation for M^8-H duality as an analog of momentum-position duality of wave mechanics considered already earlier make sense! This could also have something to do with the dual descriptions of twistorial scattering amplitudes in terms of either twistor and momentum twistors. Already the earlier work excludes the interpretation of the octonionic coordinate o as 8-momentum. Rather, $P(o)$ has this interpretation and o corrresponds to embedding space coordinate.

2. The first guess for the counterpart of the modified Dirac equation at the level of $X^4\subset M^8$ is $P\Psi = 0$, where Ψ is octonionic spinor and the octonionic polynomial P defining the space-time surface can be seen as a generalization of momentum space Dirac operator with octonion units representing gamma matrices. If associativity/co-associativity holds true, the equation becomes quaternionic/co-quaternionic and reduces to the 4-D analog of massless Dirac equation and of modified Dirac equation in H. Associativity hols true if also Ψ satisfies associativity/co-associativity condition as proposed above.

3. What about the square of the Dirac operator? There are 3 conjugations involved: quaternionic conjugation assumed in the earlier work, conjugation with respect to i, and their combination. The analog of octonionic norm squared defined as the product $o_c o_c^*$ with conjugation with respect to i only, gives Minkowskian metric $m_{kl}o^k\overline{o}^l$ as its real part. The imaginary part of the norm squared is vanishing for the projection $O_c\rightarrow M^8\rightarrow M^4$ so that time coordinate is real and spatial coordinates imaginary. Therefore Dirac equation allows solutions only for the M^4 projection X^4 and M^4 ($M8$) signature of the metric can be said to be an outcome of quaternionicity (octonionicity) alone in accordance with the duality between metric and algebraic pictures.

   Both $P^\dagger P$ and PP should annihilate Ψ. $P^\dagger P\Psi = 0$ gives $m_{kl}P^k\overline{P}^l = 0$ as the analog of vanishing mass squared in M^4 signature in both associative and co-associative cases. $PP\Psi = 0$ reduces to $P\Psi = 0$ by masslessness condition. One could perhaps interpret the projection $X_c^4\rightarrow M^8\rightarrow M^4$ in terms of Uncertainty Principle.

   There is a $U(1)$ symmetry involved: instead of the plane M^8 one can choose any plane obtained by a rotation $exp(i\phi)$ from it. Could it realize quark number conservation in M^8 picture?

   For $P = o$ having only $o = 0$ as root $Po = 0$ reduces to $o^\dagger o = 0$ and o takes the role of momentum, which is however vanishing. 6-D brane like solutions S^6 having $t = r_n$ balls $B^3\subset CD_4$ as M^4 projections one has $P = 0$ so that the Dirac equation trivializes and does not pose conditions on Ψ. o would have interpretation as space-time coordinates and $P(o)$ as position dependent momentum components P^k.

   The variation of P at mass shell of M_c^8 (to be precise) could be interpreted in terms of the width of the wave packet representing particle. Since the light-like curve at partonic 2-surface for fermion at X_L^3 is not a geodesic, mass squared in M^4 sense is not vanishing. Could one understand mass squared and the decay width of the particle geometrically? Note that mass squared is predicted also by p-adic thermodynamics [K65].

4. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. M^8-H duality [L85] suggests that this boundary is mapped to $X_L^3\subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X_L^3 gives a very powerful constraint on SH and M^8-H duality.

5. Also at 2-surfaces $X^2 \subset X^4$ an the variant Dirac equation would hold true and should commute with the corresponding chirality condition. Now $D^\dagger D\Psi = 0$ gives 2-D variant of masslessness condition with 2-momentum components represented by those of P. 2-D masslessness locates the spinor to a 1-D curve X^1_L. Its H-image would naturally contain the boundary of the string word sheet at X^3_L assumed to carry fermion quantum numbers and also the boundary of string world sheet at the light-like boundary of CD_4. The interior of string world sheet in H would not carry induced spinor field.

6. The general solution for both 4-D and 2-D cases can be written as $\Psi = P\Psi_0$, Ψ_0 a constant spinor - this in a complete analogy with the solution of modified Dirac equation in H. P depends on position: the WKB approximation using plane waves with position dependent momentum seems to be nearer to reality than one might expect.

The phenomenological picture at H-level follows from the M^8-picture

Remarkably, the partly phenomenological picture developed at the level of H is reproduced at the level of M^8. Whether the induced spinor fields in the interior of X^4 are present or not, has been long standing question since they do not seem to have any role in the physical picture. The proposed picture answers this question.

Consider now the explicit realization of $M^8 - H$-duality for fermions.

1. SH and the expected analogy with the bosonic variant of $M^8 - H$ duality lead to the first guess. The spinor modes in $X^4 \subset M^8$ restricted to X^2 can be mapped by $M^8 - H$-duality to those at their images $X^2 \subset H$, and define boundary conditions allowing to deduce the solution of the modified Dirac equation at $X^4 \subset H$. X^2 would correspond to string world sheets having boundaries X^1_L at X^3_L.

   The guess is not quite correct. Algebraic Dirac equation requires that the solutions are restricted to the 3-D and 1-D mass shells $P_kP^k = 0$ in M^8. This should remain true also in H and X^3_L and their 1-D intersections X^1_L with string world sheets remain. Fermions would live at boundaries. This is just the picture proposed for the TGD counterparts of the twistor amplitudes and corresponds to that used in twistor Grassmann approach!

   For 2-D case constant octonionic spinors Ψ_0 and gamma matrix algebra are equivalent with the ordinary Weyl spinors and gamma matrix algebra and can be mapped as such to H. This gives one additional reason for why SH must be involved.

2. At the level of H the first guess is that the modified Dirac equation $D\Psi = 0$ is true for D based on the modified gamma matrices associated with both volume action and Kähler action. This would select preferred solutions of modified Dirac equation and conform with the vanishing of super-symplectic Noether charges for SSA_n for the spinor modes. The guess is not quite correct. The restriction of the induced spinors to X^3_L requires that Chern-Simons action at X^3_L defines the modified Drac action.

3. The question has been whether the 2-D modified Dirac action emerges as a singular part of 4-D modified Dirac action assignable to singular 2-surface or can one assign an independent 2-D Dirac action assignable to 2-surfaces selected by some other criterion. For singular surfaces $M^8 - H$ duality fails since tangent space would reduce to 2-D space so that only their images can appear in SH at the level of H.

   This supports the view that singular surfaces are actually 3-D mass shells M^8 mapped to X^3_L for which 4-D tangent space is 2-D by the vanishing of $\sqrt{g_4}$ and light-likeness. String world sheets would correspond to non-singular $X^2 \subset M^8$ mapped to H and defining data for SH and their boundaries $X^1_L \subset X^3_L$ and $X^1_L \subset CD_4$ would define fermionic variant of SH.

What about the modified Dirac operator D in H?

1. For X^3_L modified Dirac equation $D\Psi = 0$ based on 4-D action S containing volume and Kähler term is problematic since the induced metric fails to have inverse at X^3_L. The only possible action is Chern-Simons action S_{CS} used in topological quantum field theories and

now defined as sum of C-S terms for Kähler actions in M^4 and CP_2 degrees of freedom. The presence of M^4 part of Kähler form of M^8 is forced by the twistor lift, and would give rise to small CP breaking effects explaining matter antimatter asymmetry [L92]. S_{C-S} could emerge as a limit of 4-D action.

The modified Dirac operator D_{C-S} uses modified gamma matrices identified as contractions $\Gamma^{\alpha}_{CS} = T^{\alpha k}\gamma_k$, where $T^{\alpha k} = \partial L_{CS}/\partial(\partial_\alpha h^k)$ are canonical momentum currents for S_{C-S} defined by a standard formula.

2. CP_2 part would give conserved Noether currents for color in and M^4 part Poincare quantum numbers: the apparently small CP breaking term would give masses for quarks and leptons! The bosonic Noether current $J_{B,A}$ for Killing vector j^k_A would be proportional to $J^{\alpha}_{B,A} = T^{\alpha}_k j_A k$ and given by $J_{B,A} = \epsilon^{\alpha\beta\gamma}[J_{\beta\gamma}A_k + A_\beta J_{\gamma k}]j^k_A$.

   Fermionic Noether current would be $J_{F,A} = \overline{\Psi}J^\alpha\Psi$ 3-D Riemann spaces allow coordinates in which the metric tensor is a direct sum of 1-D and 2-D contributions and are analogous to expectation values of bosonic Noether currents. One can also identify also finite number of Noether super currents by replacing $\overline{\Psi}$ or Ψ by its modes.

3. In the case of X^3_L the 1-D part light-like part would vanish. If also induced Kähler form is non-vanishing only in 2-D degrees of freedom, the Noether charge densities J^t reduce to $J^t = JA_k j^k_A$, $J = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma}$ defining magnetic flux. Modified Dirac operator would reduce to $D = JA_k\gamma^k D_t$ and 3-D solutions would be covariantly constant spinors along the light-like geodesics parameterized by the points 2-D cross section. One could say that the number of solutions is finite and corresponds to covariantly constant modes continued from X^1_L to X^3_L. This picture is just what twistor Grassmannian approach led to [L72].

A comment inspired by the ZEO based quantum measurement theory

I cannot resist the temptation to make a comment relating to quantum measurement theory inspired by zero energy ontology (ZEO) extending to a theory of consciousness [L91, L121, L122].

I have proposed [L111, L113] that the time evolution by "big" state function reductions (BSFRs) could be induced by iteration of real polynomial P - at least in some special cases. The foots of the real polynomial P would define a fractal at the limit of larger number of iterations. The roots of n-fold iterate $\circ^n P$ would contain the inverse images under $\circ^{-n+1}P$ of roots of P and for $P(0) = 0$ the inverse image $\circ^n P$ would consist of inverse images under $\circ^{-k}P$, $k = 0, ..., n-1$, of roots of P.

Also the mass shells for $\circ^n P$ would be unions of inverses images under $\circ^{-k}P$, $k = 0, , ..., n-1$, of roots of P. This gives rather concrete view about evolution of M^4 projections of the partonic orbits. A rough approximate expression for the largest root of real P approximated as $P(x) \simeq a_n x^n + an - 1ix^{n-1}$ for large x is $x_{max} \sim a_n/a_{n-1}$. For $\circ^n P$ one obtains the same estimate. This suggests that the size scales of the partonic orbits are same for the iterates. The mass shells would not differ dramatically: could they have an interpretation in terms of mass splitting?

The evolution by iteration would add new partonic orbits and preserve the existing ones: this brings in mind conservation of genes in biological evolution. This is true also for a more general evolution allowing general functional decomposition $Q \rightarrow Q \circ P$ to occur in BSFR.

What next in TGD?

The construction of scattering amplitudes has been the dream impossible that has driven me for decades. Maybe the understanding of fermionic $M^8 - H$ duality provides the needed additional conceptual tools. The key observation is utterly trivial but far reaching: there are 3 possible conjugations for octonions corresponding to the conjugation of commutative imaginary unit or of octonionic imaginary units or both of them. 1st norm gives a real valued norm squared in Minkowski signature natural at M^8 level! Second one gives a complex valued norm squared in Euclidian signature. 1st and 2nd norms are equivalent for octonions light-like with respect to the first norm. The 3rd conjugation gives a real-valued Euclidian norm natural at the level of Hilbert space.

1. M^8 picture looks simple. Space-time surfaces in M^8 can be constructed from real polynomials with real (rational) coefficients, actually knowledge of their roots is enough. Discrete data - roots of the polynomial!- determine space-time surface as associative or co-associative region! Besides this one must pose additional condition selecting 2-D string world sheets and 3-D light-like surfaces as orbits of partonic 2-surfaces. These would define strong form of holography (SH) allowing to map space-time surfaces in M^8 to $M^4 \times CP_2$.

2. Could SH generalize to the level of scattering amplitudes expressible in terms of n-point functions of CFT?! Could the n points correspond to the roots of the polynomial defining space-time region!

   Algebraic continuation to quaternion valued scattering amplitudes analogous to that giving space-time sheets from the data coded SH should be the key idea. Their moduli squared are real - this led to the emergence of Minkowski metric for complexified octonions/quaternions) would give the real scattering rates: this is enough! This would mean a number theoretic generalization of quantum theory.

3. One can start from complex numbers and string world sheets/partonic 2-surfaces. Conformal field theories (CFTs) in 2-D play fundamental role in the construction of scattering string theories and in modelling 2-D statistical systems. In TGD 2-D surfaces (2-D at least metrically) code for information about space-time surface by strong holography (SH) .

   Are CFTs at partonic 2-surfaces and string world sheets the basic building bricks? Could 2-D conformal invariance dictate the data needed to construct the scattering amplitudes for given space-time region defined by causal diamond (CD) taking the role of sphere S^2 in CFTs. Could the generalization for metrically 2-D light-like 3-surfaces be needed at the level of "world of classical worlds" (WCW) when states are superpositions of space-time surfaces, preferred extremals?

The challenge is to develop a concrete number theoretic hierarchy for scattering amplitudes: $R \rightarrow C \rightarrow H \rightarrow O$ - actually their complexifications.

1. In the case of fermions one can start from 1-D data at light-like boundaries LB of string world sheets at light-like orbits of partonic 2-surfaces. Fermionic propagators assignable to LB would be coded by 2-D Minkowskian QFT in manner analogous to that in twistor Grassmann approach. n-point vertices would be expressible in terms of Euclidian n-point functions for partonic 2-surfaces: the latter element would be new as compared to QFTs since point-like vertex is replaced with partonic 2-surface.

2. The fusion (product?) of these Minkowskian and Euclidian CFT entities corresponding to different realization of complex numbers as sub-field of quaternions would give rise to 4-D quaternionic valued scattering amplitudes for given space-time sheet. Most importantly: there moduli squared are real for both norms.

   It is not quite clear whether one must use the 1st Minkowskian norm requiring "time-like" scattering amplitudes to achieve non-negative probabilities or use the 3rd norm to get the ordinary positive-definite Hilbert space norm. A generalization of quantum theory (CFT) from complex numbers to quaternions (quaternionic "CFT") would be in question.

3. What about several space-time sheets? Could one allow fusion of different quaternionic scattering amplitudes corresponding to different quaternionic sub-spaces of complexified octonions to get octonion-valued non-associative scattering amplitudes. Again scattering rates would be real. This would be a further generalization of quantum theory.

There is also the challenge to relate M^8- and H-pictures at the level of WCW. The formulation of physics in terms of WCW geometry [K96, L98] leads to the hypothesis that WCW Kähler geometry is determined by Kähler function identified as the 4-D action resulting by dimensional reduction of 6-D surfaces in the product of twistor spaces of M^4 and CP_2 to twistor bundles having S^2 as fiber and space-time surface $X^4 \subset H$ as base. The 6-D Kähler action reduces to the sum of 4-D Kähler action and volume term having interpretation in terms of cosmological constant.

The question is whether the Kähler function - an essentially geometric notion - can have a counterpart at the level of M^8.

1. SH suggests that the Kähler function identified in the proposed manner can be expressed by using 2-D data or at least metrically 2-D data (light-like partonic orbits and light-like boundaries of CD). Note that each WCW would correspond to a particular CD.

2. Since 2-D conformal symmetry is involved, one expects also modular invariance meaning that WCW Kähler function is modular invariant, so that they have the same value for $X^4 \subset H$ for which partonic 2-surfaces have induced metric in the same conformal equivalence class.

3. Also the analogs of Kac-Moody type symmetries would be realized as symmetries of Kähler function. The algebra of super-symplectic symmetries of the light-cone boundary can be regarded as an analog of Kac-Moody algebra. Light-cone boundary has topology $S^2 \times R_+$ where R_+ corresponds to radial light-like ray parameterized by radial light-like coordinate r. Super symplectic transformations of $S^2 \times CP_2$ depend on the light-like radial coordinate r, which is analogous to the complex coordinate z for he Kac-Moody algebras.

   The infinitesimal super-symplectic transformations form algebra SSA with generators proportional to powers r^n . The Kac-Moody invariance for physical states generalizes to a hierarchy of similar invariances. There is infinite fractal hierarchy of sub-algebras $SSA_n \subset SSA$ with conformal weights coming as n-multiples of those for SSA. For physical states SSA_n and $[SSA_n, SSA]$ would act as gauge symmetries. They would leave invariant also Kähler function in the sector WCW_n defined by n. This would define a hierarchy of sub- WCWs of the WCW assignable to given CD.

   The sector WCW_n could correspond to extensions of rationals with dimension n, and one would have inclusion hierarchies consisting of sequences of n_i with n_i dividing n_{i+1}. These inclusion hierarchies would naturally correspond to those for hyper-finite factors of type II_1 [K126].

10.4 A vision about the role of HFFs in TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFS of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by zero energy ontology and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

10.4.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A24] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.
2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.
3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.
4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.
2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0, 1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.
3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry

mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X, μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.
2. A positive linear functional is weight with $\omega(1)$ finite.
3. A state is a weight with $\omega(1) = 1$.
4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.
5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$ For factors of type II_1 the values span the range $[0, 1]$ and for factors of type II_∞ n the range $[0, \infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.
2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \quad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{10.4.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*{}^* = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.
4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.
5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A42, A72] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \rightarrow \sigma_t^{\omega} = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_{∞} factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_{λ} this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a way to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1h_1(g_2), h_1h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1g_1(a_2), g_1g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K126] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A1] and those of factors of type III by Alain Connes [A27] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the embedding.

The basic facts proved by Jones are following [A1] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a) \ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ , \\ &b) \ \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \end{aligned} \tag{10.4.2}$$

   the numbers at right hand side are known as Beraha numbers [A60] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B49] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed. The Dynkin graphs of Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. $E_6, E_7, andE_8$ correspond to symmetry groups of tetrahedron, octahedron/cube, and icosahedron/dodecahedron. The group for octahedron/cube is missing: what could this mean?

   For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of $SU(2)$ and the interpretation proposed in [A94] is following-

   The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: A_∞ corresponding to $SU(2)$ itself, $A_{-\infty,\infty}$ corresponding to circle group $U(1)$, and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection.

One can construct also inclusions for which the diagrams corresponding to finite subgroups $G \subset SU(2)$ are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with $n = 6, 7, 8$ for tetrahedron, cube, dodecahedron. These extensions correspond to ADE type Kac-Moody algebras.

The extension is constructed by constructing first factor R as infinite tensor power of $M_2(C)$ (complexified quaternions). Sub-factor R_0 consists elements of of R of form $Id \otimes x$. $SU(2)$ preserves R_0 and for any subgroup G of $SU(2)$ one can identify the inclusion $N \subset M$ in terms of $N = R_0^G$ and $M = R^G$, where $N = R_0^G$ and $M = R^G$ consists of fixed points of R_0 and R under the action of G. The principal graph for $N \subset M$ is the extended Coxeter-Dynk graph for the subgroup G.

Physicist might try to interpret this by saying that one considers only sub-algebras R_0^G and R^G of observables invariant under G and obtains extended Dynkin diagram of G defining an ADE type Kac-Moody algebra. Could the condition that Kac-Moody algebra elements with non-vanishing conformal weight annihilate the physical states state that the state is invariant under R_0 defining measurement resolution. Besides this the states are also invariant under finite group G? Could R_0^G and R^G correspond just to states which are also invariant under finite group G.

Connes tensor product

The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} \ = \ \mathcal{M}/\mathcal{N} \otimes \mathcal{N} \ . \tag{10.4.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M}\otimes\mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^{\dagger} \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A84, A42, A72] . There are good arguments showing that in HFFs of III$_1$ appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III$_1$ and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A72] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E \in \mathcal{M}(\mathcal{O})$ can be written as WW^* with $W \in \mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W = 1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

2. Both the complement of light-cone and double light-cone define HFF of type III$_1$. Lorentz boosts induce modular automorphisms.

3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III$_1$ associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ , \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ .$$

An infinite hierarchy of inclusions of HFFs of type III$_1$s is induced by set theoretic inclusions.

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10.4.2 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?
2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and ZEO, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.
3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?
4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at embedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?
2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?
3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the Kähler-Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the Kähler-Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?
2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

ZEO and factors

The first question concerns the identification of the Hilbert space associated with the factors in ZEO. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.
2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in ZEO.

1. In ZEO M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFs is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.
2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \to 0$ limit. In quantum TGD the exponent of Kähler-Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Kähler-Dirac action can therefore be regarded as a "square root" of Kähler action.
3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in ZEO based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the Kähler-Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFs involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of Kähler-Dirac action [K127] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.
2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.
3. Quantum criticality means that Kähler-Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two super-conformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.
4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.
5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.
2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.
3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L^*_0$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.
2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem

to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K43] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

10.4.3 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

A proposal for M-matrix

The proposed general picture reduces the core of U-matrix to the construction of S-matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.
2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized eigenvalue $p^k\gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-light geodesics of embedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.
3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.
4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K116].

   The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.
5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface

the vertices would be represented by partonic 2-surfaces. In [K116] the interpretation of scattering ampiltudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are "free". At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K13] is a remnant of an "idea that came too early". The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond topartial wave in the space S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??** (`http://tgdtheory.fi/appfigures/elparticletgd.jpg` `http://tgdtheory.fi/appfigures/tgdgrpahs.jpg`) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K116] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

Quantum TGD as square root of thermodynamics

ZEO (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A84] [K126]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A84], [K126] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the Kähler-Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes.

Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the embedding space [K43]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. This implies that classical symplectic Noether charges vanish for this sub-algebra. The quantal conformal charges associated with induced spinor fields annihilate the physical states. Therefore it seems that the measured quantities are the symplectic charges and there is not need to introduce any measurement interaction term and the formalism simplifies dramatically.

The resolution increases with $h_{eff}/h = n$. Also the number of of strings connecting partonic 2-surfaces (in practice elementary particles and their dark counterparts plus bound states generated by connecting dark strings) characterizes physically the finite measurement resolution. Their presence is also visible in the geometry of the space-time surfaces through the conditions that induced W fields vanish at them (well-definedness of em charge), and by the condition that the canonical momentum currents for Kähler action define an integrable distribution of planes parallel to the string world sheet. In spirit with holography, preferred extremal is constructed by fixing string world sheets and partonic 2-surfaces and possibly also their light-like orbits (should one fix wormhole contacts is not quite clear). If the analog of AdS/CFT correspondence holds true, the value of Kähler function is expressible as the energy of string defined by area in the effective metric defined by the anti-commutators of K-D gamma matrices.

Super-symplectic algebra, whose charges are represented by Noether charges associated with strings connecting partonic 2-surfaces extends to a Yangian algebra with multi-stringy generators [K116]. The better the measurement resolution, the larger the maximal number of strings associated with the multilocal generator.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

The inclusions of super-symplectic Yangians form a hierarchy and would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra. As h_{eff} increases, infinite number of these gauge degrees of freedom become dynamical and measurement resolution is increased. This picture is definitely in conflict with the original view but the reduction of criticality in the increase of h_{eff} forces it.

Summary

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

10.4.4 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in ZEO

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.
2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.
3. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.
4. ZEO gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish ZEO allows a symmetry breaking respecting a chosen Cartan algebra.
5. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced

Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFs as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.
2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.
3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \tag{10.4.4}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{10.4.5}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \rightarrow r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{10.4.6}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \rightarrow r_2) = \mathcal{M} : \mathcal{N} \times Tr_N(SS^{\dagger}) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \ . \tag{10.4.7}$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of embedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.
2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of embedding space with larger Planck constant meaning zooming up of various quantal lengths.
3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

1. In ZEO the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.
2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.
3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.
4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A12] .
5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A12] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.
6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{10.4.8}$$

for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.
2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.
3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \quad (10.4.9)$$

for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the Kähler-Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain Kähler-Dirac gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A67] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be

between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.
2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.
3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

10.4.5 Questions about quantum measurement theory in Zero Energy Ontology

The following summary about quantum measurement theory in ZEO is somewhat out-of-date and somewhat sketchy. For more detailed view see [K70, K122, K8].

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \to P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

Quantum measurements in ZEO

ZEO based quantum measurement theory leads directly to a theory of conscious entities. The basic idea is that state function reduction localizes the second boundary of CD so that it becomes a piece of light-cone boundary (more precisely $\delta M^4_\pm \times CP_2$).

Repeated reductions are possible as in standard quantum measurement theory and leave the passive boundary of CD. Repeated reduction begins with U process generating a superposition of CDs with the active boundary of CD being de-localized in the moduli space of CDs, and is followed by a localization in this moduli space so that single CD is the outcome. This process tends to increase the distance between the ends of the CD and has interpretation as a space-time correlate for the flow of subjective time.

Self as a conscious entity corresponds to this sequence of repeated reductions on passive boundary of CD. The first reduction at opposite boundary means death of self and its re-incarnation at the opposite boundary of CD. Also the increase of Planck constant and generation of negentropic entanglement is expected to be associated with this state function reduction.

Weak form of NMP is the most plausible variational principle to characterize the state function reduction. It does not require maximal negentropy gain for state function reductions but allows it. In other words, the outcome of reduction is n-dimensional eigen space of density matrix space but this space need not have maximum possible dimension and even 1-D ray is possible in which case the entanglement negentropy vanishes for the final state and system becomes isolated from the rest of the world. Weak form of NMP brings in free will and can allow also larger negentropy gain than the strong form if n is a product of primes. The beauty of this option is that one can understand how the generalization of p-adic length scale hypothesis emerges.

10.4.6 Miscellaneous

The following considerations are somewhat out-of-date: hence the title "Miscellaneous".

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A94] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A33] .

Fusion rules are indeed something more intricate that the naïve product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A39] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k+2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naïve tensor product with something more intricate. The naïvest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naïve an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_{\pm}(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K27] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A47] .

1. The light-like 3-surfaces X_l^3 defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.
2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K6] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K38] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A47] . If the light-like CDs $X_{L,i}^3$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say 3-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

10.5 The idea of Connes about inherent time evolution of certain algebraic structures from TGD point of view

Jonathan Disckau asked me about what I think about the proposal of Connes represented in the summary of progress of noncommutative geometry in "Noncommutative Geometry Year 2000" [A30] (see `https://arxiv.org/abs/math/0011193`) that certain mathematical structures have inherent time evolution coded into their structure.

I have written years ago about Connes's proposal. At that time I was trying to figure out how to understand the construction of scattering amplitudes in the TGD framework and the proposal of Connes looked attractive. Later I had to give up this idea. However, the basic idea is beautiful. One should only replace the notion of time evolution from a one-parameter group of automorphisms to something more interesting. Also time evolution as increasing algebraic complexity is a more attractive interpretation.

The inclusion hierarchies of hyperfinite factors (HFFs) - closely related to the work of Connes - are a key element of TGD and crucial for understanding evolutionary hierarchies in TGD. Is it possible that mathematical structure evolves in time in some sense? The TGD based answer is that quantum jump as a fundamental evolutionary step - moment of subjective time evolution - is a necessary new element. The sequence of moments of consciousness as quantum jumps would have an interpretation as hopping around in the space of mathematical structures leading to increasingly complex structures.

The generalization of the idea of Connes is discussed in this framework. In particular, the inclusion hierarchies of hyper-finite factors, the extension hierarchies of rationals, and fractal inclusion hierarchies of subalgebras of supersymplectic algebra isomorphic with the entire algebra are proposed to be more or less one and the same thing in TGD framework.

The time evolution operator of Connes could corresponds to super-symplectic algebra (SSA) to the time evolution generated by $exp(iL_0\tau)$ so that the operator Δ of Connes would be identified as $\Delta = exp(L_0)$. This identification allows number theoretical universality if τ is quantized. Furthermore, one ends up with a model for the subjective time evolution by small state function reductions (SSFRs) for SSA with SSA_n gauge conditions: the unitary time evolution for given SSFR would be generated by a linear combination of Virasoro generators not annihilating the states. This model would generalize the model for harmonic oscillator in external force allowing exact S-matrix.

10.5.1 Connes proposal and TGD

In this section I develop in more detail the analog of Connes proposal in TGD framework.

What does Connes suggest?

One must first make clear what the automorphism of HFFs discovered by Connes is.

1. Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. I have described the theory earlier [K75, K44].

First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.
2. Let

$$L^{\infty}(\mathcal{M}) \equiv \mathcal{M} \quad , \quad L^2(\mathcal{M}) = \mathcal{H} \quad , \quad L^1(\mathcal{M}) = \mathcal{M}_* \quad , \tag{10.5.1}$$

   where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has ${\mathcal{M}_*}^* = \mathcal{M}$.
3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.
4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.
5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ \ , J\mathcal{M}J = \mathcal{M}' \ \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A42, A72] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \rightarrow \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

The definition of Δ^{it} reduces in eigenstate basis of Δ to the definition of complex function d^{it}. Note that is positive so that the logarithm of d is real.

In TGD framework number theoretic universality poses additional conditions. In diagonal basis $e^{log(d)it}$ must exist. A simply manner to solve the conditions is $e = exp(m/r)$ existing p-adically for an extension of rational allowing r:th root of e. This requires also quantization of as a root of unity so that the exponent reduces to a root of unity.

2. Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists of real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

3. Objections against the idea of Connes

One can represent objections against this idea.

1. Ordinary time evolution in wave mechanics is a unitary automorphism, so that in this framework they would not have physical meaning but act as gauge transformations. If outer automorphisms define time evolutions, they must act as gauge transformations. One would have an analog of gauge field theory in HFF. This would be of course highly interesting: when I gave up the idea of Connes, I did not consider this possibility. Super-symplectic algebras having fractal structure are however extremely natural candidate for defining HFF and there is infinite number of gauge conditions.

2. An automorphism is indeed in question so that the algebraic system would not be actually affected. Therefore one cannot say that HFF has inherent time evolution and time. However, one can represent in HFF dynamical systems obeying this inherent time evolution. This possibility is highly interesting as a kind of universal gauge theory.

   On the other hand, outer automorphisms affect the trace of the projector defining the identity matrix for a given factor. Does the scaling factor Λ represent some kind of renormalization operation? Could it relate to the action of scalings in the TGD framework where scalings replace time translations at the fundamental level? What the number theoretic vision of TGD could mean? Could this quantize the continuous spectrum of the scalings Λ for HFFs so that they belong to the extension? Could one have a spectrum of Λ for each extension of rationals? Are different extensions related by inclusions of HFFs?

3. The notion of time evolution itself is an essentially Newtonian concept: selecting a preferred time coordinate breaks Lorentz invariance. In TGD however time coordinate is replace by scaling parameter and the situation changes.

4. The proposal of Connes is not general enough if evolution is interpreted as an increase of complexity.

For these reasons I gave up the automorphism proposed by Connes as a candidate for defining time evolution giving rise to scattering amplitudes in TGD framework.

Two views about TGD

The two dual views about what TGD is described briefly in [L119].

1. Physics as geometry of the world of "world of classical worlds" (WCW) identified as the space of space-time surfaces in $M^4 \times CP_2$ [K96]. Twistor lift of TGD [L55] implies that the space-time surfaces are minimal surfaces which can be also regarded as extermals of the Kähler action. This implies holography required by the general coordinate invariance in TGD framework.

2. TGD as generalized number theory forcing to generalize physics to adelic physics [L53] fusing real physics as correlate of sensory experience and various p-adic physics as correlates of cognition. Now space-times are naturally co-associative surfaces in complexified M^8 (complexified octonions) defined as "roots" of octonionic polynomials determined by polynomials with rational coefficients [L109, L110, L134]. Now holography extends dramatically: finite number of rational numbers/roots of rational polynomial/points of space-time region dictate it.

$M^8 - H$ duality relates these two views and is actually a generalization of Fourier transform and realizes generalization of momentum-position duality.

The notion of time evolution in TGD

Concerning various time evolutions in TGD, the general situation is now rather well understood.

There are two quantal time evolutions: geometric one assignable to single CD and and subjective time evolution which reflects the generalization of point-like particle to a 3-surface and the introduction of CD as 4-D perceptive field of particle in ZEO [L91].

1. Geometric time evolution corresponds to the standard scattering amplitudes for which I have a general formula now in terms of zero energy ontology (ZEO) [L123, L109, L110, L134]. The analog of S-matrix corresponds to entanglement coefficients between members of zero energy state at opposite boundaries of causal diamond (CD).

2. Subjective time evolution of conscious entity corresponds to a sequence of "small" state function reductions (SSFRs) as moments of consciousness: each SSFR is preceded by an analog of unitary time evolution, call it U. SSFRs are the TGD counterparts of "weak" measurements.

   $U(t)$ is generated by the scaling generator L_0 scaling light-like radial coordinate of light-cone boundary and is a generalization of corresponding operator in superconformal and string theories and defined for super-symplectic algebras acting as isometries of the world of classical worlds (WCW) [L134]. $U(t)$ is not the exponential of energy as a generator of time translation as in QFTs but an exponential of the mass squared operator and corresponds to the scaling of radial light-like coordinate r of the light-like boundary of CD: r is analogous to the complex coordinate z in conformal field theories.

   Also "big" SFRs (BSFRs) are possible and correspond to "ordinary" SFRs and in TGD framework mean death of self in the universal sense and followed by reincarnation as time reversed subjective time evolution [L77].

3. There is also classical time evolution at the level of space-time surfaces. Here the assumption that X^4 belongs to $H = M^4 \times CP_2$ defines Minkowski coordinates of M^4 as almost unique space-time coordinates of X^4 is the M^4 projection of X^4 is 4-D. This generalizes also to the case of M^8. Symmetries make it possible to identify an essentially a unique time coordinate.

   This means enormous simplification. General coordinate invariance is a marvellous symmetry but it leads to the problem of specifying space-time coordinates that is finding preferred coordinates. This seems impossible since 3-metric is dynamical. M^4 provides a fixed reference system and the problem disappears. M^4 is dynamical by its Minkowskian signature and one can speak about classical signals.

4. There is also classical time evolution for the induced spinor fields. At the level of H the spinor field is a superposition of modes of the massless Dirac operator (massless in 8-D sense). This spinor field is free and second quantized. Second quantization of induced spinor trivializes and this is absolutely crucial for obtaining scattering amplitudes for fermions and avoiding the usual problems for quantization of fermions in curved background.

   The induced spinor field is a restriction of this spinor field to the space-time surface and satisfies modified Dirac equation automatically. There is no need for second quantization at the level of space-time surface and propagators etc.... are directly calculable. This is an enormous simplification.

There are therefore as many as 4 time evolutions and subjective time evolution by BSFRs and possibly also by SSFRs is a natural candidate for time evolution as genuine evolution as emergence of more complex algebraic structures.

Could the inherent time evolution of HFF have a physical meaning in TGD after all?

The idea about inherent time evolution defined by HFF itself as one parameter group of outer automorphisms is very attractive by its universality: physics would become part of mathematics.

1. Thermodynamic interpretation, with inverse temperature identified as an analog of time coordinate, comes first in mind but need not be the correct interpretation.

2. Outer automorphisms should act at a very fundamental level analogous to the state space of topological field theories. Fundamental group is after all in question! The assignment of the S-matrix of particle physics to the outer automorphism does not look reasonable since the time evolution would be with respect to the linear Minkowski coordinate, which is not Lorentz invariant.

For these reasons I gave up the idea of Connes when considering it for the first time. However, TGD inspired theory of consciousness as a generalization of quantum measurement theory has evolved since then and the situation is different now.

The sequence of SSFRs defines subjective time evolution having no counterpart in QFTs. Each SSFR is preceded by a unitary time evolution, which however corresponds to the scaling of the light-like radial coordinate of the light-cone boundary [L134] rather than time translation. Hamiltonian is replaced with the scaling generator L_0 acting as Lorentz invariant mass squared operator so that Lorentz invariance is not lost.

Could the time evolution assignable to L_0 correspond to the outer automorphism of Connes when one poses an infinite number of gauge conditions making inner automorphisms gauge transformations? The connection of Connes proposal with conformal field theories and with TGD is indeed suggestive.

1. Conformally invariant systems obey infinite number of gauge conditions stating that the conformal generators L_n, $n > 0$, annihilate physical states and carry vanishing Noether charges.

   These gauge conditions bring in mind the condition that infinitesimal inner automorphisms do not change the system physically. Does this mean that Connes outer automorphism generates the time evolution and inner automorphisms act as gauge symmetries? One would have an analog of gauge field theory in HFF.

2. In TGD framework one has an infinite hierarchy of systems satisfying conditions analogous to the conformal gauge conditions. The generators of the super-symplectic algebra (SCA) acting as isometries of the "world of classical worlds" (WCW) are labelled by non-negative conformal weight n and it has infinite hierarchy of algebras SCA$_k$ isomorphic to it with conformal weights given by k-multiple of those of the entire algebra, $k = 1, 2, ...$

   Gauge conditions state for SCA_k that the generators of SCA_k and its commutator with SCA annihilate physical states. The interpretation is in terms of a hierarchy of improving measurement resolutions with degrees of freedom below measurement resolution acting like gauge transformations.

   The Connes automorphism would "see" only the time evolution in the degrees of freedom above measurement resolution and as k increases, their number would increase.

In the case of hyperfinite factors of type II_1 (HFFs) the fundamental group of corresponding factor II_∞ consists of all reals: I hope I am right here.

1. The hyperfinite factors of type II_1 and corresponding factors II_∞ are natural in the TGD context. Therefore the spectrum would consist of reals unless one poses additional conditions.

2. Could the automorphisms correspond to the scalings of the lightcone proper time, which replace time translations as fundamental dynamics. Also in string models scalings take the role of time translations.

3. In zero energy ontology (ZEO) the scalings would act in the moduli space of causal diamonds which is finite-dimensional. This moduli space defines the backbone of the "world of classical worlds". WCW itself consists of a union of sub-WCs as bundle structures over CDs [?]. The fiber consists of space-time surfaces inside a given CD analogous to Bohr orbits and satisfying holography reducing to generalized holomorphy. The scalings as automorphisms scale the causal diamonds. The space of CDs is a finite-dimensional coset space and has also other symmetry transformations.

4. The number theoretic vision suggests a quantization of the spectrum of Λ so that for a given extension of rationals the spectrum would belong to the extension. HFFs would be labelled at least partially by the extensions of rationals. The recent view of $M^8 - H$ duality [L174] is dramatically simpler than the earlier view [L109, L110, ?] and predicts that the space-time regions are determined by a pair of analytic functions with rational coefficients forced by number theoretical universality meaning that the space-time surfaces have interpretation also as p-adic surfaces.

   The simplest analytic functions are polynomials with integer coefficients and if one requires that the coefficients are smaller than the degree of the polynomial, the number of polynomials is finite for a given degree. This would give very precise meaning for the concept of number theoretic evolution.

   There would be an evolutionary hierarchy of pairs of polynomials characterized by increasing complexity and one can assign to these polynomials extension of rationals characterized by ramified primes depending on the polynomials. The ramified primes would have interpretation as p-adic primes characterizing the space-time region considered. Extensions of rationals and ramified primes could also characterize HFFs. This is a rather dramatic conjecture at the level of pure mathematics.

5. Scalings define renormalization group in standard physics. Now they scale the size of the CD. Could the scalings as automorphisms of HFFs correspond to discrete renormalization operations?

Three views about finite measurement resolution

Evolution could be seen physically as improving finite measurement resolution: this applies to both sensory experience and cognition. There are 3 views about finite measurement resolution (FMR) in TGD.

1. Hyper finite factors (HFFs) and FMR

HFFs are an essential part of Connes's work and I encountered them about 15 years ago or so [K126, K44].

The inclusions of hyper-finite factors HFFs provide one of the three - as it seems equivalent - ways to describe finite measurement resolution (FMR) in TGD framework: the included factor defines an analog for gauge degrees of freedom which correspond to those below measurement resolution.

2. Cognitive representations and FMR

Another description for FMR in the framework of adelic physics would be in terms of cognitive representations [L81]. First some background about M^8-H duality.

1. There are number theoretic and geometric views about dynamics. In algebraic dynamics at the level of M^8, the space-time surfaces are roots of polynomials. There are no partial differential equations like in the geometric dynamics at the level of H.
2. The algebraic "dynamics" of space-time surfaces in M^8 is dictated by co-associativity, which means that the normal space of the space-time surface is associative and thus quaternionic. That normal space rather than tangent space must be associative became clear last year [L109, L110].
3. M^8-H duality maps these algebraic surfaces in M^8 to $H=M^4\times CP_2$ and the one obtains the usual dynamics based on variational principle giving minimal surfaces which are non-linear analogs for the solutions of massless field equations. Instead of polynomials the natural functions at the level of H are periodic functions used in Fourier analysis [L134].

At level of complexified M^8 cognitive representation would consist of points of co-associative space-time surface X^4 in complexified M^8 (complexified octonions), whose coordinates belong to extension of rationals and therefore make sense also p-adically for extension of p-adic numbers induced by extension of rationals. M^8-H duality maps the cognitive representations to H.

Cognitive representations form a hierarchy: the larger the extension of rationals, the larger the number of points in the extension and in the unique discretization of space-time surface. Therefore also the measurement resolution improves.

The surprise was that the cognitive representations which are typically finite, are for the "roots" of octonionic polynomials infinite [L109, L110]. Also in this case the density of points of cognitive representation increases as the dimension of extensions increases.

The understanding of the physical interpretation of M^8-H duality increased dramatically during the last half year.

1. X^4 in M^8 is highly analogous to momentum space (4-D analog of Fermi ball one might say) and H to position space. Physical states correspond to discrete sets of points - 4-momenta - in X^4. This is just the description used in particle physics for physical states. Time and space in this description are replaced by energy and 4-momentum. At the level of H one space-time and classical fields and one talks about frequencies and wavelengths instead of momenta.
2. M^8-H duality is a generalization of Fourier transform. Hitherto I have assumed that the space-time surface in M^8 is mapped to H. The momentum space interpretation at the level of M^8 however requires that the image must be a superposition of translates of the image in plane wave with some momentum: only the translates inside some bigger CD are allowed - this means infrared cutoff.

The total momentum as sum of momenta for two half-cones of CD in M^8 is indeed well-defined. One has a generalization of a plane wave over translational degrees of freedom of CD and restricted to a bigger CD.

At the limit of infinitely large size for bigger CD, the result is non-vanishing only when the sum of the momenta for two half-cones of CD vanishes: this corresponds to conservation of 4-momentum as a consequence of Poincare invariance rather than assumption as in the earlier approach [L134].

This generalizes the position-momentum duality of wave mechanics lost in quantum field theory. Point-like particle becomes a quantum superposition of space-time surfaces inside the causal diamond (CD). Plane wave is a plane wave for the superposition of space-time surfaces inside CD having the cm coordinates of CD as argument.

3. Inclusion hierarchy of supersymplectic algebras and FMR

The third inclusion hierarchy allowing to describe finite measurement resolution is defined by supersymplectic algebras acting as the isometries of the "world of classical worlds" (WCW) consisting of space-time surfaces are preferred extremals ("roots" of polynomials in M^8 and minimal surfaces satisfying infinite-D set of additional "gauge conditions" in H).

At a given level of hierarchy generators with conformal weight larger than n act like gauge generators as also their commutators with generators with conformal weight smaller than n correspond to vanishing Noether charges. This defines "gauge conditions".

To sum up, there are therefore 3 hierarchies allowing to describe finite measurement resolution and they must be essentially equivalent in TGD framework.

Three evolutionary hierarchies

There are three evolutionary hierarchies: hierarchies of extensions of extensions of... ofrationals...; inclusions of inclusions of .... of HFFs, and inclusions of isomorphic super symplectic algebras.

1. Extensions of rationals

The extensions of rationals become algebraically increasingly complex as their dimension increases. The co-associative space-time surfaces in M^8 are "roots" of real polynomials with rational coefficients to guarantee number theoretical universality and this means space-time surfaces are characterized by extension of rationals.

Each extension of rationals defines extensions for p-adic number fields and entire adele. The interpretation is as a cognitive leap: the system's intelligence/algebraic complexity increases when the extension is extended further.

The extensions of extensions of .... define hierarchies with Galois groups in certain sense products of extensions involved. Exceptional extensions are those which do not allow this decomposition. In this case Galois group is a simple group. Simple groups are primes of finite groups and correspond to elementary particles of cognition. Kind of fundamental, non-decomposable ideas. Mystic might speak of pure states of consciousnesswith no thoughts.

In the evolution by quantum jumps the dimension of extension increases in statistical sense and evolution is unavoidable. This evolution is due to subjective time evolution by quantum jumps, something which is in spirit with Connes proposal but replaces time evolution by a sequence of evolutionary leaps.

2. Inclusions of HFFs as a hierarchy

HFFs are fractals. They have infinite inclusion hierarchies in which sub-HFF isomorphicto entire HFFs is included to HFF.

Also the hierarchies of inclusions define evolutionary hierarchies: HFF which is isomorphic with original becomes larger and in some sense more complex than the included factor. Also now one has sequences of inclusions of inclusions of.... These sequences would correspond to sequences for extensions of extensions... of rationals. Note that the inclusion hierarchy would be the basic object: not only single HFF in the hierarchy.

3. Inclusions of supersymplectic algebras as an evolutionary hierarchy

The third hierarchy is defined by the fractal hierarchy of sub-algebras of supersymplectic algebra isomorphic to the algebra itself. At a given level of hierarchy generators with conformal weight larger than n correspond to gauge degrees of freedom. As n increases the number of physical degrees of freedom above measurement resolution increases which means evolution. This hierarchy should correspond rather concretely to that for the extensions of rationals. These hierarchies would be essentially one and the same thing in the TGD Universe.

TGD based model for subjective time development

The understanding of subjective time development as sequences of SSFRs preceded by unitary "time" evolution has improved quite considerably recently [L134]. The idea is that the subjective time development as a sequence of scalings at the light-cone boundary generated by the vibrational part $\hat{L}_0$ of the scaling generator $L_0 = p^2 - \hat{L_0}$ (L_0 annihilates the physical states). Also p-adic mass calculations use $\hat{L}_0$.

For more than 10 years ago [K75, K44], I considered the possibility that Connes time evolution operator that he assigned with thermo-dynamical time could have a significant role in the definition of S-matrix in standard sense but had to give up the idea.

It however seems that for super-symplectic algebra $\hat{L}_0$ generates an outer automorphism since the algebra has only generators with conformal with $n > 0$ and its extension to included also generators with $n \leq 0$ is required to introduce L_0: since L_0 contains annihilation operators, it indeed generates outer automorphism in SCA. The two views could be equivalent! Whereas Connes considered thermo-dynamical time evolution, in TGD framework the time evolution would be subjective time evolution by SSFRs.

1. The guess would be that the exponential of the scaling operator L_0 gives the time evolution. The problem is that L_0 annihilates the physical states. The solution of the problem would be the same as in p-adic thermodynamics. L_0 decomposes as $L_0 = p^2 - \hat{L_0}$ and the vibrational part $\hat{L}_0$ this gives mass spectrum as eigenvalues of p^2. The thermo-dynamical state in p-adic thermodynamics is $p^{\hat{L}_0\beta}$. This operator exists p-adically in the p-adic number field defined by prime p.
2. Could unitary subjective time development involve the operator $exp(i2\pi L_0\tau)$ $\tau = log(T/T_0)$? This requires $T/T_0 = exp(n/m)$ guaranteeing that exponential is a root of unity for an eigenstate of L_0. The scalings are discretized and scalings come as powers of $e^{1/m}$. This is possible using extensions of rationals generated by a root of e. The unique feature of p-adics is that e^p is ordinary p-adic number. This alone would give periodic time evolution for eigenstates of L_0 with integer eigenvalues n.

SSA and SSA_n

Supersymplectic algebra SSA has fractal hierarchies of subalgebras SSA_n. The integers in a given hierarchy are of forn $n_1, n_1n_2, n_1n_2n_3, ...$ and correspond naturally to hierarchies of inclusions of HFFs. Conformal weights are positive: $n > 0$. For ordinary conformal algebras also negative weights are allowed. Yangians have only non-negative weights. This is of utmost importance.

SSA_n with generators have radial light-like conformal weights coming as multiples of n. SSA_n annihilates physical states and $[SSA_n, SSA]$ does the same. Hence the generators with conformal weight larger than n annihilate the physical states.

What about generators with conformal weights smaller than n? At least a subset of them need not annihilate the physical states. Since L_n are superpositions of creation operators, the idea that analogs of coherent states could be in question.

It would be nice to have a situation in which L_n, $n < m$ commute. $[L_k, L_l] = 0$ effectively for $k + l \geq m$.

The simplest way to obtain a set of effectively commuting operators is to take the generators L_k, $[m/2] < k < m$, where $[m/2]$ is nearest integer larger than $m/2$.

This raises interesting questions.

1. Could the Virasoro generators $O(\{c_k\}) = \sum_{k\in[m/2],m]} c_kL_k$ as linear combinations of creation operators generate a set of coherent states as eigenstates of their Hermitian conjugates.
2. Some facts about coherent states are in order.

(a) When one adds to quantum harmonic oscillator Hamiltonian oscillator a time dependent perturbation which lasts for a finite the vacuum state evolves to an oscillator vacuum whose position is displacemented. The displacement is complex and is a Fourier component of the external force $f(t)$ corresponding to the harmonic oscillator frequency ω. Time evolution picks up only this component.

(b) Coherent state property means that the state is eigenstate of the annihilation creation operator with eivengeu $\alpha = -ig(\omega)$ where $g(omega) = \int f(u)exp(-i\omega u)du$ is Fourier transform of $f(t)$.

(c) Coherent states are not orthogonal and form an overcomplete set. The overlaps of coherent states are proportional to a Gaussian depending on the complex parameters characterizing them. One can however develop any state in terms of coherent states as a unique expansion since one can represent unitary in terms of coherent states.

(d) Coherent state obtained from the vacuum state by time evolution in presence of $f(t)$ by a unitary displacement operator $D(\alpha) = exp(\alpha a^\dagger - \overline{\alpha}a)$. (`https://en.wikipedia.org/wiki/Displacement_operator`).

The displacement operator is a unitary operator and in the general case the displacement is complex. The product of two displacement operators would be apart from a phase factor a displacement operator associated with the sum of displacements.

(e) Harmonic oscillator coherent states are indeed maximally classical since wave packets have minimal width in both q and p space. Furthermore, the classical expectation values for q and p obey classical equations of motion.

These observations raise interesting questions about how the evolution by SSFRs could be modelled.

1. Instead of harmonic oscillator in q-space, one would have time evolution in the space of scalings of causal diamond parameterized by the scaling parameter $\tau = log(T/T_0)$, where T can be identified as the radial light-like coordinate of light-cone boundary.

   The analogs of harmonic oscillator states would be defined in this space and would be essentially wave packets with ground state minimizing the width of the wave packet.

2. The role of harmonic oscillator Hamiltonian in absence of external force would be taken by the generator $\hat{L}_0$ ($L_0 = p^2 - \hat{L}_0$ acts trivially) and gives rise to mass squared quantization. The situation would be highly analogous to that in p-adic thermodynamics. The role of ω would be taken by the minimal conformal weight h_{min} such that the eigenvalues of L_0 are its multiples. It seems that this weight must be equal to $h_{min} = 1$.

   The commutations of $\hbar L_0$ with L_k, $k > 0$ would be as for L_0 so what the replacement should not affect the situation.

3. The scaling parameter τ is analogous to the spatial coordinate q for the harmonic oscillator. Can one identify the analog of the external force $f(t)$ acting during unitary evolution between two SSFRs? Or is it enough to use only the analog of $g(\omega \rightarrow h_{min} = 1)$ - that is the coefficients C_k.

   To identify $f(t)$, one needs a time coordinate t. This was already identified as τ. This one would have $q = t$, which looks strange. The space in which time evolution is the space of scalings and the time evolutions are scalings and thus time evolution means translation in this space. The analog for this would be Hamiltonian $H = i\hbar d/dq$.

   Number theoretical universality allows only the values of $\tau = r/s$ whose exponents give roots of unity. Also $exp(n\tau)$ makes sense p-adically for these values. This would mean that the Fourier transform defining g would become discrete and be sum over the values $f(\tau = r/s)$.

4. What happens if one replaces $\hat{L}_0$ with L_0. In this case one would have the replacement of ω with $h_{vac} = 0$. Also the analog of Fourier transform with zero frequency makes sense. $\hat{L_0} = p^2 - L_0$ is the most natural choice for the Hamiltonian defining the time evolution operator but is trivial. Could $\Delta^{i\tau}$ describe the inherent time evolution. It would be outer automorphism since it is not defined solely in terms of SCA. So: could one have $\Delta = exp(\hat{L_0})$ so that $\Delta^{i\tau}$ coincide with $exp(i\hat{L_0}\tau)$? This would mean the identification

$$\Delta = exp(\hat{L_0}) \ ,$$

which is a positive definite operator. The exponents coming from $exp(iL_0\tau)$ can be number theoretically universal if $\tau = log(T/T_0)$ is a rational number implying $T/T_0 = exp(r/s)$, which is possible number theoretically) and the extension of rationals contains some roots of e.

5. Could one have $\Delta = L_0$? Also now that positivity condition would be satisfied if SSA conformal weights satisfy $n > 0$.

   The problem with this operation is that it is not number theoretically universal since the exponents $exp(ilog(n)\tau)$ do not exist p-adically without introducing infinite-D extension of p-adic number making $log(n)$ well-defined.

   What is however intriguing is that the "time" evolution operator $\Delta^{i\tau}$ in the eigenstate basis would have trace equal to $Tr(\Delta^{i\tau}) \sum d(n)n^{i\tau}$, where $d(n)$ is the degeneracy of the state. This is a typical zeta function: for Riemann Zeta one has $d(n) = 1$.

   For $\Delta = exp(L_0)$ option $Tr(\Delta^{i\tau}) = \sum d(n)exp(in\tau)$ exists for $\tau = r/s$ if r:th root of e belongs to the extension of p-adics.

To sum up, one would have Gaussian wave packet as harmonic oscillator vacuum in the space of scaled variants of CD. The unitary time evolution associated with SSFR would displace the peak of the wave packet to a larger scalings. The Gaussian wave function in the space of scaled CDs has been proposed earlier.

Could this time evolution make sense and be even realistic?

1. The analogs of harmonic oscillator states are defined in the space of scalings as Gaussians and states obtained from them using oscillator operators. There would be a wave function in the moduli space of CDs analogous to a state of harmonic oscillator.
2. SSFR following the time evolutions would project to an eigenstate of harmonic oscillator having in general displaced argument. The unitary displacement operator D should commute with the operators having the members of zero energy states at the passive boundary of CD as eigenstates. This poses strong conditions. At least number theoretic measurements could satisfy these conditions.
3. SSFRs are identified as weak measurements as near as possible to classical measurements. Time evolution by the displacement would be indeed highly analogous to classical time evolution for theeharmonic oscillator.
4. The unitary displacement operator corresponds to the arbitrary external force on the harmonic oscillator and it seems that it would be selected in SSFR for the unitary evolution after SSFR. This means fixing the coefficients C_k in the operator $\sum C_k L_k$.

What is the subjective "time" evolution operator when in the case of SSA_n?

1. The scaling analog of the unitary displacement operator D as $D = \sum exp(\sum C_k L_k - \overline{C}_k L_{-k})$ is highly suggestive and would take the oscillator vacuum to a coherent state. Coefficients C_k would be proportional to τ. There would be a large number of choices for the unitary displacement operator. One can also consider complex values of τ since one has complexified M^8.
2. There should be a normalization for the coefficients: without this it is not possible to talk about a special value of τ does not make sense. For instance, the sum of their moduli squared could be equal to 1. This would give interpretation as a quantum state in the degrees of freedom considered. The width of the Gaussian would increase slowly during the unitary time evolution and be proportional to $log(T/T_0)$.

   The width of the Gaussian would increase slowly as a function of T during the unitary time evolution and be proportional to $log(T/T_0)$. The condition that c_k are proportional the same complex number times τ is too strong.
3. The arbitrariness in the choice of C_k would bring in a kind of non-determinism as a selection of this superposition. The ability to engineer physical systems is in conflict with the determinism of classical physics and also difficult to understand in standard quantum physics. Could one interpret this choice as an analog for engineering a Hamiltonian as in say quantum computation or build-up of an electric circuit for some purpose? Could goal directed action correspond to this choice?

If so engineerable degrees of freedom would correspond to intermediate degrees of freedom associated with L_k, $[m/2] \leq k \leq m$. They would be totally absent for $k = 1$ and this would correspond to a situation analogous to the standard physics without any intentional action.

10.6 The dynamics of SSFRs as quantum measurement cascades in the group algebra of Galois group

Adelic physics [L52, L54] is a proposal for the physics of both sensory experience having real physics as correlate and cognition having various p-adic physics as correlates. Adele is a book-like structure formed by real numbers and the extensions of p-adic number fields induced by a given extension of rationals with the pages of the book glued together along its back consisting of numbers belonging to the extension of rationals. This picture generalizes to space-time level. Adelic physics relies on the notion of cognitive representation as unique number theoretic discretization of the space-time surface. This discretization has also fermionic analog in terms of spinor structure associated with the group algebra of the Galois group of extension.

Adelic physics, $M^8 - H$ duality, and zero energy ontology lead (ZEO) to a proposal that the dynamics involved with "small" state function reductions (SSFRs) as counterparts of weak measurements could be basically number theoretical dynamics with SSFRs identified as reduction cascades leading to completely un-entangled state in the space of wave functions in Galois group of extension of rationals identifiable as wave functions in the space of cognitive representations. As a side product a prime factorization of the order of Galois group is obtained.

The result looks even more fascinating if the cognitive dynamics is a representation for the dynamics in real degrees of freedom in finite resolution characterized by the extension of rationals. If cognitive representations represent reality approximately, this indeed looks very natural and would provide an analog for adele formula expressing the norm of a rational as the inverse of the product of is p-adic norms.

10.6.1 Adelic physics very briefly

Number theoretic vision leading to adelic physics [L52] provides a general formulation of TGD complementary to the vision [K96] (`http://tinyurl.com/sh42dc2`) about physics as geometry of "world of classical words" (WCW).

1. p-Adic number fields and p-adic space-time sheets serve as correlates of cognition. Adele is a Cartesian product of reals and extensions of all p-adic number fields induced by given extension of rationals. Adeles are thus labelled by extensions of rationals, and one has an evolutionary hierarchy labelled by these extensions. The large the extension, the more complex the extension which can be regarded as $n - D$ space in K sense, that is with K-valued coordinates.

2. Evolution is assigned with the increase of algebraic complexity occurring in statistical sense in BSFRs, and possibly also during the time evolution by unitary evolutions and SSFRs following them. Indeed, in [L113] (`http://tinyurl.com/quofttl`) I considered the possibility that the time evolution of self in this manner could be induced by an iteration of polynomials - at least in approximate sense. Iteration is a universal manner to produce fractals as Julia sets and this would lead to the emergence of Mandelbrot and Julia fractals and their 4-D generalizations. In the sequel will represent and argument that the evolution as iterations could hold true in exact sense.

   Cognitive representations are identified as intersection of reality and various p-adicities (cognition). At space-time level they consist of points of embedding space $H = M^4 \times CP_2$ or M^8 ($M^8 - H$ duality [L49, L50, L51] allows to consider both as embedding space) having preferred coordinates - M^8 indeed has almost unique linear M^8 coordinates for a given octonion structure.

3. Given extension of given number field K (rationals or extension of rationals) is characterized by its Galois group leaving K - say rationals - invariant and mapping products to products and sums to sums. Given extension E of rationals decomposes to extension E_N of extension E_{N-1} of ... of extension E_1 - denote it by $E \equiv H_N = E_N \circ E_{N-1} ... \circ E_1$. It is represented

at the level of classical space-time dynamics in M^8 (`http://tinyurl.com/quofttl`) by a polynomial P which is functional composite $P = P_N \circ P_{N-1} \circ ... \circ P_1$. with $P_i(0) = 0$. The Galois group of $G(E)$ has the Galois group $H_{N-1} = G(E_{N-1} \circ ... \circ E_1)$ as a normal subgroup so that $G(E)/H_{N-1}$ is group.

The elements of $G(E)$ allow a decomposition to a product $g = h_{N-1} \times h_{N-1} \times ...$ and the order of $G(E)$ is given as the product of orders of H_k: $n = n_0 \times .. \times n_{N-1}$. This factorization of prime importance also from quantum point of view. Galois groups with prime order do not allow this decomposition and the maximal decomposition and are actually cyclic groups Z_p of prime order so that primes appear also in this manner.

Second manner for primes to appear is as ramified primes p_{ram} of extension for which the p-adic dynamics is critical in a well-defined sense since the irreducible polynomial with rational coefficients defining the extension becomes reducible (decomposes into a product) in order $O(p) = 0$. The p-adic primes assigned to elementary particles in p-adic calculation have been identified as ramified primes but also the primes labelling prime extensions possess properties making them candidates for p-adic primes.

Iterations correspond to the sequence $H_k = G_0^{\circ k}$ of powers of generating Galois groups for the extension of K serving as a starting point. The order of H_k is the power n_0^k of integer $n_0 = \prod p_{0i}^{k_i}$. Now new primes emerges in the decomposition of n_0. Evolution by iteration is analogous to a unitary evolution as ex^{iHt} power of Hamiltonian, where t parameter takes the role of k .

4. The complexity of extension is characterized by the orders n and the orders n_k as also the number N of the factors. In the case of iterations of extension the limit of large N gives fractal.

5. Galois group acts in the space of cognitive representations and for Galois extensions for which Galois group has same order as extensions, it is natural do consider quantum states as wave functions in $G(E)$ forming n-D group algebra. One can assign to the group algebra also spinor structure giving rise to $D = 2^{M/2}$ fermionic states where one has $N = 2M$ or $N = 2M + 1$). One can also consider chirality constraints reducing D by a power of 2. An attractive idea is that this spinor structure represents many-fermion states consisting of $M/2$ fermion modes and providing representation of the fermionic Fock space in finite measurement resolution.

10.6.2 Number theoretical state function reductions as symmetry breaking cascades and prime factorizations

The proposed picture has very important quantal implications and allows to interpret number theoretic quantum measurement as a number theoretic analog for symmetric breaking cascade and also as a factorization of an integer into primes.

1. The wave functions in $G(E)$ - elements of group algebra of $G(E)$ can be decomposed to tensor products of wave functions in $G(E)/H_{N-1}$ and H_{N-1}: these wave functions in general represent entangled states. One can decompose the wave functions in H_{N-1} in similar manner and the process can be continued so that one obtains a maximal decomposition allowing no further decomposition for any factor. These non-decomposable Galois groups have prime order since its group algebra as Hilbert space of prime dimension has no decomposition into tensor product.

2. In state function reduction of wave function $G(E)$ the density matrices associated with pairs $G(E)/H_{N-1}$ and H_{N-1} are measured. The outcome is an eigenstate or eigen-space and gives rise to symmetry breaking from $G(E) \equiv H_N$ to $E_N \times H_{N-1}$. The sequence of state function reductions should lead to a maximal symmetry breaking corresponding to a wave function as a produce of those associaetd with Galois groups of prime order. This define a prime factorization of the dimension n of Galois group/extension to $n = \prod_{i=1}^{N} p_i^k$! The moments of consciousness for self would correspond to prime factorizations! Self would be number theoretician quite universally!

Also also the fermionic cognitive representation based on finite-D Fock states defined by spinor components of $G(E)$ is involved. The interpretation of Fock state basis as a a basis of

Boolean algebra in TGD: the spinor structure of WCW could be representation for Boolean logic as a “square root” of Kähler geometry of WCW. Cognition indeed involves also Boolean logic.

10.6.3 SSFR as number theoretic state function reduction cascade and factorization of integer

A highly interesting unanswered question is following. “Small” state function reductions (SSFRs) define the life cycle of self as their sequence. What are the degrees of freedom where SSFRs occur?

1. SSFRs take place at the active boundary of CD which shifts in statistical sense towards future in the sequence of state function reductions. State at the passive boundary is not changed.
2. The idea that quantum randomness could correspond to classical chaos (or complexity) associated with the iteration of polynomials (Mandelbrot and Julia fractals) [L113] led to reconsider the hypothesis that the polynomial representing space-time decomposes to a product $P = P_2(T - r) \times P_1(r)$. T corresponds to the distance between the tips of CD and $r = t$ to the radial coordinate of M^4 assignable to the passive boundary of CD and equal to time coordinate t. $P_i(0) = 0$ is assumed to hold true.

   P_2 would change in SSFRs whereas P_1 and state at passive boundary would not. SSFRs (analogous to so called weak measurements) at active boundary would give rise to sensory input and various associations - Maya in Eastern terminology. P_1 would correspond to the unchanging part of self - “soul” or real self as one might say.

   I was also led to consider a simplified hypothesis that P_2 is obtained as iteration $P_2 = Q_1^{\circ n}$ in n:th n unitary evolution preceding SSFR. One would start from some iterate $Q_1^{\circ k}$. This would reduce quantum dynamics to iteration of polynomials and to a deep connection with Mandelbrot and Julia fractals but it was quite clear why this would be true.
3. The mere factorization $P = P_2 \times P_1$ implies that the Galois groups associated with active and passive boundary of CD commute and number theoretic state function reduction cascade for the wave functions in $G(E)$ for the extension determined by P_2 at active boundary could correspond to SSFR. Or course, also other commuting degrees of freedom are possible but number theoretic degrees of freedom could be the most important degrees of freedom involved with SSFRs.

10.6.4 The quantum dynamics of dark genes as factorization of primes

Gene level provides a fascinating application of this picture.

Thiscontribution was inspired by discussion with Bruno Marchal about his with title ”Do the laws of physics apply to the mind?” (`https://tinyurl.com/ycls2bpt`). Bruno Marchal is a representative of computationalism, which might be called idealistic and Bruno believes that physics follows from computationalism. The somewhat mystical notion of self-reference is believed to lead to consciousness . I do not share this view. The gist of the posting comes towards end where I describe how computationalism generalizes to quantum computationalism in TGD generalizing also the notion of quantum computation. What conscious problem solving is? This is the question to be discussed.

1. As found, dark photons and dark protons forming DNA codons as triplets could correspond to triplet representations for prime factor Z_3 of Galois group of Z_6. Codon and conjugate codon could in turn correspond to the prime factor Z_2 of Galois group Z_6 so that double strand would correspond to Z_6 suggested by findings of Mills [L32] and TGD inspired model color vision [L64].
2. DNA codons could correspond to extension with Galois group Z_3, and one can consider an entire hierarchy of extensions of extensions of .. .extensions with dimensions n_i satisfying thus $n = \prod_{i=1}^{N} n_i$ and having Z_6 as subgroup at the lowest level of the hierarchy. The number N of factors would be the number of polynomials in the functional composition and thus define a kind of abstraction levels (abstractions are thoughts about thoughts about..., maps of maps of ...). N is expected to increase in evolution.

3. Could this abstraction hierarchy be realized at gene level? Genes decompose into transcribed regions - exons - and introns. Could different decomposition of genes to exons and introns correspond to different values of N and n_i and to different Galois groups. Could genes themselves form larger composites?

   Could genomes form even large structures such as chromosomes with larger Galois groups. Years ago I considered the possibility of a collective gene expression based on the collective MB of organelle, organ, or even population: could this correspond to an extension associated with several genomes?

4. Could SSFR correspond to a sequence of symmetry breakings for the Galois groups of these structures decomposing them to sub-groups? Number theoretic interpretation would in terms of decompositions of integers to primes! Genome would be a quantum computer performing number theory!

5. Metabolic energy feed would increasing h_{eff} would also increase the orders $n_i = h_{eff}/h_0$ of the extensions appearing in the composition of extensions and thus the orders of polynomial factors P_i in the functional composite defining the extensions. Therefore the decompositions would be dynamical.

   Metabolic energy feed requires BSFR changing the arrow of time if metabolic energy feed is actually feed of negative energy to environment. The emergence of a new prime factorization would require BSFR. That the time evolution by iterations would not require BSFR would support the proposal that time evolution by BSFRs could be induced by iteration dynamics for the polynomial P_2 assignable to the active boundary of CD.

10.6.5 The relationship of TGD view about consciousness to computationalism

This text was inspired by discussion with Bruno Marchal about his with title "Do the laws of physics apply to the mind?" (`https://tinyurl.com/ycls2bpt`). Bruno Marchal is a representative of computationalism, which might be called idealistic and Bruno believes that physics follows from computationalism. The somewhat mystical notion of self-reference is believed to lead to consciousness.

I do not share this view. The gist of the posting comes towards end where I describe how computationalism generalizes to quantum computationalism in TGD generalizing also the notion of quantum computation. What conscious problem solving is? This is the question to be discussed.

To my view computationalism is one of the failed approaches to consciousness - it cannot cope with free will for instance. It however contains an essential aspect which is correct: the idea of deterministic program leading from A to B. Problem solving be can regarded as attempt to find this program. You fix A as initial data and try to find a program leading from A to a final state characterized by data B. The program has duration T and can be very long and it is not clear whether it exists at all. You try again and again and eventually you might find it. In the real conscious problem solving this process means making guesses so that the process cannot be deterministic.

What does this view about problem solving correspond to in ZEO? We have states A and B represented as quantum states and we try to find quantum analog of classical program leading from A to B in some time T which can be varied.

1. A and B are realized as superpositions of 3-surfaces and fermionic states at them - located at time values t=0 and t=T. T can vary. Can we find by varying T a (superposition of) deterministic time evolution(s) - preferred extremal(s) (PE) - connecting A and B?

   In ZEO and for fixed A and T PE in general does not exist. In ideal situation (infinite measurement resolution) and for given A and T, B is unique if it exists at all. One has analog of Bohr orbit and the quantum analog of classical program as the superposition of Bohr orbits starting from A and hopefully leading to B as a solution of the problem.

   Remark: These superpositions can be regarded as counterparts of functions in biology and behaviors in neuroscience. The big difference to standard physics is that time=constant snapshot in time evolution of say bio-system is replaced with quantum superposition of very special time evolutions - PEs. Darwinian selection of also behaviors in biology correlates strongly with this.

2. So: given A and B, we try to find a value of T for which superposition of PEs from A to B exists. This would be the quantum program leading from A to B, and solving our problem.

   Actually, not only ours, universe is full of conscious entities solving problems at various levels of self hierarchy. This takes place by a sequences of "small" SFRs (SSFRs, weak measurements) increasing T in statistical sense and replacing the state at B with a new one determined by state A for given value of T. At the level of conscious experience this is sensory perception and all that which is associated with it.

   Finding the solution is analogous to the halting of quantum Turing machine by ordinary state function reduction, which corresponds in ZEO to a "big" (ordinary) SFR (BSFR). This would mean death in universal sense and reincarnation with reversed arrow of time in ZEO? Or is BSFR and death failure to solve the problem? I cannot answer.

   Remark: The notion of self-reference is replaced with much more concrete notion of becoming conscious of what one was conscious of before SSFR. SSFR indeed gives rise to conscious eperience and one avoids the infinite regress associated with genuine self-reference. As an additional bonus one obtains evolution since the extension of rationals characterizing space-time surfaces can increase meaning higher level of consciousness. At the limit algebraic numbers the cognitive representation is dense subset of space-time surface.

3. Also finite measurement resolution and discreteness characterizing computation emerge from number theory.

   To be a solution classically means that the 3-surface(s) representing B to have fixed discrete cognitive representation given by finite number of embedding space points in the extension of rationals defining the adele. Quantally, quantum superpositions of these points with fixed quantum numbers represent the desired final state.

   Also Boolean logic emerges at fundamental level as square root of Kähler geometry one might say. Many-fermion state basis defines a Boolean algebra and time evolution for induced spinors is analogous to truth preserving Boolean map in which truths code for infinite number of conservation laws associated with symmetries of WCW.

4. How to find the possibly existing solution at given step (unitary evolution plus SSFR) with t=T? One performs cognitive quantum measurements at each step represented by SSFR. They reduce to cascades of quantum measurements for the states in the group algebra of Galois group - call it Gal - of Galois extension considered.

   Gal has hierarchical decomposition to inclusion hierarchy of normal subgroups implying the representation of states in group algebra of Gal as entangled states in the tensor product of the group algebras of normal sub-groups of Gal. The hope is that this Galois cascade of SFRs produces desired state as an outcome and one can shout "Eureka!".

10.7 The relation between U-Matrix and M-matrices

S-matrix is the key notion in quantum field theories. In Zero Energy Ontology (ZEO) this notion must be replaced with the triplet U-matrix, M-matrix, and S-matrix. U-matrix realizes unitary time evolution in the space for zero energy states realized geometrically as dispersion in the moduli space of causal diamonds (CDs) leaving second boundary (passive boundary) of CD and states at it fixed.

This process can be seen as the TGD counterpart of repeated state function reductions leaving the states at passive boundary unaffected and affecting only the member of state pair at active boundary (Zeno effect) [K70]. In TGD inspired theory of consciousness self corresponds to the sequence of these state function reductions [K122, K8, K98]. M-matrix describes the entanglement between positive and negative energy parts of zero energy states and is expressible as a hermitian square root H of density matrix multiplied by a unitary matrix S, which corresponds to ordinary S-matrix, which is universal and depends only the size scale n of CD through the formula $S(n) = S^n$. M-matrices and H-matrices form an orthonormal basis at given CD and H-matrices would naturally correspond to the generators of super-symplectic algebra.

The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood.

The original view about the relationship was a purely formal guess: M-matrices would define the orthonormal rows of U-matrix. This guess is not correct physically and one must consider in detail what U-matrix really means.

1. First about the geometry of CD [K75]. The boundaries of CD will be called passive and active: passive boundary correspond to the boundary at which repeated state function reductions take place and give rise to a sequence of unitary time evolutions U followed by localization in the moduli of CD each. Active boundary corresponds to the boundary for which U induces delocalization and modifies the states at it.

   The moduli space for the CDs consists of a discrete subgroup of scalings for the size of CD characterized by the proper time distance between the tips and the sub-group of Lorentz boosts leaving passive boundary and its tip invariant and acting on the active boundary only. This group is assumed to be represented unitarily by matrices Λ forming the same group for all values of n.

   The proper time distance between the tips of CDs is quantized as integer multiples of the minimal distance defined by CP_2 time: $T = nT_0$. Also in quantum jump in which the size scale n of CD increases the increase corresponds to integer multiple of T_0. Using the logarithm of proper time, one can interpret this in terms of a scaling parametrized by an integer. The possibility to interpret proper time translation as a scaling is essential for having a manifest Lorentz invariance: the ordinary definition of S-matrix introduces preferred rest system.

2. The physical interpretation would be roughly as follows. M-matrix for a given CD codes for the physics as we usually understand it. M-matrix is product of square root of density matrix and S-matrix depending on the size scale of CD and is the analog of thermal S-matrix. State function at the opposite boundary of CD corresponds to what happens in the state function reduction in particle physics experiments. The repeated state function reductions at same boundary of CD correspond to TGD version of Zeno effect crucial for understanding consciousness. Unitary U-matrix describes the time evolution zero energy states due to the increase of the size scale of CD (at least in statistical sense). This process is dispersion in the moduli space of CDs: all possible scalings are allowed and localization in the space of moduli of CD localizes the active boundary of CD after each unitary evolution.

In the following I will proceed by making questions. One ends up to formulas allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_1 of the Virasoro algebra associated with the super-symplectic algebra.

10.7.1 What can one say about M-matrices?

1. The first thing to be kept in mind is that M-matrices act in the space of zero energy states rather than in the space of positive or negative energy states. For a given CD M-matrices are products of hermitian square roots of hermitian density matrices acting in the space of zero energy states and universal unitary S-matrix $S(CD)$ acting on states at the active end of CD (this is also very important to notice) depending on the scale of CD:

   $$M^i = H^i \circ S(CD) \ .$$

   Here "$\circ$" emphasizes the fact that S acts on zero energy states at active boundary only. H^i is hermitian square root of density matrix and the matrices H^i must be orthogonal for given CD from the orthonormality of zero energy states associated with the same CD. The zero energy states associated with different CDs are not orthogonal and this makes the unitary time evolution operator U non-trivial.

2. Could quantum measurement be seen as a measurement of the observables defined by the Hermitian generators H^i? This is not quite clear since their action is on zero energy states. One might actually argue that the action of this kind of observables on zero energy states does not affect their vanishing net quantum numbers. This suggests that H^i carry no net quantum numbers and belong to the Cartan algebra. The action of S is restricted at the active boundary of CD and therefore it does not commute with H^i unless the action is in a

separate tensor factor. Therefore the idea that S would be an exponential of generators H^i and thus commute with them so that H^i would correspond to sub-spaces remaining invariant under S acting unitarily inside them does not make sense.

3. In TGD framework symplectic algebraas isometries of WCW is analogous to a Kac-Moody algebra with finite-dimensional Lie-algebra replaced with the infinite-dimensional symplectic algebra with elements characterized by conformal weights [K29, K28]. There is a temptation to think that the H^i could be seen as a representation for this algebra or its sub-algebra. This algebra allows an infinite fractal hierarchy of sub-algebras of the super-symplectic algebra isomorphic to the full algebra and with conformal weights coming as n-ples of those for the full algebra. In the proposed realization of quantum criticality the elements of the sub-algebra characterized by n act as a gauge algebra. An interesting question is whether this sub-algebra is involved with the realization of M-matrices for CD with size scale n. The natural expectation is that n defines a cutoff for conformal weights relating to finite measurement resolution.

10.7.2 How does the size scale of CD affect M-matrices?

1. In standard quantum field theory (QFT) S-matrix represents time translation. The obvious generalization is that now scaling characterized by integer n is represented by a unitary S-matrix that is as n:th power of some unitary matrix S assignable to a CD with minimal size: $S(CD) = S^n$. $S(CD)$ is a discrete analog of the ordinary unitary time evolution operator with n replacing the continuous time parameter.

2. One can see M-matrices also as a generalization of Kac-Moody type algebra. Also this suggests $S(CD) = S^n$, where S is the S-matrix associated with the minimal CD. S becomes representative of phase $exp(i\phi)$. The inner product between CDs of different size scales can n_1 and n_2 can be defined as

$$\begin{aligned} &\langle M^i(m), M^j(n)\rangle = Tr(S^{-m} \circ H^i H^j \circ S^n) \times \theta(n-m) \ , \\ &\theta(n) = 1 \ \text{for} \ n \geq 0 \ , \ \theta(n) = 0 \ \text{for} \ n < 0 \ . \end{aligned} \tag{10.7.1}$$

   Here I have denoted the action of S-matrix at the active end of CD by "$\circ$" in order to distinguish it from the action of matrices on zero energy states which could be seen as belonging to the tensor product of states at active and passive boundary.

   It turns out that unitarity conditions for U-matrix are invariant under the translations of n if one assumes that the transitions obey strict arrow of time expressed by $n_j - n_i \geq 0$. This simplifies dramatically unitarity conditions. This gives orthonormality for M-matrices associated with identical CDs. This inner product could be used to identify U-matrix.

3. How do the discrete Lorentz boosts affecting the moduli for CD with a fixed passive boundary affect the M-matrices? The natural assumption is that the discrete Lorentz group is represented by unitary matrices λ: the matrices M^i are transformed to $M^i \circ \lambda$ for a given Lorentz boost acting on states at active boundary only.

   One cannot completely exclude the possibility that S acts unitarily at both ends of zero energy states. In this case the scaling would be interpreted as acting on zero energy states rather than those at active boundary only. The zero energy state basis defined by M_i would depend on the size scale of CD in more complex manner. This would not affect the above formulas except by dropping away the "$\circ$".

Unitary U must characterize the transitions in which the moduli of the active boundary of causal diamond (CD) change and also states at the active boundary (paired with unchanging states at the passive boundary) change. The arrow of the experienced flow of time emerges during the period as state function reductions take place to the fixed ("passive") boundary of CD and do not affect the states at it. Note that these states form correlated pairs with the changing states at the active boundary. The physically motivated question is whether the arrow of time emerges statistically from the fact that the size of CD tends to increase in average sense in repeated state function reductions or whether the arrow of geometric time is strict. It turns out that unitarity conditions simplify dramatically if the arrow of time is strict.

10.7.3 What Can One Say About U-Matrix?

1. Just from the basic definitions the elements of a unitary matrix, the elements of U are between zero energy states (M-matrices) between two CDs with possibly different moduli of the active boundary. Given matrix element of U should be proportional to an inner product of two M-matrices associated with these CDs. The obvious guess is as the inner product between M-matrices

$$\begin{aligned} U^{ij}_{m,n} &= \langle M^i(m,\lambda_1), M^j(n,\lambda_2)\rangle \\ &= Tr(\lambda_1^{\dagger} S^{-m}\circ H^iH^j\circ S^n\lambda_2) \\ &= Tr(S^{-m}\circ H^iH^j\circ S^n\lambda_2\lambda_1^{-1})\theta(n-m) \ . \end{aligned} \tag{10.7.2}$$

   Here the usual properties of the trace are assumed. The justification is that the operators acting at the active boundary of CD are special case of operators acting non-trivially at both boundaries.

2. Unitarity conditions must be satisfied. These conditions relate S and the hermitian generators H^i serving as square roots of density matrices. Unitarity conditions $UU^{\dagger} = U^{\dagger}U = 1$ is defined in the space of zero energy states and read as

$$\sum_{j_1n_1} U^{ij_1}_{mn_1}(U^{\dagger})^{j_1j}_{n_1n} = \delta^{i,j}\delta_{m,n}\delta_{\lambda_1,\lambda_2} \tag{10.7.3}$$

   To simplify the situation let us make the plausible hypothesis contribution of Lorentz boosts in unitary conditions is trivial by the unitarity of the representation of discrete boosts and the independence on n.

3. In the remaining degrees of freedom one would have

$$\sum_{j_1,k\geq Max(0,n-m)} Tr(S^k\circ H^iH^{j_1})Tr(H^{j_1}H^j\circ S^{n-m-k}) = \delta^{i,j}\delta_{m,n} \ . \tag{10.7.4}$$

   The condition $k\geq Max(0,n-m)$ reflects the assumption about a strict arrow of time and implies that unitarity conditions are invariant under the proper time translation $(n,m)\rightarrow(n+r,m+r)$. Without this condition n back-wards translations (or rather scalings) to the direction of geometric past would be possible for CDs of size scale n and this would break the translational invariance and it would be very difficult to see how unitarity could be achieved. Stating it in a general manner: time translations act as semigroup rather than group.

4. Irreversibility reduces dramatically the number of the conditions. Despite this their number is infinite and correlates the Hermitian basis and the unitary matrix S. There is an obvious analogy with a Kac-Moody algebra at circle with S replacing the phase factor $exp(in\phi)$ and H^i replacing the finite-dimensional Lie-algebra. The conditions could be seen as analogs for the orthogonality conditions for the inner product. The unitarity condition for the analog situation would involve phases $exp(ik\phi_1)\leftrightarrow S^k$ and $exp(i(n-m-k)\phi_2)\leftrightarrow S^{n-m-k}$ and trace would correspond to integration $\int d\phi_1$ over ϕ_1 in accordance with the basic idea of non-commutative geometry that trace corresponds to integral. The integration of ϕ_i would give $\delta_{k,0}$ and $\delta_{m,n}$. Hence there are hopes that the conditions might be satisfied. There is however a clear distinction to the Kac-Moody case since S^n does not in general act in the orthogonal complement of the space spanned by H^i.

5. The idea about reduction of the action of S to a phase multiplication is highly attractive and one could consider the possibility that the basis of H^i can be chosen in such a way that H^i are eigenstates of of S. This would reduce the unitarity constraint to a form in which the summation over k can be separated from the summation over j_1.

$$\sum_{k\geq Max(0,n-m)} exp(iks_i-(n-m-k)s_j)\sum_{j_1} Tr(H^iH^{j_1})Tr(H^{j_1}H^j)=\delta^{i,j}\delta_{m,n} \quad . \tag{10.7.5}$$

The summation over k should gives a factor proportional to δ_{s_i,s_j}. If the correspondence between H^i and eigenvalues s_i is one-to-one, one obtains something proportional to $\delta(i,j)$ apart from a normalization factor. Using the orthonormality $Tr(H^iH^j)=\delta^{i,j}$ one obtains for the left hand side of the unitarity condition

$$exp(is_i(n-m))\sum_{j_1} Tr(H^iH^{j_1})Tr(H^{j_1}H^j)=exp(is_i(n-m))\delta_{i,j} \quad . \tag{10.7.6}$$

Clearly, the phase factor $exp(is_i(n-m))$ is the problem. One should have Kronecker delta $\delta_{m,n}$ instead. One should obtain behavior resembling Kac-Moody generators. H^i should be analogs of Kac-Moody generators and include the analog of a phase factor coming visible by the action of S.

10.7.4 How to obtain unitarity correctly?

It seems that the simple picture is not quite correct yet. One should obtain somehow an integration over angle in order to obtain Kronecker delta.

1. A generalization based on replacement of real numbers with function field on circle suggests itself. The idea is to the identify eigenvalues of generalized Hermitian/unitary operators as Hermitian/unitary operators with a spectrum of eigenvalues, which can be continuous. In the recent case S would have as eigenvalues functions $\lambda_i(\phi)=exp(is_i\phi)$. For a discretized version ϕ would have has discrete spectrum $\phi(n)=2\pi k/n$. The spectrum of λ_i would have n as cutoff. Trace operation would include integration over ϕ and one would have analogs of Kac-Moody generators on circle.

2. One possible interpretation for ϕ is as an angle parameter associated with a fermionic string connecting partonic 2-surface. For the super-symplectic generators suitable normalized radial light-like coordinate r_M of the light-cone boundary (containing boundary of CD) would be the counterpart of angle variable if periodic boundary conditions are assumed.

   The eigenvalues could have interpretation as analogs of conformal weights. Usually conformal weights are real and integer valued and in this case it is necessary to have generalization of the notion of eigenvalues since otherwise the exponentials $exp(is_i)$ would be trivial. In the case of super-symplectic algebra I have proposed that the generating elements of the algebra have conformal weights given by the zeros of Riemann zeta. The spectrum of conformal weights for the generators would consist of linear combinations of the zeros of zeta with integer coefficients. The imaginary parts of the conformal weights could appear as eigenvalues of S.

3. It is best to return to the definition of the U-matrix element to check whether the trace operation appearing in it can already contain the angle integration. If one includes to the trace operation appearing the integration over ϕ it gives $\delta_{m,n}$ factor and U-matrix has elements only between states assignable to the same causal diamond. Hence one must interpret U-matrix elements as functions of ϕ realized factors $exp(i(s_n-s_m)\phi)$. This brings strongly in mind operators defined as distributions of operators on line encountered in the theory of representations of non-compact groups such as Lorentz group. In fact, the unitary representations of discrete Lorentz groups are involved now.

4. The unitarity condition contains besides the trace also the integrations over the two angle parameters ϕ_i associated with the two U-matrix elements involved. The left hand side of the unitarity condition reads as

$$\begin{aligned}\sum_{k\geq Max(0,n-m)} & I(ks_i)I((n-m-k)s_j)\times\sum_{j_1}Tr(H^iH^{j_1})Tr(H^{j_1}H^j) \\ & = \delta^{i,j}\delta_{m,n} \quad , \\ I(s)=\frac{1}{2\pi}\times\int d\phi exp(is\phi)=\delta_{s,0} \quad . & \end{aligned} \tag{10.7.7}$$

Integrations give the factor $\delta_{k,0}$ eliminating the infinite sum obtained otherwise plus the factor $\delta_{n,m}$. Traces give Kronecker deltas since the projectors are orthonormal. The left hand side equals to the right hand side and one achieves unitarity. It seems that the proposed ansatz works and the U-matrix can be reduced by a general ansatz to S-matrix.

5. It should be made clear that the use of eigenstates of S is only a technical trick, the physical states need not be eigenstates. If the active parts of zero energy states where eigenstates of S, U-matrix would not have matrix elements between different H^i and projection operator could not change during time evolution.

10.7.5 What about the identification of S?

1. S should be exponential of time the scaling operator whose action reduces to a time translation operator along the time axis connecting the tips of CD and realized as scaling. In other words, the shift $t/T_0 = m \rightarrow m+n$ corresponds to a scaling $t/T_0 = m \rightarrow km$ giving $m+n = km$ in turn giving $k = 1+n/m$. At the limit of large shifts one obtains $k \simeq n/m \rightarrow \infty$, which corresponds to QFT limit. nS corresponds to $(nT_0)\times(S/T_0) = TH$ and one can ask whether QFT Hamiltonian could corresponds to $H = S/T_0$.
2. It is natural to assume that the operators H^i are eigenstates of radial scaling generator $L_0 = ir_M d/dr_M$ at both boundaries of CD and have thus well-defined conformal weights. As noticed the spectrum for super-symplectic algebra could also be given in terms of zeros of Riemann zeta.
3. The boundaries of CD are given by the equations $r_M = m^0$ and $r_M = T - m_0$, m_0 is Minkowski time coordinate along the line between the tips of CD and T is the distance between the tips. From the relationship between r_M and m_0 the action of the infinitesimal translation $H \equiv i\partial/\partial_{m^0}$ can be expressed as conformal generator $L_{-1} = i\partial/\partial_{rM} = r_M^{-1}L_0$. Hence the action is non-diagonal in the eigenbasis of L_0 and multiplies with the conformal weights and reduces the conformal weight by one unit. Hence the action of U can change the projection operator. For large values of conformal weight the action is classically near to that of L_0: multiplication by L_0 plus small relative change of conformal weight.
4. Could the spectrum of H be identified as energy spectrum expressible in terms of zeros of zeta defining a good candidate for the super-symplectic radial conformal weights. This certainly means maximal complexity since the number of generators of the conformal algebra would be infinite. This identification might make sense in chaotic or critical systems. The functions $(r_M/r_0)^{1/2+iy}$ and $(r_M/r_0)^{-2n}$, $n>0$, are eigenmodes of r_M/dr_M with eigenvalues $(1/2+iy)$ and $-2n$ corresponding to non-trivial and trivial zeros of zeta.

   There are two options to consider. Either L_0 or iL_0 could be realized as a hermitian operator. These options would correspond to the identification of mass squared operator as L_0 and approximation identification of Hamiltonian as iL_1 as iL_0 making sense for large conformal weights.

   (a) Suppose that $L_0 = r_M d/dr_M$ realized as a hermitian operator would give harmonic oscillator spectrum for conformal confinement. In p-adic mass calculations the string model mass formula implies that L_0 acts essentially as mass squared operator with integer spectrum. I have proposed conformal confinent for the physical states net conformal weight is real and integer valued and corresponds to the sum over negative integer valued conformal weights corresponding to the trivial zeros and sum over real parts of non-trivial zeros with conformal weight equal to 1/2. Imaginary parts of zeta would sum up to zero.

(b) The counterpart of Hamiltonian as a time translation is represented by $H = iL_0 = ir_M d/dr_M$. Conformal confinement is now realized as the vanishing of the sum for the real parts of the zeros of zeta: this can be achieved. As a matter fact the integration measure dr_M/r_M brings implies that the net conformal weight must be 1/2. This is achieved if the number of non-trivial zeros is odd with a judicious choice of trivial zeros. The eigenvalues of Hamiltonian acting as time translation operator could correspond to the linear combination of imaginary part of zeros of zeta with integer coefficients. This is an attractive hypothesis in critical systems and TGD Universe is indeed quantum critical.

10.7.6 What about Quantum Classical Correspondence?

Quantum classical correspondence realized as one-to-one map between quantum states and zero modes has not been discussed yet.

1. M-matrices would act in the tensor product of quantum fluctuating degrees of freedom and zero modes. The assumption that zero energy states form an orthogonal basis implies that the hermitian square roots of the density matrices form an orthonormal basis. This condition generalizes the usual orthonormality condition.
2. The dependence on zero modes at given boundary of CD would be trivial and induced by 1-1 correspondence $|m\rangle \rightarrow z(m)$ between states and zero modes assignable to the state basis $|m_{\pm}$ at the boundaries of CD, and would mean the presence of factors $\delta_{z_+,f(m_+)} \times \delta_{z_-,f(n_-)}$ multiplying M-matrix $M^i_{m,n}$.

To sum up, it seems that the architecture of the U-matrix and its relationship to the S-matrix is now understood and in accordance with the intuitive expectations the construction of U-matrix reduces to that for S-matrix and one can see S-matrix as discretized counterpart of ordinary unitary time evolution operator with time translation represented as scaling: this allows to circumvent problems with loss of manifest Poincare symmetry encountered in quantum field theories and allows Lorentz invariance although CD has finite size. What came as surprise was the connection with stringy picture: strings are necessary in order to satisfy the unitary conditions for U-matrix. Second outcome was that the connection with super-symplectic algebra suggests itself strongly. The identification of hermitian square roots of density matrices with Hermitian symmetry algebra is very elegant aspect discovered already earlier. A further unexpected result was that U-matrix is unitary only for strict arrow of time (which changes in the state function reduction to opposite boundary of CD).

Chapter 11

Philosophy of Adelic Physics

11.1 Introduction

I have developed during last 39 years a proposal for unifying fundamental interactions which I call "Topological Geometrodynamics" (TGD). During last twenty years TGD has expanded to a theory of consciousness and quantum biology and also p-adic and adelic physics have emerged as one thread in the number theoretical vision about TGD.

Since Quantum TGD and physical arguments have served as basic guidelines in the development of p-adic ideas, the best way to introduce the subject of p-adic physics, is by describing first TGD briefly.

In this article I will consider the p-adic aspects of TGD - the first thread of the number theoretic vision - as I see them at this moment.

1. I will describe p-adic mass calculations based on p-adic generalization of thermodynamics and super-conformal invariance [K65, K26] with number theoretical existence constrains leading to highly non-trivial and successful physical predictions. Here the notion of canonical identification mapping p-adic mass squared to real mass squared emerges and is expected to be key player of adelic physics and allow to map various invariants from p-adics to reals and vice versa.

2. I will propose the formulation of p-adicization of real physics and adelization meaning the fusion of real physics and various p-adic physics to single coherent whole by a generalization of number concept fusing reals and p-adics to larger structure having algebraic extension of rationals as a kind of intersection.

   The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far form obvious, and various constraints lead to the idea of NTU and finite measurement resolution realized in terms of number theory. Maybe the only way to overcome the problems relies on the idea that various angles and their hyperbolic analogs are replaced with their exponentials and identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

   Another challenge is the correspondence between real and p-adic physics at various levels: space-time level, embedding space level, and WCW level. Here the enormous symmetries of WCW and those of embedding space are in crucial role. Strong form of holography (SH) allows a correspondence between real and p-adic space-time surfaces induced by algebraic continuation from string world sheets and partonic 2-surface, which can be said to be common to real and p-adic space-time surfaces.

3. In the last section I will describe the role of p-adic physics in TGD inspired theory of consciousness. The key notion is Negentropic entanglement (NE) characterized in terms of number theoretic entanglement negentropy (NEN). Negentropy Maximization Principle (NMP) would force the growth of NE. The interpretation would be in terms of evolution as increase of negentropy resources - Akashic records as one might poetically say. The newest finding is

that NMP in statistical sense follows from the mere fact that the dimension of extension of rationals defining adeles increases unavoidably in statistical sense - separate NMP would not be necessary.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); World of Classical Worlds (WCW); Strong Form of GCI (SGCI); Strong Form of Holography (SH); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Quantum Criticality (QC); Hyper-finite Factor of Type II_1 (HFF); Number Theoretical Universality (NTU); Canonical Identification (CI); Negentropy Maximization Principle (NMP); Negentropic entanglement (NE); Number Theoretical Entanglement Negentropy (NEN); are the most often occurring acronyms.

11.2 TGD briefly

This section gives a brief summary of classical and quantum TGD, which to my opinion is necessary for understanding the number theoretic vision.

11.2.1 Space-time as 4-surface

TGD forces a new view about space-time as 4-surface of 8-D imbedding space. This view is extremely simple locally but by its many-sheetedness topologically much more complex than GRT space-time.

Energy problem of GRT as starting point

The physical motivation for TGD was what I have christened the energy problem of General Relativity [K129, L29].

1. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The presence of matter curves empty Minkowski space M^4 so that its isometries realized as transformations leaving the distances between points and thus shapes of 4-D objects invariant are lost. Noether's theorem states that symmetries and conservation laws correspond to each other. Hence conservation laws are lost and conserved quantities are ill-defined. Usually this is not seen a practical problem since gravitation is so weak interaction.
2. The proposed way out of the problem is based on the assumption that space-times are imbeddable as 4-surfaces to some 8-dimensional space $H = M^4 \times S$ by replacing the points of 4-D empty Minkowski space with 4-D very small internal space S. The space S is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [K129]. Isometries of space-time are replaced with those of imbedding space. Noether's theorem predicts the classical conserved charges for given general coordinate invariant (GCI) action principle.

Also now the curvature of space-time codes for gravitation. Equivalence Principle (EP) and General Coordinate Invariance (GCI) of GRT augmented with Relativity Principle (RT) of SRT remain the basic principles. Now however the number of solutions to field equations - preferred extremals (PEs) - is dramatically smaller than in Einstein's theory [K9, K15].

1. An unexpected bonus was geometrization classical fields of standard model for $S = CP_2$. Also the space-time counterparts for field quanta emerge naturally but this requires a profound generalization of the notion of space-time: the topological inhomogenities of space-time surface are identified as particles. This means a further huge reduction for dynamical field like variables at the level of single space-time sheet. By general coordinate invariance (GCI) only four imbedding space coordinates appear as variables analogous to classical fields: in a typical GUT their number is hundreds.

2. CP_2 also codes for the standard model quantum numbers in its geometry in the sense that electromagnetic charge and weak isospin emerge from CP_2 geometry: the corresponding symmetries are not isometries so that electroweak symmetry breaking is coded already at this level. Color quantum numbers correspond to the isometries of CP_2 defining an unbroken symmetry: this also conforms with empirical facts. The color of TGD however differs from that in standard model in several aspects and LHC has began to exhibit these differences via the unexpected behavior of what was believed to be quark gluon plasma [K72]. The conservation of baryon and lepton numbers follows as a prediction. Leptons and quarks correspond to opposite chiralities for imbedding space spinors.

3. What remains to be explained in standard model is family replication phenomenon for leptons and quarks. Both quarks and leptons appear as three families identical apart from having different masses. The conjecture was is that fermion families correspond to different topologies for 2-D surfaces characterized by genus telling the number g (genus) of handles attached to sphere to obtain the surface: sphere, torus, ... The 2-surfaces are identified as "partonic 2-surfaces" whose orbits are light-like 3-surface at which the signature of the induced metric of space-time surface transforms from Minkowskian to Euclidian. The partonic orbits replace the lines of Feynman diagrams in TGD Universe in accordance with the replacement of point-like particle with 2-surface.

   Only the three lowest genera are observed experimentally. A possible explanation is in terms of conformal symmetries: the genera $g \leq 2$ allow always Z_2 as a subgroup of conformal symmetries (hyper-ellipticity) whereas higher genera in general do not. The handles of partonic 2-surfaces could form analogs of unbound many-particle states for $g > 2$ with a continuous spectrum of mass squared but for $g = 2$ form a bound state by hyper-ellipticity [K26].

4. Later further arguments in favor of$H = M^4 \times CP_2$ have emerged. One of them relates to twistorialization and twistor lift of TGD [K116, K46, L31]. 4-D Minkowski space is unique space-time with Minkowskian signature of metric in the sense that it allows twistor structure. This is a problem in attempts to introduce twistors to General Relativity Theory (GRT) and a serious obstacle in the quantization based on twistor Grassmann approach, which has demonstrate its enormous power in the quantization of gauge theories. In TGD framework one can ask whether one could lift also the twistor structure to the level of H. M^4 has twistor structure and so does also CP_2: which is the only Euclidian 4-manifold allowing twistor space, which is also a Kähler manifold! This led to the notion of twistor lift of TGD inducing rather recent breakthrough in the understanding of TGD.

TGD can be also seen as a generalization of hadronic string model - not yet superstring model since this model became fashionable two years after the thesis about TGD [K4]. Later it has become clear that string like objects, which look like strings but are actually 3-D are basic stuff of TGD Universe and appear in all scales [K31, K9]. Also strictly 2-D string world sheets popped up in the formulation of quantum TGD (analogy with branes) [**?**]o that one can say that string model in 4-D space-time is part of TGD.

Concluding, TGD generalizes standard model symmetries and provides an incredibly simple proposal for a dynamics: only 4 classical field variables and in fermionic sector only quark and lepton like spinor fields. The basic objection against TGD looks rather obvious in the light of after-wisdom. One loses linear superposition of fields, which holds in good approximation in ordinary field theories, which are almost linear. The solution of the problem to be discussed later relies on the notion many-sheeted space-time [L29].

Many-sheeted space-time

The replacement of the abstract manifold geometry of general relativity with the geometry of 4-surfaces brings in the shape of surface as seen from the perspective of 8-D space-time as additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any variational principle satisfying GCI led soon to the realization that the topological structure of space-time in this framework is much more richer than in GRT.

1. Space-time decomposes into space-time sheets of finite size. This led to the identification of physical objects that we perceive around us as space-time sheets. The original identification of space-time sheet was as a surface of in H with outer boundary. For instance, the outer boundary of the table would be where that particular space-time sheet ends (what "ends" means is not however quite obvious!). We would directly see the complex topology of many-sheeted space-time! Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

   It turned that boundaries are probably excluded by boundary conditions. Rather, two sheets with boundaries must be glued along their boundaries together to get double covering. Sphere can be seen as simplest example of this kind of covering: northern and southern hemispheres are glued along equator together.

2. The original vision was that elementary particles are topological inhomogenities glued to these space-time sheets using topological sum contacts. This means drilling a hole to both sheets and connecting with a very short cylinder. 2-dimensional illustration should give the idea. In this conceptual framework material structures and shapes would not be due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in GRT.

   This view has gradually evolved to much more detailed picture. Elementary particles have wormhole contacts as basic building bricks. Wormhole contact is very small region with *Euclidian (!)* signature of the induced metric connecting two Minkowskian space-time sheets with light-like boundaries carrying spinor fields and there particle quantum numbers. Wormhole contact carries magnetic monopole flux through it and there must be second wormhole contact in order to have closed lines of magnetic flux. Particle world lines are replaced with 3-D light-like surfaces - orbits of partonic 2-surfaces - at which the signature of the induced metric changes.

   One might describe particle as a pair of magnetic monopoles with opposite charges. With some natural assumptions the explanation for the family replication phenomenon in terms of the genus g of the partonic 2-surface is not affected. Bosons emerge as fermion anti-fermion pairs with fermion and anti-fermion at the opposite throats of the wormhole contact. In principle family replication phenomenon should have bosonic analog. This picture assigns to particles strings connecting the two wormhole throats at each space-time sheet so that string model mathematics becomes part of TGD.

The notion of classical field differs in TGD framework in many respects from that in Maxwellian theory.

1. In TGD framework fields do not obey linear superposition and all classical fields are expressible in terms of four imbedding space coordinates in given region of space-time surface. Superposition for classical fields is replaced with *superposition of their effects* [K108, K129] - in full accordance with operationalism. Particle can topologically condense simultaneously to several space-time sheets by generating topological sum contacts (not stable like the wormhole contacts carrying magnetic monopole flux and defining building bricks of particles). Particle "experiences" the superposition of the effects of the classical fields at various space-time sheets rather than the superposition of the fields.

   It is also natural to expect that at macroscopic length scales the physics of classical fields (to be distinguished from that for field quanta) can be explained using only four primary field like variables. Electromagnetic gauge potential has only four components and classical electromagnetc fields give and excellent description of physics. This relates directly to electroweak symmetry breaking in color confinement which in standard model imply the effective absence of weak and color gauge fields in macroscopic scales. TGD however predicts that copies of hadronic physics and electroweak physics could exist in arbitrary long scales [K71] and there are indications that just this makes living matter so different as compared to inanimate matter.

2. The notion of induced gauge field means that one induces electroweak gauge potentials defining so called spinor connection at space-time surface (induction of bundle structure). Induction boils down locally to a projection of the imbedding space vectors representing the spinor

connection. The classical fields at the imbedding space level are non-dynamical and fixed and extremely simple: one can say that one has generalization of constant electric field and magnetic fields in CP_2. The dynamics of the 3-surface however implies that induced fields can form arbitrarily complex field patterns. This is essentially dynamics of shadows.

Induced gauge fields are not equivalent with ordinary free gauge fields. For instance, the attempt to represent constant magnetic or electric field as a space-time time surface has a limited success. Only a finite portion of space-time carrying this field allows realization as 4-surface. I call this topological field quantization. The magnetization of electric and magnetic fluxes is the outcome. Also gravitational field patterns allowing imbedding are very restricted: one implication is that topological with over-critical mass density are not globally imbeddable. This would explain why the mass density in cosmology can be at most critical. This solves one of the mysteries of GRT based cosmology [K102].

Quite generally, the field patterns are extremely restricted: not only due to imbeddability constraint but also due to the fact that by SH only very restricted set of space-time surfaces can appear solutions of field equations: I speak of preferred extremals (PEs) [K9, K15, L29]. One might speak about archetypes at the level of physics: they are in quite strict sense analogies of Bohr orbits in atomic physics: this is implies by the realization of GCI. This kind of simplicity does not conform with what we observed. The way out is many-sheeted space-time. Although fields do not superpose, particles experience the superposition of effects from the archetypal field configurations (superposition is replaced with set theoretic union).

3. The important implication is that one can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K82]. One can speak about field body or magnetic body of the system.

   Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [?] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K72]. The magnetic flux tubes of magnetic body carry monopole fluxes existing without generating currents. In cosmology the flux tubes assignable to the remnants of cosmic strings make possible long range magnetic fields in all scales impossible in standard cosmology. Also super-conductivity is proposed to rely on dark $h_{eff} = n \times h$ Cooper pairs at pairs of flux tubes carrying monopole flux.

GRT and gauge theory limit of TGD is obtained as an approximation.

1. GRT/gauge theory type description is an approximation obtained by lumping together the space-time sheets to single region of M^4, with gravitational fields and gauge potentials as sums of corresponding induced field quantities at space-time surface geometrized in terms of geometry of H. Gravitational field corresponds to the deviation of the induced metric from Minkowski metric using M^4 coordinates for space-time surface so that the description applies only in long length scale limit.

   Space-time surface has both Minkowskian and Euclidian regions. Euclidian regions are identified in terms of what I call generalized scattering/twistor diagrams. The 3-D boundaries between Euclidian and Minkowskian regions have degenerate induced 4-metric and I call them light-like orbits of partonic 2-surfaces or light-like wormhole throats analogous to blackhole horizons. The interiors of blackholes are replaced with the Euclidian regions and every physical system is characterized by this kind of region.

   Lumping of sheets together implies that global conservation laws cannot hold exactly true for the resulting GRT type space-time. Equivalence Principle (EP) as Einstein's equations stating conservation laws locally follows as a local remnant of Poincare invariance.

2. Euclidian regions are identified as slightly deformed pieces of CP_2 connecting two Minkowskian space-time regions. Partonic 2-surfaces defining their boundaries are connected to each other by magnetic flux tubes carrying monopole flux.

   Wormhole contacts connect two Minkowskian space-time sheets already at elementary particle level, and appear in pairs by the conservation of the monopole flux. Flux tube can be visualized as a highly flattened square traversing along and between the space-time sheets involved. Flux tubes are accompanied by fermionic strings carrying fermion number. Fermionic strings give rise to string world sheets carrying vanishing induced em charged weak fields (otherwise em charge would not be well-defined for spinor modes). String theory in space-time surface becomes part of TGD. Fermions at the ends of strings can get entangled and entanglement can carry information.

3. Strong form of GCI (SGCI) states that light-like orbits of partonic 2-surfaces on one hand and space-like 3-surfaces at the ends of causal diamonds on the other hand provide equivalent descriptions of physics. The outcome is that partonic 2-surfaces and string world sheets at the ends of CD can be regarded as basic dynamical objects.

   Strong form of holography (SH) states the correspondence between quantum description based on these 2-surfaces and 4-D classical space-time description, and generalizes AdS/CFT correspondence. One has huge super-symplectic symmetry algebra acting as isometries of WCW with conformal structure [K29, K96, K127], conformal algebra of light-cone boundary extending the ordinary conformal algebra, and ordinary Kac-Moody and conformal symmetries of string world sheets. This explains why 10-D space-time can be replaced with ordinary space-time and 4-D Minkowski space can be replaced with partonic 2-surfaces and string world sheets. This holography looks very much like the one we are accustomed with!

11.2.2 Zero energy ontology (ZEO)

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology (ZEO) [K75] physical states decompose to pairs of positive and negative energy states such that the net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events.

ZEO and positive energy ontology

ZEO is consistent with the crossing symmetry of QFTs meaning that the final states of the quantum scattering event can be described formally as negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem, which emerges in any theory assuming symmetries giving rise to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state of say cosmology. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in GRT based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

At the level of principle the implications are quite dramatic. In quantum jump as recreation replacing the quantum Universe with a new one it is possible to create entire sub-universes from vacuum without breaking the fundamental conservation laws. From the point of view of consciousness theory the important implication is that "free will" is consistent with the laws of physics. This makes obsolete the basic arguments in favor of materialistic and deterministic world view.

Zero energy states are located inside causal diamond (CD)

By quantum classical correspondence zero energy states must have space-time and imbedding space correlates.

1. Positive and negative energy parts of zero energy state reside at future and past light-like boundaries of causal diamond (CD) identified as intersection of future and past directed light-cones and visualizable as double cone. The analog of CD in cosmology is big bang followed by big crunch. Penrose diagrams provide an excellent 2-D visualization of the notion. CDs form a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs could also intersect.

   The interpretation of CD in TGD inspired theory of consciousness is as an imbedding space correlate for perceptive field of conscious entity: the contents of conscious experience is about the region defined by CD. At the level of space-time sheets the experience come from space-time sheets in the interior of CD. Whether the sheets can be assumed to continue outside CD is still unclear.

   Quantum measurement theory must be modified in ZEO since state function reduction can happen at both boundaries of CD and the reduced states at opposite boundaries are related by time reversal. One can also have quantum superposition of CDs changing between reductions at active boundary followed by localization in the moduli space of CDs with the tip of passive boundary fixed. Quantum measurement theory generalizes to a theory of consciousness with continuous entity identified as a sequence of state function reductions at active (changing) boundary of CD [K8].

2. By number theoretical universality (NTU) the temporal distances between the tips of the intersecting light-cones are assumed to come as integer multiples $T = m \times T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. p-Adic length scale hypothesis [K77, K125] motivates the stonger hypothesis that the distances tend to come as as octaves of T_0: $T = 2^n T_0$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K10] . This means a direct coupling between microscopic and macroscopic scales.

11.2.3 Quantum physics as physics of classical spinor fields in WCW

The notions of Kähler geometry of "World of Classical Worlds" (WCW) and WCW spinor structure are inspired by the vision about the geometrization of the entire quantum theory.

Motivations for WCW

The notion of "World of Classical Worlds" (WCW) [K57, K29, K96] was forced by the failure of both path integral approach and canonical quantization in TGD framework. The idea is that the Kähler function defining WCW Kähler geometry is determined by the real part of an action S determining space-time dynamics and receiving contributions from both Minkowskian and Euclidian regions of space-time surface X^4 (note that $\sqrt{g_4}$ is proportional to imaginary unit in Minkoskian regions).

1. If S is space-time volume both canonical quantization and path integral would make sense at least formally since in principle one could solve the time derivatives of four imbedding space coordinates as functions of canonical momentum densities (general coordinate invariance allows to eliminate four coordinates). The calculation of path integral is however more or less hopeless challenge in practice.

2. A mere space-time volume as action is however not physically attractive. This was thought to leave under consideration only Kähler action S_K - Maxwell action for the induced Kähler form expressible in terms of gauge potential defined by the induced Kähler gauge potential of CP_2. This action has however a huge vacuum degeneracy. Any space-time surface with at most 2-D CP_2 projection, which is Lagrangian sub-manifold of CP_2, is vacuum extremal. Symplectic transformations acting like U(1) gauge transformations generate new vacuum extremals. They however fail to act as symmetries of non-vacuum extremals so that gauge invariance is not in question: the deviation of the induced metric from flat metric is the reason for the failure. This degeneracy is assumed to give rise to what might be called 4-D spin glass degeneracy meaning that the landscape for the maxima of Kähler function is fractal.

3. Canonical quantization fails because by the extreme non-linearity of the action principle making it is impossible to solve time derivatives explicitly in terms of canonical momentum

densities. The problem is especially acute for the canonical imbedding of empty Minkowski space to $M^4 \times CP_2$. The action is vanishing up to fourth order in imbedding space coordinates so that canonical momentum densities vanish identically and there is no hope of defining propagator in path integral approach. The mechanical analog would be criticality around which the potential reduces to $V \propto x^4$. Quantum criticality is indeed a basic aspect of TGD Universe.

The hope held for a long time was that WCW geometry allowing to get rid of path integral would solve the problems. One could however worry about vacuum degeneracy implying that WCW metric becomes extremely degenerate for vacuum extremals and also holography becomes extremely non-unique for them. Also the expected feailure of perturbative approach around M^4 is troublesome.

WCW and twistor lift of TGD

During last year this picture has indeed changed thanks to what might be called twistor lift of TGD [K116, K46, L31] inspired by twistor Grassmann approach to supersymmetric gauge theories [B25]. Remarkably, twistor lift would provide automatically the fundamental couplings of standard model and GRT and also the scale assigned to GUTs as CP_2 radius. PEs would be both extremals of Kähler action and minimal surfaces.

1. The basic observation is E^4, and its Euclidian compactification S^4 and CP_2 are completely unique in that they allow twistor space with Kähler structure [A63]. This was discovered by Hitchin at roughly the same time as I discovered TGD! This generalizes to M^4 having a generalization of ordinary Kähler structure to what I have called Hamilton-Jacobi structure by decomposition $M^4 = M^2 \times E^2$, where M^2 allows hypercomplex structure [K116, K46]. One can consider also integral distributions of tangent decompositions $M^4 = M^2(x) \times E^2(x)$, depending on position. The twistor space has a double fibration by S^2 with base spaces identifiable as M^4 and conformal compactification of M^4 for which metric is defined only up to conformal scaling. The first fibration $M^4 \times S^2$ with a well-defined metric would correspond to the classical TGD.
2. Both Newton's constant G and cosmological constant Λ emerge from twistor lift in M^4 factor. The radius of S^2 is identified in terms of Planck length $l_P = \sqrt{G}$. For CP_2 factor, the radius corresponds to the radius of CP_2 geodesic sphere. 4-D Kähler action can be lifted to 6-D Kähler action only for $M^4 \times CP_2$ so that TGD would be completely unique both mathematically and physically. The twistor space of CP_2 is flag-manifold $SU(3)/U(1) \times U(1)$ having interpretation as the space for the choices of quantization axis of color isospin and hypercharge. This choice could correspond to a selection of Eguchi-Hanson complex coordinates for CP_2 by fixing their phase angles in which isospin and hypercharge rotations induce shifts.
3. The physically motivated conjecture is that the PEs can be lifted to their 6-D twistor bundles with S^2 serving as a fiber, that one induce the twistor structure and the outcome is equal to the twistor structure of space-time surface, and that this condition is at least part of the PE property. This would correspond to the solution of massless wave equations in terms of twistors in the original twistor approach of Penrose [B61]. The analog of spontaneous compactification would lead to 4-D action equal to Kähler action plus volume term. One could of course postulate this action directly without mentioning twistors at all.

   The coefficient of the volume term would correspond to dark energy density characterized by cosmological constant Λ being extremely small in cosmological scales. It removes vacuum degeneracy although the situation remains highly non-perturbative. This can be combined with the earlier conjecture that cosmological constant Λ behaves as $\Lambda \propto 1/p$ under p-adic coupling constant evolution so that Λ would be large in primordial cosmology.
4. The generic extremals of space-time action would depend on coupling parameters, which does not fit with the number theoretic vision inspiring speculations that space-time surface can be seen as quaternionic sub-manifolds of 8-D octonionic space-time [K107], satisfying quaternion analyticity [K46], or a 4-D generalization of holomorphy. By SH the extremals are however "preferred". What could this imply?

Intriguingly, all known non-vacuum extremals and also CP_2 type vacuum extremals having null-geodesic as M^4 projection are extremals of both Kähler action and volume term separately! The dynamics for volume term and Kähler action effectively decouple and coupling constants do not appear at all in field equations. The twistor lift would only select minimal surface amongst vacuum extremals, modify the Kähler function of WCW identifiable as exponent for the real part of action, and provide a profound mathematical and physical motivation for cosmological constant Λ remaining mysterious GRT framework. One could even hope that preferred extremals are nothing but minimal surface extremals of Kähler action with the vanishing conditions for some sub-algebra of super-symplectic algebra satisfied automatically!

The analog of decoupling of Kähler action and volume term should take place also for induced spinors. This is expected if mere analyticity properties make spinor modes solutions of modified Dirac equations. This is true in 2-D case Hamilton-Jacobi structure should guarantee this in 4-D case [K127, K46].

PEs depend on coupling parameters only via boundary conditions stating the vanishing of Noether charges for a sub-algebra of super-symplectic algebra and its commutator with entire algebra. Also the conservation conditions at 3-D light-like surfaces at which the signature of metric changes imply dependence on coupling parameters. These conditions allow the transfer of classical charges between Minkowskian and Euclidian regions necessary to understand momentum exchange between particles and environment classically only if Kähler couplings strength is complex - otherwise there is no exchange of conserved quantities since their real *resp.* imaginary at the two sides [L23]. Interestingly, also in twistor Grassmann approach the massless poles in propagators are complex.

This picture conforms with the conjecture that discrete p-adic evolution of the Kähler coupling strength in subset of primes near prime powers of two corresponds to complex zeros of zeta [L23]. This conforms also with the conjectured discreteness of p-adic coupling constant evolution by phase transitions changing the values of coupling parameters. One implication is that all loop corrections in functional integral vanish.

5. In path integral approach quantum TGD would be extremely non-perturbative around extremals for which Kähler action vanishes. Same is true also in WCW approach. The cure would be provided by the hierarchy of Planck constants $h_{eff}/h = n$, which effectively scales Λ down to Λ/n. n would be the number sheets of the M^4 covering defined by the space-time surface: the action of Galois group for the number theoretic discretization of space-time surface could give rise to this covering. The finiteness of the volume term in turn forces ZEO: the volume of space-time surface is indeed finite due to the finite size of CD.

Consider now the delicacies of this picture.

1. Should assign also to M^4 the analog of symplectic structure giving an additional contribution to the induced Kähler form? The symmetry between M^4 and CP_2 suggests this, and this term could be highly relevant for the understanding of the observed CP breaking and matter antimatter asymmetry [L57]. Poincare invariance is not lost since the needed moduli space for M^4 Kähler forms would be the moduli space of CDs forced by ZEO in any case, and M^4 Kähler form would serve as the correlate for fixing rest system and spin quantization axis in quantum measurement.

2. Also induced spinor fields are present. The well-definedness of electro-magnetic charge for the spinor modes forces in the generic case the localization of the modes of induced spinor fields at string world sheets (and possibly to partonic 2-surfaces) at which the induced charged weak gauge fields and possibly also neutral Z^0 gauge field vanish. The analogy with branes and super-symmetry force to consider two options.

   Option I: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K100].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option II: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes! In this case 4-D spinor modes do not define space-time super-symmetries.

3. Why the string world sheets coding for effective action should carry vanishing weak gauge fields? If M^4 has the analog of Kähler structure [L57], one can speak about Lagrangian sub-manifolds in the sense that the sum of the symplectic forms of M^4 and CP_2 projected to Lagrangian sub-manifold vanishes. Could the induced spinor fields for effective action be localized to generalized Lagrangian sub-manifolds? This would allow both string world sheets and 4-D space-time surfaces but SH would select 2-D Lagrangian manifolds. At the level of effective action the theory would be incredibly simple.

   Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would coupleonly to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [L1] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of weak group to electromagnetic gauge group.

   The projections of canonical currents of Kähler action to string world sheets would vanish, and the projections of the 4-D modified gamma matrices would define just the induced 2-D metric. If the induced metric of space-time surface reduces to an orthogonal direct sum of string world sheet metric and metric acting in normal space, the flow defined by 4-D canonical momentum currents is parallel to string world sheet. These conditions could define the "boundary" conditions at string world sheets for SH.

This admittedly speculative picture has revolutionized the understanding of both classical and quantum TGD during last year. [K46, L31, L29]. In particular, the construction of single-sheeted PEs as minimal surfaces allows a kind of lego like engineering of more complex PEs [L27]. The minimal surface equations generalize Laplace equation of Newton's gravitational theory to non-linear massless d'Alembert equation with gravitational self-coupling. One obtains the analog of Schwartschild solution and radiative solutions describing also gravitational radiation [L29]. An open question is whether classical theory makes sense if also the analog of Kähler form in M^4 is allowed.

Identification of WCW

The notion of WCW [K57, K29, K96] was inspired by the super-space approach of Wheeler in which 3-geometries are the basic geometric entities.

1. In TGD framework 3-surfaces take this role. Einstein's program for geometrizing classical physics is generalized to a geometrization of entire quantum physics. Hermitian conjugation corresponds to complex conjugation in infinite-dimensional context so that WCW must have Kähler geometry. The geometrization of fermionic statistics/oscillator operators is in terms of gamma matrices of WCW expressible as linear combinations of oscillator operators for second quantized induced spinor field. Formally purely classical spinor modes of WCW represent many fermion states as functionals of 3-surface. One can also interpret gamma matrices as generators of super-conformal symmetries in accordance with the fact that also SUSY involves Clifford algebra.

In ZEO the entanglement coefficients between positive and negative energy parts of zero energy states determine the S-matrix so that S-matrix would be coded by the modes of WCW spinor fields. Twistor approach to TGD [K46] suggests that the S-matrix reduces completely to the symmetries defined by the multi-local (locus corresponds to partonic 2-surface) generators of the Yangian associated with the super-symplectic algebra.

2. ZEO forces to identify 3-surfaces as pairs of 3-surfaces with members at the opposite boundaries of CD. SH reduces them to a collection of partonic 2-surfaces at boundaries of CD plus number theoretic discretization in space-time interior. Basic geometric objects are pairs of initial and final states (coordinates for both in mechanical analogy) rather than initial states with initial value conditions (coordinates and velocities in mechanical analogy) and initial value problem transforms to boundary value problem. Processes rather than states become the basic elements of ontology: this has far reaching consequences in biology and neuroscience.
3. The realization of GCI requires that the definition of WCW Kähler function assigns to a "physically" 3-surface a unique 4-surface for 4-D general coordinate transformations to act: "physically" could mean "apart from transformations acting as gauge transformations" not affecting the action and conserved classical charges. The outcome is holography.
4. Strong form of holography (SH) would emerge as follows. The condition that light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions are basic geometric entities equivalent with pairs of space-like 3-surfaces at the ends of given causal diamond CD implies SH: partonic 2-surfaces and their 4-D tangent space data should code the physics. One could also speak about almost/effective 2-dimensionality. Tangent space data could in turn be coded by string world sheets. Number theoretical discretitization of space-time interior with preferred coordinates in the extension of rationals could give meaning for "almost".
5. Kähler metric is expressible both in terms of second derivatives of Kähler function K [K57] and as anticommutators of WCW gamma matrices expressible as linear combinations of fermionic oscillator operators. This suggests a close relationship between space-time dynamics and spinor dynamics.

   Super-symplectic symmetry between the action defining space-time surfaces (Kähler action plus volume term) and modified Dirac action would realize this relationship. This is achieved if the modified gamma matrices are defined by the canonical momentum currents of 2-D action associated with string world sheets. These currents are parallel to the string world sheets. This implies the analog of AdS/CFT correspondence requiring only that induced spinor modes at string world sheets determine them in space-time interior (this is like analytic continuation). The localization of spinor modes at string world sheets is *not* required as I believed first.

The geometry of loop spaces developed by Freed [A44] serves as a model in the construction of WCW Kähler geometry [K96].

1. The existence of loop space Riemann connection requires maximal isometry group identifiable as Kac-Moody group so that Killing vector fields span the entire tangent space of the loop space.
2. In TGD framework the properties of Kähler action lead to the idea that WCW is union of homogenous or even symmetric spaces of symplectic algebra acting at the boundary of $\delta CD \subset \delta CD_+ \cup \delta CD_-$, $\delta CD_\pm \subset \delta M^4_\pm \times CP_2$. ZEO requires that the conserved quantum numbers for physical states are opposite for the positive and negative energy parts of the states at the two opposite boundary parts of CD. The symmetric spaces G/H in the union are labelled by zero modes, which do not appear in the line element as differentials but only as parameters of the metric. Conserved Noether charges of isometries and symplectic invariants of examples of zero modes as also the super-symplectic Noether charges invariant under complex conjugation of WCW coordinates.
3. Homogenous spaces of the symplectic group G are obtained by dividing by a subgroup H. An especially attractive option is suggested by the fractal structure of the symplectic algebra containing an infinite hierarchy of sub-algebras G_n for which conformal weights are $n > 0$-multiples of those for G. For this option $H = G_n$ is isomorphic to G and one could have infinite hierarchies of inclusions analogous to the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs). PE property requires almost 2-dimensionality and elimination of huge

number of degrees of freedom. The natural condition is that the Noether charges of G_n vanish at the ends of CD. A stronger condition is that also the Noether charges for $[G, G_n]$ vanish. This implies effective normal algebra property and G/G_n acts effectively like group.

The inclusion of HFFs would define measurement resolution with included factor acting like gauge algebra. Measurement resolution would be naturally determined by the number theoretic discretization of the space-time surface so that physics as geometry and number theory visions would meet each other.

4. This inclusion hierarchy can be identified in terms of quantum criticality (QC). The transitions $n \to kn$ increasing the value of $n > 0$ reduce QC since pure gauge symmetries are reduced, and new physical super-symplectic degrees of freedom emerge. QC also requires that Kähler couplings strength analogous to temperature is analogous to critical temperature so that the quantum theory is uniquely defined if their is only one critical temperature. Spectrum for α_K seems more plausible and the possibility that Kähler coupling strength depends on the level of the number theoretical hierarchy defined by the allowed extensions of rationals can be considered [L23].

WCW spinor structure

The basic idea is geometrization of quantum states by identifying them as modes of WCW spinor fields [K127, K96]. This requires definition of WCW spinors and WCW spinor structure, WCW gamma matrices and Dirac operator, etc..

The starting point is the definition of WCW gamma matrices using a representation analogous to the usual vielbein representation as linear combinations of flat space gamma matrices. The conceptual leap is the observation that there is no need to assume that the counterparts of flat space gamma matrices have vectorial quantum numbers. Instead, they are identified as fermionic oscillator operators for second quantized free induced spinor fields at space-time surface.

This allows geometrization of the fermionic statistics since WVW spinors for a given 3-surface are analogous to fermionic Fock states. One can also say that spinor structure follows as a square root of metric and also that the spinor basis defines a geometric correlate of Boolean mind [K25]. The dependence of WCW spinor field on 3-surface represents the bosonic degrees of freedom not reducible to many-fermion states. For instance, most of hadron mass would be associated with these degrees of freedom.

Quantum TGD involves Dirac equations at space-time level, imbedding space level, and level of WCW. The dynamics of the induced spinor fields is related by super-symmetry to the action defining space-time surfaces as preferred extremals. [K127, K96].

1. The gamma matrices in the equation - modified gamma matrices - are determined by contractions of the canonical momentum currents of Kähler action with the imbedding space gamma matrices. The localization at string world sheets for which only induced neutral weak fields or only em field are non-vanishing is accompanied by the integrability condition that various conserved currents run along string world sheets: one can speak of sub-flow. I
2. Modified Dirac equation can be solved exactly just like in the case of string models using holomorphy and the properties of complexified modified gamma matrices. This is expected to be true also in 4-D case by Hamilton-Jacobi structure. If the dynamics of avoidance is realized the modified Dirac equation would be essentially free Dirac equation and holomorphy would allow to solve it.

At the level of WCW one obtains also the analog of massless Dirac equation as the analog of super Virasoro conditions of Super Virasoro algebra.

1. The fermionic counterparts of super-conformal gauge conditions assignable with sub-algebra G_n of supersymplectic conformal symmetry associated with the both light-cone boundary (light-like radial coordinate), with conformal symmetries of light-cone boundary, and with string world sheets.
2. The ground states of supersymplectic representations satisfy massless imbedding space Dirac equation in imbedding space so that Dirac equations in WCW, in imbedding space, and at string world sheets are involved. In twistorialization also massless M^8 Dirac equation emerges in the tangent space M^8 of imbedding space assignable to the partonic 2-surfaces

and generalizes the 4-D light-likeness with its 8-D counterpart applying to states with M^4 mass. Here octonionic representation of imbedding space gamma matrices emerges naturally and allows to speak about 8-D analogs of Pauli's sigma matrices [K116].

11.2.4 Quantum criticality, measurement resolution, and hierarchy of Planck constants

The notions of quantum criticality (QC), finite measurement resolution, and hierarchy of Planck constants proposed to give rise to dark matter as phases of ordinary matter are central for TGD [?, K126, K43].

These notions relate closely to the strong form of holography (SH) implied by strong form of general coordinate invariance (SGCI). In adelic physics all this would relate closely to the hierarchy of extensions of rationals serving as a correlate for number theoretical evolution.

Finite measurement resolution and fractal inclusion hierarchy of super-symplectic algebras

The fractal hierarchy of isomorphic sub-algebras of supersymplectic algebra - call it g - defines an excellent candidate for the realization of finite the measurement resolution. Similar hierarchies can be assigned also for the extended super-conformal algebra assignable with light-like boundaries of CD and with Kac-Moody and conformal algebras assignable to string world sheets.

An interesting possibility is that the the conformal weights assignable to infinitesimal scaling operator of the light-like radial coordinate of light-cone boundary correspond to zeros of Riemann zeta [K125] [L22]. A kind of dual spectrum would correspond to conformal weights that correspond to logarithms for powers of primes. One can identify the conformal weight as negative of the pole of fermionic zeta $z_F = \zeta(s)/\zeta(2s)$ natural in TGD framework. The real part of conformal weight for the generators is $h_R = -1/4$ for "non-trivial" poles and positive integer $h = n > 0$ for "trivial" poles. There is also a pole for $h = -1$. Hence one obtains tachyonic ground states, which must be assumed also in p-adic mass calculations [K65].

Also the generators of Yangian algebra [K116] integrating the algebras assignable to various partonic 2-surfaces to a multi-local algebra are labelled by a non-negative integer n analogous to conformal weight and telling the number of partonic 2-surfaces involved with the action of the generator. Also this algebra has similar fractal hierarchy of sub-algebras so that the considerations that follow might apply also to it. Now that number of partonic 2-surface would play the role of measurement resolution.

As noticed, there are also other algebras, which allow conformal hierarchy if one can restrict the conformal weights to be non-negative. The first of them generates generalized conformal transformations of light-cone boundary depending on light-like radial coordinate as parameter: also now radial conformal weights for generators can have zeros of zeta as spectrum. As a special case one obtains infinite-dimensional group of isometries of light-cone boundary. Second one corresponds to ordinary conformal and Kac-Moody symmetries for induced spinor fields acting on string world sheets. Also here similar hierarchy of sub-algebras can be considered. In the following argument one restricts to super-symplectic algebra assumed to act as isometries of WCW.

Consider now how the finite measurement resolution could be realized as an infinite hierarchy of super-symplectic gauge symmetry breakings. The physical picture relies on quantum criticality of TGD Universe. The levels of the hierarchy labelled by positive integer n and a ball at the top of ball at... serves as a convenient metaphor.

1. The sub-algebra g_n for which conformal weights of generators (whose commutators give the sub-algebra) are positive integer multiples for those of the entire algebra g defines the algebra acting as pure gauge algebra defining a sub-group of symplectic group. The action of g_n as gauge algebra would mean that it affects on degrees of freedom below the measurement resolution. One can assign to this algebra a coset space G/G_n of the entire symplectic group G and of subgroup G_n. This coset space would describe the dynamical degrees of freedom. If the subgroup were a normal subgroup, the coset space would be a group. This is not the case now since the commutator $[g, g_n]$ of the entire algebra with the sub-algebra does not belong to g_n.

However, if one poses stronger - physically very attractive - gauge conditions stating that not only g_n but also the commutator algebra $[g, g_n]$ annihilates the physical states and that corresponding classical Noether charges vanish, one obtains effectively a normal subgroup and one has good hopes that coset space acts effectively as group, which is finite-dimensional as far as conformal weights are considered.

2. $n > 0$ is essential for obtaining effective normal algebra property. Without this assumption the commutator $[g, g_n]$ would be entire g. If the spectrum of supersymplectic conformal weights is integer valued it is not obvious why one should pose the restriction $n \geq 1$.

3. In this framework pure conformal invariance could reduce to a finite-dimensional gauge symmetry. A possible interpretation would be in terms of Mc-Kay correspondence [A78] assigning to the inclusions of HFFs labelled by integer $n \geq 3$ a hierarchy of simply laced Lie-groups. Since the included algebra would naturally correspond to degrees of freedom not visible in the resolution used, the interpretation as a dynamical gauge group is suggestive. The dynamical gauge group could correspond to n-dimensional Cartan algebra acting in conformal degrees of freedom identifiable as a simply laced Lie group. This would assign a infinite hierarchy of dynamical gauge symmetries to the broken conformal gauge invariance acting as symmetries of dark matter. This still leaves infinite number of degrees of freedom assignable to the imbedding space Hamiltonians and spectrum generated by zeros of zeta but this might have interpretation in terms of gauging so that additional vanishing conditions for Noether charges are suggestive.

Dark matter as large phases with large gravitational Planck constant $h_{eff} = h_{gr}$

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K101, K83].

1. The proposal is that a Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems and that only the generalizations of Bohr orbits are involved. The space-time sheets in question would carry dark matter.

2. Nottale's hypothesis would predict a gigantic value of $\hbar_{gr}$. Equivalence Principle and the independence of gravitational Compton length $\Lambda_{gr} = \hbar_{gr}/m = GM/v_0 = 2r_S/v_0$ (typically astrophysical scale) on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that $\hbar_{gr}$ would be much smaller. Large $\hbar_{gr}$ could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets, which is quantum coherent in the required time scale [K101].

   One could criticize the hypothesis since it treats the masses M and m asymmetrically: this is only apparently true [**?**].

3. It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta). The cross section of the flux tube corresponds to a sphere $S^2_i \subset CP_2$, $i = I, II$ [L31]. S^2_I is homologically non-trivial carrying Kähler magnetic monopole flux. S^2_{II} is homologically trivial carrying vanishing Kähler magnetic flux but non-vanishing electro-weak flux [L31].

   The flux tubes of type I have both Kähler magnetic energy and dark energy due to the volume action. Flux tubes of type II would have only the volume energy. Both flux tubes could be remnants of cosmic string phase of primordial cosmology. The energy of these flux quanta would be correlated for galactic dark matter and volume action and also magnetic tension would give rise to negative "pressure" forcing accelerated cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside flux tubes identifiable also as dark energy.

4. Both theoretical consistency and certain experimental findings from astrophysics [E2, E4] and biology [K24, K14] suggest the identification $h_{eff} = n \times h = h_{gr}$. The large value of h_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description) [K96]. The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also modified Dirac action is present, part of the interior degrees of freedom associated with the fermionic part of conformal algebra become physical.

   Fermionic oscillator operators could generate super-symmetries and sparticles could correspond to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to an ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) or to a bunch of n low energy gravitons.

Hierarchies of quantum criticalities, Planck constants, and dark matters

Quantum criticality is one of the corner stone assumptions of TGD. In the original approach the value of Kähler coupling strength α_K together with CP_2 radius R fixed quantum TGD and is analogous to critical temperature. Twistor lift [L31] brings in additional coupling constant Λ obeying p-adic coupling constant evolution and Planck length l_G, which like CP_2 radius would not obey coupling constant evolution (as also G). The values of these parameters should be fixed by quantum criticality. What else does quantum criticality mean is however far from obvious, and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K57, K127, K96].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could quantum criticality having classical or perhaps even thermodynamical criticality as its correlate be always accompanied by the generation of dark matter? If this were the case, the recipe would be stupifyingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale [K84, ?]. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of the particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

5. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1Z_2e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1Z_2e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

One example of criticality is super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect [D8] and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity [?].

But how does quantum criticality relate to number theory and adelic physics? $h_{eff}/h = n$ has been identified as the number of sheets of space-time surface identified as a covering space of some kind. Number theoretic discretization defining the "spine" for a monadic space-time surface [L37] defines also a covering space with Galois group for an extension of rationals acting as covering group. Could n be identifiable as the order for a sub-group of Galois group? If this is the case, the proposed rule for h_{eff} changing phase transitions stating that the reduction of n occurs to its factor would translate to spontaneous symmetry breaking for Galois group and spontaneous - symmetry breakings indeed accompany phase transitions.

TGD variant of AdS/CFT duality

AdS/CFT duality [B48] has provided a powerful approach in the attempts to understand the non-perturbative aspects of super-string theories. The duality states that conformal field theory in n-dimensional Minkowski space M^n identifiable as a boundary of $n+1$-dimensional space AdS_{n+1} is dual to a string theory in $AdS_{n+1} \times S^{9-n}$.

As a mathematical discovery AdS/CFT duality is extremely interesting but it seems that it need not have much to do with physics as such. From TGD point of view the reason is obvious: the notion of conformal invariance is quite too limited. In TGD framework conformal invariance is extended to a super-symplectic symmetry in $\delta M^4_\pm \times CP_2$, whose Lie-algebra has the structure of conformal algebra. Also ordinary super-conformal symmetries associated with string world sheets are present as well as generalization of 2-D conformal symmetries to their analogs at light-cone boundary and light-like orbits of partonic 2-surfaces. In this framework AdS/CFT duality is expected to be modified.

The matrix elements $G_{K\overline{L}}$ of Kähler metric of WCW can be expressed in two manners. As contractions of the derivatives $\partial_K\partial_{\overline{L}}K$ of the Kähler function of WCW with isometry generators or as anticommutators $\{\Gamma_K, \Gamma_{\overline{L}}\}$ of WCW gamma matrices identified as supersymplectic Noether super charges assignable to fermionic strings connecting partonic 2-surfaces. Kähler function is identified as real part of the action: if coupling parameters are real it reduces to the action for the Euclidian space-time regions with 4-D CP_2 projection and otherwise contains contributions from both Minkowskian and Euclidian regions. The action defines the modified gamma matrices appearing in modified Dirac action as contractions of canonical momentum currents with imbedding space gamma matrices.

This observation suggests that there is a super-symmetry between action and modified Dirac action. The problem is that induced spinor fields naive of SH and also well-definedness of em charge demand the localization of induced spinor modes at 2-D string world sheets. This simply cannot be true. On the other hand, SH only requires that the data about induced spinor fields and space-time surface at the string world sheets is enough to construct the modes in space-time interior.

This leaves two options if one assumes that SH is exact (recall however that the number theoretic interpretation for the hierarchy of Planck constants suggests that the number-theoretic spin of monadic space-time surface represents additional discrete data needed besides that assignable to string world sheets to describe dark matter). As found in the section 11.2.3, there are two options.

Option I: The analog of brane hierarchy is realized at the level of fundamental action. There is a separate fundamental 2-D action assignable with string world sheets - area and topological magnetic flux term - as also world line action assignable to the boundaries of string world sheets. By previous argument string tension should be determined by the value of the cosmological constant Λ obeying -adic coupling constant evolution rather than by G: otherwise there is no hope about gravitationally bound states above Planck scale. String tension would appear as an additional fundamental coupling parameter (perhaps fixed by quantum criticality). This option does not quite conform with the spirit of SH.

Option II: 4-D space-time action and corresponding modified Dirac action defining fundamental actions are expressible as effective actions assignable to string world sheets and their boundaries. String world sheet effective action action could be expressible as string area for the effective metric defined by the anti-commutators of modified gamma matrices at string world sheet. If the sum of the induced Kähler forms of M^4 and CP_2 vanishes at string world sheets the effective metric would be the induced 2-D metric: this together with the observed CP breaking could provide a justification for the introduction of the analog of Kähler form in M^4. String tension would be dynamical rather than determined by l_P and depend on Λ, l_P, R and α_K. This representation of Kähler action would be one aspect of the analog of AdS/CFT duality in TGD framework.

Both options would allow to understand how strings connecting partonic 2-surfaces give rise to the formation of gravitationally bound states. Bound states of macroscopic size are possible only if one allows hierarchy of Planck constants and this is required also by the (extremely) small value of Λ (in cosmic scales).

Consider the concerete realizations for this vision.

1. SGCI requires effective 2-dimensionality. In given UV and IR resolutions partonic 2-surfaces and string world sheets are assignable to a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). Λ would closely relate to the size scale of CD. String world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids carrying fermionic quantum numbers intersect partonic 2-surfaces at discrete points.
2. This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces, whose area by quantum classical correspondence depends on the quantum numbers of the external particles.

String tension of gravitational flux tubes

For Planckian cosmic strings only quantum gravitational bound states of length of order Planck length are possible. There must be a mechanism reducing the string tension. The *effective* string tension assignable to magnetic flux tubes must be inversely proportional to $1/h_{eff}^2$, $h_{eff} = n \times h = h_{gr} = 2\pi GMm/v_0$ in order to obtain gravitationally bound states in macroscopic length scales identified as structures for which partonic 2-surfaces are connected by flux tubes accompanied by fermionic strings.

The reason is that the size scale of (quantum) gravitationally bound states of masses M and m is given by grvitational Compton length $\Lambda_{gr} = GM/v_0$ [K101, K84, **?**] assignable to the gravitational flux tubes connecting the masses M and m. If the string tension is of order Λ_{gr}^2 this is achieved since the typical length of string would be Λ_{gr}. Gravitational string tension must be therefore of order $T_{gr} \sim 1/\Lambda_{gr}^2$. How could this be achieved? One can imagine several options and here only the option based on the assumptions

1. Twistor lift makes sense.
2. Fundamental action is 4-D for both space-time and fermionic degrees of freedom and 2-D string world sheet action is an effective action realizing SH. Note effective action makes also possible braid statistics, which does not make sense at fundamental level.

3. Also M^4 carries the analog of Kähler form and the sum of induced Kähler forms from M^4 and CP_2 vanishes at string world sheets and also weak gauge fields vanishes at string world sheets leaving only em field.

is considered since it avoids all the objections that I have been able to invent.

For the twistor lift of TGD [L31] predicting cosmological constant Λ depending on p-adic length scale $\Lambda \propto 1/p$ the gravitational strings would be naturally homologically trivial cosmic strings. These vacuum extremals of Kähler action transform to minimal surface extremals with string tension given by $\rho_{vac}S$, where ρ_{vac} the density of dark energy assignable to the volume term of the action and S the transverse area of the flux tube. One should have $\rho_{vac}S = 8\pi\Lambda S/G = 1/\Lambda_{gr}^2$ so that one would have

$$8\pi\Lambda S = \frac{G}{\Lambda_{gr}^2} \quad .$$

Λ for flux tubes (characterizing the size of CDs containing them) would depend on the gravitational coupling Mm.

11.2.5 Number theoretical vision

Physics as infinite-D spinor geometry of WCW and physics as generalized number theory are the two basic vision about TGD. The number theoretical vision involves three threads [K106, K107, K105].

1. The first thread [K106] involves the notion of number theoretical universality NTU: quantum TGD should make sense in both real and p-adic number fields (and their algebraic extensions induced by extensions of rationals). p-Adic number fields are needed to understand the space-time correlates of cognition and intentionality [K77, K47, K79].

   p-Adic mass calcuations lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [K77, K47]. One of the first applications was the calculation of elementary particle masses [K65]. The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra are involved. Not only the fundamental mass scales would reduce to number theory but also particle masses are predicted correctly under rather mild assumptions and are exponentially sensitive to the p-adic length scale predicted by p-adic length scale hypothesis. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [K65, K26].

2. Second thread [K107] is inspired by the dimensions $D = 1, 2, 4, 8$ of the basic objects of TGD and assumes that classical number fields are in a crucial role in TGD. 8-D imbedding space would have octonionic structure and space-time surfaces would have associative (quaternionic) tangent space or normal space. String world sheets could correspond to commutative surfaces. Also the notion of M^8-H-duality is part of this thread and states that quaternionic 4-surfaces of M^8 containing preferred M^2 in its tangent space can be mapped to PEs in H by assigning to the tangent space CP_2 point parametrizing it. M^2 could be replaced by integrable distribution of $M^2(x)$. If PEs are also quaternionic one has also $H - H$ duality allowing to iterate the map so that PEs form a category. Also quaternion analyticity of PEs is a highly attractive hypothesis [K116]. For instance, it might be possible to interpret string world sheets and partonic 2-surfaces appearing in strong form of holography (SH) as co-dimension 2 surfaces analogous to poles of analytic function in complex plane. Light-like 3-surfaces might be seen as analogs of cuts. The coding of analytic function by its singularities could be seen as analog of SH.

3. The third thread [K105] corresponds to infinite primes and leads to several speculations. The construction of infinite primes is structurally analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory with free particle states characterized by primes. The many-sheeted structure of TGD space-time could reflect directly the structure of infinite prime coding it. Space-time point would become infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography.

Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

11.3 p-Adic mass calculations and p-adic thermodynamics

p-Adic mass calculations carried for the first time around 1995 were the stimulus eventually leading to the number theoretical vision as a kind dual for the geometric vision about TGD. In this secton I will roughly describe the calculations [K26, K65] and the questions and challenges raised by them.

11.3.1 p-Adic numbers

Like real numbers, p-adic numbers (`http://tinyurl.com/hmgqtoh`) can be regarded as completions of the rational numbers to a larger number field [K47]. Each prime p defines a p-adic number field allowing the counterparts of the usual arithmetic operations.

1. The basic difference between real and p-adic numbers is that p-adic topology is ultra-metric. Ultrametricity means that the distance function $d(x,y)$ (the counterpart of $|x-y|$ in the real context) satisfies the inequality

$$d(x,z) \leq Max\{d(x,y), d(y,z)\} \ ,$$

(Max(a, b) denotes maximum of a and b) rather than the usual triangle inequality

$$d(x,z) \leq d(x,y) + d(y,z) \ .$$

2. The topology defined by p-adic numbers is compact-open. Hence the generalization of manifold obtained by gluing together n-balls fails because smallest open n-balls are just points and one has totally disconnected topology.
3. p-Adic numbers are not well-ordered like real numbers. Therefore one cannot assign orientation to the p-adic number line. This in turn leads to difficulties with attempts to define definite integrals and the notion of differential form although indefinite integral is well-defined. These difficulties serve as important guidelines in the attempts to understand what p-adic physics is and also how to fuse real and various p-adic physics to a larger structure.
4. p-Adic numbers allow an expansion in powers of p analogous to the decimal expansion

$$x = \sum_{n\geq 0} x_n p^n \ ,$$

and the number of terms in the expansion can be infinite so that p-adic number need not be finite as a real number. The norm of the p-adic number (counterpart of $|x|$ for real numbers) is defined as

$$N_p(x) = \sum_{n\geq 0} x_n p^n = p^{-n_0} \ ,$$

and depends only very weakly on p-adic number. The ultra-metric distance function can be defined as $d_p(x,y) = N_p(x-y)$.
5. p-Adic numbers allow a generalization of the differential calculus. The basic rules of the p-adic differential calculus are the same as those of the ordinary differential calculus. There is however one important new element: the set of the functions having vanishing p-adic derivative consists of so called pseudo constants, which are analogs of real valued piecewise constant functions. In the real case only constant functions have vanishing derivative. This implies that p-adic differential equations are non-deterministic. This non-determinism is identified as a counterpart of the non-determinism of cognition and imagination [K79].

11.3.2 Model of elementary particle

p-Adic mass calculations [K26, K65] rely heavily on a topological model for elementary particle and it is appropriate to describe it before going to the summary of calculations.

Family replication phenomenon topologically

One of the basic ideas of TGD approach to particle physics has been genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies.

With the advent of zero energy ontology (ZEO) this picture has changed somewhat.

1. The wormhole throats identified as light-like 3-surfaces at with the induced metric of the space-time surface changes its signature from Minkowskian to Euclidian correspond to the light-like orbits of partonic 2-surfaces. One cannot of course exclude the possibility that also boundary components allow to satisfy boundary conditions without assuming vacuum extremal property of nearby space-time surface.

   The intersections of the wormhole throats with the light-like boundaries of causal diamonds (CDs) identified as intersections of future and past directed light cones ($CD \times CP_2$ is actually in question but I will speak about CDs) define special partonic 2-surfaces and the conformal moduli of these partonic 2-surfaces appear in the elementary particle vacuum functionals [K26] naturally. A modification of the original simple picture came from the proposed identification of physical particles as bound states of two wormhole contacts connected by tubes carrying monopole fluxes.

2. For generalized scattering diagrams stringy trouser vertices are replaced with vertices at which the ends of light-like wormhole throats meet. This vertex is the analog of 3-vertex for Feynman diagrams in particle physics lengths scales and for the biological replication (DNA and even cell) in macroscopic length scales.

   In this picture the interpretation of the analog of trouser vertex is in terms of propagation of same particle along two different paths. This interpretation is mathematically natural since vertices correspond to 2-manifolds rather than singular 2-manifolds, which are just splitting to two disjoint components. Second complication comes from the weak form of electric-magnetic duality forcing to identify physical particles as weak strings with magnetic monopoles at their ends and one should understand also the possible complications caused by this generalization.

These modifications force to consider several options concerning the identification of light fermions and bosons and one can end up with a unique identification only by making some assumptions. Masslessness of all wormhole throats - also those appearing in internal lines - and dynamical $SU(3)$ symmetry for particle generations are attractive general enough assumptions of this kind. Bosons and their possible spartners would correspond to wormhole contacts with fermion and anti-fermion at the throats of the contact. The expectation was tht free fermions and their possible spartners correspond to CP_2 type vacuum extremals with single wormhole throat. It however turned however that dynamical $SU(3)$ symmetry forces to identify massive (and possibly topologically condensed) fermions as pairs of (g, g) type wormhole contacts. The existence of higher boson families would mean breaking of quark and lepton universality and there are indications for this kind of anomaly [K71] .

The notion of elementary particle vacuum functional

Obviously one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals (EPVFs), is made. The basic assumptions underlying the construction are the following ones [K26].

1. EPVFs depend on the geometric properties of the two-surface X^2 representing elementary particle.

2. EPVFs possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting PE as a generalized Bohr orbit [K57, K9, K15]) and determined in general coordinate invariant manner.

3. ZEO allows to select uniquely the partonic 2-surface as the intersection of the wormhole throat at which the signature of the induced 4-metric changes with either the upper or lower

boundary of $CD \times CP_2$. This is essential since otherwise one one could not specify the vacuum functional uniquely.

4. Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.

5. Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.

6. EPVFs are stable against the decay $g \to g_1 + g_2$ and one particle decay $g \to g - 1$. This process corresponds to genuine particle decay only for stringy diagrams. For generalized scattering diagrams the interpretation is in terms of propagation along two different paths simultaneously.

In [K26] the construction of EPVFs is described in detail. This requires some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces and the concept of hyper-ellipticity. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are needed. Also possible explanations for the experimental absence of the higher fermion families are considered. Concerning p-adic mass calculations, the key question is how to construct p-adic variants of EPVFs.

11.3.3 p-Adic mass calculations

p-Adic thermodynamics

Consider first the basic ideas of p-adic thermodynamics.

1. p-Adic valued mass squared is identified as as thermal mass in p-adic thermodynamics. Boltzmann weights $exp(-E/T)$ do not make sense if one just replaces exponent function with the p-adic variant of its Taylor series. The reason is that $exp(x)$ has p-adic norm equal to 1 for all acceptable values of the argument x (having p-adic norm smaller than one) so that partition function does not have the usual exponential convergence property. Nothing however prevents from consider Boltzmann weights as powers p^n making sense for integer values of n. Here the p-adic norm approaches zero for $n \to +\infty$: thus the correspondences $e^{-E/T} \leftrightarrow p^{E/T_p}$.

   The values of E/T_p must be quantized to integers. This is guaranteed if E is integer valued in suitable unit of energy and $1/T_p$ has integer valued spectrum using same unit for T_p. Super-conformal invariance guarantees integer valued spectrum of E, which in the recent case corresponds to mass squared. These number theoretical conditions are very powerful and lead to the quantization of also thermal mass squared for given p-adic prime p.

2. The p-adic mass squared is mapped to real number by canonical identification $I : \sum x_n p^n \to \sum x_n p^{-n}$ or its variant for rationals. Canonical identification is continuous and maps powers of p^n to their inverses. One modification of canonical identification maps rationals m/n in their representation in which m and n have no common divisors to $I(m)/I(n)$. The predictions of calculations depend in some cases on which variant one uses but rational option looks the most reasonable choice.

3. p-Adic length scale hypothesis states that preferred p-adic primes correspond to powers of 2: $p \simeq 2^k$, but smaller than 2^k. The values of k form with $p = 2^k - 1$ is prime - Mersenne prime - are especially favored. The nearer the prime p to 2^k, the more favored p is physically. One justification for the hypothesis is that preferred primes have been selected by an evolutionary process.

4. It turns out that p-adic temperature is $T_p = 1$ for fermions. For gauge bosons $T_p \leq 1/2$ seems to be necessary assumption for gauge bosons implying that the contribution to mass squared is very small so that super-symplectic contribution assignable to the wormhole magnetic flux tube dominates for weak bosons. For canonical identification $m/n \to I(m)/I(n)$ second order contribution to fermionic mass squared is very small.

5. The large values of p-adic prime p guarantee that the p-adic thermodynamics convervges extremely rapidly. For $m/n \to I(m)/I(n)$ already the second order contribution is extremely

small since the expansion for the real mass squared is in terms of $1/p$ and for electron with $p = M_{127}$ one has $p \sim 10^{38}$. Hence the calculations are essentially exact and errors are those of the model. It is quite possible that calculations could be done exactly using exact expressions for the super-symplectic partition functions generalized to p-adic context. The success of the p-adic mass calculations is especially remarkable because p-adic length scale hypothesis $p \simeq 2^k$ predicts exponential sensitivity of the particle mass scale on k.

Symmetries

The number theoretical existence of p-adic thermodynamics requires powerful symmetries to guarantee integer valued spectrum for the thermalized contribution to the mass squared.

1. Super-conformal symmetry with integer valued conformal weights for Virasoro scaling generator L_0 is essential because it predicts in string models that mass squared is apart from ground state contribution integer valued in suitable units. In TGD framework fermionic string world sheets are characterized by super-conformal symmetry. This gives the p-adic thermodynamics assumed in the calculations. One could however assign Super Virasoro algebra also to super-symplectic algebra having its analog as sub-algebra with positive integer conformal weights. Same applies to the extended conformal algebra of light-cone boundary.

2. TGD however predicts also generalization of conformal symmetry associated with light-cone boundary involving ordinary complex conformal weights and the conformal weight associated with the light-like radial coordinate. For the latter conformal weights for the generators of supersymmetry might be given by $h = -s_n/2$. s_n zero of zeta or pole $h = -s = -1$ of zeta.

   Also super-symplectic symmetries would have similar radial spectrum of conformal weights. Conformal confinement requiring that the conformal weights of states are real implies that the spectrum of conformal weights for physical states consists of non-negative integers as for ordinary superconformal invariance.

   It is not clear whether thermalization occurs in these degrees of freedom except perhaps for trivial conformal weights. These degrees of freedom need not therefore contribute to thermal masses of leptons and quarks but would give dominating contribution to hadron masses and weak boson masses. The negative conformal weights predicted by $h = -s/2$ hypothesis predicts that ground state weight is negative for super-symplectic representations and must be compensated for massless states.

   The assumption that ground state conformal weight is negative and thus tachyonic is essential in case of p-adic mass calculations [K65], and only for massless particles (graviton, photon, gluons) it vanishes or is of order $O(1/p)$. This could be achieved if the ground state of super-symplectic representation has $h = 0$.

3. Modular invariance [K26] assignable to partonic 2-surfaces is a further assumption similar to that made made in string models. This invariance means that for a given genus the dynamical degrees of freedom of the partonic 2-surface correspond to finite-dimensional space of Teichmueller parameters. For genus $g = 0$ this space is trivial.

   Also modular invariance for string world sheets can be considered. By SH the information needed in mass calculations should be assignable to partonic 2-surfaces: the assumption is that one can assign this information to single partonic 2-surface. Stringy contribution would be seen only in scattering amplitudes.

   This might be true only effectively: the recent view about elementary particles is that they are pairs of wormhole contacts connected by flux tubes defining a closed monopole flux and wormhole throats of contact have same genus for light states. Furthermore the quantum numbers of particle are associated with single throat for fermions and with opposite throats of single contact for bosons. The second wormhole contact would carry neutralizing weak charges to realize the finite range of weak interactions as "weak confinement".

   The number of genera is infinite and one must understand why only three quark and lepton generations are observed. An attractive explanation is in terms of symmetry. For the three lowest genera the partonic 2-surfaces are always hyper-elliptic and have thus global conformal Z_2 symmetry. For higher genera this is not true always and EPVFs constructed from the assumption of modular invariance vanish for the hyper-elliptic surfaces. This suggests that the higher genera are very massive or can be interpreted as many-particle states of handles, which are not bound states but have continuous mass squared.

Contributions to mass squared

There are several contributions to the p-adic thermal mass squared come from the degrees of freedom, which are thermalized.

Super-conformal degrees of freedom associated with string world sheets are certainly thermalized. p-Adic mass calculations strongly suggest that the number of super-conformal tensor factors is $N = 5$ but also $N = 4$ and $N = 6$ can be considered marginally.

I have considered several identifications of tensor factors and not found a compelling alternative. If one assumes that super-symplectic degrees of freedom do not contribute to the thermal mass, string world sheets should explain masses of elementary fermions. Here charged lepton masses are the test bench. One other hand, if super-symplectic degrees of freedom contribute one obtains additional tensor factor assignable to $h = -s/2$, s trivial zero of zeta). Only one tensor factor emerges since Hamiltonians correspond to the products of functions of the coordinates of light-cone boundary and CP_2).

1. $SU(2)_L \times U(1)$ gives 2 tensor factors. $SU(3)$ gives 1 tensor factor. The two transversal degrees of freedom for string world sheet suggest 2 degrees of freedom corresponding to Abelian group E^2. Rotations however transforms these degrees to each other so that 1 tensor factor should emerge. This gives 4 tensor factors. Could it correspond to the degrees of freedom parallel to string at its end assignable to wormhole throat? Could normal vibrations of partonic 2-surface? This would $N = 5$ tensor factors. Another possibility is that the fifth tensor factor comes from super-symplectic Super-Virasoro algebra defined by trivial conformal weights.
2. Super-symplectic contributions need not be present for ordinary elementary fermions. For weak bosons they could give string tension assignable to the magnetic flux tube connecting the wormhole contacts. It is not clear whether this contribution is thermalized. This contribution might be present only for the phases with $h_{eff} = n \times h$. This contribution would dominate in hadron masses.
3. Color degrees of freedom contribute to the ground state mass squared since ground state corresponds to an imbedding space spinor mode massless in 8-D sense. The mass squared contribution corresponds to an eigenvalue of CP_2 spinor d'Alembertian. Its eigenvalues correspond to color multiplets and only the covariantly constant right handed neutrino is color singlet. For the other modes the color representation is non-trivial and depends on weak quantum numbers of the fermion. The construction of the massless state from a tachyonic ground state with conformal weight $h_{vac} = -3$ must involve colored super-Kac Moody generators compensating for the anomalous color charge so that one obtains color single for leptons and color triplet for quarks as massless state.
4. Modular degrees of freedom give a contribution depending on the genus g of the partonic 2-surface. This contribution is estimated by considering p-adic variants of elementary particle vacuum functionals Ω_{vac} [K65] expressible as products of theta functions with the structure of partition function. Theta functions are expressible as sums of exponent functions $exp(X)$ with X defined as a contraction of the matrix Ω_{ij} defined by Teichmueller parameters between integer valued vectors.

   In ZEO the interpretation of Ω_{vac} is as a complex square root of partition functional (quantum theory as complex square root of thermodynamics in ZEO). The integral of $|\Omega|^2$ over allowed moduli has interpretation as partition function. The exponential $exp(Re(X)) = p^{Re(X)/log(p)}$ has interpretation as an exponential of "Hamiltonian" defined by the vacuum conformal weight defined by moduli. $T = log(p)$ is identified as p-adic temperature as in ordinary p-adic thermodynamics.

   NTU requires that the integration over the moduli parameters reduces to a sum over number theoretically universal moduli parameters. The exponents $exp(X)$ must exist p-adically. PE property alone could guarantee this. The exponentials appearing in theta functions should reduce to products $p^k p^{iy} = exp(k/log(p))p^{iy}$ with k is integer and p^{iy} a root of unity. The vacuum expectation value of $Re(X)$ contributing to the mass squared is obtained from the standard formula as logarithmic temperature derivative of the "integral" $\int |\Omega_{vac}|^2$. The formula is same as for the Super-Virasoro contributions apart from the integration reducing to a sum.

The considerations of the section 11.4.2 [L22] suggest that for given p-adic prime p the exponent $k+iy$ corresponds to a linear combinations of poles of fermionic zeta $z_F(s) = \zeta(s)/\zeta(2s)$ in the class $C(p)$ with non-negative integer coefficients. This class corresponds essentially to the conformal weights of a fractal sub-algebra of super-symplectic algebra. It could give rise also to the complex values of action so that Riemann zeta would define the core of TGD.

The general dependence of the contribution of genus g to mass squared on g follows from the functional form of EPVF as a product theta functions serving as building brick partition functions apart from overall multiplicative constant and gives a nice agreement with the observed charged lepton mass ratios. The basic feature of the formula is exponential dependence on g.

5. The super-symplectic stringy contribution assignable to the magnetic flux tube dominates for weak bosons and is analogous to the stringy contribution to the hadron masses.

p-Adic mass calculations leave open several questions. What is the precise origin of preferred p-adic primes and of p-adic length scale hypothesis? How to understand the preferred number $N = 5$ of Super-Kac-Moody tensor factors? How to calculate the contribution of super-symplectic degrees of freedom - are they thermalized? Why only 3 lowest genera are light and what are the masses of the predicted bosonic higher genera implying breaking of fermion universality.

11.3.4 p-Adic length scale hypothesis

p-Adic length scale hypothesis [K2, K77] has served as a basic hypothesis of p-adic TGD for several years. This hypothesis states that the scales $L_p = \sqrt{p}l$, $l = 1.376 \cdot 10^4\sqrt{G}$ are fundamental length scale at p-adic condensate level p. The original interpretation of the hypothesis was following:

1. Above the length scale L_p p-adicity sets on and effective course grained space-time or imbedding space topology is p-adic rather than ordinary real topology. Imbedding space topology seems to be more appropriate identification.
2. The length scale L_p serves as a p-adic length scale cutoff for the quantum field theory description of particles. This means that space-time begins to look like Minkowski space so that the QFT $M^4 \to CP_2$ becomes a realistic approximation. Below this length scale string like objects and other particle like 3-surfaces are important.
3. It is un-natural to assume that just single p-adic field would be chosen from the infinite number of possibilities. Rather, there is an infinite number of cutoff length scales. To each prime p there corresponds a cutoff length scale L_p above which p-adic quantum field theory $M^4 \to CP_2$ makes sense and one has a hierarchy of p-adic QFTs. These different p-adic field theories correspond to different hierarchically levels possibly present in the topological condensate. Hierarchical ordering $< p_1 < p_2 < ...$ means that only the surface $p_1 < p_2$ can condense on the surface p_2. The condensed surface can in practice be regarded as a point like particle at level p_2 described by the p-adic conformal field theory below length scale L_{p_2}.

The recent view inspired by adelic physics is that preferred p-adic primes correspond to so called ramified primes for the algebraic extension of rationals defining the adele [K125]. Weak form of Negentropy Maximization Principle (WNMP) [K70] in turn allows to conclude that the length scales corresponding to powers of primes are preferred. Therefore p-adic length scale hypothesis generalizes. There is evidence for 3-adic time scales in biology [**?**, **?**] and 3-adic time scales can be also assigned with Pythagorean scale in geometric theory of harmony [K91] [L19].

11.3.5 Mersenne primes and Gaussian Mersennes are special

Mersenne primes and their complex counterparts Gaussian Mersennes pop up in p-adic mass calculations and both elementary particle physics, biology [K88], and astrophysics and cosmology [K68] provide support for them.

Mersenne primes

One can also consider the milder requirement that the exponent $\lambda = 2^{\epsilon L_0}$ represents trivial scaling represented by unit in good approximation for some p-adic topology. Not surprisingly, this is the case for $L_0 = mp^k$ since by Fermat's theorem $a^p\ mod\ p = 1$ for any integer a, in particular $a = 2$. This is also the case for $L_0 = mk$ such that $2^k\ mod\ p = 1$ for p prime. This occurs if $2^k - 1$ is Mersenne prime: in this case one has $2^{L_0} = 1$ modulo p so that the sizes of the fractal sub-algebras are exponentially larger than the sizes of $L_0 \propto p^n$ algebras. Note that all scalings a^{L_0} are near to unity for $L_0 = p^n$ whereas now only $a = 2$ gives scalings near unity for Mersenne primes. Perhaps this extended fractality provides the fundamental explanation for the special importance of Mersenne primes.

In this case integrated scalings 2^{L_0} leave the states almost invariant so that even a stronger form of the breaking of the exact conformal invariance would be in question in the super-symplectic case. The representation would be defined by the generators for which conformal weights are odd multiples of n ($M_n = 2^n - 1$) and L_{-kn}, $k > 0$ would generate zero norm states only in order $O(1/M_n)$.

Especially interesting is the hierarchy of primes defined by the so called Combinatorial Hierarchy resulting from TGD based model for abstraction process. The primes are given by $2, 3, 7 = 2^3 - 1, 127 = 2^7 - 1, 2^{127} - 1$, ..: $L_0 = n \times 127$ would correspond to M_{127}-adicity crucial for the memetic code.

Gaussian Mersennes are also special

If one allows also Gaussian primes then the notion of Mersenne prime generalizes: Gaussian Mersennes are of form $(1 \pm i)^n - 1$. In this case one could replace the scaling operations by scaling combined with a twist of $\pi/4$ around some symmetry axis: $1 + i = \sqrt{2}exp(i\pi/4)$ and generalized p-adic fractality would mean that for certain values of n the exponentiated operation consisting of n basic operations would be very near to unity.

1. The integers k associated with the lowest Gaussian Mersennes are following: $2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113$. $k = 113$ corresponds to the p-adic length scale associated with the atomic nucleus and muon. Thus all known charged leptons, rather than only e and τ, as well as nuclear physics length scale, correspond to Mersenne primes in the generalized sense.

2. The primes $k = 151, 157, 163, 167$ define perhaps the most fundamental biological length scales: $k = 151$ corresponds to the thickness of the cell membrane of about ten nanometers and $k = 167$ to cell size about 2.56 μm. This observation also suggests that cellular organisms have evolved to their present form through four basic evolutionary stages. This also encourages to think that $\sqrt{2}exp(i\pi/4)$ operation giving rise to logarithmic spirals abundant in living matter is fundamental dynamical symmetry in bio-matter.

   Logarithmic spiral provides the simplest model for biological growth as a repetition of the basic operation $\sqrt{2}exp(i\pi/4)$. The naive interpretation would be that growth processes consist of $k = 151, 157, 163, 167$ steps involving scaling by $\sqrt{2}$. This however requires the strange looking assumption that growth starts from a structure of size of order CP_2 length. Perhaps this exotic growth process is associated with pair of MEs or magnetic flux tubes of opposite time orientation and energy emergenging CP_2 sized region in a mini big bang type process and that the resulting structure serves as a template for the biological growth.

3. $k = 239, 241, 283, 353, 367, 379, 457$ associated with the next Gaussian Mersennes define astronomical length scales. $k = 239$ and $k = 241$ correspond to the p-adic time scales .55 ms and 1.1 ms: basic time scales associated with nerve pulse transmission are in question. $k = 283$ corresponds to the time scale of 38.6 min. An interesting question is whether this period could define a fundamental biological rhythm. The length scale $L(353)$ corresponds to about 2.6×10^6 light years, roughly the size scale of galaxies. The length scale $L(367) \simeq \times 3.3 \times 10^8$ light years is of same order of magnitude as the size scale of the large voids containing galaxies on their boundaries (note the analogy with cells). $T(379) \simeq 2.1 \times 10^{10}$ years corresponds to the lower bound for the order of the age of the Universe. $T(457) \sim 10^{22}$ years defines a completely superastronomical time and length scale.

11.3.6 Questions

The proposed picture leaves open several questions.

1. Could the descriptions by both real and p-adic thermodynamics be possible? Could they be equivalent (possibly in finite measurement resolution) as is suggested by NTU? The consistency of these descriptions would imply temperature quantization and p-adic length scale hypothesis not possible in purely real context.
2. What could the extension of conformal symmetry to supersymplectic symmetry mean? One possible view is that super-symplectic symmetries correspond to dark degrees of freedom and that only the super-symplectic ground states with negative conformal weights affect the p-adic thermodynamics, which applies only to fermionic degrees of freedom at string world sheets. Super-symplectic degrees of freedom would give the dominant contribution to hadron masses and could contribute also to weak gauge boson masses. $N = 5$ for the needed number of tensor factors is however a strong constraint and perhaps most naturally obtained when also the super-symplectic Virasoro associated with the trivial zeros of zeta is thermalized.
3. What happens in dark sectors. Preferred extremal property is proposed to mean that the states are annihilated by super-symplectic sub-algebra isomorphic to the original algebra and its commutator with the entire algebra. The conjecture is that this gives rise to Kac-Moody algebras as dynamical symmetries - maybe ADE type algebras, whose Dynkin diagrams characterize the inclusion of HFFs. Does this give an additional tensor factor to super-Virasoro algebra?
4. Superconformal symmetry true in the sense that Super Virasoro conditions hold true. Partition function however depends on mass squared only rather than the entire scaling generator L_0 as thought erratically in the first formulation of p-adic calculation. This does not mean breaking of conformal invariance. Super Virasoro conditions hold true although partition function is for the vibrational part of L_0 determining the mass squared spectrum.

11.4 p-Adicization and adelic physics

This section is devoted to the challenges related to p-adicization and adelization of physics in which the correspondence between real and p-adic numbers via canonical identification serves as the basic building brick. Also the problems associated with p-adic variants of integral, Fourier analysis, Hilbert space, and Riemann geometry should be solved in a way respecting fundamental symmetries and their p-adic variants must be met. The notion of number theoretical universality (NTU) plays a key role here. One should also answer to questions about the origin of preferred primes and p-adic length scale hypothesis.

11.4.1 Challenges

The basic challenges encountered are construction of the p-adic variants of real number based physics, understanding their relationship to real physics, and the fusion of various physics to single coherent whole.

The p-adicization of real physics is not just a straightforward formal generalization of scattering amplitudes of existing theories but requires a deeper understanding of the physics involved. The interpretation of p-adic physics as correlate for cognition and imagination is an important guideline and will be discussed l in more detail in separate section.

Definite integral and Fourier analysis are basic elements of standard physics and their generalization to the p-adic context defines a highly non-trivial challenge. Also the p-adic variants of Riemann geometry and Hilbert space are suggestive. There are however problems.

1. There are problems associated with p-adic definite integral. Riemann sum does not make sense since it approaches zero if the p-adic norm of discretization unit approaches zero. The problems are basically due to the absence of well-orderedness essential for the definition of definite integral and differential forms and their integrals.

   Residue integration might make sense in finite angle resolution. For algebraic extension containing $e^{i\pi/n}$ the number theoretically universal approximation $i\pi = n(e^{i\pi/n} - 1)$ could be used. In twistor approach integrations reduce to multiple residue integrations and since twistor approach generalizes in TGD framework, this approach to integration is very attractive.

Positivity is a central notion in twistor Grassmannian approach [B26]. Since canonical identification maps p-adic numbers to non-negative real numbers, there is a strong temptation to think that positivity relates to NTU [L24].

2. There are problems with Fourier analysis. The naive generalization of trigonometric functions by replacing e^{ix} with its p-adic counterpart is not physical. Same applies to e^x. Algebraic extensions are needed to get roots of unity ad e as counterparts of the phases and discretization is necessary and has interpretation in terms of finite resolution for angle/phase and its hyperbolic counterpart.

3. The notion of Hilbert space is problematic. The naive generalization of Hilbert space norm square $|x|^2 = \sum x_n \overline{x}_n$ for state $(x_1, x_2, ...)$ can vanish p-adically. Also here NTU could help. State would contain as coefficients only roots of e and unity and only the overall factor could be p-adic number. Coefficients could be restricted to the algebraic numbers generating the algebraic extension of rational numbers and would not contain powers of p or even ordinary p-adic numbers expect in the overall normalization factor.

Second challenge relates to the relationship between real and p-adic physics. Canonical identification (CI) $\sum x_n p^n \to \sum x_n p^{-n}$ or some of its variants should play an important role. CI is expected to map the invariants appearing in scattering amplitudes to their real counterparts.

1. Real and p-adic variants of space-time surfaces should exist and relate to each other somehow. Is this relationship local and involve CI at space-time level or imbedding space level? Or is it only a global and non-local assignment of preferred real extremals to their p-adic counterparts? Or is between these extreme options and involves algebraic discretization of the space-time surface weakening the strong form of SH as already proposed? How do real and p-adic imbedding spaces relate to each other and can this relationship induce local correspondence between preferred extremals (PEs) [K9, K15, L31]?

2. NTU in some sense is a highly suggestive approach to these questions and would suggest that canonical identification applies to isometry invariants whereas angles and hyperbolic angles, or rather the corresponding "phases" belonging to an extension of p-adics containing roots of e and roots of unity are mapped to themselves. Note that the roots of e define extensions of rationals, which induce finite dimensional algebraic extensions of p-adic numbers. This would make possible to define imbedding space in accordance with NTU. Also the Hilbert space could be defined by requiring that its points correspond to number theoretically universal angles expressible in terms of roots of unity.

3. What about real and p-adic variants of WCW? Are they needed at all? Or could their existence be used as a powerful constraint on real physics? The representability of WCW as a union of infinite-dimensional symmetric spaces labelled by zero modes suggests that the same description applies at the level of WCW and imbedding space.

One cannot circumvent the question about how to generalize functional integral from real WCW to p-adic WCWs. In particular, what is the p-adic variant of the action defining the dynamics of space-time surfaces. In the case of exponent of action the p-adic variant could be defined by assuming algebraic universality: again the roots of e and of unity would be in central role. Also the Kähler structure of WCW implying that Gaussian and metric determinants cancel each other in functional integral, would be absolutely crucial.

One must remember that the exponents of action for scattering amplitudes for the stationary phase extremal cancel from the path integral representation of scattering amplitudes. Also now this mechanism would allow to get rid of the poorly defined exponent for single minimum. If there is sum over scattering amplitudes assignable to different maxima, normalization sould give ratios of these exponents for different extrema/maxima and only these ratios should belong to the extension of rationals.

The zero modes of WCW metric are invariants of supersymplectic group so that canonical identification could relate their real and p-adic variants. Zero modes could break NTU and would be behind p-adic thermodynamics and dependence of mass scale on p-adic prime.

The third challenge relates to the fusion of p-adic physics and real physics to a larger structure. Here a generalization of number concept obtained by glueing reals and various p-adics together along an extension of rational numbers inducing the extensions of p-adic numbers is highly suggestive. Adeles associated with the extension of rationals are highly attractive and closely related notion. Real and various p-adic physics would be correlates for sensory and cognitive aspects of the same universal physics rather than separate physics in this framework. One important implication of this view is that real entropy and p-adic negentropies characterize the same entanglement with coefficients in an extension of rationals.

NTU for hyperbolic and ordinary phases is definetely the central idea. How the invariance of angles under conformal transformations does relate to this? Could one perhaps define a discretized version of conformal symmetry preserving the phases defined by the angles between vectors assignable with the tangent spaces of discretized geometric structures and thus respecting NTU? Of should one apply conformal symmetry at Lie algebra level only?

11.4.2 NTU and the correspondence between real and p-adic physics

p-Adic real correspondence is certainly the basic problem of p-adicization and adelization. One can make several general questions about p-adic real correspondence and canonical identification inspired by p-adic mass calculations.

How generally p-adic real correspondence does apply? Could canonical identification for group invariants combined with direct identification of ordinary and hyperbolic phases identified as roots of unity and e apply at WCW and imbedding space level having maximally symmetric geometries? Could this make sense even at space-time level as a correspondence induced from imbedding space level [L37]? Does canonical identification apply locally for the discretizations of space-time surface or only globally for the parameters characterizing PEs (string world sheets and partonic 2-surfaces by SH), which are general coordinate invariant and Poincare invariant quantities?

The following vision seems to be the most feasible one found hitherto.

1. Preservation of symmetries and continuity compete. Lorenz transformations do not commute with canonical identification. This suggests that canonical identification applies only to Lorentz invariants formed from quantum numbers. This is enough in the case of scattering amplitudes. Canonical identification applies only to isometry invariants at the level of WCW and the phases/exponents of ordinary/hyperbolic angles correspond to numbers in the algebraic extension common to extensions of rationals and various p-adics.

2. Canonical identification applies at the level of momentum space and maps p-adic Lorentz invariants of scattering amplitudes to their real counterparts. Phases of angles and their hyperbolic counterparts should correspond to parameters defining extension and should be mapped as such to their p-adic counterparts.

3. The constraints coming from GCI and symmetries do not allow local correspondence but allow to consider its discretized version at space-time leve induced by the correspondence at the level of imbedding space.

   This requires the restriction of isometries and other symmetries to algebraic subgroups defined by the extension of rationals. This would imply reduction of symmetry due to finite cognitive/measurement resolution and should be acceptable. If one wants to realize the ideas about imagination, discretization must be applied also for the space-time interior meaning partial breaking of SH and giving rise to dark matter degrees freedom in TGD sense. SH could apply in real sector for realizable imaginations only. Note that the number of algebraic points of space-time surface is expected to be relatively small.

The correspondence must be considered at the level of imbedding space, space-time, and WCW.

1. At the level of imbedding space p-adic–real correspondence is induced by points in extension of rationals and is totally discontinuous. This requires that space-time dimension is smaller than imbedding space dimension.

2. At space-time level the correspondence involves field equations derivable from a local variational principle make sense also p-adically although the action itself is ill-defined as 4-D integral. The notion of p-adic PE makes sense by strong form of holography applied to 2-surfaces in the intersection. p-Adically however only the vanishing of Noether currents for a sub-algebra of the super-symplectic algebra might make sense. This condition is stronger than the vanishing of Noether charges defined by 3-D integrals.

3. Correspondence at the level of WCW can make sense and reduces to that for string world sheets and partonic 2-surfaces by SH. Real and p-adic 4-surfaces would be obtained by algebraic continuation as PEs from 2-surfaces by assuming that the space-time surface contains subset of points of imbedding space belonging to the extension of rationals [L37]. p-Adic pseudo constants make p-adic continuation easy. Real continuation need not exist always. p-Adic WCW would be considerably larger than real WCW and make possible a predictive quantum theory of imagination and cognition.

   What I have called intersection of realities and p-adicities can be identified as the set of 2-surfaces plus algebraic discretization of space-time interior. Also the values of induced spinor fields at the points of discretization must be given. The parameters characterizing the extremals (say coefficients of polynomials) - WCW coordinates - would be in extension of rationals inducing a finite-D extension of p-adic number fields.

   The hierarchy of algebraic extensions induces an evolutionary hierarchy of adeles. The interpretation could be as a mathematical correlate for cosmic evolution realized at the level of the core of WCW defined by the intersection? 2-surfaces could be called space-time genes.

4. Also the p-adic variant Kähler action or at least the exponent of Kähler action defining vacuum functional should be obtainable by algebraic continuation. The weakest condition states that the ratios of action exponents for the maxima of Kähler function to the sum of action exponents for maxima belong to the extension. Without this condition the hopes of satisfying NTU seem rather meager.

11.4.3 NTU at space-time level

What about NTU at space-time level? NTU requires a correspondence between real and p-adic numbers and the details of this corresponds have been a long standing problem.

1. The recent view about the correspondence between real PEs to their p-adic counterparts does not demand discrete local correspondence assumed in the earlier proposal [K128]. The most abstract approach would give up the local correspondence at space-time level altogether, and restrict the preferred coordinates of WCW (having maximal group of isometries) to numbers in the extension of rationals considered. WCW would be discretized.

   Intuitively a more realistic view is a correspondence at space-time level in the sense that real and p-adic space-time sheets intersect at points belonging to the extension of rationals and defining "cognitive representations". Only some p-adic space-time surfaces would have real counterpart.

2. The strongest form of NTU would require that the allowed points of imbedding space belonging an extension of rationals are mapped as such to corresponding extensions of p-adic number fields (no canonical identification). At imbedding space level this correspondence would be extremely discontinuous. The "spines" of space-time surfaces would however contain only a subset of points of extension, and a natural resolution length scale could emerge and prevent the fluctuation. This could be also seen as a reason for why space-times surfaces must be 4-D. The fact that the curve $x^n + y^n = z^n$ has no rational points for $n > 2$, raises the hope that the resolution scale could emerge spontaneously.

3. The notion of monadic geometry discussed in detail in [L37] would realize this idea. Define first a number theoretic discretization of imbedding space in terms of points, whose coordinates in group theoretically preferred coordinate system belong to the extension of rationals considered. One can say that these algebraic points are in the intersection of reality and various p-adicities. Overlapping open sets assigned with this discretization define in the real sector a covering by open sets. In p-adic sector compact-open-topology allows to assign with each point 8^{th} Cartesian power of algebraic extension of p-adic numbers. These compact open

sets define analogs for the monads of Leibniz and p-adic variants of field equations make sense inside them.

The monadic manifold structure of H is induced to space-time surfaces containing discrete subset of points in the algebraic discretization with field equations defining a continuation to space-time surface in given number field, and unique only in finite measurement resolution. This approach would resolve the tension between continuity and symmetries in p-adic–real correspondence: isometry groups would be replaced by their sub-groups with parameters in extension of rationals considered and acting in the intersection of reality and p-adicities.

The Galois group of extension acts non-trivially on the "spines" of space-time surfaces. Hence the number theoretical symmetries act as physical symmetries and define the orbit of given space-time surface as a kind of covering space. The coverings assigned to the hierarchy of Planck constants would naturally correspond to Galois coverings and dark matter would represent number theoretical physics.

This would give rise to a kind of algebraic hierarchy of adelic 4-surfaces identifiable as evolutionary hierarchy: the higher the dimension of the extension, the higher the evolutionary level.

11.4.4 NTU and WCW

p-Adic–real correspondence at the level of WCW

It has not been obvious whether one should perform p-adicization and adelization at the level of WCW. Minimalist could argue that scattering amplitudes are all we want and that their p-adicization and adelization by algebraic continuation can be tolerated only if it can give powerful enough constraints on the amplitudes.

1. The anti-commutations for fermionic oscillator operators are number theoretically universal. Supersymmetry suggests that also WCW bosonic degrees of freedom satisfy NTU. This could mean that the coordinates of p-adic WCW consist of super-symplectic invariants mappable by canonical identification to their real counterparts plus phases and their hyperbolic counterparts expressible as genuinely algebraic numbers common to all number fields. This kind of coordinates are naturally assignable to symmetric spaces [L37].
2. Kähler structure should be mapped from p-adic to real sector and vice versa. Vacuum functional identified as exponent of action should be NTU. Algebraic continuation defined by SH involves p-adic pseudo constants. All p-adic continuations by SH should correspond to the same value of exponent of action obtained by algebraic continuation from its real value. The degeneracy associated with p-adic pseudo-constants would be analogous to gauge invariance - imagination in TGD inspired theory of consciousness.
3. Ist it possible have NTU for WCW functional integration? Or is it enough to realize NTU for scattering amplitudes only. What seems clear that functional integral must reduce to a discrete sum. Physical intuition suggests a sum over maxima of Kähler function forming a subset of PEs representing stationary points. One cannot even exclude the possibility that the set of PEs is discrete and that one can sum over all of them.

   Restriction to maximum/stationary phase approximation gives rise to sum over exponents multiplied with Gaussian determinants. The determinant of Kähler metric however cancels the Gaussian determinants, and one obtains only a sum over the exponents of action.

   The breaking of strong NTU could happen: consider only p-adic mass calculations. This breaking is however associated with the parts of quantum states assignable to the boundaries of CD, not with the vacuum functional.

NTU for functional integral

Number theoretical vision relies on NTU. In fermionic sector NTU is necessary: one cannot speak about real and p-adic fermions as separate entities and fermionic anti-commutation relations are indeed number theoretically universal.

What about NTU in case of functional integral? There are two opposite views.

1. One can define p-adic variants of field equations without difficulties if preferred extremals are minimal surface extremals of Kähler action so that coupling constants do not appear in the solutions. If the extremal property is determined solely by the analyticity properties as it is for various conjectures, it makes sense independent of number field. Therefore there would be no need to continue the functional integral to p-adic sectors. This in accordance with the philosophy that thought cannot be put in scale. This would be also the option favored by pragmatist.
2. Consciousness theorist might argue that also cognition and imagination allow quantum description. The supersymmetry NTU should apply also to functional integral over WCW (more precisely, its sector defined by CD) involved with the definition of scattering amplitudes.

1. Key observations

The general vision involves some crucial observations.

1. Only the expressions for the scatterings amplitudes should should satisfy NTU. This does not require that the functional integral satisfies NTU.
2. Since the Gaussian and metric determinants cancel in WCW Kähler metric the contributions form maxima are proportional to action exponentials $exp(S_k)$ divided by the $\sum_k exp(S_k)$. Loops vanish by quantum criticality.
3. Scattering amplitudes can be defined as sums over the contributions from the maxima, which would have also stationary phase by the double extremal property made possible by the complex value of α_K. These contributions are normalized by the vacuum amplitude.

   It is enough to require NTU for $X_i = exp(S_i)/\sum_k exp(S_k)$. This requires that $S_k - S_l$ has form $q_1 + q_2 i\pi + q_3 log(n)$. The condition brings in mind homology theory without boundary operation defined by the difference $S_k - S_l$. NTU for both S_k and $exp(S_k)$ would only values of general form $S_k = q_1 + q_2 i\pi + q_3 log(n)$ for S_k and this looks quite too strong a condition.
4. If it is possible to express the 4-D exponentials as single 2-D exponential associated with union of string world sheets, vacuum functional disappears completely from consideration! There is only a sum over discretization with the same effective action and one obtains purely combinatorial expression.

2. What does one mean with functional integral?

The definition of functional integral in WCW is one of the key technical problems of quantum TGD [K125]. NTU states that the integral should be defined simultaneously in all number fields in the intersection of real and p-adic worlds defined by string world sheets and partonic 2-surfaces with WCW coordinates in algebraic extension of rationals and allowing by strong holography continuation to 4-D space-time surface. NTU is powerful constraint and could help in this respect.

1. Path integral is not in question. Rather, the functional integral is analogous to Wiener integral and perhaps allows identification as a genuine integral in the real sector. In p-adic sectors algebraic continuation should give the integral and here number theoretical universality gives excellent hopes. The integral would have exactly the same form in real and p-adic sector and expressible solely in terms of algebraic numbers characterizing algebraic extension and finite roots of e and roots of unity $U_n = exp(i2\pi/n)$ in algebraic extension of p-adic numbers.

   Since vacuum functional $exp(S)$ is exponential of complex action S, the natural idea is that only rational powers e^q and roots of unity and phases $exp(i2\pi q)$ are involved and there is no dependence on p-adic prime p! This is true in the integer part of q is smaller than p so that one does not obtain e^{kp}, which is ordinary p-adic number and would spoil the number theoretic universality. This condition is not possible to satisfy for all values of p unless the value of Kähler function is smaller than 2. One might consider the possibility that the allow primes are above some minimum value.

   The minimal solution to NTU conditions is that the ratios of action exponentials for maxima of Kähler function to the sum of these exponentials belong to the extension of rationals considered.

2. What does one mean with functional integral? TGD is expected to be an integrable in some sense. In integrable QFTs functional integral reduces to a sum over stationary points of the action: typically only single point contributes - at least in good approximation.

   For real α_K and Λ vacuum functional decomposes to a product of exponents of real contribution from Euclidian regions ($\sqrt{g_4}$ real) and imaginary contribution Minkowskian regions ($\sqrt{g_4}$ imaginary). There would be no exchange of momentum between Minkowskian and Euclidian regions. For complex values of α_K [L23] situation changes and Kähler function as real part of action receives contributions from both Euclidian and Minkowskian regions. The imaginary part of action has interpretation as analog of Morse function and action as it appears in QFTs. Now saddle points must be considered.

   PEs satisfy extremely strong conditions [K9, L31]. All classical Noether charges for a sub-algebra associated with super-symplectic algebra and isomorphic to the algebra itself vanish at both ends of CD. The conformal weights of this algebra are $n > 0$-ples of those for the entire algebra. What is fascinating that the condition that the preferred extremals are minimal surface extremals of Kähler action could solve these conditions and guarantee also NTU at the level of space-time surfaces. Supersymplectic boundary conditions at the ends of CD would however pose number theoretic conditions on the coupling parameters. In p-adic case the conditions should reduce to purely local conditions since p-adic charges are not well-defined as integrals.

3. In TGD framework one is constructing zero energy states rather calculating the matrix elements of S-matrix in terms of path integral. This gives certain liberties but a natural expectation is that functional integral as a formal tool at least is involved.

   Could one *define* the functional integral as a discrete sum of contributions of standard form for the preferred extremals, which correspond to maxima in Euclidian regions and associated stationary phase points in Minkowskian regions? Could one assume that WCW spinor field is concentrated along single maximum/stationary point.

   Quantum classical correspondence suggests that in Cartan algebra isometry charges are equal to the quantal charges for quantum states expressible in number theoretically universal manner in terms of fermionic oscillator operators or WCW gamma matrices? Even stronger condition would be that classical correlation functions are identical with quantal ones for allowed space-time surfaces in the quantum superposition. Could the reduction to a discrete sum be interpreted in terms of a finite measurement resolution?

4. In QFT Gaussian determinants produce problems because they are often poorly defined. In the recent case they could also spoil the NTU based on the exceptional properties of e. In the recent case however Gaussian determinant and metric determinant for Kähler metric cancel each other and this problem disappears. One could obtain just a sum over products of roots of e and roots of unity. Thus also Kähler structure seems to be crucial for the dream about NTU.

11.4.5 Breaking of NTU at the level of scattering amplitudes

NTU in strong sense could be broken at the level of scattering amplitudes. At space-time level the breaking does not look natural in the recent framework. Consider only p-adic mass calculations predicting that mass scale depends on p-adic prime. Also for the action strong form of NTU might fail for small p-adic primes since the value of the real part of action would be larger than than p. Should one allow this? What does one actually mean with NTU in the case of action?

Canonical identification is an important element of p-adic mass calculations and might also be needed to map p-adic variants of scattering amplitudes to their real counterparts. The breaking of NTU would take place, when the canonical real valued image of the p-adic scattering amplitude differs from the real scattering amplitude. The interpretation would be in terms of finite measurement resolution. By the finite measurement/cognitive resolution characterized by p one cannot detect the difference.

The simplest form of the canonical identification is $x = \sum_n x_n p^n \to \sum x_n p^{-n}$. Product xy and sum $x + y$ do not in general map to product and sum in canonical identification. The interpretation would be in terms of a finite measurement resolution: $(xy)_R = x_R y_R$ and $(x+y)_R = x_R + y_R$ only modulo finite measurement resolution. p-Adic scattering amplitudes are obtained by algebraic continuation from the intersection by replacing algebraic number valued parameters (such as momenta) by general p-adic numbers. The real images of these amplitudes under canonical identification are in general not identical with real scattering amplitudes the interpretation being in terms of a finite measurement resolution.

In p-adic thermodynamics NTU in the strong sense fails since thermal masses depend on p-adic mass scale. NTU can be broken by the fermionic matrix elements in the functional integral so that the real scattering amplitudes differ from the canonical images of the p-adic scattering amplitudes. For instance, the elementary particle vacuum functionals in the space of Teichmueller parameters for the partonic 2-surfaces and string world sheets should break NTU [K26].

11.4.6 NTU and the spectrum of Kähler coupling strength

During years I have made several attempts to understand coupling evolution in TGD framework. The most convincing proposal has emerged rather recently and relates the spectrum of $1/\alpha_K$ to that for the zeros of Riemann zeta [L23] and to the evolution of the electroweak U(1) couplings strength.

1. The first idea dates back to the discovery of WCW Kähler geometry defined by Kähler function defined by Kähler action (this happened around 1990) [K57]. The only free parameter of the theory is Kähler coupling strength α_K analogous to temperature parameter α_K postulated to be is analogous to critical temperature. Whether only single value or entire spectrum of of values α_K is possible, remained an open question.

   About decade ago I realized that Kähler action is *complex* receiving a real contribution from space-time regions of Euclidian signature of metric and imaginary contribution from the Minkoswkian regions. Euclidian region would give Kähler function and Minkowskian regions analog of QFT action of path integral approach defining also Morse function. Zero energy ontology (ZEO) [K129] led to the interpretation of quantum TGD as complex square root of thermodynamics so that the vacuum functional as exponent of Kähler action could be identified as a complex square root of the ordinary partition function. Kähler function would correspond to the real contribution Kähler action from Euclidian space-time regions. This led to ask whether also Kähler coupling strength might be complex: in analogy with the complexification of gauge coupling strength in theories allowing magnetic monopoles. Complex α_K could allow to explain CP breaking. I proposed that instanton term also reducing to Chern-Simons term could be behind CP breaking.

   The problem is that the dynamics in Minkowskian and Euclidian regions decouple completely and if Euclidian regions serve as space-time correlates for physical objects, there would be no exchanges of classical charges between physical objects. Should one conclude that α_K must be complex?

2. p-Adic mass calculations for 2 decades ago [K65] inspired the idea that length scale evolution is discretized so that the real version of p-adic coupling constant would have discrete set of values labelled by p-adic primes. The simple working hypothesis was that Kähler coupling strength is renormalization group (RG) invariant and only the weak and color coupling strengths depend on the p-adic length scale. The alternative ad hoc hypothesis considered was that gravitational constant is RG invariant. I made several number theoretically motivated ad hoc guesses about coupling constant evolution, in particular a guess for the formula for gravitational coupling in terms of Kähler coupling strength, action for CP_2 type vacuum extremal, p-adic length scale as dimensional quantity. Needless to say these attempts were premature and a hoc.

3. The vision about hierarchy of Planck constants $h_{eff} = n \times h$ and the connection $h_{eff} = h_{gr} = GMm/v_0$, where $v_0 < c = 1$ has dimensions of velocity [?] forced to consider very seriously the hypothesis that Kähler coupling strength has a spectrum of values in one-one correspondence with p-adic length scales. A separate coupling constant evolution associated with h_{eff} induced by $\alpha_K \propto 1/\hbar_{eff} \propto 1/n$ looks natural and was motivated by the idea that Nature is theoretician friendly: when the situation becomes non-perturbative, Mother Nature comes in rescue and an h_{eff} increasing phase transition makes the situation perturbative again.

   Quite recently the number theoretic interpretation of coupling constant evolution [K125] [L22] in terms of a hierarchy of algebraic extensions of rational numbers inducing those of p-adic number fields encouraged to think that $1/\alpha_K$ has spectrum labelled by primes and values of h_{eff}. Two coupling constant evolutions suggest themselves: they could be assigned to length scales and angles which are in p-adic sectors necessarily discretized and describable using only algebraic extensions involve roots of unity replacing angles with discrete phases.

4. Few years ago the relationship of TGD and GRT was finally understood [K121]. GRT space-time is obtained as an approximation as the sheets of the many-sheeted space-time of TGD are replaced with single region of space-time. The gravitational and gauge potential of sheets add together so that linear superposition corresponds to set theoretic union geometrically. This forced to consider the possibility that gauge coupling evolution takes place only at the level of the QFT approximation and α_K has only single value. This is nice but if true, one does not have much to say about the evolution of gauge coupling strengths.

5. The analogy of Riemann zeta function with the partition function of complex square root of thermodynamics suggests that the zeros of zeta have interpretation as inverses of complex temperatures $s = 1/\beta$. Also $1/\alpha_K$ is analogous to temperature. This led to a radical idea to be discussed in detail in the sequel.

   Could the spectrum of $1/\alpha_K$ reduce to that for the zeros of Riemann zeta or - more plausibly - to the spectrum of poles of fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ giving for $k = 1/2$ poles as zeros of zeta and as point $s = 2$? ζ_F is motivated by the fact that fermions are the only fundamental particles in TGD and by the fact that poles of the partition function are naturally associated with quantum criticality whereas the vanishing of ζ and varying sign allow no natural physical interpretation.

   The poles of $\zeta_F(s/2)$ define the spectrum of $1/\alpha_K$ and correspond to zeros of $\zeta(s)$ and to the pole of $\zeta(s/2)$ at $s = 2$. The trivial poles for $s = 2n$, $n = 1, 2, ..$ correspond naturally to the values of $1/\alpha_K$ for different values of $h_{eff} = n \times h$ with n even integer. Complex poles would correspond to ordinary QFT coupling constant evolution. The zeros of zeta in increasing order would correspond to p-adic primes in increasing order and UV limit to smallest value of poles at critical line. One can distinguish the pole $s = 2$ as extreme UV limit at which QFT approximation fails totally. CP_2 length scale indeed corresponds to GUT scale.

6. One can test this hypothesis. $1/\alpha_K$ corresponds to the electroweak U(1) coupling strength so that the identification $1/\alpha_K = 1/\alpha_{U(1)}$ makes sense. One also knows a lot about the evolutions of $1/\alpha_{U(1)}$ and of electromagnetic coupling strength $1/\alpha_{em} = 1/[cos^2(\theta_W)\alpha_{U(1)}$. What does this predict?

   It turns out that at p-adic length scale $k = 131$ ($p \simeq 2^k$ by p-adic length scale hypothesis, which now can be understood number theoretically [K125]) fine structure constant is predicted with .7 per cent accuracy if Weinberg angle is assumed to have its value at atomic scale! It is difficult to believe that this could be a mere accident because also the prediction evolution of $\alpha_{U(1)}$ is correct qualitatively. Note however that for $k = 127$ labelling electron one can reproduce fine structure constant with Weinberg angle deviating about 10 per cent from the measured value of Weinberg angle. Both models will be considered.

7. What about the evolution of weak, color and gravitational coupling strengths? Quantum criticality suggests that the evolution of these couplings strengths is universal and independent of the details of the dynamics. Since one must be able to compare various evolutions and combine them together, the only possibility seems to be that the spectra of gauge coupling strengths are given by the poles of $\zeta_F(w)$ but with argument $w = w(s)$ obtained by a global conformal transformation of upper half plane - that is Möbius transformation (see `http://tinyurl.com/gwjs85b`) with real coefficients (element of $GL(2, R)$) so that one as $\zeta_F((as + b)/(cs + d))$. Rather general arguments force it to be and element of $GL(2, Q)$, $GL(2, Z)$ or maybe even $SL(2, Z)$ ($ad - bc = 1$) satisfying additional constraints. Since TGD predicts several scaled variants of weak and color interactions, these copies could be perhaps parameterized by some elements of $SL(2, Z)$ and by a scaling factor K.

   Could one understand the general qualitative features of color and weak coupling contant evolutions from the properties of corresponding Möbius transformation? At the critical line there can be no poles or zeros but could asymptotic freedom be assigned with a pole of $cs + d$ and color confinement with the zero of $as + b$ at real axes? Pole makes sense only if Kähler action for the preferred extremal vanishes. Vanishing can occur and does so for massless extremals characterizing conformally invariant phase. For zero of $as+b$ vacuum function would be equal to one unless Kähler action is allowed to be infinite: does this make sense?. One can however hope that the values of parameters allow to distinguish between weak and color interactions. It is certainly possible to get an idea about the values of the parameters of the transformation and one ends up with a general model predicting the entire electroweak coupling constant evolution successfully.

To sum up, the big idea is the identification of the spectra of coupling constant strengths as poles of $\zeta_F((as+b/)(cs+d))$ identified as a complex square root of partition function with motivation coming from ZEO, quantum criticality, and super-conformal symmetry; the discretization of the RG flow made possible by the p-adic length scale hypothesis $p \simeq k^k$, k prime; and the assignment of complex zeros of ζ with p-adic primes in increasing order. These assumptions reduce the coupling constant evolution to four real rational or integer valued parameters (a,b,c,d). In the sequel this vision is discussed in more detail.

11.4.7 Other applications of NTU

NTU in the strongest form says that all numbers involved at "basic level" (whatever this means!) of adelic TGD are products of roots of unity and of power of a root of e. This is extremely powerful physics inspired conjecture with a wide range of possible mathematical applications.

1. For instance, vacuum functional defined as an exponent of action for preferred externals would be number of this kind. One could define functional integral as adelic operation in all number fields: essentially as sum of exponents of action for stationary preferred extremals since Gaussian and metric determinants potentially spoiling NTU would cancel each other leaving only the exponent.
2. The implications of NTU for the zeros of Riemann zeta [L22] will be discussed in more detail in the Appendix. Suffice it to say that the observations about Fourier transform for the distribution of loci of non-trivial zeros of zeta together with NTU leads to explicit proposal for the algebraic for of zeros of zeta. The testable proposal is that zeros decompose to disjoint classes $C(p)$ labelled by primes p and the condition that p^{iy} is root of unity in given class $C(p)$.
3. NTU generalises to all Lie groups. Exponents $exp(in_iJ_i/n)$ of lie-algebra generators define generalisations of number theoretically universal group elements and generate a discrete subgroup of compact Lie group. Also hyperbolic "phases" based on the roots $e^{m/n}$ are possible and make possible discretized NTU versions of all Lie-groups expected to play a key role in adelization of TGD.

   NTU generalises also to quaternions and octonions and allows to define them as number theoretically universal entities. Note that ordinary p-adic variants of quaternions and octonions do not give rise to a number field: inverse of quaternion can have vanishing p-adic variant of norm squared satisfying $\sum_n x_n^2 = 0$.

   NTU allows to define also the notion of Hilbert space as an adelic notion. The exponents of angles characterising unit vector of Hilbert space would correspond to roots of unity.

11.4.8 Going to the roots of p-adicity

The basic questions raised by the p-adic mass calculations concern the origin of preferred p-adic primes and of p-adic length scale hypothesis. One can also ask whether there might be a natural origin for p-adicity at the level of WCW.

Preferred primes as ramified primes for extensions of rationals?

Preferred primes as ramified primes for extensions of rationals?

The intuitive feeling is that the notion of preferred prime is something extremely deep and to me the deepest thing I know is number theory. Does one end up with preferred primes in number theory? This question brought to my mind the notion of *ramification of primes* (`http://tinyurl.com/hddljlf`) (more precisely, of prime ideals of number field in its extension), which happens only for special primes in a given extension of number field, say rationals. Ramification is completely analogous to the degeneracy of some roots of polynomial and corresponds to criticality if the polynomial corresponds to criticality (catastrophe theory of Thom is one application). Could this be the mechanism assigning preferred prime(s) to a given elementary system, such as elementary particle? I have not considered their role earlier also their hierarchy is highly relevant in the number theoretical vision about TGD.

1. Stating it very roughly (I hope that mathematicians tolerate this sloppy language of physicist): as one goes from number field K, say rationals Q, to its algebraic extension L, the original prime ideals in the so called *integral closure* (`http://tinyurl.com/js6fpvr`) over integers of K decompose to products of prime ideals of L (prime ideal is a more rigorous manner to express primeness). Note that the general ideal is analog of integer.

   Integral closure for integers of number field K is defined as the set of elements of K, which are roots of some monic polynomial with coefficients, which are integers of K having the form $x^n + a_{n-1}x^{n-1} + ... + a_0$. The integral closures of both K and L are considered. For instance, integral closure of algebraic extension of K over K is the extension itself. The integral closure of complex numbers over ordinary integers is the set of algebraic numbers.

   Prime ideals of K can be decomposed to products of prime ideals of L: $P = \prod P_i^{e_i}$, where e_i is the ramification index. If $e_i > 1$ is true for some i, *ramification* occurs. P_i:s in question are like co-inciding roots of polynomial, which for in thermodynamics and Thom's catastrophe theory corresponds to criticality. Ramification could therefore be a natural aspect of quantum criticality and ramified primes P are good candidates for preferred primes for a given extension of rationals. Note that the ramification make sense also for extensions of given extension of rationals.

2. A physical analogy for the decomposition of ideals to ideals of extension is provided by decomposition of hadrons to valence quarks. Elementary particles becomes composite of more elementary particles in the extension. The decomposition to these more elementary primes is of form $P = \prod P_i^{e(i)}$, the physical analog would be the number of elementary particles of type i in the state (`http://tinyurl.com/h9528pl`). Unramified prime P would be analogous a state with e fermions. Maximally ramified prime would be analogous to Bose-Einstein condensate of e bosons. General ramified prime would be analogous to an e-particle state containing both fermions and condensed bosons. This is of course just a formal analogy.

3. There are two further basic notions related to ramification and characterizing it. *Relative discriminant* is the ideal divided by all ramified ideals in K (integer of K having no ramified prime factors) and relative different for P is the ideal of L divided by all ramified P_i:s (product of prime factors of P in L). These ideals represent the analogs of product of preferred primes P of K and primes P_i of L dividing them. These two integers ideals would characterize the ramification.

In TGD framework the extensions of rationals (`http://tinyurl.com/h9528pl`) and p-adic number fields (`http://tinyurl.com/zq22tvb`) are unavoidable and interpreted as an evolutionary hierarchy physically and cosmological evolution would gradually proceed to more and more complex extensions. One can say that string world sheets and partonic 2-surfaces with parameters of defining functions in increasingly complex extensions of prime emerge during evolution. Therefore ramifications and the preferred primes defined by them are unavoidable. For p-adic number fields the number of extensions is much smaller for instance for $p > 2$ there are only 3 quadratic extensions.

How could ramification relate to p-adic and adelic physics and could it explain preferred primes?

1. Ramified p-adic prime $P = P_i^e$ would be replaced with its e:th root P_i in p-adicization. Same would apply to general ramified primes. Each un-ramified prime of K is replaced with $e = K : L$ primes of L and ramified primes P with $\#\{P_i\} < e$ primes of L: the increase of algebraic dimension is smaller. An interesting question relates to p-adic length scale. What happens to p-adic length scales. Is p-adic prime effectively replaced with e:th root of p-adic prime: $L_p \propto p^{1/2}L_1 \rightarrow p^{1/2e}L_1$? The only physical option is that the p-adic temperature for P would be scaled down $T_p = 1/n \rightarrow 1/ne$ for its e:th root (for fermions serving as

fundamental particles in TGD one actually has $T_p = 1$). Could the lower temperature state be more stable and select the preferred primes as maximimally ramified ones? What about general ramified primes?

2. This need not be the whole story. Some algebraic extensions would be more favored than others and p-adic view about realizable imaginations could be involved. p-Adic pseudo constants are expected to allow p-adic continuations of string world sheets and partonic 2-surfaces to 4-D preferred extremals with number theoretic discretization. For real continuations the situation is more difficult. For preferred extensions - and therefore for corresponding ramified primes - the number of real continuations - realizable imaginations - would be especially large.

   The challenge would be to understand why primes near powers of 2 and possibly also of other small primes would be favored. Why for them the number of realizable imaginations would be especially large so that they would be winners in number theoretical fight for survival?

Can one make this picture more concrete? What kind of algebraic extensions could be considered?

1. In p-adic context a proper definition of counterparts of angle variables as phases allowing definition of the analogs of trigonometric functions requires the introduction of algebraic extension giving rise to some roots of unity. Their number depends on the angular resolution. These roots allow to define the counterparts of ordinary trigonometric functions - the naive generalization based on Taylors series is not periodic - and also allows to defined the counterpart of definite integral in these degrees of freedom as discrete Fourier analysis. For the simplest algebraic extensions defined by $x^n - 1$ for which Galois group is abelian are are unramified so that something else is needed. One has decomposition $P = \prod P_i^{e(i)}$, $e(i) = 1$, analogous to n-fermion state so that simplest cyclic extension does not give rise to a ramification and there are no preferred primes.

2. What kind of polynomials could define preferred algebraic extensions of rationals? Irreducible polynomials are certainly an attractive candidate since any polynomial reduces to a product of them. One can say that they define the elementary particles of number theory. Irreducible polynomials have integer coefficients having the property that they do not decompose to products of polynomials with rational coefficients. It would be wrong to say that only these algebraic extensions can appear but there is a temptation to say that one can reduce the study of extensions to their study. One can even consider the possibility that string world sheets associated with products of irreducible polynomials are unstable against decay to those characterize irreducible polynomials.

3. What can one say about irreducible polynomials? Eisenstein criterion (`http://tinyurl.com/47kxjz` states following. If $Q(x) = \sum_{k=0,..,n} a_k x^k$ is n:th order polynomial with integer coefficients and with the property that there exists at least one prime dividing all coefficients a_i except a_n and that p^2 does not divide a_0, then Q is irreducible. Thus one can assign one or more preferred primes to the algebraic extension defined by an irreducible polynomial Q of this kind - in fact any polynomial allowing ramification. There are also other kinds of irreducible polynomials since Eisenstein's condition is only sufficient but not necessary.

   Furthermore, in the algebraic extension defined by Q, the prime ideals P having the above mentioned characteristic property decompose to an n:th power of single prime ideal P_i: $P = P_i^n$. The primes are maximally/completely ramified.

   A good illustration is provided by equations $x^2 + 1 = 0$ allowing roots $x_\pm = \pm i$ and equation $x^2 + 2px + p = 0$ allowing roots $x_\pm = -p \pm \sqrt{p}p - 1$. In the first case the ideals associated with $\pm i$ are different. In the second case these ideals are one and the same since $x_+ == -x_- + p$: hence one indeed has ramification. Note that the first example represents also an example of irreducible polynomial, which does not satisfy Eisenstein criterion. In more general case the n conditions on defined by symmetric functions of roots imply that the ideals are one and same when Eisenstein conditions are satisfied.

4. What is so nice that one could readily construct polynomials giving rise to given preferred primes. The complex roots of these polymials could correspond to the points of partonic 2-surfaces carrying fermions and defining the ends of boundaries of string world sheet. It must be however emphasized that the form of the polynomial depends on the choices of the complex

coordinate. For instance, the shift $x \to x+1$ transforms $(x^n-1)/(x-1)$ to a polynomial satisfying the Eisenstein criterion. One should be able to fix allowed coordinate changes in such a manner that the extension remains irreducible for all allowed coordinate changes.

Already the integral shift of the complex coordinate affects the situation. It would seem that only the action of the allowed coordinate changes must reduce to the action of Galois group permuting the roots of polynomials. A natural assumption is that the complex coordinate corresponds to a complex coordinate transforming linearly under subgroup of isometries of the imbedding space.

In the general situation one has $P = \prod P_i^{e(i)}$, $e(i) \geq 1$ so that aso now there are prefered primes so that the appearance of preferred primes is completely general phenomenon.

The origin of p-adic length scale hypothesis?

p-Adic length scale hypothesis emerged from p-adic length scale hypothesis. A possible generalization of this hypothesis is that p-adic primes near powers of prime are physically favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [?] (`http://tinyurl.com/jbh9m27`) and in music both 2-adicity and 3-adicity could be present: this is discussed in TGD inspired theory of music harmony and genetic code [K91]. See also [L38, L30].

One explanation would be that for preferred primes the number of p-adic space-time sheets representable also as real space-time sheets is maximal. Imagined worlds would be maximally realizable. Preferred p-adic primes would correspond to ramified primes for extensions with the property that the number of realizable imaginations is especially large for them. Why primes satisfying p-adic length scale hypothesis or its generalization would appear as ramified primes for extensions, which are winners in number theoretical evolution?

Also the weak form of NMP (WNMP) applying also to the purely number theoretic form of NMP [K70] might come in rescue here.

1. Entanglement negentropy for a NE [K70] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretical entanglement negentropy (NEN). If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured, which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.
2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real PE as p-adic PE (Note that $p = 1$ makes formally sense but for it the topology is discrete).
3. WNMP [K70, K122] suggests a more feasible explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator. WNMP allows "free will" so that all dimensions $n-k$, $k = 0, 1, ...n-1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.
4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n = 2^k = M_k + 1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n = 2^k - r$ near 2^k produce large entanglement negentropy and would be favored by NMP.

5. This argument suggests a generalization of p-adic length scale hypothesis so that $p = 2$ can be replaced by any prime.

11.5 p-Adic physics and consciousness

p-Adic physics as physics of cognition and imagination is an important thread in TGD inspired theory of consciousness. In the sequel I describe briefly the basic of TGD inspired theory of consciousness as generalization of quantum measurement theory to ZEO (ZEO), describe the definition of self, consider the question whether NMP is needed as a separate principle or whether it is implied is in statistical sense by the unavoidable statistical increase of $n = h_{eff}/h$ if identified as a factor of the dimension of Galois group extension of rationals defining the adeles, and finally summarize the vision about how p-adic physics serves as a correlate of cognition and imagination.

11.5.1 From quantum measurement theory to a theory of consciousness

The notion of self can be seen as a generalization of the poorly defined definition of the notion of observer in quantum physics. In the following I take the role of skeptic trying to be as critical as possible.

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. The density matrix was assumed to define the universal observable. Note that a density matrix, which is power series of a product of matrices representing commuting observables has in the generic case eigenstates, which are simultaneous eigenstates of all observables. Second aspect of self was assumed to be the integration of subsequent quantum jumps to coherent whole giving rise to the experienced flow of time.

The precise identification of self allowing to understand both of these aspects turned out to be difficult problem. I became aware the solution of the problem in terms of ZEO (ZEO) only rather recently (2014).

1. Self corresponds to a sequence of quantum jumps integrating to single unit as in the original proposal, but these quantum jumps correspond to state function reductions to a fixed boundary of causal diamond CD leaving the corresponding parts of zero energy states invariant - “small” state function reductions. The parts of zero energy states at second boundary of CD change and even the position of the tip of the opposite boundary changes: one actually has wave function over positions of second boundary (CD sizes roughly) and this wave function changes. In positive energy ontology these repeated state function reductions would have no effect on the state (Zeno effect) but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and self: self is generalized Zeno effect.

2. The first quantum jump to the opposite boundary corresponds to the act of “free will” or birth of re-incarnated self. Hence the act of “free will” changes the arrow of psychological time at some level of hierarchy of CDs. The first reduction to the opposite boundary of CD means “death” of self and “re-incarnation” of time-reversed self at opposite boundary at which the the temporal distance between the tips of CD increases in opposite direction. The sequence of selves and time reversed selves is analogous to a cosmic expansion for CD. The repeated birth and death of mental images could correspond to this sequence at the level of sub-selves.

3. This allows to understand the relationship between subjective and geometric time and how the arrow of and flow of clock time (psychological time) emerge. The average distance between the tips of CD increases on the average as along as state function functions occur repeatedly at the fixed boundary: situation is analogous to that in diffusion. The localization of contents of conscious experience to boundary of CD gives rise to the illusion that universe is 3-dimensional. The possibility of memories made possibly by hierarchy of CDs demonstrates that this is not the case. Self is simply the sequence of state function reductions at same boundary of CD remaining fixed and the lifetime of self is the total growth of the average temporal distance between the tips of CD.

One can identify several rather abstract state function reductions selecting a sector of WCW.

1. There are quantum measurements inducing localization in the moduli space of CDs with passive boundary and states at it fixed. In particular, a localization in the moduli characterizing the Lorentz transform of the upper tip of CD would be measured. The measured moduli characterize also the analog of symplectic form in M^4 strongly suggested by twistor lift of TGD - that is the rest system (time axis) and spin quantization axes. Of course, also other kinds of reductions are possible.
2. Also a localization to an extension of rationals defining the adeles should occur. Could the value of $n = h_{eff}/h$ be observable? The value of n for given space-time surface at the active boundary of CD could be identified as the order of the smallest Galois group containing all Galois groups assignable to 3-surfaces at the boundary. The superposition of space-time surface would not be eigenstate of n at active boundary unless localization occurs. It is not obvious whether this is consistent with a fixe value of n at passive boundary.

   The measured value of n could be larger or smaller than the value of n at the passive boundary of CD but in statistical sense n would increase by the analogy with diffusion on half line defined by non-negative integers. The distance from the origin unavoidably increases in statistical sense. This would imply evolution as increase of maximal value of negentropy and generation of quantum coherence in increasingly longer scales.
3. A further abstract choice corresponds to the the replacement of the roles of active and passive boundary of CD changing the arrow of clock time and correspond to a death of self and re-incarnation as time-reversed self.

Can one assume that these measurements reduce to measurements of a density matrix of either entangled system as assumed in the earlier formulation of NMP, or should one allow both options. This question actually applies to all quantum measurements and leads to a fundamental philosophical questions unavoidable in all consciousness theories.

1. Do all measurements involve entanglement between the moduli or extensions of two CDs reduced in the measurement of the density matrix? Non-diagonal entanglement would allow final states states, which are not eigenstates of moduli or of n: this looks strange. This could also lead to an infinite regress since it seems that one must assume endless hierarchy of entangled CDs so that the reduction sequence would proceed from top to bottom. It looks natural to regard single CD as a sub-Universe.

   For instance, if a selection of quantization axis of color hypercharge and isospin (localization in the twistor space of CP_2) is involved, one would have an outcome corresponding to a quantum superposition of measurements with different color quantization axis!

   Going philosophical, one can also argue, that the measurement of density matrix is only a reaction to environment and does not allow intentional free will.
2. Can one assume that a mere localization in the moduli space or for the extension of rationals (producing an eigenstate of n) takes place for a fixed CD - a kind of self measurement possible for even unentangled system? If there is entanglement in these degrees of freedom between two systems (say CDs), it would be reduced in these self measurements but the outcome would not be an eigenstate of density matrix. An interpretation as a realization of intention would be approriate.
3. If one allows both options, the interpretation would be that state function reduction as a measurement of density matrix is only a reaction to environment and self-measurement represents a realization of intention.
4. Self measurements would occur at higher level say as a selection of quantization axis, localization in the moduli space of CD, or selection of extension of rationals. A possible general rule is that measurements at space-time level are reactions as measurements of density matrix whereas a selection of a sector of WCW would be an intentional action. This because formally the quantum states at the level of WCW are as modes of classical WCW spinor field single particle states.
5. If the selections of sectors of WCW at active boundary of CD commute with observables, whose eigenstates appear at passive boundary (briefly *passive observables*) meaning that time reversal commutes with them - they can occur repeatedly during the reduction sequence and self as a generalized Zeno effect makes sense.

If the selections of WCW sectors at active boundary do not commute with passive observables then volition as a choice of sector of WCW must change the arrow of time. Libet's findings show that conscious choice induces neural activity for a fraction of second before the conscious choice. This would imply the correspondences *"big" measurement changing the arrow of time - self-measurement at the level of WCW - intentional action* and *"small" measurement - measurement at space-time level - reaction.*

Self as a generalized Zeno effect makes sense only if there are active commuting with passive observables. If the passive observables form a maximal set, the new active observables commuting with them must emerge. The increase of the size of extension of rationals might generate them by expanding the state space so that self would survive only as long at it evolves.

Otherwise there would be only single unitary time evolution followed by a reduction to opposite boundary. This makes sense only if the sequence of "big" reductions for sub-selves can give rise to the time flow experienced by self: the birth and death of mental images would give rise to flow of time of self.

A hierarchical process starting from given CD and proceeding downwards to shorter scales and stopping when the entanglement is stable is highly suggestive and favors self measurements. What stability could mean will be discussed in the next section. CDs would be a correlate for self hierarchy. One can say also something about the anatomy and correlates of self hierarchy.

1. Self experiences its sub-selves as mental images and even we would represent mental images of some higher level collective self. Everything is conscious but consciousness can be lost or at least it is not possible to have memory about it. The flow of consciousness for a given self could be due to the quantum jump sequences performed by its sub-selves giving rise to mental images.
2. By quantum classical correspondence self has also space-time correlates. One can visualize sub-self as a space-time sheet "glued" by topological sum to the space-time sheet of self. Subsystem is not described as a tensor factor as in the standard description of subsystems. Also sub-selves of selves can entangle negentropically and this gives rise to a sharing of mental images about which stereo vision would be basic example. Quite generally, one could speak of stereo consciousness. Also the experiences of sensed presence [J8] could be understood as a sharing of mental images between brain hemispheres, which are not themselves entangled. This is possible also between different brains. In the normal situation brain hemispheres are entangled.
3. At the level of 8-dimensional imbedding space the natural correlate of self would be CD (causal diamond). At the level of space-time the correlate would be space-time sheet or light-like 3-surface. The contents of consciousness of self would be determined by the space-time sheets in the interior of CD. Without further restrictions the experience of self would be essentially four-dimensional. Memories would be like sensory experiences except that they would be about the geometric past and for some reason are not usually colored by sensory qualia. For instance .1 second time scale defining sensory chronon corresponds to the secondary p-adic time scale characterizing the size of electron's CD (Mersenne prime M_{127}), which suggests that Cooper pairs of electrons are essential for the sensory qualia.

11.5.2 NMP and self

The view about Negentropy Maximization Principle (NMP) [K70] has co-evolved with the notion of self and I have considered many variants of NMP.

1. The original formulation of NMP was in positive energy ontology and made same predictions as standard quantum measurement theory. The new element was that the density matrix of sub-system defines the fundamental observable and the system goes to its eigenstate in state function reduction. As found, the localizations at to WCW sectors define what might be called self-measurements and identifiable as active volitions rather than reactions.
2. In p-adic physics one can assign with rational and even algebraic entanglement probabilities number theoretical entanglement negentropy (NEN) satisfying the same basic axioms

as the ordinary Shannon entropy but having negative values and therefore having interpretation as information. The definition of p-adic negentropy (real valued) reads as $S_p = -\sum P_k log(|P_k|_p)$, where $|.|_p$ denotes p-adic norm. The news is that $N_p = -S_p$ can be positive and is positive for rational entanglement probabilities. Real entanglement entropy S is always non-negative.

NMP would force the generation of negentropic entanglement (NE) and stabilize it. NNE resources of the Universe - one might call them Akashic records- would steadily increase.

3. A decisive step of progress was the realization is that NTU forces all states in adelic physics to have entanglement coefficients in some extension of rationals inducing finite-D extension of p-adic numbers. The same entanglement can be characterized by real entropy S and p-adic negentropies N_p, which can be positive. One can define also total p-adic negentropy: $N = \sum_p N_p$ for all p and total negentropy $N_{tot} = N - S$.

   For rational entanglement probabilities it is easy to demonstrate that the generalization of adelic theorem holds true: $N_{tot} = N - S = 0$. NMP based on N_{tot} rather than N would not say anything about rational entanglement. For extensions of rationals it is easy to find that $N - S > 0$ is possible if entanglement probabilities are of form X_i/n with $|X_i|_p = 1$ and n integer [L33]. Should one identify the total negentropy as difference $N_{tot} = N - S$ or as $N_{tot} = N$?

   Irrespective of answer, large p-adic negentropy seems to force large real entropy: this nicely correlates with the paradoxical finding that living systems tend to be entropic although one would expect just the oppositecite [L33]: this relates in very interesting manner to the work of biologists Jeremy England [**?**]. The negentropy would be cognitive negentropy and not visible for ordinary physics.

4. The latest step in the evolution of ideas NMP was the question whether NMP follows from number theory alone just as second law follows form probability theory! This irritates theoretician's ego but is victory for theory. The dimension n of extension is positive integer and cannot but grow in statistical sense in evolution! Since one expects that the maximal value of negentropy (define as $N - S$) must increase with n. Negentropy must increase in long run.

Number theoretic entanglement can be stable

Number theoretical Shannon entropy can serve as a measure for genuine information assignable to a pair of entanglement systems [K70]. Entanglement with coefficients in the extension is always negentropic if entanglement negentropy comes from p-adic sectors only. It can be negentropic if negentropy is defined as the difference of p-adic negentropy and real entropy.

The diagonalized density matrix need not belong to the algebraic extension since the probabilities defining its diagonal elements are eigenvalues of the density matrix as roots of N:th order polynomial, which in the generic case requires n-dimensional algebraic extension of rationals. One can argue that since diagonalization is not possible, also state function reduction selecting one of the eigenstates is impossible unless a phase transition increasing the dimension of algebraic extension used occurs simultaneously. This kind of NE could give rise to cognitive entanglement.

There is also a special kind of NE, which can result if one requires that density matrix serves a universal observable in state function reduction. The outcome of reduction must be an eigen space of density matrix, which is projector to this subspace acting as identity matrix inside it. This kind NE allows all unitarily related basis as eigenstate basis (unitary transformations must belong to the algebraic extension). This kind of NE could serve as a correlate for "enlightened" states of consciousness. Schrödingers cat is in this kind of state stably in superposition of dead and alive and state basis obtained by unitary rotation from this basis is equally good. One can say that there are no discriminations in this state, and this is what is claimed about "enlightened" states too.

The vision about number theoretical evolution suggests that NMP forces the generation of NE resources as NE assignable to the "passive" boundary of CD for which no changes occur during sequence of state function reductions defining self. It would define the unchanging self as negentropy resources, which could be regarded as kind of Akashic records. During the next "re-incarnation" after the first reduction to opposite boundary of CD the NE associated with the reduced state would serve as new Akashic records for the time reversed self. If NMP reduces to the statistical increase of $h_{eff}/h = n$ the consciousness information contents of the Universe increases in statistical sense. In the best possible world of SNMP it would increase steadily.

Does NMP reduce to number theory?

The heretic question that emerged quite recently is whether NMP is actually needed at all! Is NMP a separate principle or could NMP reduced to mere number theory [K70]? Consider first the possibility that NMP is not needed at all as a separate principle.

1. The value of $h_{eff}/h = n$ should increase in the evolution by the phase transitions increasing the dimension of the extension of rationals. $h_{eff}/h = n$ has been identified as the number of sheets of some kind of covering space. The Galois group of extension acts on number theoretic discretizations of the monadic surface and the orbit defines a covering space. Suppose n is the number of sheets of this covering and thus the dimension of the Galois group for the extension of rationals or factor of it.

2. It has been already noticed that the "big" state function reductions giving rise to death and reincarnation of self could correspond to a measurement of $n = h_{eff}$ implied by the measurement of the extension of the rationals defining the adeles. The statistical increase of n follows automatically and implies statistical increase of maximal entanglement negentropy. Entanglement negentropy increases in statistical sense.

   The resulting world would not be the best possible one unlike for a strong form of NMP demanding that negentropy does increaes in "big" state function reductions. n also decrease temporarily and they seem to be needed. In TGD inspired model of bio-catalysis the phase transition reducing the value of n for the magnetic flux tubes connecting reacting bio-molecules allows them to find each other in the molecular soup. This would be crucial for understanding processes like DNA replication and transcription.

3. State function reduction corresponding to the measurement of density matrix could occur to an eigenstate/eigenspace of density matrix only if the corresponding eigenvalue and eigenstate/eigenspace is expressible using numbers in the extension of rationals defining the adele considered. In the generic case these numbers belong to N-dimensional extension of the original extension. This can make the entanglement stable with respect to state the measurements of density matrix.

   A phase transition to an extension of an extension containing these coefficients would be required to make possible reduction. A step in number theoretic evolution would occur. Also an entanglement of measured state pairs with those of measuring system in containing the extension of extension would make possible the reduction. Negentropy could be reduced but higher-D extension would provide potential for more negentropic entanglement and NMP would hold true in the statistical sense.

4. If one has higher-D eigen space of density matrix, p-adic negentropy is largest for the entire subspace and the sum of real and p-adic negentropies vanishes for all of them. For negentropy identified as total p-adic negentropy SNMP would select the entire sub-space and NMP would indeed say something explicit about negentropy.

Or is NMP needed as a separate principle?

Hitherto I have postulated NMP as a separate principle [K70]. Strong form of NMP (SNMP) states that Negentropy does not decrease in "big" state function reductions corresponding to death and re-incarnations of self.

One can however argue that SNMP is not realistic. SNMP would force the Universe to be the best possible one, and this does not seem to be the case. Also ethically responsible free will would be very restricted since self would be forced always to do the best deed that is increase maximally the negentropy serving as information resources of the Universe. Giving up separate NMP altogether would allow to have also "Good" and "Evil".

This forces to consider what I christened weak form of NMP (WNMP). Instead of maximal dimension corresponding to N-dimensional projector self can choose also lower-dimensional subspaces and 1-D sub-space corresponds to the vanishing entanglement and negentropy assumed in standard quantum measurement theory. As a matter fact, this can also lead to larger negentropy gain since negentropy depends strongly on what is the large power of p in the dimension of the resulting eigen sub-space of density matrix. This could apply also to the purely number theoretical reduction of NMP.

WNMP suggests how to understand the notions of Good and Evil. Various choices in the state function reduction would correspond to Boolean algebra, which suggests an interpretation in terms of what might be called emotional intelligence [K122]. Also it turns out that one can understand how p-adic length scale hypothesis - actually its generalization - emerges from WNMP [K125].

1. One can start from ordinary quantum entanglement. It corresponds to a superposition of pairs of states. Second state corresponds to the internal state of the self and second state to a state of external world or biological body of self. In negentropic quantum entanglement each is replaced with a pair of sub-spaces of state spaces of self and external world. The dimension of the sub-space depends on which pair is in question. In state function reduction one of these pairs is selected and deed is done. How to make some of these deeds good and some bad? Recall that WNMP allows only the possibility to generate NNE but does not force it. WNMP would be like God allowing the possibility to do good but not forcing good deeds.

   Self can choose any sub-space of the subspace defined by $k \leq N$-dimensional projector and 1-D subspace corresponds to the standard quantum measurement. For $k = 1$ the state function reduction leads to vanishing negentropy, and separation of self and the target of the action. Negentropy does not increase in this action and self is isolated from the target: kind of price for sin.

   For the maximal dimension of this sub-space the negentropy gain is maximal. This deed would be good and by the proposed criterion NE corresponds to conscious experience with positive emotional coloring. Interestingly, there are $2^k - 1$ possible choices, which is almost the dimension of Boolean algebra consisting of k independent bits. The excluded option corresponds to 0-dimensional sub-space - empty set in set theoretic realization of Boolean algebra. This could relate directly to fermionic oscillator operators defining basis of Boolean algebra - here Fock vacuum would be the excluded state. The deed in this sense would be a choice of how loving the attention towards system of external world is.

2. A map of different choices of k-dimensional sub-spaces to k-fermion states is suggestive. The realization of logic in terms of emotions of different degrees of positivity would be mapped to many-fermion states - perhaps zero energy states with vanishing total fermion number. State function reductions to k-dimensional spaces would be mapped to k-fermion states: quantum jumps to quantum states!

   The problem brings in mind quantum classical correspondence in quantum measurement theory. The direction of the pointer of the measurement apparatus (in very metaphorical sense) corresponds to the outcome of state function reduction, which is now 1-D subspace. For ordinary measurement the pointer has k positions. Now it must have $2^k - 1$ positions. To the discrete space of k pointer positions one must assign fermionic Clifford algebra of second quantized fermionic oscillator operators. The hierarchy of Planck constants and dark matter suggests the realization. Replace the pointer with its space-time k-sheeted covering and consider zero energy energy states made of pairs of k-fermion states at the sheets of the n-sheeted covering? Dark matter would be therefore necessary for cognition. The role of fermions would be to "mark" the k space-time sheets in the covering.

The cautious conclusion is that NMP as a separate principle is not necessary and follows in statistical sense from the unavoidable increase of $n = h_{eff}/h$ identified as dimension of extension of rationals define the adeles if this extension or at least the dimension of its Galois group is observable.

11.5.3 p-Adic physics as correlate of cognition and imagination

The items in the following list give motivations for the proposal that p-adic physics could serve as a correlate for cognition and imagination.

1. By the total disconnectedness of the p-adic topology, p-adic world decomposes naturally into blobs, objects. This happens also in sensory perception. The pinary digits of p-adic number

can be assigned to a p-tree. Parisi proposed in the model of spin glass [B39] that p-adic numbers could relate to the mathematical description of cognition and also Khrennikov [J1] has developed this idea. In TGD framework that idea is taken to space-time level: p-adic space-time sheets represent thought bubbles and they correlate with the real ones since they form cognitive reprentations of the real world. SH allows a concrete realization of this.

2. p-Adic non-determinism due to p-adic pseudo constants suggests interpretation in terms of imagination. Given 2-surfaces could allow completion to p-adic preferred extremal but not to a real one so that pure "non-realizable" imagination is in question.

3. Number theoretic negentropy has interpretation as negentropy characterizing information content of entanglement. The superposition of state pairs could be interpreted as a quantum representation for a rule or abstracted association containing its instances as state pairs. Number theoretical negentropy characterizes the relationship of two systems and should not be confused with thermodynamical entropy, which characterizes the uncertainty about the state of single system.

The original vision was that p-adic non-determinism could serve as a correlate for cognition, imagination, and intention. The recent view is much more cautious. Imagination need not completely reduce to p-adic non-determinism since it has also real physics correlates - maybe as partial realizations of SH as in nerve pulse pattern, which does not propagate down to muscles.

A possible interpretation for the solutions of the p-adic field equations would be as geometric correlates of cognition, imagination, and perhaps even intentionality. Plans, intentions, expectations, dreams, and possibly also cognition as imagination in general could have p-adic cognitive space-time sheets as their geometric correlates. A deep principle seems to be involved: incompleteness is the characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

The most feasible view is that the intersections of p-adic and real space-time surfaces define cognitive representations of real space-time surfaces (PEs, [K15, K9, L31]). One could also say that real space-time surface represents sensory aspects of conscious experience and p-adic space-time surfaces its cognitive aspects. Both real and p-adics rather than real or p-adics.

The identification of p-adic pseudo constants as correlates of imagination at space-time level is indeed a further natural idea.

1. The construction of PEs by SH from the data at 2-surfaces is like boundary value problem with number theoretic discretization of space-time surface as additional data. PE property in real context implies strong correlations between string world sheets and partonic 2-surfaces by boundary conditions a them. One cannot choose these 2-surfaces completely independently in real context.

2. In p-adic sectors the integration constants are replaced with pseudo-constants depending on finite number of pinary digits of variables depending on coordinates normal to string world sheets and partonic 2-surfaces. The fixing of the discretization of space-time surface would allow to fix the p-adic pseudo-constants. Once the number theoretic discretization of space-time surface is fixed, the p-adic pseudo-constants can be fixed. Pseudo-constant could allow a large number of p-adic configurations involving string world sheets, partonic 2-surfaces, and number theoretic discretization but not allowed in real context.

   Could these p-adic PEs correspond to imaginations, which in general are not realizable? Could the realizable intentional actions belong to the intersection of real and p-adic WCWs? Could one identify non-realistic imaginations as the modes of WCW spinor fields for which 2-surfaces are not extendable to real space-time surfaces and are localized to 2-surfaces? Could they allow only a partial continuation to real space-time surface. Could nerve pulse pattern representing imagined motor action and not proceeding to the level of muscles correspond to a partially real PE?

   Could imagination and problem solving be search for those collections of string world sheets and partonic 2-surfaces, which allow extension to (realization as) real PEs? If so, p-adic physics would be there as an independent aspect of existence and this is just the original idea. Imagination could be realized in state function reduction, which always selects only those 2-surfaces, which allow continuation to real space-time surfaces. The distinction between only imaginable and also realizable would be the extendability by using strong form of holography.

3. An interesting question is why elementary particles are characterized by preferred p-adic primes (primes near powers of 2, in particular Mersenne primes). Could the number of realizable imaginations for these primes be especially large?

I have the feeling that this view allows respectable mathematical realization of imagination in terms of adelic quantum physics. It is remarkable that SH derivable from - you can guess, SGCI (the Big E again!), plays an absolutely central role in it.

11.6 Appendix: Super-symplectic conformal weights and zeros of Riemann zeta

Since fermions are the only fundamental particles in TGD one could argue that the conformal weight of for the generating elements of supersympelectic algebra could be negatives for the poles of fermionic zeta ζ_F. This demands $n > 0$ as does also the fractal hierarchy of supersymplectic symmetry breakings. NTU of Riemann zeta in some sense is strongly suggested if adelic physics is to make sense.

For ordinary conformal algebras there are only finite number of generating elements ($-2 \leq n \leq 2$). If the radial conformal weights for the generators of g consist of poles of ζ_F, the situation changes. ζ_F is suggested by the observation that fermions are the only fundamental particles in TGD.

1. Riemann Zeta $\zeta(s) = \prod_p (1/(1-p^{-s})$ identifiable formally as a partition function $\zeta_B(s)$ of arithmetic boson gas with bosons with energy $log(p)$ and temperature $1/s = 1/(1/2+iy)$ should be replaced with that of arithmetic fermionic gas given in the product representation by $\zeta_F(s) = \prod_p (1+p^{-s})$ so that the identity $\zeta_B(s))/\zeta_F(s) = \zeta_B(2s)$ follows. This gives

$$\frac{\zeta_B(s)}{\zeta_B(2s)} \ .$$

   $\zeta_F(s)$ has zeros at zeros s_n of $\zeta(s)$ and at the pole $s = 1/2$ of $zeta(2s)$. $\zeta_F(s)$ has poles at zeros $s_n/2$ of $\zeta(2s)$ and at pole $s = 1$ of $\zeta(s)$.

   The spectrum of $1/T$ would be for the generators of algebra $\{(-1/2+iy)/2, n > 0, -1\}$. In p-adic thermodynamics the p-adic temperature is $1/T = 1/n$ and corresponds to "trivial" poles of ζ_F. Complex values of temperature does not make sense in ordinary thermodynamics. In ZEO quantum theory can be regarded as a square root of thermodynamics and complex temperature parameter makes sense.

2. If the spectrum of conformal weights of the generating elements of the algebra corresponds to poles serving as analogs of propagator poles, it consists of the "trivial" conformal $h = n > 0$- the standard spectrum with $h = 0$ assignable to massless particles excluded - and "non-trivial" $h = -1/4 + iy/2$. There is also a pole at $h = -1$.

   Both the non-trivial pole with real part $h_R = -1/4$ and the pole $h = -1$ correspond to tachyons. I have earlier proposed conformal confinement meaning that the total conformal weight for the state is real. If so, one obtains for a conformally confined two-particle states corresponding to conjugate non-trivial zeros in minimal situation $h_R = -1/2$ assignable to N-S representation.

   In p-adic mass calculations ground state conformal weight must be $-5/2$ [K65]. The negative fermion ground state weight could explain why the ground state conformal weight must be tachyonic $-5/2$. With the required 5 tensor factors one would indeed obtain this with minimal conformal confinement. In fact, arbitrarily large tachyonic conformal weight is possible but physical state should always have conformal weights $h > 0$.

3. $h = 0$ is not possible for generators, which reminds of Higgs mechanism for which the naïve ground states corresponds to tachyonic Higgs. $h = 0$ conformally confined massless states are necessarily composites obtained by applying the generators of Kac-Moody algebra or super-symplectic algebra to the ground state. This is the case according to p-adic mass calculations [K65], and would suggest that the negative ground state conformal weight can be associated with super-symplectic algebra and the remaining contribution comes from ordinary

super-conformal generators. Hadronic masses, whose origin is poorly understood, could come from super-symplectic degrees of freedom. There is no need for p-adic thermodynamics in super-symplectic degrees of freedom.

11.6.1 A general formula for the zeros of zeta from NTU

Dyson's comment about Fourier transform of Riemann Zeta [A68] (`http://tinyurl.com/hjbfsuv`) is interesting from the point of NTU for Riemann zeta.

1. The numerical calculation of Fourier transform for the imaginary parts iy of zeros $s = 1/2+iy$ of zeta shows that it is concentrated at discrete set of frequencies coming as $log(p^n)$, p prime. This translates to the statement that the zeros of zeta form a 1-dimensional quasicrystal, a discrete structure Fourier spectrum by definition is also discrete (this of course holds for ordinary crystals as a special case). Also the logarithms of powers of primes would form a quasicrystal, which is very interesting from the point of view of p-adic length scale hypothesis. Primes label the "energies" of elementary fermions and bosons in arithmetic number theory, whose repeated second quantization gives rise to the hierarchy of infinite primes [K105]. The energies for general states are logarithms of integers.
2. Powers p^n label the points of quasicrystal defined by points $log(p^n)$ and Riemann zeta has interpretation as partition function for boson case with this spectrum. Could p^n label also the points of the dual lattice defined by iy.
3. The existence of Fourier transform for points $log(p_i^n)$ for any vector y_a in class $C(p)$ of zeros labelled by p requires $p_i^{iy_a}$ to be a root of unity inside $C(p)$. This could define the sense in which zeros of zeta are universal. This condition also guarantees that the factor $n^{-1/2-iy}$ appearing in zeta at critical line are number theoretically universal ($p^{1/2}$ is problematic for Q_p: the problem might be solved by eliminating from p-adic analog of zeta the factor $1-p^{-s}$.
   (a) One obtains for the pair (p_i, s_a) the condition $log(p_i)y_a = q_{ia}2\pi$, where q_{ia} is a rational number. Dividing the conditions for (i, a) and (j, a) gives
   $$p_i = p_j^{q_{ia}/q_{ja}}$$
   for every zero s_a so that the ratios q_{ia}/q_{ja} do not depend on s_a. From this one easily deduce $p_i^M = p_j^N$, where M and N are integers so that one ends up with a contradiction.
   (b) Dividing the conditions for (i, a) and (i, b) one obtains
   $$\frac{y_a}{y_b} = \frac{q_{ia}}{q_{ib}}$$
   so that the ratios q_{ia}/q_{ib} do not depend on p_i. The ratios of the imaginary parts of zeta would be therefore rational number which is very strong prediction and zeros could be mapped by scaling y_a/y_1 where y_1 is the zero which smallest imaginary part to rationals.
   (c) The impossible consistency conditions for (i, a) and (j, a) can be avoided if each prime and its powers correspond to its own subset of zeros and these subsets of zeros are disjoint: one would have infinite union of sub-quasicrystals labelled by primes and each p-adic number field would correspond to its own subset of zeros: this might be seen as an abstract analog for the decomposition of rational to powers of primes. This decomposition would be natural if for ordinary complex numbers the contribution in the complement of this set to the Fourier trasform vanishes. The conditions $(i, a)$and (i, b) require now that the ratios of zeros are rationals only in the subset associated with p_i.

For the general option the Fourier transform can be delta function for $x = log(p^k)$ and the set $\{y_a(p)\}$ contains N_p zeros. The following argument inspires the conjecture that for each p there is an infinite number N_p of zeros $y_a(p)$ in class $C(p)$ satisfying

$$p^{iy_a(p)} = u(p) = e^{\frac{r(p)}{m(p)}i2\pi} \ ,$$

where $u(p)$ is a root of unity that is $y_a(p) = 2\pi(m(a) + r(p))/log(p)$ and forming a subset of a lattice with a lattice constant $y_0 = 2\pi/log(p)$, which itself need not be a zero.

In terms of stationary phase approximation the zeros $y_a(p)$ associated with p would have constant stationary phase whereas for $y_a(p_i \neq p))$ the phase $p^{iy_a(p_i)}$ would fail to be stationary. The phase e^{ixy} would be non-stationary also for $x \neq log(p^k)$ as function of y.

1. Assume that for $x = qlog(p)$, where q not a rational, the phases e^{ixy} fail to be roots of unity and are random implying the vanishing/smallness of $F(x)$.
2. Assume that for a given p all powers p^{iy} for $y \notin \{y_a(p)\}$ fail to be roots of unity and are also random so that the contribution of the set $y \notin \{y_a(p)\}$ to $F(p)$ vanishes/is small.
3. For $x = log(p^{k/m})$ the Fourier transform should vanish or be small for $m \neq 1$ (rational roots of primes) and give a non-vanishing contribution for $m = 1$. One has

$$F(x = log(p^{k/m}) = \sum_{1 \leq a \leq N(p)} e^{k\frac{M(a,p)}{mN(p)}i2\pi} u(p) \ ,$$
$$u(p) = e^{\frac{r(p)}{m(p)}i2\pi} \ .$$

   Obviously one can always choose $N(a,p) = N(p)$.
4. For the simplest option $N(p) = 1$ one would obtain delta function distribution for $x = log(p^k)$. The sum of the phases associated with $y_a(p)$ and $-y_a(p)$ from the half axes of the critical line would give

$$F(x = log(p^n)) \propto X(p^n) \equiv 2cos(n\frac{r(p)}{m(p)}2\pi) \ .$$

   The sign of F would vary.
5. For $x = log(p^{k/m})$ the value of Fourier transform is expected to be small by interference effects if $M(a,p)$ is random integer, and negligible as compared with the value at $x = log(p^k)$. This option is highly attractive. For $N(p) > 1$ and $M(a,p)$ a random integer also $F(x = log(p^k)$ is small by interference effects. Hence it seems that this option is the most natural one.
6. The rational $r(p)/m(p)$ would characterize given prime (one can require that $r(p)$ and $m(p)$ have no common divisors). $F(x)$ is non-vanishing for all powers $x = log(p^n)$ for $m(p)$ odd. For $p = 2$, also $m(2) = 2$ allows to have $|X(2^n)| = 2$. An interesting ad hoc ansatz is $m(p) = p$ or $p^{s(p)}$. One has periodicity in n with period $m(p)$ that is logarithmic wave. This periodicity serves as a test and in principle allows to deduce the value of $r(p)/m(p)$ from the Fourier transform.

What could one conclude from the data (`http://tinyurl.com/hjbfsuv`)?

1. The first graph gives $|F(x = log(p^k)|$ and second graph displays a zoomed up part of $|F(x = log(p^k)|$ for small powers of primes in the range $[2, 19]$. For the first graph the eighth peak ($p = 11$) is the largest one but in the zoomed graphs this is not the case. Hence something is wrong or the graphs correspond to different approximations suggesting that one should not take them too seriously.

   In any case, the modulus is not constant as function of p^k. For small values of p^k the envelope of the curve decreases and seems to approach constant for large values of p^k (one has $x < 15$ ($e^{15} \simeq 3.3 \times 10^6$).
2. According to the first graph $|F(x)|$ decreases for $x = klog(p) < 8$, is largest for small primes, and remains below a fixed maximum for $8 < x < 15$. According to the second graph the amplitude decreases for powers of a given prime (say $p = 2$). Clearly, the small primes and their powers have much larger $|F(x)|$ than large primes.

There are many possible reasons for this behavior. Most plausible reason is that the sums involved converge slowly and the approximation used is not good. The inclusion of only 10^4 zeros would show the positions of peaks but would not allow reliable estimate for their intensities.

1. The distribution of zeros could be such that for small primes and their powers the number of zeros is large in the set of 10^4 zeros considered. This would be the case if the distribution of zeros $y_a(p)$ is fractal and gets "thinner" with p so that the number of contributing zeros scales down with p as a power of p, say $1/p$, as suggested by the envelope in the first figure.

2. The infinite sum, which should vanish, converges only very slowly to zero. Consider the contribution $\Delta F(p^k, p_1)$ of zeros not belonging to the class $p_1 \neq p$ to $F(x = log(p^k)) = \sum_{p_i} \Delta F(p^k, p_i)$, which includes also $p_i = p$. $\Delta F(p^k, p_i)$, $p \neq p_1$ should vanish in exact calculation.

   (a) By the proposed hypothesis this contribution reads as

   $$\begin{aligned} \Delta F(p, p_1) &= \sum_a cos\left[X(p^k, p_1)(M(a, p_1) + \tfrac{r(p_1)}{m(p_1)})2\pi)\right] \quad . \\ X(p^k, p_1) &= \tfrac{log(p^k)}{log(p_1)} \quad . \end{aligned}$$

   Here a labels the zeros associated with p_1. If p^k is "approximately divisible" by p^1 in other words, $p^k \simeq np_1$, the sum over finite number of terms gives a large contribution since interference effects are small, and a large number of terms are needed to give a nearly vanishing contribution suggested by the non-stationarity of the phase. This happens in several situations.

   (b) The number $\pi(x)$ of primes smaller than x goes asymptotically like $\pi(x) \simeq x/log(x)$ and prime density approximately like $1/log(x) - 1/log(x)^2$ so that the problem is worst for the small primes. The problematic situation is encountered most often for powers p^k of small primes p near larger prime and primes p (also large) near a power of small prime (the envelope of $|F(x)|$ seems to become constant above $x \sim 10^3$).

   (c) The worst situation is encountered for $p = 2$ and $p_1 = 2^k - 1$ - a Mersenne prime and $p_1 = 2^{2^k} + 1$, $k \leq 4$ - Fermat prime. For $(p, p_1) = (2^k, M_k)$ one encounters $X(2^k, M_k) = (log(2^k)/log(2^k - 1)$ factor very near to unity for large Mersennes primes. For $(p, p_1) = (M_k, 2)$ one encounters $X(M_k, 2) = (log(2^k - 1)/log(2) \simeq k$. Examples of Mersennes and Fermats are $(3, 2), (5, 2), (7, 2), (17, 2), (31, 2), (127, 2), (257, 2), ...$ Powers 2^k, $k = 2, 3, 4, 5, 7, 8, ..$ are also problematic.

   (d) Also twin primes are problematic since in this case one has factor $X(p = p_1 + 2, p_1) = \frac{log(p_1+2)}{log(p_1)}$. The region of small primes contains many twin prime pairs: (3,5), (5,7), (11,13), (17,19), (29,31),....

   These observations suggest that the problems might be understood as resulting from including too small number of zeros.

3. The predicted periodicity of the distribution with respect to the exponent k of p^k is not consistent with the graph for small values of prime unless the periodic $m(p)$ for small primes is large enough. The above mentioned effects can quite well mask the periodicity. If the first graph is taken at face value for small primes, $r(p)/m(p)$ is near zero, and $m(p)$ is so large that the periodicity does not become manifest for small primes. For $p = 2$ this would require $m(2) > 21$ since the largest power $2^n \simeq e^{15}$ corresponds to $n \sim 21$.

To summarize, the prediction is that for zeros of zeta should divide into disjoint classes $\{y_a(p)\}$ labelled by primes such that within the class labelled by p one has $p^{iy_a(p)} = e^{(r(p)/m(p))i2\pi}$ so that has $y_a(p) = [M(a, p) + r(p)/m(p))]2\pi/log(p)$.

11.6.2 More precise view about zeros of Zeta

There is a very interesting blog post by Mumford (`http://tinyurl.com/zemw27o`), which leads to much more precise formulation of the idea and improved view about the Fourier transform hypothesis: the Fourier transform or its generalization must be defined for all zeros, not only the non-trivial ones and trivial zeros give a background term allowing to understand better the properties of the Fourier transform.

Mumford essentially begins from Riemann's "explicit formula" in von Mangoldt's form.

$$\sum_p \sum_{n \geq 1} log(p)\delta_{p^n}(x) = 1 - \sum_k x^{s_k - 1} - \frac{1}{x(x^2 - 1)} \quad ,$$

where p denotes prime and s_k a non-trivial zero of zeta. The left hand side represents the distribution associated with powers of primes. The right hand side contains sum over cosines

$$\sum_k x^{s_k-1} = 2\frac{\sum_k cos(log(x)y_k)}{x^{1/2}} \quad ,$$

where y_k ithe imaginary part of non-trivial zero. Apart from the factor $x^{-1/2}$ this is just the Fourier transform over the distribution of zeros.

There is also a slowly varying term $1 - \frac{1}{x(x^2-1)}$, which has interpretation as the analog of the Fourier transform term but sum over trivial zeros of zeta at $s = -2n$, $n > 0$. The entire expression is analogous to a "Fourier transform" over the distribution of all zeros. Quasicrystal is replaced with union on 1-D quasicrystals.

Therefore the distribution for powers of primes is expressible as "Fourier transform" over the distribution of both trivial and non-trivial zeros rather than only non-trivial zeros as suggested by numerical data to which Dyson [A68] referred to (`http://tinyurl.com/hjbfsuv`). Trivial zeros give a slowly varying background term large for small values of argument x (poles at $x = 0$ and $x = 1$ - note that also $p = 0$ and $p = 1$ appear effectively as primes) so that the peaks of the distribution are higher for small primes.

The question was how can one obtain this kind of delta function distribution concentrated on powers of primes from a sum over terms $cos(log(x)y_k)$ appearing in the Fourier transform of the distribution of zeros.

Consider $x = p^n$. One must get a constructive interference. Stationary phase approximation is in terms of which physicist thinks. The argument was that a destructive interference occurs for given $x = p^n$ for those zeros for which the cosine does not correspond to a real part of root of unity as one sums over such y_k: random phase approximation gives more or less zero. To get something nontrivial y_k must be proportional to $2\pi \times n(y_k)/log(p)$ in class $C(p)$ to which y_k belongs. If the number of these y_k:s in $C(p)$ is infinite, one obtains delta function in good approximation by destructive interference for other values of argument x.

The guess that the number of zeros in $C(p)$ is infinite is encouraged by the behaviors of the densities of primes one hand and zeros of zeta on the other hand. The number of primes smaller than real number x goes like

$$\pi(x) = N(primes < x) \sim \frac{x}{log(x)}$$

in the sense of distribution. The number of zeros along critical line goes like

$$N(zeros < t) = (t/2\pi) \times log(\frac{t}{2\pi})$$

in the same sense. If the real axis and critical line have same metric measure then one can say that the number of zeros in interval T per number of primes in interval T behaves roughly like

$$\frac{N(zeros < T)}{N(primes < T)} = log(\frac{T}{2\pi}) \times \frac{log(T)}{2\pi}$$

so that at the limit of $T \to \infty$ the number of zeros associated with given prime is infinite. This asumption of course makes the argument a poor man's argument only.

11.6.3 Possible relevance for TGD

What this speculative picture from the point of view of TGD?

1. A possible formulation for NTU for the poles of fermionic Riemann zeta $\zeta_F = \zeta(s)/\zeta(2s)$ could be as a condition that is that the exponents $p^{ks_a(p)/2} = p^{k/4}p^{iky_a(p)/2}$ exist in a number theoretically universal manner for the zeros $s_a(p)$ for given p-adic prime p and for some subset of integers k. If the proposed conditions hold true, exponent reduces $p^{k/4}e^{k(r(p/m(p)i2\pi}$ requiring that k is a multiple of 4. The number of the non-trivial generating elements of super-symplectic algebra in the monomial creating physical state would be a multiple of 4. These monomials would have real part of conformal weight -1. Conformal confinement suggests that these monomials are products of pairs of generators for which imaginary parts cancel.

2. Quasi-crystal property might have an application to TGD. The functions of light-like radial coordinate appearing in the generators of supersymplectic algebra could be of form r^s, s zero of zeta or rather, its imaginary part. The eigenstate property with respect to the radial scaling rd/dr is natural by radial conformal invariance.

   The idea that arithmetic QFT assignable to infinite primes is behind the scenes in turn suggests light-like momenta assignable to the radial coordinate have energies with the dual spectrum $log(p^n)$. This is also suggested by the interpretation of ζ as square root of thermodynamical partition function for boson gas with momentum $log(p)$ and analogous interpretation of ζ_F.

   The two spectra would be associated with radial scalings and with light-like translations of light-cone boundary respecting the direction and light-likeness of the light-like radial vector. $log(p^n)$ spectrum would be associated with light-like momenta whereas p-adic mass scales would characterize states with thermal mass. Note that generalization of p-adic length scale hypothesis raises the scales defined by p^n to a special physical position: this might relate to ideal structure of adeles.

3. Finite measurement resolution suggests that the approximations of Fourier transforms over the distribution of zeros taking into account only a finite number of zeros might have a physical meaning. This might provide additional understand about the origins of generalized p-adic length scale hypothesis stating that primes $p \simeq p_1^k$, p_1 small prime - say Mersenne primes - have a special physical role.

Chapter 12

Negentropy Maximization Principle

12.1 Introduction

Quantum TGD involves "holy trinity" of time developments. There is the geometric time development dictated by the preferred extremal of Kähler action crucial for the realization of General Coordinate Invariance and analogous to Bohr orbit. There is what I originally called unitary "time development" U: $\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f$, associated with each quantum jump. This would be the counterpart of the Schrödinger time evolution $U(-t, t \rightarrow \infty)$. Quantum jump sequence itself defines what might be called subjective time development.

Concerning U, there is certainly no actual Schrödinger equation involved: situation is in practice same also in quantum field theories. It is now clear that in Zero Energy Ontology (ZEO) U can be actually identified as a sequence of basic steps such that single step involves a unitary evolution inducing delocalization in the moduli space of causal diamonds CDs) followed by a localization in this moduli space selecting from a superposition of CDs single CD. This sequence replaces a sequence of repeated state function reductions leaving state invariant in ordinary QM. Now it leaves in variant second boundary of CD (to be called passive boundary) and also the parts of zero energy states at this boundary. There is now a very attractive vision about the construction of transition amplitudes for a given CD [K116], and it remains to be see whether it allows an extension so that also transitions involving change of the CD moduli characterizing the non-fixed boundary of CD.

A dynamical principle governing subjective time evolution should exist and explain state function reduction with the characteristic one-one correlation between macroscopic measurement variables and quantum degrees of freedom and state preparation process. Negentropy Maximization Principle is the candidate for this principle. In its recent form it brings in only a single little but overall important modification: state function reductions occurs also now to an eigen-space of projector but the projector can now have dimension which is larger than one. Self has free will to choose beides the maximal possible dimension for this sub-space also lower dimension so that one can speak of weak form of NMP so that negentropy gain can be also below the maximal possible: we do not live in the best possible world. Second important ingredient is the notion of negentropic entanglement relying on p-adic norm.

The evolution of ideas related to NMP has been slow and tortuous process characterized by misinterpretations, over-generalizations, and unnecessarily strong assumptions, and has been basically evolution of ideas related to the anatomy of quantum jump and of quantum TGD itself.

Quantum measurement theory is generalized to theory of consciousness in TGD framework by replacing the notion of observer as outsider of the physical world with the notion of self. Hence it is not surprising that several new key notions are involved.

1. ZEO is in central role and brings in a completely new element: the arrow of time changes in the counterpart of standard quantum jump involving the change of the passive boundary of CD to active and vice versa. In living matter the changes of the of time are inn central role: for instance, motor action as volitional action involves it at some level of self hierarchy.

2. The fusion of real physics and various p-adic physics identified as physics of cognition to single adelic physics is second key element. The notion of intersection of real and p-adic worlds (intersection of sensory and cognitive worlds) is central and corresponds in recent view about TGD to string world sheets and partonic 2-surfaces whose parameters are in an algebraic extension of rationals. By strong form of of holography it is possible to continue the string world sheets and partonic 2-surfaces to various real and p-adic surfaces so that what can be said about quantum physics is coded by them. The physics in algebraic extension can be continued to real and various p-adic sectors by algebraic continuation meaning continuation of various parameters appearing in the amplitudes to reals and various p-adics.

   An entire hierarchy of physics labeled by the extensions of rationals inducing also those of p-adic numbers is predicted and evolution corresponds to the increase of the complexity of these extensions. Fermions defining correlates of Boolean cognition can be said so reside at these 2-dimensional surfaces emerging from strong form of holography implied by strong form of general coordinate invariance (GCI).

   An important outcome of adelic physics is the notion of number theoretic entanglement entropy: in the defining formula for Shannon entropy logarithm of probability is replaced with that of p-adic norm of probability and one assumes that the p-adic prime is that which produces minimum entropy. What is new that the minimum entropy is negative and one can speak of negentropic entanglement (NE). Consistency with standard measurement theory allows only NE for which density matrix is n-dimensional projector.

3. Strong form of NMP states that state function reduction corresponds to maximal negentropy gain. NE is stable under strong NMP and it even favors its generation. Strong form of NMP would mean that we live in the best possible world, which does not seem to be the case. The weak form of NMP allows self to choose whether it performs state function reduction yielding the maximum possible negentropy gain. If n-dimensional projector corresponds to the maximal negentropy gain, also reductions to sub-spaces with $n-k$-dimensional projectors down to 1-dimensional projector are possible. Weak form has powerful implications: for instance, one can understand how primes near powers of prime are selected in evolution identified at basic level as increase of the complexity of algebraic extension of rationals defining the intersection of realities and p-adicities.

4. NMP gives rise to evolution. NE defines information resources, which I have called Akashic records - kind of Universal library. The simplest possibility is that under the repeated sequence of state function reductions at fixed boundary of CD NE at that boundary becomes conscious and gives rise to experiences with positive emotional coloring: experience of love, compassion, understanding, etc... One cannot exclude the possibility that NE generates a conscious experience only via the analog of interaction free measurement but this option looks un-necessary in the recent formulation.

5. Dark matter hierarchy labelled by the values of Planck constant $h_{eff} = n \times h$ is also in central role and interpreted as a hierarchy of criticalities in which sub-algebra of super-symplectic algebra having structure of conformal algebra allows sub-algebra acting as gauge conformal algebra and having conformal weights coming as n-ples of those for the entire algebra. The phase transition increasing h_{eff} reduces criticality and takes place spontaneously. This implies a spontaneous generation of macroscopic quantum phases interpreted in terms of dark matter. The hierarchies of conformal symmetry breakings with $n(i)$ dividing $n(i+1)$ define sequences of inclusions of HFFs and the conformal sub-algebra acting as gauge algebra could be interpreted in terms of measurement resolution.

   n-dimensional NE is assigned with $h_{eff} = n \times h$ and is interpreted in terms of the n-fold degeneracy of the conformal gauge equivalence classes of space-time surfaces connecting two fixed 3-surfaces at the opposite boundaries of CD: this reflects the non-determinism accompanying quantum criticality. NE would be between two dark matter system with same h_{eff} and could be assigned to the pairs formed by the n sheets. This identification is important but not well enough understood yet. The assumption that p-adic primes p divide n gives deep connections between the notion of preferred p-adic prime, negentropic entanglement, hierarchy of Planck constants, and hyper-finite factors of type II_1.

6. Quantum classical correspondence (QCC) is an important constraint in ordinary measurement theory. In TGD QCC is coded by the strong form of holography assigning to the quantum

states assigned to the string world sheets and partonic 2-surfaces represented in terms of super-symplectic Yangian algebra space-time surfaces as preferred extremals of Kähler action, which by quantum criticality have vanishing super-symplectic Noether charges in the sub-algebra characterized by integer n. Zero modes, which by definition do not contribute to the metric of "world of classical worlds" (WCW) code for non-fluctuacting classical degrees of freedom correlating with the quantal ones. One can speak about entanglement between quantum and classical degrees of freedom since the quantum numbers of fermions make themselves visible in the boundary conditions for string world sheets and their also in the structure of space-time surfaces.

NMP has wide range of important implications.

1. In particular, one must give up the standard view about second law and replace it with NMP taking into account the hierarchy of CDs assigned with ZEO and dark matter hierarchy labelled by the values of Planck constants, as well as the effects due to NE. The breaking of second law in standard sense is expected to take place and be crucial for the understanding of evolution.
2. Self hierarchy having the hierarchy of CDs as embedding space correlate leads naturally to a description of the contents of consciousness analogous to thermodynamics except that the entropy is replaced with negentropy.
3. In the case of living matter NMP allows to understand the origin of metabolism. NMP demands that self generates somehow negentropy: otherwise a state function reduction to the opposite boundary of CD takes place and means death and re-incarnation of self. Metabolism as gathering of nutrients, which by definition carry NE is the way to avoid this fate. This leads to a vision about the role of NE in the generation of sensory qualia and a connection with metabolism. Metabolites would carry NE and each metabolite would correspond to a particular qualia (not only energy but also other quantum numbers would correspond to metabolites). That primary qualia would be associated with nutrient flow is not actually surprising!
4. NE leads to a vision about cognition. Negentropically entangled state consisting of a superposition of pairs can be interpreted as a conscious abstraction or rule: negentropically entangled Schrödinger cat knows that it is better to keep the bottle closed.
5. NMP implies continual generation of NE. One might refer to this ever expanding universal library as "Akaschic records". NE could be experienced directly during the repeated state function reductions to the passive boundary of CD - that is during the life cycle of sub-self defining the mental image. Another, less feasible option is that interaction free measurement is required to assign to NE conscious experience. As mentioned, qualia characterizing the metabolite carrying the NE could characterize this conscious experience.
6. A connection with fuzzy qubits and quantum groups with NE is highly suggestive. The implications are highly non-trivial also for quantum computation allowed by weak form of NMP since NE is by definition stable and lasts the lifetime of self in question.

In the sequel the formulation of NMP and various ideas involved with NMP are discussed first. The formulation of NMP for hyper-finite factors is discussed in separate section. The last section considers some consequences of NMP discussed in more detail in various books.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L17].

12.2 Basic Notions And Ideas Behind NMP

In the following the basic ideas and notions behind NMP as well as evolution of NMP are summarized. The first form of NMP was rather naïve. There was no idea about the anatomy of quantum jump and NMP only stated that the allowed quantum jumps are such that the information gain of conscious experience measured by the reduction of entanglement entropy resulting in the reduction of entanglement between the subsystem of system and its complement is maximal. Later it became clear that quantum jump has a complex anatomy. The term quantum jump is still however used about the process in question.

12.2.1 Zero Energy Ontology

Zero energy ontology (ZEO) changes considerably the interpretation of the unitary process. In zero energy ontology (ZEO) quantum states are replaced with zero energy states defined as a superpositions of pairs of positive and negative energy states identified as counterparts of initial and final states of a physical event such as particle scattering. The matrix defining entanglement between positive and negative - christened as M-matrix - is the counterpart of the ordinary S-matrix but need not be unitary. It can be identified as a "complex square root" of density matrix expressible as a product of positive square root of diagonal density matrix and unitary S-matrix. Quantum TGD can be seen as defining a "square root" of thermodynamics, which thus becomes an essential part of quantum theory.

U-matrix is defined between zero energy states and cannot therefore be equated with the S-matrix used to describe particle scattering events. Unitary conditions however imply that U-matrix can be seen as a collection of M-matrices labelled by zero energy states so that the knowledge of U-matrix implies the knowledge of M-matrices. The unitarity conditions will be discussed later. A natural guess is that U is directly related to consciousness and the description of intentional actions. For positive energy ontology state function reduction to the opposite boundary of CD would serve as a state preparation for the next quantum jump: state preparation and reduction are therefore related by time reflection.

In ZEO state function preparation and reduction can be assigned to the positive and negative energy states defining the initial and final states of the physical event. The reduction of the time-like entanglement during the state function reduction process corresponds to the measurement of the scattering matrix. In the case of negentropic time-like entanglement the reduction process is not random anymore and the resulting dynamics is analogous to that of cellular automata providing a natural description of the dynamics of self-organization in living matter. This self-organization is also 4-dimensional in ZEO: this is of utmost importance in attempts to understand living matter.

According to standard quantum measurement theory state function reductions can take place repeatedly without any change in the state. In ZEO state function reduction to a given boundary of CD can occur repeatedly without changing the corresponding part of zero energy state but affecting the part at the opposite boundary. Superposition of CDs with different sizes is possible and one can assign to the second (active) boundary a wave function in the space of moduli, which include the proper time distance between the tips of CD and discrete boosts by a subgroup of Lorentz group leaving the tip of the fixed (passive) boundary invariant. This distance must increase in average sense and this gives rise to the arrow of experienced time. Self can be identified as a sequence of quantum jumps reducing to same boundary of CD.

The simplest assumption is that there are sequences of repeated state function reductions leaving everything at the passive boundary of CD invariant. In the moduli space for active boundary (parametwrized by Lorentz boost leaving passive boundary invariant and integer shift for the proper time distance between the tips of CD given repeated reduction, which consist of a unitary evolution in the moduli space of CDs inducing delocalization followed by localization to a fixed CD.

ZEO leads to a precise identification of the subsystem at space-time level. General coordinate invariance (GCI) in 4-D sense means that 3-surfaces related by 4-D diffeomorphisms are physically equivalent. It is convenient to perform a gauge fixing by a introducing a natural choice for the representatives of the equivalence classes formed by diffeo-related 3-surfaces.

1. Light-like 3-surfaces identified as surfaces at which the Minkowskian signature of the induced space-time metric changes to Euclidian one - wormhole contacts- are excellent candidates in this respect. The intersections of these surfaces with the light-like boundaries of CD define 2-D partonic surfaces. Also the 3-D space-like ends of space-time sheets at the light-like boundaries of CDs are very natural candidates for preferred 3-surfaces.

2. The condition that the choices are mutually consistent implies effective 2-dimensionality, whose original formulation was as follows. The intersections of these surfaces defining partonic 2-surface plus the distribution of 4-D tangent spaces at its points define the basic dynamical

objects with 4-D general coordinate invariance reduced to 2-dimensional one. This effective 2-dimensionality was clear from the very beginning but is only apparent since also the data about 4-D tangent space distribution is necessary to characterize the geometry of WCW and quantum states. The quantum descriptions in terms of 3-D light-like or space-like surfaces and even in terms of 4-D surfaces are equivalent but redundant descriptions. 4-D space-time is necessary for classical part of description necessary in order to perform and interpret quantum measurements. Holography defines the correspondence between quantal (2-D surfaces) and 4-D classical degrees of freedom in space-time interior.

3. The recent formulation of effective 2-dimensionality is slightly different. Partonic 2-surfaces and string world sheets at which the modes of the induced spinor field are localized by well-definedness of em charge define the basic entities of strong form of holography [K127]. Space-time surfaces can be determined as preferred extremals from these data assuming quantum criticality meaning that clasical super-symplectic Noether charges associated with the sub-algebra of super-symplectic algebra with conformal weights coming as n-ples of those for the full algebra vanish for them.

As far as consciousness is considered effective 2-dimensionality means holography and could relate to the fact that at least our visual experience is at least effectively 2-dimensional.

12.2.2 Fusion Of Real And P-Adic Physics

The fusion of real and p-adic physics to a larger structure has been a long standing challenge for TGD. The motivations come both from elementary particle physics and TGD inspired theory of consciousness. The basic idea is that various number fields are fused to a larger structure by gluing them along rationals and common algebraic numbers. The challenge is to imagine what quantum jump and NMP could mean in this framework. The first question is how the unitary process acts.

1. U-process acts in spinorial degrees of freedom of WCW (fermionic Fock space for a given 3-surface) and in WCW degrees of freedom (the space of partonic 2-surfaces roughly).
2. WCW should decompose to sectors corresponding to space-time surfaces in varioys number fields. This suggests strongly an adelic view [K125] in which reals and various p-adic number fields form a structure analogous to a Cartesian product. These number fields froming adele (`http://tinyurl.com/64pgerm`) would have rationals in common. One can define adele also for any algebraic extension of rationals and now the algebraic extension is shared by the factors. This suggests that the various number fields are glued together like pages of a book along common back defined by the algebraic extension. Thus one has something which is not quite the Cartesian product. The implication would be a hierarchy of algebraic extensions forming an evolutionary hierarchy.
3. If one would have Cartesian proeuct, the tensor product for the fermionic Fock spaces for corresponding sub-WCWs would at the first look very natural but would lead to a situation in which each sector would contain fermionic states separately. This does not look natural. Rather, the real space-time sheets should correspond to a sensory representation of the quantum state and p-adic space-time sheets to cognitive representations of one and the same thing [K125]. Hence fermions must be localized at the back of the book.

   Fermions are indeed localized at string world sheets and partonic 2-surfaces already from the well-definedness of em charges and also the equivalence of octonionic spinor structure with the ordinary one necessary for twistorialization demands this. Also the strong form of holography is consistent with the vision that the quantum dynamics is coded by the data at these 2-D surfaces. Classical physics would be 4-D and necessary for the physical testing and interpretation of the theory. Fermions would correspond to Boolean cognition in intersection of realities and p-adicities and would be number theoretically universal as already their anti-commutation relations suggest: also the quantal version of the anti-commutation relations is number theoretically universal in the algebraic extension of rationals.
4. What can one say about the U-matrix and its satellites M and S [K75]? The earlier vision was that the transitions between different number fields are possible. The construction of transition amplitudes for them - interpreted as amplitudes for the realization of intention represented as p-adic space-time sheet as action predicted as real space-time sheet - would be

possible in the intersection but their continuation to different number fields does not seem to make sense: one should always chose on number field.

In the intersection everything is number theoretically universal. Hence the only reasonable conclusion is that these matrices exist separately in each sector and co-incide in the intersection: this is very powerful constraint and means reduction to algebraic geometry. They would give different representations of one and same thing. p-Adic mass calculations would serve as an excellent example about the usefulness of the cognitive representations - p-adic arithmetics is extremely simple as compared to the real one and the number theoretical existence fixes the physics to a high degree. This would give extremely powerful constraints also the real U-matrix.

12.2.3 Dark Matter Hierarchy

The identification of dark matter as phases having large value of Planck constant [K101, K43, K37] led to a vigorous evolution of ideas. Entire dark matter hierarchy with levels labelled by increasing values of Planck constant is predicted, and in principle TGD predicts the values of Planck constant if physics as a generalized number theory vision is accepted [K43].

The original vision was that the hierarchy of Planck constants demands a generalization of quantum TGD. This would have required a generalization of the causal diamond $CD \times CP_2$, where CD is defined as an intersection of the future and past directed light-cones of 4-D Minkowski space M^4. It however became clear that the hierarchy of Planck constants labels a hierarchy of quantum criticalities characterized by sub-algebras of super-symplectic algebras possessing a natural conformal structure. The sub-algebra for which the conformal weights come as n-ples of those for the entire algebra is isomorphic to the full algebra and acts as a conformal gauge algebra at given level of criticality.

In particular, the classical symplectic Noether charges for preferred extremals connecting 3-surfaces at the ends of CD vanish. and this defines preferred extremal property. There would be n conformal gauge equivalence classes of preferred extremals which would correspond to n sheets of a covering of the space-time surface serving as base space. There is very close similarity with the Riemann surfaces. Therefore coverings would be generated dynamically and there is no need for actual coverings of the embedding space.

The gauge degeneracy corresponds to the non-determinism associated with the criticality having interpretation in terms of non-determinism of Kähler action and with strong form of holography. The extremely strong super-symplectic gauge conditions would guarantee that the continuation of string world sheets and partonic 2-surface to preferred extremals is possible at least for some value of p-adic prime. A good guess is that this is the case for the so called ramified primes associated with the algebraic extension in question. These ramified primes would characterize physical system and the weak form of NMP would allow to understand how p-adic length scale hypothesis follows [K125].

p-Adic continuations identifiable as imaginations would be due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K79]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

A further strong prediction is that the phase transitions increasing h_{eff} and thus reducing criticality (TGD Universe is like hill at the top of the hill at....) occur spontaneously [?]. This conforms with NMP and suggests that evolution occurs spontaneously. The state function reduction increasing h_{eff} means however the death of a sub-self so that selves are fighting to stay at the criticality. The metabolic energy bringing inNE allows to satisfy the needs of NMP so that the system survives and provides a garden in which subselves can are born and die and gradually generate negentropic entanglement. Living systems are thus negentropy gatherers and each death and re-incarnation generates new negentropy.

All particles in the vertices of Feynman diagrams have the same value of Planck constant so that the particles at different pages cannot have local interactions. Thus one can speak about relative darkness in the sense that only the interactions mediated by the exchange of particles and by classical fields are possible between different pages. Dark matter in this sense can be observed, say through the classical gravitational and electromagnetic interactions. It is in principle possible to photograph dark matter by the exchange of photons which leak to another page of book, reflect, and leak back. This leakage corresponds to h_{eff} changing phase transition occurring at quantum criticality and living matter is expected carry out these phase transitions routinely in bio-control. This picture leads to no obvious contradictions with what is really known about dark matter and to my opinion the basic difficulty in understanding of dark matter (and living matter) is the blind belief in standard quantum theory. These observations motivate the tentative identification of the macroscopic quantum phases in terms of dark matter and also of dark energy with gigantic "gravitational" Planck constant.

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by the following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of h_{eff} at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

Dark matter could be a key player in quantum biology.

1. Dark matter hierarchy and p-adic length scale hierarchy would provide a quantitative formulation for the self hierarchy. To a given p-adic length scale one can assign a secondary p-adic time scale as the temporal distance between the tips of the CD. For electron this time scale is.1 second, the fundamental bio-rhythm. For a given p-adic length scale dark matter hierarchy gives rise to additional time scales coming as $h_{eff}/h = n$-multiples of this time scale.
2. The predicted breaking of second law of thermodynamics chacterizing living matter - if identified as something in the intersection of real and p-adic words - would be always below the time scale of CD considered but would take place in arbitrary long time scales at appropriate levels of the hierarchy. The scaling up of h_{eff} also scales up the time scale for the breaking of the second law.
3. The hypothesis that magnetic body is the carrier of dark matter in large h_{eff} phase has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of [J5] [K38]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [K64, K38]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [K38].

12.2.4 Quantum Classical Correspondence

Quantum classical correspondence (QCI) has served as a guideline in the evolution of the ideas and the identification of the geometric correlates of various quantum notions at the level of embedding space and space-time surfaces has been an important driving force in the progress of ideas.

1. In ZEO causal diamonds (CDs) identified roughly as intersections of future and past directed light-cones are in key role. At embedding space level CD is a natural correlate for self and sub- CDs serve as correlates of sub-selves identified as mental images. At space-time level the space-time sheets having their ends at the light-like boundaries of CD serve as correlates for self. For a system characterized by a primary p-adic length scale $L_p \propto 2^{k/2}$ the size scale of CD is secondary p-adic scale $L_{p,2} = \sqrt{p}L_p \propto 2^k$. p-Adic length scale hypothesis follows if the proper time distance between the tips of CDs is quantized in powers of 2. This quantization should relate directly to almost equivalence of octaves associated with music experience. It must be emphasized that this assumption is very probably too strong. If the distances come as integer multiples of CP_2 time, the U-matrices form a structure analogous to Kac-Moody algebra: the role of conformal weights is taken by the distances. NMP indeed selects preferred p-adic primes and thus also size scales for CDs.

2. At the level of space-time the identification of flux tubes (I called them earlier flux tubes) between space-time sheets (more precisely, between partonic 2-surfaces) as a correlate for bound state entanglement suggests itself. Flux tubes correspond typically to magnetic flux tubes in the TGD inspired quantum model of living matter. The size scale of the magnetic body of system is given by the size scale of CD and much larger than the size of the system itself.
3. The space-time sheets in the intersection of the real and p-adic WCWs characterized by the property that the mathematical representation of the partonic 2-surfaces at the ends representing holograpically the state allows interpretation in both real and p-adic sense would correspond to the correlates for negentropic entanglement. Rational and algebraic 2-surfaces defined by partonic 2-surfaces and string world sheets (in preferred coordinates) would be the common points of realities and p-adicities.

Quantum classical correspondence allows also to generate new views about quantum theory itself. Many-sheeted space-time and p-adic length scale hierarchy force to generalize the notion of sub-system. The space-time correlate for the negentropic and bound state entanglement is the formation of flux tubes connecting two space-time sheets. The basic realization is that two disjoint space-time sheets can contain smaller space-time sheets topologically condensed at them and connected by flux tubes. Thus systems un-entangled at a given level of p-adic hierarchy - that is in the measurement resolution defined by the level considered - can contain entanglement subsystems at lower level not visible in the resolution used.

In TGD inspired theory of consciousness this makes possible sharing and fusion of mental images by entanglement. The resolution dependence for the notions of sub-system and entanglement means that the entanglement between sub-systems is not "seen" in the length scale resolution of unentangled systems. This phenomenon does not result as an idealization of theoretician but is a genuine physical phenomenon. Obviously this generalized view about sub-system poses further challenges to the detailed formulation of NMP. Note that the resulting mental image should depend on whether sub-selves are entangled by bound state entanglement or NE.

12.2.5 Connection With Standard Quantum Measurement Theory

TGD allows to deduce the standard quantum measurement theory involving the notion of classical variables and their correlation with quantum numbers in an essential manner. WCW ("world of classical worlds" is a union over zero modes labelling infinite-dimensional symmetric spaces having interpretation as classical non-quantum fluctuating classical variables such as the pointer of a measurement apparatus essential for the standard quantum measurement theory [K29]. Quantum holography in its origimnal form states that partonic 2-surfaces at the light-like boundaries of CDs plus the corresponding distributions of 4-D tangent spaces of space-time surfaces at carry the information about quantum state and space-time sheet. The recent formulation talks about partonic 2-surfaces and string world sheets intersection them at discrete points with string connecting partonic 2-surfaces and string boundaries at their orbits carrying fermion number. The distribution of values of induced Kähler form of CP_2 at these surfaces defines zero modes whereas quantum fluctuating degrees of freedom correspond to the deformations of space-time surface by the flows induced by Hamiltonians associated with the degenerate symplectic structure of $\delta M^4_\pm \times CP_2$.

There exists no well-defined metric integration measure in the infinite-dimensional space of zero modes, which by definition do not contribute to the line element of WCW. This does not lead to difficulties if one assumes that a complete localization in zero modes occurs in each quantum jump. A weaker condition is that wave functions are localized to discrete subsets in the space of zero modes. An even weaker and perhaps the most realistic condition is that a localization to a finite-dimensional 2n-dimensional manifold with induced symplectic form defining a positive definite integration volume takes place.

The fundamental formulation of quantum TGD in terms of the Kähler action and Kähler-Dirac action [K127] containing measurement interaction terms guarantees quantum classical correspondence in the sense that the geometry of the space-time surface correlates with the values of conserved quantum numbers. The boundary term of Kähler-Dirac action (1-D massless Dirac action) implies that fermion line is light-like geodesic of $M^4 \times CP_2$ and carries light-like $M^4 \times E^4$ 8-momentum ($SO(4)$ quantum numbers when one uses partial wavs). The modes of embedding space spinor field carry four momentum and color ($SU(3)$) quantum numbers are also massless in 8-D sense and if the two four-momenta are identical one has Equivalence Principle (EP). The mass squared in E^4 degrees of freedom equals to the eigenvalue of spinor Laplacian in CP_2 degrees of freedom. This defines a more abstract form of EP: $SO(4)$ quantum numbers label hadrons and $SU(3)$ quantum numbers quarks and gluons so that one has dual representations.

The resulting correlation of zero modes with the values of quantum numbers can be interpreted as an abstract form of quantum entanglement reduced in quantum jump for the standard definition of the entanglement entropy.

That state function can occur at both boundaries of CD localizing the boundary in question reducing the part of zero energy state associated with it is the new element of TGD inspired quantum measurement theory and allows to understand how the arrow of experienced time emerges and precisely define self - observer - as a part of system interacting with it. Also the possibility that the arrow of time changes at some level of the self hierarchy is predicted. In living matter this is expected to occur routinely as already Fantappie speculated [J7]: the first state function reduction in the sequence of them and changing the arrow of time is indeed naturally identified as a correlate for the volitional act.

12.2.6 Quantum Jump As Moment Of Consciousness

Quantum jump between quantum histories identified as moment of consciousness was originally believed to be something irreducible and structureless. Gradually the view about quantum jump has however become more and more structured and a connection with the standard quantum measurement theory emerged. In what sense quantum jumps remains irreducible is that one cannot build any dynamical model for the non-deterministic steps appearing in quantum jump.

The general structure of quantum jump

It seems that TGD involves "holy trinity" of dynamics.

1. The dynamics defined by the preferred extremals of Kähler action corresponds to the dynamics of material existence, with matter defined as "res extensa", 3-surfaces. What preferred extremals really are has been a long standing open question. The recent formulation of the quantum theory using Kähler-Dirac action leads to the proposal that the preferred extremals are critical in the sense that they allow an infinite number of deformations for which the second variation vanishes. At the level of Kähler action this corresponds to the vanishing of classical Noether charges for a sub-algebra of super-symplectic algebra isomorphic with the entire algebra. This serves as space-time counterpart for quantum criticality of TGD Universe fixing the fundamental variational principle uniquely.
2. The dynamics defined by the sequence of state function reductions at fixed boundary of CD defining the life span of self at given level of hierarchy. This time evolution is a discrete counterpart of the ordinary Schrödinger time evolution $U \equiv U(-t,),\ t \to \infty$ and can be regarded as "informational" time development occurring at the level of objective existence. It is un-necessary and in fact impossible to assign real Schrödinger time evolution with U. U defines the S-matrix of the theory. These reductions define the dynamics of sensory perception (passive aspects of consciousness) during which external world is regarded as unchanged in standard framework. Now the part of zero energy state at the fixed boundary of CD remains unchanged and un-entangled.
3. The dynamics of state function reductions at opposite boundary of CD defines the dynamics of volition (active aspects of consciousness).

Quantum jump was originally regarded as something totally irreducible. Gradually the structure of the complex formed by state function reductions and unitary process has revealed itself and led to the understanding how one can understand basic aspects of conscious experience in terms of this structure. Let us start with the original picture.

1. The first step in quantum jump was identified as "informational time development"

$$\Psi_i \to U\Psi_i \ ,$$

where U is the counterpart of the unitary process of Penrose. The resulting state is a completely entangled multiverse state, the entire sub-universe corresponding to a given CD being in a holistic state of "oneness".

In the recent picture Universe is replaced with CD and "informational time development" corresponds to a sequence of state function reductions keeping second boundary of CDand-states associated with it fixed. Repeated measurement having no effect on quantum state is the analog in standard quantum measurement theory. Self corresponds to this sequence.

Two subsequent reductions at same boundary of CD have unitary process between them tending to increase the size CD. The challenge is to identify the unitary process U. Self experiences the flow of time, which suggests that the unitary operator followed by localization in the moduli spaces of CDs corresponds to an integer shift for the tip of the active boundary of CD. No state function reduction can occur at the active boundary of CD during this period.

2. Next comes the TGD counterpart of state function in the ordinary sense of the word:

$$U\Psi_i \rightarrow \Psi_f^0 \quad .$$

According to the recent view, the state function reduction in this sense corresponds to the state function at the opposite boundary of cD and leads to a change of the arrow of geometric time. Old self dies and new self is born. In this transition also the value of h_{eff} is expected to increase. This reduction is preceded by a scaling of by the integer ratio $h_{eff}(f)/h_{eff}(i)$ and realized as a unitary exponential of conformal scaling operator. Thus both Poincare and conformal time developments are realized.

3. The state function reduction for given CD is followed by a cascade of self measurements for sub-CDs in quantum fluctuating degrees of freedom

$$\Psi_f^0 \rightarrow .... \rightarrow \Psi_f \quad ,$$

whose dynamics is governed by the Negentropy Maximization Principle (NMP). For a generic entanglement probabilities this process leads to bound states or negentropically entangled states. This process can be regarded as an analysis or even decay process. If entanglement probabilities define projection operator, the state function reduction leads or can laed to a negentropically entangled state: this depends on what form of NMP one assumes. Entanglement coefficients correspond to unitary matrix in this case.

Quantum measurement theory involves also the correlation between quantum degrees of freedom and classical degrees of freedom (the position of the pointer of the measurement apparatus correlates with the outcome of the measurement).

1. The assumption that localization occurs in zero modes of the WCW would pose very important consistency condition: there is one-one correlation between the quantum numbers in quantum fluctuating degrees of freedom in some state basis and the values of the zero modes. This in fact has interpretation in terms of holography: classical degrees of freedom in space-time interior correlate with fermionic degrees of freedom assignable to string world sheets and partonic 2-surfaces. This together with the fact that zero modes are effectively classical variables, implies that the localization in zero modes corresponds to a state function reduction.
2. Measurement theory requires an entanglement between zero modes and quantum jumps of the physical state. The addition of a measurement interaction term to the Kähler-Dirac action coupling to four-momentum and color quantum numbers of the state and also to more general conserved quantum numbers allows an explicit realization of this coupling and induces the addition of an analogous measurement interaction term to Kähler action [K127]. This term implies the entanglement of the quantum numbers of the physical states with zero modes.

A good metaphor for quantum jump is as Djinn leaving the bottle (informational time development), fulfilling the wish (quantum jump involving choice) and returning to, possibly new, bottle (localization in zero modes and subsequent state preparation process). One could formally regard each quantum jump as a quantum computation with duration defined by the life-time of corresponding self (the increase of the average temporal distance between the tips of CD in superposition of CDs) followed by halting meaning reduction to the opposite boundary of CD.

Quantum jump to the opposite boundary could also be seen as an act of volition (or giving rise to experience of volition at some level of self hierarchy).

Is the complete localization in zero modes really necessary?

The detailed inspection of what happens in state function reductions forces to consider the possibility that state function reduction involves always a complete localization in zero modes. This was indeed the original proposal. It however seems that a localization modulo finite measurement resolution might be a more realistic assumption. Certainly it is enough to explain why the perceived Universe looks classical.

1. QFT picture strongly suggests that sub-system must be defined as a tensor factor of the space of WCW spinors at given point Y^3 of WCW . This suggests that subsystem should be defined as a function of Y^3 and should be a local concept. An important consequence of this definition is that entanglement entropy gives information about space-time geometry.
2. WCW spinor field can be formally expressed as superposition of quantum states localized into the reduced configuration space consisting of 3-surfaces belonging to light cone boundary. Hence WCW spinor field can be formally written as

$$\sum_{Y^3} C(Y^3)(n,N)|n\rangle|N\rangle$$

   for any subsystem-complement decomposition defined in Y^3. Clearly, WCW coordinates appear in the role of additional indices with respect to which entanglement coefficients are diagonal. The requirement that final state is pure state would suggest that quantum jump reducing entanglement must involve complete localization of the WCW spinor field to some Y^3 plus further quantum jump reducing entanglement in Y^3. Complete localization in WCW is however not physically acceptable option since the action of various gauge symmetries on quantum states does not commute with the complete localization operation. In particular, the requirement that physical states belong to the representations of Super Virasoro and super-symplectic algebras, is not consistent with this requirement.
3. WCW has fiber space structure. WCW metric is non-vanishing only in the fiber degrees of freedom and since the propagator for small fluctuations equals to the contravariant metric, fiber degrees of freedom correspond to genuine quantum fluctuations. WCW metric vanishes in zero modes, which can be identified as fundamental order parameters in the spirit of Haken's theory of self organization. The requirement that various local symmetries act as gauge symmetries, provides good reasons to expect that *entanglement coefficients in the fiber degrees of freedom are gauge invariants and depend on the zero modes parametrically.* The one-one correlation between quantum numbers of the state assignable to fiber degrees of freedom and classical variables identified as zero modes would encourage the assumption the a complete localization occurs in zero modes. A weaker condition is that localization occurs only modulo a finite measurement resolution.
4. The original argument was that the non-existence of metric based volume element in zero modes forces the wave functions in zero modes to have a discrete locus. There however exists a symplectic measure defined by the symplectic form in zero modes. It does not however allow a complexification to Kähler form as it does in quantum fluctuating degrees of freedom. This symplectic from could define a hierarchy of integration measures coming as restrictions of $J\wedge J...\wedge J$ with n factors to $2n$-dimensional sub-manifolds. Under some additional conditions-maybe the homological non-triviality of J and the orientability of the sub-manifold are enough, this measure would define a positive definite inner product and one would have a hierarchy finite-dimensional sub-spaces of zero modes. The maxima of Kähler function with respect to zero modes replace naturally the continuum with a discrete set of points and define the counterpart of the spin glass energy landscape consisting of the minima of free energy. Effective finite-dimensionality and even effective discreteness would be achieved.
5. The time development by quantum jumps in zero modes is effectively classical: Universe is apparently hopping around in the space of the zero modes. This looks very attractive physically since zero modes characterize the size, shape and classical Kähler fields associated with

3-surface. Therefore each quantum jump gives very precise conscious geometric information about space-time geometry and about WCW in zero modes. This also means that Haken's classical theory of self-organization generalizes almost as such to TGD context. The probability for localization to given point of zero mode space is given by the reduced probability density Q defined by the integral of the probability density R defined by WCW spinor field over fiber degrees of freedom. The local maxima of Q with respect to zero modes appear as attractors for the time development by quantum jumps. Dissipative time development could be regarded as a sequence of quantum jumps leading to this kind of local maximum.

6. Effective localization in zero modes is completely analogous to spontaneous symmetry breaking in which scalar field attains vacuum expectation value with the difference that the number of degrees of freedom is infinite unlike in typical models of symmetry breaking. Thus the general structure of the WCW spinor field together with TGD based quantum jump concept automatically implies spontaneous symmetry breaking in its TGD version (note however that particle massivation results from both p-adic thermodynamics and coupling to Higgs like field of purely geometric origin in TGD framework). TGD Universe is superposition of parallel classical universes (3-surfaces). Therefore quantum entangled state can can be regarded as a superposition of parallel entangled states, one for each 3-surface. Formally entanglement coefficients can be regarded as coefficients containing the WCW coordinates of 3-surfaces as additional index. The analogy with the spin glass also supports the localization in the zero modes.

7. Effective localization in the zero modes provides simple explanation for why the universe of conscious experience looks classical: moment of consciousness makes it classical. It also explains why the physics treating space-time as a fixed arena of dynamics has been so successful. As already found, a further important consequence is first principle description of the state function reduction.

12.2.7 NMP And Negentropic Entanglement

The evolution of NMP has been a process in which formulation has become gradually more accurate. The final outcome is surprisingly near to the original picture.

Information measures for entanglement

The attempts to formulate NMP in p-adic physics led to the realization that one can distinguish between three kinds of information measures.

1. In real physics the negative of the entanglement entropy defined by the standard Shannon formula defines a natural information measure, which is always non-positive. The formula for Shannon formula is given by $S = -\sum_n P_n log(P_n)$, where P_n are the probabilities identifiable as eigenvalues of the density matrix for a pair of system and its complement. Density matrix is defined as $\rho = C^{dagger}C$, C is the matrix defined by the entanglement coefficients for the system and its complement. In the original formulation of NMP the state function it was assumed that ρ defines the universal observable measured in state function reduction so that the sub-system (its complement) goes to an eigen state of ρ. This assumption is still kept.

2. In p-adic physics one can generalize entanglement entropy as (dis-)information measure to p-adic valued information measure by replacing the logarithms of p-adic valued probabilities with the p-based logarithms $log_p(|P|_p)$ which are integer valued and can be interpreted as p-adic numbers. This p-adic valued entanglement entropy can be mapped to a non-negative real number by the so called canonical identification $x = \sum_n x_n p^n \rightarrow \sum_n x_n p^{-n}$. In both cases a non-positive information measure results.

   When entanglement probabilities are rational numbers or at most finitely algebraically extended rational numbers one can still define logarithms of probabilities as p-based logarithms $log_p(|P|_p)$ and interpret the entropy as a rational or algebraic number. In this case the entropy can be however negative and positive definite information measure is possible. Irrespective of number field one can in this case define entanglement entropy as a maximum of number theoretic entropies S_p over the set of primes.

3. The consistency with quantum measurement theory forces to give up the most general identification of negentropic entanglement (NE). One could argue that it is not possible to distinguish between real and thus entropic entanglement and NE in any manner. One would need some signature for it. The internal consistency of quantum measurement theory indeed demands that the state function reduction occurs to an eigen space of density matrix, which in the most general case is characterized by an n-dimensional projector satisfying $P^2 = P$. Projector property of the final state density matriz would serve as a unique signature of negentropic entanglement.

It seems that the third choice is the correct one. NE would thus correspond to a density matrix proportional to a projector (identity matrix). What would be new that P is higher-dimensional projector. In real context one can argue that this situation is practically never met in reality. In TGD however hierarchy of Planck constants labelling quantum critical phases identified as dark matter would correspond to this kind of situations. Density matrix decomposes to a direct sum of terms proportional to higher-D projectors only at criticality. The interpretation could be in terms of measurement resolution: experimental resolution does not allow to discern between the n state pairs in the superposition and their probabilities are identical.

NE would result in the case of 2-particle system from entanglement coefficients defining a unitary matrix. This strongly suggests that quantum computing systems carry NE.

There has been also the question about whether NE could be identified as bound state entanglement. It is obvious that this cannot be the case for NE defined by projector treating all entangled state pairs democratically.

Does entanglement negentropy have a classical space-time correlate?

Quantum classical correspondence (QCC) suggests that number theoretic entanglement negentropy or entropy could have a classical counterpart at space-time level. The interpretation of Kähler function as the analog of thermodynamical free energy with Kähler coupling strength playing the role of critical temperature leads to ask whether the Kähler function could define the counterpart of entanglement entropy or - negentropy. The standard formula for the entropy in terms of free energy suggests that entropy is positive also now, and the interpretation as entropy would look more natural. One must of course be very cautious: also the negative of Kähler function could be identified as the analog of free energy and in this case entropy would be negative.

Kähler function is identified as Kähler action in the region of space-time with Euclidian signature and is non-negative. Kähler function is not present in GRT like theories so that it is a new concept. It is not yet clear whether also the Euclidian regions correspond to n-sheeted coverings.

What happens in Minkowkian regions, where $\sqrt{g_4}$ is imaginary and Kähler function is replaced with the analog of Morse function? In Minkowskian regions Kähler action can have also negative sign. Could Kähler action in these regions have information theoretic interpretation? If so then the magnetic flux tubes would naturally correspond to negentropic regions and electric flux quanta to entropic ones. In Minkowskian regions magnetic flux tubes with $h_{eff} = n \times h$ correspond to n-fold coverings and give rise to n-fold value of Kähler action so that the interpretation in terms of negentropy might make sense. Note however that one can ask whether the flux tubes are actually Euclidian regions connecting Euclidian regions bounded by partonic 2-surfaces. This is possible since the string world sheet associated with the string like objects can have also Euclidian signature of metric.

An interesting question is how the negentropy assignable with the inclusions of hyperfinite factors and determined by the logarithm for the index of inclusion (to be discussed later) could relate to the value spectrum of Kähler function.

Bound state entanglement and NE

It is almost trivial that bound state entanglement must be kinematically stable against NMP became obvious. One can imagine that the state function reduction proceeds step by step by reducing the state to two parts in such a way that the reduction of entanglement entropy is maximal.

1. If a resulting subsystem corresponds to a bound state having no decomposition to free subsystems the process stops for this subsystem. The natural assumption is that subsystems lose their consciousness when U process leads to bound state entanglement whereas bound state itself can be conscious.
2. If the entanglement is negentropic (and thus rational or algebraic) a more natural interpretation consistent with the teaching of spiritual practices is that subsystems experience a fusion to a larger conscious entity. The negentropic entanglement between free states is stabilized by NMP and negentropically entangled states need not reside at the bottom of potential well forbidding the reduction of entanglement. This makes possible new kinds of correlated states for which binding energy can be negative. Bound state entanglement would be like the jail of organized marriage and NE like a love marriage in which companions are free to leave but do not what it. The existence of this kind of NE is especially interesting in living matter, where metabolism (high energy phosphate bond in particular) and the stability of DNA and other highly charged polymers is poorly understood physically: NE could be responsible for stabilization making possible the transfer of metabolic energy [**?**].

Strong and weak forms of NMP

The *strong form of NMP* would state that negentropy of the universe is maximal in each state function reduction: we would live in the best possible world. This does not seem to be the case however. This leads to the *weak form of NMP* stating that in the case that maximal negentropy gain corresponds to n-dimensional projector, also the reductions to $n-k$-dimensional sub-spaces are possible and for $n-k=1$ one has ordinary reduction. Self can choose between different projector terms in ρ and for the chosen term choose lower- than maximal-dimensional sub-space.

The interpretation is that this brings to the theory of consciousness free will, ethics, and moral [K122]. Good means generation of NE, evolution. The choice in which outcome is 1-dimension sub-space means isolation, breaking of contact, as a punishment for not generating NE. The number of different choices in state function reduction for a given value of n is 2^n-1, which suggests am interpretation in terms of a Boolean algebra with n bits and an emotional realization of Boolean algebra - kind of emotional intelligence.

The weak form of NMP leads allows also to understand how preferred p-adic primes suggested by the p-adic length scale hypothesis emerged. The point is that NE per dimension of space is maximal when n is power of p. If n is power of p and $n-k$ is prime then the NE per dimension of subs-space is very large for it. For $p=2$ this would explained preferred role of Mersenne primes.

What if the eigenvalues of the density matrix go outside the algebraic extension used?

The following argument suggests that also more general algebraic entanglement could be reasonably stable against NMP, namely the entanglement for which the eigenvalues of the density matrix and eigenvectors are outside the algebraic extension associated with the parameters characterizing string world sheets and partonic 2-surfaces as space-time genes.

The restriction to a particular extension of rationals - a central piece of the number theoretical vision about quantum TGD - implies that density matrix need not allow diagonalization. In eigen state basis one would have has algebraic extension defined by the characteristic polynomial of the density matrix and its roots define the needed extension which could be quite well larger than the original extension. This would make state stable against state function reduction.

If this entanglement is algebraic, one can assign to it a negative number theoretic entropy. This negentropic entanglement is stable against NMP unless the algebraic extension associated with the parameters characterizing the parameters of string world sheets and partonic surfaces defining space-time genes is allowed to become larger in a state function reduction to the opposite boundary of CD generating re-incarnated self and producing eigenstates involving algebraic numbers in a larger algebraic extension of rationals. Could this kind of extension be an eureka experience meaning a step forwards in cognitive evolution?

If this picture makes sense, one would have both the unitary NE with a density matrix, which is projector and the algebraic NE with eigen values and NE for which the eigenstates of density matrix outside the algebraic extension associated with the space-time genes. Note that the entanglement characterized by a unitary matrix is “meditative” in the sense that any state basis is possible and therefore in this state of consciousness it is not possible to make distinctions. This strongly brings in mind koans of Zen buddhism. The more general algebraic entanglement could represent abstractions as rules in which the state pairs in the superposition represent the various instances of the rule.

Is NE experienced directly?

Does the NE at the passive boundary of CD give automatically rise to a conscious experience or must one "measure" it somehow?

1. The assumption that repeated state function reductions measure the NE and give rise to a conscious experience of it, looks natural. That NE is experienced consciously as an experience with a positive emotional coloring (love, compassion, understanding, experience of beauty,...) looks rather natural assumption since a repeated measurement of this state is in question although only the state at the active boundary changes. This experience would correspond to that part of experience which defines experiencer as something stable and unchanging (the original proposal was that self is a subsystem able to remain un-entangled and thus having self identity). The changing part of the experience would come from the active boundary of CD and give rise to an experience about flow of time due to the average increase of the distance between the tips of CD. Self would correspond to sequence of repeated reductions and would die when the first reduction to the opposite boundary of CD would occur. This would be an re-incarnation of self as a new conscious entity.
2. An alternative view is that NE as such gives rise to a conscious experience only in what is known as interaction free experiment [B2]. This idea looks un-necessary in the proposed framework. Interaction free measurement would be too complex a process to appear at the fundamental level.

It has turned that interaction free measurement could read bits (but not qubits) and might be involved with long term memory recall and reading of sensory and cognitive representations. The values of bits would remain unaffected in the interaction free measurement at idealized limit.

1. Interaction free measurement for which Elizur-Weizman bomb tester is an excellent representation (see `http://tinyurl.com/y9zenssv`) involves ordinary state function reduction. The outcome of state function reduction tells whether the bomb can act as quantum measurement apparatus or not (is it active or not) and at idealized limit the state of bomb is not changed (it does not explode). The reading of bits from memory is possible if bit 1 (say) can take the role of active state of bomb and bit 0 that of dud. In the bomb tester model the measured state corresponds to a superposition of two photon paths such that the other one traverses the bomb and induces explosion if state function reduction to this path takes place. The reduction to the other path does not induce explosion.
2. Interaction free measurement is useful if the bit can be represented as active/passive dichotomy. Active/passive dichotomy can be indeed represented in very simple manner physically. One has two state system in which lower energy state can be excited to a long lived higher energy state by photon absorption. System in higher energy state is passive and that in lower energy state active.

What happens to h_{eff} during state function reduction sequence?

What happens to h_{eff} during state function reduction sequence. Does it increase so that self would "become wiser" as it becomes older?

The natural assumption is that the value of h_{eff} stays constant during the life cycle of self and by NMP h_{eff} increasing phase transitions tend to occur as self dies and re-incarnates at opposite boundary of CD. In this process a state would be selected from the superposition of states having negentropic entanglement at the active boundary of CD.

The original idea about correlation between age and wisdom is not however wrong. h_{eff} increasing phase transitions can however occur for the sub-selves of self defining mental images of self. To these one can assign sub-CDs. Hence one can say that NE assignable to a given CD increases also during the repeated state function reductions.

It looks rather natural to assume that self does its best to stay alive by trying to gather somehow the NE needed to satisfy the demands of NMP: the easy solution is to eat other living beings! This is achieved by metabolic energy which has interpretation in terms of a transfer of NE carried by nutrients. Homeostasis in turn is a collection of mechanisms helping to stay at criticality.

Negentropic entanglement, NMP, braiding and TQC

Negentropic entanglement for which number theoretic entropy characterized by p-adic prime is negative so that entanglement carries information, is in key role in TGD inspired theory of consciousness and quantum biology.

1. The key feature of 2-particle negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) is that density matrix is projector and thus proportional to unit matrix so that the assumption that state function reduction corresponds to the measurement of density matrix does not imply state function reduction to one-dimensional sub-space. This special kind of degenerate density matrix emerges naturally for the hierarchy $h_{eff} = n \times h$ interpreted in terms of a hierarchy of dark matter phases. I have already earlier considered explicit realizations of negentropic entanglement assuming that E is invariant under the group of unitary or orthogonal transformations (also subgroups of unitary group can be considered -say symplectic group). One can however consider much more general options and this leads to a connection with topological quantum computation (TQC).
2. Entanglement matrix E equal to $1/\sqrt{n}$ factor times unitary matrix U (as a special case orthogonal matrix O) defines a density matrix given by $\rho = UU^\dagger/n = Id_n/n$, which is group invariant. One has NE respected by state function reduction if NMP is assumed. This would give huge number of negentropically entangled states providing a representation for some unitary group or its subgroup (such as symplectic group). In principle any unitary representation of any Lie group would allow representation in terms of NE. In principle any unitary representation of any Lie group would allow a representation in terms of NE.
3. In physics as generalized number theory vision, a natural condition is that the matrix elements of E belong to the algebraic extension of p-adic numbers used so that discreted algebraic subgroups of unitary or orthogonal group are selected. This realizes evolutionary hierarchy as a hierarchy of p-adic number fields and their algebraic extensions, and one can imagine that evolution of cognition proceeds by the generation of negentropically entangled systems with increasing algebraic dimensions and increasing dimension reflecting itself as an increase of the largest prime power dividing n and defining the p-adic prime in question.
4. One fascinating implication is the ability of TGD Universe to emulate itself like Turing machine: unitary S-matrix codes for scattering amplitudes and therefore for physics and negentropically entangled subsystem could represent sub-matrix for S-matrix as rules representing "the laws of physics" in the approximation that the world corresponds to n-dimension Hilbert space. Also the limit $n \to \infty$ makes sense, especially so in the p-adic context where real infinity can correspond to finite number in the sense of p-adic norm. Here also dimensions n given as products of powers of infinite primes can be formally considered.

One can consider various restrictions on E.

1. In 2-particle case the stronger condition that E is group invariant implies that unitary matrix is identity matrix apart from an overall phase factor: $U = exp(i\phi)Id$. In orthogonal case the phase factor is ± 1. For n-particle NE one can consider group invariant states by using n-dimensional permutation tensor $\epsilon_{i_1,...i_n}$.
2. One can give up the group invariance of E and consider only the weaker condition that permutation is represented as transposition of entanglement matrix: $C_{ij} \to C_{ij}$. Symmetry/antisymmetry under particle exchange would correspond to $C_{ji} = \epsilon C_{ij}$, $\epsilon = \pm 1$. This would give in orthogonal case $OO^T = O^2 = Id$ and $UU^* = Id$ in unitary case.

   In the unitary case particle exchange could be identified as hermitian conjugation $C_{ij} \to C_{ji}^*$ and one would have $U^2 = Id$. Euclidian gamma matrices γ_i define unitary and hermitian generators of Clifford algebra having dimension 2^{2m} for $n = 2m$ and $n = 2m+1$. It is relatively easy to verify that the squares of completely anti-symmetrized products of k gamma matrices representing exterior algebra normalized by factor $1/\sqrt{k!}$ are equal to unit matrix. For $k = n$ the anti-symmetrized product gives essentially permutation symbol times the product $\prod_k \gamma_k$. In this manner one can construct entanglement matrices representing negentropic bi-partite entanglement.

3. The possibility of taking tensor products $\epsilon_{ij..k...n}\gamma_i \otimes \gamma_j .. \otimes \gamma_k$ of k gamma matrices means that one can has also co-product of gamma matrices. What is interesting is that quantum groups important in topological quantum computation as well as the Yangian algebra associated with twistor Grassmann approach to scattering amplitudes possess co-algebra structure. TGD leads also to the proposal that this structure plays a central role in the construction of scattering amplitudes. Physically the co-product is time reversal of product representing fusion of particles.

4. One can go even further. In 2-dimensional QFTs braid statistics replaces ordinary statistics. The natural question is what braid statistics could correspond to at the level of NE. Braiding matrix is unitary so that it defines NE. Braiding as a flow replaces the particle exchange and lifts permutation group to braid group serving as its infinite covering.

   The allowed unitary matrices representing braiding in tensor product are constructed using braiding matrix R representing the exchange for two braid strands? The well-known Yang-Baxter equation for R defined in tensor product as an invertible element (`http://tinyurl.com/yax3j6mr`) expresses the associativity of braiding operation. Concretely it states that the two braidings leading from 123 to 321 produce the same result. Entanglement matrices constructed R as basic operation would correspond to unitary matrices providing a representation for braids and each braid would give rise to one particular NE.

   This would give a direct connection with TQC for which the entanglement matrix defines a density matrix proportional to $n \times n$ unit matrix: R defines the basic gate [B54]. Braids would provide a concrete space-time correlate for NE giving rise to "Akashic records". Note that in string theory-GRT framework this old idea of TGD has been recently introduced by Maldacena and Sussking as a proposal that wormholes connecting blackholes provide a description of entanglement.

   I have indeed proposed the interpretation of braidings as fundamental memory representations much before the vision about Akashic records. This kind of entanglement matrix need not represent only time-like entanglement but can be also associated also with space-like entanglement. The connection with braiding matrices supports the view that magnetic flux tubes are carriers of negentropically entangled matter and also suggests that this kind of entanglement between -say- DNA and nuclear or cell membrane gives rise to TQC.

Some comments concerning the covering space degrees of freedom associated with $h_{eff} = n \times h$ viz. ordinary degrees of freedom are in order.

1. Negentropic entanglement with n entangled states would correspond naturally to $h_{eff} = n \times h$ and is assigned with "many-particle" states, which can be localized to the sheets of covering but one cannot exclude similar entanglement in other degrees of freedom. Group invariance leaves only group singlets and states which are not singlets are allowed only in special cases. For instance for $SU(2)$ the state $|j, m\rangle = |1, 0\rangle$ represented as 2-particle state of 2 spin 1/2 particles is negentropically entangled whereas the states $|j, m\rangle = |1, \pm 1\rangle$ are pure.

2. Negentropic entanglement associated with $h_{eff} = n \times h$ could factorize as tensor product from other degrees of freedom. Negentropic entanglement would be localised to the covering space degrees of freedom but there would be entropic entanglement in the ordinary degrees of freedom - say spin. The large value of h_{eff} would however scale up the quantum coherence time and length also in the ordinary degrees of freedom. For entanglement matrix this would correspond to a direct sum proportional to unitary matrices so that also density matrix would be a direct sum of matrices $p_n E_n = p_n Id_n/n$, $\sum p_n = 1$ correspond ing to various values of "other quantum numbers", and state function reduction could take place to any subspace in the decomposition. Also more general entanglement matrices for which the dimensions of direct summands vary, are possible.

3. One can argue that NMP in form does not allow halting of quantum computation. This is not true. The computation halts but in different manner since negentropic entanglement tends to be generated even for weak form of NMP. Weak form of NMP allows also ordinary state function reduction. State function reduction is not need if NE can be directly experienced and self represents this mental image as a kind of abstraction or rule with the state pairs in the superposition representing the instances of the rule.

   It might be also possible to deduce the structure of negentropically entangled state by an interaction free quantum measurement replacing the state function reduction with "externalised" state function reduction. One could speak of interaction free TQC. This TQC would be reading of "Akashic records".

4. One could also counter argue that NMP allows the transfer of NE from the system so that TQC halts. NMP allows this if some another system receives at least the negentropy contained by NE. The interpretation would be as the increase of information obtained by a conscious observer about the outcome of halted quantum computation.

   Metabolism could quite concretely correspond the transfer of NE associated with the NE between nutrient molecules and some system. This would satisfy the demands of NMP and make possible for the organism to avoid the first state function reduction to the opposite boundary of CD (death) In [K84] it is suggested that this system can be of astrophysical size, say gravitational Mother Gaia with magnetic flux tubes characterized by gravitational Planck constant $\hbar_{gr} = GMm/v_0 = \hbar_{eff} = n \times \hbar$, where v_0 is a parameter with dimensions of velocity. There is experimental evidence for dark matter shell around Earth [K101] and there are highly interesting connection to the hypothesis identifying bio-photons as decay products of dark photons located at magnetic flux tubes and having $h_{eff} = h_{gr}$.

The relationship to thermodynamics

The relationship with the ordinary thermodynamics is very interesting and my views about this have been fluctuating.

1. The basic point to notice is that entanglement (neg)entropy characterize the relationship of the system to its environment whereas thermodynamical entropy characterizes single particle in an ensemble. Hence these quantities are not directly comparable and NMP need not be in conflict with the second law. Ordinary state function reductions for an ensemble of systems lead to a generation of thermodynamical entropy and this explains the second law of thermodynamics in the sector consisting of visible matter $h_{eff} = h$ provided that phase transitions generating negentropic entanglement and transforming ensembles to quantum coherent states are not too probable.

2. For the NE the outcome of the state function reduction ceases to be completely random as it is for the standard definition of entanglement entropy. For the strong form of NMP the outcome seems rather unique: the degenerate subspace for which number theoretic negentropy is maximal. For the weak form of NMP there is an additional randomness - one might speak about analog of thermodynamical fluctuations. The average increase of negentropy is positive if various choices for the dimension $n - k$ of the subspace are equally probable. Usually life is seen as a thermodynamical fluctuations, now its analog would prevent the world to be the best possible one.

   It can happen that the generation of NE transforms thermodynamical ensemble to a superposition of negentropically entangled subsystems and next state function reduction to the opposite boundary generates a negentropically entangled state. This could lead to the breaking of second law. The generation of NE means also phase transitions generating dark matter at magnetic flux tubes assumed to serve as correlates for entanglement. Since dark matter is not visible using the recent measurement technology, the breaking of second law could remain unseen. This could provide a new view to understand self-organization [K95] and evolution in living systems in which dark matter plays a key role according to the TGD inspired vision.

3. This suggests a weak formulation of the second law. In any process in which dark matter possibly created by phase transitions is observed by transforming it to ordinary matter second law holds true since the decay of dark matter to ordinary matter destroys macroscopic

quantum coherence. If it is possible to develop technologies allowing to observe dark matter without this transformation, second law does not hold in the observable Universe. The decay of dark photons to ordinary photons identified as bio-photons would represent one example of this. Note also that one can transform only a small sample of dark matter to visible matter. By a book-keeping one can detect whether ordinary matter has transformed to dark matter and with some theory can deduce by taking this kind of samples about the distribution of dark matter.

NMP in adelic approach and two interpretational problems

There have been considerable progress in the understanding of NMP in Zero Energy Ontology (ZEO) and the latest progress is discussed in detail in [K110]. In adelic approach real and various p-adic sectors are combined to adelic structure at space-time level. State space is shared by all adelic sectors and corresponds to Hilbert space with a coefficient field, which is some extension of rationals. It defines an extension of p-adic numbers for all values of p. Algebraic extensions and also extensions by roots of e correspond to finite-dimensional extensions of p-adic numbers. Together these extensions define an adele.

At the level of WCW this means that the general coordinate - and Lorentz invariant coordinates of WCW have values in the algebraic extension making sense in all number fields. Strong form of holography means that string world sheets and partonic 2-surfaces (2-surfaces) serve as "space-time genes" determining the 4-D space-time surfaces so that these conformally invariant moduli parameters for these 2-surfaces serve as WCW coordinates.

In given p-adic sector entanglement entropy (EE) is defined by replacing the logarithms of probabilities in Shannon formula by the logarithms of their p-adic norms. The resulting entropy satisfies the same axioms as ordinary entropy but makes sense only for probabilities, which must be rational valued or in an algebraic extension of rationals. The algebraic extensions corresponds to the evolutionary level of system and the algebraic complexity of the extension serves as a measure for the evolutionary level. p-Adically also extensions determined by roots of e can be considered. What is so remarkable is that the number theoretic entropy can be negative.

A simple example allows to get an idea about what is involved. If the entanglement probabilities are rational numbers $P_i = M_i/N$, $\sum_i M_i = N$, then the primes appearing as factors of N correspond to a negative contribution to the number theoretic entanglement entropy and thus to information. The factors of M_i correspond to negative contributions. For maximal entanglement with $P_i = 1/N$ in this case the EE is negative. The interpretation is that the entangled state represents quantally concept or a rule as superposition of its instances defined by the state pairs in the superposition. Identity matrix means that one can choose the state basis in arbitrary manner and the interpretation could be in terms of "enlightened" state of consciousness characterized by "absence of distinctions". In general case the basis is unique.

Metabolism is a central concept in biology and neuroscience. Usually metabolism is understood as transfer of ordered energy and various chemical metabolites to the system. In TGD metabolism could be basically just a transfer of NE from nutrients to the organism. Living systems would be fighting for NE to stay alive (NMP is merciless!) and stealing of NE would be the fundamental crime.

TGD has been plagued by a longstanding interpretational problem: can one apply the notion of number theoretic entropy in the real context or not. If this is possible at all, under what conditions this is the case? How does one know that the entanglement probabilities are not transcendental as they would be in generic case? There is also a second problem: p-adic Hilbert space is not a well-defined notion since the sum of p-adic probabilities defined as moduli squared for the coefficients of the superposition of orthonormal states can vanish and one obtains zero norm states.

These problems disappear if the reduction occurs in the intersection of reality and p-adicities since here Hilbert spaces have some algebraic number field as coefficient field. By SH the 2-D states states provide all information needed to construct quantum physics. In particular, quantum measurement theory.

1. The Hilbert spaces defining state spaces has as their coefficient field always some algebraic extension of rationals so that number theoretic entropies make sense for all primes. p-Adic numbers as coefficients cannot be used and reals are not allowed. Since the same Hilbert

space is shared by real and p-adic sectors, a given state function reduction in the intersection has real and p-adic space-time shadows.

2. State function reductions at these 2-surfaces at the ends of CD take place in the intersection of realities and p-adicities if the parameters characterizing these surfaces are in the algebraic extension considered. It is however not absolutely necessary to assume that the coordinates of WCW belong to the algebraic extension although this looks very natural.

3. NMP applies to the total EE. It can quite well happen that NMP for the sum of real and p-adic entanglement entropies does not allow ordinary state function reduction to take place since p-adic negative entropies for some primes would become zero and net negentropy would be lost. There is competition between real and p-adic sectors and p-adic sectors can win! Mind has causal power: it can stabilize quantum states against state function reduction and tame the randomness of quantum physics in absence of cognition! Can one interpret this causal power of cognition in terms of intentionality? If so, p-adic physics seems be also physics of intentionality as originally assumed.

   One could also say that Einstein was rather near to truth when he said that God does not play dice. Conscious entities play dice only whey they die and re-incarnate as time-revered selves at the opposite boundary of CD - that is perform the first state function reduction at the opposite boundary of CD.

12.2.8 Wigner's friend and Schrödinger's cat

I encountered in Facebook discussion Wigner's friend paradox (see`http://tinyurl.com/jpnvtp5` and `http://tinyurl.com/ze6bmem`). Wigner leaves his friend to the laboratory together with Schrödinger's cat and the friend measures the state of cat: the outcome is "dead" or "alive". Wigner returns and learns from his friend what the state of the cat is. The question is: was the state of cat fixed already earlier or when Wigner learned it from his friend. In the latter case the state of friend and cat would have been superposition of pairs in which cat was alive and friend new this and cat was dead also now friend new this. Entanglement between cat and bottle would have been transferred to that between cat+bottle and Wigner's friend. Recall that this kind of information transfer occur in quantum computation and quantum teleportation allows to transfer arbitrary quantum state but destroys the original.

The original purpose of Wigner was to demonstrate that consciousness is involved with the state function collapse.

TGD view is that the state function collapse can be seen as moment consciousness [K70, K8]. Or more precisely, self as conscious entity corresponds to a repeated state function reduction sequence to the same boundary of causal diamond (CD). One might say that self is generalized Zeno effect in Zero Energy Ontology (ZEO). The first reduction to the opposite boundary of CD means death of self and re-incarnation at opposite boundary as time reversed self. The experiencet flow of time corresponds to the shift of the non-fixed boundary of self reduction by reduction farther from the fixed boundary - also the state at it changes. Thus subjective time as sequence of reductions is mapped to clock time identifiable as the temporal distance between the tips of CD. Arrow of time is generated but changes in death-reincarnation.

In TGD inspired theory of consciousness the intuitive answer to the question of Wigner looks obvious. If the friend measured the state of cat, it was indeed dead or alive already before Wigner arrived. What remains is the question what it means for Wigner, the "ultimate observer", to learn about the state of the cat from his friend. The question is about what conscious communications are.

Consider first the situation in the framework of standard quantum information theory.

1. Quantum teleportation (see `http://tinyurl.com/omfkydh`) could make it possible to transfer arbitrary quantum state from the brain of Wigner's friend to Wigner's brain. Quantum teleportation involves generation of Bell state (see `http://tinyurl.com/z9g8rar` of qubits assignable with Wigner's friend (A) and Wigner (B).

2. This quantum state can be constructed by a joint measurement of component of spin in same direction at both A and B. One of the four eigenstates of (by convention) the operator $Q^z = J_x^{1)} \otimes J_y^{2)} - J_y^{1)} \otimes J_x^{2)}$ is the outcome. For spinors the actions of J_x and J_y change the

sign of J_z eigenvalue so that it becomes possible to construct the Bell states as eigenstates of Q^z.

3. After that Wigner's friend measures both the qubit representing cat's state, which is to be communicated and the qubit at A. The latter measurement does not allow to predict the state at B. Wigner's friend communicates the two bits resulting from this measurement to Wigner classically. On basis of these two classical bits his friend performs some unitary operation to the qubit at his end and transforms it to qubit that was to be communicated.

This allows to communicate the qubit representing measurement outcome (alive/dead). But what about meaning? What guarantees that the meaning of the bit representing the state of the cat is the same for Wigner and his friend? One can also ask how the joint measurement can be realized: its seems to require the presence of system containing $A \otimes B$. To answer these questions one must introduce some notions of TGD inspired theory of consciousness: self hierarchy and subself=mental image identification.

TGD inspired theory of consciousness predicts that during communication Wigner and his friend form a larger entangled system: this makes possible sharing of meaning. Directed attention means that subject and object are entangled. The magnetic flux tubes connecting the two systems would serve as a correlate for the attention. This mechanism would be at work already at the level of molecular biology. Its analog would be wormholes in ER-EPR corresponence proposed by Maldacena and Susskind. Note that directed attention brings in mind the generation of the Bell entangled pair A-B. It would make also possible quantum teleportation.

Wigner's friend could also symbolize the "pointer of the measurement apparatus" constructed to detect whether cats are dead of alive. Consider this option first. If the pointer is subsystem defining subself of Wigner, it would represent mental image of Wigner and there would be no paradox. If qubit in the brain in the brain of Wigner's friend replaces the pointer of measurement apparatus then during communication Wigner and his friend form a larger entangled system experiencing this qubit. Perhaps this temporary fusion of selves allows to answer the question about how common meaning is generated. Note that this would not require quantum teleportation protocol but would allow it.

Negentropically entangled objects are key entities in TGD inspired theory of consciousness and the challenge is to understand how these could be constructed and what their properties could be. These states are diametrically opposite to unentangled eigenstates of single particle operators, usually elements of Cartan algebra of symmetry group. The entangled states should result as eigenstates of poly-local operators. Yangian algebras involve a hierarchy of poly-local operators, and twistorial considerations inspire the conjecture that Yangian counterparts of super-symplectic and other algebras made poly-local with respect to partonic 2-surfaces or end-points of boundaries of string world sheet at them are symmetries of quantum TGD [K46]. Could Yangians allow to understand maximal entanglement in terms of symmetries?

1. In this respect the construction of maximally entangled states using bi-local operator $Q^z = J_x \otimes J_y - J_x \otimes J_y$ is highly interesting since entangled states would result by state function. Single particle operator like J_z would generate un-entangled states. The states obtained as eigenstates of this operator have permutation symmetries. The operator can be expressed as $Q^z = f^z_{ij} J^i \otimes J^j$, where f^A_{BC} are structure constants of SU(2) and could be interpreted as co-product associated with the Lie algebra generator J^z. Thus it would seem that unentangled states correspond to eigenstates of J^z and the maximally entangled state to eigenstates of co-generator Q^z. Kind of duality would be in question.

2. Could one generalize this construction to n-fold tensor products? What about other representations of SU(2)? Could one generalize from SU(2) to arbitrary Lie algebra by replacing Cartan generators with suitably defined co-generators and spin 1/2 representation with fundamental representation? The optimistic guess would be that the resulting states are maximally entangled and excellent candidates for states for which negentropic entanglement is maximized by NMP [K70].

3. Co-product is needed and there exists a rich spectrum of algebras with co-product (quantum groups, bialgebras, Hopf algebras, Yangian algebras). In particular, Yangians of Lie algebras are generated by ordinary Lie algebra generators and their co-generators subject to

constraints. The outcome is an infinite-dimensional algebra analogous to one half of Kac-Moody algebra with the analog of conformal weight N counting the number of tensor factors. Witten gives a nice concrete explanation of Yangian [B22] for which co-generators of T^A are given as $Q^A = \sum_{i<j} f^A_{BC} T^B_i \otimes T^C_j$, where the summation is over discrete ordered points, which could now label partonic 2-surfaces or points of them or points of string like object (see `http://tinyurl.com/y727n8ua`). For a practically totally incomprehensible description of Yangian one can look at the Wikipedia article (see `http://tinyurl.com/y7heufjh`).

4. This would suggest that the eigenstates of Cartan algebra co-generators of Yangian could define an eigen basis of Yangian algebra dual to the basis defined by the totally unentangled eigenstates of generators and that the quantum measurement of poly-local observables defined by co-generators creates entangled and perhaps even maximally entangled states. A duality between totally unentangled and completely entangled situations is suggestive and analogous to that encountered in twistor Grassmann approach where conformal symmetry and its dual are involved. A beautiful connection between generalization of Lie algebras, quantum measurement theory and quantum information theory would emerge.

12.3 Generalization Of NMP To The Case Of Hyper-Finite Type II_1 Factors

The intuitive notions about entanglement do not generalize trivially to the context of relativistic quantum field theories as the rigorous algebraic approach of [?] based on von Neumann algebras demonstrates. von Neumann algebras can be written as direct integrals of basic building blocks referred to as factors [A69]. Factors can be classified to three basic types labelled as type I, II, and III. Factors of type I appear in non-relativistic quantum theory whereas factors of type III_1 in relativistic QFT [?]. Factors of type II_1 [A56], believed by von Neumann to be fundamental, appear naturally in TGD framework [K126].

12.3.1 Factors

Factors of type I

The von Neuman factors of type I correspond to the algebras of bounded operators in finite or infinite-dimensional separable Hilbert spaces. In the finite-dimensional case the algebra reduces to the ordinary matrix algebra in the finite-dimensional case and to the algebra of bounded operators of a separable Hilbert space in the infinite-dimensional case. Trace is the ordinary matrix trace. The algebra of projection operators has one-dimensional projectors as basic building blocks (atoms), the notion of pure state is well-defined, and the decomposition of entangled state to a superposition of products of pure states is unique. This case corresponds to the ordinary non-relativistic quantum theory. Ordinary quantum measurement theory and also the theory of quantum computation has been formulated in terms of type I factors. Also the discussion of NMP has been formulated solely in terms of factors of type I.

Factors of type II_1

The so called hyper-finite type II_1 factors, which are especially natural in TGD framework, can be identified in terms of the Clifford algebra of an infinite-dimensional separable Hilbert space such that the unit operator has unit trace. Essentially the fermionic oscillator operator algebra associated with a separable state basis is in question. The theory of hyper-finite type II_1 factors is rich and has direct connections with conformal field theories [A94], quantum groups [A38], knot and 3-manifold invariants [A90, A47, A100], and topological quantum computation [K6], [B38].

Factors of type III

For algebras of type III associated with non-separable Hilbert spaces all projectors have infinite trace so that the very notion of trace becomes obsolete. The factors of type III_1 are associated with quantum field theories in Minkowski space.

The highly counter-intuitive features of entanglement for type III factors are discussed in [?].

1. The von Neumann algebra defined by the observables restricted to an arbitrary small region of Minkowski space in principle generates the whole algebra. Expressed in a more technical jargon, any field state with a bound energy is cyclic for each local algebra of observables so that the field could be obtained in entire space-time from measurements in an arbitrary small region of space-time. This kind of quantum holography looks too strong an idealization.

   In TGD framework the replacement of Minkowski space-time with space-time sheet seems to restrict the quantum holography to the boundaries of the space-time sheet. Furthermore, in TGD framework the situation is nearer to the non-relativistic one since Poincare transformations are not symmetries of space-time and because 3-surface is the fundamental unit of dynamics. Also in TGD framework M^4 cm degrees of 3-surfaces are present but it would seem that they appear as labels of type II_1 factors in direct integral decomposition rather than as arguments of field operators.

2. The notion of pure state does not make sense in this case since the algebra lacks atoms and projector traces do not define probabilities. The generalization of the notion of pure state as in II_1 case does not make sense since projectors have infinite trace.

3. Entanglement makes sense but has very counter-intuitive properties. First of all, there is no decomposition of density matrix in terms of projectors to pure states nor any obvious generalization of pure states. There exists no measure for the degree of entanglement, which is easy to understand since one cannot assign probabilities to the projectors as their traces.

4. For any pair of space-like separated systems, a dense set of states violates Bell inequalities so that correlations cannot be regarded as classical. This is in a sharp contrast with elementary quantum mechanics, where "de-coherence effects" are believed to drive the states into a classically correlated states.

5. No local measurement can remove the entanglement between a local system and its environment. In TGD framework local operations would correspond to operations associated with a given space-time sheet. Irreducible type II_1 entanglement between different space-time sheets, if indeed present, might have an interpretation in terms of a finite resolution at state space level due to spin glass degeneracy.

On basis of these findings, one might well claim that the axiomatics of relativistic quantum field theories is not consistent with the basic physical intuitions.

12.3.2 NMP Hyper-Finite Factors Of Type II_1

In the following hyper-finite factors of type II_1 (HFFs) will be discussed since they are certainly emerge in TGD as operator algebra defined by the fermionic oscillator operators.

The origin of hyper-finite factors of type $\mathbf{II}_1$ in TGD

Infinite-dimensional Clifford algebra corresponds in TGD framework to the super-algebra generated by complexified WCW gamma matrices creating WCW spinor s from vacuum spinor which is the counterpart of Fock vacuum [K126]. By super-conformal symmetry also WCW degrees of freedom correspond to a similar factor. For type hyper-finite II_1 factors the trace is by definition finite and normalized such that the unit operator has unit trace. As a consequence, the traces of projection operators have interpretation as probabilities.

Finite-dimensional projectors have vanishing traces so that the notion of pure state must be generalized. The natural generalization is obvious. Generalized pure states correspond to states for which density matrix reduces to a projector with a finite norm. The physical interpretation is that physical measurements are never able to resolve completely the infinite state degeneracy identifiable in TGD framework as spin glass degeneracy basically caused by the vacuum degeneracy implying non-determinism of Kähler action. An equivalent interpretation is in terms of state space resolution, which can never be complete.

In TGD framework the relevant algebra can also involve finite-dimensional type I factors as tensor factors. For instance, the entanglement between different space-time sheets could be of this kind and thus completely reducible whereas the entanglement in configuration space spin and "vibrational" degrees of freedom (essentially fermionic Fock space) would be of type II_1. The finite state-space resolution seems to effectively replace hyper-finite type II_1 factors with finite-dimensional factors of type I.

Hyper-finite factors of type II_1 and quantum measurement theory with a finite measurement resolution

The realization that the von Neumann algebra known as hyper-finite factor of type II_1 is tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

Hyper-finite factor of type II_1 has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the embedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the embedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in ZEO states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.
2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.
3. This leads also to the notion of quantum group. For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.
4. For HFFs the dimension of infinite-dimensional state space is finite and qual to $D = 1$ by convention. For included HFF $\mathcal{N} \subset \mathcal{M}$ the dimension of the tensor factor space containing only the degrees of freedom which are above measurement resolution is given by the index of inclusion $d = \mathcal{M} : \mathcal{N}$. One can say that the dimension associated with degrees of freedom below measurement resolution is $D = 1/d$. This number is never large than 1 for the inclusions and contains a set of discrete values $d = 4cos^2(2\pi/n)$, $n \geq 3$, plus the continuum above it. The fractal generalization of the formula for entanglement entropy gives $S = -log(1/D) = -log(d) \leq 0$ so that one can say that the entanglement negentropy assignable to the projection operators to the sub-factor is positive except for $n = 3$ for which it vanishes. The non-measured degrees of freedom carry information rather than entropy.
5. Clearly both HFFs of type I and II allow entanglement negentropy and allow to assign it with finite measurement resolution. In the case of factors its is not clear whether the weak form of NMP allows makes sense. Could the density matrix be expressed as a direct sum of projectors to subspaces multiplied by corresponding probabilities? Whether this is possible, is far from clear to me: in any case it would require new mathematics.

   A more natural looking option is that the decomposition of the density matrix to projectors is replaced with a particular hierarchy of inclusions and the state function reduction allows any finite sequence of inclusions. The negentropy gain would correspond to the total negentropy gain associated with the inclusion sequences obtained as sum of $S = -log(\prod_i d_i) = -\sum_i log(d_i)$. The larger the number of inclusions in the sequence, the larger the information gain. This makes sense since the measurement resolution would increase. The longer the sequence of inclusions, the higher the negentropy gain. This picture is different from that resulting from NE: in this case reduction to lower dimensional space tends to give smaller negentropy gain.

The topology of the many-sheeted space-time encourages the generalization of the notion of quantum entanglement in such a way that unentangled systems can possess entangled sub-systems. One can say that the entanglement between sub-selves is not visible in the resolution characterizing selves. This makes possible sharing and fusion of mental images central for TGD inspired theory of consciousness. These concepts find a deeper justification from the quantum measurement theory for hyper-finite factors of type II_1 for which the finite measurement resolution is basic notion.

12.4 Some Consequences Of NMP

In the sequel the most obvious consequences of self measurement and NMP are discussed from the point of view of physics, biology, cognition, and quantum computing. The recent discussion differs considerably from the earlier one since several new elements are involved. ZEO and the hierarchy of CDs, the hierarchy of Planck constants and dark matter, and - perhaps most importantly - the better understanding NE as something genuinely new and making sense in the intersection of real and various p-adic worlds at which living matter is assumed to reside.

12.4.1 NMP And P-Adic Length Scale Hypothesis

The original form of the p-adic length scale hypothesis stated that physically most interesting p-adic primes satisfy $p \simeq 2^k$, k prime or power of prime. It has however turned out that all positive integers k are possible. Surprisingly few new length scales are predicted by this generalization in physically interesting length scales. p-Adic length scale hypothesis leads to excellent predictions for elementary particle masses (note that the mass prediction is exponentially sensitive to the value of k) and explains also some interesting length scales of biology: for instance, the thicknesses of the cell membrane and of single lipid layer of cell membrane correspond to $k = 151$ and $k = 149$ respectively.

Various explanations for the origin of p-adic length scale hypothesis

The big problem of p-adic TGD is to derive this hypothesis from the basic structure of the theory.

1. One argument is based on black hole-elementary particle analogy [K80] leading to the generalization of the Hawking-Bekenstein formula: the requirement leading to the p-adic length scale hypothesis is that the radius of the so called elementary particle horizon is itself a p-adic length scale. This argument involves p-adic entropy essentially and it seems that information processing is somehow involved.

2. Zero energy ontology predicts p-adic length scale hypothesis if one accepts the assumption that the proper time distances between the tips of CDs come as powers of 2 [K80]. A more general highly suggestive proposal is that the relative position between tips forms a lattice at proper time constant hyperboloid having as a symmetry group discrete subgroup of Lorentz group (which could reduce to a subgroup of the group $SO(3)$ acting as isotropy group for the time-like direction defined by the relative coordinate between the tips of CD [K102].

   p-Adic length scale hypothesis could be understood as a resonance in frequency domain -most naturally for massless particles like photons. The secondary p-adic time scale for favored p-adic primes must be as near as possible to the proper time distance between the tips of CD. Mersenne primes $M_n = 2^n - 1$ (n is prime) satisfy this condition. Also $log(p)$ is in this case as near as possible to $log(2^n)$ and in the sense that the unit of negentropy defined as $log(2^n - m(n))/log(2^n)$ is maximized. This argument might work also for Gaussian Mersennes $G_n = (1+i)^n - 1$ (n is prime also now) if one restricts the consideration to Gaussian primes.

   A more general and more realistic looking hypothesis is that a given CD can have partonic light-like 3-surfaces ending at its boundaries for all p-adic length scales up to that associated with CD: powers of 2 would be favored by the condition of commeasurability very much analogous to frequency doubling.

3. An exciting possibility, suggested already earlier half seriously, is that evolution is present already at elementary particle level. This is the case if elementary particles reside in the

intersection of real and p-adic worlds. The success of p-adic mass calculations and the identification of p-adic physics as physics of cognition indeed forces this interpretation. In particular, one can understand p-adic length scale hypothesis as reflecting the survival of the cognitively fittest p-adic topologies.

I have discussed also other explanations.

1. A possible physical reason for the primes near prime powers of 2 is that survival necessitates the ability to co-operate, to act in resonance: this requirement might force commeasurability of the length scales for p-adic space-time sheet (p_1) glued to larger space-time sheet ($p_2 > p_1$). The hierarchy would state from 2-adic level having characteristic fractal length scales coming as powers of $\sqrt{2}$. When $p > 2$ space-time sheet is generated during cosmological evolution $L(p)$ for it must correspond to power of $\sqrt{2}$ so that one must have $p \simeq 2^n$.

2. A model for learning [K25] as a transformation of the reflective level of consciousness to proto level supports the view that evolution and learning occur already at elementary particle level as indeed suggested by NMP: the p-adic primes near power of prime powers of two are the fittest ones. The core of the argument is the characterization of learning as a map from 2^N many-fermion states to M association sequences. The number of association sequences should be as near as possible equal to 2^N. If M is power of prime: $M = p^K$, association sequences can be given formally the structure of a finite field $G(p, K)$ and p-adic length scale hypothesis follows as a consequence of $K = 1$. NMP provides the reason for why $M = p^K$ is favored: in this case one can construct realization of quantum computer with entanglement probabilities $p_k = 1/M = 1/p^K$ and the negentropy gain in quantum jump is $Klog(p)$ while for M not divisible by p the negentropy gain is zero.

Generalization of p-adic length scale hypothesis suggested by NMP

The assumption that adelic physics has as its number theoretically universal core the physics for algebraic extensions of rationals inducing those of p-adic numbers fields allows to understand preferred p-adic primes as those which are ramified [K125]. Ramified prime decomposes into a product of primes involving higher powers of prime of the extension and maximally ramified primes correspond to irreducible extensions satisfying so called Eisenstein criterion.

In strong form of holography p-adic continuations of 2-surfaces to preferred extrmals identifiable as imaginations would be easy due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K79]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes. This does not yet explain p-adic length scale hypothesis [K80, K65] stating that p-adic primes near powers of 2 are favored.

A possible generalization of this hypothesis is that primes near powers of prime are favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [?] (`http://tinyurl.com/jbh9m27`) and in music both 2-adicity and 3-adicity could be present, this is discussed in TGD inspired theory of music harmony and genetic code [K91].

The weak form of NMP might come in rescue here.

1. Entanglement negentropy for a NE [K70] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretic entanglement negentropy. If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.

2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the

final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real preferred extremal as p-adic preferred extermal (Note that $p = 1$ makes formally sense but for it the topology is discrete).

3. Weak form of NMP [K70, K122] suggests a more convincing explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator. Weak form of NMP allows free will so that all dimensions $n-k$, $k = 0, 1, ...n-1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.

4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n = 2^k = M_k + 1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n = 2^k - r$ near 2^k produce large entanglement negentropy and would be favored by NMP.

5. This argument suggests a generalization of p-adic length scale hypothesis so that $p = 2$ can be replaced by any prime.

This argument together with the hypothesis that preferred prime is ramified would correlate the character of the irreducible extension and character of super-conformal symmetry breaking. The integer n characterizing super-symplectic conformal sub-algebra acting as gauge algebra would depends on the irreducible algebraic extension of rational involved so that the hierarchy of quantum criticalities would have number theoretical characterization. Ramified primes could appear as divisors of n and n would be essentially a characteristic of ramification known as discriminant. An interesting question is whether only the ramified primes allow the continuation of string world sheet and partonic 2-surface to a 4-D space-time surface. If this is the case, the assumptions behind p-adic mass calculations would have full first principle justification.

12.4.2 NMP And Thermodynamics

The physical status of the second law has been a longstanding open issue in physics- in particular biophysics. In positive energy ontology the understanding of the origin of second law is simple. Quantum jumps involve state function reduction (or more generally, self measurement) with a random outcome and in the case of ensemble of identical system this leads to a probability distribution for the states of the members of the ensemble. This implies Boltzmann equations implying the second law. In TGD framework there are many elements which force to question this simple picture: zero energy ontology and CDs, effective four-dimensionality of the ensemble defined by states assignable to sub-CDs, hierarchy of Planck constants, and the possibility of negentropic entanglement.

Zero energy ontology and thermodynamical ensembles

Zero energy ontology means that the thermodynamics appears both at the level of quantum states and at the level of ensembles. At the level of quantum states this means that M-matrix can be seen as a complex square root of the density matrix: $\rho = MM^\dagger$, where M is expressible as a product of a positive and diagonal square root of density matrix and unitary S-matrix identifiable as the S-matrix used in quantum physics. U matrix can be seen as a collection of M-matrices as will be found later so that U-matrix fixes M-matrices contrary to what was believed originally. One can say that thermodynamics -at least in some sense- is represented at the level of single particle states. It is natural to assume that this density matrix is measured in particle physics experiment, and that this measurement corresponds to a state function reduction, which in standard physics picture corresponds to a preparation for the initial states and state function reduction for the final states.

The p-adic thermodynamics, which applies to conformal weights rather than energy, predicts successfully elementary particle masses [K76] and should reduce to this thermodynamics. That p-adic thermodynamics can be applied at all conforms with the view that even elementary particles (that is fermions serving as their basic building bricks) reside in the intersection of the real and p-adic worlds so that either p-adic thermodynamics or real thermodynamics with additional constraints on temperature implied by number theory applies.

M-matrix corresponds to a square root of density matrix, which suggests that also ordinary thermodynamics should be replaced with its square root bringing in phase factors. The imaginary part appearing in the exponent of the vacuum functional defined by Kähler action in Minkowskian regions could have interpretation in terms of a square root of thermodynamics. I have proposed this kind of description as a generalization of the model of cell membrane based on generalization of Josephson junction by bringing in dark currents flowing along magnetic flux tubes [K38].

Thermodynamical ensembles are 4-dimensional

The hierarchy of CDs within CDs defines a hierarchy of sub-systems and sub-CDs define in a natural manner 4-dimensional ensemble. If the state function reduction leads to unentangled states, the outcome is an ensemble describable by the density matrix assignable to the single particle states. The sequence of quantum jumps is expected to lead to a 4-D counterpart of thermodynamical ensemble and thermodynamics results when one labels the states by the quantum numbers assignable to their positive energy part. Entropy is assigned with entire 4-D CD rather than to its 3-dimensional time=constant snapshots. The thermodynamical time is basically the subjective time and measured in terms of quantum jumps but has a correlation with geometric time as explained in [K8] and explained briefly below.

This picture differs from the standard views, and this might explain the paradoxical situation in cosmology resulting from the fact that the initial state of the universe in the standard sense of the word looks highly entropic whereas second law would suggest the opposite [K102]. The cosmological entropy is assigned with a CD of size scale defined by the value of the age of the universe. In this kind of situation each quantum jump replaces the zero energy state with a new one and also induces a drift in the space of CDs to the direction of larger CDs with size defined by the proper time distance between the tips of CD coming as power of 2. Entropy as a function of cosmic time corresponds in TGD framework to the increase of the 4-D entropy as a function of the quantized proper time distance between the tips of the CD.

In this framework it is possible to understand second law in cosmic time scales apart from the possible effects related to NE responsible for the evolution and breaking of second law in arbitrarily long time scales caused by the transformation of thermal ensembles to quantum coherent dark matter. For instance, the number of sub-CDs increases meaning the increase of the size of the ensemble and the emergence of new p-adic length scales as the size of cosmic CD increases. What is fascinating is that the TGD counterpart of cosmic time is quantized in powers of two. This might have predictable effects such as the occurrence of the cosmic expansion in a jump-wise manner. I have discussed an explanation of the accelerated cosmic expansion in terms of quantum jumps of this kind but starting from somewhat different picture [K102].

Does NMP replace second law?

In TGD NMP defines the fundamental law of evolution. If the maximal negentropy gain corresponds to n-dimensional projector and all outcomes for $m \leq n$ are equally probable (weak NMP) the average value of the dimension associated with the projector of the reduced sub-space is $n/2$. The average negentropy gain is average over the various values of m and has sensitive dependence on the prime number decomposition of m. If m is power of prime, the negentropy gain is large. Therefore the weak form of NMP makes it possible to have larger negentropy gain for $m < n$ than for n having factors if m is prime or has small number of factors.

Second law reflecting the non-determinism of state function reduction is expected to hold true when the reduction takes place to 1-D sub-space - dark matter is not generated in state function reductions. The process can have stages involving dark matter phases labelled by non-standard value of Planck constant but the system returns back to the state in which it consists of visible matter.

The generation of NE indeed breaks second law. Particles are not anymore independent members of a thermodynamical ensemble but form larger units. Since the number of particles is reduced also thermodynamical entropy is reduced and second law can be broken in the geometric time scale considered.

How second law must be modified?

Even in this case second law as such does not certainly apply in TGD framework without special restrictions. Many of these special prerequisites hold true also in the case of NMP.

1. The hierarchy of CDs forces to introduce a fractal version of the second law taking into account the p-adic length scale hypothesis and dark matter hierarchy. This means that the idea about quantum parallel Universes generalizes to that of quantum parallel dissipating Universes. For instance, the parton model of hadrons based on quarks and gluons relies on kinetic equations and is basically thermodynamical whereas the model for hadron applied at low energies is quantum mechanical. These two views are consistent if quantum parallel dissipation realized in terms of a hierarchy of CDs is accepted. p-Adic length scale hierarchy with p-adic length scale hypothesis stating that primes near powers of two are preferred corresponds to this dissipative quantum parallellism. Dark matter hierarchy brings in a further dissipative quantum parallelism.

2. Second law should always be applied only at a given level of p-adic and dark matter hierarchy and one must always take into account two time scales involved corresponding to the time scale assignable to the system identifiable as the time scale characterizing corresponding CD and the time scale in which the system is observed. Only if the latter time scale is considerably longer than the CD time scale, second law is expected to make sense in TGD framework -this provided one restricts the consideration to the entropic entanglement. The reason is that the Boltzmann equations implying the second law require that the geometric time scale assignable to quantum jump is considerably shorter than the time scale of observation: this guarantees that the random nature of quantum jump allows to use statistical approach.

3. The reduction of entanglement entropy at single particle level implies the increase of thermodynamical entropy at the level of ensemble in the case of entropic non-binding entanglement. This applies to bound state entanglement leading to a generation of entropy at the level of binding systems and a reduction of the contribution of the bound systems to the entropy of the entire system. Note however the emission of binding energy -say in form of photons- could take care of the compensation so that entropy would be never reduced for ensemble. For NE the situation is different.

4. One must be careful in distinguishing between geometric and subjective time. In the case of subjective time the negentropy increases in statistical sense forever unless the CD disappears in some quantum jump (highly non-probable for large enough CDs). If not, then endless evolution at the level of conscious experience is possible in the intersection of real and p-adic worlds and heat death is not the fate of the Universe as in ordinary thermodynamics.

   The arrow of geometric time changes in the state function reduction to the opposite boundary (act of free will of self leading to a death of sub-self) and negentropy increases. This implies that entropy increases in opposite time direction and behaves like syntropy.

5. In thermodynamics the breaking of second law must correspond to the breaking of ergodicity. Spin glasses are non-ergodic systems and TGD Universe is analogous to a 4-D quantum spin glass by the failure of strict non-determinism of Kähler action reflecting itself as vacuum degeneracy. Does the quantum spin glass property of the TGD universe imply the breaking of the second law? Gravitation has been seen as one possible candidate for the breaking second law because of its long range nature. It is indeed classical gravitational energy which distinguishes between almost degenerate spin glass states. The huge value of gravitational Planck constant associated with space-time sheets mediating gravitational interaction and making possible perturbative quantum treatment of gravitational interaction would indeed suggest the breaking of second law in cosmological time scales. For instance, black hole entropy which is inversely proportional to $GM^2/\hbar_{gr}$ would be for the values of gravitational Planck constant involved of the order of unity.

This breaking of ergodicity implied by 4-D spin glass character of TGD might have an interpretation in terms of NE. The non-determinism implying the ergodicity is behind the hierarchy of quantum criticalities in turn realizing NE.

What do experiments say about second law?

That the status of the second law is far from settled is demonstrated by an experiment performed by a research group in Australian National University [D2]. The group studied a system consisting of 100 small beads in water. One bead was shot by a laser beam so that it became charged and was trapped. The container holding the beads was then moved from side to side 1000 times per second so that the trapped bead dragged first one way and then another. The system was monitored and for monitoring times not longer than.1 seconds second law did not hold always: entropy could also decrease.

1. What is remarkable that .1 seconds defines the duration τ of the memetic code word and corresponds to the secondary p-adic time scale $T_p(2) = \sqrt{p}L_p/c$ associated with Mersenne prime $p = M_{127}$ characterizing electron. This correspondence follows solely from the model of genetic code predicting hierarchy of codes associated with $p = 3, 7, 127$ (genetic code), $p = M_{127}$, ... τ should be the fundamental time scale of consciousness. For instance, average alpha frequency 10 Hz corresponds to this time scale and "features" inside cortex representing sensory percepts have average duration of .1 seconds.

   For electrons the CDs would have spatial size $L = 3 \times 10^7$ meters, which is slightly smaller than the circumference of Earth ($L = cT$, $T = .1$ s, the duration of sensory moment) so that they would have a strong overlap. One can of course ask whether this is an accident. For instance, the lowest Schumann frequency is around 7.8 Hz and not far from 10 Hz. What is interesting that Bohr orbit model [K101] predicts that Universe might be populated by Earth like systems having same distance from their Sun (stars with mass near that of Sun are very frequent). Bohr orbitology applied to Earth itself could also lead to the quantization of the radius of Earth.

2. The first observation was made for more than 15 years ago. Even more remarkable is the recent observation that the time scale of CD associated with electron is.1 seconds. Can one assign the breaking of the second law with the field bodies of electrons?

3. The experiment involves also a millisecond time scale. I do not know whether it is essential that the time scale is just this but one can play with the though that it is. Millisecond time scale is roughly the duration of seventh bit of the genetic codeword if its bits correspond to CDs with sizes coming as subsequent octaves of the basic time scale. Millisecond defines also the time scale for the duration of the nerve pulse and the frequency of kHz cortical synchrony.

   At the level of CDs millisecond time scale would correspond to a secondary p-adic time scale assignable to $k = 120$. Only u and d quarks, which appear with several p-adic mass scales in hadron physics and are predicted to be present as light variants also in nuclear physics as predicted by TGD, could correspond to this p-adic length scale: the prediction for their mass scale would be 5 MeV. Does this mean that the basic time scales of living matter correspond directly to the basic time scales of elementary particle physics?

4. A further interesting point is that neutrinos correspond to.1 eV mass scale. This means that the p-adic length scale is around $k = 167$ which means that the corresponding CD has time scale which is roughly 2^{40} times that for electron and corresponds to the primary p-adic length scale of 2.5 μm (size of cellular nucleus) and tothe time scale of 10^4 years. I have proposed that so called cognitive neutrino pairs consisting of neutrino and antineutrino assignable to the opposite throats of wormhole contact could play key a role in the formation of cognitive representations [K91]. This assumption looks now un-necessarily restrictive but one could quite well consider the possibility that neutrinos are responsible for the longest time scales assignable to consciousness for ordinary value of $\hbar$ (not necessarily our consciousness!). Large value of $\hbar$ could make also possible the situation in which intermediate gauge bosons are effectively massless in cell length scale so that electro-weak symmetry breaking would be absent. This would require $\hbar \simeq 2^{33}$. For this value of $\hbar$ the time scale of electronic CD is of the order of the duration of human of human life cycle. This would scale up the

Compton length of neutrino to about 10 kilometers and the temporal size of neutrino CD to a super-cosmological time scale.

12.4.3 NMP And Biology

The notion of self is crucial for the understanding of bio-systems and consciousness. It seems that the NE is the decisive element of life and that one can say that in metaphorical sense life resides in the intersection of real and p-adic worlds.

Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

NMP and negentropic entanglement demanding entanglement probabilities which are equal to inverse of integer, is the starting point. Rational and even algebraic entanglement coefficients make sense in the intersection of real and p-adic words, which suggests that in some sense life and conscious intelligence reside in the intersection of the real and p-adic worlds.

What could be this intersection of realities and p-adicities?

1. The facts that fermionic oscillator operators are correlates for Boolean cognition and that induced spinor fields are restricted to string world sheets and partonic 2-surfaces suggests that the intersection consists of these 2-surfaces.
2. Strong form of holography allows a rather elegant adelization of TGD by a construction of space-time surfaces by algebraic continuations of these 2-surfaces defined by parameters in algebraic extension of rationals inducing that for various p-adic number fields to real or p-adic number fields. Scattering amplitudes could be defined also by a similar algebraic contination. By conformal invariance the conformal moduli characterizing the 2-surfaces would defined the parameters.

This suggests a rather concrete view about the fundamental quantum correlates of life and intelligence.

1. For the minimal option life would be effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests. There are good reasons to expect that only the data from the intersection of real and p-adic string world sheets partonic two-surfaces appears in U-matrix so that the data localizable to strings connecting partonic 2-surfaces would dictate the scattering amplitudes.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving [K5]. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua. Life as a critical phenomenon in the number theoretical sense would be one aspect of quantum criticality of TGD Universe besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K95].

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

Later progress in understanding of quantum TGD allows to refine and simplify this view dramatically. The idea about p-adic-to-real transition for space-time sheets as a correlate for the transformation of intention to action has turned out to be un-necessary and also hard to realize mathematically. In adelic vision real and p-adic numbers are aspects of existence in all length scales and mean that cognition is present at all levels rather than emerging. Intentions have interpretation in terms of state function reductions in ZEO and there is no need to identify p-adic space-time sheets as their correlates.

That only algebraic extensions are possible is of course only a working hypothesis. Also finite-dimensional extensions of p-adic numbers involving transcendentals are possible and might in fact be necessary. Consider for instance the extension containing $e, e^2, .., e^{p-1}$ as units (e^p is ordinary p-adic number. Infinite number of analogous finite-dimensional extensions can be constructed by taking a function of integer variable such that $f(p)$ exists both p-adically and as a real transcendental number. The powers of $f(p)^{1/n}$ for a fixed value of n define a finite-dimensional transcendental extension of p-adic numbers if the roots do not exist p-adically.

Numbers like $log(p)$ and π cannot belong to a finite-dimensional extension of p-adic numbers [K47]. One cannot of course take any strong attitude concerning the possibility of infinite-dimensional extensions of p-adic numbers but the working hypothesis has been that they are absent. The phases $exp(i2\pi/n)$ define finite dimensional extensions allowing to replace the notion of angle in finite measurement resolution with the corresponding phase factors in finite measurement. The functions $exp(i2\pi q/n)$, where q is arbitrary p-adic integers define in a natural manner the physical counterparts of plane waves and angular momentum eigenstates not allowing an identification as ordinary p-adic exponential functions. They are clearly strictly periodic functions of q with a finite value set. If n is divisible by a power of p, these functions are continuous since the values of the function for q and $q + kp^n$ are identical for large enough values of n. This condition is essential and means in the case of plane waves that the size scale of a system (say one-dimensional box) is multiple of a power of p.

NMP and self-organization

NMP leads to new vision about self-organization about which adetailed vision is discussed in [K95]. Here only some key points are emphasized.

1. Dissipation selects the asymptotic self-organization patterns in the standard theory of self-organization and the outcomes are interesting in the presence of energy feed. The feed of energy can be generalized to feed of any kind of quantum numbers: for instance, feed of quantum numbers characterizing qualia. In fact, energy increment in quantum jump defines one particular kind of quale [K49]. Similar picture should apply now.
2. The fundamentally new element is that in ZEO basic objects are pairs of 3-D surfaces at the opposite boundaries of CDs. By holography the basic geometric objects are 4-dimensional or equivalently 3-dimensional. Strong form of holography allows also to identify the objects carrying information about quantum states as string world sheets and partonic 2-surfaces at the boundaries of CD. Self-organization leads to an asymptotic spatio-temporal pattern rather than spatial pattern, behavior or function. This picture is especially useful when one tries to understand morphogenesis and the emergence of functions and behaviors in biology and neuroscience [K88].
3. The notion of self relates very closely to self-organization in TGD framework [K95]. Self assignable to CD is a dissipative structure because it has sub-selves which dissipate quantum parallelly with it. Self as a perceiver maps the dissipation at the level of quantities in the external world to dissipation at the level of qualia in the internal world.
4. Dissipation leads to self-organization patterns and in the absence of external energy feed to thermal equilibrium. Thus thermodynamics emerges as a description for an ensemble of selves or for the time average behavior or single self when external energy feed to system is absent. One can also understand how the dissipative universe characterized by the presence of parameters like diffusion constants, conductivities, viscosities, etc.. in the otherwise reversible equations of motion, emerges. Dissipative dynamics is in a well defined sense the envelope for the sequence of reversible dynamical evolutions modelling the sequence of final state quantum histories defined by quantum jumps.

5. Quantum self-organization can be seen as iteration of the unitary process followed by state function reduction and leads to fixed point self-organization patterns analogous to the patterns emerging in Benard flow. Since selves approach "asymptotic selves", dissipation can be regarded as a Darwinian selector of both genes and memes. Thus not only surviving physical systems but also stable conscious experiences of selves, habits, skills, behaviors, etc... are a result of Darwinian selection.

6. In TGD one must distinguish between two kinds of self organizations corresponding to the entropic bound state entanglement and NE. Biological self-organization could be therefore fundamentally different from the non-biological one. The success of the p-adic mass calculations suggest that elementary particles reside in the intersection of real and p-adic worlds so that one should be very cautious in making strong conclusions. Certainly the intentional, goal-directed behavior of the system in some time scale is a signature of negentropic self-organization but it is difficult to apply this criterion in time scales vastly different from human time scales. It is the field bodies (or magnetic bodies), which can be assigned naturally to CDs which suggests that the negentropic self organization occurs at this level. TGD based vision about living matter actually assumes this implicitly.

7. What is new that even quantum jump itself can be seen as a self-organization process analogous to Darwinian selection, which yields a state containing only bound state state entanglement or NE and representing analog of the self-organization patterns. By macro-temporal quantum coherence effectively gluing quantum jumps sequences to single quantum jump this pattern replicates itself fractally in various time scales. Thus self-organization patterns can be identified as bound states and states paired by a NE and the development of the self-organization pattern as a fractally scaled up version of single quantum jump. Second new element is that dissipation is not mere destruction of order but producer of jewels. A further new element is that dissipation can occur in quantum parallel manner in various scales.

8. The failure of the determinism in standard sense for Kähler action is consistent with the classical description of dissipation. In particular, the emergence of sub-selves inside self looks like dissipation from outside but corresponds to self-organization from the point of view of self. 4-dimensional spin glass degeneracy meaning breaking of ergodicity crucial for self-organization is highly suggestive on basis of the vacuum degeneracy of Kähler action, and this alone predicts ultra-metric topology for the landscape of the maxima of Kähler function defined in terms of Kähler action so that p-adicity emerges naturally also in this manner.

One particularly interesting concrete prediction is that the time scales assignable to CDs come as powers of two. This predicts fundamental frequencies coming as powers of two, and the hierarchy of Planck constants predicts rational or at least integer multiples of these frequencies. Could these powers of two relate to frequency doubling rather generally observed in hydrodynamical self-organizing systems?

Evolution and NMP

Evolution has many facets in TGD framework.

1. A key aspect of evolution relates to the hierarchy of Planck constants labelling a hierarchy of quantum criticalities. The phase transitions reducing criticality, increasing Planck constant, and generatic NE occur spontaneously so that evolution is unavoidable. This is in sharp conflict with the standard belief that life is a thermodynamical fluctuation.

2. In the adelic vision evolution reduce to the increase of the complexity of the algebraic extension of rationals defining the preferred primes which are primes near powers of prime by NMP. The value of n characterizing Planck constant would correspond to the product of ramified primes for a given extension. Infinite primes representing bound states in arithmetic quantum field theory could code for the irreducible polynomials characterizing the basic algebraic extension whose maximal Abelian extension is represented in terms of adeles [K125].

3. A further natural characterization of evolution is in terms of p-adic topology relating naturally to cognition. p-Adic primes near powers of two are favored if CDs have the proposed discrete size spectrum. From the point of view of self this would be essentially cosmic expansion in discrete jumps. CDs and can be characterized by powers of 2 and if partonic 2-surfaces

correspond to effective p-adic p-adic topology characterized by a power of two, one obtains the commeasurability of the secondary p-adic time scale of particle and that of CD in good approximation.

4. The notion of infinite primes motivates the hypothesis that the many-sheeted structure of space-time can be coded by infinite primes [K105]. The number of primes larger than given infinite prime P is infinitely larger than the number of primes than P. The infinite prime P characterizing the entire universe decomposes in a well defined manner to finite primes and p-adic evolution at the level of entire universe is implied by local p-adic evolution at the level of selves. Therefore maximum entanglement negentropy gain for p-adic self increases at least as $log(p)$ with p in the long run. This kind of relationship might hold true for real selves of p-adic physics is physics of cognitive representations of real physics as suggested by the success of p-adic mass calculations. Thus it should be possible to assign definite p-adic prime to each partonic 2-surface.

Just for fun one can play also with numbers.

1. The highest dark matter level associated with self corresponds to its geometric duration which can be arbitrarily long: the typical duration of the memory span gives an idea about the level of dark matter hierarchy involved if one assumes that the time scale.1 seconds assignable to electrons is the fundamental time scale. If the time scale T of human life cycle corresponds to a secondary p-adic time scale then $T = 100$ years gives the rough estimate $r \equiv \hbar/\hbar_0 = 2^{33}$ if this time scale corresponds to that for dark electron. The corresponding primary p-adic time length scale corresponds to $k = 160$ and is 2.2×10^{-7} meters.
2. If human time scale -taken to be $T = 100$ years- corresponds to primary p-adic time scale of electron, one must have roughly $r = 2^{97}$.

I have already discussed the second law in TGD framework and it seems that its applies only when the time scale of perception is longer than the time scale characterizing the level of the p-adic and dark matter hierarchy. Second law as it is usually stated can be seen as an unavoidable implication of the materialistic ontology.

Stable entanglement and quantum metabolism as different sides of the same coin

The notion of binding has two meanings. Binding as a formation of bound state and binding as a fusion of mental images to larger ones essential for the functioning of brain and regarded as one the big problems of consciousness theory.

Only bound state entanglement and NE are stable against the state reduction process. Hence the fusion of the mental images implies the formation of a bound entropic state- in this case the two interpretations of binding are equivalent- or a negentropic state, which need not be bound state.

1. In the case of NE bound state need not be formed and the interesting possibility is that the NE could give rise to stable states without binding energy. This could allow to understand the mysterious high energy phosphate bond to which metabolic energy is assigned in ATP molecule containing three phosphates and liberated as ATP decays to ADP and phosphate molecule. Negentropic entanglement could also explain the stability of DNA and other highly charged biopolymers. In this framework the liberation of metabolic (negentropic) energy would involve dropping of electrons to a larger space-time sheets accompanying the process $ATP \rightarrow ADP + P_i$. A detailed model of this process is discussed in [**?**].

   In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.
2. The formation of bound state entanglement is expected to involve a liberation of the binding energy and this energy might be a usable energy. This process could perhaps be coined as

quantum metabolism and one could say that quantum metabolism and formation of bound states are different sides of the same coin. It is known that an intense neural activity, although it is accompanied by an enhanced blood flow to the region surrounding the neural activity, does not involve an enhanced oxidative metabolism [J6] (that is $ATP \rightarrow ADP$ process and its reversal). A possible explanation is that quantum metabolism accompanying the binding is involved. Note that the bound state is sooner or later destroyed by the thermal noise so that this mechanism would in a rather clever manner utilize thermal energy by applying what might be called buy now–pay later principle.

If these interpretations are correct, there would be two modes of metabolism corresponding to two different kinds of fusion of mental images.

12.4.4 NMP, Consciousness, And Cognition

As already found NMP dictates the subjective time development of self and is therefore the basic law of consciousness. If p-adic physics is the physics of cognition, the most exotic implications of NMP relate to cognition rather than standard physics.

Thermodynamics for qualia

Concerning qualia one can consider two views.

1. If only entropic entanglement is assumed, second law would hold true also at the level of conscious experience of self, which can be seen as an ensemble of its sub-selves assignable to sub-CDs. The randomness of the state function reduction process implies that conscious experience involves statistical aspects in the sense that the experienced qualia correspond to the averages of quantum number and zero mode increments over the sub-selves assignable to sub-CDs. When the number of quantum jumps in the ensemble defining self increases, qualia get more entropic and fuzzy unless macro-temporal quantum coherence changes the situation.
2. ZEO and NE means departure from this picture if sub-CDs can generate NE. This is expected to be true if they overlap if one believes on standard argument for the formation of macroscopic quantum phases. In this case the flux tubes connecting space-time sheets assignable to the sub-CDs would serve as a space-time correlate for the NE.

The basic questions are whether sensory qualia can really correspond to the increments of quantum numbers in quantum jump and whether these quantum jumps are assignable to entropic or negentropic qualia. What is clear that the sensory qualia such as colors are assigned to an object of external world rather predictably. This is not obvious if this process is based on quantum jump.

1. The original view inspired by standard view about state function reduction (positive energy ontology) was that qualia are determined basically as increments of quantum numbers [K49]. In ordinary statistical physics measured quantities would correspond to quantum numbers basically. The basic function of sensory organs would be to map quantum numbers to quantum number increments so that our sensory perception is in reasonable approximation about world rather than changes of the world.
2. Entropic entanglement is reduced to zero in state function reduction for individual sensory receptor and the outcome involves all possible values of quale, say different fundamental colors for which I have proposed a model in terms of QCD color [K49]. If the probability of particular value of quale is much larger than others, one can have statistical ensemble giving rise to predictable quale as ensemble average.

What happens when ZEO based view about state function reduction is adopted?

1. Now sensory mental image corresponds to a sub-self, which in turn corresponds to a repeated state function reduction to a fixed (passive) boundary of sub-CD. Does sub-self without any sub-sub-selves correspond to conscious experience about quantum numbers instead of only change of quantum numbers? One cannot exclude this possibility. For instance, three colored states for quarks would correspond to three fundamental colors for this option.

The alternative possibility is that quantum jumps of sub-selves give rise to the sensory mental images and the increments of quantum numbers define the qualia. Sub-selves without sub-selves would not give rise to sensory qualia. That consciousness involves always change could be seen as a support for this interpretation but one can ask what change is. Does change mean state function reduction in standard sense or does it mean a sequence of repeated state function reductions leaving the passive boundary of CD invariant but inducing sensation about flow of time and sensory experience?

2. In ZEO the increments must correspond to increments of quantum numbers for (say) positive energy part of the state. A sensation of (say) given color requires a continual feed of corresponding quantum number increment to the positive energy part of the system. Some kind of far from equilibrium thermodynamics seems to be necessary with external feed of quantum numbers generalizing the external feed of energy. The capacitor model of a sensory receptor [K49] realizes this idea in terms of generalized di-electric breakdown implying opposite charging of the capacitor plates in question. Note that in ZEO also the positive and negative energy parts of the zero energy state assignable to capacitor plates would be also analogous to a pair of oppositely charged capacitor plates and one can speak about capacitor also in time direction.

3. The flow of quantum numbers can be interpreted also in terms of feed of NE to the sub-CD of sub-self allowing it to satisfy the needs of NMP and avoid the lethal first state function reduction to the opposite boundary of its sub-CD. NE feed would accompany the feed of quantum numbers and would be accompanied by feed of metabolic energy and/or some other metabolite. Energy metabolism is indeed only one particular variant of metabolism. Metabolism would be always basically feed of NE assignable to system with quantum numbers producing the quale assignable to these quantum numbers. Each metabolite carrying NE would define its own quale. One can assign metabolites also to hearing and vision: the metabolite would be assigned with sound waves or photons and carry NE. Whether the metabolite could be identified with dark phonons or photons is an interesting question.

4. Note that in this picture quantum number increment corresponds to that for a subsystem due to the flow of quantum numbers to it rather than to the change of quantum numbers in state function reduction. Hence it is possible to assign qualia also to quantum numbers rather than their increments.

5. Also in this framework the analog of thermodynamical description is suggestive since chemical potentials provide natural thermodynamical description for the numbers of ions. In ZEO square root of thermodynamics is highly suggestive in order to take into account the macroscopic quantum coherence in living systems and I have proposed a model of cell membrane along these lines generalizing the usual thermodynamical model [K67]. Chemical potentials are in this framework replaced with the cyclotron energy differences over cell membrane.

The association of sensory qualia with the transfer of metabolites of various kinds is a powerful prediction and conforms at least in spirit with the early very naïve attempts to identify qualia in terms of biologically important charged particles assumed to form cyclotron condensates at dark magnetic flux tubes. If ATP (and GTP) are the universal carriers of metabolic energy and if energy quale must accompany any quale, one could try to identify the metabolites giving rise to qualia from the biochemistry of the sensory perception. The proposal that nutrients carry NE conforms with this picture.

3. "Final" solution to the problem of qualia

The TGD inspired theory of qualia [K49] has evolved gradually.

1. The original vision was that qualiaandother aspects of consciousness experience are determined by the change of quantum state in the reduction: the increments of quantum numbers would determine qualia. I had not yet realized that repeated state function reduction (Zeno effect) realized in ZEO is central for consciousness. The objection was that qualia change randomly from reduction to reduction.

2. Later I ended up with the vision that the rates for the changes of quantum numbers would determine qualia: this idea was realized in terms of sensory capacitor model in which qualia

would correspond to kind of generalized di-electric breakdown feeding to subsystem responsible for quale quantum numbers characterizing the quale. The Occamistic objection is that the model brings in an additional element not present in quantum measurement theory.

3. The view that emerged while writing the critics of IIT of Tononi [K110] is that qualia correspond to the quantum numbers measured in the state function reduction. That in ZEO the qualia remain the same for the entire sequence of repeated state function reductions is not a problem since qualia are associated with sub-self (sub-CD), which can have lifetime of say about .1 seconds! Only the generalization of standard quantum measurement theory is needed to reduce the qualia to fundamental physics. This for instance supports the conjecture that visual colors correspond to QCD color quantum numbers. This makes sense in TGD framework predicting a scaled variants of QCD type physics even in cellular length scales.

   This view implies that the model of sensory receptor based on the generalization of di-electric breakdown is wrong as such since the rate for the transfer of the quantum numbers would not define the quale. A possible modification of the model simple: the analog of di-electric breakdown generates Bose-Einstein condensate and the quantum numbers for the BE condensate give rise to qualia assignable to sub-self.

Questions about various kinds of entropies and negentropies

In standard positive energy ontology and in absence of dark matter second law is natural for many-particle systems. In ZEO and accepting the hierarchy of dark matters NMP replaces second law and the modification of thermodynamics replaced entropy with negentropy is highly suggestive.

Consider first the situation in positive energy ontology. There are three kinds of entropies and the basic question is how these entropies relate.

1. Does the entropy characterizing the experience of self relate to the thermodynamical entropy of some system? The fact that non-geometric sensory qualia have a statistical interpretation, suggests that the entropy associated with the qualia of the mental image corresponds to the thermodynamical entropy for a system giving rise to the qualia via the sensory mapping. The thermodynamics of quantities in the external world would thus be mapped to the thermodynamics of qualia, increments of quantities, in the inner world. Selves could also represent the fundamental thermodynamical ensembles since they define also statistical averages of quantum numbers and zero modes although these are not directly experienced.

2. Could one interpret the entropies of the space-time sheets as entropies associated with the symbolic representations of conscious experiences of selves? Could one see the entire classical reality as a symbolic representation? Does the entropy of conscious experience correspond to the thermodynamical entropy of the perceived system, which in turn would correspond to the classical space-time entropy of the system representing the perceived system symbolically? Does this conclusion generalize to the case of p-adic entropy? Quantum-classical correspondence would encourage to cautiously think that the common answer to these questions might be yes.

One can repeat these questions almost as such for ZEO option. Now one would only speak about negentropy. Each quale would corresponding to its metabolite and to a chemical potential contribution to the differentials of thermodynamical functions. The thermodynamics of qualia could allow to have quantitative model correlating qualia with chemistry.

The arrow of psychological time and thermodynamics

In positive energy ontology and standard QM the arrow of psychological time is closely related to the second law and I have considered several alternative identifications for the arrow of psychological time. In ZEO [K122, K8] NMP replaces second law and the arrow of psychological time emerges as a prediction of the model for self as sequences of state function reductions to a fixed boundary of CD. The new element is that the arrow of geometric time chan change and that the moments for these changes define increases for the distance between the tips of CD defining a discrete flow of psychological time.

The latest option favored by ZEO involves two aspects. The one related to the arrow of time coordinate assignable to the space-time sheet and the other one to the relative proper time coordinate between the tips of CD. A simple argument show that this distance should increase gradually in statistical sense since the size of CD can also change in quantum jump. This would have have interpretation in terms of a flow of "cosmic time" (CD is analogous to big bang followed by big crunch). Interestingly, CD with time scale of order 10^{11} years (age of the universe) corresponds primary p-adic length scale of only 10^{-4} meters, the size of a large neuron, and also the length scale in which the blob of water has Planck mass so that the quantization of gravitational Planck constant should become important [K101]. Could this mean that the CDs assignable to large neurons make possible to develop the idea about the cosmology and cosmology itself? Could it really be that our cognitive representations about Universe quite concretely have the size of the Universe itself as p-adic view about cognition requires?

Reductionism, holism and NMP

The fusion of sub-selves can take place in two ways: by real bound state entanglement and by NE. The resulting mental images must differ somehow, and the proposal is that the entanglement associated with the negentropic mental defines a conscious cognitive representation: kind of rule. Schrödinger cat negentropically entangled with the bottle of poison knows that it is not a good idea to open the bottle: open bottle-dead cat, closed bottle-living cat (note that thew weak form of NMP allows the cat to open the bottle so that the information is useful!). NE would generate rules and counterparts of conscious associations fundamental in brain functioning. For the mental image associated with bound state entanglement the information about bound systems would be lost. NE could give rise to stereo-consciousness essential for (say) stereo vision.

Analysis and conceptualization (synthesis) - formation of rules- could be seen as the reductionistic and holistic aspects of consciousness. The interpretation of quantum jump as a creation of a totally entangled holistic state, which is then analyzed to stable entangled pieces allows to interpret self measurement cascade as a conscious analysis. The resulting stable negentropic pieces give rise to experience of understanding and conceptualization - rules and abstractions. Perhaps the holistic character assigned to right brain hemisphere could be interpreted in terms of specialization to conceptualization and reductionist character of left brain to analysis to smallest possible pieces. This picture proposed originally in positive energy ontology makes sense also in ZEO.

Could one assume that left brain generates entropic bound state entanglement and right brain NE? This idea not so feasible as it looked originally. The reason is that only NE might be relevant for consciousness.

In ZEO the sequence of state function reductions at passive boundary of CD generates entangled holistic state at the active boundary and the reduction to opposite boundary generates the reductionistic state at it. The two boundaries of CD would seem to correspond to the reductionistic and analytic aspects of consciousness. Again one must be very cautious in making interpretations. One can also consider that holistic state corresponds to NE in long scales and with large h_{eff} whereas the reductionistic state would correspond to short scales and small values of h_{eff}.

If left and right brain work independently they should not differ unless their magnetic bodies are different in the sense that right brain correspond to a large magnetic body and to large values of h_{eff} and left brain to small values of h_{eff}. Could it be that the brain hemispheres work together quantum coherently and this allows specialization?

Could left brain produce less NE than right brain? Could left brain be the bad boy and right brain the saint? Or do both produce NE but that NE corresponds to short p-adic length scales in the case of left brain (dimension d of final state projectior is a large power of small prime p) and long p-adic length scales in the case of right brain hemisphere (d is a small power of large p)?

There are rather interesting connections with altered states of consciousness and states of macro-temporal quantum coherence.

1. Making mind empty of mental images could perhaps be interpreted as a mechanism of achieving irreducible self state. If self entangles negentropically with larger conscious entity this would lead to experiences characterized as expansion of consciousness, even cosmic consciousness. One could also consider the possibility the sub-selves representing mental images fuse to single long-lasting negentropic mental image. The absence of dissipation could relate to the reports of meditators about lowered metabolic needs.

2. The ordinary wake-up consciousness is identifiable as the analytical mode in which NE in short scales dominates. Together with weak form of NMP this would suggest that state function reductions are carried out to rather sub-spaces with rather low dimension or alternatively to sub-spaces for which the dimension is larger power of small prime defining the p-adic length scale. The reason for this could be sensory input and motor activities, which would create effective heat bath destroying holistic mental images.
3. Krishnamurti has talked a lot about states of consciousness in which no separations and discriminations occur and timelessness prevails. These states could correspond to long-lived NE with large $\hbar$ with larger conscious entities giving rise to very long effective moments of consciousness. In this kind of situation NMP does not force cognitive self measurements to occur and analysis and separations can thus be avoided.
4. Sharing and fusion of mental images by entanglement of sub-selves of separate selves makes possible quantum realization of telepathy and could be a universal element of altered states of consciousness. Also this entanglement could be bound state entanglement or NE.

Cognitive codes

p-Adic length scale hypothesis leads to the idea that each $p \simeq 2^k$, k integer, defines a hierarchy of cognitive codes with code word having duration given by the n-ary p-adic time scale $T(n,k)$ and number of bits given by any factor of k. Especially interesting codes are those for which the number of bits is prime factor or power of prime factor of k. $n=2$ seems to be in special position in ZEO. This is a strong quantitative prediction since the duration of both the code word and bit correspond to definite frequencies serving as signatures for the occurrence of commutations utilizing these codes.

If k is prime, the amount of information carried by the codon is maximal but there is no obvious manner to detect errors. If k is not prime there are several codes with various numbers of bits: information content is not maximal but it is possible to detect errors. For instance, $k=252$ gives rise to code words for which the number of bits is $k_1 = 252, 126, 63, 84, 42, 21_2, 9, 7, 6_2, 4, 3_2, 2$: the subscript $_2$ tells that there are two non-equivalent ways to get this number of bits. For instance, $126 = 42 \times 3$-bit codon can have 42 -bit parity codon: the bits of this codon would be products of three subsequent bits of 126-bit codon. This allows error detection by comparing the error codon for communicated codon and communicated error codon.

The recent view about how NMP selects preferred primes [K125] supports this idea. The values of dimension for the sub-space defining the outcome of reduction which are primes near powers of prime p are favoured by NMP. $p=2$ gives the p-adic length scale hypothesis. Large powers of small prime could give rise to cognitive codes. Not that the integers associated with code could also be in the range $[0, p^n]$. For instance, for microtubules $p=13$ is suggestive. For genetic code $p=127$ is suggestive.

Abstraction hierarchy and genetic code

Mersenne primes $M_n = 2^n - 1$, which seem to play fundamental role in elementary particle physics. This would put primes 3, 7, 31, 127, etc. in a special position. Primes appear frequently in various bio-structures and this might reflect the underlying p-adicity for the association sequences providing "plan" for the development of bio-system. For instance, we have actually 7 (!) fingers: two of them have degenerated during evolution but can be seen in the developing embryo. There are 31 subunits in our spinal chord, etc...

As already explained, the emergence of primes near powers of prime can be understood from NMP.

In the model of genetic code based on a simple model of abstraction process [K51] the so called Combinatorial Hierarchy $2, 3, 7, 127, 2^{127}-1, ...$ of Mersenne primes emerges naturally. The construction for a model of abstraction process proceeds as follows.

1. At lowest level there are two digits. The statements Yes and No.
2. At the next level one considers all Boolean statements about these two statements which can be regarded as maps from 2-element set to 2-element set. There are 4 of them. Throw one away and you get 3 statements.

3. At the next level one considers all Boolean statements about these 3 statements and the total number of them is 2^3. Throw one away and you get 7 statements. And so on.

In this case one obtains what might be interpreted as a hierarchy of statements about statements about... The mystery is why one statement must be thrown away at each level of the construction. The answer might relate to a concrete model of quantum computation. The emotional realization of this code in terms of subs-spaces of n-dimensional space allows to understand this. The reason is that the outcome of the state function reduction corresponds to any $n-k$ dimensional sub-space for a fixed choices of basis with $k = 1, ..., n-1$. $k = 0$ is obviously excluded and the number of choices is $2^n - 1$ rather than 2^n.

An open problem is how the emotional realization of Boolean algebra is mapped to its fermionic representation. The task is to map in a natural manner the outcome of the state function reduction to a corresponding many-fermion state (in ZEO it would be pair of many-fermion states at opposite boundaries of CD having opposite quantum numbers). Is it really possible to map different levels (reductions and states) to each other?

Is the sum of p-adic negentropies equal to real entropy?

I ended almost by accident to a fascinating and almost trivial theorem. Adelic theorem for information would state that conscious information represented as sum of p-adic negentropies (entropies, which are negative) is equal to real entropy. The more conscious information, the larger the chaos in the environment as everyone can verify by just looking around.

This looks bad! Luckily, it turned out that this statement is true for rational probabilities only. For algebraic extensions it cannot be true as is easy to see. That negentropic entanglement is possible only for algebraic extensions of rationals conforms with the vision that algebraic extensions of rationals characterize evolutionary hierarchy. The rationals represent the lowest level at which there is zero amount of conscious information.

It is not completely obvious that the notion of p-adic negentropy indeed makes sense for algebraic extensions of rationals. A possible problem is caused by the fact that the decomposition of algebraic integer to primes is not unique. Simple argument however strongly suggests that the various p-adic norms of the factors do not depend on the factorization. Also a formula for the difference of the total p-adic negentropy and real entropy is deduced.

1. p-Adic contribution to negentropy equals to real entropy for rational probabilities but not for algebraic probabilities

The following argument shows that p-adic negentropy equals to real entropy for rational probabilities.

1. The fusion of real physics and various p-adic physics (identified as correlates for cognition, imagination, and intentionality) to single coherent whole leads to what I call adelic physics [K125]. Adeles associated with given extension of rationals are Cartesian product of real number field with all p-adic number fields extended by the extension of rationals. Besides algebraic extensions also the extension by any root of e is possible since it induces finite-dimensional p-adic extension. One obtains hierarchy of adeles and of corresponding adelic physics interpreted as an evolutionary hierarchy.

   An important point is that p-adic Hilbert spaces exist only if one restricts the p-adic numbers to an algebraic extension of rationals having interpretation as numbers in any number field. This is due to the fact that sum of the p-adic valued probabilities can vanish for general p-adic numbers so that the norm of state can vanish. One can say that the Hilbert space of states is universal and is in the algebraic intersection of reality and various p-adicities.

2. Negentropy Maximization Principle (NMP) [K70] is the variational principle of consciousness in TGD framework reducing to quantum measurement theory in Zero Energy Ontology assuming adelic physics. One can define the p-adic counterparts of Shannon entropy for all finite-dimensional extensions of p-adic numbers, and the amazing fact is that these entropies can be negative and thus serve as measures for information rather than for lack of it. Furthermore, all non-vanishing p-adic negentropies are positive and the number of primes contributing to negentropy is finite since any algebraic number can be expressed using a generalization of

prime number decomposition of rational number. These p-adic primes characterize given systen, say elementary particle.

NMP states that the negentropy gain is maximal in the quantum jump defining state function reduction. How does one define the negentropy? As the sum of p-adic negentropies or as the sum of real negative negentropy plus the sum of p-adic negentropies? The latter option I proposed for some time ago without checking what one obtains.

3. The adelic theorem says that the norm of rational number is equal to the product of the inverses of its p-adic norms. The statement that the sum of real and p-adic negentropies is zero follows more or less as a statement that the logarithms of real norm and the product of p-adic norms for prime factors of rational sum up to zero.

   The core formula is adelic formula stating that the real norm of rational number is product of its p-adic norms. This implies that the logarithm of the rational number is sum over the logarithms of its p-adic norms. Since in p-adic entropy assigned to prime p logarithms of probabilities are replaced by their p-adic norms, this implies that for rational probabilities the real entropy equals to p-adic negentropy.

   It would seem that the negentropy appearing in the definition of NMP must be the sum of p-adic negentropies and real entropy should have interpretation as a measure for ignorance about the state of either entangled system. The sum of p-adic negentropies would serve as a measure for the information carried by a rule with superposed state pairs representingt the instances of the rule. The information would be conscious information and carried by the negentropically entangled system.

4. What about probabilities in algebraic extensions? The probabilities are now algebraic numbers. The induced p-adic norm $N_p(x)$ for n-dimensional extension of Q is defined as the determinant $det(x)$ of the linear map defined by multiplication with x. $det(x)$ is rational number. The corresponding p-adic norm is defined as the n:th root $N_p(det(x))^{1/n}$ of the ordinary p-adic norm. Root guarantees that the norm co-incides with the ordinary p-adic norm for ordinary p-adic integers. One must perform now a factorization to algebraic primes. Below an argument is given that although the factorization to primes is not always unique, the product of p-adic norms for given algebraic rational defined as ratio of algebraic integers is unique.

   The p-adic norms of probabilities are however always powers of primes so that the adelic formula *cannot* be true since on the real side one has logarithms of algebraic numbers and on the p-adic side only logarithms of primes.

What could be the interpretation?

1. If conscious information corresponds to $N - P$, it accompanies the emergence of algebraic extensions of rationals at the level of Hilbert space.

2. If N corresponds to conscious information, then at the lowest level conscious information is necessary accompanied by entropy but for algebraic extensions $N - P$ could be positive since N is maximized.

Both interpretations conform with the number theoretic vision about evolution. One expects that the value of real entropy correlates strongly with the value of negentropy. This would conform with the observation that large entropy seems to be a prerequisite for life by providing large number of states with degenerate energies providing large representative capacity. For instance, Jeremy England has made this proposal [**?**]: I have commented this proposal from [L21] (see `http://tinyurl.com/zjp3bp6`).

2. Formula for the difference of total p-adic negentropy and real entanglement entropy

Can one write an explicit formula the difference of total p-adic entanglement negentropy (positive) and real entanglement entropy using prime factorization in finite dimensional algebraic extension (note that for algebraic numbers defining infinite-dimensional extension of rationals factorization does not even exist since one can write $a = \sqrt{a}\sqrt{a} = ...$)? This requires that total p-adic entropy is uniquely defined. There is a possible problem due to the non-uniqueness of the prime factorization.

1. For Dedekind rings, in particular rings of integers, there exists by definition a unique factorization of proper ideals to prime ideals (see `http://tinyurl.com/h3oufpp`). In contrast, the prime factorization in the extensions of Q is not always unique. Already for $Q(\sqrt{-5})$ one has $6 = 2\times 3 = (1+\sqrt{-5})(1-\sqrt{-5})$ and the primes involved are not related by multiplication with units.

   Various factorizations are characterized by so called class group and class field theory (see `http://tinyurl.com/zdnw7j3`) is the branch of number theory studying factorizations in algebraic extensions of integer rings. Factorization is by definition unique for Euclidian domains. Euclidian domains allow by definition so called Euclidian function $f(x)$ having values in R_+ with the property that for any a and b one has either $a = qb$ or $a = qb + r$ with $f(r) < f(b)$. It seems that one cannot restrict to Euclidian domains in the recent situation.

2. Even when the factorization in the extension is not unique, one can hope that the product of various p-adic norms for the factors is same for all factorizations. Since the p-adic norm for the extensions of primes is induced by ordinary p-adic number this requires that the p-adic prime for which the induced p-adic norm differs from unity are same for all factorizations and that the products of p-adic norms differing from unity are same. This independence on the representative for factorization would be analogous to gauge invariance in physicist's conceptualization.

   The probabilities P_k belongs to a unique product of ideals labelled by primes of extension. The ideals are characterized by norms and if this norm is product of p-adic norms for any prime factorization as looks natural then the independence on the factorization follows. Number theorist can certainly immediately tell whether this is true. What is encouraging that for $Q(\sqrt{-5})$ $z = x+\sqrt{-5}y$ has determinant $det(z) = x^2 + 5y^2$ and for $z == 1 \pm \sqrt{-5}$ one has has $det(z) = 6$ so that for the products of p-adic norms for the factorizations $6 = 2 \times 3$ and $(1 + \sqrt{-5})(1 - \sqrt{-5})$ are equal.

3. If this this guess is true, one can write the the difference of total p-adic negentropy N and real entanglement entropy S as

$$N - S = \sum P_k log(\frac{P_k}{\prod_p N_p(P_k)}) \ . \tag{12.4.1}$$

   Here $\prod_p N_p(P_k)$ would not depend on particular factorization. The condition $\sum P_k = 1$ poses an additional condition. It would be nice to understand whether $N - S \geq 0$ holds true generally and if not, what are the conditions guaranteeing this. The p-adic numbers of numerators of rationals involved give positive contributions to $N-S$ as the example $P_k = 1/N$ in rational case shows.

3. An ansatz for entanglement probabilities guaranteeing $N - S > 0$

What about entanglement probabilities in algebraic extension of rationals? In this case real number based entanglement entropy is not maximal since entanglement probablities are different. What can one say about p-adic entanglement negentropies: are they still maximal under some reasonable conditions? The logarithms involved depend on p-adic norms of probabilities and this is in the generic case just inverse of the power of p. Number theoretical universality suggests that entanglement probabilities are of form

$$P_i = \frac{a_i}{N}$$

with $\sum_i a_i = N$ with algebraic numbers a_i not involving natural numbers and thus having unit p-adic norm.

With this assumption the p-adic norms of P_i reduce to those of $1/N$ as for maximal rational entanglement. If this is the case the p-adic negentropy equals to $log(p^k)$ if p^k divides N. The total adelic negentropy equals to $log(N)$ and is maximal and has the same value as for rational probabilities equal to $1/N$.

The real entanglement entropy is now in general however smaller than $log(N)$, which would mean that p-adic negentropy is larger than the real entropy as conjectured earlier [K73] (see `http:`

//tinyurl.com/jozwqxk). For *rational* entanglement probabilities the generation of entanglement negentropy - conscious information during evolution - would be accompanied by a generation of equal entanglement entropy measuring the ignorance about what the negentropically entangled states representing selves are.

This conforms with the observation of Jeremy England that living matter is entropy producer [L33] (see http://tinyurl.com/jff33xk). For algebraic extensions of rationals this entropy could be however smaller than the total negentropy. Second law follows as a shadow of NMP if the real entanglement entropy corresponds to the thermodynamical entropy. Algebraic evolution would allow to generate conscious information faster than the environment is polluted, one might concretize! The higher the dimension of the algebraic extension rationals, the larger the difference could be and the future of the Universe might be brighter than one might expect by just looking around! Very consolating! One should however show that the above described situation can be realized as NMP strongly suggests before opening a bottle of champaigne.

4. Cloning of maximally negentropic states is possible: DNA replication as cloning of this kind of states?

In Facebook discussion with Bruno Marchal and Stephen King the notion of quantum cloning as copying of quantum state popped up and I ended up to ask about approximate cloning and got a nice link about which more below. From Wikipedia article (see http://tinyurl.com/oyvklde) one learns some interesting facts cloning. No-cloning theorem states that the cloning of *all* states by unitary time evolution of the tensor product system is not possible. It is however possible clone *orthogonal basis of states*. Does this have some deep meaning?

As a response to my question I got a link to an article of Lamourex *et al* (see http://tinyurl.com/zq4kgda) showing that the *cloning of entanglement* - to be distinguished from the cloning of quantum state - is not possible in the general case. Separability - the absence of entanglement - is not preserved. Approximate cloning generates necessarily some entanglement in this case, and the authors give a lower bound for the remaining entanglement in case of an unentangled state pair.

The cloning of maximally entangled state is however possible. What makes this so interesting is that maximally negentropic entanglement for *rational* entanglement probabilities in TGD framework corresponds to maximal entanglement - entanglement probabilities form a matrix proportional to unit matrix- and just this entanglement is favored by Negentropy Maximization Principle. Could maximal entanglement be involved with say DNA replication? Could maximal negentropic entanglement for algebraic extensions of rationals allow cloning so that DNA entanglement negentropy could be larger than entanglement entropy?

The impossibility of cloning of entanglement in the general case makes impossible the transfer of information as any kind of entanglement. Maximal entanglement - and maybe be even negentropic entanglement maximal in p-adic sectors - could however make the communication without damaging the information at the source. Since conscious information is associated with p-adic sectors responsible for cognition, one could even allow the modification of the entanglement probabilities and thus of the real entanglement entropy in the communication process since the maximal p-adic negentropy depends only weakly on the entanglement probabilities.

NE is assigned with conscious experiences with positive emotional coloring: experience of understanding, experience of love, etc... There is an old finnish saying, which can be translated to "Shared joy is double joy!". Could the cloning of NE make possible generation of entanglement by loving attitude so that living entities would not be mere thieves trying to steal NE by killing and eating each other?

12.4.5 NMP And Quantum Computer Type Systems

In ZEO there are 3 basic matrices. U-matrix between zero energy states, M-matrices and defining entanglement coefficients between positive and negative energy states at opposite boundaries of CD. The mutually orthonormal M-matrices are assumed to be expressible as "square root" of a density matrix expressible as a product of a hermitian diagonal square root of density matrix and unitary S-matrix. Quantum theory can be see as a square root of thermodynamics in this framework.

The original mathematically attractive hypothesis that U-matrix has M-matrices as its rows turned out to be wrong. The physical picture about U-matrix as unitary matrix between states represented by M-matrices allowing also dispersion in the moduli space for the CDs with the passive boundary fixed leads with rather general assumptions to the identification of U-matrix as a representation for the unitary scalings of CDs [K75].

This is the original picture and every updating forces to challenge the earlier picture.

1. What about M-matrix? Can one really regard it as an orthonormal row of U or is M completely free? The defence for this assumption is that the orthonormality for hermitian square roots of density matrix is extremely powerful constraint. M-matrices could correspond to orthonormal basis of hermitian generators of some symmetry algebra. If symmetry algebra commutes with the S-matrix, the square roots of density matrices would be invariant under S-matrix. This assumption might be however physically unfeasible. Besides the hermitian basis one the degrees of freedom defined by the discrete moduli of CD with second (passive) boundary fixed would label M-matrices.
2. Weak form of NMP suggests that TGD Universe can be regarded as a quantum computer. CD as quantum computer is a local version of the same idea. Unitarity process U could relate closely to quantum computation. The state function reduction process represents a stepwise halting of the computation proceeding to shorter scales (sub-CDs) until the resulting states are either bound states or negentropically entangled states.
3. The question is whether it is U-matrix or M-matrix, which defines quantum computation. In other words, which kind of transitions do the repeated state functions leaving the passive boundary of CD and states at it invariant, correspond? It would seem that U-matrix is the correct identification since a repeated de-localization in the moduli space of CDs followed by a localization (but no reduction) is involved at the active boundary of CD. Note that the moduli of CD consist of discrete Lorentz boosts and proper time translations for CD. The argument of [K75] suggests that S-matrix reduces to a unitary representation for the scalings of CD by a power of S-matrix assignable to the CD with minimal size: $S(n) = S^n$, where n characterizes the size scale of CD as a temporal distance between its tips. This makes possible quantum computations.
4. In ZEO quantum computation can be associated with the sequence of repeated reductions at fixed boundary of CD defining self. NE would be realized in terms of an entanglement characterized by a multiple of unitary matrix for a two-body system at the passive boundary of CD and would be stable during computation. The computation would end with a state function reduction at opposite boundary giving the outcome of the process. It could be a state with higher entanglement negentropy but weak form of NMP allows also ordinary state function reduction. Strong form of NMP would make the halting of the quantum computation impossible. Bio-systems would be especially attractive candidates for performers of quantum computation like processes.
5. The action of U-matrix in this picture would be trivial at the passive boundary and affect only the moduli of the upper boundary and the M-matrices. U-matrix cannot be however the direct counterpart of the ordinary S-matrix since there is non-trivial action in the moduli space of CDs. The matrix elements of U-matrix are however expressible in terms of S-matrix and the orthonormal basis of hermitian square roots of density matrices.

It is useful to list the basic differences with respect to ordinary quantum computation. Ordinary quantum computation utilizes unitary evolution of quantum states in positive energy ontology. In this case quantum coherence is extremely fragile. In TGD framework NMP and ZEO allow to circumvent this problem. The outcome of the computation is however realized at the level of dark matter unless ordinary state function reduction takes place. This means that n reductions from $2^n - 1$ correspond to computations, whose outcome can be verified with the existing technology. A further new element is that the computation is conscious and this aspect might be the important one in living matter.

How quantum computation in ZEO differs from ordinary quantum computation

Quantum computation in ZEO differs in several respects from ordinary quantum computation.

1. The time parameter defining quantum computation as a unitary time evolution in standard quantum physics disappears and corresponds to the U-matrix for single repeated reduction followed by a localization in moduli of the active boundary of CD (but no reduction at the active boundary). Large number of these steps occur. This process occurs for sub-CDs of given CD and the outcome of the quantum computation as seen by CD would be determined statistically from the distribution of the outcomes of state function reductions for over sub-CDs.

   One can assign to the quantum computation a duration equal to the increase of the proper time distance between the tips of CD. For instance, .1 seconds could be the time scale assignable to quantum computations possibly assignable to electrons.

   The hierarchies of CDs and Planck constants make possible zoomed up variants of quantum computations. This kind of zooming might be essential for intelligent behavior since it is useful to simulate dynamics of the external world in the time scales natural for brain and shorter than the time scale during which it is necessary to react in order to survive. The geometric duration of the shortest possible quantum computation is of order CP_2 time about 10^4 Planck times, if the simplest estimate is correct.

2. The classical space-time correlates for the quantum computation are four-dimensional unlike in the case of ordinary quantum computation. In living matter nerve pulses and EEG frequencies would be very natural correlates of this kind. The model for DNA as topological quantum computer [K5] has as its space-time correlates magnetic flux tubes connecting DNA nucleotides and lipids of nuclear and cell membranes defining the braiding coding for the topological quantum computation. Dynamical flow of lipids defines the braiding in time direction and the memory representation is in terms of the braiding of the flux tubes induced by this flow. A good metaphor is in terms of dancers connected to a wall by threads. Dancing is the correlate for the running quantum computer program and the geometric entanglement of threads the correlate for the storage of the program to computer memory.

3. The outcome of quantum computation is described statistically in terms of a large set of quantum computations. The statistical description of the conscious experience of ensemble of sub-selves implies that mathematically the situation is very much analogous with that encountered in the standard quantum computation and it is attractive to assume that conscious experience codes for the outcome of quantum computation via the average quantities assignable to the distribution of zero energy quantum states assignable to sub-CDs.

4. A further new element is macro-temporal quantum coherence involving several aspects. One of these aspects is that the time scale of CD defines macrotemporal quantum coherence at least at the level of the field body assignable to the physical system such as electron. It is not quite clear whether electrons correspond to distinct overlapping CDs of size scale defined by.1 second time scale and of the order of Earth circumference and thus satisfying the basic criterion of quantum coherence or whether one should speak about anyonic many particle states assignable to single CD or whether both interpretations can make sense depending on situation.

   In living matter also millisecond time scale is important and would correspond naturally to the CDs assignable to u and d quarks in nuclei and perhaps also with the ends of magnetic flux tubes in the model of DNA as topological quantum computer. In the proposed model quarks and antiquarks at the ends of flux tubes represent genetic codons and their entanglement is responsible for the realization of the program at quantum level. The millisecond time scale of synchronous cortical firing and of nerve pulse could correspond to the time scale of CDs associated with u and d quarks at the ends of the flux tube. Note that larger value of $\hbar$ would scale up this time scale. Quantum parallel dissipation taking place at various size scales for CD is a further new element.

5. One must generalize the standard quantum computer paradigm since ordinary quantum computers represent only the lowest, 2-adic level of the p-adic intelligence. This suggests that qubits must be replaced by qupits since for algebraic entanglement two-state systems are naturally replaced with p-state systems. For primes of order say $p \simeq 2^{167}$ (the size of small bacterium) this means about 167 bits, which would mean gigantic quantum computational resources. The secondary p-adic time scale $T_2(127) \simeq .1$ seconds basic bit-like unit corresponds

to $M_{127} = 2^{127} - 1$ M_{127}-qupits making about 254 bits. The size of neuron corresponds to CD with time scale equal to the age of the universe and in this case the maximum the number of pinary digits is 171.

The finite measurement resolution for qubits of course poses strong limitations to the actual number of bits since the negentropic zero energy qubits must be in reasonable approximation pure qubits distinguishable from each other and could correspond CDs with time scales coming as powers of two from $n = k_{min}$ to k so that the effective number of qubits would go like 2-based logarithm of the p-adic prime. For instance, electron could correspond to six bits assignable to genetic code plus parity bit corresponding to time scale range from 1 ms to 100 ms. In any case the idea about neuron as a classical bit might be completely wrong!

6. Spin glass degeneracy also provides the needed huge number of degrees of freedom making quantum computations very effective. These degrees of freedom are associated with the flux tubes -say magnetic flux tubes- and are essentially gravitational so that a connection with Penrose-Hameroff hypothesis suggests itself. The space-time sheets mediating gravitational interaction are predicted to have a huge gravitational Planck constant $\hbar_{gr} = GMm/v_0$, $v_0/c < 1$, particles at these space-time sheets are predicted to have huge Compton wavelengths and the plausible looking identification is in terms of dark energy [K101, K83]. This would make quantum computation like activities possible in super-astronomical time scales.

Negentropic quantum computations, fuzzy qubits, and quantum groups

1. The possibility of NE is certainly the basic distinction making in the intersection of real and p-adic worlds possible a conscious process at least analogous to a quantum computation and accompanied by a conscious understanding. What makes this possible is the fact that the negentropically entangled states of N basic states have permutation of the basis states as a symmetry. For instance, states for which bit 1 appears with almost unit probability gives by permutation a state for which bit 0 appears with almost unit probability. This suggests that the outcome of quantum computation is expressed in terms of almost bits with a small mixing implying that the outcome has interpretation both as a rule and as almost bit in the ordinary sense. The conscious quantum computation would utilize states with NE in time direction. Also the analogies of bound states for time-like entanglement are possible and might make possible the counterpart of ordinary quantum computation without the higher level conscious experience about rules defined by the entangled states.

2. NE for positive and negative energy parts of bits stable and pinary digits stable under NMP means that the logic is always fuzzy. I have proposed the mathematical description of this in terms of quantum spinors for which the components do not commute anymore implying that only the probability for either spin state is is an observable [K126]. This suggests that NE might be describable in terms of quantum spinors and that it would be the unavoidable fuzziness which would make possible the representation conscious rules. What is interesting that for quantum spinors the spectrum of the probabilities for given spin is universal and depends only on the integers characterizing the quantum phase $q = exp(i2\pi/n)$. An alternative interpretation is that fuzzy logic relates to a finite measurement resolution. These interpretation need not be in conflict with each other. Since quantum groups are associated with anyonic systems, this suggests that negentropic quantum computations take place in anyonic systems assignable to phases with large value of $\hbar$. This encourages to consider the possibility that quantum phases define algebraic extensions of p-adic numbers.

3. In living systems it might be more appropriate to talk about conscious problem solving instead of quantum computation. In this framework the periods of macro-temporal quantum coherence replace the unitary time evolutions at the gates of the quantum computer as the basic information processing units and entanglement bridges between selves act as basic quantum communication units with the sharing of mental images providing a communication mode not possible in standard quantum mechanics.

12.4.6 Quantum Measurement And Quantum Computation In TGD Universe

It is interesting to test how the view about quantum computation must be modified in TGD Universe. There are considerable deviations from the standard view. Zero Energy Ontology (ZEO), weak form of NMP dictating the dynamics of state function reduction [K70], negentropic entanglement, and hierarchy of Planck constants [?] define the basic differences between TGD based and standard quantum measurement theory. TGD suggests also the importance of topological quantum computation (TQC) like processes with braids represented as magnetic flux tubes/strings along them.

The natural question is how NMP and Zero Energy Ontology (ZEO) could affect the existing view about TQC. The basic observation is that the phase transition to dark matter phase reduces dramatically the noise affecting quantum quits. This together with robustness of braiding as TQC program raises excellent hopes about TQC in TGD Universe. The restriction to negentropic space-like entanglement (NE) defined by a unitary matrix is something new but does not seem to have any fatal consequences as the study of Shor's algorithm shows.

NMP strongly suggests that when a pair of systems - the ends of braid - suffer state function reduction, the NE must be transferred somehow from the system. How? The model for quantum teleportation allows to identify a possible mechanism allowing to achieve this. This mechanism could be fundamental mechanism of information transfer also in living matter and phosphorylation could represent the transfer of NE according to this mechanism: the transfer of metabolic energy would be at deeper level transfer of negentropy.

NE defines an excellent candidate for an analog of error correcting code. If only the diagonal form of the unitary entanglement matrix carries information, the quantization of phases as roots of unity provides a scenario in which Nature itself would take care of error correction.

ZEO based quantum measurement theory

Consider first the quantum measurement theory based on ZEO.

1. Sub-system–complement pair defining larger system defines the counterpart for the pair observer-measured system in standard quantum measurement theory. In TGD framework density matrix for a sub-system–complement pair defines the universal observable. As a matter of fact, for a given system all sub-system-complement pairs defining possible splitting of this kind and the state function reduction is realized for the pair giving rise to maximum of maximal negentropy gain (NMP). A further essential assumption is that the reduction proceeds from a system inside CDE to subsystems as a cascade obeying this basic rule.

2. ZEO implies that state function reductions occur at either boundary of causal diamond (CD) - the active boundary. The sequence of reductions leaving passive boundary and state at it unaffected gives rise to a conscious entity - self. What is new that at the active boundary the state changes. Even the active boundary itself drifts to the geometric future so that the size of CD increases. This gives rise to the experience about flow of time.

   This is the TGD counterpart for the unitary time evolution and its duration corresponds to the increases of the proper time distance between the tips of CD. Eventually NMP forces the first state function reduction to the opposite boundary: this corresponds to a genuine state function reduction. The self dies and re-incarnates at the opposite boundary as time reversed self since CD increases after than at the opposite boundary to the direction of geometric past.

   In the standard quantum models for quantum computation one assumes that measurement can be realized by some interaction Hamiltonian: the state of entangled system-observer pair develops to an eigen state of the interaction Hamiltonian. The time development by this interaction Hamiltonian gives entangled state defined by the density matrix. This description can be seen as an approximation to TGD based description in which one can assign definite duration to the analog of the unitary evolution.

3. Negentropic entanglement (NE) is possible for entanglement coefficients in algebraic extension of rationals since in this case number theoretic entropy having negative values is well-defined. If the density matrix does not belong to the same algebraic extension, state function reduction requires a phase transition extending the algebraic extension of rationals used and could be

seen as kind of evolutionary jump. This kind of NE could be therefore rather stable and could be interpreted as a kind of cognitive entanglement representing a rule with instances represented as state pairs in the superposition. If the state function reduction occurs it leads to a ray of state space if density matrix is non-degenerate.

If the density matrix contains as a direct summand a higher-dimensional projector, a reduction giving rise to a projector to this sub-space is allowed by the interpretation as measurement of density matrix producing its eigen space. The state remains negentropically entangled by the unitary matrix giving rise to the projector. Weak form of NMP [K70] however allows reductions also to the subspaces of this sub-space assuming preferred state basis so that also the reduction to a ray of state space is possible as a special case. In this case any state basis is eigenbasis for the sub-space and this suggests an interpretation in terms of meditative states in which distinctions disappear.

TQC in TGD

How could (topological) quantum computation be realized in TGD framework?

1. In standard quantum theory unitary time evolution realizes the quantum computation. Unitary time evolution is engineered in terms of gates performing standardized operations for qubits. For TQC braiding defines the space-time entanglement between the systems A and B at the ends of the braid. Call this system $A \otimes B$. One can speak about evolution a kind of "space-like" topological quantum computer program with negentropically entangled "initial" and "final" states at the ends of the braid. Basic braiding operation defines the basic gate in terms of so called R-matrix and the desired NE can be build using an appropriate braiding. For the sake of concreteness the following considerations assume TQC. In fact, if there is entanglement between ends, it must be unitary entanglement since only this entanglement is respected by NMP.

2. In TQC the program is defined by braid and is robust against perturbations. The quantum states at the ends of the braid are however sensitive to noise and this requires complex error correction procedures to eliminate the errors, which are basically spin flip changing the value of qubit and change of its phase. If only phase $\pm$ 1 is allowed phase change actually reduces to spin flip in suitable basis.

   In standard quantum computation the small value of Planck constant is the basic problem. Coherence times tend to be very short and the control of external noise is a tough challenge. In TGD quantum criticality gives rise to phases of matter with effective value $h_{eff} = n \times h$ of Planck constant identified as dark matter. These phases are involved also with NE. Only systems with same value of $h_{eff} = n \times h$ have direct interactions with each other. This should dramatically reduce the noise since visible matter particle must transform to dark matter particle to interact directly with dark matter to produce noise. Also the scaling up of interaction time scales gives hopes that quantum coherence times are long enough to perform TQC.

3. The value of h_{eff} is expected to correlate with the duration of self defined as the increase ΔT of the temporal distance during the sequence of state function reductions to the same passive boundary of CD. ΔT could be interpreted as quantum coherence time. Coherence time for classical fields could be identified as the temporal distance between the tips of CD increasing during quantum computation.

4. TGD promises to guarantee the reduction of noise in terms of darkness of the particles involved with the computation: this instability is the weakness of TQC although TQC program itself is robust. TGD also promises the understanding of the role of quantum criticality in quantum measurement. The very fact that quantum measurements necessarily involve the amplification of small quantum effects to macroscopic "classical" effect, indeed strongly suggests quantum criticality.

5. The key challenge is to prepare a desired kind of negentropically entangled state - say a dark many-particle state associated with a braid system. One should be able to manipulate of dark matter, which we are not yet able to even detect! That dark matter appears at quantum criticality could be extremely helpful in the attempts to get grasp on the dark matter. A

simple clue is that the disappearance of visible matter could serve as a signature for the emergence of dark matter.

One should somehow be able to perform state function reduction of the negentropically entangled system to one of the eigenstates of the density matrix associated with an entanglement matrix proportional to a unitary matrix (in the following I will speak of unitary entanglement). This requires TGD counter part of time evolution. One can imagine two options.

1. One can couple the negentropically entangled system pair AB to a measurement apparatus C, whose function is to develop ordinary entanglement with both systems during the repeated sequence of state function reductions at fixed boundary. In the state function reduction to the opposite boundary a time reversed reduced state results and gives rise to rays of state space for both A and B. One can however argue that the situation cannot be so simple: NMP requires that entanglement negentropy increases so that NE should be transferred somewhere. This will be discussed below.
2. The measurement interaction must be able to achieve ordinary state function reduction by generating entanglement with the system formed by negentropically entangled system. One must have interaction between ordinary and dark matter and this requires transformation of ordinary matter to dark matter with the same value of h_{eff}. Quantum criticality allows the transformation of ordinary matter to dark matter so that the measuring system should be quantum critical [**?**].
3. Could one do without a third system? Weak form of NMP allows also a reduction to the lower dimensional sub-spaces of the N-D sub-space considered and also 1-D ray is possible. This process corresponds to a duration of single self, which dies when the first reduction to the opposite boundary of its CD occurs. If the braid system is not changed in the state function to the opposite boundary one can hope that a reduction to a 1-D ray can occur with some probability. By waiting long enough one can obtain state function reductions which determine the probabilities for the reduction to a given ray or sub-space. The important difference to the standard picture would be that the system does it itself. No external measurements at the end of braid would be carried out. This is however too good to be true. Only one of the two quantum measurements required by Shor algorithm can be both carried out in his manner.

The interpretation in terms of consciousness theory allows also to consider the possibility that the measurement corresponds at deeper level to transfer of negentropic entanglement.

1. One has besides AB also the third system C. The NE for AB is transferred to NE for AC and can be transferred further - say to entanglement to NE for CD. In TGD framework the iteration of this process makes possible a transfer of conscious information associated with NE for AB to that of conscious observer.
2. If the state of C is eigenstate of spin in the basis used, the final state of B is also an eigenstate of spin. Hence the transfer of NE could be thus interpreted as a measurement of the state of B or as the measurement of state of AC in Bell basis. This conforms with the fact that state function reduction for a subsystem can be interpreted as a state function reduction for its complement. Could the deeper interpretation of quantum measurement be as a transfer of NE so that essentially quantum information theory would be in question.
3. The measurement is performed for the negentropically entangled Bell states for the pair AC and performs the transfer of entanglement inducing a unitary rotation. Since in the case of NE defined by a unitary matrix any state basis is allowed, one could ask whether the outcomes are equivalent from the point of view of consciousness theory at least. The knowledge of the final state of B allows to deduce the unitary rotation needed to rotate AC state to the original AB state so that this information is enough to realize a faithful NE transfer. Since the conscious experience is dictated both by the bit telling the state of B and by the state of AC one can ask whether the conscious experience and is same for all four outcomes.

Where and how the NE could be transferred?

NMP demands that entanglement negentropy increases. An interesting question is, where and how the entanglement negentropy is transferred.

1. Does NE correspond to information transferred to the performer of quantum measurement? If so, the quantum measurement process would be basically transfer of information realized as NE. Living systems would be carrying out this all the time and ATP-ADP transformation defining the basic step of energy metabolism would be just this kind of transfer. The transfer corresponds at the level of space-time geometry the transfer of the end of magnetic flux tubes plus particles from a donor to the acceptor.

2. A possible manner to carry out the transfer of negentropic entanglement is inspired by the quantum teleportation protocol (`http://tinyurl.com/omfkydh`). In the simplest situation this protocol is as follows. Alice wants to send qubit C to Bob. A Bell state (`http://tinyurl.com/z9g8rar`) is shared between Alice and Bob by mutual agreement in advance so that both know it. Alice can achieve the teleportation by a quantum measurement in the tensor product of the qubit C with the AB Bell state.

   Alice reduces the system AC to one of the four Bell states and communicates the result classically to Bob. The factored out state of B is the original state or one of three states related to it by unitary rotation. Alice sends classically two bits telling what the measurement outcome was. If the outcome was the original state to be sent, Bob does nothing. If it was one of the three remaining states, Bob performs a unitary rotation giving as a result the original state.

3. What makes this protocol so interesting is that in the reduction the NE for AB is transferred to NE for AC as such or modified by a unitary rotation so that four different outcomes are possible. Since the states of C and AB are in 1-1-correspondence it is indeed obvious that the information about the state of B resulting from the measurement of Alice allows the rotation of the Bell state AC to the original state AB. For instance, if the state of B is the original state of C, the state is the original state AB.

   One can apply this procedure by introducing four system D - call it Doris - so that AC NE is transferred to CD NE and AB is now product state. This kind of transfer of negentropic entanglement might be a key event in in phosphorylation and in the utilization of metabolic energy coming from nutrients. The NE between phosphate P of ATP ==B and third system A would be transferred to NE between acceptor molecule and C and A. Also the NE between nutrient B and third system A could bnd third system A could be transferred to NE between phosphate and A.

Shor's algorithm from TGD point of view

Is the unitary of the entanglement matrix guaranteeing NE too strong an assumption? Just for fun I m (`http://tinyurl.com/ppxvcsd`) for the factorization of a given integer, call it N, which has been shown to work for $N = 15$. It turns out that unitary entanglement is not a problem. Furthermore, ordinary quantum measurements are needed for the two systems involved and require interaction coupling negentropically entangled pair of systems to external world so that both negentropically entangled systems generate entanglement with external world.

Consider now the Shor's algorithm. The genuinely quantal step of algorithm is that of finding the period r of the function $f(x) = a^x \ mod \ N$, for integers $1 < a < N$ and $1 < x < N$.

1. According to the Wikipedia article , the computation involves the construction of quantum function $f(x) = a^x$ as

$$\frac{1}{Q}\sum_x |x, f(x)\rangle \quad .$$

   Here Q is normalization factor. Since $a^r = 1 \ mod \ N$, $f(x)$ is not a bijection. Unless r divides Q (we do not however know $r!$), the number $N(z)$ of values of x satisfying $f(x) = z$ varies and the variation is one unit at most. Therefore the entanglement is not unitary and the density matrix of the state is not unit matrix since the norms of states

$$|Z| = \sum_x |x, f(x) = z\rangle$$

is given by $N(z)$- the number of x mapped to z and varies somewhat. NE would be obtained by normalizing the states $|Z\rangle$ to unit norm and replacing Q by the the number $N(Z)$ of points z to get

$$\frac{1}{\sqrt{N(Z)}}\sum_z \frac{1}{\sqrt{N(z)}}\sum_x |x, f(x) = z\rangle \ .$$

2. Second step in the computation is discrete quantum Fourier transform using as counterparts of plane waves powers of the root of unity defined as $\omega = exp(i2\pi/Q)$, where Q satisfies $N^2 \leq Q < 2N^2$. This operation is unitary and gives rise to entanglement matrix proportional to a unitary matrix. Since the entire entanglement matrix is product of unitary matrices, it is also unitary. The action of unitary transformation is given for given value of z by the following formula.

$$\sum_x |x, f(x) = z)\rangle \rightarrow \sqrt{1}\sqrt{N}\sum_y \sum_{z=f(x)} \omega^{xy}|y, z = f(x)\rangle \ .$$

The entire state is transformed to

$$\frac{1}{\sqrt{N(Z)}}\frac{1}{\sqrt{N}}\sum_z \frac{1}{\sqrt{N(z)}}\sum_y \omega^{xy}\sum_x |y, z = f(x)\rangle \ .$$

In this expression the state paired $|Z\rangle$ is a superposition of several values of y since the number of different values of z is smaller than those of y by a factor which in ideal situation is the sought four value of r.

3. Quantum measurement should reduce this state to a state with fixed values of y and z. This implies that the normalization factors do not matter. Weak NMP allows a self-reduction a state Z with fixed value of z. The self reduction of the system is however not able to reduce the state Z to $|y, z\rangle$.

   One must couple at least the "y" part of the system to external measurement apparatus generating ordinary or negentropic entanglement with non-degenerate density matrix belonging to the extension used and having $|y\rangle$ as eigenstates. This would force y-reduction. One can of course perform the same for both y and z. The ordinary quantum measurement theory seems to be a necessary part of the picture. In TGD framework additional constraints come from the condition that the measurement involves negentropy transfer. This requires explicit introduce of systems C and D receiving the NE.

About negentropic entanglement as an analog of error correction code

In classical computation, the simplest manner to control errors is to take several copies of the bit sequences. In quantum case no-cloning theorem prevents this. Error correcting codes (`http://tinyurl.com/oq7hl37`) code n information qubits to the entanglement of $N > n$ physical qubits. Additional contraints represents the subspace of n-qubits as a lower-dimensional sub-space of N qubits. This redundant representation is analogous to the use of parity bits. The failure of the constraint to be satisfied tells that the error is present and also the character of error. This makes possible the automatic correction of the error is simple enough - such as the change of the phase of spin state or or spin flip.

Negentropic entanglement (NE) obviously gives rise to a strong reduction in the number of states of tensor product. Consider a system consisting of two entangled systems consisting of N_1 and N_2 spins. Without any constraints the number of states in state basis is $2^{N_1} \times 2^{N_2}$ and one as $N_1 + N_2$ qubits. The elements of entanglement matrix can be written as $E_{A,B}$ $A \equiv \otimes_{i=1}^{N_1}(m_i, s_i)$, $B \equiv \otimes_{k=1}^{N_2}(m_k, s_k)$ in order to make manifest the tensor product structure. For simplicity one can consider the situation $N_1 = N_2 = N$.

The un-normalized general entanglement matrix is parametrized by 2×2^{2N} independent real numbers with each spin contributing two degrees of freedom. Entanglement matrix proportional to a unitary matrix is characterized by 2^{2N} real numbers. One might perhaps say that one has $2N$ real bits instead of almost $2N + 1$ real qubits. If the time evolution according to ZEO respects the negentropic character of entanglement, the sources of errors are reduced dramatically.

The challenge is to understand what kind of errors NE eliminates and how the information bits are coded by it. NE is respected if the errors act as unitary transformations $E \rightarrow UEU^{\dagger}$ of the entanglement matrix unitary apart from a normalization factor. One can consider two interpretations.

1. The unitary automorphisms leave information content unaffected only if they commute with E. In this case unitary automorphisms acting non-trivially would give rise genuine errors and an error correction mechanism would be needed and would be coded to quantum computer program.
2. One can also consider the possibility that the unitary automorphisms *do not affect* the information content so that the diagonal form of entanglement matrix coded by N phases would carry of information. Clearly, the unitary automorphisms would act like gauge transformations. Nature would take care that no errors emerge. Of course, more dramatic things are in principle allowed by NMP: for instance, the entanglement matrix proportional to unitary matrix could reduce to a tensor product of several unitary matrices. Negentropy could be transferred from the system and is indeed transferred as the computation halts.

   By number theoretic universality the diagonalized entanglement matrix would be parametrized by N roots of unity with each having n possible values so that n^N different NEs would be obtained and information storage capacity would be $I = log(n)/log(2) \times N$ bits for $n = 2^k$ one would have $k \times N$ bits. Powers of two for n are favored. Clearly the option for which only the eigenvalues of E matter, looks more attractive realization of entanglement matrices. If overall phase of E does not matter as one expects, the number of full bits is $k \times N - 1$. This option looks more attractive realization of entanglement matrices.

   In fact, Fermat polygons for which cosine and sine for the angle defining the polygon are expressible by iterating square root besides basic arithmetic operations for rationals (ruler and compass construction geometrically) correspond to integers, which are products of a power of two and of different Fermat primes $F_n = 2^{2^n} + 1$. l

This picture can be related to much bigger picture.

1. In TGD framework number theoretical universality requires discretization in terms of algebraic extension of rationals. This is not performed at space-time level but for the parameters characterizing space-time surfaces at the level of WCW. Strong form of holography is also essential and allows to consider partonic 2-surfaces and string world sheets as basic objects. Number theoretical universality (adelic physics) forces a discretization of phases and number theoretically allowed phases are roots of unity defined by some algebraic extension of rationals. Discretization can be also interpreted in terms of finite measurement resolution. Notice that the condition that roots of unity are in question realizes finite measurement resolution in the sense that errors have minimum size and are thus detectable.
2. Hierarchy of quantum criticalities corresponds to a fractal inclusion hierarchy of isomorphic sub-algebras of the super-symplectic algebra acting as conformal gauge symmetries. The generators in the complement of this algebra can act as dynamical symmetries affecting the physical states. Infinite hierarchy of gauge symmetry breakings is the outcome and the weakening of measurement resolution would correspond to the reduction in the size of the broken gauge group. The hierarchy of quantum criticalities is accompanied by the hierarchy of measurement resolutions and hierarchy of effective Planck constants $h_{eff} = n \times h$.
3. These hierarchies are argued to correspond to the hierarchy of inclusions for hyperfinite factors of type II_1 labelled by quantum phases and quantum groups. Inclusion defines finite measurement resolution since included sub-algebra does induce observable effects on the state. By Mac-Kay correspondence the hierarchy of inclusions is accompanied by a hierarchy of simply laced Lie groups which get bigger as one climbs up in the hierarchy. There interpretation as genuine gauge groups does make sense since their sizes should be reduced. An attractive possibility is that these groups are factor groups G/H such that the normal subgroup H (necessarily so) is the gauge group and indeed gets smaller and G/H is the dynamical group identifiable as simply laced group which gets bigger. This would require that both G and H are infinite-dimensional groups. An interesting question is how they relate to the super-symplectic group assignable to "light-cone boundary" $\delta M^4_{\pm} \times CP_2$. I have proposed this interpretation in the context of WCW geometry earlier.

4. Here I have spoken only about dynamical symmetries defined by discrete subgroups of simply laced groups. I have earlier considered the possibility that discrete symmetries provide a description of finite resolution, which would be equivalent with quantum group description.

Summarizing, these arguments boil down to the conjecture that discrete subgroups of these groups act as effective symmetry groups of entanglement matrices and realize finite quantum measurement resolution. A very deep connection between quantum information theory and these hierarchies would exist.

Gauge invariance has turned out to be a fundamental symmetry principle, and one can ask whether entanglement matrices proportional to unitary matrices assuming that only the eigenvalues matter, could give rise to a simulation of discrete gauge theories. The reduction of the information to that provided by the diagonal form be interpreted as an analog of gauge invariance?

1. The hierarchy of inclusions of hyper-finite factors of type II_1 suggests strongly a hierarchy of effective gauge invariances characterizing measurement resolution realized in terms of hierarchy of normal subgroups and dynamical symmetries realized as coset groups G/H. Could these effective gauge symmetries allow to realize entanglement matrices proportional to unitary matrics invariant under these symmetries?

2. A natural parametrization for single qubit errors is as rotations of qubit. If the error acts as a rotation on *all* qubits, the rotational invariance of the entanglement matrix defining the analog of S-matrix is enough to eliminate the effect on information processing.

   Quaternionic unitary transformations act on qubits as unitary rotations. Could one assume that complex numbers as the coefficient field of QM is effectively replaced with quaternions? If so, the multiplication by unit quaternion for states would leave the physics and information content invariant just like the multiplication by a complex phase leaves it invariant in the standard quantum theory.

   One could consider the possibility that quaternions act as a discretized version of local gauge symmetry affecting the information qubits and thus reducing further their number and thus also errors. This requires the introduction of the analog of gauge potential and coding of quantum information in terms of SU(2) gauge invariants. In discrete situation gauge potential would be replaced with a non-integrable phase factors along the links of a lattice in lattice gauge theory. In TGD framework the links would correspond the fermionic strings connecting partonic two-surfaces carrying the fundamental fermions at string ends as point like particles. Fermionic entanglement is indeed between the ends of these strings.

3. Since entanglement is multilocal and quantum groups accompany the inclusion, one cannot avoid the question whether Yangian symmetry crucial for the formulation of quantum TGD [K116] could be involved.

12.5 p-Adic physics and consciousness

p-Adic physics as physics of cognition and imagination is an important thread in TGD inspired theory of consciousness. In the sequel I describe briefly the basic of TGD inspired theory of consciousness as generalization of quantum measurement theory to ZEO (ZEO), describe the definition of self, consider the question whether NMP is needed as a separate principle or whether it is implied is in statistical sense by the unavoidable statistical increase of $n = h_{eff}/h$ if identified as a factor of the dimension of Galois group extension of rationals defining the adeles, and finally summarize the vision about how p-adic physics serves as a correlate of cognition and imagination.

12.5.1 From quantum measurement theory to a theory of consciousness

The notion of self can be seen as a generalization of the poorly defined definition of the notion of observer in quantum physics. In the following I take the role of skeptic trying to be as critical as possible.

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. The density matrix was assumed to define the universal observable. Note that a density matrix, which is power series of a product of matrices representing commuting observables has in the generic case eigenstates, which are simultaneous eigenstates of all observables. Second aspect of self was assumed to be the integration of subsequent quantum jumps to coherent whole giving rise to the experienced flow of time.

The precise identification of self allowing to understand both of these aspects turned out to be difficult problem. I became aware the solution of the problem in terms of ZEO (ZEO) only rather recently (2014).

1. Self corresponds to a sequence of quantum jumps integrating to single unit as in the original proposal, but these quantum jumps correspond to state function reductions to a fixed boundary of causal diamond CD leaving the corresponding parts of zero energy states invariant - "small" state function reductions. The parts of zero energy states at second boundary of CD change and even the position of the tip of the opposite boundary changes: one actually has wave function over positions of second boundary (CD sizes roughly) and this wave function changes. In positive energy ontology these repeated state function reductions would have no effect on the state (Zeno effect) but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and self: self is generalized Zeno effect.
2. The first quantum jump to the opposite boundary corresponds to the act of "free will" or birth of re-incarnated self. Hence the act of "free will" changes the arrow of psychological time at some level of hierarchy of CDs. The first reduction to the opposite boundary of CD means "death" of self and "re-incarnation" of time-reversed self at opposite boundary at which the the temporal distance between the tips of CD increases in opposite direction. The sequence of selves and time reversed selves is analogous to a cosmic expansion for CD. The repeated birth and death of mental images could correspond to this sequence at the level of sub-selves.
3. This allows to understand the relationship between subjective and geometric time and how the arrow of and flow of clock time (psychological time) emerge. The average distance between the tips of CD increases on the average as along as state function functions occur repeatedly at the fixed boundary: situation is analogous to that in diffusion. The localization of contents of conscious experience to boundary of CD gives rise to the illusion that universe is 3-dimensional. The possibility of memories made possibly by hierarchy of CDs demonstrates that this is not the case. Self is simply the sequence of state function reductions at same boundary of CD remaining fixed and the lifetime of self is the total growth of the average temporal distance between the tips of CD.

One can identify several rather abstract state function reductions selecting a sector of WCW.

1. There are quantum measurements inducing localization in the moduli space of CDs with passive boundary and states at it fixed. In particular, a localization in the moduli characterizing the Lorentz transform of the upper tip of CD would be measured. The measured moduli characterize also the analog of symplectic form in M^4 strongly suggested by twistor lift of TGD - that is the rest system (time axis) and spin quantization axes. Of course, also other kinds of reductions are possible.
2. Also a localization to an extension of rationals defining the adeles should occur. Could the value of $n = h_{eff}/h$ be observable? The value of n for given space-time surface at the active boundary of CD could be identified as the order of the smallest Galois group containing all Galois groups assignable to 3-surfaces at the boundary. The superposition of space-time surface would not be eigenstate of n at active boundary unless localization occurs. It is not obvious whether this is consistent with a fixe value of n at passive boundary.

   The measured value of n could be larger or smaller than the value of n at the passive boundary of CD but in statistical sense n would increase by the analogy with diffusion on half line defined by non-negative integers. The distance from the origin unavoidably increases in statistical sense. This would imply evolution as increase of maximal value of negentropy and generation of quantum coherence in increasingly longer scales.
3. A further abstract choice corresponds to the replacement of the roles of active and passive boundary of CD changing the arrow of clock time and correspond to a death of self and re-incarnation as time-reversed self.

Can one assume that these measurements reduce to measurements of a density matrix of either entangled system as assumed in the earlier formulation of NMP, or should one allow both options. This question actually applies to all quantum measurements and leads to a fundamental philosophical questions unavoidable in all consciousness theories.

1. Do all measurements involve entanglement between the moduli or extensions of two CDs reduced in the measurement of the density matrix? Non-diagonal entanglement would allow final states states, which are not eigenstates of moduli or of n: this looks strange. This could also lead to an infinite regress since it seems that one must assume endless hierarchy of entangled CDs so that the reduction sequence would proceed from top to bottom. It looks natural to regard single CD as a sub-Universe.

   For instance, if a selection of quantization axis of color hypercharge and isospin (localization in the twistor space of CP_2) is involved, one would have an outcome corresponding to a quantum superposition of measurements with different color quantization axis!

   Going philosophical, one can also argue, that the measurement of density matrix is only a reaction to environment and does not allow intentional free will.

2. Can one assume that a mere localization in the moduli space or for the extension of rationals (producing an eigenstate of n) takes place for a fixed CD - a kind of self measurement possible for even unentangled system? If there is entanglement in these degrees of freedom between two systems (say CDs), it would be reduced in these self measurements but the outcome would not be an eigenstate of density matrix. An interpretation as a realization of intention would be approriate.

3. If one allows both options, the interpretation would be that state function reduction as a measurement of density matrix is only a reaction to environment and self-measurement represents a realization of intention.

4. Self measurements would occur at higher level say as a selection of quantization axis, localization in the moduli space of CD, or selection of extension of rationals. A possible general rule is that measurements at space-time level are reactions as measurements of density matrix whereas a selection of a sector of WCW would be an intentional action. This because formally the quantum states at the level of WCW are as modes of classical WCW spinor field single particle states.

5. If the selections of sectors of WCW at active boundary of CD commute with observables, whose eigenstates appear at passive boundary (briefly *passive observables*) meaning that time reversal commutes with them - they can occur repeatedly during the reduction sequence and self as a generalized Zeno effect makes sense.

   If the selections of WCW sectors at active boundary do not commute with passive observables then volition as a choice of sector of WCW must change the arrow of time. Libet's findings show that conscious choice induces neural activity for a fraction of second before the conscious choice. This would imply the correspondences *"big" measurement changing the arrow of time - self-measurement at the level of WCW - intentional action* and *"small" measurement - measurement at space-time level - reaction.*

   Self as a generalized Zeno effect makes sense only if there are active commuting with passive observables. If the passive observables form a maximal set, the new active observables commuting with them must emerge. The increase of the size of extension of rationals might generate them by expanding the state space so that self would survive only as long at it evolves.

   Otherwise there would be only single unitary time evolution followed by a reduction to opposite boundary. This makes sense only if the sequence of "big" reductions for sub-selves can give rise to the time flow experienced by self: the birth and death of mental images would give rise to flow of time of self.

A hierarchical process starting from given CD and proceeding downwards to shorter scales and stopping when the entanglement is stable is highly suggestive and favors self measurements. What stability could mean will be discussed in the next section. CDs would be a correlate for self hierarchy. One can say also something about the anatomy and correlates of self hierarchy.

1. Self experiences its sub-selves as mental images and even we would represent mental images of some higher level collective self. Everything is conscious but consciousness can be lost or at least it is not possible to have memory about it. The flow of consciousness for a given self could be due to the quantum jump sequences performed by its sub-selves giving rise to mental images.
2. By quantum classical correspondence self has also space-time correlates. One can visualize sub-self as a space-time sheet "glued" by topological sum to the space-time sheet of self. Subsystem is not described as a tensor factor as in the standard description of subsystems. Also sub-selves of selves can entangle negentropically and this gives rise to a sharing of mental images about which stereo vision would be basic example. Quite generally, one could speak of stereo consciousness. Also the experiences of sensed presence [J8] could be understood as a sharing of mental images between brain hemispheres, which are not themselves entangled. This is possible also between different brains. In the normal situation brain hemispheres are entangled.
3. At the level of 8-dimensional embedding space the natural correlate of self would be CD (causal diamond). At the level of space-time the correlate would be space-time sheet or light-like 3-surface. The contents of consciousness of self would be determined by the space-time sheets in the interior of CD. Without further restrictions the experience of self would be essentially four-dimensional. Memories would be like sensory experiences except that they would be about the geometric past and for some reason are not usually colored by sensory qualia. For instance .1 second time scale defining sensory chronon corresponds to the secondary p-adic time scale characterizing the size of electron's CD (Mersenne prime M_{127}), which suggests that Cooper pairs of electrons are essential for the sensory qualia.

12.5.2 NMP and self

The view about Negentropy Maximization Principle (NMP) [K70] has co-evolved with the notion of self and I have considered many variants of NMP.

1. The original formulation of NMP was in positive energy ontology and made same predictions as standard quantum measurement theory. The new element was that the density matrix of sub-system defines the fundamental observable and the system goes to its eigenstate in state function reduction. As found, the localizations at to WCW sectors define what might be called self-measurements and identifiable as active volitions rather than reactions.
2. In p-adic physics one can assign with rational and even algebraic entanglement probabilities number theoretical entanglement negentropy (NEN) satisfying the same basic axioms as the ordinary Shannon entropy but having negative values and therefore having interpretation as information. The definition of p-adic negentropy (real valued) reads as $S_p = -\sum P_k log(|P_k|_p)$, where $|.|_p$ denotes p-adic norm. The news is that $N_p = -S_p$ can be positive and is positive for rational entanglement probabilities. Real entanglement entropy S is always non-negative.

   NMP would force the generation of negentropic entanglement (NE) and stabilize it. NNE resources of the Universe - one might call them Akashic records- would steadily increase.
3. A decisive step of progress was the realization is that NTU forces all states in adelic physics to have entanglement coefficients in some extension of rationals inducing finite-D extension of p-adic numbers. The same entanglement can be characterized by real entropy S and p-adic negentropies N_p, which can be positive. One can define also total p-adic negentropy: $N = \sum_p N_p$ for all p and total negentropy $N_{tot} = N - S$.

   For rational entanglement probabilities it is easy to demonstrate that the generalization of adelic theorem holds true: $N_{tot} = N - S = 0$. NMP based on N_{tot} rather than N would not say anything about rational entanglement. For extensions of rationals it is easy to find that $N - S > 0$ is possible if entanglement probabilities are of form X_i/n with $|X_i|_p = 1$ and n integer [L33]. Should one identify the total negentropy as difference $N_{tot} = N - S$ or as $N_{tot} = N$?

   Irrespective of answer, large p-adic negentropy seems to force large real entropy: this nicely correlates with the paradoxical finding that living systems tend to be entropic although one would expect just the oppositecite [L33]: this relates in very interesting manner to the work of biologists Jeremy England [**?**]. The negentropy would be cognitive negentropy and not visible for ordinary physics.

4. The latest step in the evolution of ideas NMP was the question whether NMP follows from number theory alone just as second law follows form probability theory! This irritates theoretician's ego but is victory for theory. The dimension n of extension is positive integer and cannot but grow in statistical sense in evolution! Since one expects that the maximal value of negentropy (define as $N - S$) must increase with n. Negentropy must increase in long run.

Number theoretic entanglement can be stable

Number theoretical Shannon entropy can serve as a measure for genuine information assignable to a pair of entanglement systems [K70]. Entanglement with coefficients in the extension is always negentropic if entanglement negentropy comes from p-adic sectors only. It can be negentropic if negentropy is defined as the difference of p-adic negentropy and real entropy.

The diagonalized density matrix need not belong to the algebraic extension since the probabilities defining its diagonal elements are eigenvalues of the density matrix as roots of N:th order polynomial, which in the generic case requires n-dimensional algebraic extension of rationals. One can argue that since diagonalization is not possible, also state function reduction selecting one of the eigenstates is impossible unless a phase transition increasing the dimension of algebraic extension used occurs simultaneously. This kind of NE could give rise to cognitive entanglement.

There is also a special kind of NE, which can result if one requires that density matrix serves a universal observable in state function reduction. The outcome of reduction must be an eigen space of density matrix, which is projector to this subspace acting as identity matrix inside it. This kind NE allows all unitarily related basis as eigenstate basis (unitary transformations must belong to the algebraic extension). This kind of NE could serve as a correlate for "enlightened" states of consciousness. Schrödingers cat is in this kind of state stably in superposition of dead and alive and state basis obtained by unitary rotation from this basis is equally good. One can say that there are no discriminations in this state, and this is what is claimed about "enlightened" states too.

The vision about number theoretical evolution suggests that NMP forces the generation of NE resources as NE assignable to the "passive" boundary of CD for which no changes occur during sequence of state function reductions defining self. It would define the unchanging self as negentropy resources, which could be regarded as kind of Akashic records. During the next "re-incarnation" after the first reduction to opposite boundary of CD the NE associated with the reduced state would serve as new Akashic records for the time reversed self. If NMP reduces to the statistical increase of $h_{eff}/h = n$ the consciousness information contents of the Universe increases in statistical sense. In the best possible world of SNMP it would increase steadily.

Does NMP reduce to number theory?

The heretic question that emerged quite recently is whether NMP is actually needed at all! Is NMP a separate principle or could NMP reduced to mere number theory [K70]? Consider first the possibility that NMP is not needed at all as a separate principle.

1. The value of $h_{eff}/h = n$ should increase in the evolution by the phase transitions increasing the dimension of the extension of rationals. $h_{eff}/h = n$ has been identified as the number of sheets of some kind of covering space. The Galois group of extension acts on number theoretic discretizations of the monadic surface and the orbit defines a covering space. Suppose n is the number of sheets of this covering and thus the dimension of the Galois group for the extension of rationals or factor of it.

2. It has been already noticed that the "big" state function reductions giving rise to death and reincarnation of self could correspond to a measurement of $n = h_{eff}$ implied by the measurement of the extension of the rationals defining the adeles. The statistical increase of n follows automatically and implies statistical increase of maximal entanglement negentropy. Entanglement negentropy increases in statistical sense.

The resulting world would not be the best possible one unlike for a strong form of NMP demanding that negentropy does increaes in "big" state function reductions. n also decrease temporarily and they seem to be needed. In TGD inspired model of bio-catalysis the phase transition reducing the value of n for the magnetic flux tubes connecting reacting bio-molecules allows them to find each other in the molecular soup. This would be crucial for understanding processes like DNA replication and transcription.

3. State function reduction corresponding to the measurement of density matrix could occur to an eigenstate/eigenspace of density matrix only if the corresponding eigenvalue and eigenstate/eigenspace is expressible using numbers in the extension of rationals defining the adele considered. In the generic case these numbers belong to N-dimensional extension of the original extension. This can make the entanglement stable with respect to state the measurements of density matrix.

   A phase transition to an extension of an extension containing these coefficients would be required to make possible reduction. A step in number theoretic evolution would occur. Also an entanglement of measured state pairs with those of measuring system in containing the extension of extension would make possible the reduction. Negentropy could be reduced but higher-D extension would provide potential for more negentropic entanglement and NMP would hold true in the statistical sense.

4. If one has higher-D eigen space of density matrix, p-adic negentropy is largest for the entire subspace and the sum of real and p-adic negentropies vanishes for all of them. For negentropy identified as total p-adic negentropy SNMP would select the entire sub-space and NMP would indeed say something explicit about negentropy.

Or is NMP needed as a separate principle?

Hitherto I have postulated NMP as a separate principle [K70]. Strong form of NMP (SNMP) states that Negentropy does not decrease in "big" state function reductions corresponding to death and re-incarnations of self.

One can however argue that SNMP is not realistic. SNMP would force the Universe to be the best possible one, and this does not seem to be the case. Also ethically responsible free will would be very restricted since self would be forced always to do the best deed that is increase maximally the negentropy serving as information resources of the Universe. Giving up separate NMP altogether would allow to have also "Good" and "Evil".

This forces to consider what I christened weak form of NMP (WNMP). Instead of maximal dimension corresponding to N-dimensional projector self can choose also lower-dimensional sub-spaces and 1-D sub-space corresponds to the vanishing entanglement and negentropy assumed in standard quantum measurement theory. As a matter fact, this can also lead to larger negentropy gain since negentropy depends strongly on what is the large power of p in the dimension of the resulting eigen sub-space of density matrix. This could apply also to the purely number theoretical reduction of NMP.

WNMP suggests how to understand the notions of Good and Evil. Various choices in the state function reduction would correspond to Boolean algebra, which suggests an interpretation in terms of what might be called emotional intelligence [K122]. Also it turns out that one can understand how p-adic length scale hypothesis - actually its generalization - emerges from WNMP [K125].

1. One can start from ordinary quantum entanglement. It corresponds to a superposition of pairs of states. Second state corresponds to the internal state of the self and second state to a state of external world or biological body of self. In negentropic quantum entanglement each is replaced with a pair of sub-spaces of state spaces of self and external world. The dimension of the sub-space depends on which pair is in question. In state function reduction one of these pairs is selected and deed is done. How to make some of these deeds good and some bad? Recall that WNMP allows only the possibility to generate NNE but does not force it. WNMP would be like God allowing the possibility to do good but not forcing good deeds.

   Self can choose any sub-space of the subspace defined by $k \leq N$-dimensional projector and 1-D subspace corresponds to the standard quantum measurement. For $k = 1$ the state function reduction leads to vanishing negentropy, and separation of self and the target of the action.

Negentropy does not increase in this action and self is isolated from the target: kind of price for sin.

For the maximal dimension of this sub-space the negentropy gain is maximal. This deed would be good and by the proposed criterion NE corresponds to conscious experience with positive emotional coloring. Interestingly, there are $2^k - 1$ possible choices, which is almost the dimension of Boolean algebra consisting of k independent bits. The excluded option corresponds to 0-dimensional sub-space - empty set in set theoretic realization of Boolean algebra. This could relate directly to fermionic oscillator operators defining basis of Boolean algebra - here Fock vacuum would be the excluded state. The deed in this sense would be a choice of how loving the attention towards system of external world is.

2. A map of different choices of k-dimensional sub-spaces to k-fermion states is suggestive. The realization of logic in terms of emotions of different degrees of positivity would be mapped to many-fermion states - perhaps zero energy states with vanishing total fermion number. State function reductions to k-dimensional spaces would be mapped to k-fermion states: quantum jumps to quantum states!

   The problem brings in mind quantum classical correspondence in quantum measurement theory. The direction of the pointer of the measurement apparatus (in very metaphorical sense) corresponds to the outcome of state function reduction, which is now 1-D subspace. For ordinary measurement the pointer has k positions. Now it must have $2^k - 1$ positions. To the discrete space of k pointer positions one must assign fermionic Clifford algebra of second quantized fermionic oscillator operators. The hierarchy of Planck constants and dark matter suggests the realization. Replace the pointer with its space-time k-sheeted covering and consider zero energy energy states made of pairs of k-fermion states at the sheets of the n-sheeted covering? Dark matter would be therefore necessary for cognition. The role of fermions would be to "mark" the k space-time sheets in the covering.

The cautious conclusion is that NMP as a separate principle is not necessary and follows in statistical sense from the unavoidable increase of $n = h_{eff}/h$ identified as dimension of extension of rationals define the adeles if this extension or at least the dimension of its Galois group is observable.

12.5.3 p-Adic physics as correlate of cognition and imagination

The items in the following list give motivations for the proposal that p-adic physics could serve as a correlate for cognition and imagination.

1. By the total disconnectedness of the p-adic topology, p-adic world decomposes naturally into blobs, objects. This happens also in sensory perception. The pinary digits of p-adic number can be assigned to a p-tree. Parisi proposed in the model of spin glass [B39] that p-adic numbers could relate to the mathematical description of cognition and also Khrennikov [J1] has developed this idea. In TGD framework that idea is taken to space-time level: p-adic space-time sheets represent thought bubbles and they correlate with the real ones since they form cognitive reprentations of the real world. SH allows a concrete realization of this.

2. p-Adic non-determinism due to p-adic pseudo constants suggests interpretation in terms of imagination. Given 2-surfaces could allow completion to p-adic preferred extremal but not to a real one so that pure "non-realizable" imagination is in question.

3. Number theoretic negentropy has interpretation as negentropy characterizing information content of entanglement. The superposition of state pairs could be interpreted as a quantum representation for a rule or abstracted association containing its instances as state pairs. Number theoretical negentropy characterizes the relationship of two systems and should not be confused with thermodynamical entropy, which characterizes the uncertainty about the state of single system.

The original vision was that p-adic non-determinism could serve as a correlate for cognition, imagination, and intention. The recent view is much more cautious. Imagination need not completely reduce to p-adic non-determinism since it has also real physics correlates - maybe as partial realizations of SH as in nerve pulse pattern, which does not propagate down to muscles.

A possible interpretation for the solutions of the p-adic field equations would be as geometric correlates of cognition, imagination, and perhaps even intentionality. Plans, intentions, expectations, dreams, and possibly also cognition as imagination in general could have p-adic cognitive space-time sheets as their geometric correlates. A deep principle seems to be involved: incompleteness is the characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

The most feasible view is that the intersections of p-adic and real space-time surfaces define cognitive representations of real space-time surfaces (PEs, [K15, K9, L31]). One could also say that real space-time surface represents sensory aspects of conscious experience and p-adic space-time surfaces its cognitive aspects. Both real and p-adics rather than real or p-adics.

The identification of p-adic pseudo constants as correlates of imagination at space-time level is indeed a further natural idea.

1. The construction of PEs by SH from the data at 2-surfaces is like boundary value problem with number theoretic discretization of space-time surface as additional data. PE property in real context implies strong correlations between string world sheets and partonic 2-surfaces by boundary conditions a them. One cannot choose these 2-surfaces completely independently in real context.
2. In p-adic sectors the integration constants are replaced with pseudo-constants depending on finite number of pinary digits of variables depending on coordinates normal to string world sheets and partonic 2-surfaces. The fixing of the discretization of space-time surface would allow to fix the p-adic pseudo-constants. Once the number theoretic discretization of space-time surface is fixed, the p-adic pseudo-constants can be fixed. Pseudo-constant could allow a large number of p-adic configurations involving string world sheets, partonic 2-surfaces, and number theoretic discretization but not allowed in real context.

   Could these p-adic PEs correspond to imaginations, which in general are not realizable? Could the realizable intentional actions belong to the intersection of real and p-adic WCWs? Could one identify non-realistic imaginations as the modes of WCW spinor fields for which 2-surfaces are not extendable to real space-time surfaces and are localized to 2-surfaces? Could they allow only a partial continuation to real space-time surface. Could nerve pulse pattern representing imagined motor action and not proceeding to the level of muscles correspond to a partially real PE?

   Could imagination and problem solving be search for those collections of string world sheets and partonic 2-surfaces, which allow extension to (realization as) real PEs? If so, p-adic physics would be there as an independent aspect of existence and this is just the original idea. Imagination could be realized in state function reduction, which always selects only those 2-surfaces, which allow continuation to real space-time surfaces. The distinction between only imaginable and also realizable would be the extendability by using strong form of holography.
3. An interesting question is why elementary particles are characterized by preferred p-adic primes (primes near powers of 2, in particular Mersenne primes). Could the number of realizable imaginations for these primes be especially large?

I have the feeling that this view allows respectable mathematical realization of imagination in terms of adelic quantum physics. It is remarkable that SH derivable from - you can guess, SGCI (the Big E again!), plays an absolutely central role in it.

12.6 Negentropy Maximization Principle and Second Law

This article was inspired by a birthday gift. The gift was a highly inspiring book "Lifespan" by David Sinclair and Matthew LaPlante [?]. The book tells about the recent understanding concerning aging. The general vision about aging represented in the book can be summarized as follows.

1. The key idea is that genes do not determine everything. DNA has also continuum degrees of freedom characterized by its shape. These degrees of freedom are related to epigenesis which is higher level control activity controlling what genes are expressed. The basic mechanisms are prevention and allowance of gene expression. Acetylization [?], methylation

[?] and many other modifications affect the gene expression by attaching to proteins known as histones forming kind of pearls in the necklace defined by DNA: genes follow nucleosomes in the DNa strand. Also the reversals of these processes - for instance, deacetylation [?, ?]and demethylation [?] are essential for the control of gene expressions.

2. Aging involves the emergence of various diseases. Usually the attention is directed to dealing with these diseases. Now the view is however more general. Aging is seen as a gradual approach to chaos manifesting as various diseases. In order to prevent the diseases one should slow down the approach to chaos.

3. Epigenetic chaos hypothesis suggests that some control systems associated with information molecules and related to the control of DNA transcription and translation by epigenesis must approach chaos. This is seen as the gradual randomization of DNA conformations causing problems in the transcription of DNA: DNA becomes loopy. The DNA coding for the ribosome acting as the translation machinery of DNA is of special importance and becomes also loopy. What comes first in physicist's mind, is an approach to thermal equilibrium. Is there some system controlling epigenesis which approaches thermal equilibrium with the environment? In standard chemistry it is difficult to identify this kind of system.

4. Biology has invented ingenious mechanisms to slow down aging. For instance, there are molecules having two functions. There are proteins making the translation of the genes related to cell replication possible.

   When the DNA coding for the ribosome gets loopy their function changes. The translation of genes ceases as the proteins leave the histone molecules and enter the damaged DNA and freeze it so that it can be prepared. This however slows down cell replication and also causes other problems leading to various diseases related to aging. One can say that a cell is like a hospitalized patient.

   The slowing down of aging would be in this framework basically fighting against the thermodynamical arrow of time. Is it really possible to understand the processes involved in the framework of standard bio-chemistry with a single arrow of time?

   Even the understanding of the biocatalysis is difficult: how the reacting molecules are able to find each other in the molecular soup and how the huge increase for the rate of these processes is possible. The TGD based solution of these problems will be discussed later.

What could TGD inspired theory of consciousness and quantum biology rely on zero energy ontology (ZEO) inspired biology allow to say about the mechanism behind aging?

1. Negentropy Maximization Principle (NMP) as the variational principle of consciousness replaces the second law and implies it for ordinary matter. State function reduction (SFR) means a reduction of the entanglement for a pair $S_a - S_b$ of sub-system S_a and its complement S_b in S. Measurement cascade proceeding from long to short scales decomposes at each step a system to a pair of unentangled subsystems is in question. NMP as a variational principle of consciousness states that negentropy gain in these reductions is maximized and selects the pair $S_a - S_b$ at given step.

   In adelic physics [L53, L54] the negentropy $N = -S_1 - S_2$ is sum of real and various p-adic negentropies but p-adic negentropy can be positive so that for non-trivial extensions of rationals one can have $N > 0$. This kind of entanglement is stable against NMP so that the process stops. One can assign positively colored emotions to this kind of entanglement and it distinguishes between living and inanimate matter and also between dark and ordinary matter.

2. TGD inspired theory of consciousness is basically an extension of quantum measurement theory allowing to get rid of the basic paradox of quantum measurement theory. There are two kinds of state function reductions (SFRs) "big" SFR and "small" SFR (briefly BSFR and SSFR) [L91].

   SSFRs are counterparts of "weak" measurements which are much like classical measurements and do not involve any dramatic changes. The sequence of SSFRs gives rise to a conscious entity -self - as a sequence of moments of consciousness. Subjective time as a sequence of SSFRs correlates with the geometric time. BSFRs are counterparts of ordinary quantum measurements and have a dramatic effect: in a very general sense one can say that self dies and reincarnates with an opposite arrow of geometric time.

3. There is a hierarchy of magnetic bodies carrying dark matter as phases of ordinary matter with effective value $h_{eff} = nh_0$ of Planck constant. n corresponds to the dimension of an extension of rationals. The extensions define evolutionary hierarchies with increasing complexity. n serves as a measure of algebraic complexity and as a universal IQ, and also characterizes the scale of quantum coherence. For instance, genes are characterized by the value of h_{eff} associated with their MB.

   Since MBs have higher universal IQ than ordinary biomatter, they control the biochemistry. In particular, they would control DNA and DNAs MB would actually realize genetic codons in terms of dark proton triplets. Also dark photon triplets would provide this kind of realization crucial for control of and communication with ordinary biomatter.

4. ZEO implies a theory of self-organization [L86] and of self-organized quantum criticality (SOQC) relying on time reversal [L177]. The dissipation of a system looks like in reversed time direction extraction of energy from the environment. Also SOQC becomes possible since criticality, since a state, which is a repeller, becomes an attractor in reversed time direction. The system seems to tend to criticality for an observer with an opposite arrow of time.

5. In this framework the aging could be seen as the approach of the system formed by MBs of the information molecules and of ordinary biomatter to a thermal equilibrium. The temperature of MB gradually grows and eventually reaches a maximal temperature (due to the stringy character of flux loops) known as Hagedorn temperature and identifiable as the physiological temperature. System dies.

12.7 Very brief summary about some aspects of aging?

The book of Sinclair and LaPlante [?] is about aging and how to slow down it. The basic hypothesis is that aging need not mean getting sicker and sicker all the time. Biology has developed molecular tools for slowing down aging and there are longevity genes analogs to SPs taking care of this. It might be possible to help them by a healthy lifestyle.

The books represents a vision involving the following pieces.

1. Aging is information loss and molecular level, in particular DNA level. Ribosomal DNA seems to be in a special role assignable to nucleolus. Physicist could try to understand this from the second law: entropy un-avoidably increases . Entropy increases for isolated system but it is perhaps not so simple.

2. It has been learned that damage to DNA alone cannot explain aging. There must be additional degrees of freedom assignable to epigenesis as a control of genome. Besides genetic code there should exist additional continuous information carrying degrees of freedom.

   Epigenesis involves these degrees of freedom and DNAs conformation (involving coilings of coilings of ..) represents these degrees of freedom. Histones appear tangles along the DNA double strand selecting which genes are expressed. Acetyl tag in the histome allows gene expression to take place. When acetyl is absent, nothing happens. The effect of acetyl tag can be also silenced.

3. There are enzymes Sirn, $n = 1, 2, ..7$, called sirtuins [?, ?] (`https://cutt.ly/Hjkh0ia`). In particular, Sir2 silences so-called mating type genes so that the cell replicates normally. If Sir2 is not present in mating type genes, they are expressed and replication does not take place normally. I understood that for yeast, the cell loses its sexual identity and does not replicate.

   During this non-replication period the cell would concentrate on maintenance. Under stress situations this would occur quite generally and make survival possible. If you cannot eat and replicate, sleep, and generate metabolic energy from thermal energy for instance. Also SPs would be at work. On the basis of [L177], one could guess that a kind of hibernation state with a reversed arrow of time could be in question. To live longer it is good to die sufficiently often!

4. This is not the only function of Sir2. When DNA double strand is broken, Sir2 must leave its job and hurry to the broken DNA and catalyze expression of SIRn coding for histone deacetylase HDAC, which removes acetyl tags from histones and deactivates DNA. After this the broken DNA is prepared. This is like putting a victim of a traffic accident to a hospital bed or even artificial coma.

   As cells get older, this kind of DNA breaks occur more often and Sir2 must leave its basic job more often and the cell loses its ability to replicate more often. It can also happen that Sir2 does not find its original position in the mating gene and starts to silence a wrong gene. This leads to epigenetic noise inducing aging.

5. In particular, ribosomal DNA in the nucleolus, the largest structure of the nucleus, can end up with chaos. Loops are formed and recombination between portions of the same strand can occur (remember the reconnection mechanism and time reversal). Ribosome plays a fundamental role in translation so that there is no wonder that difficulties emerge. Important class of damage consists of breaking DNA double strands. This leads to a chaotic conformation involving loops. Sir2 must rush to the nucleolus and this means that mating genes activate and the replication stops. When these accidents occur too often, the cell becomes senescent.

The first cognitive measurement leads to a product decomposition in $F(G/(G_2...G_n)) \times F(G_2..G_n)$ if the entanglement coefficients between $G_1 = G/(G_2...G_n)$ and $F(G_2..G_n)$ are in the extension of rationals considered. Same can happen at the next step and leads to a similar decomposition of $F(G_2,...G_n)$. The maximal cognitive measurement cascade leads to a product of wave functions in $F(G_i)$ but it can happen that there is no measurement cascade at all.

Suppose that the time evolution following SSFR for individual mutually unentangled subsystems is in good approximation unitary (their interactions with other such subsystems can be neglected) so that they do not entangle, the density matrix of an individual system suffers a unitary automorphism so that entanglement entropies remain unaffected and the negentropy gain vanishes. One could speak of "asymptotic freedom" as a condition for the cognitive fixed point.

The cognitive fixed point would define the "silent wisdom" of the re-incarnate having the formerly active boundary of CD as a passive boundary of CD. What would be learned during life would help during the next life cycle.

12.8 Negentropy Maximization Principle (NMP) and Second Law

The natural expectation is that second law relates to aging. This motivates a section devoted to the recent view about Negentropy Maximization Principle (NMP) [K70] defining the variational principle of consciousness in the TGD framework and implying in adelic physics [L54, L53] second law in the case of non-negentropic entanglement (in standard physics entanglement is always non-negentropic).

Mathematically NMP is analogous to the second law in that it is not deterministic like the variational principles of classical physics. For a given entangled system NMP allows state function reduction (SFR) for that sub-system-complement pair for which the negentropy gain is maximal. The state function reduction can occur to any eigenstate of the density matrix of the selected subsystem in accordance with standard quantum measurement theory. This would lead to a product of pure states and the negative entanglement negentropy of the initial state would become vanishing in the final state so that negentropy would increase. The inclusion of p-adic contribution to negentropy identifiable in terms of cognitive information assignable to entanglement changes the situation and the entanglement can be stable against NMP and state function reduction cascade stops to entangled state representing cognitive fixed point. Since negentropy gain is not anymore popssible in SSFRs, death is bound to take place.

12.8.1 General observations about second law

First some general observations about second law.

1. Second law is an empirical fact. Second law forces the increase of entropy in statistical sense. Thermo-dynamical equilibrium is the most probable equilibrium. Second law in the standard form assumes a fixed arrow of time. Zero energy ontologuy (ZEO) forces to give up this assumption and allow both arrows of time.

2. Quantum physics is certainly behind second law. If you have an entangled system state, SFRs occur for subsystems with reduction probabilities determined by its entanglement with the environment. This eventually leads to a loss of entanglement and quantum coherence and one must apply statistical using density matrix for individual sub-system and eventually justifies thermo-dynamical description. It is important to notice that in SFR the entanglement entropy of an individual system is reduced in SFR but that in the case of ensemble of identical systems this generates entanglement entropy identical to the entanglement entropy of single particle giving thermo-dynamical entropy as a special case.

   One can consider two interpretations: a) the generation of entanglement generates single particle entropy although actually the entropy of the entire system does not increase in unitary evolution or b) the transformation of this entropy to ensemble entropy corresponds to second law. Option b) looks more realistic.

This is however only a description for what happens. One can ask what is behind second law. Is there some deeper principle as one might suspect because quantum measurement is so poorly defined theory. For instance, von Neumann proposed that only humans cause SFRs. It is often assumed that decoherence occurs without making any proposal how this happens. What is known and well-tested is that reduction probabilities for a measurement reducing the entanglement are coded by the measured density matrix, and one can say that the system goes to an eigenstate of the density matrix as the entanglement is reduced. For an ensemble of identical particles this process transforms entanglement entropy to ensemble entropy with the same value.

Negentropy Maximization Principle (NMP) is the TGD based proposal for the variational principle behind SFRs.

12.8.2 The new physics elements involved with NMP

NMP involves several new physics elements.

1. What is new is the hierarchy of systems having the hierarchy of space-time sheets as a geometric correlate. At the level of consciousness theory it would have self hierarchy as a correlate. Quantum measurements are assumed to correspond to SFR cascades proceeding from higher to lower levels of the hierarchy.

2. ZEO brings in the notions of "small" SFR (SSFR) as counterpart of "weak" measurement and "big" SFR (BSFR) as counterpart of ordinary quantum measurement [L91] and forces giving up the assumption about a fixed arrow of time. This modifies standard thermodynamics and leads to a new view about self-organization self-organized quantum criticality [L86, L177].

3. In the standard physics framework there is no definition of negentropy as a measure of conscious information. Negentropy can be only defined as the negative of ordinary entropy and is therefore non-positive. The best that one could have would be vanishing negentropy. This failure is understandable since standard physics does not even try to describe cognition. One manner to solve the problem is to claim that only entropy gradients, whose sign can be also negative matter and thus consider only information flows. In TGD different view is adopted.

4. To bring in conscious information one must introduce cognition. In the TGD framework it is assumed to be described by adelic physics [L53, L54]. This brings in p-adic space-time surfaces as correlates of cognition. Real space-time surfaces are replaced with their adelic counterparts forming a kind of Cartesian product of real and various p-adic space-time surfaces obeying the same algebraic equations.

   By M^8-H duality [L109, L110] one can regard space-time surfaces as surfaces in M^8 or in $H=M^4\times CP_2$. M^8 is a subspace of the space of complexified octonions O_c and space-time surface is determined as a 4-D "root" of a real polynomial algebraically continued to an octonionic polynomial. If the coefficients of the polynomial are rational numbers, the polynomial makes sense for both real and p-adic number fields implying number theoretical universality. The dynamical principle is simple: the normal space of the space-time surface is associative/quaternionic.

$M^8 - H$-duality maps these 4-surfaces to 4-surfaces in H. In both cases one has minimal surfaces. Also the notion of cognitive representation emerges and is essential for the number theoretical universality. It is also crucial for the construction of the scattering amplitudes [L124, L109, L110].

12.8.3 Detailed formulation of NMP

Consider now the formulation of NMP [K70] in this conceptual framework.

1. In adelic physics cognition is described in terms of p-adic degrees of freedom. Entropy is a sum of two terms: $S = S_1 + S_2$.

   S_1 is the ordinary entropy describing the amount of ignorance of the observer about the state of either entangled system - say Schrödinger cat and the bottle of poison.

   S_2, as the p-adic variant of entropy (also real valued) assignable to cognitive information has an analogous formula and similar defining properties but can be *negative*(!) and is interpreted as a measure for the information carried by entanglement.

   Thr possibility of having negative sign is basically due to the fact that the logarithms $log(p_k)$ of probabilities p_k in the Shannon formula $S = -\sum_k p_k log(p_k)$ for entropy are replaced by the logarithms of their p-adic norms $|p_k|_p$ given by p^{-n} for $p_k = p^n(a_0 + a_1 p + ...)$ (note that the exponent changes sign!): $log(p_k) \to log(|p_k|_p)$. Entropy remains additive since the p-adic norm of product is product of p-adic norms.

   A more general formula for the real Shannon entropy S_1 is as $S_1 = -Tr(\rho log(\rho))$ (ρ is the density matrix). Even in the case that the matrix elements of ρ are in the extension of rationals used, this formula need not generalize for S_2 since also $log(\rho)$ should have this property. The manner to avoid the problem is to diagonalize ρ. This is possible if the eigenvalues of ρ - having an interpretation as entanglement probabilities p_k (equivalently reduction probabilities) - belong to the extension of rationals considered.

   At the fundamental level this extension is defined by the space-time surface determined by a polynomial with rational coefficients ($M^8 - H$ duality [L109, L110]): the roots of the polynomial determine the extension and space-time surface (number theoretic holography). If the entanglement probabilities are not in the extension, one might argue that the entanglement is stable - note however that NMP alone could make it stable.

   Quantum coherence involves stable entanglement carrying cognitive information measured by S_2. The destruction of coherence if allow by NMP destroys information defined as the sum $N = -S = -S_1 - S_2$. In absence of cognition one would have $N = -S_1$ and NMP would transform to second law.

2. The cascade of "small" state function reductions (SSFRs) eventually leads to a state in which the remaining entanglement is stable. There is no subsystem-complement pair for which SSFR could take place in such a way that negentropy $N = -S = -S_1 - S_2$ would increase. The resulting states are analogous to bound states.

3. Remarkably, in its adelic formulation NMP states that the total entropy, which tends to be negative for extensions of rationals, gets smaller and negative: information is generated! The pessimistic second law transforms to an optimistic NMP! The gloomy character of second law would be due to the neglect of cognition from physics.

   Cognitive entropy gets more and more negative but real entropy which is closely related to it but tending to have a smaller magnitude than p-adic entropy for extensions of rationals also increases [K70] [L91]. Hence their sum tends to increase with the dimension $n = h_{eff}/h_0$ of the extension.

What makes entanglement stable against SSFR? One can consider two mechanisms.

1. Adelic physics allows negentropic entanglement, which tends to be stable against SSFRs since it can only become even more negentropic.

2. One can also consider another stabilization mechanism. The rule would state that if the density matrix of the subsystem-complement pair does not allow eigenvalues in the extension of rationals considered, the reduction is not possible. For a stable entanglement density matrix would not allow eigenvalues in the extension of rationals considered. One can of course criticize this rule as somewhat *ad hoc* and the first option might be enough. One can also ask whether this mechanism is equivalent with the first mechanism.

3. What could be the interpretation of the negentropic entanglement? I have assigned positive emotions like love to this entanglement, also experience of understanding, etc...

One can raise an objection against entanglement coefficients in an extension of rationals, call it E.

1. Entanglement coefficients would be in E characterizing the polynomial determining space-time surface in M^8 and by M^8-H-duality this would be the case also in $H=M^4\times CP_2$. The problem is that this does not seem to allow smooth time evolution of entanglement coefficients.

2. At the level of M^8 the construction of scattering amplitudes relies on discretization - cognitive representation [L109, L110]. M^8 is analogous to momentum space and dynamics is purely algebraic at the level of M^8. The algebraic view about solutions of Dirac equation is analogous to momentum space view about ordinary Dirac equation: the spinor mode is localized to a point of momentum space. Many-fermion state corresponds naturally to a cognitive representation. This allows getting rid of the problems since no such evolutions exist at M^8 level.

3. What about the situation at the level of $H=M^4\times CP_2$? Should one only assume that entanglement coefficients for many-fermion state are transformable to a number theoretic entanglement by a unitary matrix leaving the eigenvalues of the density matrix representing entanglement invariant? Since the induced spinors in H satisfy partial differential equations, this seems to be a reasonable option.

   Algebraic entanglement coefficients are needed only at the level of the scattering amplitudes. M^8-H duality allows to map cognitive representation in M^8 to H. This also applies to entanglement coefficients so that they would be in an extension of rationals at the points of cognitive representation containing point-like fermions at both sides.

NMP implies increase of ordinary entanglement entropy

NMP implies increase of the ordinary entanglement entropy. The hasty conclusion would be that this implies also increase of thermal entropy and thus second law. Here one must be however cautious.

1. Second law as an increase of ordinary entropy would still hold true but the increase of cognitive information would be larger than the increase of the real entropy for non-trivial extensions of rationals (this is always the case).

   The asymptotic states with maximum negentropy and with stable entanglement would have maximal real and minimal p-adic entanglement entropy and their sum would be negative - and $N=-S$ would therefore serve as a measure for the amount of conscious information.

2. One might argue that intelligent systems tend to pollute their environment: they are entanglement entropy generators and by witnessing what has been happening to our environment, it would be easy to agree.

   One must be however extremely cautious with formulas. The stability of negentropic entanglement means that the real entanglement entropy cannot transform to ensemble entropy and cannot therefore actualize! Is this what distinguishes loving attention as something unique and positive: the entanglement is stable and cannot transform to ordinary entropy?

Could NMP allow the failure of second law in some situations?

The dream about eternal youth seems to be in conflict with the second law. For physicist second law is usually the absolute authority. Working with the details of NMP however force to challenge this view.

A generalization of second law taking into account time reversals is required in ZEO and already this implies apparent breakings of second law. Furthermore, NMP implies second law as the increase of entanglement entropy. NMP does not allow SFRs transforming negentropic entanglement entropy to thermodynamic ensemble entropy unless the SFRs occurs at higher level of hierachy so that the local reduction of negentropy is compensated by its increase in a longer scale. The implications of this fact remain to be understood.

Could NMP break the second law? Can this be consistent with empirical facts? Could the breaking of second law occur at the level of dark matter only? Second law would apply only to the entropy transformable to thermal entropy. The sum $N = -S_1 - S_2$ is what matters: for a trivial extension one has $N = 0$ so that this transformation is possible. $N = 0$ can be however true also for non-trivial extensions. Could the total entanglement negentropy assignable to the ordinary matter satisfy $N = 0$ and be therefore transformable to thermal entropy whereas "dark" entanglement negentropy satisfying $N > 0$ would not allow this. Could one identify dark/living matter as negentropic matter and ordinary/inanimate matter as non-negentropic thermalizable matter? Note that also the phases with $h_{eff}/h_0 = n$ could in principle have $N = 0$. The stability of dark entanglement could directly relate to the failure to observe dark matter.

Comparison with the proposal of Jeremy England

Jeremy England [?] has noticed that living systems increase entropy and has proposed it as a basic principle of biology. England's proposal is discussed from TGD point of view in [?]. I did not however realize in this article the fact, that negentropic entanglement entropy need not allow a transformation to thermal entropy.

One can represent several objections against England's idea.

1. Second law cannot force or even allow the generation of life. Second law relates to the occurrence of SFRs but we do not have a real theory of quantum measurement.
2. Second law assumes preferred arrow of time and there is a lot of support for its violation in living matter as realized first by Fantappie [J7]: in particular, self-organization processes could involve dissipation with reversed arrow of time.
3. To understand life one must take it seriously. Living system is somehow different from inanimate matter. The emergence of life means the generation of conscious information but in the framework of standard physics there is no definition of conscious information.

These objections raise several questions. Why the emergence of life would be accompanied by a generation of entropy? What could serve as a measure for conscious information? How to describe cognition? To these questions adelic physics provides a possible answer. If entropy that England talks about is identified as the entropy produced in SFRs of systems having $N = 0$, TGD view is consistent with the proposal of England.

Cognitive fixed point instead of thermal equilibrium?

The analogy with the second law strongly suggests that the system approaches a cognitive fixed point (negentropy maximum) during the sequence of SSFRs followed by the analog of unitary time evolution. SSFRs cannot generate negentropy anymore. Since the system does not learn anymore, BSFR is bound to occur. A possible number theoretic formulation for the fixed point could be following.

1. The time evolution following SSFR generates entanglement. This entanglement is maximally reduced in measurements of observables, which correspond to operators, whose action does not affect the states at the passive boundary.

2. Cognitive measurements define an important class of such measurements [L121]. The cognitive quantum states correspond to wave functions in the Galois group G of the extension - that is elements of the group algebra $F(G)$ of G. G can be decomposed to a product $G = \prod G_i$ of subgroups defined by the hierarchy of normal subgroups of G defined by the representation of the extension as an extension of an extension of ... of rationals.

   Elements of $F(G)$ decompose to superpositions of products of functions in G_i and the factors are entangled. Note that the order of G_i matters and is induced by the inclusion hierarchy for the extensions considered: the largest extension is at the top of the hierarchy. One has "ordered" entanglement. This is analogous to the directedness of attention which is difficult to understand in the standard physics framework.

   Eastern philosophies speak also of states of consciousness in which there is no distinction between observer and observed and not division. Could this kind of attention involve negentropic entanglement between systems, which correspond to the same extension of rationals so that the. attention cannot be directed? Or could it correspond to negentropic cognitive entanglement allowing cognitive SSFRs?

12.9 TGD based model for aging

In this section the TGD based view about aging is discussed. The king idea is that the magnetic bodies (MBs) of information molecules and linear molecules formed from them (DNA,RNA, amino-acids,tRNA) are at very low temperature in the beginning. The temperature of MB starts to raise and approach the physiological temperature. The entropy of MB increases. Since the MB of the molecule controls the molecule, the control by MB starts to fail and this leads to the diseases accompanying aging.

12.9.1 Aging as approach of MB and BB to thermal equilibrium

Ordinary entropy increases for an isolated system, it approaches thermal equilibrium - thermalizes. Aging must correspond to thermalization in some sense. There are two views about this.

1. The weak form of the proposal making sense in the standard physics context would identify aging as thermalization. For ordinary biomatter, which already is in thermal equilibrium in good approximation, this idea does not lead to anything interesting.

2. Ordinary matter and the dark matter at MBs carrying dark matter as phases of ordinary matter with $h_{eff} = nh_0$ have widely different temperatures at the moment of birth. Aging means that these systems approach thermal equilibrium in the sense that temperatures become identical . MB has infinite number of degrees of freedom and therefore maximal temperature known as Hagedorn temperature identifiable naturally as physiological temperature [L177]. This option will be considered in the sequel.

   Consider the situation in TGD.

1. What are the continuous degrees of freedom whose entropy growth would lead to aging. In the TGD framework they would be naturally the geometric degrees of freedom associated with the flux tubes of dark DNA controlling ordinary DNA. Their number is infinite implying that temperature is below Hagedorn temperature around physiological temperature. One can assign a temperature to the flux tubes and also to these degrees of freedom and this is below Hagedorn temperature. When temperature at flux tubes increases, the geometric shape starts to thermally fluctuate and the overall size increases. Cells indeed increase in aging as do also we!

2. For information molecules the temperature of MB must be very low: dark DNA flux tubes have a very precise shape and therefore also ordinary DNA. For SPs the situation is different and this makes possible their basic functions.

3. Aging could simply mean that the dark genome approaches thermal equilibrium with ordinary biomatter at physiological (Hagedorn) temperature and entropy of dark genes and magnetic flux tubes increases. Flux tubes get more and more irregular shaped and induce a development of loops for ordinary DNA and breaks DNA double strand. Nucleosomes are loop like structures associated with histones and also these are known to be lost. Epigenetic chaos is induced. When thermal equilibrium is achieved death as heat death occurs and changes the arrow of time at the level of the entire body which is left in the geometric past of the standard observer to continue life with an opposite arrow of time.

4. There is a connection to the article [L175] about DNA and arrow of time. One can argue as follows. As the electric field along DNA double strand decreases by the shortening of the sticky ends, the string tension as density of electric energy per length decreases, the stiffness of DNA decreases, and the fluctuations start to develop. Second possibility is that the shortening of telomeres and sticky ends is a controlled process causing a programmed aging.

5. There are molecules devoted to preparation of the damaged DNA. The epigenetic tags on histones of mating genes could control the arrow of time for the gene involved. If the tag is present, the gene is expressed. If not or if inhibited by say Ser2, the arrow of time is reversed.

Objections related to metabolism

Metabolic energy feed is needed to keep the distribution of h_{eff}:s and even increase the values of h_{eff} - defining universal IQ and characterizing quantum coherence scale. This relates to the second important aspect of life: quantum coherence in long length scales is needed to generate the coherent behavior of ordinary biomatter and is not possible in standard bio.chemistry framework. Ageing would be a gradual reduction of this quantum coherence by thermalization of MBs of the basic information molecules, in particular the dark variants of the basic biomolecules. If you want to live long, take care of your personal quantum coherence!

One can develop some objections against the vision about ageing as thermalization of the MBs of information molecules.

1. Aging is viewed as changes of the body after birth. What about the processes before birth? When sperm and gametes inoculate and divide and divide and form some distinguished organs, this process needs a high amount of energy; that is why mothers get to eat more during pregnancy.

   Fetus generates new structures - parts of MBs containing dark matter as $h_{eff} = n \times h_0$ phases of ordinary matter with increasing value of h_{eff}. This requires high metabolic energy feed provided by mother. Information molecules are however still very far from thermal equilibrium and the gradual increase of the temperature of MBs has practically no effects. The situation remains the same also at the young age. At later age MBs approach thermal equilibrium and problems with the bio-control by MB emerge.

2. Reactive Oxygen Species (ROS) cause also damage for DNA: the more ROS, the shorter the life of the cell. That is why food with low calories content or fastings or low carbohydrates (which need less oxygen to burn) diets are good for longevity.

   ROS have been also seen as a cause of aging and one could argue that they should cause a lot of damage during the fetus period involving intense metabolism. The repair mechanisms of MB work almost optimally for fetus and at young age and allow handling of the problems due to ROS. The authors of [**?**] argue that it is now known that ROS are not the basic reason for aging. As a matter of fact, ROS are essential for the demethylation [**?**].

Is apoptosis as programmed cell death consistent witht he proposal?

Also programmed cell death - apoptosis - could be treated as an objection against aging as approach to thermal equilibrium. Apoptosis as a programmed cell death cannot be purely thermal event. It could be induced by MB at the higher level of hierarchy as BSFRs. Perhaps by MB of cell group as in the development of fingers from the cell mass. Apoptosis could have several motivations.

1. The basic prediction is that giving systems are intentional systems having free will at all levels. MB at the higher level could act like dictator and destroy quantum coherence locally leading to the death of the cells but generating quantum coherence and generation of structures in longer scales which would also take the metabolic energy resources used by the dead cells.

   Negentropy Maximization Principle (NMP) would be the deeper principle behind the second law. Apoptosis would be consistent with the NMP which implies second law as a by-product and as its name tells, implies negentropy increase. Therefore thermalization would not the only cause of cell death.

   Apoptosis would indeed generate more complex structures when fingers develop from tissue. Destruction of lower level structure would be the price paid for the generation of a higher level structure and negentropy gain in longer scales.

2. Evolution means steps in which h_{eff} increases in BSFRs and longer scales of quantum coherence at the level of MB emerge. Extinction of cells and sauri would be part of evolution.

A controlled BSFR causing the analog of death/hibernation of a subsystem could be also represented as an objection. The BSFR would have the survival of a larger system as a motivation [L177]. In fact, all motor actions can be seen as BSFRs at some lower level so that life is continual dying! Death/hibernation of a subsystem means savings of metabolic energy and can be seen as one manner to fight against second law since the dead subsystem lives with the opposite arrow of time: living system is basically 4-D entity in ZEO - not just the time slice which corresponds to conscious experience!

Is the biochemical approach trying to describe dissipation as a controlled process?

A general comment related to the distinction of the standard approach and the TGD approach relying on ZEO is in order. Standard approach does its best to identify control mechanisms leading from state A to state B. Huge amount of information exists about reaction pathways and one can only admire these data mountains.

This approach is very natural as long as time reversal is not involved. If this is the case, there are processes, basically healing and repair processes, that occur in a reversed time direction as dissipative processes and each BSFR leading to a time reversed state involves its own reaction pathways. The tragedy would be that the standard approach tries desperately to understand loss of order as a controlled process inventing endlessly reaction pathways!

Of course, this work would not be useless. The problem is however that a deeper understanding is missing and prevents seeing how incredibly simple the picture is at the fundamental level.

1. The increase of entropy in reverse time direction apparently breaks second law in the standard time direction. Stress proteins (SPs) discussed in [L177] are involved with this battle. The magnetic bodies of SPs can extract heat energy from the environment in heat shock and heat DNA and proteins in cold shock, and also act as heat engines for molecular motors.

2. As also the authors of [?] emphasize: diseases are a consequence of a loss of information. Disorder increases as quantum coherence is lost, and manifests as numerous diseases. Quite concretely, the h_{eff} distribution flattens in the sequence of SSFRs. System gets less intelligent and is unable to cope with the hard reality! Second law would eventually win although this process can be slowed down by BSFRs of subsystems.

3. Things can go wrong in very many ways: as Tolstoy said, families can be unhappy in myriads of different ways but there are very few ways to be a happy family! Therefore the disease centered thinking of medicine is perhaps not the best approach. One should do something which helps to prevent all diseases simultaneously. One can avoid diseases by choosing a healthy lifestyle. Also a medicine relying on the idea that BSFRs for subsystems could help. BSFR could be seen also as falling sleep and resting and gathering metabolic energy - even from thermal energy.

4. Time reversed dissipative evolution looks like healing in the reversed arrow of time. If this is the case, the tragedy of biology would be the attempt to understand time reversed dissipation in terms of complex control actions based on complex reaction pathways or even as some kind of computer programs.

5. MB would be in a key role since most diseases would be problems in the control performed by MB and basically due to the reduction of h_{eff} and therefore of information contents. About detailed criteria for when one has a disease this approach cannot say much.

Loss of the control of housekeeping genes causes vicious circle leading to death

The basic problem from the point of view of longevity would be that during aging MB gradually loses control of not only methylation, acetylation and their reversals but also other modification processes.

A possible explanation for hypermethylation is that the control actions inducing demethylation fail. The observed hypomethylation in the complement of CpG islands could be due the failure of methylation so that the state becomes stable. More generally, this suggests that the loss of control of all modifications is the mechanism leading to the situation in which the modifications cannot be changed. For instance, the differential methylation of hippocampus is known to be relevant for memory recall, which could relate to the emergence of memory problems at the old age.

CpG:s which are hypermethylated appear in the promoter regions of almost all housekeeper genes so that housekeeping [?] (`https://cutt.ly/2jQgOSD`), in particular transcription and translation machineries, metabolism, functioning of stress proteins, etc.... becomes difficult.

The enzymes responsible for the methylation and demethylation are especially important for housekeeping genes [?] whose promoter regions contain CpG islands. Metabolism related enzymes like Cytochrome P450 are involved also with demethylation as enzymes. Methylation of the promoter region of housekeeping genes means also methylation of genes coding for demethylase. This vicious circle - not so positive positive feedback - leads to death.

What causes the loss of the control of these modifications? The mechanisms leading to the loss of control would relate to modifications of the chromatin and DNA organization. These include reduced global heterochromatin, nucleosome remodeling and loss, changes in histone marks, global DNA hypomethylation with CpG island hypermethylation, and the relocalization of chromatin modifying factors [?, ?, ?]. In the TGD framework these changes would be caused by the thermalization of the MBs of DNA and chromosomes.

Also cancer induces these changes about which the appearance of additional chromosomes in the mitochondrial DNA in cancer is an example. It has been found that a very weak oscillating magnetic field with strength in nanotesla range and with oscillation frequency around 60 Hz (Schumann frequency) leads to the disappearance of additional chromosomes [?]. The loss of quantum coherence is the general explanation but it is not clear whether this can be due to thermalization at the level of DNA in this case. A possible explanation is that the control by MB at a higher level of hierarchy is lost and the presence of magnetic field re-establishes a connection with this MB in turn re-establishing quantum coherence [L63].

MB controls the conformations of DNA and chromosomes. MB is identifiable as a flux tube network and its control relies on its motor actions involving reconnections and shortenings of the flux tubes by a temporary reduction of h_{eff}. These motor actions of MB would become fuzzy by the thermal motion. The precise motor performance of MB is crucial for the realization of modifications occurring at the promoter regions near histones and at histone tails. Therefore the thermalization of the flux tube degrees of freedom of MB could be the basic reason for the problems.

When does death occur?

Aging could simply mean that the dark genome approaches thermal equilibrium with ordinary biomatter at physiological (Hagedorn) temperature [L177] and entropy of dark genes and magnetic flux tubes increases. Flux tubes get more and more irregular shaped and induce a development of loops for ordinary DNA and breaks DNA double strand. Epigenetic chaos is induced. When thermal equilibrium is achieved death as an analog of heat death occurs and changes the arrow of time at the level of the entire body which is left in the geometric past of the standard observer to continue life with an opposite arrow of time.

BSFR means death and death is bound to occur. But when? TGD proposes a general criterion: at a given step either SSFR or BSFR occurs and the SFR that means maximum negentropy gain takes place. This SFR is not unique. It can be either SSFR or BSFR and in both cases there are a lot of options for the final state.

When would BSFR win in the comparison?

1. Total entropy can be defined as the sum of p-adic and real Shannon entropies. p-Adic Shannon entropies involving p-adic norms $N_p(p_k)$ of probabilities p_k in the logarithmic factors $log(N_p(p_k))$ can be negative. In this case they characterize the information (associated with cognition) assignable to the entanglement.

2. Real entropy characterizes the lack of information about the state of either entangled system and is associated with sensory input (is the cat dead or alive?). The sum of the real and p-adic entropies can be negative for non-trivial extensions of rationals so that one would have genuine cognitive information. One could also speak of mere cognitive information as the p-adic contribution to the entropy and this can be negative.

3. Intuitively it seems obvious that thermalization meaning that the temperature difference between MBs and systems such as genes is reduced, means loss of information defined in this manner. Information molecules cease to be information molecules at least in the geometric degrees of freedom.

4. BSFR becomes the winner if SSFRs can give only very small negentropy gain or if the negentropy gain becomes negative. The fact that we do not learn much anymore at the old age, could reflect the reduction of the negentropy gain in SSFRs.

Also the distribution of $h_{eff} = n \times h_0$ values could reduce IQ. As found in [L121], a complete cognitive measurement inducing maximal reduction of entanglement for an extension with dimension n would reduces the state to a product state with state space with dimensions n. which are factors of n and thus smaller than n: instead of single MB with high IQ several with lower IQs. It might happen that the next SSFRs are not anymore able to regenerate larger values of n and the system becomes less intelligent.

An objection against this picture is that there are also situations when resurrection seems to occur: this has happened for people having had near-death experiences. One can also slow down the process of aging by a appropriate diet.

1. The slowing down of the aging process is possible by the reversal of the arrow of time at lower levels so that time reversed dissipative processes at these levels look like self-organization and generate order from the point of view of the organism. This would be a general. mechanism used by living matter to slow down the approach of MB to thermal equilibrium with MB.

2. This does not however explain resurrection. The opposite BSFR can however occur at the level of the entire organism but with a suitable stimulation like resuscitation opposite BSFR can take place. Taking into account the fact that also the organism is only one level in the hierarchy of conscious entities, this reduces to the first option.

   The analog of resuscitation occurs at the atomic level in the experiments of Minev *et al* [L77]. Although the deterministic process apparently leading to the final state of BSFR had already occurred, it could be stopped by a suitable stimulus. In TGD framework the interpretation is that the BSFR had already occurred and the time reversed time evolution apparently leading to the final was observed. A suitable stimulation however induced the opposite BSFR so that the process apparently stopped [L77].

DNA and the arrow of time

There is a connection with the article about DNA and arrow of time by Rastmanesh and Pitkänen [L176]. The proposal is as follows. As the electric field along DNA double strand (with dark DNA strands included) decreases by the shortening of the sticky ends, the string tension as density of electric energy per unit length decreases, the stiffness of DNA is reduced, and thermal fluctuations start to develop.

Biologist might wonder how various biological and homeostatic maneuvers like weakening of acetylation/phosphorylation/methylation eventually translate to a decrease of the electric field strength along DNA! In the TGD framework one can see the situation in a different manner: chemistry is not the boss now but is controlled by MB.

1. Modifications (or rather, the loss of the control of modifications) are not the primary cause of the weakening of the electric field. What happens at the control level, at MB, is the primary cause. The weakening of the electric field along DNA would correlate with the shortening of the sticky ends carrying electric charges creating the longitudinal electric field.

   This would also correlate with the reduction of the level of consciousness at the level of DNA if one is ready to generalize Becker's findings [J3] about the correlation of the strength of the longitudinal electric field along the body axis with the level of consciousness. Similar correlation with consciousness can be assigned to the electric field directed from visual cortex to frontal lobes.

2. The reduction of the electric field strength reduces energy density of DNA and therefore string tension. DNA begins to fluctuate geometrically, which generates epigenetic noise. Initially dark DNA is like a tense guitar string but transforms gradually to spaghetti. Basically the reduction of string tension reflects the dissipation accompanying the approach of MB to thermal equilibrium with the ordinary bio-matter.

   One could perhaps say that the reduction of string tension of MB flux tubes forces the reduction of electric field strength and the internal consistency (Maxwell equations) requires reduction of the sticky end lengths proportional to the charges generating the electric field along DNA. Note that also charge separations tend to disappear in the approach to thermal equilibrium.

3. An interesting question is whether hyper-methylation accompanying aging [**?**, **?**, **?**] could be seen as an attempt to minimize the effects of DNA damage - analogous to an amputation of a leg to prevent necrosis. Hyper-methylation accompanies also cancer [**?**, **?**].

   Second view is that hypermethylation is due to the loss of control of MB caused by the approach to thermal equilibrium. Hyper-methylation could be seen as the failure of demethylation caused by the low level of demethylase activated by MB and caused by methylation of the genes coding for demethylase! This positive feedback loop would lead to the failure of the control of MB.

Is the shortening of the telomeres a controlled process or due to thermalization? The first option could be argued to be realistic since otherwise the population would end up to fight about metabolic resources. Second law could of course solve the problem without any need for a controlled action. If the length of telomeres correlates with the charges of the sticky ends proportional to its length which in turn would be proportional to the length of the telomere as proposed in [L176], the conclusion would be that the shortening is not a controlled process.

12.10 Epigenesis and aging in TGD framework

In the TGD framework epigenesis would be control of the biological body by MB consisting of ordinary biomatter. The basic control tool would be dark photon 3N-plets coupling resonantly to the dark proton sequences of proteins serving as enzymes and RNAs serving as ribozymes. The coupling would be precise and based on the addresses defined by dark proton 3N-sequences defining emitting dark 3N-photons.

These would in turn catalyze the basic biochemical processes and here TGD suggests a mechanism explaining why the reactants find each other and where the energy needed to overcome the energy barrier to make reaction fast enough come. The reduction of h_{eff} for flux tubes would be the needed mechanism.

Also other catalysts than enzymes and ribozymes can be considered. For these catalysts and organic and non-organic molecules in general, the coupling with MB could be single photon resonant coupling transforming 3N-photon to bio-photon.

12.10.1 How MB could control biochemistry

How does the general biochemistry picture involving biomolecules and reaction pathways relate to the multi-resonance vision about how MB controls ordinary biomatter? Can one reduce this picture to a description in terms of multir-resonance frequencies - that is to the level of MB and MB-BB communications alone.

1. Suppose that MB of DNA, RNA, or protein controls DNA, RNA or protein by signals from dark genes using multi-resonance mechanism allowing to select the target and use modulation of dark photon signal to code control signals. Also the MB of DNA, RNA, or protein can be controlled by a higher level of the hierarchy.

   If all control takes place in this manner, epigenetic control would be control of proteins acting as enzymes, of RNA, in particular ribozymes, and of DNA. MB would also activate genes coding for various enzymes, in particular housekeeping enzymes.

   The controlled proteins would be naturally enzymes catalyzing various biochemical reactions.

2. Could the MB of DNA just change its geometric conformation inducing change of DNA conformation and changing also the epigenetic patterns determined by methylation, etc...? This would represent something new: in TGD one has a network of molecules connected by flux tubes, in biochemistry approach one has only molecules.

   The first basic mechanism for the change of the conformation would be the reduction of h_{eff} leading to the shortening of the flux tubes and liberation of energy and its reversal. The reduction of h_{eff} is crucial in the TGD based model of bio-catalysis. The opposite process would feed metabolic energy to MB. The formation of reconnection would be another key process and allow to change the topology of the flux tube network. This would be the basic mechanism of the immune system and also of biocatalysis in which the U-shaped flux tubes associated with the reacting molecules would reconnect.

   For instance, the actions of cells during say catastrophic events mean typically that proteins like Sir2 come in rescue by travelling along flux tubes or pairs of them serving as highways: these highways do not exist in standard biology. The existing pattern of flux tubes determines the road network. MB would control the topology of this network by reconnecting and by controlling the lengths of the flux tubes by h_{eff} changing transitions: motor actions of MB would be in question.

3. TGD leads to a view about emotions as sensory perceptions of MB. The model for genetic code emerging from a model of bio-harmony [L19, L75, L117, L133] based on icosahedral and tetrahedral geometries and the observation that music expresses and induces emotions leads to the proposal that the bio-harmony characterized by 64 allowed 3-chords in one-one correspondence with DNA codons has 3N-resonances assignable to the 3-chords of the harmony as a correlate. These resonant interactions induce transitions of selected bio-molecules and possibly also specific transitions of a given biomolecule characterizing the harmony. Could epigenesis be regarded as expressions of emotions by music of light?

BSFR would create a superposition of deterministic time evolutions leading to the geometric past. It would define an average time evolution described in terms of reaction pathways. Could the final state of MB in BSFR dictate also the epigenetic patterns - say bio-harmony determined by frequencies of cyclotron transitions of protons? They are indeed determined by the strengths of the magnetic fields at flux tubes. This would conform with the proposal that the outcome of volitional action as BSFR dictates what happens in the brain of geometric past explaining the findings of Libet [J2] [L77].

SSFRs give rise to an approximately classical time evolution and generation of entropy, and therefore aging. h_{eff} distribution becomes flatter and MBs of information molecules and ordinary matter approaches thermal equilibrium. The distribution for the conformations of the magnetic flux tubes thermalizes and cell size increases. Basically string tensions decrease since the electric fields involved weaken and the electric and also magnetic contribution to the tension weakens.

What could be the general mechanism of bio-catalysis? The MB of the enzyme could activate the enzyme when the value of h_{eff} of a flux tube connecting it to other reactants is reduced and induces the shortening of the flux tube and liberation of energy.

Depending in what direction energy flows, one can imagine two scenarios for what happens.

1. The energy could flow from higher levels of hierarchy to lower levels. The flux tube at the highest level would be shortened and liberate energy transferred to a lower level. At the lowest level enzyme would be excited and return to the ground state and liberate the energy needed to overcome the potential wall making reaction slow. Now the shortening of the flux tube of enzyme's MB does not seem to be necessary energetically.

   The process would proceed from higher to lower levels in the hierarchy of MBs by this kind of excitations and de-excitations transferring the energy to the lower level, somewhat like in photosynthesis. The flow of money from top towards bottom in a big project serves as a second metaphor for the course of events.

2. If the process is a generalized motor action involving BSFRs and time reversals, the higher levels in the hierarchy of MBs extract energy from shorter scales (very much like higher levels extract work of lower levels in the social hierarchies!). One could also say that negative energy is sent to the lower levels of the hierarchy.

   The lower level would provide the energy by reducing its h_{eff} so that its energy is reduced and energy is liberated and taken by the higher level. This would induce the shortening of magnetic flux tubes at all levels. The cascade would proceed down to the level of MBs of proteins and also the U-shaped flux tubes connecting the protein to the other reactants would shorten and bring the reactants together. The reduction of the flux tube length should provide energy to overcome the potential wall, not only the energy going to the higher level of hierarchy.

It is not quite clear which option is realized. Motor actions involve transfer of metabolic energy from short to length scales giving rise to macroscopic coherent motion: time reversal would be natural from this point of view.

Methylation and acetylation

The figures of `https://cutt.ly/Qjgrko3` illustrate the effect of methylation and acetylation of DNA or of histone tail.

1. The nucleosomes [?] surround a given gene but nucleosomes can roll along DNA downstream towards the gene to be transcribed and opens the DNA double strand. The modification of the histone tail can prevent or facilitate the opening of the double strand.

2. The portion of DNA between the nucleosome and gene corresponds to the promoter part of DNA initiating the gene expression. Proteins initiating the transcription bind to it or alternatively it can be transcribed to RNA.

The methylation [?] and acetylation [?] of the histone tail serves as the first example. The binding of the methyl, acetyl, or some other group to the histone tail has an indirect effect on the gene. Histone is positively charged. Since DNA is negatively charged, DNA and histone bind together.

The addition of a modifier can increase or reduce the charge of the histone and tighten or loosen the binding between histone and DNA. Methyl is positively charged and tightens the binding and makes the opening of DNA necessary for transcription more difficult. Acetyl is negatively charged and loosens the binding so that the transcription becomes easier.

Consider next the methylation of a promoter region (`https://cutt.ly/3jj8ohZ`).

1. A promoter is a sequence of DNA to which proteins bind that initiate transcription of a single RNA from the DNA downstream of it. This RNA may encode a protein, or can have a function in and of itself, such as tRNA, mRNA, or rRNA. Promoter region has therefore two - it seems alternative - functions.

2. The methylation of cytosin occurs at CpG islands associated with the promoter region of gene. Promoter region is the region to which proteins initiating the transcription of gene bind. Methylation occurs also for the promoter regions of CpG-islands [?].

3. How methylation silences the gene transcription? Methylation decreases the charge of DNA locally and loosens the binding to histone. This would favor the transcription of the promoter region instead of the transcription of the gene requriing the binding of RNA polymerase to the promoter region.

4. If methyl is always positively charged, the direct binding to DNA reduces DNA charge locally and reduces the interaction between histone and help opening of DNA in the promoter region: this would not facilitate the transcription of gene but transcription of non-translated RNA or protein. The binding would also prevent the binding of RNA polymerase to the promoter region. The start codon of the non-protein coding gene could be in the promoter region.

More facts about DNA methylation [?] (`https://cutt.ly/pjj3G45`) are needed to develop a TGD based view about the situation.

1. DNA methylation reprogramming occurs during gameto-genesis and early embryogenesis. The methylation patterns are erased and regenerated. This requires that the memory about the methylation pattern is stored. In the TGD framework MB could serve as the temporary information storage.

2. DNA methylation occurs also in highly transcribed gene bodies and must be distinguished from the methylation of promoter regions. The methylation of gene bodies seems to relate to splicing and could prevent the transcription of intronic portions of the gene.

3. In general, the level of DNA methylation is very low. The level of methylation is however high in promoter regions. In particular, in CpG islands [?] (`https://cutt.ly/sjj3STW`) accompanying the promoter regions of genes, especially those coding for housekeeping proteins.

4. Usually the methylation of C in CpG leads to a mutation replacing C with T. This could have led to CpG loss in DNA except in CpG islands, where some stabilization mechanism should prevent the mutations: presumably an energy barrier somehow caused by CpG is involved.

   Hypermethylation accompanies cancer and also aging [?, ?] and could be seen in the TGD framework as reflecting the approach to epigenetic chaos basically due to the reduction of the scale of quantum coherence in turn caused by the reduction of the values of h_{eff}.

5. CpG loss is believed to be induced by transposable elements (TEs) attaching to DNA and hopping around it. TEs are methylated and lose CpG as C transforms to T.

6. So called housekeeping proteins [?] are enzymes crucial for various functions including general gene expression, and the control of various housekeeping functions takes place via the control of the expression of housekeeping genes. Therefore CpG islands, which are stable against mutations and allow both methylation and demethylation are needed. Dynamical and differential methylation is also known to relate to memory recall in the case of hippocampus.

   For CpG islands $C \to T$ the mutations induced by methylation are prevented by some mechanism. The loss of CpG makes sense outside CpG islands since this stabilizes the genes against C $\to$ T mutations.

MB uses enzymes and ribozymes as a tool in the control of the basic biochemical processes. DNA methyltransferases [?] catalyze methylation and MB would control the process by activation of this enzyme. In the case of demethylation the enzymes used are demethylases [?].

The mechanism of demethylation can be taken as an example since the failure of demethylation might lead to hypermethylation of CpG islands known to accompany aging [?, ?] and in TGD framework it could be due to the approach of the dark genome and proteome to thermal equilibrium.

1. Oxidative demethylation [?] (`https://cutt.ly/Gjj2Ilz`) replaces CH_3 group with hydrogen. This requires the presence of a reactive oxygen species (ROS). ROS include superoxide O_2^-, hydrogen peroxide H_2O_2 and hydroxyl radical OH.(`https://cutt.ly/yjj2Ubv`).

   Superoxide is produced in aerobic metabolism via $O_2 + e^- \rightarrow O_2^-$. This in turn leads to reactions $2H^+ + 2O_2^- \rightarrow H_2O_2 + O_2$ followed by $H_2O_2 + e^- \rightarrow HO^- + OH$ and $2H^+ + 2e^- + H_2O_2 \rightarrow H_2O$.

2. Demethylation is catalyzed by demethylases in presence of O_2. N-methyl groups are oxidized with oxygen coming from ROS O_2 and CH_2O splits out so that the net reaction is $R_2N - CH_2 \rightarrow R_2N - H + CH_2O$.

   Enzymes known as alpha-ketoglutarate-dependent hydroxylases act as DNA demethylases. Also Cytochrome P450 [?] (`https://cutt.ly/ujj2OuK`) catalyzes demethylation in histones and some forms of DNA (cytocine associated with CpG). "450" refers to "450 nm"", which is the wavelength at which cytochrome P450 has maximum absorption. The wavelength corresponds to blue light near UV range and the photon energy is 2.76 eV. CYPs is a very large class of enzymes catalyzing metabolic processes.

What TGD view could be?

1. Suppose that MB controls bio-matter by expressing its moods coded by bio-harmonies in terms of dark photons 3N-plets (say) with the frequency patterns correlating with mood and affecting matter in mood dependent manner via a transformation to bio-photons.

2. 60 per cent of promoter regions of human genes contain CpG islands of length about 100-1000 codons and almost all housekeeping genes have CpG islands in their promoter regions. Why?

3. MB would induce both methylation and demethylation and other modification using various enzymes which they could activate by dark 3N-photons using as address the dark proton sequence associated with the enzyme. After activation the reaction would proceed by the proposed general mechanism of biocatalysis.

4. One can imagine several alternative courses of events after the activation. Even the question whether the energy transfer is from short to long length scales associated with MB or vice versa is not fully settled: this depends on the arrow of time assignable to this process.

   In the case of CYP450, one cannot avoid the temptation to ask whether a biophoton with 450 nm wavelength could be formed in a decay of 3N-dark photon to ordinary photon.

Methylation, aging, and memory

According to the abstract of [?] (`https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3482848/`), the general trends, supported by an increasing body of both in vitro and in vivo work, are the establishment of global hypomethylation (non-CpG islands) and regions of hypermethylation (primarily CpG islands) with age. CpG islands are located in the promoter regions of genes, in particular those of housekeeping genes.

Hypomethylation outside CpG islands could be due to the spontaneous mutation $C \rightarrow T$ but also the failure of the control of the methylation by MB could be involved. Hypermetylation of promoter regions implies that the promoter region transcribing RNA is transcribed instead of gene. This could be due to the failure of demethylation caused by the loss of the control.

In both cases the loss of control could have the same reason. The MBs of the genes coding for housekeeping genes and possibly also the MBs of the housekeeping enzymes approach thermal equilibrium with the ordinary bio-matter.

How methylation could relate to aging in TGD framework?

1. Methylation could become irreversible during aging and lead to hypermethylation if MB loses the control of demethylation. If enzymes are the control tools of MB, the reduced transcription of DNAs coding for demethylases would lead to a failure of the control. The approach of MBs of DNA and enzymes to thermal equilibrium with ordinary biomatter could be the basic reason for the failure.

2. Housekeeping proteins are an especially important class of proteins since they catalyze basic biological functions necessary for the transcription of genes - also the genes coding form them. Their promoter regions are also almost always CpG islands. Therefore one can say that the methylation of their promoter regions would be a natural cause of various problems with housekeeping activities caused by aging.

3. CYP450 catalyzes generation of ROS in turn catalyzing demethylation and a large number of metabolic processes crucial for the functioning of the organism. In particular, demethylation could become less effective with aging due to the reduced level of CYP450. CYP450 is a housekeeping protein and the promoter regions of genes coding for CYP450 would be methylated. Methylation slows down transcription of CYP450 and this in turn slows down demethylation. This positive feedback loop eventually leads to a kind of death spiral.

4. Differential methylation of the hippocampus is known to be crucial for the memory recall (here memories are understood as learned behaviors rather than episodal memories) [J10]. Differential methylation is not possible without demethylation. If methylation becomes irreversible the formation of recallable memories becomes more difficult. Short term memory recall as also memory recall in longer time scales indeed become less effective during aging.

12.10.2 How epigenetic information is inherited?

There is evidence for the inheritance of epigenetic information.

1. Epigenetic inheritance takes place in mitosis and sometimes also in meiosis. The methylation related epigenetic disorder increases with aging.

2. How could the epigenetic information be inherited in this picture? It could be represented by the geometry of MB - flux tube network - and at the genetic level by both control genes. Emotional aspects, something new, would have bio-harmony as a correlate, and bio-harmony is determined by cyclotron frequencies determined by the magnetic field strengths of the flux tubes. Not only cell but also MB replicates and the replication of MB induces replication at cell and DNA level.

   Both genetic and epigenetic information could be inherited in the replication if MB replicates geometrically like a particle in the decay A→ A+A. Usually particles are regarded as pointlike and Feynman diagram expresses this. The line A decays to two lines A+A. This makes sense also for 3-surfaces, in particular magnetic bodies, replacing the point like particles. That replication occurs at the fundamental physics is a new element in TGD based vision.

   At the level of causal diamonds (CDs) A→A+A would look like follows. The CD of A in the initial state and the CDS of A+A in the final state would intersect and contain the vertex region. Could the moods of A be inherited by A:s in A+A under some conditions - in other words, are cyclotron frequency spectra of flux tubes of A inherited?: this is true i the flux tubes would replicate as such.

   Some methylation patterns are inherited in meiosis but not all. If these patterns are determined by the bio-harmony, magnetic flux tubes are copied faithfully in some cases even in meiosis but not always.

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Chapter 13

What Scattering Amplitudes Should Look Like?

13.1 Introduction

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant [K126, K43] a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the zero energy ontology applied already earlier in TGD inspired cosmology [K1]. ZEO motivated the replacement of the term "S-matrix" with "M-matrix". This led to the discovery that rather stringy formulas for M-matrix elements emerge in TGD framework.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix scattered in the chapters of books about TGD and often drowned into details and plagued by side tracks and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction is suggested in the chapters about twistors and TGD.

My hope is that this chapter might provide a kind of bird's eye of view and help the reader to realize how fascinating and profound and near to physics the mathematics of hyper-finite factors is.

The goal is to sketch an overall view about the ideas which have led to the recent view about the construction of M-matrix. First the basic philosophical ideas are discussed. These include the basic ideas behind TGD inspired theory of consciousness [K111]. the identification of p-adic physics as physics of cognition forcing the central idea of number theoretic universality, quantum classical correspondence, and the crucial notion of zero energy ontology.

The understanding of the fundamental variational principles of TGD is so detailed that one can sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to Euclidian regions of 4-D surfaces - preferred extremals - defined by orbits of wormhole contacts plus the string world sheets connecting them and carrying spinor modes. Fermioaction contains also a part associated with the boundaries of string world sheets at partonic orbits. As a consequence, fundamental fermions propagate as particles with momenta which are light-like in 8-D sense along the light-like geodesics defined by the boundaries of string world sheets at which spinor modes are localized. This strongly suggests 8-D generalization of twistor approach.

The topological identification of the basic interaction vertices is as partonic 2-surfaces at which the orbits of partonic 2-surfaces meet. Fermions behave like free massless (in 8-D sense) particles during propagation along boundaries of string world sheets but interact at partonic surfaces and associated wormhole contacts by classical induced gauge fields. The naive guess would be that the conformal scaling generator L_0 for super-symplectic algebra could serving as propagator mediating the interaction between fermions at opposite wormhole throats.

The notion of preferred extremal does not favor ordinary Feynman diagrammatics resulting from path integral approach. The picture suggested by twistorialization looks more natural. Scattering amplitudes would be analogous to a minimal sequences of calculations transforming a given initial state to a given final state located at boundaries of CD. I proposed this vision for many years ago in terms of bi-algebras and related structures but gave it up as too speculative, and the only remnant of the enthusiasism period is a little appendix [K13]. The basic operations would be product and co-product in the Yangian associated with the super-symplectic algebra. Interaction vertices would correspond product and co-product for the generators of the Yangian algebra. The generators of this algebra would be Noether super charges associated with strings connecting partonic two surfaces.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L17].

13.2 General Vision Behind Matrices

In the following I summarize the basic notions and ideas discussed in previous chapters.

13.2.1 Basic Principles

My original intention was to summarize the basic principles of Quantum TGD first. The problem is however where to start from since everything is so tightly interwoven that linear representation proceeding from principles to consequences seems impossible. Therefore it might be a good idea to try to give a summary with emphasis on what has happened during the few months in turn of 2008 to 2009 assuming that the reader is familiar with the basic concepts discussed in previous chapters. This summary gives also a bird's eye of view about what I believe M-matrix to be. Later this picture is used to answer the questions raised in the earlier version of this chapter.

Zero energy ontology

One of the key notions underlying the recent developments is zero energy ontology.

1. Zero energy ontology leads naturally to the identification of light-like 3-surfaces interpreted as a generalization of Feynman diagrams as the most natural dynamical objects (equivalent with space-like 3-surface by holography).

2. The fractal hierarchy of causal diamonds (CD) with light like boundaries of CD interpreted as carriers of positive and negative energy parts of zero energy state emerges naturally. If the scales of CDs come as powers of 2, p-adic length scale hypothesis follows as a consequence.

3. The identification of M-matrix as time-like entanglement coefficients between zero energy states identified as the product of positive square root of the density matrix and unitary S-matrix emerges naturally and leads to the unification of thermodynamics and quantum theory.

4. The identification of M-matrix in terms of Connes tensor product means that the included algebra $\mathcal{N} \subset \mathcal{M}$ acts effectively like complex numbers and does not affect the physical state. The interpretation is that $\mathcal{N}$ corresponds to zero energy states in size scales smaller than the measurement resolution and thus the insertion of this kind of zero energy state should not have any observable effects. The uniqueness of Connes tensor product gives excellent hopes that the M-matrix could be unique apart from the square root of of density matrix.

5. The unitary U-matrix between zero energy states assignable to quantum jump has nothing to do with S-matrix measured in particle physics experiments. A possible interpretation

is in terms of consciousness theory. For instance, U-matrix could make sense even for p-adic-to-real transitions interpreted as transformations of intentions to actions making sense since zero energy state is generated ("Everything is creatable from vacuum" is the basic principle of zero energy ontology) [K70] . One can express U-matrix as a collection of M-matrices labeled by zero energy states and unitaritity conditions for U-matrix boil down to orthogonality conditions for the zero energy states defined by M-matrices.

The notion of finite measurement resolution

The notion of finite measurement resolution as a basic dynamical principle of quantum TGD might be seen by a philosophically minded reader as the epistemological counterpart of zero energy ontology.

1. As far as length scale resolution is considered, finite measurement resolution implies that only CDs above some size scale are allowed. This is not an approximation but a property of zero energy state so that zero energy states realize finite measurement resolution in their structure. One might perhaps say that quantum states represent only the information that we can becomes conscious of.

2. In the case of angle resolution the hierarchy of Planck constants accompanied by a hierarchy of algebraic extensions of rationals by roots of unity, and realized in terms of the book like structures assigned with CD and CP_2, is a natural outcome of this thinking.

3. Number theoretic braids implying discretization at parton level can be seen as a space-time correlate for the finite measurement resolution. Zero energy states should contain in their construction only information assignable to the points of the braids. Note however that there is also information about tangent space of space-time surface at these points so that the theory does not reduce to a genuinely discrete theory. Each choice of M^2 and geodesic spheres defines a selection of quantization axis and different choice of the number theoretic braid. Hence discreteness does not reduce to that resulting from the assumption that space-time as the arena of dynamics is discrete but reflects the limits to what we can measure, perceive, and cognize in continuous space-time. Zero energy state corresponds to wave-function in the space of these choices realized as the union of copies of the page $CD \times CP_2$. Quantum measurement must induce a localization to single point in this space unless one is ready to take seriously the notion of quantum multiverse.

4. Finite measurement resolution allows a realization in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 (HFFs) about which the WCW Clifford algebra provides standard example. Also the factor spaces $\mathcal{M}/\mathcal{N}$ are suggestive and should correspond to quantum variants of HFFs with a finite quantum dimension. p-Adic coupling constant evolution can be understood in this framework and corresponds to the inclusions of HFFs realized as inclusions of spaces of zero energy states with two different scale cutoffs.

Number theoretical compactification and $M^8 - H$ duality

The closely related notions of number theoretical compactification and $M^8 - H$ duality have had a decisive impact on the understanding of the mathematical structure of quantum TGD.

1. The hypothesis is that TGD allows two equivalent descriptions using either M^8- the space of hyper-octonions- or $H = M^4 \times CP_2$ as embedding space so that standard model symmetries have a number theoretic interpretation. The underlying philosophy is that the world of classical worlds and thus H is unique so that the symmetries of H should be something very special. Number theoretical symmetries indeed fulfil this criterion.

2. In M^8 description space-time surfaces decompose to hyper-quaternionic and co-hyperquaternionic regions. The map assigning to $X^4 \subset M^8$ the image in $X^4 \subset H$ must be a isometry and also preserve the induced Kähler form so that the Kähler action has same value in the two spaces. The isometry groups of E^4 and CP_2 are different, and the interpretation is that the low energy description of hadrons in terms of $SO(4)$ symmetry and high energy description in terms of $SU(3)$ gauge group reflect this duality.

3. Number theoretic compactification implies very detailed conjectures about the preferred extremals of Kähler action implying dual slicings of the M^4 projection of space-time surface to string world sheets Y^2 and partonic 2-surfaces X^2 for Minkowskian signature of induced metric. This occurs for the known extremals of Kähler action of this kind [K15, K107, K125]. These slicings allow to understand how Equivalence Principle emerges via its stringy variant in TGD framework through dimensional reduction. The tangent spaces of Y^2 and X^2 define local planes of physical and un-physical polarizations and M^2 defines also the plane for the four-momentum assignable to the braid strand so that gauge symmetries are purely number theoretical interpretation.

4. Also a slicing of $X^4(X_l^3)$ to light-like 3-surfaces Y_l^3 parallel to X_l^3 giving equivalent space-time representations of partonic dynamics is predicted. This implies holography meaning an effective reduction of space-like 3-surfaces to 2-D surfaces. Number theoretical compactification leads also to a dramatic progress in the construction of quantum TGD in terms of the second quantized induced spinor fields. The holography seems however to be not quite simple as one might think first. Kac-Moody symmetries respecting the light-likeness of X_l^3 and leaving X^2 fixed act as gauge transformations and all light-like 3-surfaces with fixed ends and related by Kac-Moody symmetries would be geometrically equivalent in the sense that WCW Kähler metric is identical for them. These transformations would also act as zero modes of Kähler action.

5. A physically attractive realization of the braids - and more generally- of slicings of space-time surface by 3-surfaces and string world sheets, is discussed in [K60] by starting from the observation that TGD defines an almost topological QFT of braids, braid cobordisms, and 2-knots. The boundaries of the string world sheets at the space-like 3-surfaces at boundaries of CDs and wormhole throats would define space-like and time-like braids uniquely.

   The idea relies on a rather direct translation of the notions of singular surfaces and surface operators used in gauge theory approach to knots [A48] to TGD framework. It leads to the identification of slicing by 3-surfaces as that induced by the inverse images of $r = constant$ surfaces of CP_2, where r is $U(2)$ invariant radial coordinate of CP_2 playing the role of Higgs field vacuum expectation value in gauge theories. $r = \infty$ surfaces correspond to geodesic spheres and define analogs of fractionally magnetically charged Dirac strings identifiable as preferred string world sheets. The union of these sheets labelled by subgroups $U(2) \subset SU(3)$ would define the slicing of space-time surface by string world sheets. The choice of $U(2)$ relates directly to the choice of quantization axes for color quantum numbers characterizing CD and would have the choice of braids and string world sheets as a space-time correlate.

WCW spinor structure

The construction of WCW ("world of classical worlds", configuration space) spinor structure in terms of second quantized induced spinor fields is certainly the most important step made hitherto towards explicit formulas for M-matrix elements.

1. Number theoretical compactification ($M^8 - H$ duality) states that space-time surfaces can be equivalently regarded as 4-dimensional surfaces of either $H = M^4 \times CP_2$ or of 8-D Minkowski space M^8, and consisting of hyper-quaternionic and co-hyper-quaternionic regions identified as regions with Minkowskian and Euclidian signatures of induced metric. Duality preserves induced metric and Kähler form. This duality poses very strong constraints on the geometry of the preferred extremals of Kähler action implying dual slicings of the space-time surface by string worlds sheets and partonic 2-surfaces as also by light-like 1-surfaces and light-like 3-surfaces. These predictions are consistent what is known about the extremals of Kähler action. The predictions of number theoretical compactification lead to dramatic progress in the construction of configurations space spinor structure and geometry.

2. The construction of WCW geometry and spinor structure in terms of induced spinor fields leads to the conclusion that finite measurement resolution is an intrinsic property of quantum states basically due to the vacuum degeneracy of Kähler action. This gives a justification for the notion of number theoretic braid effectively replacing light-like 3-surfaces. Hence the

infinite-dimensional WCW is replaced with a finite-dimensional space $(\delta M^4_\pm \times CP_2)^n/S_n$. A possible interpretation is that the finite fermionic oscillator algebra for given partonic 2-surface X^2 represents the factor space $\mathcal{M}/\mathcal{N}$ identifiable as quantum variant of Clifford algebra. $(\delta M^4_\pm \times CP_2)^n/S_n$ would represent its bosonic analog.

3. The isometries of the WCW corresponds to X^2 local symplectic transformations $\delta M^4_\pm \times CP_2$ depending only on the value of the invariant $\epsilon^{\mu\nu}J_{\mu\nu}$, where $J_{\mu\nu}$ can correspond to the Kähler form induced from $\delta M^4_\pm$ or CP_2. This group parameterizes quantum fluctuating degrees of freedom. Zero modes correspond to coordinates which cannot be made complex, in particular to the values of the induced symplectic form which thus behaves as a classical field so that WCW allows a slicing by the classical field patterns $J_{\mu\nu}(x)$ representing zero modes.

4. By the effective 2-dimensionality of light-like 3-surfaces X^3_l (holography) the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. Quantum classical correspondence suggests that gauge fixing in Kac-Moody degrees of freedom takes place and implies correlation between the quantum numbers of the physical state and X^3_l or equivalently any light-like 3-surface Y^3_l parallel to X^3_l. There would be no path integral over X^3_l and only functional integral defined by WCW geometry over partonic 2-surfaces.

5. The condition that the Noether currents assignable to the modified Dirac equation are conserved requires that space-time surfaces correspond to extremals for which second variation of Kähler action vanishes. A milder condition is that the rank of the matrix defined by the second variation of Kähler action is less than maximal. Preferred extremals of Kähler action can be identified as this kind of 4-surface and the interpretation is in terms of quantum criticality.

   For given preferred extremal one expects the existence of an infinite number of deformations with a vanishing second variation of Kähler action. These deformations act as conformal gauge symmetries realizing quantum criticality at space-time level. The natural assumption is that the number, call it n, of conformal gauge equivalence classes of space-time surfaces with fixed 3-surfaces at their ends at the boundaries of CD is finite. This integer would characterize the effective value of Planck constant $h_{eff} = n \times h$.

6. The physically most transparent formulation of criticality as a hierarchy of broken super-symplectic conformal symmetries emerged rather recently. Super-symplectic algebra has an infinite fractal hierarchy of isomorphic sub-algebras with conformal weights coming as multiple of integer n for a given sub-algebra. The natural hypothesis is that the sub-algebra labelled by n acts as a conformal gauge algebra. This gives rise to infinite number of hierarchies of super-symplectic breakings labelled by sequences of integers $n_{i+1} = \prod_{k<i+1} m_k$. In a given symmetry breaking criticality is reduced as gauge degrees of freedom transform to physical ones. At quantum level the gauge sub-algebra labelled by n annihilates the physical states. At space-time level the corresponding super-symplectic Noether charges vanish. This defines precisely what it means to be a preferred extremal in zero energy ontology (ZEO).

Hierarchy of Planck constants

The hierarchy of Planck constants realized as a replacement of CD and CP_2 of $CD \times CP_2$ with book like structures labeled by finite subgroups of $SU(2)$ assignable to Jones inclusions is now relatively well understood as also its connection to dark matter, charge fractionization, and anyons [K43, K85].

1. This notion leads also to a unique identification of number theoretical braids as intersections of CD (CP_2) projection of X^3_l and the back M^2 (the backs S^2_I and S^2_{II}) of M^4 (CP_2) book. The spheres S^2_I and $S^2_{II}$are geodesic spheres of CP_2 orthogonal to each other).

2. The formulation of M-matrix should involve the local data from the points of number theoretic braids at partonic 2-surfaces. This data involves information about tangent space of $X^4(X^3)$ so that the theory does not reduce to 2-D theory. The hierarchy of CDs within CDs means that the improvement of measurement resolution brings in new CDs with smaller size.

3. The points of number theoretical braids are by definition quantum critical with respect to the phase transitions changing Planck constant and meaning leakage between different pages of the books in question. This quantum criticality need not be equivalent with the quantum criticality in the sense of the degeneracy of the matrix like entity defined by the second variation of Kähler action. Note that the entire partonic 2-surface at the boundary of CD cannot be quantum critical unless it corresponds to vacuum state with only topological degrees of freedom excited (that is have as its CD (CP_2) projection at the back of CD (CP_2) book or both) since Planck constant would be ill-defined in this kind of situation.

Super-conformal symmetries

The attempts to understand super-conformal symmetries has been unavoidably a guess work and produced several alternative scenarios. The consistency with p-adic mass calculations requiring five tensor factors to Super-Virasoro algebra has been the basic experimental constraint. The work with Kähler-Dirac equation has helped dramatically in the attempts to understand of super-conformal symmetries. Also the understanding of Super-Kac-Moody symmetries acting as gauge symmetries and made possible by the non-determinism of Kähler action has helped a lot.

There have been a considerable progress also in the understanding of super-conformal symmetries [K127, K28].

1. Super-symplectic algebra corresponds to the isometries of WCW constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW .

2. One expects also gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

3. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

4. A further step of progress relates to the understanding of the fusion rules of symplectic field theory [K22]. These fusion rules makes sense only if one allows discretization that is number theoretic braids. An infinite hierarchy of symplectic fusion algebras can be identified with nice number theoretic properties (only roots of unity appear in structure constants). Hence there are good hopes that symplecto-conformal N-point functions defining the vertices of generalized Feynman diagrams can be constructed exactly.

5. The possible reduction of the fermionic Clifford algebra to a finite-dimensional one means that super-conformal algebras must have a cutoff in conformal weights. These algebras must reduce to finite dimensional ones and the replacement of integers with finite field is what comes first in mind.

6. The conserved fermionic currents implied by vanishing second variations of Kähler action for preferred extremal define a hierarchy of super-conformal algebras assignable to zero modes. These currents are appear in the expression of measurement interactions added to the Kähler-Dirac action in order to obtain stringy propagators and the coding of super-conformal quantum numbers to space-time geometry.

13.2.2 Various Inputs To The Construction Of M-Matrix

It is perhaps wise to summarize briefly the vision about M-matrix.

Zero energy ontology and interpretation of light-like 3-surfaces as generalized Feynman diagrams

1. Zero energy ontology is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X_l^3 connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

2. Light-like 3-surfaces are the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

Identification of TGD counterpart of S-matrix as time-like entanglement coefficients

1. The TGD counterpart of S-matrix -call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD. One can also assign to quantum jump between zero energy states a matrix-call it U-matrix - which is unitary and assumed to be expressible in terms of M-matrices. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several good arguments suggesting that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.

2. In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive

square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW .

Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

Conformal symmetries and stringy diagrammatics

The Kähler-Dirac equation has rich super-conformal symmetries helping to achieve concrete vision about the structure of M-matrix in terms of generalized Feynman diagrammatics.

Both super-conformal symmetries and the effective reduction of space-time sheet to string world sheets at Minkowskian regions as a consequence of finite measurement resolution suggest that the generalized Feynman diagrams have as vertices N-point functions of a conformal field theory assignable to the partonic 2-surfaces at which the lines of Feynman diagram meet. The vertices can be assigned with wormhole contacts with Euclidian signture of induced metric. In

Minkowskian regions fundamental fermions propagate like massless particles along boundaries of string world sheets. One can say that a hybrid of Feynman and stringy diagrammatics results.

Finite measurement resolution means that this conformal theory is defined in the discrete set defined by the intersections of braids defined by boundaries of string worlds sheets with partonic two-surfaces. The presence of symplectic invariants in turn suggest a symplectic variant of conformal field theory leading to a concrete construction of symplectic fusion rules relying in crucial manner to discretization.

TGD as almost topological QFT

The idea that TGD could be regarded as almost topological QFT has been very fruitful although the hypothesis that Chern-Simons term for induced Kähler gauge potential assignable to light-like 3-surfaces identified as regions of space-time where the Euclidian signature of induced metric assignable to the interior or generalized Feynman diagram changes to Minkowskian one turned out to be too strong. The reduction of WCW and its Clifford algebra to finite dimensional structures due to finite measurement resolution however realizes this idea but in different manner.

1. There is functional integral over the small deformations of Feynman cobordisms corresponding to the maxima of Kähler function which is finite-dimensional if finite measurement resolution is taken into account. Almost topological QFT property of quantum suggests the identification of M-matrix as a functor from the category of generalized Feynman cobordisms (generalized Feynman diagrams) to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces.

2. The limit at which momenta vanish is well-defined for M-matrix since the Kähler-Dirac action contains measurement interaction term and at this limit one indeed obtains topological QFT.

3. Almost TQFT property suggests that braiding S-matrices should have important role in the construction. It is indeed possible to assign the with the lines of the generalized Feynman diagram. The reduction of quantum TGD to topological QFT should occur at quantum criticality with respect to the change of Planck constant since in this situation the M-matrix should not depend at all on Planck constant. Factoring QFTs in 1+1 dimensions give examples of this kind of theories.

Heuristic picture about generalized Feynman rules

Concerning the understanding of the relationship between HFFs and M-matrix the basic implications are following.

1. General visions do not allow to provide explicit expressions for M-matrix elements. Therefore one must be humble and try to feed in all understanding about quantum TGD and from the quantum field theoretic picture. In particular, the dependence of M-matrix on Planck constant should be such that the addition of loop corrections as sub- CDs corresponds to an expansion in powers of $1/\hbar$ as in quantum field theory whereas for tree diagrams there is no dependence on $\hbar$.

2. The vacuum degeneracy of Kähler action and the identification of Kähler function as Dirac determinant strongly suggest that fermionic oscillator operators define what could be interpreted as a finite quantum-dimensional Clifford algebra identifiable as a factor space $\mathcal{M}/\mathcal{N}$, $\mathcal{N} \subset \mathcal{M}$. One must be however very cautious since also an alternative option in which excitations of labeled by conformal weight are present cannot be excluded. Finite-dimensionality would mean an enormous simplification, and together with the unique identification of number theoretic braids as orbits of the end points of string world sheets this means that the dynamics is finite-quantum-dimensional conforming with the fact effective finite-dimensionality is the defining property of HFFs. Physical states would realize finite measurement resolution in their structure so that approximation would cease to be an approximation.

3. An interesting question is whether this means that M-matrix must be replaced with quantum M-matrix with operator valued matrix elements and whether the probabilities should be determined by taking traces of these operators having interpretation as averaging over $\mathcal{N}$ defining the degrees of freedom below measurement resolution. This kind of picture would conform with the basic properties of HFFs.

4. To the strands of number theoretic braids one would attach fermionic propagators. Since bosons correspond to fermion pairs at the throats of wormhole contact, all propagators reduce to fermionic ones. As found, the addition of measurement interaction term fixes fermionic propagator completely and gives it a stringy character.

5. Similar correlation function in WCW degrees of freedom would be given in lowest order - and perhaps exact - approximation in terms of the contravariant metric of the configuration space proportional to g_K^2. Besides this the exponent of Kähler action would be involved. For elementary particles it would be the exponent of Kähler action for CP_2 type vacuum extremal. In this manner something combinatorially very similar to standard perturbation theory would result and there are excellent hopes that p-adic coupling constant evolution in powers of 2 is consistent with the standard coupling constant evolution.

6. Vertices correspond to n-point functions. The contribution depending on fermionic fields defines the quantum number dependent part of the vertices and comes from the fermion field and their conjugats attached to the ends of propagator lines identified as braid strands. Besides this there is a symplecto-conformal contribution to the vertex.

7. The stringy variant of twistor Grassmannian approach is highly suggestive since the necessary conditions are satisfied. In particular, the fundamental fermions propagate in the internal lines effectively as massless on-mass shell states but with non-physical polarization. M^4 *resp.* CP_2 is the unique 4-D manifold *resp.* compact manifold with Minkowskian *resp.* Euclidian signature of metric allowing twistor space with Kähler structure [A63]. This suggests that a generalization of twistorialization to 8-D context makes sense. The twistor space for CP_2 is 6-dimensional flag manifold $SU(3)/U(1)\times U(1)$ parameterizing the choice of color quantization axes and has popped up earlier in TGD inspired theory of consciousness.

The expansion of M-matrix in powers of $\hbar$

One should understand how the proportionality of gauge couplings to g_K^2 emerges and how loops give rise to powers of α_K. In zero energy ontology one does not calculate M-matrix but tries to construct zero energy state in the hope that QFT wisdom yields cold help to construct Connes tensor product correctly.

1. The basic rule of quantum field theory is that each loop gives $\alpha = g^2/4\pi$ and thus $1/\hbar$ factor whereas in tree diagrams only g^2 appears so that they correspond to the semiclassical approximation.

2. This rule is obtained if one assumes loops correspond to a hierarchy of sub- CDs and that in loop one can distinguish one line as "base line" and other lines as radiative corrections. To each internal line one must one must assign the factor $r^{-1/2} = (\hbar_0/\hbar)^{1/2}$ and factor g_K^2 except to the portion of base line appearing in loop since otherwise double counting would result. This dictates the expansion of M-matrix in powers of $r^{-1/2}$. It would not be too surprising to have this kind of expansion.

3. g_K^2 factor comes from the functional integral over the partonic 2-surface selected by stationary phase approximation using the exponent of Kähler action. The functional integral over the WCW degrees of freedom is carried out using contravariant Kähler metric as a propagator and this gives g_K^2 factor in the lowest non-trivial order since one must develop a perturbation theory with respect to the deformations at the partonic 2-surfaces at the ends of line.

   If the analogs of radiative corrections to this functional integral vanish - as suggested by quantum criticality and required by number theoretic universality - the resulting dependence on g_K^2 is exact and completely analogous to the free field theory propagator. The numerical factors give the appropriate gauge coupling squared.

4. Besides this one must assign to the ends of the propagator line positive and negative energy parts of quantum state representing the particle in question. These give a contribution which is zeroth order in $\hbar$. For instance, gauge bosons correspond to fermionic bilinears. Essentially fermion currents formed from spinor fields at the two light-like wormhole throats of the wormhole contact at which the signature of the induced metric changes are in question. Correct dimension requires the presence of $1/\hbar$ factor in boson state and $1/\sqrt{\hbar}$ factor in fermion state. The correlators between fermionic fields at the end points of the line are proportional to $\hbar$ so that normalization factors cancel the $\hbar$ dependence. Besides this one would expect N-points function of symplecto-conformal QFT with $N = N_{in} + N_{out}$ having no dependence on $\hbar$.

13.2.3 But What About The Concrete Feynman Rules?

The skeptic reader can say that all this is just an endless list of general principles. I dare however claim that the only manner to proceed is to try to identify the general principles first. At this moment the understanding of the fundamental variational principled of TGD understood at such level of detail that one can indeed sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to the 4-D surfaces defined by the Euclidian regions defined by wormhole contacts plus the string world sheets connecting them and carrying spinor modes. One might also talk about combination of Feynman diagrams and stringy diagrams or even about generalization of Wilson loops. The lines of these diagrams form also braids.

1. The boundaries of string world sheets at which the modes of induced spinor field are localized (by well-definedness of em charge) carry fermion number and are identifiable as braid strands within partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. 1-D Dirac action for induced metric and its bosonic counterpart - must be assigned with partonic orbits in order to obtain non-trivial fermionic propagator. Massles fermion propagator emerges if light-like portions of string world sheet boundary contain 1-D Dirac action in induced metric. The bosonic part of this action implied by supersymmetry implies that light-like geodesic of embedding space is in question and there is a conserved light-like four-momentum associated with the fermion line.

2. The fundamental interaction is the scattering of fermions at opposite wormhole throats of wormhole contact. With string model based intuition one can argue that this interaction must correspond essentially to the stringy propagator $1/L_0$ so that one would obtain a combination of Feynman rules and stringy rules. The vertices correspond topologically to a fusion of 4-D lines along the 3-surfaces at their ends and this means deviation from string model picture: stringy diagrams correspond at topological level to what happens when particle travels between A and B along two different routes and has nothing to do with particle decay.

   One can criticize this idea about ad hoc character. Furthermore, super-symmetry requires also the presence of super-generator G and its hermitian conjugate. In TGD however these operators carry baryon or lepton number and cannot appear as propagators unless they appear as pairs $GG^{dagger} \propto L_0$.

   The vision about scattering amplitudes as sequences of algebraic operations with 3-vertices identified as product and co-products in super-sympelectic Yangian of super-symplectic algebra looks much more feasible option [K116].

3. Physical particles are bound states of massless fundamental fermions and correspond to pairs of wormhole contacts: a pair is required since wormhole throats behave effectively as magnetic monopoles and closed flux tube consisting of pieces at the two space-time sheets and wormhole contacts is required. This resolves the infrared difficulties of twistor approach. Twistor Grassmann approach strongly suggests that the residue integral over the virtual four-momenta reduces the propagators of fundamental fermions to their inverses at mass-shell so that only non-physical fermion helicities appear as virtual fermions.

The reader wishing for a brief summary of TGD might find the three articles about TGD, TGD inspired theory of consciousness, and TGD based view about quantum biology helpful [L4, L3, L2].

13.3 How To Define Generalized Feynman Diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in ZEO (ZEO) [K94] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K116] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the "world of classical worlds" (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

2. ZEO (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also embedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K116] . Kähler-Dirac gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D embedding space parameterized by quaternionic space-time surfaces.

5. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K127]. It took long time to realize that in ZEO the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of

3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

6. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

7. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K127, K116] .

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B62] automatically satisfied as in the case of ordinary Feynman diagrams.

2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of Kähler-Dirac operator.

2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K57] in infinite-dimensional context already in the case of much simpler loop spaces [A44] .

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II_1 defining the finite measurement resolution.

2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

13.3.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to this goal is by making questions.

What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the embedding space. This would be true at least in the intersection of real and p-adic worlds.

3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the Kähler-Dirac action [K127] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

   A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

   In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. The light-like 8-momenta p^k have same M^4 and CP_2 mass squared and latter correspond to the the eigenvalues of the CP_2 spinor d'Alembertian by quantum-classical correspondence.

5. One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.

6. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

1. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the Kähler-Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.

2. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.

3. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub-CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adizication would thus give a further good reason why for ZEO.

4. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.

5. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

13.3.2 Generalized Feynman Diagrams At Fermionic And Momentum SpaceLevel

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. ZEO encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_\pm$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and $X_\pm$ might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the Kähler-Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^{\alpha}p_{\alpha} + \hat{\Gamma}^{\alpha}D_{\alpha} \ , \\ p_{\alpha} &= p_k\partial_{\alpha}h^k \ . \end{aligned} \quad (13.3.1)$$

The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is Kähler-Dirac gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D Kähler-Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

4. Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [?] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N-4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B5] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_{\pm}$ pairs ($X_{\pm}$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion-$X_{\pm}$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_{\pm}$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and Kähler-Dirac operator.

2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [?] .

3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \to F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).

4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K38] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

13.3.3 Harmonic Analysis In WCW As a way To Calculate WCWFunctional Integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding “radial” coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the “radial” coordinates only and the possible generalization involves the identification the counterparts of the “radial” coordinates in the case of WCW.

Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of “kinetic” terms and interaction term.

2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

3. Momentum conservation correlates the eigenvalue spectra of the Kähler-Dirac operator D at propagator lines [K127] . G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for

each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \tag{13.3.2}$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \tag{13.3.3}$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the Kähler-Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \tag{13.3.4}$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K29, K127]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K) J d^2x \ , \\ J &= \epsilon^{\alpha\beta} J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = K J_{12} \ . \end{aligned} \tag{13.3.5}$$

works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/ 4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of embedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

   Recall that also the construction of number theoretic braids and symplectic QFT [K27] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

   This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2 x_\pm \quad . \tag{13.3.6}$$

   Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

   The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J_{kl}^- \ , \\ J_\pm^{kl} &= (1 + K_\pm) \partial_\alpha s^k \partial_\beta s^l J_\pm^{\alpha\beta} \quad . \end{aligned} \tag{13.3.7}$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the Kähler-Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1+K)J$ with $X\partial(s^1, s^2)/\partial(x^1_\pm, x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1+K)J\delta^2(x, y)$ would be replaced with $X\delta^2(s^+, s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

6. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.

2. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α^0_K and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α^0_K could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional

integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

1. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.

2. One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.

3. In ZEO this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the Kähler-Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

13.4 A More Detailed View About The Construction Of Scattering Amplitudes

The following represents an update view about construction of scattering amplitudes at the level of "world of classical worlds" (WCW).

13.4.1 Basic Principles

In order to facilitate the challenge of the reader I summarize basic ideas behind the construction of scattering amplitudes.

Construction of scattering amplitudes as functional integrals in WCW

The decomposition of space-time surface to Minkowskian and Eucldian regions is the basic distinction from ordinary quantum field theories since it replaces path integral with mathematically well-defined functional integral over WCW .

1. Space-time surface decomposes to regions with Minkowskian or Euclidian signature of the induced metric. The regions with Euclidian metric are identified as lines of generalized Feynman diagrams. The boundaries between two kinds of regions - to be called parton orbits - can be regarded as carriers of elementary particle quantum numbers such as fermion number assignable to the boundaries of string world sheets at them. Induced spinor fields are localized at them from the well-definedness of electromagnetic charge requiring that induced W boson fields vanish. Hence strings emerge from TGD. Note that at boundary between Euclidian and Minkowskian regions the metric determinant vanishes. Unlike the name would suggest, generalized Feynman diagrams are analogous to twistor diagrams, and instead of infinite number of superposed diagrams there might just single diagram.

2. Weak form of electric magnetic duality together with the assumption that the term $j^\alpha A_\alpha$ in Kähler action vanishes imply that Kähler action reduces to 3-D Chern-Simons term. This hypothesis is inspired by TGD as almost topological quantum field theory conjecture. In Minkowskian regions this conjecture is very natural. In the Euclidian region the contribution to Kähler action need not reduce to a mere Chern-Simons term associated with its boundary. This would be due to the non-triviality of the U(1) bundle defined by Kähler form giving also Chern-Simons terms inside the CP_2 type vacuum extremal.

3. Scattering amplitude is a functional integral over space-time surfaces: the data about these space-time surfaces are coded by their ends about the opposite light-like boundaries of causal diamond (CD) of given scale. The weight function in the functional integral is exponential of Kähler function of "world of classical worlds" coming from Euclidian regions of the space-time surface representing lines of generalized Feynman diagram and being deformation of CP_2 type vacuum extremals representing wormhole contacts connecting two space-time sheets with Minkowskian signature of induced metric. Kähler function is the exponent of Kähler action from Euclidian regions. The real exponent takes care that the functional integral is obtained instead of path integral so that the outcome is mathematically well-defined.

4. Euclidian region would give only the analog of thermodynamics but there is also an imaginary exponential coming from the exponential of the imaginary Kähler action from Minkowskian regions. Space-time surfaces are extremals of Kähler action and for very general ansatz Minkowskian contribution to Kähler action reduces to imaginary Chern-Simons term at the light-like 3-D boundary between regions at which the 4-D metric is degenerate. This term makes possible interference of different contributions to the functional integral which is absolutely essential in quantum field theory.

5. The details of the theory in fermionic sector have turned out to be crucial. From the well-definedness of the electric charge for the modes of the induced spinor field - and also by number theoretic arguments - spinor modes are localized at 2-D string world sheets carrying vanishing W gauge fields. Preferred extremals can be constructed by fixing first partonic 2-surfaces, string world sheets, and possibly also the light-like orbits of partonic 2-surfaces and

posing the condition that the canonical momentum densities have no components normal to string world sheets. Also the condition that a sub-algebra of super-symplectic algebra gives rise to vanishing Noether charges at the space-like ends of preferred extremal is natural.

This construction would conform with the strong form of holography. The boundaries of string world sheets at the light-like orbits of partonic 2-surfaces carry 1-D Dirac action for induced gamma matrices. The bosonic counterpart of this action gives as solutions light-like geodesics of embedding space - light-likeness in 8-D sense. 1-D Dirac equation for induced gamma matrices is satisfied. A very twistorial picture emerges and suggests 8-D generalization of twistor approach. M^4 and CP_2 are indeed twistorially completely unique.

6. The generators of super-symplectic algebra can be represented as Noether charges for the fermionic strings and the supercharges identifiable as WCW gamma matrices are natural identification for fermionic oscillator operators. Since one expects that a given partonic 2-surface is connected to a large number of partonic 2-surfaces a generalization to Yangian [A25] [B29, B22, B23] of super-symplectic algebra seems necessary and is in spirit with twistorialization. It seems possible to identify the fundamental vertices assignable to partonic 2-surfaces at which three lines of diagram meet in terms of product and co-product for Yangian so that there are hopes about realizing the already forgotten TGD inspired dream about reduction of scattering amplitudes to sequences of algebraic operations of Yangian with minimal length and connecting chosen initial and final states at the boundaries of CD. Universe would be Yangian algebraist!

So what one expects vertices and propagators to be? Fermionic propagators would be massless in 8-D sense and they should be contracted with the legs of the vertices defined by product tor co-product involving three Yangian generators. Structure constants would define the coupling constants. Each Yangian generator would involve a collection of fermions fields associated with strings and with each fermion field propagator would contract. The only modification of the ordinary vertex is that partonic 2-surfaces carry many-fermion states and the vertices involve 3 multi- fermion states. Fermion lines can also turn backwards in time: this gives rise to virtual bosons.

Why it might work?

There are many reasons encouraging the hopes about calculable theory.

1. The theory has huge super-conformal symmetries dramatically reducing the dynamical degrees of freedom by the choice of conformal gauge. This implies that both the space-like 3-surfaces at the ends of space-time surface and partonic orbits satisfy classical Super conformal conditions for generalizations of ordinary super-conformal algebras perhaps extending to multilocal Yangian with loci identified as strings connecting partonic 2-surfaces at the light-like boundary of CD. This algebra extends also to include both boundaries of CD. Fermionic anticommutation relations which allow by 2-dimensionality of string world sheet also quantum group variant determine the anticommutations between all generators.

   Yangian symmetry in turn gives excellent hopes about twistorialization: in fact, $M^4 \times CP_2$ is completely unique choice for the embedding space by twistorial considerations and the product of the twistor spaces of M^4 and CP_2 allows to constructed the twistor spaces of space-time surfaces as liftings of the extremals of Kähler action to 6-D sphere bundles over space-time surface.

2. The integrand in the functional integral represents the analog of ordinary Feynman diagrams involving only fermions and 1-D lines. Indeed, by bosonic emergence all bosons (in fact all elementary particles) can be regarded as composites of fundamental fermions. The only purely fermionic vertices are 2-fermion vertices. 3-vertices correspond to space-time surfaces meeting along common 3-surface and are thus purely topological, and as already mentioned could correspond to product and co-product for Yangian. This is of course excellent news from the point of view of finiteness. The fermionic vertices are represented by the discontinuity of the Kähler-Dirac operator associated with the string boundary line at partonic 2-surface so that there are no coupling constants involved. The only fundamental coupling parameter

is Kähler strength whose value is dictated by quantum criticality as the analog of critical temperature.

One must have a view about what elementary particles - as opposed to fundamental fermions - are, how the ordinary view about scattering based on exchanges of elementary particles emerges from this picture and how say BFF vertex reduces to a diagram at for fundamental fermions involving only 2-fermion vertices.

13.4.2 Elementary Particles In TGD Framework

The notion of elementary particles involves two aspects: elementary particles as space-time surfaces and elementary particles as many-fermion states with fundamental fermions localized at the wormhole throats and defining elementary particles as their bound states (including physical fermions).

Let us first summarize what kind of picture ZEO suggests about elementary particles.

1. Kähler magnetically charged wormhole throats are the basic building bricks of elementary particles. The lines of generalized Feynman diagrams are identified as the Euclidian regions of space-time surface. The weak form of electric magnetic duality forces magnetic monopoles and gives classical quantization of the Kähler electric charge. Wormhole throat is a carrier of many-fermion state with parallel momenta and the fermionic oscillator algebra gives rise to a badly broken large $\mathcal{N}$ SUSY [**?**].

2. The first guess would be that elementary fermions correspond to wormhole throats with unit fermion number and bosons to wormhole contacts carrying fermion and anti-fermion at opposite throats. The magnetic charges of wormhole throats do not however allow this option. The reason is that the field lines of Kähler magnetic monopole field must close. Both in the case of fermions and bosons one must have a pair of wormhole contacts (see **Fig.** `http://tgdtheory.fi/appfigures/wormholecontact.jpg` or **Fig. ??** in the appendix of this book) connected by flux tubes. The most general option is that net quantum numbers are distributed amongst the four wormhole throats. A simpler option is that quantum numbers are carried by the second wormhole: fermion quantum numbers would be carried by its second throat and bosonic quantum numbers by fermion and anti-fermion at the opposite throats. All elementary particles would therefore be accompanied by parallel flux tubes and string world sheets.

3. A cautious proposal in its original form was that the throats of the other wormhole contact could carry weak isospin represented in terms of neutrinos and neutralizing the weak isospin of the fermion at second end. This would imply weak neutrality and weak confinement above length scales longer than the length of the flux tube. This condition might be un-necessarily strong.

   The realization of the weak neutrality using pair of left handed neutrino and right handed antineutrino or a conjugate of this state is possible if one allows right-handed neutrino to have also unphysical helicity. The weak screening of a fermion at wormhole throat is possible if ν_R is a constant spinor since in this case Dirac equation trivializes and allows both helicities as solutions. The new element from the solution of the Kähler-Dirac equation is that ν_R would be interior mode de-localized either to the other wormhole contact or to the Minkowskian flux tube. The state at the other end of the flux tube is sparticle of left-handed neutrino.

   It must be emphasized that weak confinement is just a proposal and looks somewhat complex: Nature is perhaps not so complex at the basic level. To understand this better, one can think about how M_{89} mesons having quark and antiquark at the ends of long flux tube returning back along second space-time sheet could decay to ordinary quark and antiquark.

13.4.3 Scattering Amplitudes

The basic challenge is to introduce vertices and fermionic propagators. The recent based on stringy realization of Yangian algebra allows to do this.

Fermionic propagators

How fermionic propagators emerge? The first explanation coming in mind is based on the discontinuity associated with the Dirac operator at the partonic 2-surfaces defining vertices.

Discontinuities can be of two different types. Fermionic lines has discontinuous tangent at the partonic 2-surfaces meaning local non-conservation of light-like 8-momentum. Also second kind of discontinuity in which two lines belonging to orbits of distinct partonic 2-surfaces emerge at single point. Their 8-momenta need not be opposite if one requires only global momentum conservation. If it is assumed one can say that fermionic line turns backwards in time. These kind of pairs of lines forming closed curves with peaks at ends are associated with bosonic propagators-say those describing boson exchange between two fermions.

The discontinuities of the induced spinor along the fermionic line making a turn at the partonic 2-surface give rise to delta function singularities under the action of 1-D Dirac operator. This would give Dirac equation with a source term and its solution would be given by Dirac propagator convoluted with the discontinuity.

Vertices

Vertices can be considered at both space-time level and fermionic level.

1. At space-time level vertices correspond to the fusion of space-surfaces representing particles along common 3-surface defining the vertex. At the parton level 3-light-like parton orbits fuse together along partonic 2-surface. In these vertices particle number changes this change correspond the change of particle number for elementary particles.

2. At fermion level vertices are localized at the partonic 2-surfaces. The above argument would suggest that vertices corresponds to the discontinuity of the Kähler Dirac operator at the corner of the line representing the boundary of string world sheet. The creation of fermion pair from vacuum corresponds to an corner of string boundary at which the boundaries of string world sheets associated with two outgoing or incoming sheets meet. The creation/annihilation of a fermion pair is essential for the realization of say tree diagrams describing fermion scattering by virtual boson exchange.

The identification of vertex as a product or co-product in Yangian looks the most promising approach. The charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator at each leg plus vertex factor. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge - essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices. This is very much like the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormfhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place. One obtains integration of the light-like 8-momenta of fermions in natural manner and something resembling very strongly standard QFT. The integration interpreted as residue integral should give only inverse of the propagator actin on on mass shell states with wrong helicity. Virtual fermions would have wrong helicity unlikes incoming ones.

13.4.4 What One Should Obtain At QFT Limit?

After functional integration over WCW of one should obtain a scattering amplitude in which the fermionic 2- vertices defined as discontinuities of the Kähler-Dirac operator at partonic 2-surfaces should boil down to a contraction of an M^8 vector with gamma matrices of M^8. This vector has dimension of mass. This basic parameter should characterize many different physical situations. Consider only the description of massivation of elementary particles regarded as bound states of fundamental massless fermions and the mixing of left and right-handed fermions. Also CKM mixing should involve this parameter. These vectors should also appear in Higgs couplings, which in QFT description contain Higgs vacuum expectation as a factor.

In twistor approach virtual particles have complex light-like 8-momenta. Fundamental fermions have most naturally real and light-like momenta. $\mathcal{N} = 4$ SUSY describes gauge bosons which correspond to bound states of fundamental fermions in TGD. This suggests that the four-momenta of bound states of massless fermions - be they hadrons, leptons, or gauge bosons - can be taken to be complex.

There is an intriguing connection with TGD based notion of space-time. In TGD one obtains at space-time level complexified light-like 8-momenta since the 8-momentum from Minkowskian/Euclidian region is real/imaginary. In the case of physical particle necessary involving two wormhole contacts and two flux tubes connecting them the total complexifies four momentum would be sum of two real and two imaginary contributions. Every elementary particle should have also imaginary part in its 8-momentum and would be massless in complexified sense allowing mass in real sense given by the length of the imaginary four-momentum. In twistor approach complex light-like momenta indeed appear in BCFW bridge.

TGD predicts Higgs boson although Higgs expectation does not have any role in quantum TGD proper. Higgs vacuum expectation is however a necessary part of QFT limit (Higgs decays to WW pairs require that vacuum expectation is non-vanishing). Higgs vacuum expectation must correspond in TGD framework to a quantity with dimensions of mass. In TGD Higgs cannot be scalar but a vector in CP_2 degrees of freedom. The problem is that CP_2 does not allow covariantly constant vectors. The imaginary part of classical four-momentum gives a parameter which has interpretation as a vector in the tangent space of which is same as that of $M^4 \times CP_2$. Could $M^8 - H$ duality be realized at the level of tangent space and for relate four-momentum and color quantum numbers to the E^4 part of 8-momentum?

Elementary particles of course need not be eigenstates of the CP_2 part of 8-momentum. For a fixed mass one can have wave functions in the space of CP_3 part of 8-momentum analogous to S^3 spherical harmonics at the sphere of E^4 with radius defined by the length of imaginary four-momentum (mass). These harmonics are characterized by $SO(4)$ quantum numbers. Could one interpret this complexification in terms of M^8-duality and say that SO(4) defines the symmetries for the low energy dual of WCW defining high energy description of QCD based on SU(3) symmetry. SO(4) would corresponds to the symmetry group assigned to hadrons in the approach based on conserved vector currents and partially conserved axial currents. SO(4) would be much more general and associated also with leptons.

The anomalous color hyper-charge of leptonic spinors would imply that one can have also in the case of leptons a wave function in S^3. Higher harmonics would correspond to color excitations of leptons and quarks. If one considers gamma matrices, complexification of M^4 means introduction of gamma matrix algebra of complexified M^4 requiring 8 gamma matrices. This suggests a connection with $M^8 - H$ duality. All elementary particles have also imaginary part of four-momentum and the 8-momentum can be interpreted as M^8-momentum combining the four-momentum and color quantum numbers together.

Chapter 14

TGD View about Coupling Constant Evolution?

14.1 Introduction

Atyiah has recently proposed besides a proof of Riemann Hypothesis also an argument claiming to derive the value of the structure constant (see `http://tinyurl.com/y8xw8cey`). The mathematically elegant arguments of Atyiah involve a lot of refined mathematics including notions of Todd exponential and hyper-finite factors of type II (HFFs) assignable naturally to quaternions. The idea that $1/\alpha$ could result by coupling constant evolution from π looks however rather weird for a physicist.

What makes this interesting from TGD point of view is that in TGD framework coupling constant evolution can be interpreted in terms of inclusions of HFFs with included factor defining measurement resolution [K126, K44]. An alternative interpretation is in terms of hierarchy of extensions of rationals with coupling parameters determined by quantum criticality as algebraic numbers in the extension [L53, L54].

In the following I will explain what I understood about Atyiah's approach. My critics includes the arguments represented also in the blogs of Lubos Motl (see `http://tinyurl.com/ycq8fhsy`) and Sean Carroll (see `http://tinyurl.com/y87f8psg`). I will also relate Atyiah's approach to TGD view about coupling evolution. The hasty reader can skip this part although for me it served as an inspiration forcing to think more precisely TGD vision.

There are two TGD based formulations of scattering amplitudes.

1. The first formulation is at the level of infinite-D "world of classical worlds" (WCW) [K96] uses tools like functional integral. The huge super-symplectic symmetries generalizing conformal symmetries raise hopes that this formulation exists mathematically and that it might even allow practical calculations some day. TGD would be an analog of integrable QFT.

2. Second - surprisingly simple - formulation [L68] is based on the analog of micro-canonical ensemble in thermodynamics (quantum TGD can be seen as complex square root of thermodynamics). It relates very closely to TGD analogs of twistorialization and twistor amplitudes [K116, L55].

   During writing I realized that this formulation can be regarded as a generalization of cognitive representations of space-time surfaces based on algebraic discretization making sense for all extensions of rationals to the level of scattering amplitudes. In the adelization the key question is whether it is necessary to define the p-adic counterparts of action exponentials. The number theoretical constraints seem hopelessly strong. One solution would be that the action exponentials for allow space-time surfaces equal to one. This option fails. The solution of the problem is however trivial. Kähler function can have only single minimum for given values of zero modes and the action exponentials cancel from scattering amplitudes completely in this case. This formulation allows a continuation to p-adic sectors and adelization [L53, L54]. Note that no conditions on α_K are obtained contrary to the first beliefs.

One can also understand the relationship of the two formulations in terms of M^8-H duality. This view allows also to answer to a longstanding question concerning the interpretation of the surprisingly successful p-adic mass calculations [K76]: as anticipated, p-adic mass calculations are carried out for a cognitive representation rather than for real world particles and the huge simplification explains their success for preferred p-adic prime characterizing particle as so called ramified prime for the extension of rationals defining the adeles.

The understanding of coupling constant evolution has been one of most longstanding problems of TGD and I have made several proposals during years. TGD view about cosmological constant turned out to be the solution of the problem.

1. The formulation of the twistor lift of Kähler action led to a rather detailed view about the interpretation of cosmological constant as an approximate parameterization of the dimensionally reduced 6-D Kähler action (or energy) allowing also to understand how it can decrease so fast as a function of p-adic length scale. In particular, a dynamical mechanism for the dimensional reduction of 6-D Kähler action giving rise to the induction of the twistor structure and predicting this evolution emerges.

   In standard QFT view about coupling constant evolution ultraviolet cutoff length serves as the evolution parameter. TGD is however free of infinities and there is no cutoff parameter. It turned out cosmological constant replaces this parameter and coupling constant evolution is induced by that for cosmological constant from the condition that the twistor lift of the action is not affected by small enough modifications of the moduli of the induced twistor structure. The moduli space for them corresponds to rotation group $SO(3)$. This leads to explicit evolution equations for α_K, which can be studied numerically.

2. I consider also the relationship to a second TGD based formulation of coupling constant evolution in terms of inclusion hierarchies of hyper-finite factors of type II_1 (HFFs) [K126, K44]. I suggest that this hierarchy is generalized so that the finite subgroups of $SU(2)$ are replaced with Galois groups associated with the extensions of rationals. An inclusion of HFFs in which Galois group would act trivially on the elements of the HFFs appearing in the inclusion: kind of Galois confinement would be in question.

Ramified primes are conjecture to correspond to the preferred p-adic primes characterizing particles. Ramified primes are special in the sense that their expression as a product of primes P_i of extension contains higher than first powers and the number P_i is smaller than the maximal number n defined by the dimension of the extension. It is not quite clear why ramified primes appear as preferred p-adic primes and in the following Dedekind zeta functions and what I call ramified zeta functions inspired by the interpretation of zeta function as analog of partition function are used in attempt to understand why ramified primes could be physically special.

The intuitive feeling is that quantum criticality is what makes ramified primes so special. In $O(p)=0$ approximation the irreducible polynomial defining the extension of rationals indeed reduces to a polynomial in finite field F_p and has multiple roots for ramified prime, and one can deduce a concrete geometric interpretation for ramification as quantum criticality using M^8-H duality.

14.2 Criticism of Atyiah's approach

The basic idea of Atyiah is that π and the inverse of the fine structure constant $1/\alpha = 137.035999....$ are related by coupling constant evolution - that is renormalization - which is a basic operation in quantum field theory and has physical interpretation. For a physicist it is easy to invent objections.

1. In quantum field theory fine structure constant and all coupling strengths obey a continuous evolution as function of mass scale or length scale and one should predict the entire evolution rather than say its value at electron length scale. In TGD framework the coupling constant evolution becomes discrete and would basically labelled by the hierarchy of extensions of rationals.

2. π is purely geometric constant - kind of Platonic transcendental having very special role in the mathematical world order - whereas fine structure constant is a dynamical coupling

parameter. Atyiah does not have any proposal for why these constants would be related in this manner. Also no explanation for what it would mean that the circumference of unit circle would grow from 2π to $2/\alpha$ is given.

Remark: In TGD actually the coverings labelled by the value $h_{eff}/n_0 = n$ identified as the order of Galois group of extension of rationals defining given level of the hierarchy of evolutionary levels (entanglement coefficients would belong to this extension as also S-matrix elements). The full angle using M^4 rotation angle as coordinate increases effectively to $n \times 2\pi$ for the covering spaces of extensions introducing n:th root of unity. In TGD would however have n instead of $1/(\alpha\pi)$.

3. That $1/\alpha \sim 137$ should have interpretation as renormalized value of angle π looks rather weird to me. The normalization would be very large and it is extremely difficult to see why $1/\pi$ have a role of fine structure constant say at high energy limit if one accepts coupling constant evolution and identifies $1/\alpha$ as the value of $1/\alpha$ at zero momentum transfer.

In fact, Atyiah proposes a discrete evolution of π to $1/\alpha$ defined by approximations of HFF as a finite-D algebra. Forgetting π as the starting point of the evolution, this idea looks beautiful. At first the idea that all numbers suffer a renormalization evolution, looks really cute. Coupling constant evolution is however not a sequence of approximations but represents a genuine dependence of coupling constants on length scale.

Remark: In TGD framework I propose something different.The length scale evolution of coupling constants would correspond to a hierarchy of inclusions of HFFs rather than a sequence of finite-D approximations approaching HFF. The included factor would represent measurement resolution. Roughly, the transformations of states by operations defined in included factor would leave state invariant in the measurement resolution defined by the included factor. Different values of coupling constant would correspond to different measurement resolutions.

1. Atyiah mentions as one of his inspirers the definition of 2π via a limiting procedure identifying it as the length of the boundary of n-polygon inside unit circle. Amusingly, I have proposed similar definition of 2π in p-adic context, where the introduction of π would give rise to infinite extension.

   Atyiah generalizes this definition to the area of quaternionic sphere so that the limiting procedure involves two integers. For sphere tessellations as analogs of lattices allow only Platonic solids. For torus one could have infinite hierarchy of tessellations [L61] allowing to define the area of torus in this manner. The value of n defined by the extension of rationals containing root of unity $exp(i2\pi/n)$ such that n is maximal. The largest n for the roots of unity appearing in the extension of p-adics would determine the approximation of 2π used.

2. Atyiah suggests a concrete realization for the coupling constant evolution of numbers, not only coupling constants. The evolution would correspond to a sequence of approximation to HFF converging to HFF. One can of course define this kind of evolution but to physicist it looks like a formal game only.

3. HFF is interpreted as an infinite tensor product of 2×2 complex Clifford algebras $M_2(C)$, which can be also interpreted as complexified quaternions. One defines the trace by requiring that the trace of infinite tensor product of unit matrices equals to 1. The usual definition of schoolbooks would given infinite power of 2, which diverges. The inner product is the product of the usual inner products for the factors of the tensor product labelled by n but divided by power $2^{-n_{max}}$ to guarantee that the trace of the identity matrix is unity as product of traces for factors otherwise equal to 2^n. In fact, fermionic Fock algebra familiar to physicist is HFF although in hidden manner.

   Remark: The appearance of quaternions is attractive from TGD point of view since in $M^8 - H$ duality the dynamics at the level of M^8 is determined by associativity of either tangent or normal space of 4-surface in M^8 and associativity is equivalent with quaternionicity [L48]. The hierarchy of HFFs is also basic piece of quantum TGD and realizable in terms of quaternions.

4. Atyiah tells there is an algebra isomorphism from complex numbers C to the subset of commuting matrices in HFF. One can define the map to C as either eigenvalue of the matrix and obtains to isomorphisms: t_+ and t_-. One can define the renormalization map $C \rightarrow C$ in terms of the inverse of $t_- \circ t_+^{-1}$ or its inverse. This would assign to a complex numbers z its normalized value.

   HFFs allow an excellent approximation by finite number of tensor factors and one can perform an approximation taking only finite number of tensor factors and at the limit of infinite number of factors get the desired normalization map. The approximation would be $t_-(n) \circ t_+(n)$. I must confess that I did not really understand the details of this argument.

   In any case, to me this does not quite correspond to what I understand with renormalization flow. Rather this is analogous to a sequence of approximations defining scattering amplitude as approximation containing only contributions up to power g^n. I would argue than one must consider the infinite sequence of inclusions of HFFs instead of a sequence of approximations defining HFF.

   In this manner one would the renormalization map would be $t_-(n+1) \circ t_+^{-1}(n)$, where n now labels the hierarchy of HFFs in the inclusion hierarchy. $t_\pm n$ is now the exact map from commuting sub-algebra to complex numbers.

   There is however a rather close formal resemblance since simple inclusions correspond to inclusions of the sub-algebra with one M_C^2 factor replaced with mere identity matrix.

5. The proposal of Atyiah is that this renormalization of numbers is mediated by so called Todd exponentiation used in the construction of the characteristic classes. This map would be defined in terms of generating function $G(x) = x/(1 - exp{-x})$ applied to $x = \pi$. If I understood anything about the explanation, this map is extended to infinite number of tensor factors defining the HFF and the outcome would be that $x = \pi$ for single tensor factor would be replaced with $1/\alpha$. Why Todd exponentiation? Atyiah also argues that one has $T(\pi)/\pi = T(\gamma)/\gamma$, where γ is Euler's constant. My mathematical education is so limited that I could not follow these arguments.

6. Atyiah also claims that the approximation $1/\alpha = 137$ assumed by Eddington to be exact has actually deeper meaning. There are several formulas in this approximation such as $1/\alpha = 2^0+2^3+2^7 = 1+8+128$. If I understood correctly, Atyiah tells that the numbers 1, 8, and 128 appear in the Bott periodicity theorem as dimensions of subsequent stable homotopy groups. My own favorite formula is in terms of Mersenne primes: $1/\alpha = M_2+M_3+M_7 = 3+7+127$. The next Mersenne prime would be M_{127} and corresponds to the p-adic length scale of electron.

   Remark: A fascinating numerological fact is that $p \simeq 2^k$, $k \simeq 137$, corresponds to the p-adic length scale near to Bohr radius: kind of cosmic joke one might say. Fine structure constant indeed emerged from atomic physics!

It would be of course marvellous if the renormalization would not depend on physics at all but here physicist protests.

1. The coupling constant evolutions for the coupling strengths of various interactions are different and depend also on masses of the particles involved. One might however hope that this kind of evolution might make sense for fundamental coupling constants of the theory. In TGD Kähler coupling strength $1/\alpha_K$ would be such parameter.

2. The quantum criticality of TGD Universe suggests that Atyiah's claim is true in a weaker sense. Quantum criticality is however a dynamical notion. I have actually proposed a model for the evolution of $1/\alpha_K$ based on the complex zeros of Riemann Zeta [L23] and also a generalization to other coupling strengths assuming that the argument of zeta is replaced with its Möbius transform.

   Very strong consistency conditions should be met. Preferred primes would be primes near prime power of 2 and ramified primes of extension, and also the zero of zeta in question should belong to the extension in question. I am of course the first to admit that this model is motivated more by mathematical aesthetics than concrete physical calculations.

3. The idea about renormalization evolution in this manner could - actually should - generalize. One can consider a maximal set of commuting set of observables in terms of tensor product of HFFs and define for them map to diagonal $n \times n$ matrices with complex eigenvalues. One would have infinite sum over the eigenvalues of diagonal matrices over factors: just as one has for many particle state in QFT containing contribution from all tensor factors which are now however ordered by the label n. The length scale evolution of these observables could be defined by the above formula for inclusion. Fine structure constant basically reduces to charge as eigenvalue of charge operator so that this could make sense.

   The beauty of this view would be that renormalization could be completely universal. In TGD framework quantum criticality (QC) indeed strongly suggests this universality in some sense. The hierarchy of extensions of rationals would define the discrete coupling constant evolution.

14.3 About coupling constant evolution in TGD framework

It is often forgotten that fine structure constant depends on length scale. When Eddington was working with the problem, it was not yet known that fine structure constant is running coupling constant. For continuous coupling constant evolution there is not much point to ponder why its value is what it is at say electron length scale. In TGD framework - adelic physics - coupling parameters however obey discrete length scale evolution deriving from the hierarchy of extensions of rationals. In this framework coupling constants are determined by quantum criticality implying that they do not run at all in the phase assignable to given extension of rational. They are analogous to critical temperature and determined in principle by number theory.

Two approaches to quantum TGD

There are two approaches to TGD: geometric and number theoretic. The "world of classical worlds" (WCW) is central notion of TGD as a geometrization of quantum physics rather than only classical physics.

1. WCW consists of 3-surfaces and by holography realized by assigning to these 3-surfaces unique 4-surfaces as preferred extremals. In zero energy ontology (ZEO) these 3-surfaces are pairs of 3-surfaces, whose members reside at opposite boundaries of causal diamond (CD) and are connected by preferred extremal analogous to Bohr orbit. The full quantum TGD would rely on real numbers and scattering amplitudes would correspond to zero energy states having as arguments these pairs of 3-surfaces. WCW integration would be involved with the definition of inner products.

2. The theory could be seen formally as a complex square root of thermodynamics with vacuum functional identified as exponent of Kähler function. Kähler geometry would allow to eliminate ill-defined Gaussian determinants and metric determinant of Kähler metric and they would simply disappear from scattering amplitudes. WCW is infinite-D space and one might argue that this kind of approach is hopeless. The point is however that the huge symmetries of WCW - super-symplectic invariance - give excellent hopes of really construction the scattering amplitudes: TGD would be integrable theory.

3. A natural interpretation would be that Kähler action as the analog of Hamiltonian defines the Kähler function of WCW and functional integral defined by it allows definition of full scattering amplitudes.

The number theoretic approach could be called adelic physics [L52, L54] providing also the physics of cognition.

1. At space-time level p-adicization as description of cognition requires discretization. Cognitive representations at space-time level consist of finite set of space-time points with preferred coordinates M^8 in extension of rationals inducing the extensions of p-adic number fields. These representations would realize the notion of finite measurement resolution. p-Adicization and adelization for given extension of rationals are possible only in this manner since these points can be interpreted as both real and p-adic numbers.

2. What about cognitive representations at the level of WCW? The discrete set of space-time points would replace the space-time surface with a finite discrete set of points serving also as its WCW coordinates and define the analog of discretization of WCW using polynomials in M^8 fixed by their values at these points [L48]. If the space-time surface is represented by a polynomial, this representation is all that is needed to code for the space-time surface since one can deduce the coefficients of a polynomial from its values at finite set of points. Now the coefficients belong to extension of rationals. If polynomials are replaced by analytic functions, polynomials provide approximation defining the cognitive representation.

While writing this I realized that what I have micro-canonical ensemble [L68] as kind of complex square root of its counterpart in thermodynamics can serve as a cognitive representation of scattering amplitudes. Cognitive representations of space-time surfaces would thus give also cognitive representations of WCW and micro-canonical ensemble would realize cognitive representations for the scattering amplitudes. Cognitive representations define only a hierarhcy of approximations. The exact description would involve the full WCW, its Kähler geometry, and vacuum functional as exponent of Kähler function.

The idea of micro-canonical ensemble as a subset of space-time surfaces with the same vanishing action would select a sub-set of surfaces with the same values of coupling parameters so that the fixing the coupling parameters together with preferred extremal property selects the subset with same value of action. There are two options to consider.

1. The real part of the action vanishes and imaginary part is multiple of 2π so that the action exponential is equal to unity. For the twistor lift this actually implies the vanishing of the entire action since volume term and Kähler term have the same phase (that of $1/\alpha_K$). The role of coupling parameters would be analogous to the role of temperature and applied pressure. In principle this condition is mathematically possible. The electric part of Kähler action in Minkowskian regions has sign opposite to magnetic part and volume term (actually magnetic S^2 part of 6-D Kähler action) so that these two contributions could cancel. The problem is that Kähler function would be constant and therefore also the Kähler metric.

2. I have also proposed [L68] that the analog of micro-canonical ensemble makes sense meaning that all space-time surfaces contributing to the scattering amplitude have the same action. As a consequence, the action exponential and the usual normalization factor would cancel each other and one would obtain just a sum over space-time surfaces with same action: otherwise action exponential would not appear in the scattering amplitudes - this is the case also in perturbative QFTs. This is crucial for the p-adicization and adelization since these exponential factors belong to the extension of rationals only under very strong additional conditions.

   This option has analog also at the level of WCW since Kähler function should have for give values of zero modes only single minimum so that localization in zero modes would mean that the action exponential cancels in the normalization of the amplitudes. It seems that this option is the only possible one.

   Note that the cancellation of the metric determinant and Gaussian determinant possible for Kähler metric with the exponent of Kähler function serving as vacuum functional reduces the perturbative integrations around the minima of Kähler action to a sum over exponents, and if only single minimum contributes for given values of the zero modes, the sum contains only single term.

14.3.1 Number theoretic vision about coupling constant evolution

Let us return to the question about the coupling constant evolution.

1. Each extension of rationals corresponds to particular values of coupling parameters determined by the extension so that it indeed makes sense to ponder what the spectrum of values for say fine structure constant is. In standard QFT this does not make sense.

2. Coupling constant evolution as a function of momentum or length scales reduces to p-adic coupling constant evolution in TGD as function of p-adic prime. Particles are characterized

by preferred p-adic primes - for instance, electron corresponds to $M_{127} = 2^{127} - 1$ - the largest Mersenne prime which does not correspond to super-astronomical Compton length - and the natural identification is as so called ramified primes of extension.

Why the interpretation of p-adic primes as ramified primes?

1. As one increases length scale resolution particle decomposes to more elementary particles.

2. Particles correspond in TGD to preferred p-adic primes. This suggests that when a prime (ideal) of given extension is looked at improved precision determined by an extension of the orignal extension it decomposes into a product of primes. This indeed happens.

   The number of primes of the larger extension appearing in the decomposition to product equals to the dimension of extension as extension of the original extension. All these primes appear and only once in the generic case. Ramified primes of ordinary extension are however odd-balls. Some primes of extension are missing and some appear as higher powers than 1 in their decomposition.

3. Ramified primes are analogous to critical systems. Polynomial with a multiple root - now prime of extension appearing as higher power - corresponds to a critical system. TGD is quantum critical so that one expects that ramified primes are preferred physically and indeed correspond to quantum critical systems.

4. Only the momenta belonging to the extension of rationals are considered and one can identify them as real-valued or p-adic valued momenta. Coupling constants do not depend on the values of the momenta for given extension of rationals and are thus analogous to critical temperature.

   This involves interesting not totally resolved technical question inspired by p-adic mass calculations for which the p-adic mass squared value is mapped to its real value by canonical identification $S \sum x_n p^n \rightarrow \sum x_n p^{-n}$. The correspondence is continuous and can be applied to Lorentz invariants appearing in scattering amplitudes [K77].

   Could this correspondence be applied also to momenta rather than only mass squared values and Lorentz invariants? $M^8 - H$ correspondence [L48] selects fixed Poincare frame as moduli space for octonionic structures and at M^8 level this could make sense.

14.3.2 Cosmological constant and twistor lift of Kähler action

Cosmological constant Λ is one of the biggest problems of modern physics. Surprisingly, Λ turned out to provide the first convincing solution to the problem of understanding coupling constant evolution in TGD framework. In QFTs the independence of scattering amplitudes on UV cutoff length scale gives rise to renormalization group (RG) equations. In TGD there is however no natural cutoff length scale since the theory is finite. Cosmological constant should however evolve as a function of p-adic length scales and cosmological constant itself could give rise to the length scale serving in the role of cutoff length scale. Combined with the view about cosmological constant provided by twistor lift of TGD this leads to explicit RG equations for α_K and scattering amplitudes.

Cosmological constant has two meanings.

1. Einstein proposed non-vanishing value of Λ in Einstein action as a volume term at his time in order to get what could be regarded as a static Universe. It turned out that Universe expanded and Einstein concluded that this proposal was the greatest blunder of his life. For two decades ago it was observed that the expansion of the Universe acclerates and the cosmological constant emerged again. Λ must be extremely small and have correct sign in order to give accelerating rather decelerating expansion in Robertson-Walker cooordinate. Here one must however notice that the time slicing used by Einstein was different and fr this slicing the Universe looked static.

2. Λ can be however understood in an alternative sense as characterizing the dynamics in the matter sector. Λ could characterize the vacuum energy density of some scalar field, call it

quintessense, proportional to 3- volume in quintessence scenario. This Λ would have sign opposite to that in the first scenario since it would appear at opposite side of Einstein's equations.

Cosmological constant in string models and in TGD

It has turned out that Λ could be the final nail to the coffin of superstring theory.

1. The most natural prediction of M-theory and superstring models is Λ in Einsteinian sense but with wrong sign and huge value: for instance, in AdS/CFT correspondence this would be the case. There has been however a complex argument suggesting that one could have a cosmological constant with a correct sign and even small enough size.

   This option however predicts landscape and a loss of predictivity, which has led to a total turn of the philosophical coat: the original joy about discovering the unique theory of everything has changed to that for the discovery that there are no laws of physics. Cynic would say that this is a lottery win for theoreticians since theory building reduces to mere artistic activity.

2. Now however Cumrun Vafa - one of the leading superstring theorists - has proposed that the landscape actually does not exist at all [B58] (see `http://tinyurl.com/ycz7wvng`). Λ would have wrong sign in Einsteinian sense but the hope is that quintessence scenario might save the day. Λ should also decrease with time, which as such is not a catastrophe in quintessence scenario.

3. Theorist D. Wrase *et al* has in turn published an article [B21] (see `http://tinyurl.com/ychrhuxk`) claiming that also the Vafa's quintessential scenario fails. It would not be consistent with Higgs Higgs mechanism. The conclusion suggesting itself is that according to the no-laws-of-physics vision something catastrophic has happened: string theory has made a prediction! Even worse, it is wrong.

   Remark: In TGD framework Higgs is present as a particle but p-adic thermodynamics rather than Higgs mechanism describes at least fermion massivation. The couplings of Higgs to fermions are naturally proportional their masses and fermionic part of Higgs mechanism is seen only as a way to reproduce the masses at QFT limit.

4. This has led to a new kind of string war: now inside superstring hegemony and dividing it into two camps. Optimistic outsider dares to hope that this leads to a kind of auto-biopsy and the gloomy period of superstring hegemony in theoretical physics lasted now for 34 years would be finally over.

String era need not be over even now! One could propose that both variants of Λ are present, are large, and compensate each other almost totally! First I took this as a mere nasty joke but I realized that I cannot exclude something analogous to this in TGD. It turned that this is not possible. I had made a delicate error. I thought that the energy of the dimensionally reduced 6-D Kähler action can be deduced from the resulting 4-D action containing volume term giving the negative contribution rather than dimensionally reducing the 6-D expression in which the volume term corresponds to 6-D magnetic energy and is positive! A lesson in non-commutativity!

The picture in which Λ in Einsteinian sense parametrizes the total action as dimensionally reduced 6-D twistor lift of Kähler action could be indeed interpreted formally as sum of genuine cosmological term identified as volume action. This picture has additional bonus: it leads to the understanding of coupling constant evolution giving rise to discrete coupling constant evolution as sub-evolution in adelic physics. This picture is summarized below.

The picture emerging from the twistor lift of TGD

Consider first the picture emerging from the twistor lift of TGD.

1. Twistor lift of TGD leads via the analog of dimensional reduction necessary for the induction of 8-D generalization of twistor structure in $M^4 \times CP_2$ to a 4-D action determining space-time surfaces as its preferred extremals. Space-time surface as a preferred extremal defines

a unique section of the induced twistor bundle. The dimensionally reduced Kähler action is sum of two terms. Kähler action proportional to the inverse of Kähler coupling strength and volume term proportional to the cosmological constant Λ.

Remark: The sign of the volume action is negative as the analog of the magnetic part of Maxwell action and *opposite* to the sign of the area action in string models.

Kähler and volume actions should have opposite signs. At M^4 limit Kähler action is proportional to $E^2 - B^2$ In Minkowskian regions and to $-E^2 - B^2$ in Euclidian regions.

2. Twistor lift forces the introduction of also M^4 Kähler form so that the twistor lift of Kähler action contains M^4 contribution and gives in dimensional reduction rise to M^4 contributions to 4-D Kähler action and volume term.

   It is of crucial importance that the Cartesian decomposition $H = M^4 \times CP_2$ allows the scale of M^4 contribution to 6-D Kähler action to be different from CP_2 contribution. The size of M^4 contribution as compared to CP_2 contribution must be very small from the smallness of CP breaking [L57] [L55].

   For canonically imbedded M^4 the action density vanishes. For string like objects the electric part of this action dominates and corresponding contribution to 4-D Kähler action of flux tube extremals is positive unlike the standard contribution so that an almost cancellation of the action is in principle possible.

3. What about energy? One must consider both Minkowskian and Euclidian space-time regions and be very careful with the signs. Assume that Minkowskian and Eucidian regions have *same time orientation*.

   (a) Since a dimensionally reduced 6-D Kähler action is in question, the sign of energy density is positive Minkowskian space-time regions and of form $(E^2 + B^2)/2$. Volume energy density proportional to Λ is positive.

   (b) In Euclidian regions the sign of g^{00} is negative and energy density is of form $(E^2 - B^2)/2$ and is negative when magnetic field dominates. For string like objects the M^4 contribution to Kähler action however gives a contribution in which the electric part of Kähler action dominates so that M^4 and CP_2 contributions to energy have opposite signs.

   (c) 4-D volume energy corresponds to the magnetic energy for twistor sphere S^2 and is therefore positive. For some time I thought that the sign must be negative. My blunder was that I erratically deduced the volume contribution to the energy from 4-D dimensionally reduced action, which is sum of Kähler action and volume term rather than deducing it for 6-D Kähler action and then dimensionally reducing the outcome. A good example about consequences of non-commutativity!

The identification of the observed value of cosmological constant is not straightforward and I have considered several options without making explicit their differences even to myself. For Einsteinian option cosmological constant could correspond to the coefficient Λ of the volume term in analogy with Einstein's action. For what I call quintessence option cosmological constant Λ_{eff} would approximately parameterize the total action density or energy density.

1. Cosmological constant - irrespective of whether it is identified as Λ or Λ_{eff} - is extremely small in the recent cosmology. The natural looking assumption would be that as a coupling parameter Λ or Λ_{eff} depends on p-adic length scale like $1/L_p^2$ and therefore decreases in average sense as $1/a^2$, where a is cosmic time identified as light-cone proper time assignable to either tip of CD. This suggests the following rough vision.

   The increase of the thickness of magnetic flux tubes carrying monopole flux liberates energy and this energy can make possible increase of the volume so that one obtains cosmic expansion. The expansion of flux tubes stops as the string tension achieves minimum and the further increase of the volume would increase string tension. For the cosmological constant in cosmological scales the maximum radius of flux tube is about 1 mm, which is biological length scale. Further expansion becomes possible if a phase transition increasing the p-adic length scale and reducing the value of cosmological constant is reduced. This phase transition liberates volume energy and leads to an accelerated expansion. The space-time surface would expand by jerks in stepwise manner. This process would replace continuous cosmic expansion of GRT. One application is TGD variant of Expanding Earth model explaining Cambrian Explosion, which is really weird event [L65].

One can however raise a serious objection: since the volume term is part of 6-D Kähler action, the length scale evolution of Λ should be dictated by that for $1/\alpha_K$ and be very slow: therefore cosmological constant identified as Einsteinian Λ seems to be excluded.

2. It however turns that it possible to have a large number of embedding of the twistor sphere into the product of twistor spheres of M^4 and CP_2 defining dimensional reductions. This set is parameterized by rotations sphere. The S^2 part of 6-D Kähler action determining Λ can be arbitrarily small. This mechanism is discussed in detail in [L71, L72] and leads also to the understanding of coupling constant evolution. The cutoff scale in QFT description of coupling constant evolution is replaced with the length scale defined by cosmological constant.

Second manner to increase 3-volume

Besides the increase of 3-volume of M^4 projection, there is also a second manner to increase volume energy: many-sheetedness. The phase transition reducing the value of Λ could in fact force many-sheetedness.

1. In TGD the volume energy associated with Λ is analogous to the surface energy in superconductors of type I. The thin 3-surfaces in superconductors could have similar 3-surface analogs in TGD since their volume is proportional to surface area - note that TGD Universe can be said to be quantum critical.

   This is not the only possibility. The sheets of many-sheeted space-time having overlapping M^4 projections provide second mechanism. The emergence of many-sheetedness could also be caused by the increase of $n = h_{eff}/h_0$ as a number of sheets of Galois covering.

2. Could the 3-volume increase during deterministic classical time evolution? If the minimal surface property assumed for the preferred extremals as a realization of quantum criticality is true everywhere, the conservation of volume energy prevents the increase of the volume. Minimal surface property is however assumed to fail at discrete set of points due to the transfer of conserved charged between Kähler and volume degrees of freedom. Could this make possible the increase of volume during classical time evolution so that volume and Kähler energy could increase?

   Remark: While writing this for the first time, I did not yet realize that if the action contains also parts associated with string world sheets and their light-like boundaries as $M^8 - H$ duality suggests, then the transfer of conserved quantities between space-time interior and string world sheets and string world sheets and their boundaries is possible, and implies the failure of the minimal surface property at these surfaces. One can however formulated precisely the proposed option and it implies that also string world sheets are quantum critical and therefore minimal surfaces: the question whether this occurs everywhere or only for the portions of string world sheets near the boundaries of causal diamonds remains open [L84].

3. ZEO allows the increase of average 3-volume by quantum jumps. There is no reason why each "big" state function reduction changing the roles of the light-like boundaries of CD could not decrease the average volume energy of space-time surface for the time evolutions in the superposition. This can occur in all scales, and could be achieved also by the increase of $h_{eff}/h_0 = n$.

4. The geometry of CD suggests strongly an analogy with Big Bang followed by Big Crunch. The increase of the volume as increase of the volume of M^4 projection does not however seem to be consistent with Big Crunch. One must be very cautious here. The point is that the size of CD itself increases during the sequence of small state function reductions leaving the members of state pairs at passive boundary of CD unaffected. The size of 3-surface at the active boundary of CD therefore increases as also its 3-volume.

The increase of the volume during the Big Crunch period could be also due to the emergence of the many-sheetedness, in particular due to the increase of the value of n for space-time sheets for sub-CDs. In this case, this period could be seen as a transition to quantum criticality accompanied by an emergence of complexity.

Is the cosmological constant really understood?

The interpretation of the coefficient of the volume term as cosmological constant has been a long-standing interpretational issue and caused many moments of despair during years. The intuitive picture has been that cosmological constant obeys p-adic length scale scale evolution meaning that Λ would behave like $1/L_p^2 = 1/p \simeq 1/2^k$ [L31].

This would solve the problems due to the huge value of Λ predicted in GRT approach: the smoothed out behavior of Λ would be $\Lambda \propto 1/a^2$, a light-cone proper time defining cosmic time, and the recent value of Λ - or rather, its value in length scale corresponding to the size scale of the observed Universe - would be extremely small. In the very early Universe - in very short length scales - Λ would be large.

A simple solution of the problem would be the p-adic length scale evolution of Λ as $\Lambda \propto 1/p$, $p \simeq 2^k$. The flux tubes would thicken until the string tension as energy density would reach stable minimum. After this a phase transition reducing the cosmological constant would allow further thickening of the flux tubes. Cosmological expansion would take place as this kind of phase transitions (for a mundane application of this picture see [L65]).

This would solve the basic problem of cosmology, which is understanding why cosmological constant manages to be so small at early times. Time evolution would be replaced with length scale evolution and cosmological constant would be indeed huge in very short scales but its recent value would be extremely small.

I have however not really understood how this evolution could be realized! Twistor lift seems to allow only a very slow (logarithmic) p-adic length scale evolution of Λ [L70]. Is there any cure to this problem?

1. The magnetic energy decreases with the area S of flux tube as $1/S \propto 1/p \simeq 1/2^k$, where $\sqrt{p}$ defines the transversal length scale of the flux tube. Volume energy (magnetic energy associated with the twistor sphere) is positive and increases like S. The sum of these has minimum for certain radius of flux tube determined by the value of Λ. Flux tubes with quantized flux would have thickness determined by the length scale defined by the density of dark energy: $L \sim \rho_{vac}^{-1/4}$, $\rho_{dark} = \Lambda/8\pi G$. $\rho_{vac} \sim 10^{-47}$ GeV^4 (see `http://tinyurl.com/k4bwlzu`) would give $L \sim 1$ mm, which would could be interpreted as a biological length scale (maybe even neuronal length scale).

2. But can Λ be very small? In the simplest picture based on dimensionally reduced 6-D Kähler action this term is not small in comparison with the Kähler action! If the twistor spheres of M^4 and CP_2 give the same contribution to the induced Kähler form at twistor sphere of X^4, this term has maximal possible value!

   The original discussions in [K116, L31] treated the volume term and Kähler term in the dimensionally reduced action as independent terms and Λ was chosen freely. This is however not the case since the coefficients of both terms are proportional to $(1/\alpha_K^2)S(S^2)$, where $S(S^2)$ is the area of the twistor sphere of 6-D induced twistor bundle having space-time surface as base space. This are is same for the twistor spaces of M^4 and CP_2 if CP_2 size defines the only fundamental length scale. I did not even recognize this mistake.

The proposed fast p-adic length scale evolution of the cosmological constant would have extremely beautiful consequences. Could the original intuitive picture be wrong, or could the desired p-adic length scale evolution for Λ be possible after all? Could non-trivial dynamics for dimensional reduction somehow give it? To see what can happen one must look in more detail the induction of twistor structure.

1. The induction of the twistor structure by dimensional reduction involves the identification of the twistor spheres S^2 of the geometric twistor spaces $T(M^4) = M^4 \times S^2(M^4)$ and of T_{CP_2} having $S^2(CP_2)$ as fiber space. What this means that one can take the coordinates

of say $S^2(M^4)$ as coordinates and embedding map maps $S^2(M^4)$ to $S^2(CP_2)$. The twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ have in the minimal scenario same radius $R(CP_2)$ (radius of the geodesic sphere of CP_2. The identification map is unique apart from $SO(3)$ rotation R of either twistor sphere possibly combined with reflection P. Could one consider the possibility that R is not trivial and that the induced Kähler forms could almost cancel each other?

2. The induced Kähler form is sum of the Kähler forms induced from $S^2(M^4)$ and $S^2(CP_2)$ and since Kähler forms are same apart from a rotation in the common S^2 coordinates, one has $J_{ind} = J + RP(J)$, where R denotes a rotation and P denotes reflection. Without reflection one cannot get arbitrary small induced Kähler form as sum of the two contributions. For mere reflection one has $J_{ind} = 0$.

   Remark: It seems that one can do with reflection if the Kähler forms of the twistor spheres are of opposite sign in standard spherical coordinates. This would mean that they have have opposite orientation.

   One can choose the rotation to act on (y,z)-plane as $(y,z) \rightarrow (cy + sz, -sz + cy)$, where s and c denote the cosines of the rotation angle. A small value of cosmological constant is obtained for small value of s. Reflection P can be chosen to correspond to $z \rightarrow -z$. Using coordinates $(u = cos(\Theta), \Phi)$ for $S^2(M^4)$ and (v, Ψ) for $S^2(CP_2)$ and by writing the reflection followed by rotation explicitly in coordinates (x,y,z) one finds $v = -cu - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = arctan[(su/\sqrt{1-u^2}cos(\Phi) + ctan(\Phi)]$. In the lowest order in s one has $v = -u - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = \Phi + scos(\Phi)(u/\sqrt{1-u^2})$.

3. Kähler form J^{ind} is sum of unrotated part $J(M^4) = du \wedge d\Phi$ and $J(CP_2) = dv \wedge d\Psi$. $J(CP_2)$ equals to the determinant $\partial(v,\Psi)/\partial(u,\Phi)$. A suitable spectrum for s could reproduce the proposal $\Lambda \propto 2^{-k}$ for Λ. The S^2 part of 6-D Kähler action equals to $(J^{ind}_{\theta\phi})^2/\sqrt{g_2}$ and in the lowest order proportional to s^2. For small values of s the integral of Kähler action for S^2 over S^2 is proportional to s^2.

   One can write the S^2 part of the dimensionally reduced action as $S(S^2) = s^2F^2(s)$. Very near to the poles the integrand has $1/[sin(\Theta) + O(s)]$ singularity and this gives rise to a logarithmic dependence of F on s and one can write: $F = F(s, log(s))$. In the lowest order one has $s \simeq 2^{-k/2}$, and in improved approximation one obtains a recursion formula $s_n(S^2,k) = 2^{-k/2}/F(s_{n-1}, \log(s_{n-1})$ giving renormalization group evolution with k replaced by anomalous dimension $k_{n,a} = k + 2log[F(s_{n-1}, log(s_{n-1})]$ differing logarithmically from k.

4. The sum $J^{ind} = J + RP(J)$ defining the induced Kähler form in $S^2(X^4)$ is covariantly constant since both terms are covariantly constant by the rotational covariance of J.

5. The embeddings of $S^2(X^4)$ as twistor sphere of space-time surface to both spheres are holomorphic since rotations are represented as holomorphic transformations. Also reflection as $z \rightarrow 1/z$ is holomorphic. This in turn implies that the second fundamental form in complex coordinates is a tensor having only components of type $(1,1)$ and $(-1,-1)$ whereas metric and energy momentum tensor have only components of type $(1,-1)$ and $(-1,1)$. Therefore all contractions appearing in field equations vanish identically and $S^2(X^4)$ is minimal surface and Kähler current in $S^2(X^4)$ vanishes since it involves components of the trace of second fundamental form. Field equations are indeed satisfied.

6. The solution of field equations becomes a family of space-time surfaces parameterized by the values of the cosmological constant Λ as function of S^2 coordinates satisfying $\Lambda/8\pi G = \rho_{vac} = J \wedge (*J)(S^2)$. In long length scales the variation range of Λ would become arbitrary small.

7. If the minimal surface equations solve separately field equations for the volume term and Kähler action everywhere apart from a discrete set of singular points, the cosmological constant affects the space-time dynamics only at these points. The physical interpretation of these points is as seats of fundamental fermions at partonic 2-surface at the ends of light-like 3-surfaces defining their orbits (induced metric changes signature at these 3-surfaces). Fermion orbits would be boundaries of fermionic string world sheets.

One would have family of solutions of field equations but particular value of Λ would make itself visible only at the level of elementary fermions by affecting the values of coupling constants. p-Adic coupling constant evolution would be induced by the p-adic coupling constant evolution for the relative rotations R combined with reflection for the two twistor spheres. Therefore twistor lift would not be mere manner to reproduce cosmological term but determine the dynamics at the level of coupling constant evolution.

8. What is nice that also $\Lambda = 0$ option is possible. This would correspond to the variant of TGD involving only Kähler action regarded as TGD before the emergence of twistor lift. Therefore the nice results about cosmology [K102] obtained at this limit would not be lost.

14.3.3 Does p-adic coupling constant evolution reduce to that for cosmological constant?

One of the chronic problems if TGD has been the understanding of what coupling constant evolution could be defined in TGD.

Basic notions and ideas

Consider first the basic notions and ideas.

1. The notion of quantum criticality is certainly central. The continuous coupling constant evolution having no counterpart in the p-adic sectors of adele would contain as a sub-evolution discrete p-adic coupling constant evolution such that the discrete values of coupling constants allowing interpretation also in p-adic number fields are fixed points of coupling constant evolution.

   Quantum criticality is realized also in terms of zero modes, which by definition do not contribute to WCW metric. Zero modes are like control parameters of a potential function in catastrophe theory. Potential function is extremum with respect to behavior variables replaced now by WCW degrees of freedom. The graph for preferred extremals as surface in the space of zero modes is like the surface describing the catastrophe. For given zero modes there are several preferred extremals and the catastrophe corresponds to the regions of zero mode space, where some branches of co-incide. The degeneration of roots of polynomials is a concrete realization for this.

   Quantum criticality would also mean that coupling parameters effectively disappear from field equations. For minimal surfaces (generalization of massless field equation allowing conformal invariance characterizing criticality) this happens since they are separately extremals of Kähler action and of volume term.

   Quantum criticality is accompanied by conformal invariance in the case of 2-D systems and in TGD this symmetry extends to its 4-D analogas isometries of WCW.

2. In the case of 4-D Kähler action the natural hypothesis was that coupling constant evolution should reduce to that of Kähler coupling strength $1/\alpha_K$ inducing the evolution of other coupling parameters. Also in the case of the twistor lift $1/\alpha_K$ could have similar role. One can however ask whether the value of the 6-D Kähler action for the twistor sphere $S^2(X^4)$ defining cosmological constant could define additional parameter replacing cutoff length scale as the evolution parameter of renormalization group.

3. The hierarchy of adeles should define a hierarchy of values of coupling strengths so that the discrete coupling constant evolution could reduce to the hierarchy of extensions of rationals and be expressible in terms of parameters characterizing them.

4. I have also considered number theoretical existence conditions as a possible manner to fix the values of coupling parameters. The condition that the exponent of Kähler function should exist also for the p-adic sectors of the adele is what comes in mind as a constraint but it seems that this condition is quite too strong.

   If the functional integral is given by perturbations around single maximum of Kähler function, the exponent vanishes from the expression for the scattering amplitudes due to the presence of normalization factor. There indeed should exist only single maximum by the Euclidian signature of the WCW Kähler metric for given values of zero modes (several extrema would mean extrema with non-trivial signature) and the parameters fixing the topology of 3-surfaces at the ends of preferred extremal inside CD. This formulation as counterpart also in terms of the analog of micro-canonical ensemble (allowing only states with the same energy) allowing only discrete sum over extremals with the same Kähler action [L68].

5. I have also considered more or less ad hoc guesses for the evolution of Kähler coupling strength such as reduction of the discrete values of $1/\alpha_K$ to the spectrum of zeros of Riemann zeta or actually of its fermionic counterpart [L23]. These proposals are however highly ad hoc.

Could the area of twistor sphere replace cutoff length?

As I started once again to consider coupling constant evolution I realized that the basic problem has been the lack of explicit formula defining what coupling constant evolution really is.

1. In quantum field theories (QFTs) the presence of infinities forces the introduction of momentum cutoff. The hypothesis that scattering amplitudes do not depend on momentum cutoff forces the evolution of coupling constants. TGD is not plagued by the divergence problems of QFTs. This is fine but implies that there has been no obvious manner to define what coupling constant evolution as a continuous process making sense in the real sector of adelic physics could mean!

2. Cosmological constant is usually experienced as a terrible head ache but it could provide the helping hand now. Could the cutoff length scale be replaced with the value of the length scale defined by the cosmological constant defined by the S^2 part of 6-D Kähler action? This parameter would depend on the details of the induced twistor structure. It was shown above that if the moduli space for induced twistor structures corresponds to rotations of S^2 possibly combined with the reflection, the parameter for coupling constant restricted to that to $SO(2)$ subgroup of $SO(3)$ could be taken to be taken $s = sin(\epsilon)$.

3. RG invariance would state that the 6-D Kähler action is stationary with respect to variations with respect to s. The variation with respect to s would involve several contributions. Besides the variation of $1/\alpha_K(s)$ and the variation of the $S(^2)$ part of 6-D Kähler action defining the cosmological constant, there would be variation coming from the variations of 4-D Kähler action plus 4-D volume term . This variation vanishes by field equations. As matter of fact, the variations of 4-D Kähler action and volume term vanish separately except at discrete set of singular points at which there is energy transfer between these terms. This condition is one manner to state quantum criticality stating that field equations involved no coupling parameters.

   One obtains explicit RG equation for α_K and Λ having the standard form involving logarithmic derivatives. The form of the equation would be

$$\frac{dlog(\alpha_K)}{ds} = -\frac{S(S^2)}{(S_K(X^4)/Vol(X^4)) + S(S^2)}\frac{dlog(S(S^2))}{ds} \quad . \tag{14.3.1}$$

   It should be noticed that the choices of the parameter s in the evolution equation is arbitrary so that the identification $s = sin(\epsilon)$ is not necessary. Note that one must use Kähler action per volume.

   The equation contains the ratio $S(S^2)/(S_K(X^4) + S(S^2))$ of actions as a parameter. This does not conform with idea of micro-locality. One can however argue that this conforms with the generalization of point like particle to 3-D surface. For preferred extremal the action is indeed determined by the 3 surfaces at its ends at the boundaries of CD. This implies that the construction of quantum theory requires the solution of classical theory.

   In particular, the 4-D classical theory is necessary for the construction of scattering amplitudes, and one cannot reduce TGD to string theory although strong form of holography states that the data about quantum states can be assigned with 2-D surfaces. Even more: $M^8 - H$ correspondence implies that the data determining quantum states can be assigned with discrete set of points defining cognitive representations for given adel This set of points depends on the preferred extremal!

4. How to identify quantum critical values of α_K? At these points one should have $dlog(\alpha_K)/ds = 0$. This implies $dlog(S(S^2)/ds = 0$, which in turn implies $dlog(\alpha_K)/ds = 0$ unless one has $S_K(X^4) + S(S^2) = 0$. This condition would make exponent of 6-D Kähler action trivial and the continuation to the p-adic sectors of adele would be trivial. I have considered also this possibility [L70].

   The critical values of coupling constant evolution would correspond to the critical values of S and therefore of cosmological constant. The basic nuisance of theoretical physics would determine the coupling constant evolution completely! Critical values are in principle possible. Both the numerator $J^2_{u\Phi}$ and the numerator $1/\sqrt{det(g)}$ increase with ϵ. If the rate for the variation of these quantities with s vary it is possible to have a situation in which the one has

$$\frac{dlog(J^2_{u\Phi})}{ds} = -\frac{dlog(\sqrt{det(g)})}{ds} \quad . \tag{14.3.2}$$

5. One can make highly non-trivial conclusions about the evolution at general level. For the extremals with vanishing action and for which α_K is critical (vanishing derivate), also the second derivative of $d^2S(S^2)/ds^2 =$ holds true at the critical point. The QFT analogs of these points are points at which beta function develops higher order zero. The tip of cusp catastrophe is second analogy.

   The points at which that the action has minimum are also interesting. For magnetic flux tubes for which one has $S_K(X^4) \propto 1/S$ and $S_{vol} \propto S$ in good approximation, one has $S_K(X^4) = S_{vol}$ at minimum (say for the flux tubes with radius about 1 mm for the cosmological constant in cosmological scales). One can write

$$\frac{dlog(\alpha_K)}{ds} = -\frac{1}{2}\frac{dlog(S(S^2))}{ds} \quad , \tag{14.3.3}$$

   and solve the equation explicitly:

$$\frac{\alpha_{K,0}}{\alpha_K} = \frac{S(S^2)}{S(S^2)_0})^x \quad , \quad x = 1/2 \quad . \tag{14.3.4}$$

   A more general situation would correspond to a model with $x \neq 1/2$: the deviation from $x = 1/2$ could be interpreted as anomalous dimension. This allows to deduce numerically a formula for the value spectrum of $\alpha_{K,0}/\alpha_K$ apart from the initial values.

6. One can solve the equation also for fixed value of $S(X^4)/Vol(X^4)$ to get

$$\frac{\alpha_{K,0}}{\alpha_K} = \frac{S(S^2)}{S(S^2)_0})^x \quad , \quad x = 1/2 \quad . \tag{14.3.5}$$

$$\frac{\alpha_K}{\alpha_{K,0}} = \frac{S_K(X^4)/Vol(X^4)) + S(S^2)}{S_K(X^4)/Vol(X^4))} \quad . \tag{14.3.6}$$

   At the limit $S(S^2) =\rightarrow 0$ one obtains $\alpha_K \rightarrow \alpha_{K,0}$.

7. One should demonstrate that the critical values of s are such that the continuation to p-adic sectors of the adele makes sense. For preferred extremals cosmological constant appears as a parameter in field equations but does not affect the field equations expect at the singular points. Singular points play the same role as the poles of analytic function or point charges in electrodynamics inducing long range correlations. Therefore the extremals depend on parameter s and the dependence should be such that the continuation to the p-adic sectors is possible.

   A naïve guess is that the values of s are rational numbers. Above the proposal $s = 2^{-k/2}$ motivated by p-adic length scale hypothesis was considered but also $s = p^{-k/2}$ can be considered. These guesses might be however wrong, the most important point is that there is that one can indeed calculate $\alpha_K(s)$ and identify its critical values.

8. What about scattering amplitudes and evolution of various coupling parameters? If the exponent of action disappears from scattering amplitudes, the continuation of scattering amplitudes is simple. This seems to be the only reasonable option. In the adelic approach [L52] amplitudes are determined by data at a discrete set of points of space-time surface (defining what I call cognitive representation) for which the points have M^8 coordinates belong to the extension of rationals defining the adele.

   Each point of $S^2(X^4)$ corresponds to a slightly different X^4 so that the singular points depend on the parameter s, which induces dependence of scattering amplitudes on s. Since coupling constants are identified in terms of scattering amplitudes, this induces coupling constant evolution having discrete coupling constant evolution as sub-evolution.

Could the critical values of α_K correspond to the zeros of Riemann Zeta?

Number theoretical intuitions strongly suggests that the critical values of $1/\alpha_K$ could somehow correspond to zeros of Riemann Zeta. Riemann zeta is indeed known to be involved with critical systems.

The naïvest ad hoc hypothesis is that the values of $1/\alpha_K$ are actually proportional to the non-trivial zeros $s = 1/2 + iy$ of zeta [L23]. A hypothesis more in line with QFT thinking is that they correspond to the imaginary parts of the roots of zeta. In TGD framework however complex values of α_K are possible and highly suggestive. In any case, one can test the hypothesis that the values of $1/\alpha_K$ are proportional to the zeros of ζ at critical line. Problems indeed emerge.

1. The complexity of the zeros and the non-constancy of their phase implies that the RG equation can hold only for the imaginary part of $s = 1/2 + it$ and therefore only for the imaginary part of the action. This suggests that $1/\alpha_K$ is proportional to y. If $1/\alpha_K$ is complex, RG equation implies that its phase RG invariant since the real and imaginary parts would obey the same RG equation.

2. The second - and much deeper - problem is that one has no reason for why $dlog(\alpha_K)/ds$ should vanish at zeros: one should have $dy/ds = 0$ at zeros but since one can choose instead of parameter s any coordinate as evolution parameter, one can choose $s = y$ so that one has $dy/ds = 1$ and criticality condition cannot hold true. Hence it seems that this proposal is unrealistic although it worked qualitatively at numerical level.

It seems that it is better to proceed in a playful spirit by asking whether one could realize quantum criticality in terms of of the property of being zero of zeta.

1. The very fact that zero of zeta is in question should somehow guarantee quantum criticality. Zeros of ζ define the critical points of the complex analytic function defined by the integral

$$X(s_0, s) = \int_{C_{s_0 \to s}} \zeta(s)ds \ , \tag{14.3.7}$$

   where $C_{s_0 \to s}$ is any curve connecting zeros of ζ, a is complex valued constant. Here s does not refer to $s = sin(\epsilon)$ introduced above but to complex coordinate s of Riemann sphere.

By analyticity the integral does not depend on the curve C connecting the initial and final points and the derivative $dS_c/ds = \zeta(s)$ vanishes at the endpoints if they correspond to zeros of ζ so that would have criticality. The value of the integral for a closed contour containing the pole $s = 1$ of ζ is non-vanishing so that the integral has two values depending on which side of the pole C goes.

2. The first guess is that one can define S_c as complex analytic function $F(X)$ having interpretation as analytic continuation of the S^2 part of action identified as $Re(S_c)$:

$$
\begin{array}{ll}
S_c(S^2) = F(X(s,s_0)) \ , & X(s,s_0) = \int_{C_{s_0\rightarrow s}} \zeta(s)ds \ , \\
S(S^2) = Re(S_c) = Re(F(X)) \ , & \\
\zeta(s) = 0 \ , & Re(s_0) = 1/2 \ .
\end{array} \tag{14.3.8}
$$

$S_c(S^2) = F(X)$ would be a complexified version of the Kähler action for S^2. s_0 must be at critical line but it is not quite clear whether one should require $\zeta(s_0) = 0$.

The real valued function $S(S^2)$ would be thus extended to an analytic function $S_c = F(X)$ such that the $S(S^2) = Re(S_c)$ would depend only on the end points of the integration path $C_{s_0\rightarrow s}$. This is geometrically natural. Different integration paths at Riemann sphere would correspond to paths in the moduli space $SO(3)$, whose action defines paths in S^2 and are indeed allowed as most general deformations. Therefore the twistor sphere could be identified Riemann sphere at which Riemann zeta is defined. The critical line and real axis would correspond to particular one parameter sub-groups of $SO(3)$ or to more general one parameter subgroups.

One would have

$$
\frac{\alpha_{K,0}}{\alpha_K} = (\frac{S_c}{S_0})^{1/2} \ . \tag{14.3.9}
$$

The imaginary part of $1/\alpha_K$ (and in some sense also of the action $S_c(S^2)$) would determined by analyticity somewhat like the real parts of the scattering amplitudes are determined by the discontinuities of their imaginary parts.

3. What constraints can one pose on F? F must be such that the value range for $F(X)$ is in the value range of $S(S^2)$. The lower limit for $S(S^2)$ is $S(S^2) = 0$ corresponding to $J_{u\Phi} \rightarrow 0$.

The upper limit corresponds to the maximum of $S(S^2)$. If the one Kähler forms of M^4 and S^2 have same sign, the maximum is $2 \times A$, where $A = 4\pi$ is the area of unit sphere. This is however not the physical case.

If the Kähler forms of M^4 and S^2 have opposite signs or if one has RP option, the maximum, call it S_{max}, is smaller. Symmetry considerations strongly suggest that the upper limit corresponds to a rotation of 2π in say (y, z) plane ($s = sin(\epsilon) = 1$ using the previous notation).

For $s \rightarrow s_0$ the value of S_c approaches zero: this limit must correspond to $S(S^2) = 0$ and $J_{u\Phi} \rightarrow 0$. For $Im(s) \rightarrow \pm\infty$ along the critical line, the behavior of $Re(\zeta)$ (see `http://tinyurl.com/y7b88gvg`) strongly suggests that $|X| \rightarrow \infty$. This requires that F is an analytic function, which approaches to a finite value at the limit $|X| \rightarrow \infty$. Perhaps the simplest elementary function satisfying the saturation constraints is

$$
F(X) = S_{max} tanh(-iX) \ . \tag{14.3.10}
$$

One has $tanh(x+iy) \rightarrow \pm 1$ for $y \rightarrow \pm\infty$ implying $F(X) \rightarrow \pm S_{max}$ at these limits. More explicitly , one has $tanh(-i/2-y) = [-1+exp(-4y)-2exp(-2y)(cos(1)-1)]/[1+exp(-4y)-2exp(-2y)(cos(1)-1)]$. Since one has $tanh(-i/2+0) = 1-1/cos(1) < 0$ and $tanh(-i/2+\infty) = 1$, one must have some finite value $y = y_0 > 0$ for which one has

$$tanh(-\frac{i}{2}+y_0) = 0 \ . \tag{14.3.11}$$

The smallest possible lower bound s_0 for the integral defining X would naturally to $s_0 = 1/2 - iy_0$ and would be below the real axis.

4. The interpretation of $S(S^2)$ as a positive definite action requires that the sign of $S(S^2) = Re(F)$ for a given choice of $s_0 = 1/2 + iy_0$ and for a propertly sign of $y - y_0$ at critical line should remain positive. One should show that the sign of $S = a\int Re(\zeta)(s = 1/2 + it)dt$ is same for all zeros of ζ. The graph representing the real and imaginary parts of Riemann zeta along critical line $s = 1/2 + it$ (see `http://tinyurl.com/y7b88gvg`) shows that both the real and imaginary part oscillate and increase in amplitude. For the first zeros real part stays in good approximation positive but the amplitude for the negative part increase be gradually. This suggests that S identified as integral of real part oscillates but preserves its sign and gradually increases as required.

A priori there is no reason to exclude the trivial zeros of ζ at $s = -2n$, $n = 1, 2, ...$

1. The natural guess is that the function $F(X)$ is same as for the critical line. The integral defining X would be along real axis and therefore real as also $1/\alpha_K$ provided the sign of S_c is positive: for negative sign for S_c not allowed by the geometric interpretation the square root would give imaginary unit. The graph of the Riemann Zeta at real axis (real) is given in MathWorld Wolfram (see `http://tinyurl.com/55qjmj`).

2. The functional equation

$$\zeta(1-s) = \zeta(s)\frac{\Gamma(s/2)}{\Gamma((1-s)/2)} \tag{14.3.12}$$

allows to deduce information about the behavior of ζ at negative real axis. $\Gamma((1-s)/2)$ is negative along negative real axis (for $Re(s) \leq 1$ actually) and poles at $n + 1/2$. Its negative maxima approach to zero for large negative values of $Re(s)$ (see `http://tinyurl.com/clxv4pz`) whereas $\zeta(s)$ approaches value one for large positive values of s (see `http://tinyurl.com/y7b88gvg`). A cautious guess is that the sign of $\zeta(s)$ for $s \leq 1$ remains negative. If the integral defining the area is defined as integral contour directed from $s < 0$ to a point s_0 near origin, S_c has positive sign and has a geometric interpretation.

3. The formula for $1/\alpha_K$ would read as $\alpha_{K,0}/\alpha_K(s = -2n) = (S_c/S_0)^{1/2}$ so that α_K would remain real. This integration path could be interpreted as a rotation around z-axis leaving invariant the Kähler form J of $S^2(X^4)$ and therefore also $S = Re(S_c)$. $Im(S_c) = 0$ indeed holds true. For the non-trivial zeros this is not the case and $S = Re(S_c)$ is not invariant.

4. One can wonder whether one could distinguish between Minkowskian and Euclidian and regions in the sense that in Minkowskian regions $1/\alpha_K$ correspond to the non-trivial zeros and in Euclidian regions to trivial zeros along negative real axis. The interpretation as different kind of phases might be appropriate.

What is nice that the hypothesis about equivalence of the geometry based and number theoretic approaches can be killed by just calculating the integral S as function of parameter s. The identification of the parameter s appearing in the RG equations is no unique. The identification of the Riemann sphere and twistor sphere could even allow identify the parameter t as imaginary coordinate in complex coordinates in $SO(3)$ rotations around z-axis act as phase multiplication and in which metric has the standard form.

Some guesses to be shown to be wrong

The following argument suggests a connection between p-adic length scale hypothesis and evolution of cosmological constant but must be taken as an ad hoc guess: the above formula is enough to predict the evolution.

1. p-Adicization is possible only under very special conditions [L52], and suggests that anomalous dimension involving logarithms should vanish for $s = 2^{-k/2}$ corresponding to preferred p-adic length scales associated with $p \simeq 2^k$. Quantum criticality in turn requires that discrete p-adic coupling constant evolution allows the values of coupling parameters, which are fixed points of RG group so that radiative corrections should vanish for them. Also anomalous dimensions Δk should vanish.

2. Could one have $\Delta k_{n,a} = 0$ for $s = 2^{-k/2}$, perhaps for even values $k = 2k_1$? If so, the ratio c/s would satisfy $c/s = 2^{k_1} - 1$ at these points and Mersenne primes as values of c/s would be obtained as a special case. Could the preferred p-adic primes correspond to a prime near to but not larger than $c/s = 2^{k_1} - 1$ as p-adic length scale hypothesis states? This suggest that we are on correct track but the hypothesis could be too strong.

3. The condition $\Delta d = 0$ should correspond to the vanishing of dS/ds. Geometrically this would mean that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

14.3.4 An alternative view about the coupling constant evolution in terms of cosmological constant

The above view about the evolution of cosmological constant relies crucially on the identification of $M^4 \times S^2$ as twistor space of M^4, and the assumption that the radii of twistor spheres $S^2(M^4$ and $S^2(CP_2)$ assignable to the twistor bundle of CP_2 are same.

One can however argue that the standard twistor space CP_3 of M^4 with Minkowskian signature (3,-3) is a more feasible candidate for the twistor space of M^4. Accepting this, one ends up to a modification of the above vision about coupling constant evolution [L96, L99]. The progress in understanding SUSY in TGD framework led also to a dramatic progress in the understanding of the coupling constant evolution [L92].

Getting critical about geometric twistor space of M^4

Let us first discuss the recent picture and how to modify it so that it is consistent with the hierarchy of CDs. The key idea is that the twistor space and its base space represents CD so that one obtains scale hierarchy of twistor spaces as a realization of broken scale invariance giving rise to the p-adic length scale hierarchy.

1. I have identified the twistor space of M^4 simply as $T(M^4) = M^4 \times S^2$. The interpretation would be at the level of octonions as a product of M^4 and choices of M^2 as preferred complex sub-space of octonions with S^2 parameterizing the directions of spin quantization axes. Real octonion axis would correspond to time coordinate. One could talk about the space of of light-like directions. Light-like vector indeed defines M^2. This view could be defended by the breaking of both translation and Lorentz invariance in the octonionic approach due to the choice of M^2 and by the fact that it seems to work.

   Remark: $M^8 = M^4 \times E^4$ is complexified to M^8_c by adding a commuting imaginary unit i appearing in the extensions of rationals and ordinary M^8 represents its particular sub-space. Also in twistor approach one uses often complexified M^4.

2. The objection is that it is ordinary twistor space identifiable as CP_3 with (3,-3) signature of metric is what works in the construction of twistorial amplitudes. CP_3 has metric as compact space and coset space. Could this choice of twistor space make sense after all as geometric twistor space?

Here one must pause and recall that the original key idea was that Poincare invariance is symmetry of TGD for $X^4 \subset M^4 \times CP_2$. Now Poincare symmetry has been transformed to a symmetry acting at the level of M^8 in the moduli space of octonion structures defined by the choice of the direction of octonionic real axis reducing Poincare group to $T \times SO(3)$ consisting of time translations and rotations. Fixing of M^2 reducrs the group to $T \times SO(2)$ and twistor space can be seen as the space for selections of quantization axis of energy and spin.

3. But what about the space H? The first guess is $H = M^4_{conf} \times CP_2$. According to [B10] (see `http://tinyurl.com/y35k5wwo`) one has $M^4_{conf} = U(2)$ such that $U(1)$ factor is time- like and $SU(2)$ factor is space-like. One could understand $M^4_{conf} = U(2)$ as resulting by addition and identification of metrically 2-D light-cone boundaries at $t = \pm\infty$. This is topologically like compactifying E^3 to S^3 and gluing the ends of cylinder $S^3 \times D^1$ together to the $S^3 \times S^1$.

   The conformally compactified Minkowski space M^4_{conf} should be analogous to base space of CP_3 regarded as bundle with fiber S^2. The problem is that one cannot imagine an analog of fiber bundle structure in CP_3 having $U(2)$ as base. The identification $H = M^4_{conf} \times CP_2$ does not make sense.

4. In ZEO based breaking of scaling symmetry it is CD that should be mapped to the analog of M^4_{conf} - call it cd_{conf}. The only candidate is $cd_{conf} = CP_2$ with one hypercomplex coordinate. To understand why one can start from the following picture. The light-like boundaries of CD are metrically equivalent to spheres. The light-like boundaries at $t = \pm\infty$ are identified as in the case of M^4_{conf}. In the case of CP_2 one has 3 homologically trivial spheres defining coordinate patches. This suggests that cd_{conf} is simply CP_2 with second complex coordinate made hypercomplex. M^4 and E^4 differ only by the signature and so would do cd_{conf} and CP_2.

   The twistor spheres of CP_3 associated with points of M^4 intersect at point if the points differ by light-like vector so that one has and singular bundle structure. This structure should have analog for the compactification of CD. CP_3 has also bundle structure $CP_3 \to CP_2$. The S^2 fibers and base are homologically non-trivial and complex analogs of mutually orthogonal line and plane and intersect at single point. This defines the desired singular bundle structure via the assignment of S^2 to each point of CP_2.

   The M^4 points must belong to the interior of cd and this poses constraints on the distance of M^4 points from the tips of cd. One expects similar hierarchy of cds at the level of momentum space.

5. In this picture $M^4_{conf} = U(2)$ could be interpreted as a base space for the space of CDs with fixed direction of time axis identified as direction of octonionic real axis associated with various points of M^4 and therefore of M^4_{conf}. For Euclidian signature one would have base and fiber of the automorphism sub-group $SU(3)$ regarded as $U(2)$ bundle over CP_2: now one would have CP_2 bundle over $U(2)$. This is perhaps not an accident, and one can ask whether these spaces could be interpreted as representing local trivialization of $SU(3)$ as $U(2) \times CP_2$. This would give to metric cross terms between $U(2)$ and CP_2.

6. The proposed identification can be tested by looking whether it generalizes. What the twistor space for entire M^8 would be? $cd = CD_4$ is replaced with CD_8 and the discussion of the preceding chapter demonstrated that the only possible identification of the twistor space is now is as the 12-D hyperbolic variant of HP_3 whereas $CD_{8,conf}$ would correspond to 8-D hyperbolic variant of HP_2 analogous to hyperbolic variant of CP_2.

The outcome of these considerations is surprising.

1. One would have $T(H) = CP_3 \times F$ and $H = CP_{2,H} \times CP_2$ where $CP_{2,H}$ has hyperbolic metric with metric signature $(1, -3)$ having M^4 as tangent space so that the earlier picture can be understood as an approximation. This would reduce the construction of preferred extremals of 6-D Kähler action in $T(H)$ to a construction of polynomial holomorphic surfaces and also the minimal surfaces with singularities at string world sheets should result as bundle

projection. Since M^8-H duality must respect algebraic dynamics the maximal degree of the polynomials involved must be same as the degree of the octonionic polynomial in M^8.

2. The hyperbolic variant Kähler form and also spinor connection of hyperbolic CP_2 brings in new physics beyond standard model. This Kähler form would serve as the analog of Kähler form assigned to M^4 earlier, and suggested to explain the observed CP breaking effects and matter antimatter asymmetry for which there are two explanations [L92].

Some comments about the Minkowskian signature of the hyperbolic counterparts of CP_3 and CP_2 are in order.

1. Why the metric of CP_3 could not be Euclidian just as the metric of F? The basic objection is that propagation of fields is not possible in Euclidian signature and one completely loses the earlier picture provided by $M^4\times CP_2$. The algebraic dynamics in M^8 picture can hardly replace it.

2. The map assigning to the point M^4 a point of CP_3 involves Minkowskian sigma matrices but it seems that the Minkowskian metric of CP_3 is not explicitly involved in the construction of scattering amplitudes. Note however that the antisymmetric bi-spinor metric for the spin $1/2$ representation of Lorentz group and its conjugate bring in the signature. $U(2,2)$ as representation of conformal symmetries suggests $(2,2)$ signature for 8-D complex twistor space with 2+2 complex coordinates representing twistors.

   The signature of CP_3 metric is not explicitly visible in the construction of twistor amplitudes but analytic continuations are carried out routinely. One has also complexified M^4 and M^8 and one could argue that the problems disappear. In the geometric situation the signatures of the subspaces differ dramatically. As already found, analytic continuation could allow to define the variants of twistor spaces elegantly by replacing a complex coordinate with a hyperbolic one.

   Remark: For E^4 CP_3 is Euclidian and if one has $E^4_{conf}=U(2)$, one could think of replacing the Cartesian product of twistor spaces with $SU(3)$ group having $M^4_{conf}=U(2)$ as fiber and CP_2 as base. The metric of $SU(3)$ appearing as subgroup of quaternionic automorphisms leaving $M^4\subset M^8$ invariant would decompose to a sum of M^4_{conf} metric and CP_2 metric plus cross terms representing correlations between the metrics of M^4_{conf} and CP_2. This is probably mere accident.

How the vision about coupling constant evolution would be modified?

The above described vision about coupling constant evolution in case of $T(M^4)=M^4\times S^2$ would be modified since the interference of the Kähler form made possible by the same signature of $S^2(M^4)$ and $S^2(CP_2)$. Now the signatures are opposite and Kähler forms differ by factor i (imaginary unit commuting with octonion units) so that the induced Kähler forms do not interfere anymore. The evolution of cosmological constant must come from the evolution of the ratio of the radii of twistor spaces (twistor spheres).

1. M^8-H duality has two alternative forms with $H=CP_{2,h}\times CP_2$ or $H=M^4\times CP_2$ depending on whether one projects the twistor spheres of $CP_{3,h}$ to $CP_{2,h}$ or M^4. Let us denote the twistor space $SU(3)/U(1)\times U(1)$ of CP_2 by F.

2. The key idea is that the p-adic length scale hierarchy for the size of 8-D CDs and their 4-D counterparts is mapped to a corresponding hierarchy for the sizes of twistor spaces $CP_{3,h}$ assignable to M^4 by M^8-H-duality. By scaling invariance broken only by discrete size scales of CDs one can take the size scale of CP_2 as a unit so that $r=R^2(S^2(CP_{3,h})/R(S^2(F))$ becomes an evolution parameter.

   Coupling constant evolution must correspond to a variation for the ratio of $r=R^2(S^2(CP_{3,h})/R(S^2(F))$ and a reduction to p-adic length scale evolution is expected. A simple argument shows that Λ is inversely proportional to constant magnetic energy assignable to $S^2(X^4)$ divided by $1/\sqrt{g_2(S^2)}$ in dimensional reduction needed to induce twistor structure. Thus one has $\Lambda\propto 1/r^2\propto 1/L_p^2$. Preferred p-adic primes would be identified as ramified primes of extension of rationals defining the adele so that coupling constant evolution would reduce to number theory.

3. The induced metric would vanish for $R(S^2(CP_{3,h}) = R(S^2(F))$. Λ would be infinite at this limit so that one must have $R(S^2(CP_{3,h}) \neq R(S^2(F))$. The most natural assumption is that one $R(S^2(CP_{3,h}) > R(S^2(F))$ but one cannot exclude the alternative option. Λ behaves like $1/L_p^2$. Inversions of CDs with respect to the values of the cosmological time parameter $a = L_p$ would produce hierarchies of length scales, in particular p-adic length scales coming as powers of $\sqrt{p}$. CP_2 scale and the scale assignable to cosmological constant could be seen as inversions of each other with respect to a scale which is of order 10^{-4} meters defined by the density of dark energy in the recent Universe and thus biological length scale.

4. The above model for the length scale evolution of coupling parameters would reduce to that along paths at $S^2(CP_2)$ and would depend on the ends points of the path only, and also now the zeros of Riemann zeta could naturally correspond to the quantum critical points.

TGD vision about SUSY and coupling constant evolution

TGD view about SUSY leads to radical modification and re-interpretation of SUSY [L96, L92], and to a dramatic progress in the understanding of coupling constant evolution.

Quarks would be the only fundamental fermion fields, and leptons would be spartners of quarks identified as local composites of 3 quarks. Embedding space coordinates would have an expansion in terms of local super-monomials of quarks and antiquarks with vanishing baryon number and appearing as sums of monomial and its conjugate to guarantee hermiticity. Super-spinors would have similar expansion involving only odd quark numbers. This picture is forced by the requirement that propagators are consistent with the statistics of the spartner. Theta parameters would be replaced by creation and annihilation operators for quarks so that super-symmetrization would mean also second quantization. Number theoretic vision requires that only a finite number of Wick contractions of oscillator operators can vanish. These conditions have interpretation as conservation for the Noether currents of some symmetries.

This picture leads to a concrete view about S-matrix for the preferred extremals of a SUSY-variant of the basic action principle relying on the notion of super-variant of embedding space and super-variant of the modified Dirac action. Coupling constant evolution discretizes and would reduce to an increase of the finite number of non-vanishing Wick contractions interpreted as radiative corrections as the dimension of the extension of rationals defining the adele increases. This evolution reflects directly the corresponding evolution at the level of M^8 in terms of octonionic polynomials determining the extension of rationals involved. Whether this view is consistent with the above general vision remains to be seen.

14.3.5 Generalized conformal symmetry, quantum criticality, catastrophe theory, and analogies with thermodynamics and gauge theories

The notion of quantum criticality allows two realizations: as stationarity of S^2 part of the twistor lift of Kähler action and in terms of zeros of zeta are key elements in the explicit proposal for discrete coupling constant evolution reducing to that for cosmological constant.

Quantum criticality from different perspectives

Quantum criticality is however much more general notion, and one must ask how this view relates to the earlier picture.

1. At the real number side continuous coupling constant evolution makes sense. What does this mean? Can one say that quantum criticality makes possible only adelic physics together with large $h_{eff}/h_0 = n$ as dimension for extension of rationals. This hierarchy is essential for life and cognition.

   Can one conclude that living systems correspond to quantum critical values of $S(S^2)$ and therefore α_K and in-animate systems correspond to other values of α_K? But wouldn't his mean that one gives up the original vision that α_K is analogous to critical temperature. The whole point was that this would make physics unique?

From mathematical view point also continuous α_K can make sense. α_K can be continuous if it corresponds to a higher-dimensional critical manifold at which two or more preferred extremals associated with the same parameter values co-incide - roots of polynomial $P(x,a,b)$ depending on parameters a,b serves as the canonical example. The degree of quantum criticality would vary and there would be a hierarchy of critical systems characterized by the dimension of the critical manifold. One would have a full analog of statistical physics. For mathematician this is the only convincing interpretation.

2-D cusp catastrophe serves as a basic example helping to generalize [A50]. Cusp corresponds to the roots of $dP_4/dx = 0$ of third order polynomial $P_4(x,a,b)$, where (a,b) are control variables. The projection of region with 3 real roots to (a,b)-plane is bounded by critical lines forming a roughly V-shaped structure. d^2P_4/dx^2 vanishes at the edges of V, where two roots co-incide and d^3P_4/dx^3 vanishes at the tip of V, where 3 roots co-incide.

2. A hierarchy of quantum criticalities has been actually assumed. The hierarchy of representations for super-symplectic algebra realizing 4-D analog of super-conformal symmetries allows an infinite hierarchy of representations for which infinite-D sub-algebra isomorphic to a full algebra and its commutator with the full algebra annihilate physical states. Also classical Noether charges vanish. What is new is that conformal weights are non-negative integers. The effective dimensions of these systems are finite - at least in the sense that one one has finite-D Lie algebra (or its quantum counterpart) or corresponding Kac-Moody algebra as symmetries. This realization of quantum criticality generalize the idea that conformal symmetry accompanies 2-D criticality.

   This picture conforms also with the vision about hierarchy of hyper-finite-factors with included hyper-finite factor defining measurement resolution [K126]. Hyper-finiteness indeed means finite-dimensionality in excellent approximation.

TGD as catastrophe theory and quantum criticality as prerequisite for the Euclidian signature of WCW metric

It is good to look more precisely how the catastrophe theoretic setting generalizes to TGD.

1. The value of the twistor lift of Kähler action defining Kähler function very probably corresponds to a maximum of Kähler function since otherwise metric defined by the second derivatives could have non-Euclidian signature. One cannot however exclude the possibility that in complex WCW coordinates the (1,1) restriction of the matrix defined by the second derivatives of Kähler function could be positive definite also for other than minima.

   It would seem that one cannot accept several roots for given zero modes since one cannot have maximum of Kähler function for all of them. This would allow only the boundary of catastrophe region in which 2 or more roots co-incide. Positive definiteness of WCW metric would force quantum criticality.

   For given values of zero modes there would be single minimum and together with the cancellation of Gaussian and metric determinants this makes perturbation theory extremely simple since exponents of vacuum functional would cancel.

2. There is an infinite number of zero modes playing the role of control variables since the value of the induce Kähler form is symplectic invariant and there are also other symplectic invariants associated with the M^4 degrees of freedom (carrying also the analog of Kähler form for the twistor lift of TGD and giving rise to CP breaking). One would have catastrophe theory with infinite number of control variables so that the number of catastrophes would be infinite so that standard catastrophe theory does not as such apply.

3. Therefore TGD would not be only a personal professional catastrophe but a catastrophe in much deeper sense. WCW would be a catastrophe surface for the functional gradient of the action defining Kähler function. WCW would consists of regions in which given zero modes would correspond to several minima. The region of zero mode space at which some

roots identifiable as space-time surfaces co-incide would be analogous to the V-shaped cusp catastrophe and its higher-D generalizations. The question is whether one allows the entire catastrophe surface or whether one demands quantum criticality in the sense that only the union of singular sets at which roots co-incide is included.

4. For WCW as catastrophe surface the analog of V in the space of zero modes would correspond to a hierarchy of sub-WCWs consisting of preferred extremals satisfying the gauge conditions associated with a sub-algebra of supersymplectic algebra isomorphic to the full algebra. The sub-WCWs in the hierarchy of sub-WCWs within sub-WCWs would satisfy increasingly stronger gauge conditions and having decreasing dimension just like in the case of ordinary catastrophe. The lower the effective dimension, the higher the quantum criticality.

5. In ordinary catastrophe theory the effective number of behavior variables for given catastrophe can be reduced to some minimum number. In TGD framework this would correspond to the reduction of super-symplectic algebra to a finite-D Lie algebra or corresponding Kac-Moody (half-)algebra as modes of supersymplectic algebra with generators labelled by non-negative integer n modulo given integer m are eliminated as dynamical degrees of freedom by the gauge conditions: this would effectively leave only the modes smaller than m. The fractal hierarchy of these supersymplectic algebras would correspond to the decomposition of WCW as a catastrophe surface to pieces with varying dimension. The reduction of the effective dimension as two sheets of the catastrophe surface co-incide would mean transformation of some modes contributing to metric to zero modes.

RG invariance implies physical analogy with thermodynamics and gauge theories

One can consider coupling constant evolution and RG invariance from a new perspective based on the minimal surface property.

1. The critical values of Kähler coupling strength would correspond to quantum criticality of the S^2 part $S(S^2)$ of 6-D dimensionally reduced Kähler action for fixed values of zero modes. The relative S^2 rotation would serve as behavior variable. For its critical values the dimension of the critical manifold would be reduced, and keeping zero modes fixed a critical value of α_K would be selected from 1-D continuum.

   Quantum criticality condition might be fundamental since it implies the constancy of the value of the twistor lift of Kähler action for the space-time surfaces contributing to the scattering amplitudes. This would be crucial for number theoretical vision since the continuation of exponential to p-adic sectors is not possible in the generic case. One should however develop stronger arguments to exclude the continuous evolution of Kähler coupling strength in S^2 degrees of freedom for the real sector of the theory.

2. The extremals of twistor lift contain dependence on the rotation parameter for S^2 and this must be taken into account in coupling constant evolution along curve of S^2 connecting zeros of zeta. This gives additional non-local term to the evolution equations coming from the dependence of the embedding space coordinates of the preferred extremal on the evolution parameter. The derivative of Kähler action with respect to the evolution parameter is by chain rule proportional to the functional derivatives of action with respect to embedding space coordinates, and vanish if 4-D Kähler action and volume term are *separately* stationary with respect to variations. Therefore minimal surface property as implied by holomorphy guaranteeing quantum criticality as universality of the dynamics would be crucial in simplifying the equations! It does not matter whether there is coupling between Kähler action and volume term.

Could one find interpretation for the miminal surface property which implies that field equations are separately satisfied for Kähler action and volume term?

1. Quantum TGD can be seen as a "complex" square root of thermodynamics. In thermodynamics one can define several thermodynamical functions. In particular, one can add to energy E as term pV to get enthalpy $H = E + pV$, which remains constant when entropy and pressures are kept constant. Could one do the same now?

In TGD V replaced with volume action and p would be a coupling parameter analogous to pressure. The simplest replacement would give Kähler action as outcome. The replacement would allow RG invariance of the modified action only at critical points so that replacement would be possible only there. Furthermore, field equations must hold true separately for Kähler action and volume term everywhere.

2. It seems however that one must allow singular sets in which there is interaction between these terms. The coupling between Kähler action and volume term could be non-trivial at singular sub-manifolds, where a transfer of conserved quantities between the two degrees of freedom would take place. The transfer would be proportional to the divergence of the canonical momentum current $D_\alpha(g^{\alpha\beta}\partial_\beta h^k)$ assignable to the minimal surface and is conserved outside the singular sub-manifolds.

   Minimal surfaces provide a non-linear generalization of massless wave-equation for which the gradient of the field equals to conserved current. Therefore the interpretation could be that these singular manifolds are sources of the analogs of fields defined by M^4 and CP_2 coordinates.

   In electrodynamics these singular manifolds would represented by charged particles. The simplest interpretation would be in terms of point like charges so that one would have line singularity. The natural identification of world line singularities would be as boundaries of string world sheets at the 3-D light-like partonic orbits between Minkowskian and Euclidian regions having induced 4-metric degenerating to 3-D metric would be a natural identification. These world lines indeed appear in twistor diagrams. Also string world sheets must be assumed and they are are natural candidates for the singular manifolds serving as sources of charges in 4-D context. Induced spinor fields would serve as a representation for these sources. These strings would generalize the notion of point like particle. Particles as 3-surfaces would be connected by flux tubes to a tensor network and string world sheets would connected fermion lines at the partonic 2-surfaces to an analogous network. This would be new from the standard model perspective.

   Singularities could also correspond to a discrete set of points having an interpretation as cognitive representation for the loci of initial and final states fermions at opposite boundaries of CD and at vertices represented topologically by partonic 2-surfaces at which partonic orbits meet. This interpretation makes sense if one interprets the embedding space coordinates as analogs of propagators having delta singularities at these points. It is quite possible that also these contributions are present: one would have a hierarchy of delta function singularities associated with string worlds sheets, their boundaries and the ends of the boundaries at boundaries of CD, where string world sheet has edges.

3. There is also an interpretation of singularities suggested by the generalization of conformal invariance. String world sheets would be co-dimension 2 singularites analogous to poles of analytic function of two complex coordinates in 4-D complex space. String world sheets would be co-dimension 2 singularities analogous to poles at light-like 3-surfaces. The ends of the world lines could be analogous of poles of analytic function at partonic 2-surfaces.

   These singularities could provide to evolution equations what might be called matter contribution. This brings in mind evolution equations for n-point functions in QFT. The resolution of the overall singularity would decompose to 2-D, 1-D and 0-D parts just like cusp catastrophe. One can ask whether the singularities might allow interpretation as catastrophes.

4. The proposal for analogs of thermodynamical functions suggests the following physical picture. More general thermodynamical functions are possible only at critical points and only if the extremals are miminal surfaces. The singularities should correspond to physical particles, fermions. Suppose that one considers entire scattering amplitude involving action exponential plus possible analog of pV term plus the terms associated with the fermions assigned with the singularities. Suppose that the coupling constant evolution from 6-D Kähler action is calculated *without* including the contribution of the singularities.

   The derivative of n-particle amplitude with respect to the evolution parameter contains a term coming from the action exponential receiving contributions only from the singularities and a term coming from the operators at singularities. RG invariance of the scattering amplitude would require that the two terms sum up to zero. In the thermodynamical picture the presence of particles in many particle scattering amplitude would force to add the analog of pressure term to the Kähler function: it would be determined by the zero energy state.

One can of course ask how general terms can be added by requiring minimal surface property. Minimal surface property reduces to holomorphy, and can be true also for other kinds of actions such as the TGD analogs of electroweak and color actions that I considered originally as possible action candidates.

This would have interpretation as an analog for YM equations in gauge theories. Space-time singularities as local failure of minimal surface property would correspond to sources for H coordinates as analogs of Maxwell's fields and sources currents would correspond to fermions currents.

14.3.6 TGD view about inclusions of HFFs as a way to understand coupling constant evolution

The hierarchy of inclusions of HFFs is an alternative TGD inspired proposal for understanding coupling constant evolution and the intuitive expectation is that the inclusion hierarchies of extensions and their Galois groups contain the inclusion hierarchies of HFFs as special case. The included factor would define measurement resolution in well-defined sense. This notion can be formulated more precisely using Connes tensor product [A41, A94].

How Galois groups and finite subgroups of could $SU(2)$ relate

The hierarchy of finite groups associated with the inclusions of HFF corresponds to the finite subgroups of $SU(2)$. The set of these groups is very small as compared to the set of Galois groups - if I have understood correctly, any finite group can appear as Galois group. Should the hierarchy of inclusions of HFFs be replaced by much more general inclusion hierarchy? Is there a map projecting Galois groups to discrete subgroup of $SU(2)$?

By M^8-H duality quaternions appear at M^8 level and since $SO(3)$ is the automorphism group of quaternions, the discrete subgroups of $SU(2)$ could appear naturally in TGD. In fact, the appearance of quaternions as a basic building brick of HFFs in the simplest construction would fit with this picture.

It would seem that the elements of the discrete subgroups of $SU(2)$ must be in the extension of rationals considered. The elements of finite discrete subgroups G of $SU(2)$ are expressible in terms of rather small subset of extensions of rationals. Could the proper interpretation be that the hierarchy of extensions defines a hierarchy of discrete groups with elements in extension and the finite discrete subgroups in question are finite discrete subgroups of these groups. There would be correlation with the inclusion and extension. For instance, the fractal dimension of extension is an algebraic number defined in terms of root of unity so that the extension must contain this root of unity.

For icosahedron and dodecahedron the group action can be expressed using extension of rationals by $cos(\pi/n)$ and $sin(\pi/n)$ for $n=3,5$. For tetrahedron and cube one would have $n=2,3$. For tetrahedon, cube/octahedron and icosahedron basic geometric parameters are also expressible in terms of algebraic numbers in extension but in case of dodecahedron it is not clear for me whether the surface area proportional to $\sqrt{25+20\sqrt{5}}$ allows this (see `http://tinyurl.com/p4rwc7`).

It is very feasible that the finite sub-groups of also other Lie groups than $SU(2)$ are associated with the inclusions of HFFs or possibly more general algebras. In particular, finite discrete subgroups of color group $SU(3)$ should be important and extension of rationals should allow to represent these subgroups.

Once again about ADE correspondence

For a non-mathematician like me Mc-Kay correspondence is an inspiring and frustrating mystery (see `http://tinyurl.com/y8jzvogn`). What could be its physical interpretation?

Mac-Kay correspondence assigns to the extended Dynkin diagrams of ADE type characterizing Kac-Moody algebras finite subgroups of $SU(2)$, more precisely the McKay diagrams describing the tensor product decomposition rules for the fundamental representation of the finite subgroup of $SU(2)$. In the diagram irreps χ_i and χ_j are connected by n_{ij} arrows if χ_j appears n_{ij} times in the tensor product $V \otimes \chi_i$, where V is but need not be fundamental representation.

One can assign also to inclusions of HFFs of index $d \geq 4$ with ADE type Dynkin diagrams. To inclusions with index $d < 4$ one can assign subset of ADE type diagrams for Lie groups (rather than Kac-Moody groups) and they correspond to sub-groups of $SU(2)$. The correspondence generalizes to subgroups of other Lie groups.

1. As explained in [B49] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dim(g) - r)/r$. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed. The Dynkin graphs of Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. E_6, E_7, and E_8 correspond to symmetry groups of tetrahedron, octahedron/cube, and icosahedron/dodecahedron. The group for octahedron/cube is missing: what could this mean?

   For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of $SU(2)$ and the interpretation proposed in [A94] is following.

   The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: A_∞ corresponding to $SU(2)$ itself, $A_{-\infty,\infty}$ corresponding to circle group $U(1)$, and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection.

   One can construct also inclusions for which the diagrams corresponding to finite subgroups $G \subset SU(2)$ are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with $n = 6, 7, 8$ for tetrahedron, cube, dodecahedron. These extensions correspond to ADE type Kac-Moody algebras.

   The extension is constructed by constructing first factor R as infinite tensor power of $M_2(C)$ (complexified quaternions). Sub-factor R_0 consists elements of of R of form $Id \otimes x$. $SU(2)$ preserves R_0 and for any subgroup G of $SU(2)$ one can identify the inclusion $N \subset M$ in terms of $N = R_0^G$ and $M = R^G$, where $N = R_0^G$ and $M = R^G$ consists of fixed points of R_0 and R under the action of G. The principal graph for $N \subset M$ is the extended Coxeter-Dynk graph for the subgroup G.

   Physicist might try to interpret this by saying that one considers only sub-algebras R_0^G and R^G of observables invariant under G and obtains extended Dynkin diagram of G defining an ADE type Kac-Moody algebra. Could the condition that Kac-Moody algebra elements with non-vanishing conformal weight annihilate the physical states state that the state is invariant under R_0 defining measurement resolution. Besides this the states are also invariant under finite group G? Could R_0^G and R^G correspond just to states which are also invariant under finite group G.

   Could this kind of inclusions generalize so that Galois groups would replace G. If this is possible it would assign to each Galois group an inclusion of HFFs and give a precise number theoretic formulation for the notion of measurement accuracy.

2. At M^8-side of $M^8 - H$ duality the construction of space-time surfaces reduces to data at finite set of points of space-time surface since they are defined by an octonionic extension of a polynomial of real variable with coefficients in extension of rationals. Space-time surfaces would have quaternionic tangent space or normal space. The coordinates of quaternions are restricted to extension of rationals and the subgroup of automorphisms reduce to a subgroup for which matrix elements belong to an extension of rationals.

   If the subgroup is finite, only the subgroups appearing in ADE correspondence are possible and the extension must be such that it allows the representation of this group. Does this mean that the extension can is obtained from an extension allowing this representation? For $\mathcal{M} : \mathcal{N} = 4$ case this sub-group would leave the states invariant.

14.3.7 Entanglement and adelic physics

In the discussion about fine structure constant I asked about the role entanglement in coupling constant evolution. Although entanglement does not have direct relationship to coupling constant evolution, I will discuss entanglement from number theoretic point of view since it enlightens the motivations of adelic physics.

1. For given extension of rationals determining the values of coupling parameters by quantum criticality, the entanglement coefficients between positive and negative energy parts of zero energy states are in the extension of rationals. All entanglement coefficients satisfy this condition.

2. Self the counterpart of observer in the generalization of quantum measurement theory - as conscious entity [L56] corresponds to sequence of unitary evolutions followed by weak measurements. The rule for weak measurements is that only state function for which the eigenvalues of the density matrix is in the extension of rationals can occur. In general they are in a higher-D extension as roots of N:th order polynomials, N the dimension of density matrix. Therefore state function reduction cannot occur in the generic case. State cannot decohere and entanglement is stable under weak measurements except in special situations when the eigenvalues of density matrix are in original extension.

3. The extension can change only in big state function reductions in which the arrow of clock time changes: this can be seen as an evolutionary step. From the point of view of consciousness theory big state function reduction means what might be called death and reincarnation of system in opposite time direction.

4. The number theoretical stabilization of entanglement at the passive boundary of CD makes possibility quantum computation in longer time scales than possible in standard quantum theory. $h_{eff}/h_0 = n$ equals to the dimension of extension of rationals and is therefore directly related to this.

   This could have profound technological implications.

1. Ordinary quantum computation as single unitary step is replaced by a sequence of them followed by the analog of weak measurement.

2. ZEO allows also quantum computations in opposite time direction. This might allow shorten dramatically the duration of quantum computations from the perspective of the observed since most of the computation could be done with opposite arrow of clock time.

   The philosophy of adelic physics is discussed in article in book published by Springer [L54, L53] (see `http://tinyurl.com/ybzkfevz` and `http://tinyurl.com/ybqpkwg9`).

14.4 Trying to understand why ramified primes are so special physically

Ramified primes (see `http://tinyurl.com/m32nvcz` and `http://tinyurl.com/y6yskkas`) are special in the sense that their expression as a product of primes of extension contains higher than first powers and the number of primes of extension is smaller than the maximal number n defined by the dimension of the extension. The proposed interpretation of ramified primes is as p-adic primes characterizing space-time sheets assignable to elementary particles and even more general systems.

In the following Dedekind zeta functions (see `http://tinyurl.com/y5grktvp`) as generalization of Riemann zeta [L70, L79] are studied to understand what makes them so special. Dedekind zeta function characterizes given extension of rationals and is defined by reducing the contribution from ramified reduced so that effectively powers of primes of extension are replaced with first powers.

If one uses the naïve definition of zeta as analog of partition function and includes full powers $P_i^{e_i}$, the zeta function becomes a product of Dedekind zeta and a term consisting of a finite number of factors having poles at imaginary axis. This happens for zeta function and its fermionic analog having zeros along imaginary axis. The poles would naturally relate to Ramond and N-S boundary conditions of radial partial waves at light-like boundary of causal diamond CD. The additional factor could code for the physics associated with the ramified primes.

The intuitive feeling is that quantum criticality is what makes ramified primes so special. In $O(p) = 0$ approximation the irreducible polynomial defining the extension of rationals indeed reduces to a polynomial in finite field F_p and has multiple roots for ramified prime, and one can deduce a concrete geometric interpretation for ramification as quantum criticality using $M^8 - H$ duality.

$M^8 - H$ duality central concept in following and discussed in [L48, L85, L81, L82] [L96]. Also the notion of cognitive representation as a set of points of space-time surface with preferred embedding space coordinates belonging to the extension of rationals defining the adele [L53] is important and discussed in [L88, L87, L95].

14.4.1 Dedekind zeta function and ramified primes

One can take mathematics and physical intuition guided by each other as a guideline in the attempts to understand ramified primes.

1. Riemann zeta can be generalized to Dedekind zeta function ζ_K for any extension K of rationals (see `http://tinyurl.com/y5grktvp`). ζ_K characterizes the extension - maybe also physically in TGD framework since zeta functions have formal interpretation as partition function. In the recent case the complexity is not a problem since complex square roots of partition functions would define the vacuum part of quantum state: one can say that quantum TGD is complex square root of thermodynamics.

   ζ_K satisfies the same formula as ordinary zeta expect that one considers algebraic integers in the extensions K and sums over non-zero ideals a - identifiable as integers in the case of rationals - with n^{-s} replaced with $N(a)^{-s}$, where $N(a)$ denotes the norm of the non-zero ideal. The construction of ζ_K in the extension of rationals obtained by adding i serves as an illustrative example (see `http://tinyurl.com/y563wcwv`). I am not a number theorists but the construction suggests a poor man's generalization strongly based on physical intuition.

2. The rules would be analogous to those used in the construction of partition function. $log(N(a))$ is analogous to energy and s is analogous to inverse temperature so that one has Boltzmann weight $exp(-log(N(a)s)$ for each ideal. Since the formation of ideals defined by integers of extension is analogous to that for forming many particle states labelled by ordinary primes and decomposing to primes of extension, the partition function decomposes to a product over partition functions assignable to ordinary primes just like in the case of Riemann zeta. Let K be an extension of rationals Q.

3. Each rational prime p decomposes in the extension as $p = \prod_{i=1,...g} P_i^{e_i}$, where n is the dimension of extension and e_i is the ramification degree. Let f_i be so called residue degree of P_i defined as the dimension of K mod P_i interpreted as extension of rational integers $Z \ mod \ p$. Then one has $\sum_1^g e_i f_i = n$.

   Remark: For Galois extensions for which the order of Galois group equals to the dimension n of the extension so that for given prime p one has $e_i = e$ and $f_i = f$ and $efg = n$.

4. Rational (and also more general) primes can be divided into 3 classes with respect to this decomposition.

   For ramified primes dividing the discriminant D associated with the polynomial ($D = b^2 - 4c$ for $P(x) = x^2 + bx + c$) one has $e_i > 1$ at least for one i so that $f_i = 0$ is true at least for one index. A simple example is provided by rational primes (determined by roots of $P(x) = x^2 + 1$ with discriminant -4): in this case $p = 2$ corresponds to ramified prime since on has $(1 + i)(1 - i) = 2$ and $1 + i$ and $1 - i$ differ only by multiplication by unit $-i$.

5. Split primes have n factors P_i and thus have ($e_i = 1, f_i = 1, g = n$) . They give a factor $(1 - p^{-s})^{-n}$. The physical analogy is n-fold degenerate state with original energy energy $nlog(p)$ split to states with energy $log(p)$.

Inert primes are also primes of extension and there is no splitting and one has ($e_1 = e = 1, g = 1, f_1 = f = n$). In this case one obtains factor $1/(1-p^{-ns})$. The physical analogy is n-particle bound state with energy $nlog(p)$.

6. For ramified primes the situation is more delicate. Generalizing from the case of Gaussian primes $Q[i]$ (see `http://tinyurl.com/y563wcwv`) ramified primes p_R would give rise to a factor

$$\prod_{i=1}^{g} \frac{1}{1-p_R^{-f_i s}} \ .$$

g is the number of *distinct* ideals P_i in the decomposition of p to the primes of extension.

For Gaussian primes $p=2$ has $g=1$ since one can write $(2) = (1+i)(1-i) \equiv (1+i)^2$. This because $1+i$ and $1-i$ differ only by multiplication with unit $-i$ and thus define same ideal in $Q[i]$. Only the number g of distinct factors P_i in the decomposition of p matters.

One could understand this as follows. For the roots of polynomials ramification means that several roots co-incide so that the number of distinct roots is reduced. $e_i > 1$ is analogous to the number co-inciding roots so that number if distinct roots would be 1 instead of e_i. This would suggests $k_i = 1$ always. For ramified primes the number of factors Z_p the number $\sum_{i=1}^{g} f_i k_i = n$ for un-ramified case would reduce from to $\sum_{i=1}^{g} f_i k_i = n_d$, which is the number of distinct roots.

7. Could the physical interpretation be that there are g types of bound states with energies $f_i log(p)$ appearing with degeneracy $e_i = 1$ both in ramified and split case. This should relate to the fact that for ramified primes p L/p contains non-vanishing nilpotent element and is not counted. One can also say that the decomposition to primes of extension conserves energy: $\sum_{i=1,...,g} e_i f_i log(p) = n_d log(p)$.

For instance, for Galois extensions ($e_i = e, f_i = f, g = n_d/ef$) for given p the factor is $1/(1-p^{-es})^{fg}$: $efg = n_d$. If there is a ramification then all P_i are ramified. Note that e, f an g are factors of n_d.

8. One can can extract the factor $1/(1-p^{-s})$ from each of the 3 contributions and organize these factors to give the ordinary Riemann zeta. The number of ramified primes is finite whereas the numbers of split primes and inert primes are infinite. One can therefore extract from ramified primes the finite product

$$\zeta^1_{R,K} = \prod_{p_R}(1-p_R^{-s}) \times \zeta^2_{R,K} \ \ , \ \ \zeta^2_{R,K} = \prod_{p_R}[\prod_{i=1}^{g} \frac{1}{1-p^{-f_i s}}] \ .$$

One can organize the remaining part involving infinite number of factors to a product of ζ and factors $(1-p^{-s})/(1-\prod p^{-s}))^n$ and $(1-p^{-s})/(1-p^{-ns})$ giving rise to zeta function -call it $\zeta_{si,K}$ - characterizing the extension. Note that $\zeta^2_{R,K}$ has interpretation as partition function and has pole of order n_d at origin.

One therefore can write the ζ_L as

$$\zeta_K = \zeta^1_{R,K} \times \zeta_{si,K} \times \zeta \ .$$

where $\zeta_{si,K}$ is the contribution of split and inert primes multiplied by $(1-p^{-s})$

ζ_L has pole only at $s=1$ and it carries in no obvious manner information about ramified primes. The naïve guess for ζ_L would be that also ramified primes p_R would give rise to a factor

$$\prod_{i=1}^{g} \frac{1}{(1-p_R^{-f_i s})^{e_i}} \ .$$

One could indeed argue that at the limit when e_i prime ideals P_i of extension co-incide, one should obtain this expression. The resulting ζ function would be product

$$\zeta_{naive,K} = \zeta_{R,K}\zeta_K \ , \qquad \zeta_{R,K} = \prod_{p_R} X(p_R)$$
$$X(p_R) = \prod_{i=1}^{g} \frac{1}{(1-p_R^{-f_i s})^{e_i-1}} \ .$$

Note that the parameters e_i, f_i, g depend on p_R and that for Galois extensions one has $e_i = d, f_i = f$ for given p_R. $\zeta_{R,L}$ would have poles at along imaginary axis at points $s = -n2\pi/log(p)$. Ramified primes would give rise to poles along imaginary axis. As far as the proposed physical interpretation of ramified primes is considered, this form looks more natural.

Fermionic counterparts of Dedekind zeta and ramified ζ

One can look the situation also for the generalization of fermionic zeta as analog of fermionic partition function, which for rationals has the expression

$$\zeta^F(s) = \prod_p (1 + p^{-s}) = \frac{\zeta(s)}{\zeta(2s)} \ .$$

Supersymmetry of supersymmetric arithmetic QFT suggest the product of fermionic and bosonic zetas. Also the supersymmetry of infinite primes for which first level of hierarchy corresponds to irreducible polynomials suggests this. On the other hand, the appearance of only fermions as fundamental particles in TGD forces to ask whether the ramified part of fermionic zeta might be fundamental.

1. By an argument similar to used for ordinary zeta based on interpretation as partition function, one obtains the decomposition of the fermionic counterpart of ζ_K^F Dirichlet zeta to a product $\zeta_K^F = \zeta_{R,K}^F \zeta_{si,K}^F \zeta^F$ of ramified fermionic zeta $\zeta_{R,K}^F$, $\zeta_{si,K}^F$, and ordinary fermionic zeta ζ^F. The basic rule is simple: replace factors $1/(1-p^{-ks}$ appearing in ζ_K with $(1+p^{-ks})$ in ζ_K^F and extract ζ^F from the resulting expression. This gives

$$\zeta_{R,K}^{F,1} = \prod_{p_R}(1-p_R^{-s})\zeta_{R,K}^F \ , \quad \zeta_{R,K}^F = \prod_{p_R}[\prod_{i=1}^{g}(1+p_R^{-f_i s})] \ .$$

   where p_R is ramified prime dividing the discriminant. $\zeta_{R,K}^F$ is analogous to a fermionic partition function for a finite number of modes defined by ramified primes p_R of extension.

2. Also now one can wonder whether one should define ζ_K^F as a product in which ramified primes give factor

$$\prod_{p_R}[\prod_{i=1}^{g}(1+p_R^{-f_i s})^{e_i}]$$

   so that one would have

$$\zeta_{naive,K}^F = \zeta_{R,K}^F \zeta_K^F \ , \qquad \zeta_R^F = \prod_{p_R} Y(p_R) \ ,$$
$$Y(p_R) = \prod_{i=1}^{g}(1+p_R^{-f_i s})^{e_i-1}$$

   $\zeta_F(naïve, K)$ would have zeros along imaginary axis serving as signature of the ramified primes.

About physical interpretation of $\zeta_{R,K}$ and $\zeta_{R,K}^F$

$\zeta_{R,K}$ and $\zeta_{R,K}^F$ are attractive from the view point of number theoretic vision and the idea that ramified primes are physically special. TGD Universe is quantum critical and in catastrophe theory the ramification for roots of polynomials is analogous to criticality. Maybe the ramification for p-adic primes makes them critical. $K/(p_R)$ has nilpotent elements, which brings in mind on mass shell massless particles.

1. $\zeta_{R,K}$ has poles at

$$s = i\frac{2n\pi}{log(p)f_i}$$

and $p_R^s = exp(in2\pi/f_i)$ is a root of unity, which conforms with the number theoretical vision. Only P_i with $e_i > 1$ contribute.

2. $Z^F_{R,K}$ has zeros

$$s = i\frac{(2n+1)\pi}{log(p)f_i}$$

and $p_R^s = exp(i(2n+1)\pi/f_i)$ is a root of unity. Zeros are distinct from the poles of $Z_{R,K}$.

3. The product $\zeta_{R,tot,K} = \zeta_{R,K}\zeta^F_{R,K}$ has the poles and zeros of $\zeta_{R,K}$ and $\zeta^F_{R,K}$. In particular, there is n:th order pole of $Z_{R,K}$ at $s = 0$. The zeros of $z_{F,K}$ along imaginary axis at $p^{iy} = -1$ also survive in $\zeta_{R,tot,K}$.

   $\zeta^F_{R,K}$ has only zeros and since fundamental fermions are primary fields in TGD framework, one could argue that only it carries physical information. On the other hand, supersymmetric arithmetic QFT [K105] and the fact that TGD allows SUSY [L92] suggests that the product $\zeta_{R,K} \times Z^F_{R,K}$ is more interesting.

From TGD point of view the ramified zeta functions $\zeta_{R,K}$, $\zeta^F_{R,K}$ and their product $\zeta_{R,K}\times\zeta^F_{R,K}$ look interesting.

1. $\zeta_{R,K}$ behaves like s^{-n_d}, $n_d = \sum_1^g(e_i - 1)$ near the origin. Could n_d-fold pole at $s = 0$ be interpreted in terms of a massless state propagating along light-cone boundary of CD in radial direction? This would conform with the proposal that zeros of zeta correspond to complex radial conformal weights for super-symplecti algebra. That ramified primes correspond to massless particles would conform with the identification of ramified prime as p-adic primes labelling elementary particles since in ZEO their mass would result from p-adic thermodynamics from a mixing with very massive states [L82].

   Besides this there would be stringy spectrum of real conformal weights along negative real axis and those coming as non-trivial zeros and these could correspond to ordinary conformal weights.

2. The zeros of $\zeta^F_{R,K}$ along imaginary axis might have interpretation as eigenvalues of Hamiltonian in analogy with Hilbert-Polya hypothesis. Maybe also the poles of $\zeta_{R,K}$ could have similar interpretation. The real part of zero/pole would not produce troubles (on the other hand, for waves along light-cone boundary it can be however absorbed to the integration measure.

3. A possible physical interpretation of the imaginary conformal weights could be as conformal weights associated with radial waves assignable to the radial light-like coordinate r of the light-cone boundary: r indeed plays the role of complex coordinate in conformal symmetry in the case of super-symplectic algebra suggested to define the isometries of WCW. Poles and zero could correspond to radial modes satisfying periodic/anti-periodic boundary conditions.

   The radial conformal weights s defined by the zeros of $\zeta^F_{R,K}$ would be number theoretically natural since one could pose boundary condition $p^{is(r/r_0)} = -1$ at $r = r_0$ requiring $p^{is} = -1$ at the corner of cd (maximum value of r in $CD = cd \times CP_2$.

   For the poles of $\zeta_{R,K}$ the periodic boundary condition $p^{is(r/r_0)} = 1$ is natural. The two boundary conditions could relate to Ramond an N-S representations of super-conformal algebras (see `http://tinyurl.com/y49y2ouj`). With this interpretation $s = 0$ would correspond to a radial plane-wave constant along light-like radial direction and therefore light-like momentum propagating along the boundary of CD. Other modes would correspond to other massless modes propagating to the interior of CD.

4. I have earlier considered an analogous interpretation for a subset zeros of zeta satisfying similar condition. The idea was that for given prime p as subset of $s = 1/2 + iy_i$ of non-trivial zeros ζ $p^s = p^{1/2+iy_i}$ is an algebraic number so that p^{iy_i} would be a root of unity. Zeros would decompose to subsets labelled by primes p. Also for trivial zeros of ζ (and also poles) the same holds true as for the zeros and poles ζ_R. This encourages the conjecture that the property is true also for L-functions.

The proposed picture suggests an assignment of "energy" $E = nlog(p)$ to each prime and separation of "ramified" energy $E_d = n_d log(p)$, $n_d = \sum_1^g f_i(e_i - 1)$, to each ramified prime. The interpretation as partition function suggests that that one has g types of states of f_i identical particles and energy $E_i = f_i log(p)$ and that this state is e_i-fold degenerate with energies $E_i = f_i log(p)$. For inert primes one would have $f_i = f = n$. For split primes one would have $e_i = 1, f_i = 1$. In case of ramified primes one can separate one of these states and include it to the Dedekind zeta.

Can one find a geometric correlate for the picture based on prime ideals?

If one could find a geometric space-time correlate for the decomposition of rational prime ideals to prime ideals of extensions, it might be also possible to understand why quantum criticality makes ramified primes so special physically and wha this means.

What could be correlate for f_i fundamental fermions behaving like single unit and what degeneracy for $e_i > 1$ does mean? One can look the situation first at the level of number fields Q and K and corresponding Galois group $Gal(K/Q)$, finite fields $F = Q/p$ and $F_i = K/P_i$, and corresponding Galois group $Gal(F_i/F)$. Appendix summarizes the basic terminology.

1. Inertia degree f_i is the number of elements of F_i/F_p ($F_i = K/P_i$ is extension of finite field $F_p = Q/p$). The Galois group $Gal(F_i/F_p)$ is identifiable as factor group D_i/I_i, where the *decomposition group* D_i is the subgroup of Galois group taking P_i to itself and the *inertia group* I_i leaving P_i point-wise invariant. The orbit under $Gal(F_i/F_p)$ in F_i/F_p would behave like single particle with energy $E_i = f_i log(p)$.

   For inert primes with $f_i = n$ inertia group would be maximal. For split primes the orbits of ideals would consist of $f_i = 1$ points only and isotropy group would be trivial.

2. Ramification for primes corresponds intuitively to that for polynomials meaning multiple roots as is clear also from the expression $p = \prod P_i^{e_i}$. In accordance with the intuition about quantum criticality, ramification means that the irreducible polynomial reduced to a reducible polynomial in finite field Q/p has therefore a multiple roots with multiplicities e_i (see Appendix). For Galois extensions one has ($e_i = e, f_i = f$) Criticality would be seen at the level of finite field $F_p = Q/p$ associated with ramified prime p.

The interpretation of roots of corresponding octonionic polynomials as n-sheeted covering space like structures encourages to ask whether the independent tensor factors labelled by i suggested by the interpretation as a partition function could be assigned with the sheets of covering so that ramification with $e_i > 1$ would correspond to singular points of cognitive representation for which e_i sheets co-incide in some sense, maybe in finite field approximation ($O(p) = 0$). Galois groups indeed act on the coordinates of point of cognitive representation belonging to the extension K. In general the action does not take the point to a point belonging to a cognitive representation but one can consider quantum superpositions of cognitive representations.

This suggests an interpretation in terms of space-time surfaces accompanied by cognitive representation under Galois group. Quantum states would be superpositions of preferred extremals at orbits of Galois group and for cognitive representations the situation would be discrete.

1. To build a concrete connection between geometric space-time picture and number theoretic picture, one should find geometric counterparts of integers, ideals, and prime ideals. The analogs of prime ideals should be associated with the discretizations of space-time surfaces/cognitive representations in $O(p) = 0$ or $O(P_i) = 0$ approximation. Could one include only points of cognitive representations differing from zero in $O(p) = 0$ approximation and form quantum states as quantum superpositions of these points of cognitive representation?

in $O(p) = 0$ approximation and for ramified primes irreducible polynomials would have multiple roots so that e_i sheets would co-incide at these points in $O(p) = 0$ approximation. Th conjecture that elementary particles correspond to this kind of singularities has been speculated already earlier with inspiration coming from quantum criticality.

2. In M^8 picture the octonionic polynomials obtained as continuation of polynomials with rational coefficients would be reduced to polynomials in finite field F_p. One can study corresponding discrete algebraic surfaces as discrete approximations of space-time surfaces.

3. One would like to have only single embedding space coordinate since the probability that all embedding space coordinates correspond to the same P_i is small. $M^8 - H$ duality reduces the number of embedding space coordinates characterizing partonic 2-surfaces containing vertices for fundamental fermions to single one identifiable as time coordinate.

   At the light-like boundary of 8-D CD in M^8 the vanishing condition for the real or imaginary part (quaternion) of octonionic polynomial $P(o)$ reduces to that for ordinary polynomial, and one obtains n roots r_n, which correspond to the values of M^4 time $t = r_n$ defining 6-spheres as analogs of branes. Partonic 2-surfaces corresponde to intersections of 4-D roots of $P(o)$ at partonic 2-surfaces. Galois group of the polynomial naturally acts on r_n labelling these partonic 2-surfaces by permuting them. One could form representations of Galois group using states identified as quantum superpositions of these partonic 2-surfaces corresponding to different values of $t = r_n$. Galois group leaves invariant the degenerate roots $t = r_n$.

4. The roots can be reduced to finite field F_p or K/P_i. Ramification would take place in this approximation and mean that e_i roots $t = r_n$ are identical in $O(p) = 0$ approximation. e_i time values $t = r_n$ would nearly co-incide. This gives more concrete contents to the statement of TGD inspired theory of consciousness that these time values correspond to very special moments in the life of self. Since this is the situation only approximately, one can argue that one must indeed count each root separately so that partition function must be defined as product of the contribution form ramified primes an Dedekind zeta.

   The assignment of fundamental fermions to the points of cognitive representations at partonic 2-surfaces assignable to the intersections of 4-D roots and universal 6-D roots of octonionic polynomials (brane like entities) conforms with this picture.

5. The analogs of 6-branes would give rise to additional degrees of freedom meaning effectively discrete non-determinism. I have speculated with this determinism with inspiration coming from the original identification of bosonic action as Kähler action having huge 4-D spin glass degeneracy. Also the number theoretic vision suggest the possibility of interpreting preferred extremals as analogs of algebraic computations such that one can have several computations connecting given states [L45]. The degree n of polynomial would determine the number of steps and the degeneracy would correspond to n-fold degeneracy due to the discrete analogs of plane waves in this set.

What extensions of rationals could be winners in the fight for survival?

It would seem that the fight for survival is between extensions of rationals rather than individual primes p. Intuition suggests that survivors tend to have maximal number of ramified primes. These number theoretical speciei can live in the same extension - to "co-operate".

Before starting one must clarify some basic facts about extensions of rationals.

1. Extension of rationals are defined by an irreducible polynomial with rational coefficients. The roots give n algebraic numbers which can be used as a basis to generate the numbers of extension ast their rational linear combinations. Any number of extension can be expressed as a root of an irreducible polynomial. Physically it is is of interest, that in octonionic picture infinite number of octonionic polynomials gives rise to space-time surface corresponding to the same extension of rationals.

2. One can define the notion of integer for extension. A precise definition identifies the integers as ideals. Any integer of extension are defined as a root of a monic polynomials $P(x) =$

$x^n + p_{n-1}x^{n-1}x^{n-1} + ... + p_0$ with integer coefficients. In octonionic monic polynomials are subset of octonionic polynomials and it is not clear whether these polynomials could be all that is needed.

3. By definition ramified primes divide the discriminant D of the extension defined as the product $D = \prod_{i\neq j}(r_i - r_j)$ of differences of the roots of (irreducible) monic polynomial with integer coefficients defining the basis for the integers of extension. Discriminant has a geometric interpretation as volume squared for the fundamental domain of the lattice of integers of the extension so that at criticality this volume interpreted as p-adic number would become small for ramified primes an vanish in $O(p)$ approximation. The extension is defined by a polynomial with rational coefficients and integers of extension are defined by monic polynomials with roots in the extension: this is not of course true for all monic polynomials polynomial (see `http://tinyurl.com/k3ujjz7`).

4. The scaling of the $n-1$-tuple of coefficients $(p_{n-1}, ..., p_1)$ to $(ap_{n-1}, a^2p_{n-1}....., a^np_0)$ scales the roots by a: $x_n \rightarrow ax_n$. If a is rational, the extension of rationals is not affected. In the case of monic polynomials this is true for integers k. This gives rational multiples of given root.

   One can decompose the parameter space for monic polynomials to subsets invariant under scalings by rational $k \neq 0$. Given subset can be labelled by a subset with vanishing coefficients $\{p_{i_k}\}$. One can get rid of this degeneracy by fixing the first non-vanishing p_{n-k} to a non-vanishing value, say 1. When the first non-vanishing p_k differs from p_0, integers label the polynomials giving rise to roots in the same extension. If only p_0 is non-vanishing, only the scaling by powers k^n give rise to new polynomials and the number of polynomials giving rise to same extension is smaller than in other cases.

   Remark: For octonionic polynomials the scaling symmetry changes the space-time surface so that for generic polynomials the number of space-time surfaces giving rise to fixed extension is larger than for the special kind polynomials.

Could one gain some understanding about ramified primes by starting from quantum criticality? The following argument is poor man's argument and I can only hope that my modest technical understanding of number theory does not spoil it.

1. The basic idea is that for ramified primes the minimal monic polynomial with integer coefficients defining the basis for the integers of extension has multiple roots in $O(p) = 0$ approximation, when p is ramified prime dividing the discriminant of the monic polynomial. Multiple roots in $O(p) = 0$ approximation occur also for the irreducible polynomial defining the extension of rationals. This would correspond approximate quantum criticality in some p-adic sectors of adelic physics.

2. When 2 roots for an irreducible rational polynomial co-incide, the criticality is exact: this is true for polynomials of rationals, reals, and all p-adic number fields. One could use this property to construct polynomials with given primes as ramified primes. Assume that the extension allows an irreducible olynomial having decomposition into a product of monomials $= x - r_i$ associated with roots and two roots r_1 and r_2 are identical: $r_1 = r_2$ so that irreducibility is lost.

   The deformation of the degenerate roots of an irreducible polynomial giving rise to the extension of rationals in an analogous manner gives rise to a degeneracy in $O(p) = 0$ approximation. The degenerate root $r_1 = r_2$ can be scaled in such a way that the deformation $r_2 = r_1(1+q))$, $q = m/n = O(p)$ is small also in real sense by selecting $n >> m$.

   If the polynomial with rational coefficients gives rise to degenerate roots, same must happen also for monic polynomials. Deform the monic polynomial by changing $(r_1, r_2 = r_1)$ to $(r_1, r_1(1+r))$, where integer r has decomposition $r = \prod_i p_i^{k_i}$ to powers of prime. In $O(p) = 0$ approximation the roots r_1 and r_2 of the monic polynomial are still degenerate so that p_i represent ramified primes.

   If the number of p_i is large, one has high degree of ramification perhaps favored by p-adic evolution as increase of number theoretic co-operation. On the other hand, large p-adic primes are expected to correspond to high evolutionary level. Is there a competition between large ramified primes and number of ramified primes? Large $h_{eff}/h_0 = n$ in turn favors large dimension n for extension.

3. The condition that two roots of a polynomial co-incide means that both polynomial $P(x)$ and its derivative dP/dx vanish at the roots. Polynomial $P(x) = x^n + p_{n-1}x^{n-1} + ..p_0$ is parameterized by the coefficients which are rationals (integers) for irreducible (monic) polynomials. $n-1$-tuple of coefficients $(p_{n-1}, ..., p_0)$ defines parameter space for the polynomials. The criticality condition holds true at integer points $n-1-D$ surface of this parameter space analogous to cognitive representation.

   The condition that critical points correspond to rational (integer) values of parameters gives an additional condition selecting from the boundary a discrete set of points allowing ramification. Therefore there are strong conditions on the occurrence of ramification and only very special monic polynomials are selected.

   This suggests octonionic polynomials with rational or even integer coefficients, define strongly critical surfaces, whose p-adic deformations define p-adically critical surfaces defining an extension with ramified primes p. The condition that the number of rational critical points is non-vanishing or even large could be one prerequisite for number theoretical fitness.

4. There is a connection to catastrophe theory, where criticality defines the boundary of the region of the parameter space in which discontinuous catastrophic change can take place as replacement of roots of $P(x)$ with different root. Catastrophe theory involves polynomials $P(x)$ and their roots as well as criticality. Cusp catastrophe is the simplest non-trivial example of catastrophe surface with $P(x) = x^4/4 - ax - bx^2/2$: in the interior of V-shaped curve in (a, b)-plane there are 3 roots to $dP(x) = 0$, at the curve 2 solutions, and outside it 1 solution. Note that now the parameterization is different from that proposed above. The reason is that in catastrophe theory diffeo-invariance is the basic motivation whereas in M^8 there are highly unique octonionic preferred coordinates.

If p-adic length scale hypothesis holds true, primes near powers of 2, prime powers, in particular Mersenne primes should be ramified primes. Unfortunately, this picture does not allow to say anything about why ramified primes near power of 2 could be interesting. Could the appearance of ramified primes somehow relate to a mechanism in which $p = 2$ as a ramified prime would precede other primes in the evolution. $p = 2$ is indeed exceptional prime and also defines the smallest p-adic length scale.

For instance, could one have two roots a and $a + 2^k$ near to each other 2-adically and could the deformation be small in the sense that it replaces 2^k with a product of primes near powers of 2: $2^k = \prod_i 2^{k_i} \to \prod_i p_i$, p_i near 2^{k_i}? For the irreducible polynomial defining the extension of rationals, the deforming could be defined by $a \to a + 2^k$ could be replaced by $a \to a + 2^k/N$ such that $2^k/N$ is small also in real sense.

14.4.2 Appendix: About the decomposition of primes of number field K to primes of its extension L/K

The followings brief summary lists some of the basic terminology related to the decomposition of primes of number field K in its extension.

1. A typical problem is the splitting of primes of K to primes of the extension L/K which has been already described. One would like to understand what happens for a given prime in terms of information about K. The splitting problem can be formulated also for the extensions of the local fields associated with K induced by L/K.

2. Consider what happens to a prime ideal p of K in L/K. In general p decomposes to product $p = \prod_{i=1}^{g} P_i^{e_i}$ of powers of prime ideals P_i of L. For $e_i > 1$ ramification is said to occur. The finite field K/p is naturally imbeddable to the finite field L/P_j defining its extension. The degree of the residue field extension $(L/P_i)/(K/p)$ is denoted by f_i and called inertia degree of P_i over p. The degree of L/K equals to $[L : K] = \sum e_i f_i$.

If the extension is Galois extension (see http://tinyurl.com/zu5ey96), one has $e_i = e$ and $f_i = f$ giving $[L : K] = efg$. The subgroups of Galois group $Gal(L/K)$ known as decomposition group D_i and inertia group I_i are important. The Galois group of F_i/F equals to D_i/I_i.

For Galois extension the Galois group $Gal(L/K)$ leaving p invariant acts transitively on the factors P_i permuting them with each other. Decomposition group D_i is defined as the subgroup of $Gal(L/K)$ taking P_i to itself.

The subgroup of $Gal(L/K)$ inducing identity isomorphism of P_i is called inertia group I_i and is independent of i. I_i induces automorphism of $F_i = L/P_i$. $Gal(F_i/F)$ is isomorphic to D_i/I_i. The orders of I_i and D_i are e and ef respectively. The theory of Frobenius elements identifies the element of $Gal(F_i/F) = D_i/I_i$ as generator of cyclic group $Gal(F_i/F)$ for the finite field extension F_i/F. Frobenius element can be represented and defines a character.

3. Quadratic extensions $Q(\sqrt{n})$ are simplest Abelian extensions and serve as a good starting point (see http://tinyurl.com/zofhmb8) the discrimant $D = n$ for $p\ mod\ 4 = 1$ and $D = 4n$ otherwise characterizes splitting and ramification. Odd prime p of the extension not dividing D splits if and only if D quadratic residue modulo p. p ramifies if D is divisible by p. Also the theorem by Kronecker and Weber stating that every Abelian extension is contained in cyclotomic extension of Q is a helpful result (cyclotonic polynomials has as it roots all n roots of unity for given n)

Even in quadratic extensions L of K the decomposition of ideal of K to a product of those of extension need not be unique so that the notion of prime generalized to that of prime ideal becomes problematic. This requires a further generalization. One ends up with the notion of ideal class group (see http://tinyurl.com/hasyllh): two fractional ideals I_1 and I_2 of L are equivalent if the are elements a and b such that $aI_1 = bI_2$. For instance, if given prime of K has two non-equivalent decompositions $p = \pi_1\pi_2$ and $p = \pi_3\pi_4$ of prime ideal p associated with K to prime ideals associated with L, then π_2 and π_3 are equivalent in this sense with $a = \pi_1$ and $b = \pi_4$. The classes form a group J_K with principal ideals defining the unit element with product defined in terms of the union of product of ideals in classes (some products can be identical). Factorization is non-unique if the factor J_K/P_K - ideal class group - is non-trivial group. $Q(\sqrt{-5})$ gived a representative example about non-unique factorization: $2\times 3 = (1+\sqrt{-5})(1-\sqrt{-5})$ (the norms are 4×9 and 6×6 for the two factorizations so that they cannot be equivalent.

This leads to class field theory (see http://tinyurl.com/zdnw7j3 and http://tinyurl.com/z3s4kjn).

1. In class field theory one considers Abelian extensions with Abelian Galois group. The theory provides a one-to-one correspondence between finite abelian extensions of a fixed global field K and appropriate classes of ideals of K or open sub-groups of the idele class group of K. For example, the Hilbert class field, which is the maximal unramified abelian extension of K, corresponds to a very special class of ideals for K.

2. Class field theory introduces the adele formed by reals and p-adic number fields Q_p or their extensions induced by algebraic extension of rationals. The motivation is that the very tough problem for global field K (algebraic extension of rationals) defines much simpler problems for the local fields Q_p and the information given by them allows to deduce information about K. This because the polynomials of order n in K reduce effectively to polynomials of order $n\ mod\ p^k$ in Q_p if the coefficients of the polynomial are smaller than p^k. One reduces monic irreducible polynomial f characterizing extension of Q to a polynomial in finite field F_p. This allows to find the extension Q_p induced by f.

   An irreducible polynomial in global field need not be irreducible in finite field and therefore can have multiple roots: this corresponds to a ramification. One identifies the primes p for which complete splitting (splitting to first ordinary monomials) occurs as unramified primes.

3. Class field theory also includes a reciprocity homomorphism, which acts from the idele class group of a global field K, i.e. the quotient of the ideles by the multiplicative group of K, to the Galois group of the maximal abelian extension of K. Wikipedia article makes the

statement "*Each open subgroup of the idele class group of K is the image with respect to the norm map from the corresponding class field extension down to K*". Unfortunately, the content of this statement is difficult to comprehend with physicist's background in number theory.

14.5 Appendix: Explicit formulas for the evolution of cosmological constant

What is needed is induced Kähler form $J(S^2(X^4)) \equiv J$ at the twistor sphere $S^2(X^4) \equiv S^2$ associated with space-time surface. $J(S^2(X^4))$ is sum of Kähler forms induced from the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$.

$$J(S^2(X^4) \equiv J = P[J(S^2(M^4)) + J(S^2(CP_2))] \ , \tag{14.5.1}$$

where P is projection taking tensor quantity T_{kl} in $S^2(M^4)\times S^2(CP_2)$ to its projection in $S^2(X^4)$. Using coordinates y^k for $S^2(M^4)$ or $S^(CP_2)$ and x^μ forS^2, P is defined as

$$P : T_{kl} \rightarrow T_{\mu\nu} = T_{kl}\frac{\partial y^k}{\partial x^\mu}\frac{\partial y^l}{\partial x^\nu} \ . \tag{14.5.2}$$

For the induced metric $g(S^2(X^4)) \equiv g$ one has completely analogous formula

$$g = P[g(J(S^2(M^4)) + g(S^2(CP_2))] \ . \tag{14.5.3}$$

The expression for the coefficient K of the volume part of the dimensionally reduced 6-D Kähler action density is proportional to

$$L(S^2) = J^{\mu\nu}J_{\mu\nu}\sqrt{det(g)} \ . \tag{14.5.4}$$

(Note that $J_{\mu\nu}$ refers to S^2 part 6-D Kähler action). This quantity reduces to

$$L(S^2) = (\epsilon^{\mu\nu}J_{\mu\nu})^2\frac{1}{\sqrt{det(g)}} \ . \tag{14.5.5}$$

where $\epsilon^{\mu\nu}$ is antisymmetric tensor density with numerical values +,-1. The volume part of the action is obtained as an integral of K over S^2:

$$S(S^2) = \int_{S^2} L(S^2) = \int_{-1}^{1} du \int_0^{2\pi} d\Phi \frac{J^2_{u\Phi}}{\sqrt{det(g)}} \ . \tag{14.5.6}$$

$(u,\Phi) \equiv (cos(\Theta,\Phi)$ are standard spherical coordinates of $S^2)$ varying in the ranges $[-1,1]$ and $[0,2\pi]$.

This the quantity that one must estimate.

14.5.1 General form for the embedding of twistor sphere

The embedding of $S^2(X^4) \equiv S^2$ to $S^2(M^4)\times S^2(CP_2)$ must be known. Dimensional reduction requires that the embeddings to $S^2(M^4)$ and $S^2(CP_2)$ are isometries. They can differ by a rotation possibly accompanied by reflection

One has

$$\begin{aligned} &(u(S^2(M^4)), \Phi(S^2(M^4)) = (u(S^2(X^4), \Phi(S^2(X^4)) \equiv (u,\Phi) \ , \\ &\big[u(S^2(CP_2)), \Phi(S^2(CP_2))\big] \equiv (v,\Psi) = RP(u,\Phi) \end{aligned}$$

where RP denotes reflection P following by rotation R acting linearly on linear coordinates (x,y,z) of unit sphere S^2). Note that one uses same coordinates for $S^2(M^4)$ and $S^2(X^4)$. From this action one can calculate the action on coordinates u and Φ by using the definite of spherical coordinates.

The Kähler forms of $S^2(M^4)$ resp. $S^2(CP_2)$ in the coordinates $(u = cos(\Theta), \Phi)$ resp.(v, Ψ) are given by $J_{u\Phi} = \epsilon = \pm 1$ resp. $J_{v\Psi} = \epsilon = \pm 1$. The signs for $S^2(M^4)$ and $S^2(CP_2)$ are same or opposite. In order to obtain small cosmological constant one must assume either

1. $\epsilon = -1$ in which case the reflection P is absent from the above formula (RP $\rightarrow$ R).

2. $\epsilon = 1$ in which case P is present. P can be represented as reflection $(x, y, z) \rightarrow (x, y, -z)$ or equivalently $(u, \Phi) \rightarrow (-u, \Phi)$.

Rotation R can represented as a rotation in (y,z)-plane by angle ϕ which must be small to get small value of cosmological constant. When the rotation R is trivial, the sum of induced Kähler forms vanishes and cosmological constant is vanishing.

14.5.2 Induced Kähler form

One must calculate the component $J_{u\Phi}(S^2(X^4)) \equiv J_{u\Phi}$ of the induced Kähler form and the metric determinant $det(g)$) using the induction formula expressing them as sums of projections of M^4 and CP_2 contributions and the expressions of the components of $S^2(CP_2)$ contributions in the coordinates for $S^2(M^4)$. This amounts to the calculation of partial derivatives of the transformation R (or RP) relating the coordinates (u, Φ) of $S^2(M^4)$ and to the coordinates (v, Ψ) of $S^2(CP_2)$.

In coordinates (u, Φ) one has $J_{u\Phi}(M^4) = \pm 1$ and similar expression holds for $J(v\Psi)S^2(CP_2)$. One has

$$J_{u\Phi} = 1 + \frac{\partial(v, \Psi)}{\partial(u, \Phi)} \quad . \tag{14.5.7}$$

where right-hand side contains the Jacobian determinant defined by the partial derivatives given by

$$\frac{\partial(v,\Psi)}{\partial(u,\Phi)} = \frac{\partial v}{\partial u}\frac{\partial \Psi}{\partial \Phi} - \frac{\partial v}{\partial \Phi}\frac{\partial \Psi}{\partial u} \quad . \tag{14.5.8}$$

14.5.3 Induced metric

The components of the induced metric can be deduced from the line element

$$ds^2(S^2(X^4) \equiv ds^2 = P[ds^2(S^2(M^4)) + ds^2(S^2(CP_2))] \quad .$$

where P denotes projection. One has

$$P(ds^2(S^2(M^4))) = ds^2(S^2(M^4)) = \frac{du^2}{1-u^2} + (1-u^2)d\Phi^2 \quad .$$

and

$$P[ds^2(S^2(CP_2))] = P[\frac{(dv)^2}{1-v^2} + (1-v^2)d\Psi^2] \quad ,$$

One can express the differentials $(dv, d\Psi)$ in terms of $(du, d\Phi)$ once the relative rotation is known and one obtains

$$P[ds^2(S^2(CP_2))] = \frac{1}{1-v^2}[\frac{\partial v}{\partial u}du + \frac{\partial v}{\partial \Phi}d\Phi]^2 + (1-v^2)[\frac{\partial \Psi}{\partial u}du + \frac{\partial \Psi}{\partial \Phi}d\Phi]^2 \quad .$$

This gives

$$
\begin{aligned}
&P[ds^2(S^2(CP_2))] \\
&= [(\frac{\partial v}{\partial u})^2 \frac{1}{1-v^2} + (1-v^2)(\frac{\partial \Psi}{\partial u})^2]du^2 \\
&+[(\frac{\partial v}{\partial \Phi})^2 \frac{1}{1-v^2} + (\frac{\partial \Psi}{\partial \Phi})^2 1-v^2]d\Phi^2 \\
&+2[\frac{\partial v}{\partial u}\frac{\partial v}{\partial \Phi}\frac{1}{(1-v^2)} + \frac{\partial \Psi}{\partial u}\frac{\partial \Psi}{\partial \Phi}(1-v^2)]dud\Phi \ .
\end{aligned}
$$

From these formulas one can pick up the components of the induced metric $g(S^2(X^4)) \equiv g$ as

$$
\begin{aligned}
g_{uu} &= \frac{1}{1-u^2} + (\frac{\partial v}{\partial u})^2 \frac{1}{1-v^2} + (1-v^2)(\frac{\partial \Psi}{\partial u})^2] \ , \\
g_{\Phi\Phi} &= 1-u^2 + (\frac{\partial v}{\partial \Phi})^2 \frac{1}{1-v^2} + (\frac{\partial \Psi}{\partial \Phi})^2 (1-v^2) \\
g_{u\Phi} &= g_{\Phi u} = \frac{\partial v}{\partial u}\frac{\partial v}{\partial \Phi}\frac{1}{(1-v^2)} + \frac{\partial \Psi}{\partial u}\frac{\partial \Psi}{\partial \Phi}(1-v^2) \ .
\end{aligned}
\tag{14.5.9}
$$

The metric determinant $det(g)$ appearing in the integral defining cosmological constant is given by

$$det(g) = g_{uu}g_{\Phi\Phi} - g_{u\Phi}^2 \ . \tag{14.5.10}$$

14.5.4 Coordinates (v, Ψ) in terms of (u, Φ)

To obtain the expression determining the value of cosmological constant one must calculate explicit formulas for (v, Ψ) as functions of (u, Φ) and for partial derivations of (v, Ψ) with respect to (u, Φ).

Let us restrict the consideration to the RP option.

1. P corresponds to $z \to -z$ and to

$$u \to -u \ . \tag{14.5.11}$$

2. The rotation R $(x, y, z) \to (x', y', z')$ corresponds to

$$x' = x, \quad y' = sz + cy = su + c\sqrt{1-u^2}sin(\Phi) \ , \quad z' = v = cu - s\sqrt{1-u^2}sin(\Phi) \tag{14.5.12}$$

Here one has $(s, c) \equiv (sin(\epsilon), cos(\epsilon)$, where ϵ is rotation angle, which is extremely small for the value of cosmological constant in cosmological scales.

From these formulas one can pick v and $\Psi = arctan(y'/x)$ as

$$v = cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \tag{14.5.13}$$

3. RP corresponds to

$$v = -cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[-\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \tag{14.5.14}$$

14.5.5 Various partial derivatives

Various partial derivates are given by

$$\begin{aligned}
\frac{\partial v}{\partial u} &= -1 + s\frac{u}{\sqrt{1-u^2}}sin(\Phi) \ \ , \\
\frac{\partial v}{\partial \Phi} &= -s\frac{u}{\sqrt{1-u^2}}cos(\Phi) \ \ , \\
\frac{\partial \Psi}{\partial \Phi} &= (-s\frac{u}{\sqrt{1-u^2}}sin(\Phi) + c)\frac{1}{X} \ \ , \\
\frac{\partial \Psi}{\partial u} &= \frac{scos(\Phi)(1+u-u^2)}{(1-u^2)^{3/2}}\frac{1}{X} \ \ , \\
X &= cos^2(\Phi) + [-s\frac{u}{\sqrt{1-u^2}} + csin(\Phi)]^2 \ \ .
\end{aligned} \tag{14.5.15}$$

Using these expressions one can calculate the Kähler and metric and the expression for the integral giving average value of cosmological constant. Note that the field equations contain S^2 coordinates as external parameters so that each point of S^2 corresponds to a slightly different space-time surface.

14.5.6 Calculation of the evolution of cosmological constant

One must calculate numerically the dependence of the action integral S over S^2 as function of the parameter $s = sin(\epsilon))$. One should also find the extrema of S as function of s.

Especially interesting values are very small values of s since for the cosmological constant becomes small. For small values of s the integrand (see Eq. 14.5.6) becomes very large near poles having the behaviour $1/\sqrt{g} = 1/(sin(\Theta) + O(s))$ coming from $\sqrt{g}$ approaching that for the standard metric of S^2. The integrand remains finite for $s \neq 0$ but this behavior spoils the analytic dependence of integral on s so that one cannot do perturbation theory around $s = 0$. The expected outcome is a logarithmic dependence on s.

In the numerical calculation one must decompose the integral over S^2 to three parts.

1. There are parts coming from the small disks D^2 surrounding the poles: these give identical contributions by symmetry. One must have criterion for the radius of the disk and the natural assumption is that the disk radius is of order s.

2. Besides this one has a contribution from S^2 with disks removed and this is the regular part to which standard numerical procedures apply.

One must be careful with the expressions involving trigonometric functions which give rise to infinite if one applies the formulas in straightforward manner. These infinities are not real and cancel, when one casts the formulas in appropriate form inside the disks.

1. The limit $u \to \pm 1$ at poles involves this kind of dangerous quantities. The expression for the determinant appearing in $J_{u\Phi}$ remains however finite and $J^2_{u\phi}$ vanishes like s^2 at this limit. Also the metric determinant $1/\sqrt{g}$ remains finite expect at $s = 0$.

2. Also the expression for the quantity X in $\Psi = arctan(X)$ contains a term proportional to $1/cos(\Phi)$ approaching infinity for $\Phi \to \pi/2, 3\pi/2$. The value of $\Psi = arc(tan(X)$ remains however finite and equal to $\pm\Phi$ at this limit depending on on the sign of us.

Concerning practical calculation, the relevant formulas are given in Eqs. 14.5.5, 14.5.6, 14.5.7, 14.5.8, 14.5.9, 14.5.10, and 14.5.15.

The calculation would allow to test/kill the key conjectures already discussed.

1. There indeed exist extrema satisfying $dS(S^2)/ds = 0$.

2. These extrema are in one-one correspondence with the zeros of zeta.

There are also much more specific conjctures to be killed.

1. These extrema correspond to $s = sin(\epsilon) = 2{-}k$ or more generally $s = p^{-k}$. This conjecture is inspired by p-adic length scale hypothesis but since the choice of evolution parameter is to high extent free, the conjecture is perhaps too spesific.

2. For certain integer values of integer k the integral $S(S^2)$ of Eq. 14.5.6 is of form $S(S^2) = xs^2$ for $s = 2^{-k}$, where x is a universal numerical constant.

   This would realize the idea that p-adic length scales realized as scales associated with cosmological constant correspond to fixed points of renormalization group evolution implying that radiative corrections otherwise present cancel. In particular, the deviation from $s = 2^{-d/2}$ would mean anomalous dimension replacing $s = 2^{-d/2}$ with $s^{-(d+\Delta d)/2}$ for $d = k$ the anomalies dimension Δd would vanish.

   The condition $\Delta d = 0$ should be equivalent with the vanishing of the dS/ds. Geometrically this means that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

Chapter 15

Some questions about coupling constant evolution

15.1 Introduction

In this article questions related to the notions of the p-adic CCE and hierarchy of Planck constants will be considered.

15.1.1 How p-adic primes are determined?

p-Adic length scale (PLS) hypothesis plays a central role in TGD in all length scales. For instance, it makes it possible to use simple scaling arguments to deduce quantitative predictions for the masses of new particles predicted by TGD.

PLS hypothesis states that the size scales of space-time surfaces correspond to PLSs $L_p = \sqrt{p}R(CP_2)$. The additional hypothesis is $p \simeq m^k$, $m = 2, 3, ...$ a small prime. The success of p-adic mass calculations [K65] supports $p \simeq 2^k$ hypothesis [K80] seriously. There also exists empirical evidence for a possible generalization to small primes, in particular $m = 3$, in biology [?, ?].

The physical and mathematical identification of the origin of the p-adic prime p defining the PLS is however a problem.

The proposal has been that the p-adic prime p defining the PLS corresponds to a ramified prime of the extension of rationals (EQ) associated with the polynomial defining space-time region in M^8 picture. Ramified primes appear as factors of the discriminant of the polynomial defining EQ. I have not been able to find any really convincing explanation for why p should correspond to a ramified prime so that p-dic prime might emerge in some other way.

In p-adic thermodynamics Boltzmann weights $exp(-E/T)$ must be replaced with $p^{L_0/T}$, where L_0 is scaling generator. The exponent $\Omega = exp(K)$ of the Kähler function K of WCW defines vacuum functional. Could Ω be number-theoretically universal and thus exist as a p-adic number for some prime p determining naturally the PLS. This is the case if one has $\Omega = p^n$, n integer.

As such, this idea does not make sense but one consider a subsystem defined by sub-CD defining self in zero energy ontology (ZEO) based theory of consciousness [L91, L138] [K130]?

p-Adic prime defines naturally the scale of CD for trivial extension of rationals and this scale is scaled up by factor n for an extension of dimension n. This also conforms with the assumption that p-adic CCE and "dark" CCE are independent.

15.1.2 Trying to understand p-adic CCE

TGD leads to a number theoretic vision about CCE [L70]. Number theoretic universality plays a key role in this picture. CCE certainly involves the hierarchy of extensions of rationals (EQs) possibly involving non-rational extensions by roots of e, which induce finite extensions of p-adics. It would be nice if the EQ alone would determine the values of the coupling constants.

1. The starting point is that the continuous CCE with respect to length scale reduces to a discrete PLS evolution with respect to L_p, $p \simeq 2^k$. There is also dark evolution with respect to $n = h_{eff}/h_0$. These evolutions are separate since the scaling of the roots of the polynomial do not affect the purely algebraic properties of the extension. The natural assumption is that these evolutions factorize so that one has $\alpha_K = g_K^2(p)/2h_{eff}$.

2. p-Adic CCE would be roughly logarithmic with respect to L_p. The observation that α is near $\alpha = 1/137$ for p-adic length scale $L(137)$ suggests that for α_K defining the fundamental coupling strength one has

$$\alpha_K = \frac{g_K^2(max)}{2kh_{eff}} \quad .$$

Since $1/\alpha_K(137) = 137$ is prime for ordinary matter with $h_{eff} = h$, one must have

$$\frac{g_K^2(max)}{2h} = 1 \quad .$$

giving $h = g_K^2(max)/2$. The value h need not however be the minimal value h_0 of h_{eff} since one can have $h = n_0 h_0$ $\alpha_K(max) = 2n_0$ so that one can write

$$\alpha_K = \frac{1}{knn_0} \quad . \tag{15.1.1}$$

$n_0 > 1$ would mean that the ordinary matter would be actually dark in the sense that the order of the extension of rationals associated with the ground state would be n_0.

For h_0 that value of α_K could be so large that the perturbation series does not converge except in very long length scales for which k is expected to be large. Exotic phases with $h_{eff} < h$ could become possible in these scales.

15.1.3 How p-adic prime is defined at the level of WCW geometry?

The p-adic prime p should emerge from the dynamics defined by Kähler function.

1. The Kähler function K of the "world of classical worlds" (WCW), or more generally the generalization of $exp(K)$ to a vacuum functional possibly involving also a genuine state dependent part is a central quantity concerning scattering amplitudes. Suppose that one can consider a subsystem defined by CD and the contribution ΔK from CD to K.

   Number theoretical universality suggests that the exponential $exp(\Delta K)$ or its appropriate generalization exists in all p-adic number fields or at least in an extension of the p-adic number field corresponding to the p-adic prime p. Could this condition fix p dynamically?

2. Suppose that for some prime p one can write

$$e^{\Delta K} = p^{\frac{\Delta K}{log(p)}}$$

   such that $\Delta K/log(p)$ is integer. The exponential would be a power of p just as the p-adic analog of Boltzmann weight in p-adic thermodynamics [K65]. This would select a unique p-adic prime p defining the PLS and this prime need not be a ramified prime. In p-adic thermodynamics [K65] $X = \Delta K/log(p)$ has interpretation as an eigenvalue of the scaling generator L_0 of conformal algebra and one can even consider the possibility that there is a connection.

15.1.4 What about the evolution of the gravitational fine structure constant?

Nottale hypothesis [E1] predicts gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ ($\beta_0 = v_0/c$ is velocity parameter), which has gigantic values so that the above picture fails. Gravitational fine structure constant is given by $\alpha_{gr} = \beta_0/4\pi$.

Kepler's law $\beta^2 = GM/r = r_S/2r$ suggests length scale evolution $\beta^2 = xr_S/2L_N = \beta^2_{0,max}/N^2$, where x is proportionality constant, which can be fixed. Phase transitions changing β_0 are possible at $L_N/a_{gr} = N^2$ and these scales correspond to radii for the gravitational analogs of the Bohr orbits of gravitational Bohr atom. PLS hierarchy is replaced by that for the radii of Bohr orbits.

What could be the interpretation of N? The safest assumption is that the CCE of β_0 is analogous to that of the other coupling constants and induced from that of α_K.

15.1.5 What is the minimal value of h_{eff}?

The formula $h_{eff} = nh_0$ involves the minimal value h_0 of h_{eff}. The simplest explanation for the findings of Randell Mills [D3] is that one has $h = 6h_0$. h_0 could be also smaller [L32].

What is the value of h_0? A possible answer to this question came from the observation made already during the first 10 years of TGD. The observation was that the imbeddings of spherically symmetric stationary metrics (see the Appendix) suggest that CP_2 radius R is of order Planck length l_P rather than by factor about $10^{7.5}$ longer. Could one have $h = n_0h_0$, $n_0 \sim 10^{7.5}$ so that the ordinary matter would be actually dark?

CP_2 radius would be Planck length apart from numerical constant not far from unity. The p-adic mass calculations would give correct results for $h_{eff} = h_0$. R could be interpreted as $R^2 = n_0 l_P^2$. The perturbative expansion for $h_{eff} < h$ would not converge except in long p-adic length scales, where the p-adic evolution reduces the value of α_K.

Gauge coupling strengths are predicted to be practically zero at gravitational flux tubes with very large h_{eff} so that only gravitational interaction is effectively present. This conforms with the view about dark matter.

15.2 Number theoretical universality of vacuum functional and p-adic CCE

The Kähler geometry of WCW is defined by a Kähler function $K(X^4(X^3))$ identified as the action of preferred extremal consisting of volume term and Kähler action. The vacuum functional is of form $\Omega = exp(K + iS)$. Here K is the real Kähler function and S is the counterpart of real action in the path integral of QFTs.

$exp(iS)$ could be interpreted as a dynamical part of vacuum functional, which depends on state rather than being "God-given". The reason why this would be the case would be that it is possible. For $exp(K)$ there is no choice since the Kähler geometry of WCW is expected to be unique merely from its existence as already in the case of loop spaces [A44].

Number theoretical universality is a challenge for this general picture.

1. In the p-adic context the notion of WCW geometry is highly questionable. The integration associated with definition of volume term and Kähler action is the tough problem.

   This has inspired the proposal that the exponent of the action completely disappears from the scattering amplitudes. This indeed happens in quantum field theory based on path integral around stationary point.

2. The classical nondeterminism suggests a weaker formulation. The sum over the contributions of stationary points would be replaced by integral over preferred extremals consisting of 3-surfaces at PB plus sum over the paths of the tree resulting from classical non-determinism.

   The sum over the paths of the tree-like structure remains in the superposition of amplitudes for sub-CD and it might be possible to define the deviation $\Delta K + i\Delta S$ of the action for each of them and separate $exp(\Delta K + i\Delta S)$ from the entire exponent of action, which would therefore disappear from the expression of the scattering amplitudes for given X^3 and given CD. Otherwise, the knowledge of the entire WCW Kähler function would be needed.

A possible interpretation is in terms of a decomposition to an unentangled tensor product corresponding to sub-CD and its environment so that one can separate the physics inside sub-CD from that of environment and code it by $exp(\Delta K + i\Delta S)$.

3. The simplest option, very probably too simple, would be that one has $\Delta K = 0$. Kähler function would be same for all paths of the tree and one would obtain a discretized analog of path integral. This would require that all the branches of the tree have same value of action. This does not however require the same value of volume and Kähler action separately.

   It will be foud that $\Delta K \neq 0$ assuming that $exp(\Delta K)$ reduces to an integer power of p for some prime identifiable as p-adic prime defining the PLS, is more interesting option since it would reduce p-adic thermodynamics to the level of WCW and also allow to the understand of PLS evolution of couping constants.

   The number theoretical existence of the phases $exp(i\Delta S)$ would require that they belong to the EQ defining the space-time region inside CD.

4. This picture suggests that the number theoretically universal part is associated with the sub-CDs and with the discrete physics of the tree-like structure whereas the Kähler function for 3-surfaces would be defined only in real framework. This would neatly separate the physics of sensory and Boolean cognition as something number theoretically universal from the physics proper, so to say.

   Since conscious experience gives all information about physics, one can ask whether the adelic physics associated with various sub-selves could together be enough to represent all that is representable from the physics proper. This could result as somekind of limiting case (EQ approaches algebraic numbers).

If this view is correct, then one expects that various notions shared by QFTs and TGD, in particular CCE, could have number theoretic descriptions as indeed suggested [L70]. In the sequel I will discuss some speculations in this framework.

15.2.1 The recent view about zero energy ontology

Zero energy ontology (ZEO) [K130] [L91, L138] plays a key role in the formulation of TGD based quantum measurement theory.

1. The concept of causal diamond (CD) is central. CD serves as a correlate for the perceptive field of conscious entity: this in the case that one has sub-CD so that the space-time surfaces inside CD continue outside it.

   The scale size scale of the CD identifiable as the temporal distance T between its tips could be proportional the p-adic prime p at the lowest level of dark matter hierarchy and to np at dark sectors. p-Adic length scales L_p characterizing the sizes scale of 3-surfaces are proportional to $\sqrt{p}$ and the proposal is that the relation between T and L_p is same as the relationship between diffusion time T and the root mean square distance R travelled by diffusion.

2. The twistor lift of TGD predicts that the action principle defining space-time surfaces is the sum of a volume term characterized by length scale dependent cosmological constant Λ and Kähler action and induced from 6-D Kähler action whose existence fixes the embedding space uniquely to $M^4 \times CP_2$. The reason is that the required Kähler structure exists only for the twistor spaces of M^4, E^4, and CP_2 [A63].

3. The recent progress in the understanding of zero energy ontology (ZEO) [L138] leads to rather detailed view about the dynamics of the space-time surfaces inside sub-CD.

   Space-time surfaces are analogs of soap films spanned by a frame having the 3-surfaces at its ends located at the boundary of CD as fixed part of frame and the dynamically generated parts of frame in the interior of CD. Outside the frame preferred extremal is an analog of a complex surface and a simultaneous extremal of both volume term and Kähler action since the field equations reduce to conditions expressing the analogy of holomorphy [L55, L72]. The field equations reduce to contractions of tensors of type (1,1) with tensors of type (2,0)+(0,2) and are therefore trivially true.

The minimal surface property fails at the frame, and only the full field equations are true. The divergences of isometry currents associated with volume term have delta function singularities which however cancel each other to guarantee field equations and conservation laws. This is expected to give rise to a failure of determinism, which is however finite in the sense that the space-time surfaces associated with given 3-surface X^3 at the passive boundary of CD (PB) form a finite set which is a tree-like structure (for a full determinism only single space-time surface as analog of Bohr orbit would be realized). Therefore the non-determinism of classical dynamics for a fixed X^3 is extremely simple and quantum dynamics and classical dynamics are very closely related since quantum states are superpositions of the paths of the tree.

4. One also ends up to quite precise identification sub-CD or space-time surface inside sub-CD as a correlate of perceptive field of a conscious entity. The essential element of the picture that for sub-CD the 3-surface X^3 at PB is fixed but due to the non-determinism the end at active boundary (AB) is not completely fixed and there is finite non-determinism in the state space defined by superpositions of the paths of the tree.

   For the highest level in the hierarchy of CDs associated with self, the space-time surfaces inside CD do not continue outside it and this CD God-like entity, whose dynamics is not restricted by the boundary conditions.

This view provides additional perspectives on discreteness of adelic physics unifying the physics of sensory experience and cognition [L54, L53].

1. Discreteness is essential in the number theoretic universality since in these case real structures and their p-adic counterparts correspond naturally to each other. This has led to the notion of cognitive representation as a set of points of space-time surface with preferred embedding space coordinates having values in the EQ defined by the polynomial defining the space-time surface in complexified M^8 and mapped to H by M^8-H duality [L109, L110]. The finite-dimensionality of the state space associated with the tree structure conforms with this vision.

2. Discreteness is natural for the dynamics of concious experience and cognition. Mental images as sub-selves correspond to the sub-CDs inside CD. Sub-CDs are naturally located at the loci of non-determinism defined by the fixed part of the frame dynamically and generated frames in the interior and at AB.

   Attention would fix the 3-surfaces at the PB of a sub-CD as a perceptive sub-field and all CDs in the hierarchy would be fixed in this manner. The loci of non-determinism would serve as targets of attention. Sensory perception, memory recall, and other functions would reduce to directed attention inside CD.

   Fermionic degrees of freedom at boundaries of CD are are additional discrete degrees of freedom and responsible for Boolean cognition whereas the discrete dynamics of frame would correspond to sensory experience and sensory aspects of cognition.

3. This picture inspires the question whether the number theoretically universal parts of adelic physics might relate to the physics due to the non-determinism in the interior of sub-CD. This physics would be basically the physics that can be observed. This would mean enormous simplification.

   This idea is not new. The amazing success of p-adic thermodynamics based mass calculations [K65] could be understood if p-adic physics is seen as a physics of cognitive representation of real number based physics.

In the sequel some speculations are discussed by taking the above picture as a basis.

15.2.2 Number theoretical constraints on $exp(\Delta K)$

Number theoretical universality suggests that the exponents $exp(\Delta K+i\Delta S)$ for X^4 inside sub-CD is well-defined at least for some p-adic number fields or their extensions.

It has been already found that number theoretical universality requires that the phases $exp(i\Delta S)$ belong to the EQ associated with the space-time surfaces considered.

The condition that the phase is a root of unity is more general than the condition of semi-classical approximation of wave mechanics stating that the action is quantized as a multiple of Planck constant h. The analog of this condition would imply $exp(i\Delta S) = 1$. This quantization condition would make S obsolete.

What about the number theoretical universality of $exp(\Delta K)$? One can consider three options.

1. p-Adic exponent function $exp(x)$ exists if the p-adic norm of x is smaller than 1. The problem is that the p-adic exponent function and its real counterpart behave very differently [K77]. In particular, $exp(x)$ is not periodic. Integer powers of e^p are however ordinary p-adic number by its Taylor series and roots of e define finite-D extensions of p-adic number fields. Therefore $exp(\Delta K)$ could make sense as an integer power for a root of e.

   If ΔK is integer, $exp(\Delta K)$ exists p-adically for primes p dividing ΔK.

2. Also $p^{\Delta K/log(p)}$ could exist p-adically if $\Delta K/log(p)$ is integer. This implies strong conditions. ΔK must be of form $\Delta K = log(p)m$, m integer. If ΔK corresponds to Kähler function of WCW, p is fixed and would define the sought-for preferred p-adic prime p defining the PLS.

3. Since the powers p^n converge to zero for $n \to \infty$, one can formally replace $exp(\Delta K)$ with $exp(\Delta K) = p^{\Delta K)/log(p)}$ and require that the exponent is an integer. The replacement of the ordinary Boltzman weights with powers of p is indeed carried out in p-adic thermodynamics [K65]. This suggests that the Boltzman factors of p-adic thermodynamics reduce to exponents $p^{\Delta K}$ at the level of WCW.

15.3 Hierarchy of Planck constants, Nottale's hypothesis, and TGD

15.3.1 Nottale's hypothesis

Nottale's hypothesis [E1] and its generalization to TGD [K101, K11] has non-relativistic and relativistic forms.

1. The non-relativistic formula for $\hbar_{gr}$ as given by the Nottale's formula

$$\begin{aligned} \hbar_{gr} &= \frac{GMm}{\beta_0} \ , \\ \alpha_{gr} &= \frac{GMm}{4\pi\hbar_{gr}} = \frac{\beta_0}{4\pi} \ . \end{aligned} \tag{15.3.1}$$

   The formula makes sense only $\hbar_{gr}/\hbar > 1$.

2. The relativistically invariant formula for $\hbar_{gr}$ reads for four momenta $P = (M, 0)$ $p = (E, p_3)$ as:

$$\hbar_{gr} = \frac{GP \cdot p}{\beta_0} = \frac{GME}{\beta_0} = \frac{r_s E}{2\beta_0} \ , \tag{15.3.2}$$

   where r_s is Schwartschild radius. Adelic physics implies that momentum components belong to an extension of rationals defining the adele so that the spectrum of E and of $\hbar_{gr}$ are discretized.

Nottale's hypothesis and biology

Nottale's hypothesis involves a lot of uncertainties also at the conceptual level. Hence it is important to see whether basic facts from TGD inspired biology support the Nottale's hypothesis.

1. The cyclotron frequencies in an "endogenous" magnetic field $B_{end} = 2B_E/5$, where $B_E = .5$ Gauss is the nominal value of the Earth's magnetic emerge in the explanation of the findings of Blackman and other [J4] showing that ELF photons have effects on vertebrate brain. B_{end} is assigned with the monopole flux tubes of B_E. Also lower and higher values of B_{end} can be considered and the models of hearing [K91] and genetic code [L19] suggests that the values of B_{end} correspond to the notes of 12-note scale. This suggests that also the Z^0 magnetic field might be involved.

2. Biophoton energies are in visible and UV range and in the TGD based model they are assumed to result in the transformations of dark photons with much smaller frequency but same energy to ordinary photons. For instance, photons with 10 Hz frequency can transform to biophotons. By $E = h_{eff}f$, requires $h_{eff} = h_{gr}$. The implication is that cyclotron energies do not depend on particle mass. Furthermore, Schwartschild radius $r_S = .9$ cm of Earth defines universal gravitational Compton length for $\beta_0 = 1/2$.

Assume that $\hbar_{gr}$ corresponds to Earth mass and $\beta_0 = 1/2$ and consider cyclotron states in $B_{end} = .2$ Gauss.

1. The value of $r = \hbar_{gr}/\hbar$ for proton is given as the ratio r_s/L_p, where L_p is the Compton radius of proton. This gives $r = .833 \times 10^{13}$. For ions with mass number A the value of r is scaled to Ar.

2. What is the cyclotron energy associated with the 10 Hz frequency in this case? The energy of a photon with frequency f is for $\hbar_{gr}(m_p)$ given by $E_c/eV = r \times 1.24 \times (f/(3 \times 10^{14} Hz)$. Proton's cyclotron frequency is $f_c = 300$ Hz in B_{end} and corresponds to 10 eV, which is in the UV region and rather large.

3. All cyclotron frequencies of charged particles correspond to $E_c = 10$ eV cyclotron energy, which seems rather large. If $\hbar_{gr}$ is reduced by factor 1/4 as required to explain the findings of Mills at least partially, the cyclotron energy becomes 2.5 eV, which is in the visible range. Scaling by factor 1/2 gives cyclotron energy 5 eV in UV.

4. Smaller values of E_c would require smaller fields. The Z_0 charge of proton is roughly a fraction 1/50 of its em charge and since Kähler field contributes also to Z^0 field one would obtain energy about .2 eV in the IR region.

10 Hz alpha frequency which is of special interest concerning understanding of conscious experience and it is interesting to look for concrete numbers.

1. $f = 10$ Hz is alpha frequency and the cyclotron frequency $f_c = 10$ eV Fe^{2+} ion with mass number $A = 56$. Fe^{2+} ions play a central role in biology.

2. For $f = 10$ Hz the energy $\hbar_{gr}(m_p)$ (proton) is .333 eV to be compared with the metabolic energy currency $\sim .5$ eV and is below the visible range.

3. In the TGD inspired biology, 3 proton units represent dark genetic codons and for $\hbar_{gr}(3m_p)$ the energy corresponds to $E \times 1$ eV, which is still slightly below the visible range [L178, L118, L133]. In the dark variant of double DNA strand parallel to the ordinary double strand, the 2 dark codons form a pair by the dark variant of the base pairing so that one has effective $A06$ and $E = 2$ eV, which corresponds to red light.

4. The energy $E = 2$ eV of the codon pair for $f = 10$ Hz corresponds formally to $A = 6$ and would characterize ^6Li. Litium's cyclotron frequency is around $f_c = 50$ Hz is known to have biological significance. Li is used in the treatment of depression [L63]. One might imagine that the coupling of Lithium to dark codon pairs might be involved.

5. For higher mass numbers, the energies for 10 Hz and $\hbar_{gr}(Am_p)$ belong to the UV region. For oxygen one with $A = 16$ has $E = 5.3$ eV, which could correspond to some important molecular transition energy. Molecular bond dissociation energies (`https://cutt.ly/3QoZxY9`) vary in the range .03 -10 eV. O-H, O=O ad O=CO bond energies are somewhat above 5 eV. The idea indeed is that the transformation of dark photons to ordinary bio-photons allows a control of molecular biochemistry.

6. DNA codons have charge proportional to mass and in a good approximation one has $f_c(DNA) = 1$ Hz independently of the length of the DNA strand. For $\hbar_{gr}(Fe^{++})$ $f_c(DNA)$ would correspond to $E = 1.86$ eV in the range of visible energies.

15.3.2 Trying to understand $\hbar_{eff}$ and $\hbar_{gr}$

Although $\hbar_{eff}$and $\hbar_{gr}$ have become an essential part of quantum TGD, there are still many poorly understood aspects related to them.

Should one introduce a hierarchy of poly-local Planck constants?

The ordinary Planck constant is a universal constant and single-particle entity and serves as a quantization unit for local charges. $\hbar_{gr}$ depends on the masses of the members of the interacting systems and a bi-local character. This suggests that one should not mix these notions.

Both $\hbar_{gr}$ and its possible generalization to gauge interactions such as $\hbar_{em}$, would depend on the charges of the interacting particles. If they serve as charge units, the charges must be bilocal.

Should one introduce a hierarchy of poly-local Planck constants? Later a possible interpretation in terms of Yangian symmetries [A25] [B36, B22], which involve poly-local charges, will be considered. Each multi-local contribution to charge would involve its own Planck constant determined number theoretically.

Standard quantization rules for observables use $\hbar$ as a basic unit. Should one modify these rules by replacing $\hbar$ with (say) $\hbar_{em} = q_1q_2e^2/\alpha$ for $q_1q_2\alpha \geq 1$? Could these rules hold true at magnetic flux tubes characterized by $\hbar_{em}$? Could the charge units for the matter in the non-perturbative phase be q_1q_2-multiples of the ordinary basic units? Could one find empirical evidence for the scaling up of the quantization unit in non-perturbative phases?

In order to avoid total confusion, one must distinguish clearly between the single particle Planck constant and its 2-particle and n-particle variants as Yangian picture suggests. One must also distinguish between p-adic CCE as a discrete counterpart of ordinary CCE and dark coupling constant evolution.

The counterpart of $\hbar_{gr}$ for gauge interactions

The gauge couplings g_i for various interactions disappear completely from the basic formulation of TGD since they are automatically absorbed into the definition of the induced gauge potentials. Hence $\beta_0/4\pi \equiv \alpha_K$ appears as a coupling parameter in the perturbative expansion based on the exponent of Kähler function. $\hbar$ or $\hbar_{eff}$ appear as charge unit only in the definition of conserved charges as Noether charges but not in the action exponential.

The generalization of the Nottale formula to other interactions is not quite obvious. Two-particle Planck constant $h_{eff}(2)$ is in question and GMm would be replaced with the product $q_1q_2g_i^2$. Since α_K determines all other coupling strengths so that it is enough to consider it.

The parameter $\beta_0/4\pi$ is analogous to fine structure constant since gravitational perturbative expansion is in powers of it [K101] [L137].

β_0 is the gravitatoinal counterpart of the dimensionless coupling strength α_K defined in the QFT framework as a derived quantity $\alpha_K = g_K^2/4\pi$ but identified in the TGD context as the fundamental parameter appearing in Kähler action.

In TGD e does not appear as gauge coupling at the fundamental level (as opposed to QFT limit) but one can *define* e^2 as $e^2 = 4\pi\alpha\hbar$. α would obey p-adic CCE and $\hbar$ would be universal constant at single particle level. For dark phases, for which one has $h_{eff} > h$, $\alpha(1) \propto 2/n$, n dimension of the extension would hold true.

Consider the analog of the Nottale formula for em interactions. The coupling strength would be $q_1q_2e^2$ and for $q_1q_2e^2\alpha > 1$, one would have

$$\hbar_{em}(2) = \frac{q_1 q_2 e^2}{\beta_0} \ . \tag{15.3.3}$$

This would give $\alpha_{em}(2) = \beta_0$. For $\beta_0 = \alpha$, one would obtain a coupling parameter α instead of $q_1 q_2 \alpha$ and the interpretation would be in terms of a transition to non-perturbative phase.

Does this phase transition correspond to a transition to dark phase? Could one interpret the phase transition by saying the dimension of extension is scaled by $n = \hbar_{em}(2)/\hbar$ identified as scaling of the dimension of extension of rationals?

Number theoretic vision predicts that in the dark evolution h_{eff} scales as n, the dimension of extension of rationals for all values of particle number in the definition of $h_{eff}(h)$ so that the single particle coupling constant strength would behave like $1/n$.

Charge fractionalization and the value of $\hbar_{eff}$

$\hbar_{eff} < \hbar$ implies charge fractionalization at the level of embedding space. This inspires the question whether an analog of fractional quantum Hall effect could be in question. This is not the case.

1. The TGD based model for anyons [K85] relies on the observation that the unit for the fractional quantization of transverse conductance in fractional quantum Hall effect (FQHE) as

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{15.3.4}$$

   The proposal is that FQHE could be understood as integer quantum Hall effect corresponding to $n \to kn$ for $h_{eff} = km\hbar$. $k = 1$ is the simplest possibility. Interestingly, the observed values of m are primes [D1]: they would correspond to simple Galois groups Z_p in the TGD framework.

2. The fractionalization of charges could be understood at space-time level by noticing that n-sheetedness can be realized as analog of analytic function $z^{1/n}$. n full 2π turns are needed to return to the original point at space-time level so that it is possible to have fractional spin as multiples of $\hbar/n$. The many-particle states however have half-integer spin always since they correspond to representations of the Lorenz group as a symmetry group of $M^4 \times CP_2$. The action of rotations by multiples of 2π would correspond to the action of the Galois group.

   These two apparently conflicting mechanisms of charge fractionization correspond to two views about symmetries: either their act at the level of the embedding space or of space-time.

3. For $GMm/v_0 < \hbar$ one would have formally $\hbar_{eff} < \hbar$. Could this option make sense and give rise to a charge fractionalization? One can argue that for $\hbar_{gr} < \hbar_0$, $\hbar_0$ serves as the quantization unit and holds at the level of ordinary matter. This would give a condition $GMm \leq \beta_0$ to the product Mm of the masses involved.

   A stronger condition would hold true at single particle level and state $M/M_{Pl} \geq \sqrt{\beta_0}$ (or $M/M(CP_2) \geq \sqrt{\beta_0}$) for both masses involved. Dark quantum gravity would hold true only above Planck masses. In applications to elementary particle level this would require quantum coherent states of particles with total mass not smaller than Planck mass. Interestingly, a water blob with the size of a large cell has this size for $\beta_0 = 1/2$ [L100].

What does the dependence of $\hbar_{gr}$ on particle masses mean?

$\hbar_{gr}$ depends on two masses. How could one interpret this geometrically?

1. The interpretation has been that a particle with energy E (and mass m) experiences the gravitational field of mass M via gravitational flux tubes characterized by $h_{gr} = GME/v_0$ so that every particle has its specific gravitational flux tubes.

2. Could the thickness of the gravitational flux tubes correspond to the ordinary Compton length λ_c or gravitational Compton length $\lambda_{gr} = GM/v_0$? λ_c decreases with mass and λ_{gr} looks a more reasonable option concerning gravitational interaction.

3. At least static gravitational fields are analogous to static electric fields and in many-sheeted space-time the voltages as analogs of gravitational potential difference are the same along different space-time sheets. The same should hold for gravitational potential.

   Could one assume that gravitational potential has almost copies at all parallel sheets of the many-sheeted space-time (parallel with respect to M^4). Could these sheets correspond to different particle masses so that a particle with a given mass would have its own space-time sheet to represent its interactions with the central mass M.

4. These classical fields would be somehow represented by Kähler magnetic flux tubes carrying generalized Beltrami fields [B11, B60, B35, B41] having also an electric part.

   Could these flux tubes somehow also represent the classical gravitational field? Could the electric part for the induced M^4 Kähler form predicted by the twistor lift of TGD [L55, L72] giving rise to CP breaking, give a representation for the gravitational potential? Could this concretely realize the analogy between gravitation and electromagnetism?

5. A possible realization of this picture would be a fractal structure consisting of flux tubes within flux tubes emanating from the central mass. The radii of the flux tubes would decrease with m as long as $GMm/\beta > \hbar$ holds true. For smaller masses, the flux tube radius would correspond to Compton length.

   Fractal structures known as fractons (`https://cutt.ly/WRuXnrC`) are the recent hot topic of condensed matter physics (`https://cutt.ly/YQjqyjJ`. The explanation requires the replacement of the time evolution as a time translation with a scaling and condensed matter lattice would be replaced with fractal. These phases have exotic properties: in particular, thermal equilibrium need not be possible. There are also long range correlations due to fractality, which makes these phases ideal for quantum computation.

   In the TGD framework, the time evolutions between SSFRs are indeed generated by the scaling operator L_0 of super-conformal algebra and many-sheeted space-time is both p-adic and dark fractal. The hierarchy of Planck constants makes possible quantum coherence in all scales.

Yangian symmetry and poly-local Planck constants

The product structure of $\hbar_{gr}$ and $\hbar_{em}$ has remained a mystery since it suggests that it characterizes the interaction of 2 space-time sheets whereas the ordinary Planck constant serves as a quantization unit for single particle states. Instead of a Galois group for a single space-time sheet, one would have a product of Galois groups for the two space-time sheets determined as roots for the polynomials in the product. Therefore one should write $\hbar_{gr} = \hbar_{eff,2}$ to distinguish it from a single particle Planck constant $\hbar_{eff}(1) \equiv \hbar_{eff}$.

1. In the TGD framework, wormhole contacts connecting two space-time sheets with Minkowskian signature are indeed building bricks of elementary particles and fundamental fermions appearing as building bricks of elementary particles would be associated with the throats of the wormhole contact.

   Could the two Minkowskian sheets be microscopically k-sheeted entities with sheets parallel to M^4 and perhaps determined as roots of a polynomial of degree k and having Galois group with order m? The maximal Galois group would be S_k with $m = k!$.

The scaling of $\hbar_0 \rightarrow \hbar(2)$ would mean that the pairs of these space-time surface sheets decompose to $\hbar(2)/\hbar_0$ pairs as orbit of $Gal \times Gal$ contributing to various quantum numbers a contribution proportional $\hbar_{gr}(2)/\hbar_0 = n_1 n_2 = k_1 k_2 m^2$.

The quantization unit would be $\hbar_0(2)$ for 2-particle quantities such as relative angular momentum. Spin is however thought to be single particle observable. The ordinary phase has a single-particle Planck constant as $\hbar(1)/\hbar_0 = m$.

2. There is no obvious reason for excluding the values of single particle $h_{eff}(1)/h_0$, which are considerably smaller than m or even equal to the minimal value $h_{eff}(1) = h_0$: they would correspond to Galois groups with smaller orders than $m = k!$ of say S_k.

   These exotic particles would have charge and spin units considerably smaller than $\hbar = mh_0$. Why have they not been observed (the findings of Mills are a possible exception and anyonic charge fractionization seems to be a different phenomenon)? Are these space-time sheets somehow unstable? Does gravitation somehow select the Galois group of stable ground state space-time surface so that R as a fundamental length scale is replaced with l_P as effective fundamental length?

3. Yangian algebras [A25] [B36, B22] involve besides single particle observables also $n > 1$-particle observables. Conserved charges have poly-local components which depend on n particles. Note that interaction energy represented as a potential energy is the simplest example about non-local 2-particle contribution to conserved energy.

   Yangian algebras are proposed to be central for TGD [L45] and would reflect the replacement of the space-time locality with locality at the level of "world of classical worlds" (WCW) due to the replacement of a point like with a 3-surface, which can also consist of disjoint parts. Yangian picture suggests that single-particle $\hbar$ has n-particle generalization. The possible number theoretical rule could be

$$\frac{\hbar_{gr,n}}{\hbar_0} = \prod_k n_k \quad , \tag{15.3.5}$$

   where n_k correspond to the orders of Galois groups associated with the space-time sheets involved.

15.3.3 Do Yangians and Galois confinement provide $M^8 - H$ dual approaches to the construction of the many-particle states?

The construction of many-particle states as zero energy states defining scattering amplitudes and S-matrix is one of the basic challenges of TGD. TGD suggests two approaches implied by physics as geometry and physics as number theory views to TGD. Geometric vision suggests Yangians of the symmetry algebras of the "world of classical worlds" (WCW) at the level of $H = M^4 \times CP_2$. Number theoretic vision suggests Galois confinement at the level of complexified M^8. Could these approaches be $M^8 - H$ duals of each other?

Yangian approach

The states would be constructed from fermions and antifermions as modes of WCW spinor field. An idea taking the notion of symmetry to extreme is that this could be done purely algebraically using generators of symmetries.

Consider first the construction of TGD analogs of single particle states as representations of symmetries.

1. For a given vacuum state assignable to a partonic 2-surface and identifiable as a ground state of Kac-Moody type representation, the states would be generated by Kac-Moody algebra. Also super-Kac-Moody algebra could be used to construct states with nonvanishing fermion and antifermion numbers. In the case of super symplectic algebra the generators would correspond to super Noether charges form the isometries of WCW and would have both fermionic and might also have bosonic parts.

2. The spaces of states assignable to partonic 2-surfaces or to a connected 3-surface is however still rather restricted since it assumes in the spirit of reductionism that the symmetries are local single particle symmetries. The first guess for many-particle states in this approach is as free states and one must introduce interactions in an ad hoc manner and the problems of quantum field theories are well-known.

3. In the TGD framework there is a classical description of interactions in terms of Bohr-orbit like preferred extremals and one should generalize this to the quantum context using zero energy ontology (ZEO). Classical interactions have as space-time correlates flux tubes and "massless extremals" connecting 3-surfaces as particle and topological vertices for the partonic 2-surfaces.

4. The construction recipe of many-particle states should code automatically for the interactions and they should follow from the symmetries as a polylocal extension of single particle symmetries. They should be coded by the modification of the usual tensor product giving only free many-particle states. One would like to have interacting many-particle states assignable to disjoint connected 3-surfaces or many-parton states assignable to single connected space-time surfaces inside causal diamond (CD).

Yangian algebras are especially interesting in this respect.

1. Yangian algebras have a co-algebra structure allowing to construct multi fermion representations for the generators using comultiplication operation, which is analogous to the time reversal of a Lie-algebra commutator (super algebra anticommutator) regarded as interaction vertex with two incoming and one outgoing particle. The co-product is analogous to tensor product and assignable to a decay of a particle to two outgoing particles.

2. What is new is that the generators of Yangian are poly-local. The infinitesimal symmetry acts on several points simultaneously. For instance, they could allow a more advanced mathematical formulation for n-local interaction energy lacking from quantum field theories, in particular potential energy. The interacting state could be created by a bi-local generator of Yangian. The generators of Yangian can be generated by applying coproducts and starting from the basic algebra. There is a general formula expressing the relations of the Yangian.

3. Yangian algebras have a grading by a non-negative integer, which could count the number of 3-surfaces (say all connected 3-surfaces appearing at the ends of the space-time surface at the boundaries of causal diamond (CD)), or the number of partonic 2-surfaces for a given 3-surface. There would also be gradings with respect to fermion and antifermion numbers.

There are indications that Yangians could be important in TGD.

1. In TGD, the notion of Yangian generalizes since point-like particles correspond to disjoint 3-surfaces, for a given 3-surface to partonic 2-surfaces, and for a partonic 2-surface to point-like fermions and antifermions. In the TGD inspired biology, the notion of dark genes involves communications by n-resonance. Two dark genes with N identical codons can exchange cyclotron 3N-photon in 3N-resonance. Could genes as dark N-codons allow a description in terms of Yangian algebra with N-local vertex? Could one speak of 3N-propagators for 3N cyclotron-photons emitted by dark codons.

2. In quantum theory, Planck constant plays a central role in the representations of the Lie algebras of symmetries. Its generalization assignable to n-local Lie algebra generators could make sense for Yangians. The key physical idea is that Nature is theoretician friendly. When the coupling strength proportional to a product of total charges or masses becomes so large that perturbation series fails to converge, a phase transition increasing the value of h_{eff} takes place. Could this transition mean a formation of bound states describable in terms of poly-local generators of Yangian and corresponding poly-Planck constant?

   For instance, the gravitational Planck constant $\hbar_{gr}$, which is bilocal and proportional to two masses to which monopole flux tube is associated, could allow an interpretation in terms of Yangian symmetries and be assignable to a bi-local gravitational contribution to energy momentum. Also other interaction momenta could have similar Yangian contributions and characterized by corresponding Planck constants.

It is not clear whether $\hbar_{gr}$ and its generalization can be seen as a special case of the proposal $h_{eff} = nh_0$ generalizing the ordinary single particle Planck constant or whether it is something different. If so, the hierarchy of Planck constant would correspond to a hierarchy of polylocal generators of Yangian.

Galois confinement

The above discussion was at the level of $H = M^4 \times CP_2$ and "world of classical worlds" (WCW). $M^8 - H$ duality predicts that this description has a counterpart at the level of M^8. The number theoretic vision predicting the hierarchy of Planck constants strongly suggests Galois confinement as a universal mechanism for the formation of bound states of particles as Galois singlets.

1. The simplest formulation of Galois confinement states that the four-momenta of particles have components which are algebraic integers in the extension of rationals characterizing a polynomial defining a 4-surface in complexified M^8, which in turn is mapped to a space-time surface in $H = M^4 \times CP_2$, when the momentum unit is determined by the size of causal diamond (CD).

   The total momentum for the bound state would be Galois singlet so that its components would be ordinary integers: this would be analogous to the particle in box quantization. Each momentum component "lives" in n-dimensional discrete extension of rationals with coefficient group, which consists of integers.

   In principle one has a wave function in this discrete space for all momentum components as a superposition of Galois singlet states. The condition that total momentum is Galois singlet forces an entanglement between these states so that one does not have a mere product state.

2. Galois confinement poses strong conditions on many-particle states and forces entanglement. Could Galois confinement be $M^8 - H$ dual of the Yangian approach?

15.3.4 h/h_0 as the ratio of Planck mass and CP_2 mass?

Could one understand and perhaps even predict the value of h_0? Here number theory and the notion of n-particle Planck constant $h_{eff}(n)$ suggested by Yangian symmetry could serve as a guidelines.

1. Hitherto I have found no convincing empirical argument fixing the value of $r = \hbar/\hbar_0$: this is true for both single particle and 2-particle case.

   The value $h_0 = \hbar/6$ [L32] as a maximal value of $\hbar_0$ is suggested by the findings of Randell Mills [D3] and by the idea that spin and color must be representable as Galois symmetries so that the Galois group must contain $Z_6 = Z_2 \times Z_3$. Smaller values of h_0 cannot be however excluded.

2. A possible manner to understand the value r geometrically would be following. It has been assumed that CP_2 radius R defines a fundamental length scale in TGD and Planck length squared $l_P^2 = \hbar G = x^{-2} \times 10^{-6} R^2$ defines a secondary length scale. For Planck mass squared one has $m_{Pl}^2 = m(CP_2, \hbar)^2 \times 10^6 x^2$, $m(CP_2, \hbar)^2 = \hbar/R^2$. The estimate for x from p-adic mass calculations gives $x \simeq 4.2$. It is assumed that CP_2 length is fundamental and Planck length is a derived quantity.

   But what if one assumes that Planck length identifiable as CP_2 radius is fundamental and CP_2 mass corresponds the minimal value h_0 of $h_{eff}(2)$? That the mass formula is quadratic and mass is assignable to wormhole contact connecting two space-time sheets suggests in the Yangian framework that $h_{eff}(2)$ is the correct Planck constant to consider.

One can indeed imagine an alternative interpretation. CP_2 length scale is deduced indirectly from p-adic mass calculation for electron mass assuming $h_{eff} = h$ and using Uncertainty Principle. This obviously leaves the possibility that $R = l_P$ apart from a numerical constant near unity, if the value of h_{eff} to be used in the mass calculations is actually $h_0 = (l_P/R)^2\hbar$. This would fix the value of $\hbar_0$ uniquely.

The earlier interpretation makes sense if $R(CP_2)$ is interpreted as a dark length scale obtained scaling up l_P by $\hbar/\hbar_0$. Also the ordinary particles would be dark.

h_0 would be very small and $\alpha_K(\hbar_0) = (\hbar/\hbar_0)\alpha_K$ would be very large so that the perturbation theory for it would not converge. This would be the reason for why $\hbar$ and in some cases some smaller values of h_{eff} such as $\hbar/2$ and $\hbar/4$ [D3] [L32] seem to be realized.

For $R = l_P$ Nottale formula remains unchanged for the identification $M_P = \hbar/R$.

For $R = l_P$ Nottale formula remains unchanged for the identification $M_P = \hbar/R$ (note that one could consider also $\hbar_0/R^2$ as natural unit of mass squared in the p-adic mass calculations).

Various options

Number theoretical arguments allow to deduce precise value for the ratio $\hbar/\hbar_0$. Accepting the Yangian inspired picture, one can consider two options for what one means with $\hbar$.

1. $\hbar$ refers to the single particle Planck constant $\hbar_{eff}(1)$ natural for point-like particles.

2. $\hbar$ refers to $h_{eff}(2)$. This option is suggested by the proportionality $M^2 \propto \hbar$ in string models due to the proportionality $M^2 \propto \hbar/G$ in string models. At a deeper level, one has $M^2 \propto L_0$, where L_0 is a scaling generator and its spectrum has scale given by $\hbar$.

   Since M^2 is a p-adic thermal expectation of L_0 in the TGD framework, the situation is the same. This also due the fact that one has In TGD framework, the basic building bricks of particles are indeed pairs of wormhole throats.

One can consider two options for what happens in the scaling $h_{eff} \rightarrow kh_{eff}$.

Option 1: Masses are scaled by k and Compton lengths are unaffected.

Option 2: Compton lengths are scaled by k and masses are unaffected.

The interpretation of $M_P^2 = (\hbar/\hbar_0)M^2(CP_2)$ assumes Option 1 whereas the new proposal would correspond to Option 2 actually assumed in various applications.

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For Option 1 $m_{Pl}^2 = (\hbar_{eff}/\hbar)M^2(CP_2)$. The value of $M^2(CP_2) = \hbar/R^2$ is deduced from the p-adic mass calculation for electron mass. One would have $R^2 \simeq (\hbar_{eff}/\hbar)l_P^2$ with $\hbar_{eff}/\hbar = 2.54 \times 10^7$. One could say that the real Planck length corresponds to R.

Quantum-classical correspondence favours Option 2)

In an attempt to select between these two options, one can take space-time picture as a guideline. The study of the embeddings of the space-time surfaces with spherically symmetric metric carried out for almost 4 decades ago suggested that CP_2 radius R could naturally correspond to Planck length l_P. The argument is described in detail in Appendix and shows that the $l_P = R$ option with $h_{eff} = h$ used in the classical theory to determine α_K appearing in the mass formula is the most natural.

Deduction of the value of $\hbar/\hbar_0$

Assuming Option 2), the questions are following.

1. Could $l_P = R$ be true apart from some numerical constant so that CP_2 mass $M(CP_2)$ would be given by $M(CP_2)^2 = \hbar_0/l_P^2$, where $\hbar_0 \simeq 2.4 \times 10^{-7}\hbar$ ($\hbar$ corresponds to $\hbar_{eff}(2)$) is the minimal value of $\hbar_{eff}(2)$. The value of h_0 would be fixed by the requirement that classical theory is consistent with quantum theory! It will be assumed that $\hbar_0$ is also the minimal value of $\hbar_{eff}(1)$ both $\hbar_{eff}(2)$.

2. Could $\hbar(2)/\hbar_0(2) = n_0$ correspond to the order of the product of identical Galois groups for two Minkowskian space-time sheets connected by the wormhole contact serving as a building brick of elementary particles and be therefore be given as $n_0 = m^2$?

Assume that one has $n_0 = m^2$.

1. The natural assumption is that Galois symmetry of the ground state is maximal so that m corresponds to the order a maximal Galois group - that is permutation group S_k, where k is the degree of polynomial.

   This condition fixes the value k to $k = 7$ and gives $m = k! = 7! = 5040$ and gives $n_0 = (k!)^2 = 25401600 = 2.5401600 \times 10^7$. The value of $\hbar_0(2)/\hbar(2) = m^{-2}$ would be rather small as also the value of $\hbar_0(1)\hbar(1)$. p-Adic mass calculations lead to the estimate $m_{Pl}/m(CP_2) = \sqrt{m}m(CP_2) = 4.2 \times 10^3$, which is not far from $m = 5040$.

2. The interpretation of the product structure $S_7 \times S_7$ would be as a failure of irreducibility so that the polynomial decomposes into a product of polynomials - most naturally defined for causally isolated Minkowskian space-time sheets connected by a wormhole contact with Euclidian signature of metric representing a basic building brick of elementary particles.

   Each sheet would decompose to 7 sheets. $\hbar_{gr}$ would be 2-particle Planck constant $h_{eff}(2)$ to be distinguished from the ordinary Planck constant, which is single particle Planck constant and could be denoted by $h_{eff}(1)$.

   The normal subgroups of $S_7 \times S_7$ $S_7 \times A_7$ and $A_7 \times A_7$, S_7, A_7 and trivial group. A_7 is simple group and therefore does not have any normal subgroups expect the trivial one. S_7 and A_7 could be regarded as the Galois group of a single space-time sheet assignable to elementary particles. One can consider the possibility that in the gravitational sector all EQs are extensions of this extension so that $\hbar$ becomes effectively the unit of quantization and m_{Pl} the fundamental mass unit. Note however that for very small values of α_K in long p-adic length scales also the values of $h_{eff} < h$, even h_0, are in principle possible.

   The large value of $\alpha_K \propto 1/\hbar_{eff}$ for Galois groups with order not considerably smaller than $m = (7!)^2$ suggests that very few values of $h_{eff}(2) < h$ are realized. Perhaps only $S_7 \times S_7$ $S_7 \times A_7$ and $A_7 \times A_7$ are allow by perturbation theory. Now however that in the "stringy phase" for which super-conformal invariance holds true, h_0 might be realized as required by p-adic mass calculations. The alternative interpretation is that ordinary particles correspond to dark phase with R identified dark scale.

3. A_7 is the only normal subgroup of S_7 and also a simple group and one has $S_7/A_7 = Z_2$. $S_7 \times S^7$ has $S_7 \times S^7/A_7 \times A_7 = Z_2 \times Z_2$ with $n = n_0/4$ and $S_7 \times S^7/A_7 \times S_7 = Z_2$ with $n = n_0/2$. This would allow the values $\hbar/2$ and $\hbar/4$ as exotic values of Planck constant.

   The atomic energy levels scale like $1/\hbar^2$ and would be scaled up by factor 4 or 16 for these two options. It is not clear whether $\hbar \to \hbar/2$ option can explain all findings of Randel Mills [D3] in TGD framework [L32], which effectively scale down the principal quantum number n from n to $n/2$.

4. The product structure of the Nottale formula suggests

   $$n = n_1 \times n_2 = k_1 k_2 m^2 \quad . \tag{15.3.6}$$

   Equivalently, n_i would be a multiple of m. One could say that $M_{Pl} = \sqrt{\hbar/\hbar_0}M(CP_2)$ effectively replaces $M(CP_2)$ as a mass unit. At the level of polynomials this would mean that polynomials are composites $P \circ P_0$ where P_0 is ground state polynomial and has a Galois group with degree n_0. Perhaps S_7 could be called the gravitational or ground state Galois group.

15.3.5 Connection with adelic physics and infinite primes

The structure of $\hbar_{gr}$ and its electromagnetic counterpart $\hbar_{em}$ characterize 2-particle states whereas $\hbar$ characterizes single particle state. Yangian picture suggests that the notion of $\hbar_{eff}(n)$, $n = 1, 2, ..$ makes sense.

One can decompose a state consisting of N particles in several ways to partitions consisting of m subsets with n_i, $i = 1, ..., n$ in a given subset of particles. Could these subsets correspond to gravitationally bound states so that one can take these sets as basic entities characterized by masses and assume that gravitational interactions reduce to gravitational interactions between them and are quantal for $GM_iM_j/v_0 \geq \hbar$. Same question applied to electromagnetic, weak and color interactions.

Connection with adelic physics

This picture would have analog at the level of adelic physics [L109, L110, L134].

1. In the M^8 picture space-time surfaces correspond to "roots" of complexified octonionic polynomials obtained from irreducible real polynomials with rational (or perhaps even algebraic) coefficients. The dynamics realizes associativity of the normal space of the complexified space-time surface having 4-D space-time surface as real part mapped from M^8 to $H = M^4 \times CP_2$ by $M^8 - H$ correspondence.

2. One can consider irreducible polynomials of several variables such that the additional variables are interpreted as parameters [L128]. The parametrized set of polynomials defines a parametrized set of space-time surfaces and one can have a superposition of quantum states corresponding to irreducible polynomial of degree n and products of irreducible polynomials with sum of degrees n_i equal to n. This kind of parametrized set could define sub-spaces of the "world of classical worlds" (WCW).

3. Irreducibility fails for some parameter values forming lower-dimensional manifolds of the parameter space. The failure of the irreducibility means decomposition to a product of polynomials in which the set of roots decomposes to subsets R_i, which are roots of a rational polynomial with a lower degree n_i. Spacetime surface as a coherent structure decomposes to uncorrelated space-time surfaces with a discrete set of points as intersections. In this manner one obtains a decomposition of the parameter space to subsets of decreasing dimension. The generic situation has maximal dimension and dimension equal to that of the parameter space.

4. The catastrophe theory [A50] founded by Rene Thom studies these situations. In catastrophe theory, the failure of the irreducibility is of very special nature and means that some roots of the polynomial co-incide and become multiple roots. For polynomials with rational coefficients, they would become multiple rational roots so that the degree of the polynomial determining the extension would be reduced by two units. This is discussed in detail from TGD point of view in [L128]. For polynomials with rational coefficients, typically complex conjugate roots become rational and the dimension of the algebraic extension is reduced.

5. The quantum state defined by the polynomial of several variables would be a superposition of space-time surfaces labelled by the points of the parameter space. It would decompose to subsets defining what is known as a stratification. The subsets for which the polynomial fails to be irreducible would have lower dimension. For polynomials with rational coefficients these sets would be discrete and it is not clear whether the lower-dimensional sets are non-empty in the generic case.

6. The decomposition to k irreducible polynomials with degrees n_i, $i = 1, ..., k$ would correspond to a decomposition of the space-time surface to separate space-time surfaces with $h_{gr,i} = n_i h_0 = GM_i m/v_0$ (same applies to h_{em}) satisfying $\sum n_i = n$. These would correspond to different decompositions of the total energy to a sum of energies E_i: $E = \sum E_i$. The irreducible polynomials with degree n_i could be interpreted as bound states for a subset of basic units. Maximal decomposition would correspond to $n_i = 1$ and have interpretation as a set of elementary particles with $h_{eff} = h_0$ (note that $h = 6h_0$ in the proposal inspired by the findings of Randel Mills [L32]).

Connection with infinite primes

The notion of infinite prime [K105] resonates with this picture.

1. The hierarchy of infinite primes has an interpretation as a repeated second quantization of supersymmetric arithmetic QFT. Polynomial primes of variable polynomials of single variable with rational coefficients follow ordinary primes in the hierarchy. Higher levels correspond to polynomial primes for polynomials of several variables and second quantization corresponds to the formation of polynomials of single variable with coefficients as polynomials of $n-1$ variables.

   Irreducible polynomials of higher than first order have interpretation as bound states whereas polynomials reducing to products of monomials correspond to Fock states of free particles.

2. The beatiful feature would be a number theoretic description of also bound states. The description of the particle decays as a failure of the irreducibility of the polynomials corresponding to infinite primes would extend this picture to the dynamics.

3. Second beautiful feature is the number theoretic description of particle reactions. Particle reactions with unentagled final states would naturally correspond to a situation in which the initial (prepared) and final (state function reduced) states are products of polynomials. Interaction period would correspond to an irreducible polynomial.

   This picture conforms with the proposal inspired originally by a model of "cold fusion". unnelling phenomenon crucial for nuclear reactions would correspond to a formation of dark phase in which the value of h_{eff} increases [L89, L44, L114]. This picture generalizes to all particle reactions.

15.4 How to understand coupling constant evolution?

In this section, the evolutions of Kähler coupling strength α_K and gravitational fine structure constant α_{gr} are discussed. The reason for restricting to α_K is that it is expected to induce the evolution of various gauge couplings, and could also induce the evolution of α_{gr}.

15.4.1 Evolution of Kähler coupling strength

The evolution of Kähler coupling strength $\alpha_K = g_K^2/2h_{eff}$ gives the evolution of α_K as a function of dimension n of EQ: $\alpha_K = g_K^2/2nh_0$. If g_K^2 corresponds to electroweak U(1) coupling, it is expected to evolve also with respect to PLS so that the evolutions would factorize.

Note that the original proposal that g_K^2 is renormalization group invariant was later replaced with a piecewise constancy: α_K has indeed interpretation as piecewise constant critical temperature

1. In the TGD framework, coupling constant as a continuous function of the continuous length scale is replaced with a function of PLS so that coupling constant is a piecewise constant function of the continuous length scale.

   PLSs correspond to p-adic primes p, and a hitherto unanswered question is whether the extension determines p and whether p-adic primes possible for a given extension could correspond to ramified primes of the extension appearing as factors of the moduli square for the differences of the roots defining the space-time surface.

   In the M^8 picture the moduli squared for differences $r_i - r_j$ of the roots of the real polynomial with rational coefficients associated with the space-time surfaces correspond to energy squared and mass squared. This is the case of p-adic prime corresponds to the size scale of the CD.

   The scaling of the roots by constant factor however leaves the number theoretic properties of the extension unaffected, which suggests that PLS evolution and dark evolution factorize in the sense that PLS reduces to the evolution of a power of a scaling factor multiplying all roots.

2. If the exponent $\Delta K/log(p)$ appearing in $p^{\Delta K/log(p))} = exp(\Delta K)$ is an integer, $exp(\Delta K)$ reduces to an integer power of p and exists p-adically. If ΔK corresponds to a deviation from the Kähler function of WCW for a particular path in the tree inside CD, p is fixed and $exp(\Delta K)$ is integer. This would provide the long-sought-for identification of the preferred

p-adic prime. Note that p must be same for all paths of the tree. p need not be a ramified prime so that the trouble-some correlation between n and ramified prime defining padic prime p is not required.

3. This picture makes it possible to understand also PLS evolution if ΔK is identified as a deviation from the Kähler function. $p^{\Delta K/log(p))} = exp(\Delta K)$ implies that ΔK is proportional to $log(p)$. Since ΔK as 6-D Kähler action is proportional to $1/\alpha_K$, $log(p)$-proportionality of ΔK could be interpreted as a logarithmic renormalization factor of $\alpha_K \propto 1/log(p)$.

4. The universal CCE for α_K inside CDs would induce other CCEs, perhaps according to the scenario based on Möbius transformations [L70].

Dark and p-adic length scale evolutions of Kähler coupling strength

The original hypothesis for dark CCE was that $h_{eff} = nh$ is satisfied. Here n would be the dimension of EQ defined by the polynomial defining the space-time surface $X^4 \subset M^8_c$ mapped to H by $M^8 - H$ correspondence. n would also define the order of the Galois group and in general larger than the degree of the irreducible polynomial.

Remark: The number of roots of the extension is in general smaller and equal to n for cyclic extensions only. Therefore the number of sheets of the complexified space-time surface in M^8_c as the number of roots identifiable as the degree d of the irreducible polynomial would in general be smaller than n. n would be equal to the number of roots only for cyclic extensions (unfortunately, some former articles contain the obviously wrong statement $d = n$).

Later the findings of Randell Mills [D3], suggesting that h is not a minimal value of h_{eff}, forced to consider the formula $h_{eff} = nh_0$, $h_0 = h/6$, as the simplest formula consistent with the findings of Mills [L32]. h_0 could however be a multiple of even smaller value of h_{eff}, call if h_0 and the formula $h_0 = h/6$ could be replaced by an approximate formula.

The value of $h_{eff} = nh_0$ can be understood by noticing that Galois symmetry permutes "fundamental regions" of the space-time surface so that action is n times the action for this kind of region. Effectively this means the replacement of α_K with α_K/n and implies the convergence of the perturbation theory. This was actually one of the basic physical motivations for the hierarchy of Planck constants. In the previous section, it was argued that $\hbar/h_0$ is given by the ratio R^2/l_P^2 with R identified as dark scale equals to $n_0 = (7!)^2$.

The basic challenge is to understand p-adic length scale evolutions of the basic gauge couplings. The coupling strengths should have a roughly logarithmic dependence on the p-adic length scale $p \simeq 2^{k/2}$ and this provides a strong number theoretic constraint in the adelic physics framework.

Since Kähler coupling strength α_K induces the other CCEs it is enough to consider the evolution of α_K.

p-Adic CCE of α from its value at atomic length scale?

If one combines the observation that fine structure constant is rather near to the inverse of the prime $p = 137$ with PLS, one ends up with a number theoretic idea leading to a formula for α_K as a function of p-adic length scale.

1. The fine structure constant in atomic length scale $L(k = 137)$ is given $\alpha(k) = e^2/2h \simeq 1/137$. This finding has created a lot of speculative numerology.

2. The PLS $L(k) = 2^{k/2}R(CP_2)$ assignable to atomic length scale $p \simeq 2^k$ corresponds to $k = 137$ and in this scale α is rather near to $1/137$. The notion of fine structure constant emerged in atomic physics. Is this just an accident, cosmic joke, or does this tell something very deep about CCE?

   Could the formula

$$\alpha(k) = \frac{e^2(k)}{2h} = \frac{1}{k}$$

   hold true?

There are obvious objections against the proposal.

1. α is length scale dependent and the formula in the electron length scale is only approximate. In the weak boson scale one has $\alpha \simeq 1/127$ rather than $\alpha = 1/89$.

2. There are also other interactions and one can assign to them coupling constant strengths. Why electromagnetic interactions in electron Compton scale or atomic length scales would be so special?

The idea is however plausible since beta functions satisfy first order differential equation with respect to the scale parameter so that single value of coupling strength determines the entire evolution.

p-Adic CCE from the condition $\alpha_K(k = 137) = 1/137$

In the TGD framework, Kähler coupling strength α_K serves as the fundamental coupling strength. All other coupling strengths are expressible in terms of α_K, and in [L70] it is proposed that Möbius transformations relate other coupling strengths to α_K. If α_K is identified as electroweak $U(1)$ coupling strength, its value in atomic scale $L(k = 137)$ cannot be far from 1/137.

The factorization of dark and p-adic CCEs means that the effective Planck constant $h_{eff}(n, h, p)$ satisfies

$$h_{eff}(n, h, p) = h_{eff}(n, h) = nh \quad . \tag{15.4.1}$$

and is independent of the p-adic length scale. Here n would be the dimension of the extension of rationals involved. $h_{eff}(1, h, p)$ corresponding to trivial extension would correspond to the p-adic CCE as the TGD counterpart of the ordinary evolution.

The value of h need not be the minimal one as already the findings of Randel Mills [D3] suggest so that one would have $h = n_0h_0$.

$$h_{eff} = nn_0h \quad , \quad \alpha_{K,0} = \frac{g^2_{K,max}}{2h_0} = n_0 \quad . \tag{15.4.2}$$

This would mean that the ordinary coupling constant would be associated with the non-trivial extension of rationals.

Consider now this picture in more detail.

1. Since dark and p-adic length scale evolutions factorize, one has

$$\alpha_K(n) = \frac{g^2_K(k)}{2h_{eff}} \quad , \quad h_{eff} = nh_0 \quad . \tag{15.4.3}$$

   $U(1)$ coupling indeed evolves with the p-adic length scale, and if one assumes that $g^2_K(k, n_0)$ $(h = n_0h_0)$ is inversely proportional to the logarithm of p-adic length scale, one obtains

$$\begin{array}{l} g^2_K(k, n_0) = \frac{g^2_K(max)}{k} \quad , \\ \alpha_K = \frac{g^2_K(max)}{2kh_{eff}} \quad . \end{array} \tag{15.4.4}$$

2. Since $k = 137$ is prime (here number theoretical physics shows its power!), the condition $\alpha_K(k = 137, h_0) = 1/137$ gives

$$\frac{g^2_K(max)}{2h_0} = \alpha_K(max) = (7!)^2 \quad . \tag{15.4.5}$$

The number theoretical miracle would fix the value of $\alpha_K(max)$ to the ratio of Planck mass and CP_2 mass $n_0 = M_P^2/M^2(CP_2) = (7!)^2$ if one takes the argument of the previous section seriously.

The convergence of perturbation theory could be possible also for $h_{eff} = h_0$ if the p-adic length scale $L(k)$ is long enough to make $\alpha_K = n_0/k$ small enough.

3. The outcome is a very simple formula for α_K

$$\alpha_K(n,k) = \frac{n_0}{kn} \quad , \tag{15.4.6}$$

$$\tag{15.4.7}$$

which is a testable prediction if one assumes that it corresponds to electroweak $U(1)$ coupling strength at QFT limit of TGD. This formula would give a practically vanishing value of α_K for very large values of n associated with h_{gr}. Here one must have $n > n_0$.

For $h_{eff} = nn_0h$ characterizing extensions of extension with $h_{eff} = h$ one can write

$$\alpha_K(nn_0,k) = \frac{1}{kn} \quad . \tag{15.4.8}$$

4. The almost vanishing of α_K for the very large values of n associated with $\hbar_{gr}$ would practically eliminate the gauge interactions of the dark matter at gravitational flux tubes but leave gravitational interactions, whose coupling strength would be $\beta_0/4\pi$. The dark matter at gravitational flux tubes would be highly analogous to ordinary dark matter.

15.4.2 The evolution of the gravitational fine structure constant

Nottale [E1] introduced the notion of gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ ($\beta_0 = v_0/c$ is velocity parameter), which has gigantic values so that the original proposal $h_{gr} = nh_0$ would predict very large values for n. If p-adic and dark evolutions are independent this is not a problem since p-adic length scales need not be gigantic.

Evolution of the parameter β_0

Gravitational fine structure constant is given by $\alpha_{gr} = GMm/4\pi\hbar_{gr} = \beta_0/4\pi$. The basic challenge is to understand the value spectrum of β_0.

1. Kepler's law $\beta^2 = GM/r = r_S/2r$ suggests length scale evolution of form

$$\beta_N = \sqrt{\frac{r_S}{2L(N)x}} = \frac{\beta_{0,max}}{N} \quad . \tag{15.4.9}$$

The coefficient x has been included in the formula because otherwise a conflict with Bohr model for planetary orbits results.

2. How to identify N?

   (a) $N = n = h_{gr}/h_0$ would give a gigantic value of N and this would give extremely small value for β_0. Actually $N = n$ for n in $h_{gr} = nh_0$ is impossible as is clear from the defining equation.

   (b) It is not clear whether N be identified as a dimension for some factor in the composition of extension to simple factors rather than as n. This would conform with the vision that there are evolutionary hierarchies of extensions of extensions of... for which the dimension is product of dimensions of the extensions involved.

(c) The simplest option is that p-adic length scale evolution determines N as in case of the gauge interactions, and it corresponds to k in $p \simeq 2^k$. $log_2(p)$ exists also for a general prime p in real sense. In p-adic sense it exists for all primes except $p = 2$ as integer valued function. $p = 2$ could be chosen to be the exceptional prime.

This would conform with the idea that gravitational sector and gauge interaction sector correspond to different factors in the decomposition of extension of rationals. Perhaps the gravitational part of EQ extends its gauge part. This would conform with the idea that gravitation does not differentiate between states with different gauge quantum numbers.

What can one say about the value of $\beta_{0,max}$ and its length scale evolution?

1. The value of $\beta_{0,max} = 1/2$ would give for the length scale $L = GM/\beta_{0,max} = r_S$. If one requires that the scale L is not smaller than Scwartschild radius, $\beta_{0,max} \leq 1/2$ follows. $\beta_{0,max} = 1/2$ is the first guess but it turns that number theoretical constraintss exclude it and suggest $\beta_{0,max} = \pi/6$ as the simplest guess.

2. Gravitational Bohr radius a_{gr} given by

$$a_{gr} = \frac{\hbar_{gr}}{\alpha_{gr} m} per. \quad (15.4.10)$$

defines a good candidate for the minimal value of L_n as $L_1 = a_{gr}$.

3. The analogs of p-adic length scales would be equal to the radii of gravitational Bohr atom as n^2- multiples of the gravitational Bohr radius a_{gr}:

$$L_n = n^2 a_{gr} \quad , \quad a_{gr} = \frac{4\pi GM}{\beta_0^2} \quad . \quad (15.4.11)$$

This expression realizes the condition $\beta_0^2 = xGM/r$ inspired by the Kepler's law with $x = 4\pi$.

4. One must fix a_{gr} as a multiple $a_{gr} = kr_S$ of r_S. Substitution to the above equation gives

$$\beta_{0,max} = \sqrt{\frac{2\pi}{k}} \quad .$$

The condition $\beta_{0,max} = 1/2$ would give $k = 8\pi$ and $a_{gr} = 8\pi r_S$ as a minimal radius for a Bohr orbit. The condition $\beta_{0,max} < 1$ gives $k \geq 2\pi$ and $a_{gr} \geq 2\pi r_S$.

Just as in the case of hydrogen atom, the falling of the orbiting system to the blackhole like entity (in TGD frameworkd blackholes are replaced with what might be called flux tube spaghettis [L78, L71]) is prevented. This should have obviously consequences for the view about the dynamics around blackhole like objects. The circular orbits have as analogs s-waves and of these are realized, the falling to blackhole like entity is possible.

5. The proposed formula does not force the condition $\beta_0 < 1$ and it is not clear whether it holds true at the relativistic limit. The replacement $\beta_0 \to sinh(\eta) = \beta_0/\sqrt{1-\beta_0^2}$, where η is the hyperbolic angle, forces the condition $\beta_0 < 1$, and would give

$$\beta_0 \to \frac{\beta_0}{\sqrt{1-\beta_0^2}} = \sqrt{\frac{2\pi}{k}} \quad .$$

The condition $\beta_{0,max} = 1/2$ gives $k/2\pi = 3$. This would correspond to the minimal Bohr radius $a_{gr} = 6\pi r_S \simeq 18.84 r_S$.

Number theoretical universality as a constraint

Also number theoretical universality could be also used as a constraint. The condition would be that only finite-dimensional extensions are allowed. π defines an infinite-D transcendental extension so that it should disappear in central formulas.

1. The appearance of 4π in the formula $a_{gr} = 4\pi G/\beta_{0,max}^2$ creates number-theoretical worries. Suppose that a_{gr} is a rational number.

2. I have proposed that G is dynamically determined and relates to the CP_2 radius via the formula $G = R^2/\hbar_{grav} = 2\pi R^2/h_{grav}$, where $h_{grav}/h_0 \sim 10^7$ holds true [K11].

   This gives

$$a_{gr} = \frac{4\pi G}{\beta_{0,max}^2} = \frac{8\pi^2 R^2}{h_{grav}\beta_{0,max}^2} \tag{15.4.12}$$

   .

3. Since $\beta_0/4\pi$ appears as coupling strength in the perturbation theory, it should also be rational. $\beta_{0,max} = \pi/6$ would realize the condition $\beta_{0,max} = 1/2$ approximately.

4. With this assumption the rationality of a_{gr} requires that h_{gr} is proportional to π so that also G would be rational. This implies that $\hbar_{eff} = h_{eff}/2\pi$ is rational. Also α_K would be rational if g_K^2 is rational. This would be true also for the other coupling constants.

5. $\beta_0 = \pi/6$ would realize the condition $\beta_0 = 1/2$ approximately. This also implies that α_{gr} is rational. The condition $k/2\pi = 1/\beta_{0,max}^2$ implies $k \propto 1/\pi$. $a_{gr} = kr_s = kGM$ is rational, and this requires $M \propto \pi$. This guarantees the rationality of GM/β_0. Gravitational fine structure constant α_{gr} would be an inverse integer multiple of $\alpha_{gr}(max) = 1/24$. It would seem that the system is consistent.

   The alternative condition $\beta_0^2/(1-\beta_0^2) = 2\pi/k$ is excluded because it implies that k is a rather complex transcendental.

What makes this interesting is that 24 is one of the magic numbers of mathematics (`https://cutt.ly/Rn0x0Tr`) and it appears in the bosonic string model as the number of space-like dimensions.

1. Euclidian string world sheet with torus topology has a conformal equivalence class defined by the ratio ω_2/ω_1 of the complex vectors spanning the parallelogram defining torus as an analog of a unit cell. String theory must be invariant under modular group $SL(2, Z)$ leaving the periods and thus the conformal equivalence class of torus invariant. Same applies to higher genera. In TGD these surfaces correspond to partonic 2-surfaces.

2. Modular invariance raises elliptic functions (doubly periodic analytic functions in complex plane) in a special role. In particular, Weierstrass function, which satisfies the differential equation $(d\mathcal{P}/dz)^2 = 4\mathcal{P}^3 - g_2\mathcal{P} - g_3$ has a key role in the theory of elliptic functions (`https://cutt.ly/Bn0xrMS`).

   The discriminant $\Delta = g_2^3 - 3g_2^3$ of the polynomial at the r.h.s can be locally regarded as a function of the ratio of $\tau = \omega_2/\omega_1$ of the periods of $\mathcal{P}$ defining the conformal equivalence class of torus.

   $\Delta(\tau)$ is not a genuine modular invariant function of τ. Rather, Δ defines a modular form of weight 12 transforming as $\Delta(a\tau + b/(c\tau + d)) \to (c\tau + d)^{12}\Delta(\tau)$ under $SL(2, Z)$. The number 24 comes from the fact that one can express Δ as 24^{th} power of the Dedekind η function: $\Delta = (2\pi)^{12}\eta^{24}$.

3. In dimension $D = 24$ there are 24 even positive definite unimodular lattices, called the Niemeier lattices, and the so-called Leech lattice is one of them. Interestingly, in dimension 4 there exists a 24-cell analogous to Platonic solid having 24 octahedrons as its 3-D "faces".

This encourages the question whether there might be a connection between TGD and string theory based views of quantum gravitation.

Test cases for the proposal

Phase transitions changing β_0 are possible at $r_n/a_{gr} = n^2$ at the Bohr orbits. For instance, in the Bohr orbit model the orbit of Earth is such an orbit. It can be regarded as $n = 5$ orbital with $\beta_0 \simeq 2^{-11}$ and is nearly circular so that the phase transition with $n = 1$ orbital with $\beta_0 \rightarrow \beta_0/5$ is possible. The outer planets indeed have $\beta_0/5$.

p-Adic length scale hierarchy is replaced union of hierarchies with $\beta_0 = \beta_{0,max}/n = 1/2n$, each of which is a subset of the set of Bohr orbits for $\beta_0 = \beta_{0,max}$. One can test this hypothesis for the proposed applications [L143].

1. In the Bohr orbit model the 4 inner planets Mercury, Venus, and Earth, and Mars identifiable correspond to $n = 3, 4, 5, 6$ orbitals for $\beta_0 \simeq 2^{-11}$. Solar radius is $R_{Sun} \simeq .7$ Gm. The orbital radius of Mercury is $R_M \simeq 58$ Gm $= 82.9 \times R_{Sun}$. This gives $a_{gr} = R_M/9 \simeq 9.2R_{sun}$. This gives $\beta_0 = \sqrt{2\pi R_S/a_{gr}} \simeq 17.1 * 10^{-4}$.

   The approximation used hitherto has been $\beta_0 = 2^{-11)} \simeq 5 \times 10^{-4}$ and is by a factor about 1/3 smaller. Using $a_{gr} = R_M$ instead of $a_{gr} = R_M/9$ would give roughly correct value.

   One could indeed regard Mercury as $n = 1$ orbit for $v_0 = v_0/3$ in which case one would have $a_{gr} = R_M$ and one would obtain $\beta_0 = .57$ which is not far from the valued used. Mercury would therefore correspond to $n = 3$ dark matter gravitationally whereas Venus must correspond to $n = 1, 2$ or $n = 4$.

2. The transition $\beta_0 \rightarrow \beta_0/5$ possible for Earth and required for outer planets could be interpreted as the increase of n having interpretation as increase of dimension of extension of rationals $n \rightarrow 5n$.

For the Earth one has $R_E = 6.371 \times 10^6$ m and $r_S = 10^{-2}$ m. The model of the superfluid fountain effect [K34] [L143] suggests $\beta_0 = 1/2$ for which one would have $GM/v_0 = 1/2$. The value of $a_{gr} = 6\pi r_S$ for the relativistic form of the Nottale condition. The principal quantum number n for the Bohr orbit of the super-fluid would be $n \simeq R_E/a_{gr} = R_E/6\pi r_S \simeq 3.4 \times 10^7$. This would correspond to the large quantum number limit. The difference of radii between nearby Bohr orbits would be $\Delta r = 2R_E/n \simeq 19$ cm, which makes sense.

The levels in the hierarchy of gravitationally dark matters are labelled by $h_{gr} = GMm/\beta_0$ with $\beta_0 = \beta_{0,max}/n$, where n is the dimension of EQ, and each level defines a hierarchy of atomic orbitals. The sets of orbital radii at various levels form a nested hierarchy and phase transitions can occur at least between the states with the same angular momentum and orbital radius.

The quantum variant of the similar picture is expected to apply in the case of the hydrogen atom and the fact that there is evidence for dark valence electrons suggests that these phase transitions indeed take place.

What about long cosmic strings thickened to flux tubes explaining galactic dark matter in the TGD framework? In this case the Kepler law gives $\beta^2 = TG$ so that the all orbiting stars would correspond to the same value of β_0 and n.

15.5 Appendix: Embedding of spherically symmetric stationary symmetric metric as a guideline

There are two basic questions to be answered.

1. Is $R = l_P$ or $R = m^2 l_P$, $m = 7!$ realized?
2. Should one assume that $g_K^2 \propto \hbar_{eff}$ or $\alpha_K \propto 1/\hbar_{eff}$?

For the first option α_K is the same for dark phases but would be subject to p-adic CCE. This would conform with the notion of gravitational Planck constant predicting that the parameter. The *effective* value of α_K would be however given by α_K/n for dark phases since the Galois symmetry is n-fold multiple of the action for a "fundamental region" for the Galois group.

Second option would predict that α_K behaves like $1/n$ so that effective α_K would behave like $1/n^2$. It seems that this option is excluded and one can concentrate on the first question. The increase of g_K^2 with n is not a problem since it does not appear as a parameter of perturbative expansion since g_K is automatically absorbed to a scaling of the induced gauge potentials.

Quantum-classical correspondence suggests that classical theory theory, in particular spherically symmetric stationary embeddings, could help to answer the first question. Even the extremal property is not absolutely necessary.

The action is a sum of Kähler action and volume term proportional to length scale dependent cosmological constant approaching zero in long length scale and in equilibrium both give contributions of the same order of magnitude. This suggests that Kähler action corresponding to $\Lambda = 0$ could serve as a guideline.

I studied the embedding of a stationary spherically symmetric metric as a space-time surface during the first 10 years of TGD and the results suggested that the $R = l_P$ option looks more realistic. p-Adic mass calculations based on the definition of the Compton length as $\hbar/M$ however led to the conclusion that the one must have $r \sim 10^{7.6} l_P$. If one replaces $\hbar$ with $\hbar_0$, $R = l_P$ is natural.

The spherically symmetric ansatz assumes that space-time surfaces has a projection to a geodesic sphere S^2 of CP_2 which can be either homologically trivial or non-trivial. Using spherical coordinates (Θ, Φ) for S^2 and spherical Minkowski (t, r, θ, ϕ) coordinates for M^4, the ansatz reads

$$\begin{aligned} & s \equiv sin(\Theta) = f(r) \ , \quad \Phi = \omega t \ , \\ & g_{tt} = 1 - k^2 s^2 \ , \qquad k^2 = R^2\omega^2 \ . \end{aligned} \tag{15.5.1}$$

In far-away region one can approximate s as

$$s = s_0 + \tfrac{r_1}{r}, \quad s_0 = sin(\Theta_0) \ . \tag{15.5.2}$$

The induced metric has component g_{tt} given by

$$g_{tt} = 1 - k^2 s_0^2 - 2k^2 s_0 \frac{r_1}{r} \ , \tag{15.5.3}$$

by taking $u = t\sqrt{2 - k^2 s_0^2}$ as a new time coordinate can expresses g_{tt} in terms of the parameters of Scwartshild metric

$$\begin{aligned} & g_{uu} = 1 - 2k^2 s_0 \tfrac{r_1}{r} \equiv 1 - \tfrac{r_s}{r} \ , \\ & r_s = 2GM = \tfrac{2k^2 s_0 r_1}{1-k^2 s_0^2} \ , \\ & r_1 = \tfrac{1-k^2 s_0^2}{2k^2 s_0} r_s \equiv k_1 r_s \ . \end{aligned} \tag{15.5.4}$$

The approximation makes sense for $s \leq 1$, which gives the condition

$$r \geq r_{min} = (1 - s_0) r_1 = (1 - s_0) k_1 r_s = (1 - s_0) \frac{1 - k^2 s_0^2}{2k^2 s_0} r_s \equiv y_1 r_s \ . \tag{15.5.5}$$

Remark: The radial component of the metric goes to zero much faster than for Schwartschild metric. The shift of time coordinates depending on the radial coordinate allows to correct this problem. This is however not essential for the recent argument. Schwartchild metric however implies that $\sqrt{g}$ in the calculation of mass gives just the volume element of the flat metric since $g_{tt} g_{rr} = 1$ is true. This is assumed in the following.

One can estimate the mass of the system as Kähler electric energy. Assume that the contribution to the mass comes only from the region $r > y_1 r_s$. The Kähler electric mass $M = r_s/2G$ is given by the expression

$$M = \frac{r_s}{2G}$$
$$= \frac{\hbar_{eff}}{2\alpha_K}\frac{s_0^2}{1-s_0^2}r_1^2\omega^2\int_{r_{min}}^{\infty}\frac{dr}{r^2} = \frac{\hbar_{eff}}{2\alpha_K}\frac{(1-k^2s_0^2)s_0}{2(1-s_0)}r_s\frac{1}{R^2} \quad . \tag{15.5.6}$$

This gives a consistency condition relating R and l_P

$$R^2 = \frac{\hbar_{eff}}{\hbar}Xl_P^2 \quad ,$$
$$X = \frac{(1-k^2s_0^2)s_0}{\alpha_K(1-s_0)} \quad . \tag{15.5.7}$$

One can consider two cases.

1. For $\hbar_{eff} = \hbar$ the condition reduces to

$$R^2 = Xl_P^2 \quad . \tag{15.5.8}$$

   $l_P = R$ gives $X = (1-k^2s_0^2)s_0/\alpha_K(1-s_0) = 1$. One should have $s_0 \simeq \alpha_K$ so that the value of $1/\alpha_K$ as an analog of critical temperature would be coded to the geometry of the space-time surface.

   $R = (7!)^2)l_P$ would require $X = \hbar/\hbar_0$, one should have $1 - s_0 \sim 10^{-5}$ for $\alpha_K \sim 10^{-2}$.

2. For $\hbar_{eff} = \hbar_0$ the condition reduces to

$$R^2 = X\frac{\hbar_0}{\hbar}\times l_P^2 \quad . \tag{15.5.9}$$

   $l_P = R$ gives $X = \hbar/\hbar_0$. One might of course argue that α_K decreases in long scales in the discrete p-adic length scale evolution but this option does not look plausible.

To sum up, intuitively $\hbar$ option with $R = l_P$ looks the most reasonable option.

Chapter 16

The Recent View about SUSY in TGD Universe

What SUSY is in TGD framework is a longstanding question, which found a rather convincing answer rather recently. In twistor Grassmannian approach to $\mathcal{N} = 4$ SYM [B29, B22, B23, B25, B65, B30, B15] twistors are replaced with supertwistors and the extreme elegance of the description of various helicity states using twistor space wave functions suggests that super-twistors are realized both at the level of M^8 geometry and momentum space.

In TGD framework $M^8 - H$ duality allows to geometrize the notion of super-twistor in the sense that at the level of M^8 different components of super-field correspond to components of super-octonion each of which corresponds to a space-time surfaces satisfying minimal surface equations with string world sheets as singularities - this is geometric counterpart for masslessness.

16.0.1 New view about SUSY

The progress in understanding of $M^8 - H$ duality [L85] throws also light to the problem whether SUSY is realized in TGD [L92] and what SUSY breaking cold mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them. Super-octonion components of polynomials have different orders so that also the extension of rational assignable to them is different and therefore also the ramified primes so that p-adic prime as one them can be different for the members of SUSY multiplet and mass splitting is obtained.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and perhaps also hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge could appear as a space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with an improved understanding of quantum criticality and the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials: the criticality is criticality for the polynomials interpreted as p-adic polynomials in $O(p) = 0$ approximation meaning the presence of multiple roots in this approximation.

16.0.2 Connection of SUSY and second quantization

The linear combinations monomials of theta parameters appearing in super-fields are replaced in case of hermitian H super coordinates consisting of combinations of monomials with vanishing quark number. For super-spinors of H the monomials carry odd quark number with quark number 1. Monomials of theta parameters are replaced by local monomials of quark oscillator operators labelled besides spin and weak isospin also by points of cognitive representation with embedding space coordinates in an extension of rationals defining the adele. Discretization allows anti-commutators which are Kronecker deltas rather than delta functions. If continuum limit makes sense, normal ordering must be assumed to avoid delta functions at zero coming from the contractions. The monomials (not only the coefficients appearing in them) are solved from generalized classical field equations and are linearly related to the monomials at boundary of CD playing the role of quantum fields and classical field equations determine the analogs of propagators.

The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals. The polynomial composition hierarchies correspond to inclusion hierarchies for isomorphic sub-algebras of super-symplectic algebra having interpretation in terms of inclusions of hyper-finite factors of type II_1.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

16.0.3 Proposal for S-matrix

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces.

1. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

2. Super-Dirac equation implies that super-Dirac action vanishes on-mass-shell. The proposed construction however allows to get also scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation however makes possible to express derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution follows from the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

3. S-matrix is trivial unless CD contains the images of 6-D analogs of branes as universal special solutions of the algebraic equations determining space-time surfaces at the level of M^8. 4-D space-time surfaces representing particle orbits meet at the partonic 2-surfaces associated with the 3-D surfaces at $t = r_n$ hyper-surfaces of M^4. The values of $t = r_n$ correspond to the roots of the real polynomial with rational coefficients determining the space-time surface. These transitions are analogs of weak measurements, and in TGD theory of consciousness they give rise to the experience flow of time and can be said to represent "very special moments" in the life of self [L81].

4. The creation and annihilation operators at vertices associated with the monomials would be connected to the points assignable to cognitive representations at opposite boundaries of CD and also to partonic 2-surfaces in the interior of CD possibly accompanied by sub-CDs. This would give analogs of twistor Grassmannian diagrams containing finite number of partonic 2-surfaces as topological vertices containing in turn finite number ordinary vertices defined by the monomials. The diagrams would be completely classical objects in accordance with the fact that quantum TGD is completely classical theory apart from state function reduction.

5. This view allows also a formulation of continuum theory since the monomials appearing in the action density in the interior of CD are linear superposition of the monomials at the points of boundary of CD involving 3-D integral so that contractions of oscillator operators only reduce one integration without introducing divergence. One can also normal order the monomials at boundary of CD serving as initial values. If preferred extremals are analogs of Bohr orbits, one can express extremals using either boundary as the seat of initial data.

16.1 How to formulate SUSY at the level of $H = M^4 \times CP_2$?

In the following I will represent the recent trial for constructing SUSY at the level of $H = M^4 \times CP_2$. The first trial replaced theta parameters of SUSY with quark oscillator operators labelled by spin and isospin and had rather obvious shortcomings: in particular, one did not obtain many-quark states with large quark numbers. The second trial allows quark oscillator operators to have as labels also the points of space-time surface in cognitive representation and thus having coordinates of H belonging to an extension of rationals defining the adele [**?**]

16.1.1 First trial

If SUSY is realized at the level of M^8, it should have a formulation also at the level of H. The basic elements of the first trial form part of also second trial. The basic modification made in the second trial is that finite number of theta parameters replaced with the fermionic oscillator operators labelled by the points of cognitive representations so that they are analogous to fermion fields in lattice, and only local composites of the oscillator operators appear in the super coordinates and super-spinors. This means that SUSY is essential element of the second quantization of fermions in TGD.

1. $M^8 - H$ duality is non-local and means that the dynamics at the level of H is not strictly local but dictated by partial differential equations for super-fields having interpretation as describing purely local many-fermion states made of fundamental fermions with quantum numbers of leptons and quarks (quarks do not possess color as spin like quantum number) ad their antiparticles.

2. Classical field equations and modified Dirac equation must result from this picture. Induction procedure for the spinors of H must generalize so that spinors are replaced by super-spinors Ψ_s having multi-spinors as components multiplying monomials of theta parameters θ. The determinant of metric and modified gamma matrices depend on embedding space coordinates h replaced with super coordinates h_s so that monomials of θ appear in two different ways. Hermiticity requires that sums of monomial and its hermitian conjugate appear in h_s. Monomials must also have vanishing fermion numbers. Otherwise one can obtain fermionic states propagating like bosons. For Dirac action one must assume that Ψ_s involves only odd

monomials of θ with quark number 1 involving monomials appearing in h_s to get only states with quark number 1 and correct kind of propagators.

3. One Taylor expands both bosonic action density (6-D Kähler action dimensionally reducing to 4-D Kähler action plus volume term) and Super-Dirac action with respect to the super-coordinates h_s. In Super-Dirac action one has also the expansion of super-spinor in odd monomials with total quark number 1. The coefficients of the monomials of θ:s are obtained are partial derivatives of the action. Since the number of θ parameters is finite and corresponds to the number of spin-weak-isopin states of quarks and leptons, the number of terms is finite if the θ parameters anti-commute to zero. If not, one can get an infinite number of terms from the Taylor series for the action to the coefficient given monomial. Number theoretical considerations do not favor this and there should exist a cancellation mechanism for the radiative corrections coming from fermionic Wick contractions if thetas correspond to fermionic oscillator operators as it seems to be.

4. One can interpret the superspace as the exterior algebra of the spinors of H. This reminds of the result that the sections of the exterior algebra of Riemann manifold codes for the Riemann geometry (see `http://tinyurl.com/yxrcr8xv`). This generalizes the observation that one can hear the shape of a drum since the sound spectrum is determined by its frequency spectrum defined by Laplacian.

   Super-fields define a Clifford algebra generated by θ parameters as a kind of square root of exterior algebra which corresponds to the Clifford algebra of gamma matrices. Maybe this algebra could code also for the spinor structure of embedding space or even that of space-time surface so that the super-fields could be seen as carriers of geometric information about space-time surface as a preferred extremal. In 8-D case there is also $SO(1,8)$ triality suggesting that corresponding three Clifford algebras correspond to exterior algebra fermionic and anti-fermionic algebras.

What about the situation at the level of M^8?

1. At M^8 level the components of super-octonion correspond to various derivatives of the basic polynomial $P(t)$ so that space-time geometry correlates with the quantum numbers assignable to super-octonion components - this is in accordance with QCC (quantum-classical correspondence). This is highly desirable at the level of H too.

   Could the space-time surface in M^8 be same for super-field components with degree $d < d_{max}$ in some special cases? The polynomial associated with super octonion components are determined by the derivatives of the basic polynomial $P(t)$ with order determined by the degree of the super-monomial. If they have decomposition $P(t) = P_1^k(t)$, the monomials with degree $d < k$ the roots corresponding to the roots $P_1(t)$ co-incide. Besides this there are additional roots of $d^r P_1/dt^r$ for super-octonion component with r θ parameters.

   A possible interpretation could be as quantum criticality in which there is no SUSY breaking for components having $d < k$ (masses in p-adic thermodynamics could be the same since the extension defined by P_1 and corresponding ramified primes would be same). This would conform with the general vision about quantum criticality.

2. Usual super-field formalism involves Grassmann integration over θ parameters to give the action. M^8 formalism does not involve the θ integral at all. Should this be the case also at the level of H? This would guarantee that different components of H- coordinates as super-field would give rise to different space-time surface and QCC would be realized. θ integration produces SUSY invariants naturally involved with the definition of vertices involving components of super-fields. Also vertices involving fermionic and bosonic states emerge since bosonic super-field components appear in super-coordinates in super-Dirac action.

This approach does not say anything about second quantization. There is a strong temptation to replace the theta parameters with fermionic oscillator operators. One cannot however obtain second quantization of fermions in this manner since the maximal quark number (and lepton number if leptons are present as fundamental fermions) of the states is 4. To achieve second quantization, one must replace the theta parameters with fermionic oscillator operators labelled besides spin and weak isospin by the coordinates of points of 3-surface, most naturally the points belonging to a cognitive representation characterizing space-time surface for given extension of rationals.

16.1.2 Second trial

I have already earlier considered a proposal for how SUSY could be realized in TGD framework. As it often happens, the original proposal was not quite correct. The following discussion gives a formulation solving the problems of the first proposal and suggests a concrete formulas for the scattering amplitudes in ZEO based on super-counterparts of preferred extremals. In the sequel I will talk about super Kähler function as functional of 3-surfaces and - super Kähler function action. By holography allowing to identify 3-surfaces with corresponding space-time surfaces as analogs of Bohr orbits, these notions have the same meaning.

Could the exponent of super-Kähler function as vacuum functional define S-matrix as its matrix elements

Consider first the key ideas - some of them new - formulated as questions.

1. Could one see SUSY in TGD sense as a counterpart for the quantization in the sense of QFT so that oscillator operators replace theta parameters and would become fermionic oscillator operators labelled by spin and electroweak spin - as proposed originally - and by selected points of 3-surface of light-cone boundary with embedding space coordinates in extension of rationals? One would have analog of fermion field in lattice identified as a number theoretic cognitive representation for given extension of rationals. The new thing would be allowance of local composites of oscillator operators having interpretation in terms of analogs for the components of super-field.

   SUSY in TGD sense would be realized by allowing local composites of oscillator operators containing 4+4 quark oscillator operators at most. At continuum limit normal ordering would produce delta functions at origin unless one assumes normal ordering from beginning. For cognitive representations one would have only Kronecker deltas and one can consider the possibility that normal ordering is not present. The vanishing of normal ordering terms above some number of them suggested to be the dimension for the extension of rationals would give rise to a discrete coupling constant evolution due to the contractions of fermionic oscillator operators.

2. What is dynamical in the superpositions of oscillator operator monomials? Are the coefficients dynamical? Or are the oscillator operators themselves dynamical - this would mean a QFT type reduction to single particle level? The latter option seems to be correct. Oscillator operators are labelled by points of cognitive representation and in continuum case define an analog of quantum spinor field, call it q. This suggests that this field satisfies the super counter part of modified Dirac equation and must involve also super part formed from the monomials of q and $\overline{q}$. This however requires the replacement of q with q_s in super-Dirac operator and super-coordinates h_s and one ends up with an iteration $q \rightarrow q_s \rightarrow ...$

   The only solution to the paradoxical situation is that one has self-referential equation $q = q_s$ having interpretation in terms of quantum criticality fixing the coefficients of terms in $q = q_s$. Analogous condition $h_s = (h_s)_s$ must be satisfied by h_s under substitution $h_s \rightarrow (h_s)_s$. These conditions fix coefficients of terms in H super-coordinate h_s and q_s interpreted as coupling constants so that quantum criticality implying a discrete coupling constant evolution as function of extension of rationals follows. Also super-Dirac equation $D_s q_s = 0$ and field equations $D_{s,\alpha}\Gamma^{\alpha,s} = 0$ for Kähler action guaranteeing hermiticity are satisfied.

3. Could one interpret the time reversal operation taking creation- and annihilation operators to each other as time reflection permuting the points at the opposite boundaries of CD? The positive *resp.* negative energy parts of zero energy states would be created by creation *resp.* annihilation operators from respective vacuums assigned to the opposite boundaries of CD.

4. Could one regard preferred extremal regarded as 4-surface in super embedding space parameterized by the hermitian embedding coordinates plus the coefficients of the monomials of quarks and antiquarks with vanishing quark number, whose time evolution follows from dimensionally reduced 6-D super-Kähler action? Could one assume similar interpretation for super spinors consisting of monomials with total quark number equal to 1 and appearing in super-Dirac action?

5. In WKB approximation the exponent of action defines wave function. In QFTs path integral is defined by an exponent of action and scattering operator can be formally defined as action exponential. Could the matrix elements for the exponent of the super counterpart of Kähler function plus super Dirac action between states at opposite boundaries of CD between positive and negative energy parts of zero energy states define S-matrix? Could the positive and negative energy parts of zero energy states be identified as many particles states formed from the monomials associated with embedding space super-coordinates and super-spinors?

6. Could the construction of S-matrix elements as matrix elements of super-action exponential reduce to classical theory? Super-field monomials in the interior of CD would be linear superpositions of super-field monomials at boundary of CD. Note that oscillator operator monomials rather than their coefficients would be the basic entities and the dynamics would reduce to that for oscillator operators as in QFTs. The analogs of propagators would relate the monomials to those at boundary ly to the monomials at the boundary of CD, and would be determined by classical field equations so that in this sense everything would be classical. Note however that the fixed point condition $q = q_s$ and super counterpart of modified Dirac equation are non-linear.

   Vertices would be defined by monomials appearing in super-coordinate and super-spinor field appearing in terms of those at boundary of CD. If two vertices at interior points x and y of CD are connected there is line leading from x to a point z at boundary of CD and back to y and one would have sum over points z in cognitive representation. This applies also to self energy corrections with $x = y$. At the possibly existing continuum limit integral would smoothen the delta function singularities and in presence of normal ordering at continuum would eliminate them.

   In the expressions for the elements of S-matrix annihilation operators appearing in the monomials would be connected to the passive boundary P of CD and creation operators to the active boundary. If no partonic 2-surfaces appear as topological vertices in the interior of CD, this would give trivial S-matrix!

   $M^8 - H$ duality however predicts the existence of brane like entities as universal 6-D surfaces as solutions of equations determining space-time surfaces. Their M^4 projection is $t = r_n$ hyperplane, where r_n corresponds to a root of a real polynomial with algebraic coefficients giving rise to octonion polynomial, and is mapped to similar surface in H. 4-D space-time surfaces representing incoming and outgoing lines would meet along their ends at these partonic 2-surfaces.

   Partonic 2-surfaces at these hyper-surfaces would contain ordinary vertices as points in cognitive representation. Given vertex would have at most 4+4 incoming and outgoing lines assignable to the monomial defining the vertex. This picture resembles strongly the picture suggested by twistor Grassmannian approach. In particular the number of vertices is finite and their seems to be no superposition over different diagrams. In this proposal, the lines connecting vertices would correspond to 1-D singularities of the space-time surfaces as minimal surfaces in H. Also stringy singularities can be considered but also these should be discretized.

   By fixing the set of monomials possibly defining orthonormal state basis at both boundaries one would obtain given S-matrix element. S-matrix elements would be matrix elements of the super-action exponential between states formed by monomials of quark oscillator operators. Also entanglement between the monomials defining initial and finals states can be allowed. Note that this in principle allows also initial and final states not expressible using monomials but that monomials are natural building bricks as analogs of field operators in QFTs.

7. The monomials associated with embedding space coordinates are embedding space vectors constructible from Dirac currents (left- or right-handed) with oscillator operators replacing the induced spinor field and its conjugate. The proposed rules for constructing S-matrix would give also scattering amplitudes with odd quark number at boundaries of CD. Could the super counterpart of the bosonic action (super Kähler function) be all that is needed to construct the S-matrix?

   In fact, classically Dirac action vanishes on mass shell: if this is true also for super-Dirac action then the addition of Dirac action would not be needed. The super-Taylor expansion of super- Kähler action gives rise to the analogs of perturbation theoretic interaction terms so that one has perturbation theory without perturbation theory as Wheeler might state it. The detailed study of the structure of the monomials appearing in the super-Kähler action shows that they have interpretation as currents assignable to gauge bosons and scalar and pseudo-scalar Higgs.

   Super Dirac action is however needed. Super-Dirac equation for q and $D_{\alpha,s}\Gamma^{\alpha}_{s} = 0$ allow to reduce ordinary divergences $\partial_{\alpha} j^{\alpha}$ of fermionic currents appearing in super-Kähler action to commutators $[A_{\alpha,s} j^{\alpha}]$. Therefore no information about q at nearby points is needed and one avoids lattice discretization: cognitive representation is enough.

8. Topological vertices represent discontinuities of the space-time surface bringing strongly in mind the non-determinism of quantum measurement, and one can ask whether the 3-branes and associated partonic 2-surfaces. Could the state function reductions analogous to weak measurements correspond to these discontinuities? Ordinary state function reductions would change the arrow of time and the roles of active and passive boundaries of CD [L77]. In TGD inspired theory of consciousness these time values would correspond to "very special moments" in the life of self [L81].

9. The unitarity of S-matrix can be understood from the structure of the exponent of Kähler action. The exponent decomposes to a sum of real and purely imaginary parts. The exponent of the hermitian imaginary part is a unitary operator for a given space-time surface. Real exponent containing also radiative corrections from the normal ordering gives exponent of Kähler function as vacuum functional in WCW (sum in the case of cognitive representations) and by choosing the normalization factor of the state appropriately one obtains unitary S-matrix.

16.1.3 More explicit picture

The following sketch tries to make the picture of the second trial more explicit.

1. The construction of S-matrix should reduce to super-geometry coded by super Kähler function determined by the 6-D Kähler action for twistor lift by dimensional reduction. This might be possible since zero energy states have vanishing total conserved charges and exponent of super-Kähler function has matrix elements only between states at opposite boundaries of CD having same total charges.

2. Construction should reduce to preferred extremals and their super-deformations determined by variational principle with boundary conditions. The boundary values of super-deformations at either boundary could be also interpreted as initial values for preferred extremals analogous to Bohr orbits. The expectations for the super action with fixed initial values between positive and negative energy parts would give the scattering amplitudes assignable to a given space-time surface. There would be functional integral over space-time surfaces using exponent of Kähler function as weight. In number theoretic vision this would reduce to sum over preferred extremals labelled by cognitive representations serving as WCW coordinates.

3. Number theoretic vision suggests a discretization in terms of cognitive representation consisting of points with coordinates in extension of rationals defining the adele. This representation could be associated with the boundaries of CD and possibly with M^4 time=constant hyper-planes assignable with the universal special solutions in M^8. At the partonic 2-surfaces associated with these hyper-planes 4-D extremals would meet along their ends: topological

particle vertices would be in question. Is string world sheets and partonic 2-surfaces correspond to singularities, the boundaries of strings world sheets as intersections of the string world sheets and orbits of partonic 2-surfaces should represent fermion lines.

4. Creation operators would be assigned with the passive boundary of CD - call it P - and annihilation operators as their conjugates would act as creation operators at the opposite boundary, active boundary - call it A. Time reversal symmetry of CD suggests that annihilation operator as conjugate of creation operator labelled by the a point of boundary of CD corresponds to the same point in common coordinates for light-cone boundary. This would conform also with the basic character of the half-algebras associated with super-symplectic symmetries.

   The original proposal was that oscillator operators have only spin and electroweak spin as indices but the standard view about spin and statistics requires that also the points of the 3-surface must label them. Also the fact that the total quark number can be larger than 4 of course requires this too. Algebraically the only difference with respect to this proposal is that one allows also the points of 3-surface at the boundary of CD as labels.

5. Number theoretical vision requires that only points of 3-surface having embedding space coordinates in the extension of rationals defining the adelic physics are allowed. In the generic case the number of points in the cognitive representation would be finite and would increase with the dimension of extension so that at the limit of algebraic numbers they form a dense set of 3-surface.

   Since action has infinite expansion in powers of super coordinates the contractions of oscillator operators would give rise to a renormalization of the coefficients of the monomials and of classical action. For cognitive representations one would avoid normal ordering problems sine the number of contractions is limited by the number of points in cognitive representation. This would give rise to discrete coupling constant evolution as function of the extension of rationals.

6. In continuum theory all points of 3-D boundary would label quark oscillator operators and one must normal order the oscillator operators in given local monomial. Also now the idea about connecting creation and annihilation operators to opposite boundaries of CD would allow to get rid of infinities due to contractions.

The action exponential would lead to a rather concrete proposal for the coefficients of the monomials appearing in super-fields.

1. The deformations of embedding space coordinates would be expressible as WCW-local superpositions of isometry generators or as WCW-global superpositions of Hamiltonian currents contracted with the coordinate deformations. The latter would conform with super-symplectic symmetries of WCW. CP_2 Hamiltonian currents would give color quantum numbers. S^2 Hamiltonian currents would be also present. One could see space-time local Kac-Moody symmetries assignable to light-like partonic orbits and string world sheets as a dual representations at space-time level of symplectic symmetries at embedding space level.

2. Spinor modes would be expressible as superpositions of embedding space spinor modes having expansion as super-Taylor series at the boundaries of CD. This would give spin and electroweak quantum numbers.

Does one really obtain description of gauge bosons and gravitons by using the exponent?

1. Could the coefficients of super-monomials at boundary of CD allow interpretation in terms of gauge bosons? These entities could have well-defined quantum numbers so that this might be possible. Quark spin and isospin would represent additional spin degrees of freedom. The Hamiltonians of H of CP_2 expressible for given 3-surface as local superpositions of SU(3) Killing vector fields would represent color degrees of freedom.

   For string world sheets one would naturally have transversal M^4 super-coordinates and CP_2 super-coordinates as analogs of fields. Could this allow to get gauge bosons as excitations of strings as in string theories.

2. Gauge bosons could be also bi-local composites of fermion and anti-fermion at opposite boundaries of wormhole contact or at opposite wormhole contacts of wormhole flux tube. Gravitons could be 4-local composites. Baryons and mesons could be this kind of non-local composites. One can consider also the analog of monopole phase of QFTs in which particles would be multilocal composites.

3. The bosonic action is for induced metric and induced Kähler form. QFT wisdom would suggest that their super-analogs could correspond to external particles. One could indeed take the induced gauge potentials or -fields at boundary and form their contractions with Killing vectors of isometries to obtain general coordinate invariant quantities and form their super-analogs as normal ordered local composites. One can consider the same idea for induced gravitational field or its deviation from Minkowski metric.

   Formally this would correspond to an addition to the action exponential of perturbative terms of type jA appearing in QFTs representing coupling to external currents and take the limit $j \to 0$. In QFT picture this works since various gauge fields are functionally independent but in TGD framework this is not the case. Second problem is to to construct a complete orthonormalized set of states in this manner. Therefore it seems this description can make sense only at QFT limit of TGD.

Dimensionally reduced 6-D Kähler action as an analog of SYM action

The 6-D dimensionally reduced Kähler action reduces to a sum of 4-D Kähler action and volume term and will be simply referred to as Kähler action. The super variant of this action is obtained by replacing embedding space coordinates with their super counterparts. Super-Kähler action is analogous to pure SYM action.

1. Space-time would be super-surface in super counterpart of $H = M^4 \times CP_2$ with coordinates h^k having super components proportional to multi-spinors multiplying the monomials of oscillator operators. The ocillator operator monomials rather than only the multi-spinor coefficients of the oscillator monomials transforming like vectors of H are regarded as analogs of quantum fields expressible by classical field equations as linear superpositions of their values at the boundary of CD for preferred extremals. The dynamics of monomials would reduce to that for oscillator operators labelled by points of cognitive representation and having interpretation as restriction of quantized quark field satisfying super-Dirac equation and the quantum criticality condition $q = q_s$.

2. Fermionic creation operators and annihilation operators labelled not only by spin and weak isospin as in the original proposal but also by the finite number of points of the cognitive representation. Therefore oscillator operators are analogous to the values of fermion field in discretization obeying super variant of modified Dirac equation. Both leptonic and quark like oscillator operators corresponding to two different H-chiralities and having different couplings to Kähler gauge potential could be present but octonionic triality allows only quarks. The vacuum expectation value of the action action exponentials contains only monomials with vanishing B (and L if leptons are present as fundamental fields). The matrix elements between positive and negative energy parts of zero energy states gives S-matrix.

   Real super-coordinates can be assumed to be hermitian and thus contain only sums of monomials and their conjugates having vanishing fermion numbers. This guarantees super-symmetrization respecting bosonic statistics at the level of propagators since all kinetic terms involve two covariant derivatives - one can indeed transform ordinary derivatives of monomials coming from the Taylor expansion to covariant derivatives involving also the coupling to Kähler form since the total Kähler charge of terms vanishes.

The lack of anti-commutativity of fermionic oscillator operators implies the presence of terms resulting in contractions.

1. The super-Taylors series would involve a finite number of partial derivatives of action. Wick contractions of oscillator operators would give rise to an infinite number of terms in continuum case. The appearance of infinite Taylor series defining the coefficients of super-polynomial

is however troublesome from the point of view of number theoretic vision since there is no guarantee that the coefficients are rational functions. The finite number of points in the cognitive representation implying finite number of oscillator operators however allows only finite number of terms in the super-Taylor expansion.

The monomials appearing in action in the interior of CD can be expressed as linear superpositions of those at boundary also in continuum case. Therefore each monomial is 3-D integral over the monomials at the boundary of CD. As a consequence, the contractions giving delta functions only eliminate one integration but do not give rise to infinities. A general solution to the divergence problems emerges.

This is actually nothing new: one of the key ideas behind the notion of WCW is that path integral over space-time surfaces is replaced by a functional integral over 3-surfaces in WCW holographically equivalent with preferred extremals as analogs of Bohr orbits. The non-locality of the theory due to the replacement of point-like particles with 3-surfaces would solve the divergence problems.

An interesting possibility in line with the speculations of Nima-Arkani Hamed and others is that the action defining space-time as a 4-surface of embedding space could emerge from the anti-commutators of the oscillator operator monomials as radiative corrections so that the bosonic action would vanish when the super-part of h_s vanishes.

Super-Dirac action

Before doing anything one can recall what happens in the case of modified Dirac action.

1. One has separate modified Dirac actions $\overline{\Psi} D\Psi$, $D = \Gamma^\alpha D_\alpha$ for quarks and leptons (later it will be found that modified Dirac action for quarks might be enough) and the covariant derivatives differ since there is a coupling to n-ple of included Kähler potential. For leptons one has $n = -3$ and for quarks $n = 1$. This guarantees that em charges come out correctly. This coupling appears in the covariant derivative D_α of fermionic super field.

2. One obtains modified Dirac equations for quarks and leptons by variation with respect to spinors. The variation with respect to the embedding space coordinates gives quantized versions of classical conservation laws with respect to isometries. One also obtains and infinite number of super-currents as contractions of modes of the modified Dirac operator with Ψ.

3. Classical field equations for the space-time surface emerge as a consistency condition guaranteeing that the modified Dirac operator is hermitian: canonical momentum currents of classical action must be conserved and define conserved quantum when contracted with Killing vectors of isometries. Quantum-classical correspondence (QQC) requires than for Cartan algebra of symmetry algebra the classical Noether charges are same as the fermionic Noether charges.

   It turns out that the super-symmetrization of modified Dirac equation gives only fermions and they fermionic superpartners in this manner if one requires that propagators are consistent with statistics.

Consider first the situation without the quantum criticality condition $q = q_s = \Psi_s$. H coordinates are super-symmetrized and induced spinor field becomes a super-spinor $\Psi_s = \Psi^N O_N(q, \overline{q})$ with Ψ_N depending on h_s (summation over N is understood).

1. As in the case of bosonic action the vacuum expectation value gives modified Dirac action conserving fermion numbers but one could assume that the monomials in the leptonic (quark) modified Dirac action have either non-vanishing L (B) and vanishing B (L). It seems that the lepton (baryon -) number of monomials can vary from 1 to maximum value. A more restrictive condition would be that the value is 1 for all terms.

2. Super-Dirac spinor is expanded in monomials $O_N(q, \overline{q})$ of q and its conjugate $\overline{q}$, whose anti-commutator is non-trivial. One can equally well talk about quark like oscillator operators.

The sum $\Psi = \Psi^N O_N$ defining super-spinor field. The multi-spinors Ψ_N are functions of space-time coordinates, which are ordinary numbers. Quark oscillator operators are same as appearing in the embedding space super-coordinates. Only monomials O_N having total quark number equal to 1 are allowed. Super-spinor field however contains terms involving quark pairs giving rise to spartners of multiquark states with fixed quark number. The conjugate of super-spinor is defined in an obvious manner.

3. The metric determinant and modified gamma matrices appearing in the Dirac action are expanded as Taylor series in hermitian super-coordinate $h_s + \overline{h}_s$ with $h = h^N O_N$. This as as in the case of bosonic action.

   There are also couplings to gauge potentials defined by the spinor connection of CP_2 and the expansion of them with respect to the embedding space coordinates gives at the first step rise covariant derivatives of gauge potentials giving spinor curvature. At next steps one obtains covariant derivatives of spinor curvature, which however vanish so that the number of terms coming from the dependence of spinor connection on CP_2 coordinates is expected to be finite. Constant curvature property of CP_2 is therefore be essential (not that also M^4 would have covariantly constant spinor curvature in twistor lift and give rise to CP breaking).

   The super-coordinate expansion of the metric determinant $\sqrt{g}$ and modified gamma matrices Γ^α and covariant derivatives D_α involving dependence on H coordinates give additional monomials of q parameters appear as hermitian monomials. Classical field equations correspond to $D_\alpha \Gamma^\alpha = 0$ guaranteeing the hermiticity of $D = \Gamma^\alpha D_\alpha$.

4. When super-coordinates of H are replaced with ordinary embedding space coordinates the only Wick contractions are between O^N and $\overline{O}^N$ in the vacuum expectation of Dirac action, and the action reduces to super-Dirac action with components satisfying modified Dirac equation. Propagator is Dirac propagator for all terms and the presence of only odd components in Ψ with quark number 1 and even components in h^s guarantees that Fermi statistics is not violated at the level of propagators. The dependence on h_s induces coupling between different components of the super-spinor. The components of super-spinor are interpreted as second quantized objects.

5. The terms in the action would typically involve n-tuples of partial derivatives $L_{k_1\alpha_1\ldots k_n 1\alpha_n}$ defined earlier for $L = \sqrt{g}$ coming from super-Taylor expansions. Similar derivatives come from the modified gamma matrices Γ^α.

   Also now one obtains loops from the self contractions in the terms coming from the expression of action and gamma matrices. These terms should vanish and as already found this would requires vanishing of currents perhaps identifiable as Noether currents of symmetries. This guarantees that the Taylor expansion contains only finite number of terms as required by number theoretic vision.

The multi-fermion vertices defined by the action would be non-trivial but involve always contraction of all fermion indices between monomials formed from oscillator operators in Ψ and their conjugates in $\overline{\Psi}$ if the loop contractions sum up to zero. One could interpret these super-symmetric vertices as a redistribution of fermions of a local many-fermion state between external local many-fermion states particles represented by the monomials appearing in the vertices. The fermions making the initial state would be same as in final state and all distributions of fermion number between sfermion lines would be allowed. The action obtained by contraction would has SUSY as symmetry but the propagation of different sfermions is fermionic and does not look like that for ordinary spartners.

The quantum criticality condition $q = q_s$ makes the situation non-linear and should fix the coefficients of various terms in super-Taylor expansions as fixed point values of coupling constants.

Could super-Kähler action alone give fermionic scattering amplitudes?

The concrete study of the super-counterpart of Kähler action led to a realization of an astonishing possibility: super-Kähler action alone could give also fermionic scattering amplitudes.

1. In principle this is possible if in S-marix one has contractions of quark creation operator and annihilation operator appearing in quark-antiquark bilinear with different partonic 2-surfaces. This would give fermionic line connecting the points of the cognitive representation at the boundary of CD with points at partonic 2-surfaces in $t = r_n$ hyper-planes in the interior of CD or at the opposite boundary of CD.

   As a matter of fact, this must be the case if the exponent for the sum of super-Kähler and super-Dirac action gives the scattering amplitudes as its matrix elements! The reason is that super-Dirac action vanishes or its solutions.

   The super-Dirac equation must be however present and corresponding variational principle must be satisfied. The hermiticity of the modified Dirac operator requires the vanishing of the covariant derivatives of the modified gamma matrices meaning that bosonic field equations are satisfied. This must be true also for the super variants of the modified gamma matrices.

   If super-Dirac equation is satisfied, the action of modified Dirac operator without connection (ordinary rather than covariant derivative) terms on the discretized quark fields can be expressed in terms of spinor connection as $\Gamma^\alpha - s\partial_\alpha \Psi = \Gamma^\alpha_s A_{\alpha,s} \Psi$ and there is no need for explicit information about the behavior of quark field in the nearby points so that cognitive representation is enough. Otherwise one must have the usual lattice type discretization.

2. The super expansion of super-Kähler action contains only ordinary derivatives of 4-currents defined by quark bi-linears. If the quark field operators with continuous arguments are behind those with discretized arguments and satisfy modified Dirac equation, one can transform the action on quark and antiquark fields to a multiplication with induced gauge potential. This gives nothing but the coupling terms to the gauge potentials in the standard perturbation theory, where one assumes free solutions of Dirac action as approximate solutions. One therefore obtains on mass shell variant of the perturbation theory! Perturbation theory without perturbation theory, might Wheeler say. Or more concretely: the fact that one can treat super-coordinates only perturbatively.

3. The natural guess is that all terms in the expansion of super-Kähler can be transformed to interaction terms and super-Kähler action gives the analog of perturbation theory as a discretized version. The leptonic terms associated with $(3, 3)$ term in super-Kähler action should transfrom to the analog of interaction terms for leptonic Dirac action. Whether Kähler gauge potential and spinor connection are developed in super-Taylor series in ordinary manner or remains an open questions.

16.1.4 What super-Dirac equation could mean and does one need super-Dirac action at all?

What does super-Dirac equation actually mean? Super Dirac action vanishes on mass shell and super-Kähler action would give all scattering amplitudes. Are super-Dirac action and super-spinor field needed at all? Should one interpret the oscillator operators defining analog of quark field q as the super-Dirac field Ψ_s as conceptual economy suggests. But doesn't this imply $q = q_s$?

One can consider 3 options as an attempt to answer these questions. Options I and II are not promising. Option III leads to very nice concrete realization of quantum criticality.

Option I: No super-Dirac action and constant oscillator operators

1. If oscillator operators can be regarded as constant, the super Taylor expansion for super Kähler action would give ordinary divergences of the fermionic currents and the action of derivative would be on modified gamma matrices and charge matrix A commutator of $[A_\alpha, \Gamma^\alpha Q]$ and the outcome would be non-vanishing so that one would obtain the coupling terms also now. Could the commutator $[A_\alpha, \Gamma^\alpha]$ be interpreted in terms of gravitational interaction and the commutator $[A_\alpha, Q]$ as electro-weak interaction? In any case, there would be no need for super-Dirac action!

2. There is however an objection. Quark oscillator operators are labelled by the points of cognitive representation and in continuum case they are analogous to the values of quantized

spinor field. Should one identify this spinor field with super-spinor field and solve it using a generalization of modified Dirac equation to super-Dirac equation? Can one argue that oscillator operators labelled by points represent superpositions of constant oscillator operators involving integration over 3-D surface at light-cone boundary and are indeed constant?

This option does not look promising.

Option II: q satisfies ordinary Dirac equation

1. Could one assume that the solution q_0 of ordinary Dirac equation defines the solution to be used as q in the super-Kähler action. The coupling terms of super-Kähler action obtained using $D_0q_0 = 0$ would be proportional to the classical spinor connection. Classical Kähler action does not involve gauge potentials so that internal consistency would not be lost at this level. The super-variant of Kähler action however involves derivatives of the analogs of fermion currents and there transformation to purely local objects requires the introduction of electroweak gauge potentials so that the symmetry between super-Kähler and super-Dirac would be lost.

2. This would save from developing gauge potentials A_k to super Taylor series - as found this would give only 2 terms by the covariant constancy of spinor curvature. The divergence would reduce to a term involving only a commutator $[A_{alpha}, Q]$, where A_α is purely classical. If Q is Kähler charge, this commutator would vanish, which looks strange since electroweak hyper-charge is proportional to Q_K. This could be seen as a failure. If Kähler gauge potential is replaced with its super-variant $A_\alpha + J_{\alpha l}\delta h^l_s$ the commutator is non-vanishing as it should be.

3. Leptons would not appear in $q = q_0$ but since the exponent of super-Kähler action would define the scattering amplitudes by the vanishing of (super-)Dirac action, one could say that leptons emerge as 3-quark composites. SUSY would be dynamical after all!

Mathematically this option looks awkward and must be dropped from consideration.

Option III: q is a solution of super-Dirac equation

It is best to start from an objection.

1. Assume that q is given Super-Dirac equation

$$D_s(q)q = 0 \ .$$

This non-linear equation involves powers of q and its conjugate. The problem is that super-Dirac equation is non-linear in q and there are actually 7 separate equations for the part of q with quark number one. 7 equations is too much. The only manner to solve the problem is to replace q with q_s to get $D_sq_s = 0$. But this would require replacing q with q_s in $D_s(q)$ and it would seem that one has an infinite recursion.

2. Could q be self-referential in the sense that one has

$$q_s = q \ . \tag{16.1.1}$$

q would be invariant under iteration $q \to q_s$. This would give excellent hopes of fixing q uniquely. This allows also physical interpretation. The fixed points of iteration give typically fractals and quantum criticality means indeed fractality. This condition could therefore realize quantum criticality, and would give hopes about unique solution for $q = q_s$ for given extension of rationals.

Also h_s should satisfy similar self-referentiality condition expressing quantum criticality:

$$h_s = (h_s)_s \ . \tag{16.1.2}$$

The general ansatz for h_s involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

The general ansatz for h_s^k involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

3. Many consciousness theorists love the idea of self-referentiality described by Douglas Hofstadter in fascinating manner in his book "Gödel, Escher, Bach". They might get enthusiastic about the naïve identification of q_s and h_s with field of consciousness. In TGD inspired theory of consciousness the self-referentiality of consciousness is understood in different manner but $q = q_s$ and $h_s = (h_s)_s$ as quantum correlated for the self-referentiality is certainly a fascinating possibility.

Consider now a more detailed picture.

1. What does one really mean with q_s? q_s could contain parts with quark number 1 and 3 but a very natural requirement is that it has well-defined fermion number and thus has only a part with quark number 1. The part with quark number 3 is not needed since super-Kähler action would contain it: leptons would emerge as local 3-quark composites from super-Kähler action.

2. Super-Dirac equation would be given by

$$\begin{array}{rcl} D_s(q)q & = & 0 \ , \\ D_s(q) & = & \Gamma^{\alpha,s}(q)D_{\alpha,s}(q) \ . \end{array} \tag{16.1.3}$$

$D_s(q)$ is super-Dirac operator and

$$\Gamma^\alpha_s = T^{\alpha k}_s \gamma_k \tag{16.1.4}$$

are super counterparts of the modified gamma matrices $\Gamma^\alpha = T^{\alpha k}\gamma_k$ defined by the contractions of canonical momentum currents of Kähler action with the gamma matrices γ_k of H:

$$T^\alpha_k = \frac{\partial L_K}{\partial(\partial_\alpha h^k)} \ . \tag{16.1.5}$$

One would have $\gamma_{k,s} = \gamma_k$ by covariant constancy. L_K denotes Kähler action density, which is sum of 4-D Kähler action and volume term. The field equations of super Kähler action give

$$D_{\alpha,s}\Gamma^{\alpha}_{s} = 0 \tag{16.1.6}$$

guaranteeing the hermiticity of the super Dirac operator.

3. The basic equations would thus reduce to

$$\begin{aligned} q &= q_s \ , \\ D_{\alpha,s}\Gamma^{\alpha}_{s} &= 0 \ , \\ D_s(q)q &= 0 \ . \end{aligned} \tag{16.1.7}$$

In the continuum case one could think of solving the field equations iteratively.

1. One would first by solve $q = q_0$ for classical modified Dirac operator $D(h_0)$ defined by the ordinary coordinates h_0 of H. Next one would solve $q_1 = q_0 + \Delta q_1$ for the super version $D_1 = D(q_0)$. This would allow to solve next iterate $h_1 = h_0 + \Delta h_1$ using $D(q_1)$. One could continue this process in the hope that the iteration converges. At each step one have group of equations $D_n q_n = 0$ for q_n and for h_{n+1}.

2. An objection is that the iteration could lead outside the extension of rationals if it involves infinite number of iterates. This could occur for space-time surface itself if the normal ordering terms affect the classical action and force to modify the preferred extremal and also cognitive representation at each step. Remaining inside the extension of rationals could also mean that the coefficients of the monomials at points of cognitive representation belong to the extension.

   It is not of course completely clear whether these equations make sense in the interior of CD or can be solved unlike the lowest equation. It however seems that for each independent monomial m_n the equation would be of form $D_0 m_n = ...$ so that other terms would define kind of sources term and the equation super-Dirac equation could be written as non-linear equation $D_0 q = -\Delta D(q)q$.

3. Each order of bosonic monomials would give its own group of equations making sense also for the cognitive representations and the same would be true for quark monomials and monomials of different orders would be coupled but different quark numbers in q (quarks and leptons) would decouple. These equations are analogous to those appearing in QFT in a gauge theory involving gauge fields and fermion fields.

For cognitive representations the situation is much simpler.

1. All that is needed is the transformation of the ordinary divergences of fermionic currents to a form in which derivative ∂_α is replaced with the linear action of super-gauge potential $A_{\alpha,s}$. Therefore there is no need to solve the non-linear modified Dirac equation in this case and it would become necessary only at the continuum limit. The full solution of non-linear super-Dirac equation would be necessary only in the continuum theory.

2. Could one think that q has vanishing derivatives at the points of cognitive representation: $\partial_\alpha q = 0$ implying $\Gamma^\alpha A_\alpha q = 0$ If the condition holds true then q would be effectively constant for cognitive representations and the situation would effectively reduce to that for option I. This condition is is diffeo-invariant but not gauge invariant. If the points of cognitive representation correspond to singularities of the space-time surface at which several roots of the octonionic polynomial co-incide, the tangent space at the level of M^8 parameterized by a point of CP_2 is not unique and the singular point is mapped to several points in H, and the conditions $\partial_\alpha q = 0$ would make sense at the level of M^8 at least.

3. If one assumes that the quarks correspond to singular points defined by intersections of roots also in the continuum case, one obtains discretization also in this case irrespective of whether one assumes $\partial_\alpha q = 0$ at singularities. Allowing analytic functions with rational Taylor coefficients one obtains also now roots which can be however transcendental and one can identify intersections of roots in the similar manner.

To sum up, there are many uncertainties involved but to my opinion the most satisfactory option is Option III. If one assumes that condition at continuum case, one would obtain also now the discretization.

What information is needed to solve the scattering amplitudes?

One can look the situation also from a more practical point of view. Are there any hopes of actually calculating something? Is it possible to have the information needed?

1. The condition that super-Dirac equation is satisfied would remove the need to have a lattice and cognitive representation would be enough. If the condition $\partial_\alpha q = 0$ holds true, the situation simplifies even more but this condition is not essential. The condition that the points of the cognitive representation assignable to quark oscillator operators correspond to singularities of space-time surface at which several space-time sheets intersect, would make the identification of these points of cognitive representation easier. Note that the notion of singular point makes sense also at the continuum limit giving cognitive representation even in this case in terms of possibly transcendental roots of octonion analytic functions.

   If the singular points correspond to solution to 4 polynomial conditions on octonionic polynomials besides the 4 conditions giving rise to the space-time surfaces. The intersections for two branches representing two roots of polynomial equation for space-time surface indeed involve 4 additional polynomial conditions so that the points would have coordinates in an extension of rationals, which is however larger than for the roots $t = r_n$. One could of course consider an additional condition requiring that the points belong to the extension defined by r_n but this seems un-necessary.

   The octonionic coordinates used at M^8-side are unique apart from a translation of real coordinate and value of the radial light-like coordinate $t = r_n$ corresponds to a root of the polynomial defining the octonionic polynomial as its algebraic continuation. At this plane the space-time surfaces corresponding to polynomials defining external particles as space-time surfaces would intersect at partonic 2-surfaces containing the shared singular points defined as intersections.

2. The identification of cognitive representations goes beyond the recent knowhow in algebraic geometry. I have considered this problem in [L88] in light of some recent number theoretic ideas. If the preferred extremals are images of octonionic polynomial surfaces and $M^8 - H$ duality the situation improves, and one might hope of having explicit representation of the images surfaces in H-side as minimal surfaces defined by polynomials.

16.1.5 About super-Taylor expansion of super-Kähler and super-Dirac actions

The study of the details of of the general vision reveals several new rather elegant features and clarifies the connections with QFT picture.

About the structure of bosonic and fermionic monomials

The super part of the embedding space coordinates is H-vector and this allows to pose strong conditions on the form of the monomials.

1. One can construct the simplest monomials as bilinears of quarks and anti-quarks. Since oscillator operators are analogs of quark fields, one can construct analogs of left- and right-handed electroweak currents $\overline{q}(1 \pm \gamma_5)\gamma^k Qq$ involving charge matrix Q naturally assignable to electroweak interactions. The charge matrices Q should reflect the structure of CP_2 spinor

connection so that analogs of electroweak currents would be in question. One can multiply the objects Hamiltonians HA_A of the isometries and even symplectic transformations at the boundary of CD.

2. One can obtain higher monomials of q and $\overline{q}$ by multiplying these vectorial currents by bi-linears, which are scalars and pseudo-scalars obtained by contracting some symmetry related vector field j^k_A of H with gamma matrices of H to give $\overline{q}(1\pm\gamma_5)j^k_A Q\gamma_k q$ giving rise to analogs of scalar and pseudoscalar Higgs. The Killing vector fields of isometries of H and symplectic vector fields assignable to the Hamiltonians of $\delta CD \times CP_2$ are a natural choice for j^k_A.

   One can construct also scalar currents for which gamma matrices contract with gradient of Hamiltonian to give $\overline{q}(1\pm\gamma_5)\gamma^k\partial_k H_A Q\gamma_k q$ as kind of duals of symplectic currents. These do not define symplectic transformations.

   These vector fields make sense at the boundaries of CD and this is enough (they could make sense also at shifted boundaries) since the field equations would allow to express monomials as linear superpositions of the monomials at boundary of CD. Oscillator would always be assigned with the boundaries of CD.

3. If the spin of graviton is assigned with spinor indices, the vector nature of the monomials excludes the analog of graviton. One can however consider also the possibility that the second spin index of graviton like state corresponds to the Hamilton of a symplectic isometry of S^2: for small enough size scales of CD this angular momentum would look like spin. In CP_2 degrees this would give rise to an analog of gluon. Also gluon with spin zero would be obtained.

   An alternative option is to assume that graviton corresponds to a non-local state with vectorial excitations at opposite throats of wormhole contact or at different wormhole contacts of closed flux tube. All these states are in principle possible and the question is which of them correspond to ordinary gravitons.

The super counterpart of Dirac spinor consists of odd monomials of quark spinor. Well-defined fermion number allows only monomials with quark number 1 and with definite H-chirality. Quark spinors allow leptons like stats as local 3-quark composites appearing in the super-Kähler action determining the scattering amplitudes since super-Dirac action vanishes at mass shell.

1. In the bosonic case one has vectorial entities and now it is natural to require that one has an object transforming like spinor of H. This poses strong conditions on the monomials since one should have spin 1/2-isospin 1/2 representation.

2. The lowest monomial correponds to quark-antiquark current. What about leptonic analog. The number of oscillator operators at given point is 4+4=8. Leptonic part of super-Kähler action must have 3+3 indices. Therefore also leptonic bilinear seems to be possible and pairs of quarks and lepton like states are possible.

   Intuitively it is clear that leptonic term exists and corresponds to an entity completely anti-symmetric in spin-isospin index pairs (s_3, i_3) of quark spinors. The construction of baryons without color symmetry indeed gives proton and neutron. In order to obtain Δ resonance from u and d quarks, one must have color degrees of freedom and perform anti-symmetrization in these.

   The general condition is that the tensor product of 3 8-D spin representation of $SO(1,7)$ contains 8-D representation in its decomposition. The existence of lepton representation is clear from the fact that the completely antisymmetric representation formed from 4 quarks is $SO(1,7)$ singlet and is product of lepton representation with 3 fold tensor product which must therefore contain spin-isospin 4-plet . The coupling to Kähler gauge potential would correspond to leptonic coupling, which is 3 times the quark coupling.

3. Quarks and lepton monomials have also satellites obtained by adding scalars and pseudo-scalars constructible as quark-anti-quark bi-linears in the manner already discussed. The interpretation as analogs of Higgs fields might make sense.

Normal ordering terms from contractions of oscillator operators

Normal ordering terms from contractions of oscillator operators is a potential problem. In the discretization based on cognitive representations this problem disappears.

1. Contraction terms could induce discrete coupling constant evolution by renormalizing the local monomials. Infinite number these terms would spoil number theoretical vision since a sum over infinite number of terms in general leads outside the extension of rationals involved. If the number of contractions is finite, there are no problems. This is the case in the number theoretical vision since contraction involves always a pair of points. If the rule for construction of S-matrix holds true these points are at opposite boundaries of CD. In the general case they can be at the same boundary. The number of contracted points cannot be larger than the number of points in cognitive representation, which is finite in the generic situation.

   This would give discrete coupling constant evolution as function of extension of rationals since the contractions renormalize the coefficients of the 4+4 terms in the local composites of oscillator operators. The original proposal that additional symmetries are needed to obtain discrete coupling constant evolution is not needed.

2. One could argue that algebraic numbers as a limit for extension is enough to get the continuum limit since the points of cognitive representation would be dense subset of 3-surface. For continuum theory 3-D delta functions would replace Kronecker deltas in anti-commutators implying in ordinary QFT divergences coming as powers of 3-D delta function at zero.

   In the proposed vision one can allow contractions even in the continuum case. The monomials in the interior are linear multilocal composites of those at either boundary of CD involving 3-D integration over boundary points. Contractions associated with two monomials in the interior means an appearance of delta function cancelling the second integration so that there is no divergence.

About the super-Taylor expansions of spinor connection and -curvature

There are also questions related to the details of the expansion of of spinor connection and -curvature in powers of monomials of quark oscillator operators.

1. The rule is that one develops Kähler function as Taylor series with argument shifted by super-part of the super-coordinate. This involves expansion in powers of coordinate gradients and also the expansion of Kähler gauge potential. In the case of modified Dirac action one must expand also the spinor connection of CP_2.

   A potential problem is that the Taylor expansions of Kähler gauge potential and spinor connection have infinite number of terms. Since the monomials in the interior can be expressed linearly in terms of those at boundary of CD by classical field equations, number theoretic discretization based on cognitive representation implies that only a finite number of terms are obtained by using normal ordering and the fact that the number of oscillator operators at same point is 4+4=8. Normal ordering terms would represent radiative corrections giving rise to renormalization depending on the extension of rationals.

2. Is this enough or should one modify the Taylor expansion of Kähler gauge potential A? The idea that $A_k dh^k$ is the basic entity suggests that one must form super Taylor series for both A_k and dh^k. This would give $A_k dh^k \rightarrow A_k \partial_k \delta h^k + A_l \partial(\delta h^l))dh^k$. By performing an infinitesimal super gauge transformation $A_l \rightarrow A_l + \partial_l(A_l \delta h^k)$ one obtains $A_k \rightarrow A_k + J_{kl} \Delta h_s^k$, where Δh_s^k denotes super part of super-coordinate. The next term would vanish by covariant constancy of J_{kl}.

   The same trick could be applied to spinor connection and since also spinor curvature is covariantly constant, one would obtain only 2 terms in the expansion also in the continuum case. This provides an additional reason for why S $(= CP_2)$ must be constant curvature space.

   This applies also to M^4: in fact, twistor approach strongly suggests that also M^4 has the analog of covariantly constant Kähler form. This conforms with the breakdown of Poincare symmetry at M^8 level forced by the selection of the octonion structure. Poincare invariance is gained by integrating over the moduli space of octonion structures in the construction of scattering amplitudes. What is remarkable that one could use the irreps of Lorentz group at boundaries of CD, which for obvious reasons are much more natural than than those of Poincare group.

3. In the case of embedding metric the same trick would give only the c-number term and only the gradients of embedding space coordinates would contribute to the super counterpart of the induced metric. In this case general gauge super-coordinate transformation would allow to treat the components of metric as constants.

What is the role of super-symplectic algebra?

This picture is not the whole story yet. Super-symplectic approach predicts that the super-symplectic algebra (SSA) generated essentially by the Hamiltonians of $S^2 \times CP_2$ assignable to the representations of $SO(3) \times SU(3)$ localized with the respect to the light-like radial coordinate of light-cone boundary characterize the states besides electro-weak quantum numbers. Color quantum numbers would correspond to Hamiltonians in octet representation. This would predict huge number of additional states.

There are however gauge conditions stating that sub-algebra of SSA having radial conformal weights coming as n-ples of SSA and isomorphic to SSA and its commutator with SSA annihilate physical states. This reduces the degrees of freedom considerably but the number of symplectic Hamiltonians is still infinite: measurement resolution very probably makes this number to finite.

16.2 Other aspects of SUSY according to TGD

In this section other aspects of SUSY according to the present proposal are discussed.

16.2.1 $M^8 - H$ duality and SUSY

$M^8 - H$ duality and $h_{eff}/h_0 = n$ hypothesis pose strong constraints on SUSY in TGD sense.

1. $h_{eff}/h_0 = n$ interpreted as dimension of extension of rationals gives constraints. Galois extensions are defined by irreducible monic polynomials $P(t)$ extended to octonionic polynomials, whose roots correspond to 4-D space-surfaces and in special case 6-spheres at 7-D light-cones of M^8 taking the role of branes.

   The condition that the roots of extension defined by Q are preserved for larger extension $P \circ Q$ is satisfied if P has zero as root:

$$P(0) = 0 \ .$$

   This simple observation is of crucial importance, and suggests an evolutionary hierarchy $P \circ Q$ with simplest possible polynomials Q at the bottom of the hierarchy are very naturally assignable to elementary particles. These polynomials have degree two and are of form $Q = x^2 \pm n$. Discriminant equals to $D = 2n$ and has the prime factors of n as divisors defining ramified primes identified as p-adic primes assignable to particles.

   Remark: Also polynomials $P(t) = t - c$ are in principle possible. The corresponding space-time surfaces at the level of H would be M^4 and CP_2 and they are extremals of Kähler action but do not have particle interpretation.

   It turns out the normal ordering of oscillator operators renormalizes the coefficients of P. In particular P can be shifted by a constant term and this deforms the roots of the real polynomial. Also the action principle to be discussed allows $RE(P) = c$ and $IM(P) = c$ surfaces as solutions.

2. The key idea is that the powers o^n of octonion are associative. If the coefficients of $P(o)$ are real or possibly even complex rationals $m + in$ commuting with octonions, associativity is

not lost. Octonion o would be replaced by super-octions o_s with (possibly complex-) rational coefficients. o_s is octonion shifted by oscillator operator polynomial analogous to a real number. The conjugate octonion $\overline{o}$ would be treated analogously. Associativity would be preserved.

3. One could assign oscillator operators to both leptons and quarks but the option identifying leptons as local 3-quark local composites and in this sense spartners of quarks allows only baryon number zero composites of quarks and anti-quarks to appear in the octonionic polynomial, which is also hermitian. This would conform with $SO(1,7)$ triality.

   Remark: Anti-leptons are spartners of quarks in the sense of being their local composites but not in the sense that they would appear as local composites in q_s. Leptonic currents can appear in super-Kähler action so that anti-leptons are spartners of quarks in this sense.

   Oscillator operators would transform like components of 8-D spinor *resp.* its conjugate and have interpretation as quark *resp.* anti-quark like spinors. $SO(1,7)$ triality allows only leptonic or quark-like spinors and quark-like spinors are the only physical choice. Also the super-quark q_s which must satisfy self-referential condition $q_s = q$ must have components behaving like $8-D$ spinors with quark number 1. o_s should satisfy analogous condition $o_s = (o_s)_s$.

4. Super-polynomial $P_s(o)$ would be defined by super-analytic continuation as $P(o_s)$ by Taylor expanding it with respect to the super-part of o_s. The outcome is super-polynomial with coefficients of oscillator operator monomials containing k quark-antiquark pairs given by ordinary octonionic polynomials $P_{n-k}(o)$. Each $P_{n-k}(o)$ obtained by algebraically continuing the k:th derivative of the real polynomial $P(t)$ would define 4-surface by requiring that the imaginary or real part of $P_{n-k}(o)$ (in quaternionic sense) vanishes or is constant. Normal ordering of oscillator operators renormalizes the coefficients of P_{n-k}. The interpretation would be as radiative corrections.

   Octonionic super-polynomials obtained from octonionic polynomials of degree n as super-Taylor series decompose to a sum of products of octonionic polynomials $P_k(o)$ with degree $k = n - d$ with oscillator operator monomials consisting of d quark-antiquark pairs. If the degree n of the octonionic polynomial is smaller than the maximal number $N = 4$ of oscillator operator pairs in super-polynomial, only a fraction of spartners are possible. SUSY is realized only partially and one can say that part of spartners are absent at the lowest levels of evolutionary hierarchy. At the lowest level of hierarchy corresponding to $n = 2$ only fermions (quarks) would be present as local states and would form non-local states such as baryons and mesons. Gauge bosons and Higgs like state would be bi-local states and graviton 4-local state.

   Remark: Gauge bosons and Higgs like states as local fermion-anti-fermion composites at level $n = 2\times 2$. For the option involving only quarks (color is not spin like quantum number). Note that the value of $n_0 = 3 \times 2 = 6$ in $h = n_0 \times h_0$ suggested by the findings of Randel Mills [L32, L64] would allow the known elementary particles.

5. The geometric description of SUSY would be in terms of super-octonions and polynomials and the components of SUSY multiplet would correspond to components of a real polynomial continued to that of super-octonion and would in general give rise to minimal space-time surfaces as their roots: one space-time sheet for each component of the super-polynomial.

   The components would have different degrees so that the minimal extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles could be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected. The components of super-polynomial would have different degrees so that the extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles would be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected.

16.2.2 Can one construct S-matrix at the level of M^8 using exponent of super-action?

The construction of S-matrix in H picture in terms of exponential of action defining Kähler function of WCW forces to ask whether M^8 really is an alternative picture as the term "duality" would suggest or is it only part of a description necessitating both M^8 and H. If the duality holds true in strict sense the proposed construction of S-matrix at the level of H should make sense also at the level of M^8. Is this possible at all or could it be that S-matrix emerges the level of H and that M^8 level provides only a tool to describe preferred extremals in H by using what I have called M^8 duality? In the sequel I will look what one obtains if the duality holds true in strict sense.

1. The original idea was to identify space-time-surfaces in M^8 as roots of polynomial equations generalizing ordinary polynomial conditions. Could this makes sense also when octonions are replaced by super-octonions and what super-octonions and quark oscillator operators could mean?

2. The oscillator operators are interpreted as a discretized version of second quantized quark field q allowing local composites of q defining analogs of SUSY multiplets. One can indeed define second quantization for cognitive representations also now. Quark oscillator operators would be analogs of complex coefficients commuting with octonionic units ($i = \sqrt{-1}$ commute with them). The gamma matrices appearing in the quark-antiquark bi-linears would be ordinary gamma matrices of M^8.

   Remark: I have also considered the possibility that M^8 spinors correspond to octonionic spinors with octonionic units defining sigma matrices.

3. One could define simplest contribution the octonionic super-coordinate o_s as sum of M^8 octonion and super-part defined as contraction of 8-component quark current $\overline{q}\gamma^k q$ with contracted with octonionic units e_k to give $\Delta o_s = \overline{q}\gamma^k Q q e_k$. Charge matrices Q are linear combinations of sigma matrices of M^8 in the currents. Gamma matrices should be ordinary gamma matrices and q would transform like ordinary M^8 spinor. The entity $o_s = o + \Delta o_s$ would replace octonionic coordinate o in polynomial equations expressing the vanishing of the real or imaginary part (in quaternionic sense) for $P(0_s)$.

   The contractions of Killing vector fields of translations with gamma matrices would give scalars $j^k\gamma_k$ giving in turn scalars $S = \overline{q}j^k\gamma_k Q q$ and these could be used to build higher monomials. Octonion analyticity in the proposed sense does not allow to use Killing vector fields of rotations and symplectic currents. On the other hand, for cognitive representations these vector fields are restricted to single point of cognitive representation: could this mean that one can allow also the more general scalars.

   Leptons should emerge from o_s. This is the case if one allows also higher monomials in o_s. Also leptonic tri-linears and their conjugate could be built and these would give leptonic bi-linears $\overline{L}\gamma^k Q L$. Therefore all (covariantly) constant contributions to super-octonion are possible. The coefficients of various monomials in o_s would be derivatives of polynomial P since they are obtained as super-Taylor series and the coefficients of these polynomials would have interpretation as coupling constants.

4. At the level of H one can construct much larger number of monomials of quark oscillator operators transforming like vector in H. The scalars and pseudo-scalars constructed from the Killing vector fields and symplectic currents can be used to build higher monomials. At the level of H the super-symplectic Hamiltonian currents except those associated with isometries could however annihilate physical states.

   The quark currents defined by symplectic isometries are however not constant so that there seems to be a slight inconsistency. Could one assume that also color isometries at the level of H annihilate states quite generally as also S^2 isometries associated with the "heavenly" sphere S^2 in the decomposition $\delta M^4_+ = S^2 \times R_+$? Or can one argue that the restriction to translations is enough because one considers only points of cognitive representation?

5. What about quantum super-spinors q_s (analog of quantized quark field). q would be ordinary rather than octonionic spinor. q_s would be constructed using q and the scalars already discussed. These monomials would carry information about couplings constants. If they are identifiable as the spinors appearing in o_s, one must have $q = q_s$ realizing quantum criticality in quark sector. This would pose strong conditions on the coefficients of the monomials appearing in q interpreted as coupling constants. The conditions would depend on the extension of rationals defined by the polynomia $P(o)$.

   The discretization by cognitive representations at the level of H is made possible by super-Dirac equation. At M^8 level there is no need to get rid of partial derivatives acting on currents and super-Dirac equation is not needed.

6. The polynomial equations are purely local algebraic equations and the notions of propagation and boundary value problem do not make sense at the level of M^8. $M^8 - H$ correspondence should lead to the emergence of these notions by mapping surfaces to minimal surfaces natural by quantum criticality. Octonion analyticity and associativity of tangent or normal space inducing dynamics should induce M^8 analog of propagation.

Could one imagine a counterpart for the action exponential and a construction of S-matrix similar to that in the case of H?

1. The action principle should be purely local involving no derivatives of the super-octonionic polynomial $P(o_s)$. It should produce $RE(P) = 0$ and $IM(P) = 0$ as solutions. One might allow also solution $RE(P) = c$, where c is rational number. This would shift of the real polynomial continued algebraically to octonionic polynomial modifying the roots. One should obtain also 6-spheres as universal solutions and identifiable as subsets of 7-D light cones. Now one would have $IM(P) = 0$, $RE(P) = c$ modifying the roots $t = r_n$ defining hyper-surfaces in M^4.

2. Action should be sum over contributions over the points of cognitive representation, perhaps identifiable as the set of singular points at which two roots co-incide.

   (a) Could one minimize the action with respect to the components of $RE(P)$ or $IM(P)$? If this were the case one obtains one would have either $RE(P) = 0$ or $IM(P) = 0$. Surfaces with associative tangent and normal space should have different action and this does not look nice.

   (b) Could one require stationarity of the action with respect to the small deformations of the points of cognitive representation so that they would represent local extrema of action density? These points indeed change, when the polynomial is modified. Since only the deformations of these points are the visible trace of variation for cognitive representations, one could require that the value of action is stationary against these variations rather than variations of the values of $RE(P)$ and or $IM(P)$. This would give rise a condition involving derivatives of $RE(P)$ and $IM(P)$ at singular points with respect to space-time components of octonion. This option will be considered in the sequel.

3. The action density should be finite, and allow both solution types. One can imagine two options.

   Option I: If one requires that the action density is dimensionless, the simplest guess for the "action density" L is

$$L = \frac{(RE, IM)}{[(RE, RE) + (IM, IM)]} \ ,$$

   where one has $RE \equiv RE(P(o))$ and $IM \equiv IM(P(o))$ and the inner product is quaternionic inner product. The problem is that denominator gives infinite series giving rise to infinite number of normal ordering terms which may lead out of extension. For exceptional solutions $RE = 0, IM = 0$ the denominator also diverges.

Option II: The alternative avoiding these problems is analogous to the action density of completely local free field theory given by

$$L = K(RE, IM) \ . \tag{16.2.1}$$

K is constant with dimensions of inverse length squared and should relate to the CP_2 length squared. This is not dimensionless but can remain bounded if the quantity (RE, IM) remains bounded for large values of $(RE, RE) + (IM, IM)$.

4. For **Option I** L is a generalization of conformally invariant action from 2-D complex case, in which L reduces to $L = w_1 w_2/(w_1^2 + w_2^2) = sin(\phi)cos(\phi)$, $w_1 = Re(w(z))$,$w_2 = Im(w(z))$. (ϕ) is the conformally invariant direction angle associated with w.

   The variation of 2-D action with respect to position of the point of cognitive representation gives

   $$\frac{[(\partial_u w_1 w_2 + w_1 \partial_u w_2)(w_1^2 + w_2^2) + w_1 w_2 (w_1 \partial_u w_1 + w_2 \partial_u w_2)]}{(w_1^2 + w_2^2)^2} \ , \ \ u \in \{x, y\} \ .$$

   The general solutions are $w_i = c_i \neq 0$, where c_i are constant rational numbers.

   The criticality of the action density (maybe it could be seen as a manifestation of quantum criticality) is essential and means that the graph of L as function of w_1 and w_2 is analogous to saddle $w_1 w_2/((w_1^2 + w_2^2)$. The condition that L is well-defined requires $c_1 \neq 0$. c_1 could in principle depend on point of cognitive representation. **Option II** gives the same equations in complex case.

5. For **Option II** one obtains 8 equations in the octonionic case and the outcome is that the derivatives of RE or IM or both with respect to components of o vanish. One can have $RE(P(o)) = c_1 \neq 0$ or $IM(P(o)) = c_2 \neq 0$, where c_i is rational. Both conditions are true for the special 6-D solution at 7-D light-cone boundary. Also now both options give the same equations.

What about the super variant of the variational principle?

1. Super-Taylor expansion must be carried out and normal ordering reduces the action to 5 independent terms according to the number $k \in \{0, ..., 4\}$ of quark pairs involved. It seems that only **Option II** is free of number theoretical problems due to normal ordering. Also in this case one has renormalization corrections to various terms in RE and IM. Inner product does not however give rise to additional terms. The degree of the polynomial $P_{n-k}(o_s)$ is equal to $n - k$ and decreases as the degree h of the monomial increases and normal ordering terms are present.

2. One can decompose action action density as $L = \sum L_k$ corresponding to different numbers k of quark pairs. The stationarity conditions hold true for the polynomial coefficient $P_{n-k}(o)$ of each oscillator operator monomial appearing in RE and IM. One has both $RE(P_{n-k}) = c_k \neq$ and $IM(P_{n-k}) = c_k \neq 0$ options. Both conditions are true for the special solutions. Without further conditions the option can depend on k and on the point of cognitive representation. $c_k \neq 0$ for some values of k guarantees that L to be non-vanishing so that the exponential of S can define a non-trivial S-matrix.

   Since an approximation of continuous case should be in question, the options should be same all points of the cognitive representation. In the lowest order approximation one obtains $k = 0$ solution obtained without super-symmetry. Normal ordering terms however modify the coefficients of $P(o)$ so that this solution is not exact.

3. Each monomial $P_{n-k}(o)$ defines its own space-time surface and conditions should hold true independently for each super-component L_k. Second option would be to consider vacuum expectation value of the action in which case one would have only single surface.

4. One would have purely local free field theory and the construction of S-matrix would be extremely simple. One could introduce CDs and the identification of hermitian conjugates of fermionic oscillator operators labelled by points at given boundary of CD as creation operators at time reflected points at opposite boundary. If one can talk about sub-CDs assignable to partonic 2-surfaces in M^8 picture one obtains similar identification for them. Also leptons would emerge from S-matrix.

To sum up, the second trial has a generalization although octonionic picture allows only the Killing vectors of translations of E^8 in the construction of o_s and q_s. The action principle replaces the earlier ansatz with solution in which one has roots of polynomials of $RE(P)$ and $IM(P)$ shifted by rational number. Also a renormalization of P takes place.

16.2.3 How the earlier vision about coupling constant evolution would be modified?

In [L79, L70] I have considered a vision about coupling constant evolution assuming twistor space $T(M^4) = M^4 \times S^2$. In this model the interference of the Kähler form made possible by the same signature of $S^2(M^4)$ and $S^2(CP_2)$ gives rise to a length scale dependent cosmological constant appearing defining the running mass squared scale of coupling constant evolution.

For $T(M^4)$ identified as $CP_3(3,h)$ the signatures of twistor spheres are opposite and Kähler forms differ by factor i (imaginary unit commuting with octonion units) so that the induced Kähler forms do not interfere anymore. The evolution of cosmological constant must come from the evolution of the ratio of the radii of twistor spaces (twistor spheres). This forces to modify the earlier picture.

1. $M^8 - H$ duality has two alternative forms with $H = CP_{2,h} \times CP_2$ or $H = M^4 \times CP_2$ depending on whether one projects the twistor spheres of $CP_{3,h}$ to $CP_{2,h}$ or M^4. Let us denote the twistor space $SU(3)/U(1) \times U(1)$ of CP_2 by F.

2. The key idea is that the p-adic length scale hierarchy for the size of 8-D CDs and their 4-D counterparts is mapped to a corresponding hierarchy for the sizes of twistor spaces $CP_{3,h}$ assignable to M^4 by $M^8 - H$-duality. By scaling invariance broken only by discrete size scales of CDs one can take the size scale of CP_2 as a unit so that $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ becomes an evolution parameter.

   Coupling constant evolution must correspond to a variation for the ratio of $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ and a reduction to p-adic length scale evolution is expected. A simple argument shows that Λ is inversely proportional to constant magnetic energy assignable to $S^2(X^4)$ divided by $1/\sqrt{g_2(S^2)}$ in dimensional reduction needed to induce twistor structure. Thus one has $\Lambda \propto 1/r^2 \propto 1/L_p^2$. Preferred p-adic primes would be identified as ramified primes of extension of rationals defining the adele so that coupling constant evolution would reduce to number theory.

3. The induced metric would vanish for $R(S^2(CP_{3,h}) = R(S^2(F))$. Λ would be infinite at this limit so that one must have $R(S^2(CP_{3,h}) \neq R(S^2(F))$. The most natural assumption is that one $R(S^2(CP_{3,h}) > R(S^2(F))$ but one cannot exclude the alternative option. Λ behaves like $1/L_p^2$. Inversions of CDs with respect to the values of the cosmological time parameter $a = L_p$ would produce hierarchies of length scales, in particular p-adic length scales coming as powers of $\sqrt{p}$. CP_2 scale and the scale assignable to cosmological constant could be seen as inversions of each other with respect to a scale which is of order 10^{-4} meters defined by the density of dark energy in the recent Universe and thus biological length scale.

4. The original model for the length scale evolution of coupling parameters [L79] would reduce to that along paths at $S^2(CP_2)$ and would depend on the ends points of the path only. This picture survives as such. Also in the modified picture the zeros of Riemann zeta could

naturally correspond to the quantum critical points as fixed points of evolution defining the coupling constants for a given extension of rationals.

Space-time surfaces the level of M^8 would be determined by octonionic polynomials determined by real polynomials with rational coefficients. The non-critical values of couplings might correspond to the values of the couplings for space-time surfaces associated with octonion analytic functions determined by real analytic functions with rational Taylor coefficients.

16.2.4 How is the p-adic mass scale determined?

p-Adic prime identified as a ramified prime of extension of rationals is assumed to determine the p-adic mass scale. There are however several ramified primes and somehow the quantum numbers of particle should dictate with ramified prime is chosen. There are two options to consider depending on whether both the extension and ramified prime are same for all spartners Option 1) or whether spartners can have different ramified primes (Option 2)). There also options depending on whether both leptons and quarks appear in their own super-Dirac actions (Option a) or whether only quarks appear in super-Dirac action (Option b implied by quark number conservation) . Call the 4 composite options Option 1a), 2a), 1b), 2b) respectively.

1. Consider first Options 1a) and 1b). The ramified prime is same for all states corresponding to the same degree of θ monomial and thus same value of $F + \overline{F}$. At the lowest $k = 2$ level containing only fermions as local states the p-adic thermal masses of quarks and leptons are same for Option 1a) at least for single generation and for all generations if Q_2 does not depend on the genus g of the partonic 2-surface. For Option 1b) the masses would *not* be same for leptons and quarks since they would correspond to different degrees of super-octonionic polymials. For both options would have $n = n(g)$.

2. For Option 2 ramified prime depends on the state of the SUSY multiplet. This would require that for fermions with $k = 2$ the integer n in $Q_2(x) = x^2 \pm n$ has the p-adic primes assignable to leptons and quarks as factors.

   There are 6 different quarks and 6 different leptons with different p-adic mass scales. For Option 2a) n should have 12 prime factors which are near to power of 2. For leptons the factors correspond to Mersenne primes M_k, $k \in \{107, 127\}$ and Gaussian Mersenne $k = 113$. Gaussian Mersenne is complex integer. TGD requires complexification of octonions with imaginary unit i commuting with octonionic units so that also Gaussian primes are possible. This would resolve the question whether $P(t)$ can have complex coefficients $m + in$.

   For option 2b) quarks and leptons as local proton and neutron would have different extensions since the polynomials would be different. The p-adic primes for 6 quark states quarks would depend on genus. The value of n need not depend on genus g since the ramified primes p depends on g: $p = p(g)$.

   Since the polynomials describing higher levels of the dark hierarchy would be composites $P \circ Q_2$ with $P(0) = 0$, Q_2 would be a really fundamental polynomial in TGD Universe. For Option 2b) it would be associated with quarks and would code for the elementary particles physics. The higher levels such as leptons would represent dark matter levels.

3. The crucial test is whether the mass scales of gauge bosons can be understood. If one assumes additivity of p-adic mass squares so that the masses for 2-local bosons would be p-adically sums of mass squared at the "ends" of the flux tube. If the discriminant $D = 2n$ of Q_2 contains high enough number of factors this is possible. The value of the factor p for photon would be rather larger from the limits on photon mass. For graviton the value p would be even larger.

To sum up, the vision about dark phases suggests that the monopole phase is possible already for the minimal value $n = 2$ involving only fundamental quarks for Option 2b), which is the simplest one and could solve the probelm of matter antimatter asymmetry. Bosons and leptons as purely local composites of quarks are possible for $n = 6$. Rather remarkably, also empirical constraints [L32, L64] led to the conclusion $h = 6h_0$. The condition is actually weaker: $h/h_0 \ mod \ 6 = 0$.

16.2.5 Super counterpart for the twistor lift of TGD

Twistor lift of TGD is now relatively well understood. I have made somewhat adhoc attempts to construct TGD analog of the Grassmannian approach so super-twistors. The proposed formalism for constructing scattering amplitudes seems to generalize as such to the twistor lift of TGD.

Could twistor Grassmannian approach make sense in TGD?

By M^8-H duality [L48] there are two levels involved: M^8 and H. These levels are encountered both at the space-time level and momentum space level. Do super-octonions and super-twistors make sense at M^8 level?

1. At the level of M^8 the high uniqueness and linearity of octonion coordinates makes the notion of super-octonion natural. By $SO(8)$ triality octonionic coordinates (bosonic octet 8_0), octonionic spinors (fermionic octet 8_1), and their conjugates (anti-fermionic octet 8_{-1}) would for triplet related by triality. A possible problem is caused by the presence of separately conserved B and L. Together with fermion number conservation this would require $\mathcal{N}=4$ or even $\mathcal{N}=4$ SUSY, which is indeed the simplest and most beautiful SUSY.

2. At the level of the 8-D momentum space octonionic twistors would be pairs of two quaternionic spinors as a generalization of ordinary twistors. Super octo-twistors would be obtained as generalization of these.

Also Grassmannian is replaced with super-Grassmannian and super-coordinates as matrix elements of super matrices are introduced.

1. The integrand of the Grassmannian integral defining the amplitude can be expanded in Taylor series with respect to θ parameters associated with the super coordinates C as rows of super $G(k,n)$ matrix.

2. The delta function $\delta(C,Z)$ factorizing into a product of delta functions is also expanded in Taylor series to get derivatives of delta function in which only coordinates appear. By partial integration the derivatives acting on delta function are transformed to derivatives acting on integrand already expanded in Taylor series in θ parameters. The integration over the θ parameters using the standard rules gives the amplitudes associated with different powers of θ parameters associated with Z and from this expression one can pick up the scattering amplitudes for various helicities of external particles.

The super-Grassmannian formalism is extremely beautiful but one must remember that one is dealing with quantum field theory. It is not at all clear whether this kind of formalism generalizes to TGD framework, where particle are 3-surfaces [L48]. The notion of cognitive representation effectively reducing 3-surfaces to a set of point-like particles strongly suggests that the generalization exists.

The progress in understanding of M^8-H duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It seems now clear that sparticles are predicted and SUSY remains in the simplest scenario exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The increased undestanding of what twistorialization leads to an improved understanding of what twistor space in TGD could be. It turns out that the hyperbolic variant $CP_{3,h}$ of the standard twistor space CP_3 is a more natural identification than the earlier $M^4\times S^2$ also in TGD framework but with a scale corresponding to the scale of CD at the level of M^8 so that one obtains a scale hierarchy of twistor spaces [L99]. Twistor space has besides the projection to M^4 also a bundle projection to the hyperbolic variant $CP_{2,h}$ of CP_2 so that a remarkable analogy between M^4 and CP_2 emerges. One can formulate super-twistor approach to TGD using the same formalism as will be discussed in this article for the formulation at the level of H. This requires introducing besides

6-D Kähler action and its super-variant also spinors and their super-variants in super-twistor space. The two formulations are equivalent apart from the hierarchy of scales for the twistor space. Also M^8 allows analog of twistor space as quaternionic Grassmannian HP_3 with signature (6,6). What about super- variant of twistor lift of TGD? consider first the situation before the twistorialization.

1. The parallel progress in the understanding SUSY in TGD framework [L92] leads to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.

2. In fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of embedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.

3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

Super-counterpart of twistor lift using the proposed formalism

The construction of super-coordinates and super-spinors suggests a straightforward twistorialization. One would only replace the super-embedding space and super-spinors with super-twistor space and corresponding super-spinors. Dimensional reduction should give essentially the 4-D theory apart from the variation of the radius of the twistor space predicting variation of cosmological constant. The size scale of CD would correspond to the size scale of the twistor space for M^4 and for CP_2 the size scale would serve as unit and would not vary.

1. Replace the coordinates of twistor space with superspinors expressed in terms of quark and anti-quark spinors lifted to the corresponding spinors of twistor space. Express 6-D Kähler action in terms of super-coordinates.

2. Replace H-spinors with the spinors of 12-D twistor space and assume only quark chirality. By the bundle property of the twistor space one can express the spinors as tensor products of spinors of the twistor spaces $T(M^4)$ and $T(CP_2)$. One can express the spinors of $T(M^4)$ tensor products of spinors of M^4 - and S^2 spinors locally and spinors of $T(CP_2)$ as tensor products of CP_2 - and S^2 spinors locally. Chirality conditions should reduce the number of 2 spin components for both $T(M^4)$ and $T(CP_2)$ to one so that there are no additional spin degrees of freedom.

   The dimensional reduction can be generalized by identifying the two S^2 fibers for the preferred extremals so that one obtains induced twistor structure. In spinorial sector the dimensional reduction must identify spinorial degrees of freedom of the two S^2s by the proposed chirality conditions also make them non-dynamical. The S^2 spinors covariantly constant in S^2 degrees of freedom.

   Define the twistor counterpart of the analog of modified Dirac action using same general formulas as in case of H.

3. Identify super spinors as sum of odd monomials of theta parameters with quark number 1 identified as oscillator operators. Identify super-Dirac action for twistor space by replacing $T(H)$ coordinates with their super variants and Dirac spinors with their super variants.

16.3 Are quarks enough to explain elementary particle spectrum?

TGD based SUSY involves super-spinors and super-coordinates. Suppose that one has a cognitive representation defined by the points of space-time surface with coordinates in an extension of rationals defining adele and belonging to the partonic 2-surfaces defined by the intersections of 6-D roots of octonionic polynomials with 4-D roots. This representation has H counterpart.

Cognitive representation gives rise to a tensor product of these algebras and the oscillator operators define a discretized version of fermionic oscillator operator algebra of quantum field theories. One would have interpretation as many-fermion states but the local many-fermion states would have particle interpretation. This would replace fermions of the earlier identification of elementary particles with SUSY multiplets in the proposed sense. This brings in large number of new particles. One can however ask whether the return to the original picture in which single partonic 2-surface corresponds to elementary particle could be possible. Certainly it would simplify the picture dramatically.

Could this picture explain elementary particle spectrum and how it would modify the recent picture?: these are the questions.

16.3.1 Attempt to gain bird's eye of view

Rather general arguments suggest that SYM action plus Super-Dirac action could explain elementary particle spectrum. Some general observations help to get a bird's eye of view about the situation.

1. The antisymmetric tensor products for fermions and anti-fermions produce states with same spectrum of electro-weak quantum numbers irrespectively of whether the fermion and anti-fermion are at same point or at different points. Which option is correct or are these options correspond analogous to two different phases of lattice gauge theory in which nodes *resp.* links determine the states? Only multi-local states containing fermions with identical spin and weak isospin at different points are not possible as local states.

   There is no point in denying the existence of either kind of states. What suggests itself is the generalization of electric-magnetic duality relating perturbative Coulomb phase in which ordinary particles dominate and the non-perturbative phase in which magnetic monopoles dominate. I have considered what I have called weak form of electic-magnetic duality already earlier [K127] but as a kind of self-duality stating that for homologically charged partonic 2-surfaces electric and magnetic fluxes are identical. The new picture would conform with the view of ordinary QFT about this duality.

2. The basic distinction between TGD and standard model is that color is not spin-like quantum number but represented as color partial waves basically reducing to the spinor harmonics plus super-symplectic generators carrying color quantum numbers. Spinor harmonics as such have non-physical correlation between color and electro-weak quantum numbers [K65] although quarks and leptons correspond to triality $t = 1$ and triality $t = 0$ states.

3. It turns out that one could understand quarks, leptons, and electro-weak gauge bosons and their spartners as states involving only single partonic 2-surface [K26]: this would give essentially the original topological model for family replication in which partonic 2-surfaces were identified as boundary components of 3-surface. In principle one can allow also quarks and gluons with unit charge matrix with color partial waves defining Lie-algebra generator as bosonic states. Could these states correspond to free partons for which perturbative QCD applies at high energies?

   Also color octet partial waves of electro-weak bosons and Higgs and the predicted additional pseudo-scalar - something totally new - are possible as both local and bi-local states. There would be no mixing of $U(1)_Y$ state and neutral $SU(2)_w$ states for color octet gluon. In this sense electro-weak symmetry breaking would be absent.

4. Electro-weak group as holonomy group of CP_2 can be mapped to the Cartan group of color group, and electro-weak and color quantum numbers would relate like spin and angular

momentum to each other. This encourages to think that there are deep connections between electro-weak physics and color physics, which have remained hidden in standard model.

The conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) of hadron physics suggests a strong connection between color physics and electro-weak physics. There is also evidence for so called X bosons with mass 16.7 MeV [?] [L39] suggesting in TGD framework that weak physics could have fractally scaled down copy in hadronic and even nuclear scales.

Could ordinary gluons be responsible for CVC whereas colored variants of weak bosons and Higgs/pseudo-scalar Higgs would be responsible for PCAC? Usually strong force in hadronic sense is assigned with pion exchange. This approach does not work perturbatively. Could one assign strong force with the exchange of pseudo-scalar, and colored variants of gluons, pseudo-scalar, and Higgs?

5. Hitherto it has been assumed that homology charges (Kähler magnetic charges) characterize flux tubes connecting the two wormhole throats associated with the monopole flux of elementary particle. Could one understand the bi-local or multi-local objects of this kind as exotic phase analogous to magnetic monopole dominated phase of gauge theories as dual of Coulomb phase?

   Hadrons would certainly be excellent candidates for monopole dominated phase. Gluons would be pairs of quarks associated with homologically charged partonic 2-surfaces with opposite homology charges. Gluons would literally serve as "glue" in the spirit of lattice QCD. Gluons and hadrons would be multi-local states made from quarks and gluons as homologically trivial configurations with vanishing total homology charge.

6. Is there a correlation between color hyper-charge and homology charge forcing quarks and gluons to be always in this phase and forcing leptons to be homologically neutral? This could provide topological realization of color confinement. The simplest option is that valence quarks have homology charges $2, -1, -1$ summing up to zero. This was one of the first ideas in TGD about 38 years ago.

   One can also imagine that the homological quark charges $(3, -2, -1)$ summing up to zero define a classical correlate for the color triplet of quarks, a realization of Fermi statistics, and allow to understand color confinement topologically. The color partial waves in H would emerge at the embedding space level and characterize the ground states of super-symplectic representations. Color triplets of quarks and antiquarks could thus correspond to homology charges $(3, -2, -1)$ and $(-3, 2, 1)$ and neutral gluons could be superpositions of pairs of form $(q, -q)$, $q = 3, -1, -1$. Charged gluons as flux tubes would not be possible in the confined phase.

7. Is monopole phase possible also for leptons as general QFT wisdom suggests? For instance, could Cooper pairs could be flux tubes having members of Cooper pair - say electrons - at its ends and photons in this phase be superposition of fermion and anti-fermion at the ends of the flux tube and monopole confinement would make the length of flux tube short and photon massive in superconducting phase.

16.3.2 Comparing the new and older picture about elementary particles

The speculative view held hitherto about elementary particles in TGD Universe correspond to the TGD analog of the magnetic monopole dominated phase of QFTs. This view is considerably more complicated than the new view and involves unproven assumptions.

1. Identification of elementary particles

   Old picture: Ordinary bosons (and also fermions) are identified as multilocal many-fermion states. The fermions and anti-fermions would reside at different throats of the 2 wormhole contacts associated with a closed monopole flux tube associated with the elementary particle and going through wormhole contact to second space-time sheet. All elementary particles are analogous to hadron-like entities involving closed monopole flux tubes.

One can raise objections against this idea. Leptons are known to be very point-like. One must also assume that the topologies of monopole throats are same for given genus in order that p-adic mass calculations make sense. The assumption that quarks correspond to monopole pairs makes things unnecessarily complex: it would would be enough to assume that they correspond to partonic 2-surfaces with monopole charge at the "ends" of flux tubes at given space-time sheet.

One must assume that the genus of the 4 throats is same for known elementary particles: this assumption looks rather natural but can be criticized. The correlations forced by preferred extremal property should of course force the genera of wormhole throats to be identical.

New picture: Elementary fermions would be partonic 2-surfaces. Leptons would have vanishing homology charge. Elementary bosons could be simply pairs of fermion anti-fermion located at the opposite ends of flux tubes. This would dramatically simplify the topological description of particle reactions. In the case of quarks however the homological space-time correlate of color confinement is attractive and would force monopole flux tubes. It turns out that this picture corresponds to the simplest level in the $h_{eff} = nh_0$ hierarchy. One could also see leptons and quarks as analogs of perturbative and non-perturbative monopole dominated phases of gauge theories.

Flux tubes could allow to understand phases like super-conductivity involving massivation of photons (Meissner effect). For instance, Cooper pairs could correspond closed flux tubes involving charged fermions at their "ends". In high Tc super-conductivity Cooper pairs in this sense would be formed at higher critical temperature and at lower critical temperature they would form quantum coherent phase [K89, K90]. Flux tube picture could also allow to understand strongly interacting phases of electrons.

2. Electroweak massivation

   Old picture: Electro-weak massivation has been assumed to involve screening of electro-weak isospin by a neutrino pair at the second wormhole contact. The screening is not actually necessary in p-adic thermodynamics in its recent form since the thermal massivation is due to the mixing of different mass eigenstates.

   New picture: There is no need to add pairs of right- and left-handed neutrino to screen the weak charges in the scale of flux tube.

3. Identification of vertices

   Old picture: In old picture one could do almost without vertices: in the simplest proposal particle reactions would correspond to re-arrangements of fermions and antifermions so that fermion and antifermion number would be conserved separately. Therefore one needs an analog of vertex in which partonic 2-surface turns back in time in order to describe creation of particle pairs and emission of bosons identified as fermion-antifermion pairs.

   New picture: In vertices fermions and antifermions assignable to super spinor component would be redistributed between different orbits of partonic 2-surfaces meeting along their ends at the 6-D braney object in M^8 picture or turn backwards in time - the interpretation for this might be in terms of interaction with classical induce gauge field. What is new are the new vertices corresponding to the monomials of oscillator operators in the super-spinor. The original identification of particles (given up later) as single partonic 2-surface predicts genus-generation correspondence without additional assumptions. Both old and new picture predict also higher gauge boson genera for which some evidence exists: TGD predictions for the masses are correct [K71].

16.3.3 Are quarks enough as fundamental fermions?

For the first option - call it Option a) - quarks and leptons would define their own super-spinors. Whether only quark or lepton-like spinors are enough remains still an open question.

1. I have also considered the possibility that quarks are actually anti-leptons carrying homology charge and have anomalous em charge equal to $-1/3$ units. One might perhaps say that quarks are kind of anyonic states [K85]. It is however difficult to understand how the coupling

to Kähler form could be dynamical and have values $n = -3$ and $n = 1$ for homologically neutral and charged states respectively. This would mean that only lepton like θ parameters appear in super-coordinates and only leptonic Dirac action is needed.

2. For this option proton would be bound state of homologically charged leptons. This in principle allows decays of type $p \rightarrow e^+ ...$ and $p \rightarrow e^+ + e^+ + \overline{\nu}$ requiring that the 3 partonic 2-surfaces fused with non-trivial homology charges fuse to single homologically trivial 2-surface. This form of proton instability would be different from that of GUTs. The topology changing process is expected to be slow. Is the introduction of two super-octonionic θ parameters natural assignable to B and L or is single parameter enough?

3. The coupling to Kähler form is not explicitly visible on the bosonic action but is visible in modified Dirac action. Could leptonic modified Dirac action transform to quark type modified Dirac action? This does not seem plausible.

The super-Dirac action for quarks however suggests another option, call it Option b). Leptons could be local 3-quark states.

1. Could one identify leptons as local 3 quark composites - essentially anti-baryons as far as quantum numbers are considered - but with different p-adic scale and emerging from the super-Dirac action for quarks as purely local states with super-degree $d = 3$? Could one imagine totally new approach to the matter antimatter asymmetry?

   Leptons would be purely local 3-quark composites and baryons non-local 3-quark composites so that charge neutrality alone would would guarantee matter-antimatter symmetry at fundamental level. Anti-quark matter would slightly prefer to be purely local and quark matter 3-local. The small CP violation due to the M^4 part of Kähler action forced by twistor lift should explain this asymmetry.

   Leptons and anti-leptons would drop from thermal equilibrium with quarks at some stage in very early cosmology. The reason would be the slowness of the reactions producing local 3-quark composites from quarks. This slowness is required also by the stability of proton. Opposite matter anti-matter asymmetries at the level of both leptons and quarks would have been generated at this stage by CP violation and would have become visible after annihilation.

2. The local baryons would have much simpler spectrum and would correspond for given genus g (lepton generation) to the baryons formed from u and d quarks having however no color. There would be no counterparts for higher quarks. This would suggests that (L, ν_L) could be local analog of (p, n).

   For ordinary baryons statistics is a problem and this led to the introduction of quark color absent for local states. The isospin structure of the local analogs of p and n is not a problem. In uud (udd) type states allowed by statistics the spins of the u (d) quarks must have opposite spin. The analogs of Δ resonances are not possible so that one would obtain only the analogs of p and n!

3. The widely different mass scales for leptons and quarks would be due to locality making possible different ramified primes for the extension of rationals. The widely differing p-adic length scales of leptons and neutrinos could be undersood if the ramified prime for given extension can be different for the particles super-multiplets with same degree of octonionic polynomial. This could be caused by electroweak symmetry breaking. The vanishing electroweak quantum numbers of right-handed neutrino implies a dynamics in sharp contrast with that of neutron, whose dynamics would be dictated by non-locality.

   Also local pions are possible. The lepto-pions of lepto-hadron hypothesis [K119] could correspond to either local pions or to pion-like bound states of lepton and anti-leptons. There is evidence also for the muon- and tau-pions.

4. This idea might provide a mathematically extremely attractive solution to the matter antimatter asymmetry: matter and antimatter would be staring us directly into eyes. The alternative TGD inspired solution would be that small CP breaking would induce opposite matter-antimatter asymmetries inside long cosmic strings and in their exteriors so that annihilation period would lead to the observed asymmetry.

The decay $p \to e^+ + X$ could in principle take place and also the reverse decay $e^+ \to p + X$ can be considered in higher energy collisions of electron. The life-time for the decay modes predicted by GUTs is extremely long - longer than 1.67×10^{34} years (see `http://tinyurl.com/nqco2j7`). This fact provides a killer test for the proposal.

One should estimate the life-time of proton in number theoretic approach. The corresponding SUSY vertex corresponds to a Wick contraction involving 4 terms in super-Dirac action: the trilinear term for quarks and 3 linear terms.

1. The vertex would associated with a partonic 2-surface at which 3 incoming quark space-time sheets and outgoing electron space-time sheet meet. At quark level the vertex means an emanation of 3 quark lines from single 3-quark line at a point of partonic 2-surface in the intersection of the ends of 4 space-time surfaces with 6-sphere $t = r_n$ defining a universal root of octononic polynomial $P(o)$. t is M^4 time coordinate [L85]. The vertex itself does not seem to be small.

2. A fusion of 3 homologically non-trivial partonic 2-surfaces to single partonic 2-surface with trivial homology charge cannot occur since partonic 2-surfaces with different homology charge cannot co-incide.

   The reaction $p \to e^+ + ..$ can occur only if the quark-like partonic 2-surface fuse first to single homologically trivial partonic 2-surface: this would correspond to de-confinement phase transition for quarks. After that the 3 quark lines would fuse to single e^+ line.

   (a) To gain some intuition consider two oppositely oriented circles around a puncture of a plane with opposite homology charges. The circles can reconnect to homologically trivial circle. Instead of circles one would now have 3 homologically trivial quark-like 2-surfaces at three light-like boundaries between Minkowskian and Euclidian regions of the space-time surface representing proton. First 2 quark-like 2-surfaces would touch and develop a wormhole contact connecting them. After that the resulting di-quark 2-surface and third quark 2-surface would fuse. The 3 quarks would be now analogous to de-confined quarks.

   (b) At the next step the 3 separate quark lines would fuse to single one. This process must occur in single step since di-quark cannot correspond to single point because the Dirac super-polynomial is odd in oscillator operators and has quark number 1. The fusion point would correspond to 3 degenerate roots of the octonionic polynomial associated with the partonic 2-surface. This partonic 2-surface would be associated with $t = r_n$ hyperplane of M^4 and it would become leptonic 3-surface.

   (c) 3 4-D sheets defined by the roots of the octonionic polynomial should meet at the vertex assignable to $t = r_n$ hyper-plane. This gives 2 additional conditions besides the conditions defining space-time sheets. This for both the protonic and positronic space-time sheets. One would have double quantum criticality. The tip of a cusp catastrophe serves as an analog. Since the coefficients of the octonionic polynomial are rational numbers, it might be possible to estimate the probability for this to occur: the probability could be proportional to the ratio N_2/N_0 of the number N_2 of doubly critical points to the number N_0 of all points with coordinates in the extension. This could make the process very rare.

It must be however emphasized that also the option in which also leptons are fundamental fermions cannot be excluded.

16.3.4 What bosons the super counterpart of bosonic action predicts?

It has been already noticed that the spectra of fermion-antifermion states are identical for local and bi-local states if one assumes that the wave function in the relative coordinate of fermion and anti-fermion is symmetric. This does not yet imply that the particle spectrum is realistic in the case of the bosonic action.

The situation is simplified considerably by the facts that color is not spin-like quantum number but analogous to momentum and can therefore be forgotten, family replication can be explained topologically, and depending B and L are separately conserved for Option a) but for Option b) L reduces to B since leptons would be local 3-quark composites. Let us restrict first the considered to Option b).

1. What kind of spectrum would be predicted? Consider first quark Clifford algebra formed by the oscillator operators defining the spartners of quark without any conditions on total quark number of the monomial Forgetting color, one has 8 states coming from left and right handed weak doublet and their anti-doublets. The numbers of elements $N(k)$ in Clifford algebra with given quark number $B = k = N(q) - N(\overline{q})$ is given by $N(k) = \sum 0 \leq q \leq 4 - kB(4, q+k) \times B(4, q)$ in terms of binomial coefficients.

   For $B = 0$ one obtains $N(0) = \sum 0 \leq q \leq 4B(4, q)^2 = 70$ states. The states corresponding to the same degree of oscillator operator polynomial and therefore having fixed $q + \overline{q} = B + \overline{B}$ have same masses. For $q - \overline{q} = 0$ bosonic state having $q = \overline{q} = 0$ with fixed k one has $q+\overline{q} = 4+k$ so that one has $N(k) = B(4, k)^2$ ($N(k)$ states with same mass even after p-adic massivation). The numbers $N(k)$ are $(1, 4^2 = 16, 6^2 = 36, 4^2 = 16, 1)$.

2. The number of $q\overline{q}$ type states in super-Kähler action is 16. If one considers super-symmetrization of the bosonic action, these states would correspond to bosons. Could these states allow an interpretation in terms of the known gauge bosons and Higgs? Weak bosons correspond to 4 helicity doublets giving 8 states. Higgs doublet corresponds to doublet and its conjugate. There is also a pseudo-scalar doublet and its conjugate.

   Gluon cannot belong to this set of states, which actually conforms with the fact that gluon corresponds to CP_2 isometries rather than holonomies and gluon corresponds to CP_2 partial wave since color is not spin-like quantum number. Known particle would give 8+2+2=12 states and pseudo-scalar doublets the remaining 4. This kind of pseudo-scalar states are predicted both as local and the bi-local states. As already explained, one can however also understand gluons in this picture as octet color partial waves. Also color octet variants of $SU(2)_w$ weak bosons are predicted.

3. There are actually some indications for a Higgs like state with mass 96 GeV (see `http://tinyurl.com/yxnmy8c7`) . Could this be the pseudo-scalar state. Higgs mass 125 GeV is very nearly the minimal mass for $k = 89$. The minimal mass for $k = 90$ would be 88 GeV so that the interpretation as pseudo-scalar with $k = 90$ might make sense. The proposal that gluons could have also weak counterparts suggests that also the pseudo-scalar could have this kind of counterpart. The scaling of the mass of the Higgs like state with $k = 90$ to $k = 112$ ($k = 113$ corresponds to nuclear p-adic scale) would give mass $m(107) = 37.5$ MeV. Kh.U. Abraamyan *et al* have found evidence for pion like boson with mass 38 MeV [?, ?, ?] (see `http://tinyurl.com/y7zer8dw`).

4. For Option b) only monomials with $N(q)-N(\overline{q}) = k = 1$ are allowed in q_s and leptons would be local 3-quark states and currents formed from them would appear in super-Kähler action. One would obtain $N(k = 1) = \sum 0 \leq q \leq 3B(4, q+1) \times B(4, q) = 56$ statesi quark multiplet. There would be no doubling gauge bosons since only one H-chirality would be present. The observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.

Option b) involving only quarks as fundamental fermions does not predict unobserved gauge bosons whereas Option a) involving both leptons and quarks as fundamental fermions does so.

1. For Option a) taking into account quarks and restricting to electro-weak bosonic states to those with ($B = L = 0$) leads to a doubling of bosonic states at $k = 2$ level. The couplings of gauge bosons require that the states are superpositions of quark and lepton pairs with coefficients proportional to the coupling parameters. There are two orthogonal superpositions of quark and lepton pairs having orthogonal charge matrices with inner product defined by trace for the product. Ordinary gauge bosons correspond to the first combination.

   The orthogonality of charge matrices gives a condition on them. The charged matrices having vanishing trace can be chosen that they have opposite signs for opposite H-chiralities. For charge matrices involving unit matrix one must have charge matrices proportional to (-3,1) for (L,q) one must have (1,3) for second state. For gluons there is no condition if one treats color octet as Lie algebra generator with vanishing trace. The problem is that there is no experimental evidence for these bosons.

2. For Option b) leptons would be local 3-quark states and spartners of quarks. There would be no doubling gauge bosons since only one H-chirality would be present. The observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.

3. Option b) predicts that given quark with given isospin and M^4 helicity L or R), say u_L, has 5 spartners with same quantum numbers given by $u_L u_R \overline{u}_L$, $u_L d_R \overline{d}_L$, $u_L d_L \overline{d}_R$; $u_R d_L \overline{d}_L$; and $d_L d_R \overline{u}_L$. These 6 states cannot correspond to quark families and SUSY breaking due to the possibility of having different p-adic scale (ramified prime) making the mass scale of the spartners large is suggestive.

There would be two phases of matter corresponding to local and bi-local states (baryons would be 3-local states).

1. For both phases electro-weak bosons and also gluons with electro-weak charge matrix 1 to bosonic super action as states involving only single partonic 2-surface. As already mentioned, also color counterparts of $SU(2)_w$ bosons are possible. Also graviton could correspond to spartner for bosonic super-action. This would give essentially the original model for family replication. 2-surfaces would be homologically trivial in this phase analogous to Coulomb phase.

2. In the dual phase the bi-local states would correspond to non-vanishing homology charges for quarks at least. In this phase one should assign also to leptons 2 wormhole contacts. In super-conducting phase it could the second electron of Cooper pair. Massive photons in this phase would consist of homologically charged fermion pairs. Lepton could also involve screening lepton-neutrino pair at second wormhole contact.

The universality of gauge boson couplings provides a test for the model.

1. In bi-local model gauge bosons would correspond to representations of a dynamical symmetry group $SU(3)_g$ associated with the 3 genera [K26]. Bosons would correspond to octet and singlet representations and one expects that the 3 color neutral states are light. This would give 3 gauge boson generations. Only the couplings of the singlet representation of $SU(3)_g$ would be universal and higher generations would break universality both for both gluons and electro-weak bosons. There is evidence the breaking of universality as also for second and third generation of some weak bosons and the mass scales assigned with Mersenne primes above M^{89} are correct [K71].

2. If also fermions correspond to closed flux tubes with 2 wormhole contacts, the fermion boson couplings would correspond to the gluing of two closed flux tube strings along their both "ends" defined by wormhole contacts. A pair of 3-vertices for Feynman diagrams would be in question. If fermions are associated with single wormhole contact, its is not so easy to imagine how the closed bosonic flux tube could transform to single wormhole contact in the process. The wormhole contacts that meet and have opposite fermion numbers should disappear. This is allowed in the scenario involving 6-branes if the magnetic flux is trivial as it must be. For quarks and gluons the homology charges must be opposite if wormhole contact is to disappear.

3. If gauge bosons correspond to local fermion pairs, the most natural boson states have fixed value of g apart from topological mixing giving rise to CKM mixing just like fermions and universality is not natural. One can of course assume topological mixing guaranteeing it. Ordinary gauge bosons should be totally de-localized in the space of 3 lowest genera [K26] (analogous to constant plane waves) in order to have universality. The vertices could be understood as a fusion of partonic 2-surfaces. One should however understand why the mixing is so different for fermions and bosons. SUSY would suggest identical mixings.

The simplest model corresponds to quarks as fundamental fermions. Leptons and various bosons would be local composites in perturbative phase. In monopole dominate phase hadronic quarks would have homology charges and gluons would be pairs of quark and anti-quark at opposite throats of closed monopole flux tube. Basically particle reaction vertices would correspond to gluing of 3-surfaces along partonic 2-surfaces at 3-spheres defining $t = r_n$ hyperplanes of M^4.

16.4 Is it possible to have leptons as (effectively) local 3-quark composites?

The idea about leptons as composites of 3 quarks is strongly suggested by the mathematical structure of TGD. In [L92] a proposal that leptons are local composites of quarks. In [L123, L109, L110] a more general idea that leptons look like local composites of quarks in scale longer than CP_2 scale defining the scale of partonic 2-surface assignable to the particle.

A strong mathematical motivation for the proposal is that quark oscillator operators are enough to construct the gamma matrices of the "world of classical worlds" (WCW) and leptonic oscillator operators corresponding to opposite chirality for $H = M^4 \times CP_2$ spinors are somehow superfluous.

The proposal has profound consequences. One might say that SUSY in the TGD sense has been below our nose for more than a century. The proposal could also solve matter-antimatter asymmetry since the twistor-lift of TGD predicts the analog of Kähler structure for Minkowski space and a small CP breaking, which could make possible a cosmological evolution in which quarks prefer to form baryons and antiquarks to form leptons.

The objection against the proposal is that the leptonic analog of Δ might emerge. One must explain why this state is at least experimentally absent. In [L92] I did not develop a detailed argument for the intuition that one indeed avoids the leptonic analog of Δ. In this article the construction of leptons as effectively local 3 quark states allowing effective description in terms of the modes of leptonic spinor field in $H = M^4 \times CP_2$ having H-chirality opposite to quark spinors is discussed in detail.

16.4.1 Some background

Some background is necessary.

1. In TGD color is not spin-like quantum number but corresponds to color partial waves in CP_2 for H-spinors describing fundamental fermions distinguished from fermions as elementary particles.

   Different chiralities of H-spinors were identified in the original model as leptons and quarks. If quarks couple to $n = 1$ Kähler gauge potential of CP_2 and leptons to its $n = 3$ multiple, ew quantum numbers of quarks and leptons come out correctly and lepton and quark numbers are separately conserved.

2. Few years ago emerged the idea that fundamental leptons to be distinguished from physical leptons are bound states of 3-quarks. They could be either local composites or look like local composites in scales larger than CP_2 size scale assignable to partonic 2-surface associated with the lepton.

3. The spin, ew quantum numbers associated with $SU(2)_L \times U(1)_R$ are additive and these quantum numbers should come out correctly for states with leptonic spin and ew numbers.

   Fundamental leptons/quarks are not color singlets/triplets although have vanishing triality. The color quantum numbers also correlate with ew quantum numbers and M^4 helicity/handedness. Only the right-handed neutrino ν_R is a color singlet. The mass squared values of the resulting states deducible from the massless Dirac equation in H are non-vanishing since CP_2 partial waves carry mass of order CP_2 mass.

   The application of color octet generators of super-symplectic algebra (SSA) of super-Kac-Moody algebra (SKMA) with non-vanishing conformal weight contributing to mass squared can guarantee that color quantum numbers are those of physical leptons and quarks. In p-adic mass calculations one must assume negative half-integer valued ground state conformal weight $h_{vac} < 0$.

There are two challenges.

1. One must construct leptons as local of the effectively local 3-quark composites. The challenge is to prove that the resulting states with spin and ew quantum numbers possess the color quantum numbers of fundamental leptons.

2. A priori one cannot exclude leptonic analog of Δ resonance obtained in the quark model of baryons as states for which the wave functions in spin and ew spin degrees of freedom are completely symmetric. The color wave function would be indeed completely antisymmetric also for the leptonic Δ. The challenge is to explain why they do not exist or are not observed.

16.4.2 Color representations and masses for quarks and leptons as modes of $M^4 \times CP_2$ spinor field

It would be also highly desirable to obtain for the masses of 3-quark states the same expressions as embedding space Dirac operator predicts for leptonic masses. The masses depend on ew spin but are same for right and left-handed modes except in the case of right-handed neutrino. This could fixes the value of h_{vac} for leptons if it is assumed to be representable as 3-quark state. Empirical data are consistent with its absence from the spectrum.

The color representations associated with quark and lepton modes of $M^4 \times CP_2$ spinor fields were originally discussed by Hawking and Pope [A62] and are considered from TGD point of view in [K65].

Consider first quarks. For U_R the representations $(p+1,p)$ with triality 1 are obtained and $p=0$ corresponds to color triplet 3. For D_R the representations $(p,p+2)$ are obtained and color triplet is missing from the spectrum ($p = 0$corresponds to $\overline{6}$). The representations and masses are the same for the left handed representations in both cases since the left handed modes are obtained by applying CP_2 Dirac operator to the right-handed modes.

The CP_2 contributions to the quark masses are given by the formula

$$\begin{aligned} m^2(U,p) &= \frac{m_1^2}{3}\left[p^2+3p+2\right] \quad , \quad p\geq 0 \ , \\ m^2(D,p) &= \frac{m_1^2}{3}\left[p^2+4p+4\right] = \frac{m_1^2}{3}(p+2)^2 \quad , \quad p\geq 0 \ , \\ m_1^2 &\equiv 2\Lambda \ . \end{aligned} \tag{16.4.1}$$

Here Λ is cosmoloigal constant characterizing the CP_2 metric. The mass squared splitting between U and D type states is given by

$$\Delta m^2(D,U) = m^2(D,p) - m^2(U,p) = \frac{m_1^2}{3}(p+2) \ . \tag{16.4.2}$$

Consider next leptons. Right handed neutrino ν_R corresponds to (p,p) states with $p \geq 0$ with mass spectrum

$$m^2(\nu) = \frac{m_1^2}{3}\left[p^2+2p\right] \quad , \quad p\geq 0 \ . \tag{16.4.3}$$

Charged handed charged leptons L correspond to $(p,p+3)$ states with mass spectrum

$$m^2(L) = \frac{m_1^2}{3}\left[p^2+5p+6\right] \quad , \quad p\geq 0 \ . \tag{16.4.4}$$

$(p, p+3)$ instead of (p, p) reflects the fact that leptons couple to 3-multiple of Kähler gauge potential. Right-handed neutrino has however vanishing total coupling.

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p, p) = (0, 0)$ state) annihilates it.

The mass splitting between charged leptons and neutrinos is given by

$$\Delta m^2(L, \nu) = m^2(L, p) - m^2(\nu, p) = m_1^2(p + 2) = 3\Delta m^2(D, U) \ , \tag{16.4.5}$$

and is 3 times larger than the corresponding mass splitting. The mass splitting for leptons as states of type UUD and UDD is however different. If mass squared is additive as assumed in p-adic mass calculations one has $\Delta m^2(UDD, UUD) = \Delta m^2(D, U)$. The condition that the mass splitting for lepton states is the same as predicted by the identification as 3-quark states requires that the scale factor m_1^2 for 3 quarks states is 3 times larger than for quarks:

$$m_1^2(L) = 3m_1^2(q) \ . \tag{16.4.6}$$

16.4.3 Additivity of mass squared for quarks does not give masses of lepton modes

It would be natural that the same values for the leptons as 3-quark composites are same as for leptons as fundamental fermions. It is interesting to see whether the additivity of the mass squared values conforms with this hypothesis.

The sums of mass squared values for UUD (charged lepton) and UDD (neutrino) type states are given by

$$\begin{aligned} m^2(UUD) &= 2m(U)^2 + m(D)^2 = 3p^2 + 10p + 8 \ , \\ m^2(UDD) &= 2m(D)^2 + m(U)^2 = 3p^2 + 11p + 10 \ . \end{aligned} \tag{16.4.7}$$

These mass squared values are not consistent with the values proportional to the mass squared values proportional to $p^2 + 5p + 6$ for L and to $p(p + 2)$ for neutrinos. Covariantly constant right handed neutrino is not possible as a 3-quark state and this conforms with empirical facts.

The working hypothesis that mass squared is additive can be of course given up and a more general condition could be formulated in terms of four-momenta:

$$\begin{aligned} & p_1(U) + p_2(U) + p(D))^2 \\ & = 2m(U)^2 + m(D)^2 + 2\sum[p_1(U) \cdot p_2(U) + (p_1(U) + p_2(U)) \cdot p(D)] = km(L)^2 \ , \\ & (p(U) + p_1(D) + p_2(D))^2 \\ & = m(U)^2 + 2m(D)^2 + 2\sum[p_1(D) \cdot p_2(D) + (p_1(D) + p_2(D)) \cdot p(U)] = km(\nu)^2 \ . \end{aligned} \tag{16.4.8}$$

k is proportionality constant. These condition give single constraint in the 9-dimensional 3-fold Cartesian power of 3-D mass shells. The constraint is rather mild.

16.4.4 Can one obtain observed leptons and avoid leptonic Δ?

The antisymmetry of the wave function under exchange of quark states gives a strong constraint and fixes the allowed states. Does one obtain states with the quantum numbers of observed leptons as color singlets, and can one avoid the leptonic analogue of Δ?

1. For ordinary leptons complete color antisymmetry would require a complete symmetry under permutations of spin-ew quantum numbers: there are four states altogether. Antisymmetrization would be completely analogous to that occurring for baryons as 3-quark states and would require that fundamental leptons are antisymmetric color singlets.

2. The standard quark model picture natural for strong isospin does not conform with spin-ew symmetries and the resulting states need not allow an interpretation as effective modes of fundamental leptonic spinors. For $SU(2)_L \times U(1)_R$ the situation changes since right-handed helicities are $SU(2)_L$ singlets. The states of form $U_L D_R U_R$ (L_R) and $D_L D_R U_R$ (ν_R) could correspond to right-handed leptons and states of form $U_L D_R U_R$ (L_L) and $D_L D_R U_R$ (ν_L) to left-handed leptons.

3. The manipulation of Yang Tableaux (`https://cutt.ly/Ik9SGuU`) allow to see when a color singlet is contained in all 3-fold tensor products - that is $3 \otimes 3 \times 3$, $3 \times 3 \times \overline{6}$, $3 \times \overline{6} \times \overline{6}$, and $\overline{6} \times \overline{6} \times \overline{6}$ - formed from the representations 3 and $\overline{6}$.

   One has $3 \otimes 3 = \overline{3} + 6$ and $\overline{6} \otimes \overline{6} = 6 + 15_1 + 15_2$. Both $\overline{3} \otimes 3 = 1 \oplus 2 \times 8 \oplus 10$ and $\overline{6} \otimes 6 = 1 \oplus 8 \oplus 27$ contain singlet and octet.

   Therefore both $3 \otimes 3 \times 3$ (UUU) and $\overline{6} \times \overline{6} \times \overline{6}$ (DDD) contain 1 and 8. $3 \otimes 3 \otimes \overline{6}$ (UUD corresponding to charged lepton) contains $6 \otimes \overline{6}$ and therefore both 1 and 8. However, $3 \otimes \overline{6} \otimes \overline{6}$ (neutrino as UDD) contains neither singlet nor octet.

4. The singlet contained in $\overline{6} \otimes 6$ should be also antisymmetric under the permutations of the color partial waves of quarks in 6. The singlet state has representation of the form $B_{KLM} A^K A^L A^M$, where $A^K = A^K_{rs} q^r q^s$ is the representation of $\overline{6}$ in terms of color triplet q^i. The tensor G_{KLM} should be antisymmetric. Since the singlet comes from Yang diagram as a vertical column, which corresponds to an anti-symmetric representation of S_3t, it seems that it is indeed antisymmetric.

   If this is the case, UUU and DDD singlets are indeed antisymmetric with respect to the exchange of quarks, and the state in spin-ew degrees of freedom can be totally symmetric.

5. As found, $\overline{6} \otimes 3 \times 3)$ (charged lepton as UUD) contains both 1 and 8 and 1 is antisymmetric as a full vertical column in the Yang diagram. If charged lepton corresponds to 1 it is analogous to proton in these degrees of freedom.

   $\overline{6} \otimes \overline{6} \times 3)$ (neutrino as DDU) contains neither 1 nor 8. In both cases an entanglement between color and spin-ew degrees of freedom is implied.

   Remark: Baryonic quarks reside at distinct partonic 2-surfaces and allow separate color neutralization by SSA or SKMA generators and are color triplets so that the standard picture about color confinement prevails in the baryonic sector.

6. If the 3-quark state is not a color octet, the operators needed to cancel the negative conformal weight must consist of at least two SSA or SKMA operators, which are color octets. UUD contains 8 and 1 but UDD does not. For neutrinos which cannot be color octets or singlets, at least 2 color octet generators are required to neutralize the color. For color singlet charged lepton this is not needed since p-adic thermodynamics allows a massless ground state. The difference charged leptons and neutrinos might relate to the fact that the long p-adic length scales for neutrinos are so long as compared to those for charged leptons.

   As has become clear, the neutral Δ type state UDD is not possible since color singlet and octet are not allowed and the neutralization of the negative conformal weight using at least two color generators as in the case of neutrino. Also for other components of Δ color singlet-ness requires at least two generators whereas octet requires only one generator. For color octets a complete symmetry in spin-ew degrees of freedom is not possible.

The conclusion is that charged lepton and charged components of Δ allow for color singlet completely symmetric wave function in spin-ew degrees of freedom unentangled from color. Neutrino and neutral Δ require entanglement between color and spin-ew degrees of freedom.

16.4.5 Are both quarks and leptons or only quarks fundamental fermions?

One of the longstanding open problems of TGD has been which of the following options is the correct one.

1. Quarks and leptons are fundamental fermions having opposite H-chiralities. This predicts separate conservation of baryon and lepton numbers in accordance with observations.
2. Leptons correspond to bound states of 3 quarks in CP_2 scale. This option is simple but an obvious objection is that they are expected to have mass of order CP_2 mass. Baryons could decay to 3 leptons. Also GUTs have this problem. This scenario also allows the existence of exotic leptons as analogs of Delta resonances for baryons.

I haven't been able to answer this question yet and several arguments supporting the quarks + leptons option have emerged.

Consider first what is known.

1. Color is real and baryons are color singlets like leptons.
2. In QCD, it is assumed that quarks are color triplets and that color does not correlate with electroweak quantum numbers, but this is only an assumption of QCD. Because of quark confinement, we cannot be sure of this.

The TGD picture has two deviations from the QCD picture, which could also cause problems.

1. The fundamental difference is that color and electroweak quantum numbers are correlated for the spinor harmonics of H in both the leptonic and quark sector. In QCD, they are not assumed to be correlated. Both u and d quarks are assumed to be color triplets in QCD, and charged lepton L and ν_L are color singlets.
   (a) Could the QCD picture be wrong? If so, the quark confinement model should be generalized. Color confinement would still apply, but now the color singlet baryons would not be made up of color triplet quark states, but would be more general irreducible representations of the color group. This is possible in principle, but I haven't checked the details.
   (b) Or can one assume, as I have indeed done, that the accompanying color-Kac Moody algebra allows the construction of "observed" quarks as color triplet states. In the case of leptons, one would get color singlets. I have regarded this as obvious. One should carefully check out which option works or whether both might work.
2. The second problem concerns the identification of leptons. Are they fundamental fermions with opposite H-chirality as compared to quarks or are they composites of three antiquarks in the CP_2 scale (wormhole contact). In this case, the proton would not be completely stable since it could decay into three antileptons.
   (a) If leptons are fundamental, color singlet states must be obtained using color-Kac-Moody. It must be admitted that I am not absolutely sure that this is the case.
   (b) If leptons are states of three antiquarks, then first of all, other electroweak multiplets than spin and isospin doublets are predicted. There are 2 spin-isospin doublets (spin and isospin 1/2) and 1 spin-isospin quartets (spin and isospin 3/2). This is a potential problem. Only one duplicate has been detected.
   (c) Limitations are brought by the antisymmetrization due to Fermi statistics, which drops a large number of states from consideration. In addition, masses are very sensitive to quantum numbers, so it will probably happen that the mass scale is the CP_2 mass scale for the majority of states, perhaps precisely for the unwanted states.

It is good to start by taking a closer look at the tensor product of the irreducible representations (irreps) of the color group [K65].

1. The irreps are labeled by two integers (n_1, n_2) by the maximal values of color isospin and hypercharge. The integer pairs (n_1, n_2) are not additive in the tensor product, which splits into a direct sum of irreducible representations. There is however a representation for which the weights are obtained as the sum of the integer pairs (n_1, n_2) for the representations appearing in the tensor product.

   Rotation group presentations simplified example. We get the impulse moment $j_1 + j_2, ...|j_1 - j_2|$. Further, three quarks make a singlet.

2. On basis of the triality symmetry, one expects that, by adding Kac-Moody octet gluons, the states corresponding to (p,p+3)-type and (p,p)-type representations can be converted to each other and even the conversion to color singlet (0,0) is possible. This is the previous assumption that I took for granted and there is no need to give it up.

Let's look at quarks and baryons first.

1. U type spinor harmonics correspond to (p+1,p) type color multiplets, while D type spinor harmonics correspond to (p,p+2) type representations. From these, quark triplets can be obtained by adding Kac-Moody gluons and the QCD picture would emerge. But is this necessary? Could one think of using only quark spinor harmonics?

2. The three-quark state UUD corresponds to irreducible representations in the decomposed tensor product. The maximum weight pair is (3p+2,3p+2) if p is the same for all quarks, while UDD with this assumption corresponds to the maximum weights (3p+1,3p+1+3). The value of p may depend on the quark, but even then we get (P,P) and (P,P+3) as maximal weight pairs. UUU and DDD states can also be viewed.

   Besides these, there are other pairs with the same triallity and an interesting question is whether color singlets can be obtained without adding gluons. This would change the QCD picture because the fundamental quarks would no longer be color triplets and the color would depend on the weak isospin.

3. The tensor product of a (p,p+3)-type representation and (possibly more) gluon octets yields also (p,p)-type representations. In particular, it should be possible to get (0,0) type representation.

Consider next the identification of leptons.

1. For leptons, neutrino nu$_L correspondstoa(p,p) - typerepresentationandchargedlepton$L$toa(p+3,p) - typerepresentation.$

2. Could the charged antilepton correspond to a representation of the type UDD and antineutrino to a representation of the type UUD?

   Here comes the cold shower! This assumption is inconsistent with charge additivity! UDD is neutral and corresponds to (p,p+3) rather than (p,p). You would expect the charge to be 1 if the correspondence for color and electroweak quantum numbers is the same as for the lepton + quark option!

   UUD corresponds to (p,p) rather than (p,p+3) and the charge is 1. You would expect it to be zero. Lepton charges cannot be obtained correctly by adding charge +1 or -1 to the system.

   In other words, the 3-quark state does not behave for its quantum numbers like a lepton, i.e. an opposite spinor with H-chirality as a spinor harmonic.

   Therefore bound states of quarks cannot be approximated in terms of spinor modes of H for purely group-theoretic reasons. The reason might be that leptonic and quark spinors correspond to opposite H-chiralities. Of course, it could be argued that since the physical leptons are color singlets, this kind of option could be imagined. Aesthetically it is an unsatisfactory option.

To sum up, the answers to the questions posed above would therefore be the following:

1. Quark spinor harmonics can be converted into color triplets by adding gluons to the state (Kac-Moody). Even if this is not done, states built from three non-singlet quarks can be converted into singlets by adding gluons.

2. The states of the fundamental leptons can be converted into color singlets by adding Kac-Moody gluons. Therefore the original scenario, where the baryon and lepton numbers are preserved separately, is group-theoretically consistent.

3. Building of analogs of leptonic spinor harmonics from antiquarks is not possible since the correlation between color and electroweak quantum numbers is not correct. I should have noticed this a long time ago, but I didn't. In any case, there are also other arguments that support the lepton + quark option. For example, symplectic *resp.* conformal symmetry representations could involve only quarks *resp.* leptons.

16.5 Appendix: Still about the topology of elementary particles and hadrons

In its recent form TGD allows several options for the model of elementary particles [L92]. I wrote this piece of text because I got worried about details of the definition of wormhole contact appearing as basic building brick of elementary particle.

1. Wormhole contacts in 4-D sense (having Euclidian signature of induced metric) modellable as deformed pieces of CP_2 type extremals connecting Minkowskian space-time sheets (representable as graphs of a map $M^4 \to CP_2$) are identified basic building bricks of elementary particles. 3-D light-like orbits of 2-D wormhole throats- partonic 2-surfaces - at which the signature of induced metric changes from Euclidian to Minkowskian - partonic orbits - are assumed to be carriers of elementary particle quantum numbers localized at points representing intersections of fermionics string world sheets with the partonic 2-surfaces.

2. One can identify simplest wormhole contact as topological sum: two surfaces touch each other. Remove 3-D regions from both space-time sheets and connecting the topologically identical boundaries with a cylinder $X^2 \times D^1$, where X^2 has the topology of the boundary characterized by genus. The assumption that X^2 is boundary requires that its projection to CP_2 is homologically trivial.

   This is not consistent with the assumption that the flux tube carries monopole flux. These wormhole contacts are unstable and must be distinguished from wormhole contacts mediating monopole flux. I have not however defined the notion precisely enough.

3. One can consider two situations in which homologically non-trivial wormhole contact appears.

   Option I: Assume that the 3-D time=constant sections of two Minkowskian space-time sheets are glued together along their boundaries to form a closed 2-sheeted surface and the throats of wormhole contact - partonic 2-surfaces - serve as magnetic charges creating opposite fluxes. One can say that the two throats have opposite homology charges and therefore form a homologically trivial 2-surface to which one can glue the wormhole contact along its boundaries. The flux at sheet B could be seen as return flux from sheet A and the throat could be seen as very short monopole flux tube.

   Option II: Assume no gluing along boundaries for the 3-D time=constant sections of two Minkowskian space-time sheets. In this case one must assume at least two wormhole contacts to get vanishing homology charges at both sheets. At both space-time sheets the throats of the contacts with opposite homology charges would be connected by monopole fluxes flowing through the wormhole contacts identifiable as a very short monopole flux tube. This makes sese also for the Option I and might be required since is not clear whether space-time having boundaries carrying monopole flux can be glued together.

   Remark: One can also consider the light-like orbit of partonic 2-surface connecting its ends (the minimal distance between partonic 2-surfaces vanishes). The homology charges of ends are opposite in ZEO.

The proper identification of the model of elementary particles remains still open [L92] [K71]. What relevance do these two options this picture have to the model of elementary particles?

1. For Option I leptons and gauge bosons could be identified as single wormhole contact carrying non-trivial homology flux. The size scale of the closed space-time sheet would correspond to the Compton wavelength of the particle. This model is the simplest one at the level of scattering diagrams and was re-considered in [L92].

   Even Euclidian regions of single space-time sheet with vanishing homology charge can be considered as a model for leptons and gauge bosons. In this case it is however not clear how to understand how the size scale of the particle as Compton length could be understood at space-time level. This model was one of the first models. I have also considered the identification of the particle as boundary component of Minkowskian space-time surface.

2. Option II was assumed in the model following the original model for leptons and gauge bosons. It was also proposed that electroweak confinement as dual description of massivation takes place in the sense that the weak charges associated with the two wormhole contacts cancel each other. The size scale of flux tube at given sheet would correspond to the Compton length assignable to the particle. In this case scattering amplitudes are more complex topologically.

What about baryons?

1. The simplest model assumes that quarks do not differ from leptons and gauge bosons in any manner. The contribution of the quarks to masses of hadrons is very small fraction of total mass, which suggests that color flux tubes carrying also homology charge are present and give the dominating contribution.

   One can also consider a structure formed by color magnetic monopole flux tubes carrying most of the hadron mass with Minkowskian signature carrying flux of 2 units branching to two flux tubes carrying 1 unit each. The flux tubes would have length given by hadronic p-adic length scale. The ends of flux tubes would be wormhole throats connected by wormhole contacts to the mirror image of this structure. One can say that homology charges 2,-1,-1 assignable to the throats of single space-time sheet sum up to zero. This brings in mind color hypercharge. Could color confinement have vanishing of homology charge as classical space-time correlate?

2. In this article I have considered two alternative identification of leptons. Leptons and quarks could correspond to the different chiralities of $M^4 \times CP_2$ spinors and lepton and baryon numbers would be separately conserved. For second option leptons would b local 3-quark composites and therefore analogous to spartners of quarks: this option is possible only in TGD framework and the reason is that color is not spin-like quantum number in TGD framework. Baryon and lepton numbers would not be separately conserved.

   One can ask what could be the simplest mechanism inducing the decay of baryon as 3-quark composite involving only 3 wormhole contacts and giving lepton as a local 3-quark composite plus something. Wormhole throats of 3 quarks carrying the quark quantum numbers should fuse together to form a leptonic wormhole throat, and the 3 quark lines representing boundaries of string world sheets should fuse to single line. If the sum of quark homology charges is vanishing, lepton must have a vanishing homology charge unless the reaction involves also a step taking care of the conservation of homology charge as a decay of the resulting wormhole contact with vanishing monopole flux to two wormhole contacts with opposite monopole fluxes. Already the first step of the decay process is quite complex, and one can hope that the rate for the reaction is slow enough.

Chapter 17

Could ZEO provide a new approach to the quantization of fermions?

17.1 Introduction

The exact details of the quantization of fermions have remained open in TGD framework. The basic problem is the possibility of divergences coming from anti-commutators of fermions expected to involve delta functions in continuum case. In standard framework normal ordering saves from these divergences for the "free" part of the action but higher order terms give the usual divergences of quantum field theories. In supersymmetric theories the normal ordering divergences however cancel.

What happens in TGD?

1. The replacement of point like particles with 3-surfaces replaces the dynamics of fields with that of surfaces. The resulting non-locality in the scale of 3-surfaces gives excellent hopes about the cancellation of divergences in the bosonic sector. The situation is very similar to that in super-string models.

2. What about fermions? The TGD counterpart of Dirac action - modified Dirac action - is dictated uniquely by the bosonic action which is induced from twistor lift of TGD as sum of Kähler action analogous to Maxwell action and of volume term [K127, K96]. Supersymmetry in TGD sense is proposed in [L92] overcomes the problems of standard SUSY - in particular Majorana spinors are avoided. The key idea is theta parameters are replaced with fermionic creation operatos and the spartners correspond to states created by the local composites of these.

   In the second quantization based on cognitive representations [L53, L76] as unique discretization of the space-time surface for an adele defined by extension of rationals superpartners would correspond to local composites of quarks and anti-quarks. TGD variant of super-space of SUSY approach so that space-time as 4-surface is replaced with its super-variant identified as union of surfaces associated with the components of super coordinates. Fermions are correlates of quantum variant of Boolean logic which can be seen as square root of Riemann geometry. There is no need for Majorana fermions.

   This approach replaced the earlier view in which right-handed neutrinos served a role as generators of $\mathcal{N} = 2$ SUSY [**?**, K100]. In the approach to be discussed one is forced to ask whether their counterparts as local 3-quarks composites could make comeback in a more precise formulation of the picture first discussed in [L92]. The answer turns out to be negative (see Appendix).

   The simplest option involves only quarks as fundamental fermions and leptons would be local composites of 3 quarks: this is possibly by the TGD based view about color. Quark oscillator operators are enough for the construction of gamma matrices of "world of classical worlds" (WCW [K96]) and they inherit their anti-commutators from those of fermionic oscillator operators. Even the super-variant of WCW can be considered. The challenge is to fix these anti-commutation relations for oscillator operator basis at 3-D surface: the modified Dirac equation would dictate the commutation relations later. This is not a trivial problem. One can also wonder whether one avoid the normal ordering divergences.

3. In a discretization the anti-commutators of fermions and antifermions by cognitive representations [L72, L84, L87, L76] do not produce problems but in the continuum variant of this approach one obtains normal ordering divergences. Adelic approach [L53] suggests that also continuum variant of the theory must exists as also that of WCW so that one should find a way to get rid of the divergences by defining the quantization of fermions in such a way that one gets rid of divergences.

One can start by collecting a list of reasonable looking conditions possibly leading to the understanding of the fermionic quantization, in particular anticommutation relations.

1. The quantization should be consistent with the number theoretic vision implying discretization in terms of cognitive representations [L72, L84, L87, L76]. Could one assume that anti-commutators for the quark field for discretization is just Kronecker delta so that the troublesome squares of delta function could be avoided already in Dirac action and expressions of conserved quantities unless one performs normal ordering which is somewhat ad hoc procedure.

   The anti-commutators of induced spinor fields located at opposite boundaries of CD and quite generally, at points of $H = M^4xCP_2$ (or in M^8 by M^H duality) with non-space-like separation should be determined by the time evolution of induced spinor fields given by modified Dirac equation.

   In the case of cognitive representation could fix the anti-commutators for given time slice in $M^4 \times CP_2$ as usual Kronecker delta for the set of points with algebraic coordinates so that if anti-commutators of fermionic operators between opposite boundaries of CD were not needed, everything would be well-defined. By solving the modified Dirac equation for the induced spinors one can indeed express the induced spinor field at the opposite boundary of CD in terms of its values at given boundary. Doing this in practice is however difficult.

2. Situation gets more complex if one requires that also the continuum variant of the theory exists. One encounters problems with fermionic quantization since one expects delta function singularities giving rise to at least normal ordering singularities. The most natural way to quantize quarks fields is as a free field in $H = M^4 \times CP_2$ expanded as harmonics of H. This however implies 7-D delta functions and bad divergences from them. Can one get rid of these divergences by changing the standard quantization recipes based on ordinary ontology in which one has initial value problem in time= constant snapshot of space-time to a quantization more appropriate in zero energy ontology (ZEO)?

Induction procedure plays a key role in the construction of classical TGD. The longstanding question has been whether the induction of spinor structure could be generalized to the induction of second quantization of free fermions at the level of 8-D embedding space to the level of space-time so that induced spinor field $\Psi(x)$ would be identified as $\Psi(h(x))$, where $h(x)$ corresponds to the embedding space coordinates of the space-time point. One would have restrictions of free fermion theory from embedding space H to space-time surface.

The problem is that the anti-commutators are 8-D delta functions in continuum case and could induce rather horrible divergences. It will be found that zero energy ontology (ZEO) [K75] [L91] and the new view about space-time and particles allow to modify the standard quantization procedure by making modified Dirac action bi-local so that one gets rid of divergences. The rule is simple: given partonic 2-surface contains either creation operators or annihilation operators but not both. Also the multi-local Yangian algebras proposed on basis of physical intuition to be central in TGD [L45] emerge naturally.

17.2 Induction of quantum spinor structure in ZEO and second quantization of fermions

In the following it will be argued that the quantization based on ZEO as induction of second quantization at the level of embedding space $M^4 \times CP_2$ (or M^8 by$M^8 - H$ duality [L85, L81, L82]) could provide a solution to the divergence problem for fermions.

17.2.1 How could ZEO help?

Consider first what ZEO is.

1. In ZEO one replaces time=constant snapshots with pairs of 3-surfaces at opposite boundaries of causal diamond (CD $=cd \times CP_2$, where cd is the intersection of future and past directed light-cones of M^4). In ordinary classical ontology these snapshots correspond to initial and final states of classical deterministic time evolution. Note that 3-surfaces can be seen as unions of 3-surfaces (with several disjoint components) at the opposite boundaries of CD.

   General Coordinate Invariance (GCI) implies holography so that one can equivalently talk about space-time surfaces as preferred extremals analogous to Bohr orbits. Strong form of holography (SHF) requires that either the data provided 3-D light-like parton orbits or by 3-surfaces at the ends of CD is enough to code for quantum states.

   The action is given by the dimensional reduction of the 6-D Kähler action for the twistor lift of TGD. The product of twistor spaces of M^4 and CP_2 replaces embedding space H. The existence of 6-D Kähler action requires that these twistor spaces allow Kähler structure and this is indeed the case and actually fixes the theory completely [A63]: standard model symmetries lead to the same outcome as also $M^8 - H$ duality based on classical number fields.

   Dimensional reduction is required to get 6-D surface as an analog of twistor space of space-time surface as a sphere bundle with space-time surface as base space. [L55, L31] A central physical prediction is length scale dependent cosmological constant as coefficient of volume term in the dimensionally reduced action. Quantum states are replaced by superpositions of pairs of 3-surfaces or equivalently of space-time surfaces.

2. The analog of massless Dirac action - modified Dirac action - is needed and is dictated by supersymmetry from the bosonic action [K127, K96]. This means tha the gamma matrices appearing in modified Dirac operator D are obtained as contractions of with embedding space gamma matrices with canonical momentum currents defined by the bosonic action. This is required by hermiticity of the D. The volume term in the bosonic action guarantees that the 4 gamma matrices are in general linearly independent.

3. Dirac action can be also super-symmetrized as also embedding space-time coordinates using local quark multi-linears [L92]. I have discussed this at the level of cognitive representations, which are unique number theoretical discretizations of space-time surfaces consisting of points of embedding space with coordinates in extension of rationals defining the adele in question [L53, L54]. The p-adic number fields in the fundamental rational adele are replaced with the their extensions induced by the extensions of rationals and the hierarchy of adeles defines an evolutionary hierarchy.

How ZEO could help in attempt to make quantization of fermions precise and computable and free of divergences? It is best to proceed by making questions.

1. Could fermionic bi-linear in the Dirac action be replaced by bi-linears of form

   $$\overline{\Psi}_1 D_2^{\rightarrow} \Psi_2 - \overline{\Psi}_1 D^{\leftarrow} \Psi_2 \ ,$$

   where Ψ_1 would be spinor field at the 3-surface associated passive boundary CD and Ψ_2 induced spinor field at the opposite active boundary of CD? Bi-locality would allow to get rid of divergences.

One can also construct a slicings of CD by light-cones parallel to passive and active boundaries respectively. One would obtain Dirac equation in the interior of space-time surface.

The generalized Dirac action would involve two 4-D integrals over space-time surface so that integration is 8-D. The slicing of M^4 by light-cones would allow to define time ordering for the slices, and the definition of action would naturally involve hermitian conjugation of action permuting the roles of Ψ_i as in time order product of spinor fields in quantum field theories. The Dirac equation would be obtained in the same manner as usually by varying with respect to $\overline{\Psi_1}$ and Ψ_2.

2. This is not yet the most general definition of the bilinearity and modified Dirac action. The cancellation of normal ordering divergences allows also to consider the bi-linears associated with different components of 3-surface at either boundary. Even more generally, by strong form of holography one could allow pairing of partonic 2-surfaces for given connected 3-surface. In particular opposite throats of wormhole contacts can be paired.

   These terms would have interpretation in term of bi-linears of fermion and anti-fermion oscillators whereas the pairing with members associated with opposite boundaries of CD would correspond to a pairing of fermionic (anti-fermionic) creation operators and anti-fermionic (fermionic) annihilation operators. One would obtains geometric counterpart for the decomposition of Dirac action and canonical momentum currenrs to various oscillator operator bi-linears.

   One could allow both creation and annihilation operators at the same boundary of CD provided they are not associated with the same 3-surface. If the action can have terms associated with string world sheets, partonic 2-surfaces and 3-D their light-like orbits, even the condition that they do not correspond to same partonic 2-surfaces or string world sheet is enough to get rid of divergences. One could go even further and allow creation and annihilation operators at different points of partonic orbit if they reside at 1-D light-like boundaries of different string world sheets.

3. What could be the minimum condition for avoiding divergences. The super-symmetrization to be discussed in more details below allows even local composites of quark and antiquark creation (annihilation) operators in the super-field since these have vanishing anti-commutators.

4. The modified Dirac action would not involve anti-commutators of $\Psi_1^\dagger$ and Ψ_2 at the same point so that the action would be finite even if one identifies the anti-commutations relations as those induced by the identification $\Psi(x) = \Psi(h(x))$, where $Psi(h)$ is second quantized embedding space spinor field written as superposition over all its modes with oscillator operators obeying standard anti-commutation relations in $H = M^4 \times CP_2$. ZEO would make everything unique , finite, and calculable! One would obtain also direct connection with the p-adic mass calculations in which spinor modes of H-spinor fields define ground states of the super-symplectic representations.

Consider now what conditions that super-symmetrization in terms of local composites of fermionic oscillator operators [L92] gives.

1. In the proposed discretization based on cognitive representations super-symmetrization replaces embedding space coordinates h with super-coordinates h_S having a local expansion in powers of hermitian local composites of Ψ with vanishing fermion number and same quantum numbers as tangent space coordinates of H. In the case of super quark field all super components have same quantum numbers as quark itself. Leptonic super-field would consist of local composites of 3-quarks with leptonic quantum numbers and could be regarded as superpartner of quark field.

2. The local composites contain in the general case both Ψ and its conjugate $\overline{\Psi}$ and here one encounters a problem if the anti-commutations are induced from those for second quantized embedding space spinor field. This is the case even if the quantization is carried out using discretized 4-momenta ,which is of course number theoretically well-motivated, the problem remains.

3. Quark creation operators and antiquark annihilation operators cannot be allowed at the same point. This would however allow local composites involving both types of creation operators at given boundary of CD. The minimum option would be this. The quantization at embedding space level indeed allows unique decomposition of positive and negative energy parts unlike quantization in general curved space-time. For the most general option avoiding divergences the only condition is that annilation and creation operators do not appear at the same point of light-like curve as boundary of string world sheet at the light-like partonic orbit. It is however essential that modified Dirac action contains terms assignable to 2-D and possibly also 3-D surfaces. This point will be discussed later.

4. This approach forces to generalize the view about fermionic conserved quantities. Usually conserved quantities have leading term given by a term bilear in creation and annihilation operators acting on fermionic vacuum. Accepting the proposed bi-linearity, the conserved quantities would contain only nonlocal bi-linear terms of various kinds. How does one define the notion of eigenstate?

   Assume that both kinds of oscillator operators at both boundaries of CD but that given point of partonic 2-surface carries only creation or annihilation type operators. This allows to get rid of divergences and define the notion of eigenstate at given boundary of CD by assuming that given boundary of CD corresponds to same fermionic vacuum. Bi-local composites of creation and annihilation operators appearing in the hermiatian operator representing observable would shift fermions between points of the partonic 2-surface but delocalization would allow to consruct eigenstates.

17.2.2 Fermionic quantization as induced quantization

Consider now the details of fermionic quantization as induced quantization.

1. The fermionic propagators for a pair of points of space-time surface would be induced from those for free fermions in $H = M^4 \times CP_2$. Induced spinor field at space-time surface would be a restriction of embedding space spinor field and fermionic oscillator operators at 3-surface at the boundary of CD would be Fourier components of H spinor fields at 4-surface with respect to the basic of spinor spinor modes at 4-surface restricted to 3-surface.

   For 4-D surfaces the generalization of Dirac action involves 2 4-D integrals corresponding to Ψ_1 and Ψ_2. Since modified gamma matrices Γ^α have dimension $d = -1$, Ψ_1 and Ψ_2 must have dimension $d = -7/2$. Remarkably, also embedding space spinor fields have this dimension! Therefore it is natural to identify the space-time propagators as induced propagators. This works only for 4-D space-time surface for 8-D H so that space-time dimension would be fixed.

2. There is however an objection. Does one need modified Dirac equation for induced spinor fields at all? Could one do using only second quantized embedding space spinor fields to define correlation functions needed in S-matrix? Could one construct fermionic multi-linears using just these.

   One would lose all the nice theory related to the induced spinor structure and the super-symmetry based connection with the bosonic action. One would lose also quantum classical correspondence between eigenvalues of fermionic conserved charges in Cartan algebra and classical conserved quantities defined by the bosonic part of the action.

   For what purposes the modified Dirac action would be needed? Could one see space-time description and embedding space description as duals of each other. Could space-time description give interpretation for classical Noether charges for bosonic action in terms of fermionic charges given basically by modified Dirac action and embedding space anti-commutation relations?

17.2.3 How to generalize the Dirac action for string world sheets and partonic 2-surfaces

The description of 4-D Dirac action is not yet the whole story. One must assign modified Dirac action also to string world sheets and partonic 2-surfaces and avoid divergences and justify the proposed picture about second quantization. Consider first the physical picture.

1. One has also string world sheets, partonic 2-surfaces, and their light-like orbits as regions at which the signature of the induced metric changes from Minkowskian to Euclidian. Wormhole contacts have interpretation as building bricks of particles and if the wormhole contact carries homology charge (Kähler magnetic charge) also second wormhole contact is necessary to give closed flux lines traversing along first Minkowskian sheet going through second wormhole contact and returning back along second space-time sheet and returning through first wormhole contact.

   One can also consider the possibility that wormhole contacts carry no homology charge but in this case they are not stable. Whether all elementary particles have two homologically charged wormhole contacts as building brick or whether only quarks carry homology charges is not yet settled in the general case. If leptons are local 3-quark composites, one could assume non-vanishing homology charge for them.

2. M^8-picture [L85] predicts 6-D surfaces having topology of 6-sphere S^6 and analogous to branes in string theory. S^6 has 3-ball B^3 represented as $t = constant$ intersection of $cd \subset M^4$ as M^4 projection so that radial M^4 coordinate r_M satisfies $r_M \leq t$ (t is linear time of M^4 and corresponds to octonionic real coordinate). t corresponds to a root of a polynomial P having rational (or possibly even algebraic) coefficients so that one obtains connection with number theory.

   These 6-sphere S^6 is a special solution of the algebraic equations stating vanishing of either real or imaginary part of the octonionic polynomial obtained as an algebraic continuation of P: both real and imaginary part in quaternionic sense vanish for these roots. The other solutions are 4-D regions of space-time surface.

   In $M^8 = M^4 \times E^4$ the point of B^3 with distance $r_M \leq t$ from origin corresponds to the sphere of E^4 with radius r_M since point of 8-D light-cone boundary is in question.

   The physical picture is that the space-time surfaces consist of 4-D roots of octonionic polynomials glued together at partonic 2-surfaces which appear as 2-D intersection of 6-spheres and 4-D surfaces. These partonic 2-surfaces would define a generalization of vertices appearing in topological diagrams having light-like partonic orbits as lines. Space-time surfaces need not intersect along 3-D B^3 although this cannot be excluded. String world sheets would have their light-like boundaries at these orbits and strings would thus connect partonic 2-surfaces.

   In TGD inspired theory of consciousness these 6-spheres represent "very special moments in the life of self" [L81].

Modified Dirac action for 2-surfaces, partonic orbits and string world sheet boundaries involving self-pairing

How to identify the counterpart of Dirac action for string world sheets and their 1-D light-like boundaries and partonic 2-surfaces and their light-like orbits? Self-pairing in the sense that dimensions of paired objects are same, comes first in mind.

1. If the bi-local Dirac action couples two 2-surfaces the action must contain dimensional constants if one wants spinors to be induced from embedding space spinors withe dimension $d = -7/2$. Since the modified Dirac action is determined by the 2-D part of corresponding bosonic action argued to emerge automatically at the 2-D singularities, one can argue that the emerging dimensional scaling factor guarantees that the action is dimensionless. For $d = 2$ dimensional objects the scaling factor would have length scale dimension 4 and scaling factor would be apart from numerical constant R^4, R CP_2 radius.

2. This works also for the pairing of two partonic orbits $d = 3$: now the scaling factor proportional to R^2. Chern-Simons action is rather plausible possibility. For $d = 1$ the scale parameter would be proportional R^6. In both cases these parts of actions would be related to singularities.

These action would have rather satisfactory features. In Appendix also the possibility that 2-D surfaces pair with special 6-D solution of polynomial equations in M^8 with topology of S^6 is discussed. This surface has 5-D sub-manifold with which partonic orbits could pair. It turns however that these pairings lead to non-physical predictions.

17.2.4 This is not enough!

In optimistic mood one might think that the reduction of fermion propagation at space-time level to the level of embedding space could be enough. Unfortunately this is not the case: a nice advance in quantitative understanding has occurred but is not enough. One must consider two pictures for the propagation of physical (elementary) particles to be distinguished from fundamental fermions.

About the massivation of elementary particles in H-picture

Consider first the existing ideas about propagation of elementary particles - as opposed to fundamental fermions - when space-time surface is regarded as a surface in $H = M^4 \times CP_2$.

1. p-Adic mass calculations carried out for about 25 years ago were based on this picture. One has super-conformal invariance and super-Kac-Moody representations accompanied by Super-Virasoro representations. Ground states would be characterized in the case of fermion states by spinor harmonics of H [K65]. The masses of these harmonics can be calculated and the mass scale is determined by CP_2 radius $R \sim 10^{-4} l_{Planck}$ and is huge. There is electroweak symmetry breaking and only the right-handed neutrino is massless.

2. To get physical masses, being extremely small in CP_2 mass scale and depending only slightly on electroweak quantum numbers, one must have massless ground states as the first approximation. This requires tachyonic ground state with large tachyonic mass - this is highly analogous to Higgs mechanism. The conformal weight of ground state would be negative and equal to $h_{vac} = -5/2$ and would be dictated the tachyonic ground state mass. Physical particles would be Kac-Moody excitations of the tachyonic ground states and massless in the first approximation. The origin of the tachyonic ground state conformal weight has remained poorly understood in H-picture.

   The idea is that the thermal mixing of states with different mass squared - conformal weight - gives rise to the observed mass squared. p-Adic thermodynamics for mass squared- essentially Virasoro generator L_0 representing scaling - leads to excellent predictions for the masses of elementary particles. The only free parameters are integer valued. p-Adic temperature quantized by number theoretical conditions to $T = 1/n$ and having value $T = 1$. The p-adic prime p, which by p-adic length scale hypothesis is near power 2^k of 2 with Mersenne primes and ordinary primes associated with Gaussian Mersennes favored. The masses are exponentially sensitive to the value of k and family replication phenomenon explained in terms of genus of partonic 2-surface is also essential and predicts mass ratios of leptons correctly [K26].

3. One can consider the situation also classically [K9]. Classical particles are assumed to have wormhole throats of wormhole contacts as building bricks. Wormhole contact has Euclidian signature of metric but the model as CP_2 type extremal predicts light-like geodesic line as M^4 projection. The boundary of the wormhole contact is light-like 3-surface and analogous to massless particle. QCC encourages suggests that physical particles are massless in good approximation. This gives justification for the idea about negative ground state conformal weight.

   String world sheets are also present as singularities of minimal surfaces representing space-time surfaces and are themselves minimal surfaces [L85] Strings generate correlations between fundamental fermions at wormhole throats. This could give a justification for the p-adic thermodynamics for Super Virasoro representations.

   A possible physical picture is that the p-adic thermodynamics is associated with the strings connecting opposite wormhole throats. Should one assume that the fermion is de-localized along this short string having Euclidian signature inside the wormhole contact? Could the

Euclidian signature explaining tachyonic ground state weight and tachyonic mass squared and could the mass of fermion give rise to the positive contributions allowing massless states?

Massivation and propagation in M^8 picture

What about massivation in M^8 picture?

1. In M^8 picture one has decomposition $M^8 \times M^4 \times CP_2$. This decomposition is highly non-unique and the Lorentz group $SO(1,7)$ gives new decomposition having interpretation as a particular octonionic sructure involving choice of real axis as time axis and preferred imaginary axis defining quantization axis for angular momentum and meaning choices of preferred M^2 crucial for M^8-H duality. Therefore the choice of M^4 is not unique [L49, L50, L51, L85]. The decomposition $H = M^4 \times CP_2$ however does not involve this non-uniqueness of M^4.

   $M^8 - H$ duality suggests that the space-time surfaces in H could somehow represent the non-uniqueness of the choice of $M^4 \subset M^8$. M^4 allows warped embeddings to $M^4 \times CP_2$ with metric components related by scalings to the metric of canonically imbedded M^4. Warped embeddings correspond to maps of M^4 to a geodesic circle of CP_2 given by $\Phi = k \cdot m$, k 4-D wave vector, which can be light-like or space-like. The metric is given by $g_{kl} = m_{kl} + R^2 k_k k_l$. Could these warped embeddings correspond to the non-standard choices of M^4?

2. The mass squared in M^8 vanishes but M^4 mass squared is non-vanishing and equal to the E^4 mass squared. By a suitable choice of the rest frame in M^8 M^4 momentum equals to M^8 momentum and the mass squared vanishes [L82]. It seems that this choice gives mass squared which corresponds to the H-mass squared, which would vanish in excellent approximation.

3. The superposition of states with different M^8-momenta such that the choice of M^4 giving masslessness is different for these states forces massivation and p-adic thermodynamics would describe it. The natural condition is that the choice of M^4 is such that dominant contribution to the state is massless the other states give thermal contribution to the mass squared [L82].

Common features of H- and M^8 pictures

M^8 - and H-pictures share several features.

1. CD as product $cd \times CP_2$ is replaced with 8-D CD cd_8 identified as intersection of future and past directed light-cone. S^6 is preserved and can be regarded as $t = constant$ section of cd as in case of H.

2. Also now both Euclidian and Minkowskian regions are expected to be possible and CP_2 type extremal should correspond to 4-surface having 4-D E^4 projection and 1-D projection light-like geodesic as M^4 projection. Here one must however notice that M^4 projection is not light-like for a general choice of M^4!

3. Space-time surfaces in M^8 0are analogous to complex manifolds which are minimal surfaces: what is done is that complex numbers are replaced by complexified octonions so that one has analogs of polynomials of complex variable replaced with complexified octonions.

   This encourages the conjecture that these surfaces are minimal surfaces in E^4. Closed minimal surfaces are not in general possible because minimal surfaces have vanishing external curvatures. Rather, this surface should be analogous to a soap film spanned by frame. The frames would naturally correspond to the 3-D ends of this surface at the boundaries of CD_8. This would give one further motivation for ZEO. Also the gluing of regions of space-time surfaces together along partonic surfaces at 6-D branes S^6 could have interpretation in terms of frames.

   The counterpart for the orbit of wormhole contact connecting two Minkowskian space-time sheets as deformed CP_2 type extremal would be minimal surface with Euclidian signature glued to Minkowskian regions of space-time surface along 3-D light-like surfaces. Also the wormhole throats would play the role of soap film frames.

4. The bi-local picture about modified Dirac action in H is preserved. The situation however simplifies dramatically since now the electroweak splitting of the masses for the modes of modified Dirac operator is absent.

17.2.5 Connection with the Yangian symmetries

There is a connection also with the Yangian picture proposed on basic of twistorialization and symmetry arguments.

1. For Yangian algebra the generators are multi-local. This picture strongly suggests Yangian algebras since fermionic oscillator operators are superpositions of contributions from several 3-surface at given boundary of CD [L45]. Allowing operators, which are multi-local having fermions at several components of 3-surface such that quarks and antiquarks are at opposite boundaries of CD.

   One could say that quarks and leptons as local composites of 3- quarks are fundamental fermions if particle corresponds to partonic 2-surface. If it corresponds to 3-surface then also spartners are possible as composites of partonic 2-surfaces - in particular pairs of wormhole throats. These conditions conditions are consistent with the phenomenological picture that has guided the development of TGD based view about elementary particles. This would allow also to understand SUSY breaking.

2. Yangian picture could also generalize to the level of WCW an even super variant of WCW since 3-surface would be pair of 3-surfaces consisting of several components at opposite boundaries of CD. WCW gamma matrices and their conjugates would be associated with disjoint partonic 2-surface of CD and all anti-commutators and commutators would be well-defined and expressible in terms of anti-commutators of second quantized free spinor fields in H. This is just the original dream about generalization of induction of spinor structure to quantum realm that has waited its realization for four decades.

17.2.6 Connection with quantum classical correspondence

The notion of quantum classical correspondence (QCC) deserves some comments.

1. What QCC means is not completely clear. A rather stringent form of QCC would state that the classical expressions for the conserved quantities from the classical action defining space-time surfaces are equal to the eigenvalues of their fermionic counterparts in Cartan algebra of symmetries. This condition might make sense for fermionic conserved quantities identified as bi-linears of fermionic oscillator operators defined by the bi-linears formed from spinor fields at opposite boundaries of CD or more generally - at different partonic 2-surfaces. Everything would be finite and expressible in terms of anti-commutators at the level of embedding space.

2. The charges in the complement of Cartan algebra should vanish classically corresponding to the vanishing of matrix elements of non-Cartan algebra generators for eigenstates: this would generalize the definition of the rest system as system in which 3-momentum vanishes to all conserved charges. This condition would be analogous to Einstein's equations.

3. What could be the interpretation of these condition? Could one interpret that as stating the condition that the sums of classical conserved quantities and fermionic conserved quantities vanish for eigenstates. One encounters conservation laws also for WCW spinor fields to which Noether theorem for isometries of WCW applies. This gives expressions for conserved quantities in terms of super-symplectic algebra of isometries and includes also the algebra of embedding space isometries. These operators act on WCW points and WCW spinor fields.

4. Super-symmetrization of the bosonic action by introducing super-coordinates suggests a weaker form of QCC. Both bosonic action and modified Dirac action contribute to the total action and the quantal Noether charges in Cartan algebra can contain also c-number term. If this term is no-vanishing, one can speak about central extension analogous to that induced by addition of constant term to the Hamiltonians of Cartan algebra of symplectic algebra of isomehries of CP_2. The condition that central extension term vanishes would give the strong form of QCC.

17.3 Appendix: Could the pairing $d = 2, 3$ objects with $d = 6, 5$ objects make sense?

The emergence of 6-sphere S^6 as a brane like objects forces to ask whether a generalized pairing in which 2-surfaces would pair with S^6 so that the modified bi-local Dirac action would involve no dimensional constants. S^6 has also 5-D object as $\delta cd \times S^2$, where $S^2 \subset CP_2$ if homologically non-trivial geodesic sphere, and this could pair with the $d = 3$ light-like partonic orbits.

Bosonic action is needed to define the gamma matrices in the modified Dirac action and 4-D action cannot induce this action. Therefore this option might fail. The pairing idea turns out to have physically questionable implications. The considerations showing this however led to a possible identification of the counterpart of $S^6 \in M^8$ as an object in H.

A further objection is that the pairing of 1-D light-like boundaries of string world sheets is not possible since no 7-D object exists as special solution in M^8. String world sheet boundaries however play a key role as carriers of fermion quantum numbers.

Since self-pairings are well-defined and do not require introduction of new dimensional constants, it seems that they are the correct choice.

17.3.1 Pairings of 2-D and 3-D object

Pairing of 2-D objects with S^6

These pairings looked at first highly interesting since S^6 as analog of brane would gain further good reason for its existence. Again the induction idea and the condition that the action has no dimensionless parameters can be used as a constraint.

1. The pairing at the level of M^8 would easy to understand. One should assign to these 2-surfaces - string world sheets and partonic 2-surfaces - modified Dirac action as a bi-linear pairing given 2-surface with some other surface. One would use same formulas as for 4+4 pairing. If one does not want to introduce dimensional constants, the bi-linear action involving these 2-D surface pairing with 6-D object. It would be very natural to pair string world sheets and partonic 2-surfaces with brane-like 6-spheres in the modified Dirac action at the level of M^8.

2. What about the pairing at the level of $H = M^4 \times CP_2$? The challenge is to map the 6-sphere of M^8 to 6-D surfaces in H. What is clear that the projection to B^3 of S^6 to cd must be mapped to H as such. Also a group-theoretically natural proposal is that the radial B^3 coordinate r_M is mapped to a radial CP_2 coordinate r_{CP_2} labelling $U(2)$ 3-spheres of CP_2 and define as $r^2_{CP_2} = |\xi_1|^2 + |\xi_2|^2$ so that rotational symmetries of B^3 and CP_2 correspond to each other. Therefore one would have $r_{CP_2} = f(r_M)$ and one should be able to identify the function f.

   At the limit $r \rightarrow t$ one must have light-like 3-surfaces in H as orbit of partonic 2-surfaces. This requires that CP_2 projection becomes 2-D. This is achieved for $r_{CP_2} \rightarrow \infty$ at this limit. This formal 3-sphere is actually homologically charged geodesic 2-sphere of CP_2 at which two patches of CP_2 coordinate covering are glued together. At the level of space-time surfaces this gluing would correspond to the gluing of wormhole contact with Euclidian signature of induced metric and its complement with Minkowskian signature of metric.

3. The modified Dirac operator must annihilate the S^6 solution. The modified Dirac operator has 2 parts corresponding to metric part and Kähler parts from M^4 and from CP_2. Since the induced metric has Euclidian signature a good guess is that the solutions are covariantly constant.

   The condition $r_{CP_2} = r_M$ means that taking r_M as coordinate one has spherical coordinates for $S^2 \subset B^3$ and $S^3 \subset CP_2$ as coordinates plus radial coordinate which can be taken as radial coordinate r_{CP_2} of CP_2, call it R to minimize notational complexity. g_{RR} is sum of M^4 and CP_2 contributions: $g_{RR} = -(\partial_{r_M} f)^2 + s_{RR}$. The condition is that covariantly constant mode expected to have trivial dependence on the coordinates of $S^2 \times S^2$ exists. What is required is that the component A_R of the induced spinor connection vanishes for the mode. This poses a condition on the function $R = f(r_M)$ and might fix it.

4. The existence of covariantly constant mode suggests large isometry group. Maximal isometry group would be that of metric S^6 having also interpretation as octonionic 6-sphere. Covariantly constant modes are assigned with super-symmetries and the existence of this mode would give rise to the analog of supersymmetry. Thus the existence of this supersymmetry in generalized $M^8 - H$ correspondence would make possible to assign modified Dirac action to 2-surfaces.

   Local 3-quark composite with quantum numbers of right-handed neutrino could correspond to the covariantly constant mode at S^6 for the super variant of modified Dirac equation [L92]. What would happen that the coupling to both CP_2 and M^4 spinor connections would reduce to couplings to Kähler form and vanish for a proper choice of the couplings and appropriate choice of function f.

The question whether a counterpart of S^6 surface in H exists and what it is, is rather natural and the considerations indeed lead to its identification. The trivialization of the fermionic dynamics at S^6 however suggests that they are not needed.

The pairing for light-like 3-surfaces with 5-D objects

Consider next the pairing 3-D light-like partonic orbits with possibly existing 5-D objects.

The first question is whether the 3-D modes of Dirac equation needed at all The physical picture is that fermions at partonic orbits are at *boundaries* of string world sheets at light-like orbits of partonic 2-sheets and the modified Dirac action for string world sheets dictates the dynamics at these boundaries. One might do without spinors restricted to the partonic orbits but this is not obvious.

Are these 3-D spinor modes possible?

1. The 5-D object should be sub-manifold of S^6. At light-like 3-D surfaces the signature of the induced metric changes. This region would be naturally $X_5 = \delta cd \times S^2$ at which the CP_2 projection reduces to homologically charged geodesic sphere $s^2 \subset CP_2$.

2. What about the induced metric of X_5? The induced metric degenerates at δCD to 2-D metric effectively and this means that contravariant modified gamma matrices Γ^α can be ill-defined. The general manner to overcome the problem is to use modified gamma matrices associated with the action defined by Chern-Simons term associated with CP_2 Kähler form and M^4 Kähler form, which is required by twistor lift of TGD. The induced metric appearing in the 3-D permutation symbol and in volume element cancel each other and one obtain a finite result.

   Note also that the contribution from Kähler actions of M^4 and CP_2 to the induced metric are non-vanishing and could be enough: as a matter fact they could reduce to the Chern-Simons terms by boundary conditions.

3. There is however a delicate problem. The Chern-Simons form of CP_2 (M^4) is non-vanishing only if the CP_2 (M^4) projection is 3-D: this is not possible for the partonic 2-surface or string world sheet. Therefore the modified Dirac operator can act only at the paired 5-surface. CP_2 projection is however 2-D homologically non-trivial geodesic sphere and Chern-Simons term vanishes identically leaving only the modified Dirac operator defined by induced metric.

   The M^4 projection of the 5-surface is 3-D δcd and the corresponding Chern-Simons form is non-vanishing. The modified Dirac operator of S^2 couples to M^4 Kähler form. If the coupling is correct, one expects that covariantly constant modes are possible. A coupling of fermionic degrees of freedom at partonic 2-surfaces and boundary of CD would emerge.

4. For $S^2 \subset CP_2$ covariantly constant right-neutrino solutions are possible but not for quarks. Should one allow also leptons as fundamental fermions? The construction of WCW spinor structure does not require both leptons and quarks and the construction of quarks from leptons does not look plausible idea although I have considered also this option. Quarks seems to be the only elegant option.

There is however a loophole. The local 3-quark composites have electroweak quantum numbers of leptons and they should couple to electroweak gauge fields like leptons so that covariantly constant right-handed neutrino like mode would be possible for super-variant of modified Dirac equation! A long-standing proposal has indeed been that right-handed neutrino and its antineutrino generate $\mathcal{N}=2$ SUSY [K100]. This picture could realize this proposal also when quarks are the only fundamental fermions.

Summary about pairings

The fact that the modified Dirac action is bi-local means that the propagation of physical particles is more complex than propagation of free fermions in H. One must distinguish between different dimension $d \in \{4,3,2\}$ for the geometric objects carrying induced spinor fields.

1. For $d=4$ there would be pairing of 3-D surfaces such as wormhole contacts. The notion of eigenstates would require de-localization of fermions: wave function would be superposition of contribution at throats. Situation would be 3-dimensional.

   Self-pairing with objects of same dimension seems the most plausible option.also for $d=2,3$ but one can consider also the pairing between different dimensions.

2. For quarks at $d=2$ string world sheets and partonic 2-surfaces there would be only pairing with 6-sphere S^6. For string world sheets one obtains propagating states and one expects stringy mass spectrum for these states in the manner suggested by p-adic mass calculations. This is of course true also for self-pairing.

   Does the state at S^6 or $\delta CD \times S^2$ correspond to a local 3-quark composite representing covariantly constant right handed neutrino.

3. For $d=3$ one would have 3-D propagation at light-like partonic 2-surface. The modes are now massless and also now the contribution of the state at $\delta cd \times S^2$ has vanishing momentum.

17.3.2 Could the analog of standard SUSY makes sense in TGD?

For a long time I tried to find whether right-handed neutrinos could give rise to the analog of standard SUSY [?, K100]. I also proposed that many fermion states at partonic 2-surface give rise to larger but badly broken SUSY. Last summer (2019) I made a breakthrough in the understanding of SUSY in TGD frame.

The members of SUSY multiplets would be replaced with local composites of creation operators and only quarks would be needed to build also leptons. I gave up the idea about right-handed neutrino as generators of SUSY. The pairing of objects with different dimensions in modified Dirac action however forces to reconsider the situation. The conclusion are not however changed.

Breaking of analog of standard SUSY for 4-D fermions

Consider first the analog of standard SUSY and its breaking in TGD framework.

1. In the case of M^8 spinor harmonics there is no symmetry breaking as in the case of CP_2 and M^4 masses assignable to spinor harmonics do not depend on E^4 spin. This seems to be the case in excellent approximation also for the physical particles in H. If M^8-H duality holds true, the proposed mechanism explaining this would be a description for something which is true from the beginning in M^8 picture.

2. One would have the analog of $\mathcal{N}=4$ SUSY corresponding to covariantly constant spinors of E^4 having 4 spin states corresponding to right- and left-handed isospin doublets. As a matter of fact, the $SO(4)$ symmetry could be interpreted as the symmetry of hadron physics not so fashionable nowadays and $SU(3)$ as symmetry for quarks and $SO(4)$ as symmetry for hadrons would have interpretation in terms of M^8-H duality [L85, L82]. H-picture would correspond to an analog for the breaking of $\mathcal{N}=4$ SUSY down to $\mathcal{N}=2$ or even $\mathcal{N}=1$: the latter option might be due to possibility of only second spin state for $B^3 \subset S^6$ and δCD.

3. What could the breaking of $\mathcal{N} = 4$ SUSY to $\mathcal{N} = 2$ SUSY correspond to? One can have canonically imbedded M^4 as a minimal surface extremal in $M^4 \times CP_2$ and for it the induced Dirac operator would reduce to massless Dirac operator in M^4 have all spinor modes as massless states and also covariantly constant modes would be allowed.

   For deformations of M^4 different M^4 chiralities mix and right- and left-hand chiralities mix. This is a signature for massivation and breaking of SUSY. Also inside Euclidian wormhole contacts representing elementary particles SUSY is broken and only right-handed neutrino is massless in analogy $\mathcal{N} = 2$ SUSY.

Is the analog of standard SUSY possible for lower dimensional fermions?

Pairing with brane like objects suggests strongly the analogs of covariantly constant right-handed neutrino spinors as local 3-quark composites. The earlier proposal was that covariantly constant right-handed neutrino generates analog of $\mathcal{N} = 2$ SUSY. The basic objection against the idea is that for the generic space-time surface modified gamma matrices induces mixing of left- and right-handed neutrinos and leads to massivation and loss of these modes.

1. For S^6 and $\delta cd \times S^2$ there are however good reasons to expect that only these modes are possible. Could this make possible the standard SUSY?

2. Could the modes with vanishing M^4 momentum associated with pairings 2-D *resp.* 3-D fermions with 6-D *resp.* 5-D objects be described as analogs of Majorana-like degrees of freedom described as oscillator operators reducing to theta parameters? The non-conservation of fermion number would not be seen in the physics of the momentum carrying degrees of freedom if the transfer of neutrinos to right-handed neutrinos at these surfaces does not happen. SUSY in this sense would not be present for 4-D fermions at wormhole contacts. Could the breaking of $\mathcal{N} = 1$ be due to the mixing of 2- or 3-D fermions with 4-D fermions?

3. If this picture is correct, standard SUSY would not have been found because it would have been searched in wrong place. One should be able to study fermionic states with $d = 2, 3$.

The first objection is that in standard picture the vertices for particle and sparticle couple differently since they have different spins. Vertices as partonic 2-surfaces would be sub-manifolds of S^6. Could the presence of right-handed neutrinos at S^6 affect the vertex and their spin could be seen in the vertex? This does not seem plausible.

Second objection is that the pairing between different dimensions leads also to severe problems with conserved charges for $d \in \{2, 3\}$, the reason is that objects with dimension $d = 6, 5$ have space-like induced metric.

1. Since the bi-local Dirac operator D is hermitian, the notion of eigenstate should make sense for the Noether charges assignable to the bi-local Dirac operator also for $d \in \{2, 3\}$. The action of conserved charge creates (annihilates) quark quark with $d = 2$ *resp.* $d = 3$ and annihilates (creates) ν_R at brane-like surface with $d = 6$ *resp.* $d = 5$. Suppose that one has a state which is of form

$$|\Psi\rangle = |q\rangle|vac\rangle + \epsilon|vac\rangle|\nu_R\rangle \ .$$

   This state is superposition of states with quark number one and anti-lepton number 1, which corresponds to quark number 3.

2. Consider a hermitian conserved charge associated with D. The action on $|\Psi\rangle$ is given by

$$O|\Psi\rangle = o_1|vac\rangle|\nu_R\rangle + \overline{o}_1\epsilon|q\rangle|vac\rangle = o\Psi \ .$$

   The condition gives $o = \epsilon\overline{o}_1$ and $o\epsilon = o_1$ solved by $\epsilon = o_1/\overline{o}_1$ and $o = |o_1|$.

The ill-definedness of quark number is analogous to the fact that for Majorana spinors fermion number is conserved only modulo 2. Also the electroweak and color and Poincare quantum numbers are ill-defined in the general case. Only if $d = 2, 3$ surface contains right-handed neutrino standard model quantum numbers are well-defined. This would reduce the number of allowed states dramatically. This kind of formal eigenstates look unphysical. It seems that self-pairing is the only reasonable option.

Chapter 18

Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory

18.1 Introduction

I have worked with the problem of understanding the construction of scattering amplitudes in the framework provided by Topological Geometrodynamics (TGD) for about four decades. It soon became clear that the naïve generalization of the path integral approach to a path integral over space-time surfaces did not work because of the horrible non-linearities involved. Around 1985 I started to work with the notion that I later called the "world of classical worlds" (WCW). Eventually I apprehended that the realization of general coordinate invariance (GCI) forces to assign to a 3-surface possibly unique space-time surface (X^4) at which the general coordinate transformations act [K57, K29]. Holography would reduce to GCI. The intuitive expectation is that either space-like 3-surfaces or light-like partonic orbits defining boundaries between Minkowskian and Euclidian space-time regions should be enough to determine X^4 as an analog of Bohr orbit. This leads to strong form of holography (SH) stating that data at partonic 2-surfaces and string world sheets code for X^4.

It should be possible to geometrize the entire quantum physics in terms of WCW geometry and associated spinor structure identifying WCW spinors as fermionic Fock states. A geometrization of the hermitian conjugation essential in quantum theory is needed. This fixed the WCW geometry to be Kähler geometry determined by Kähler function and defining Kähler form providing a realization of the imaginary unit as an antisymmetric tensor [K57]. The existence of Riemann connection fixes the Kähler geometry uniquely already in the case of loop spaces [A44]: maximal isometry group is required. In TGD framework it would correspond supersymplectic transformations of $\delta M^4_\pm \times CP_2$, where $\delta M^4_\pm$,denotes future or past light-cone [K29].

Classical physics becomes an exact part of quantum physics if the space-time surfaces are preferred extremals for some action and therefore analogous to Bohr orbits. Spinor fields should obey the modified Dirac equation (MDE). Modified Dirac action (MDA) is determined by the bosonic action via supersymmetry condition. Kähler function identified as the action for the preferred extremal associated with the 3-surface defines in complex coordinates the Kähler metric and Kähler form via its second derivatives of type $(1,1)$.

The natural looking identification of the action was as Kähler action - a non-linear generalization of Maxwell action replacing Maxwell field and metric with induced Kähler form and metric. It possessed a huge vacuum degeneracy interpreted as spin glass degeneracy and for a long time I looked this feature as something positive despite the fact that the WCW metric becomes degenerate at the vacuum extremals and classical determinism is lost. The addition of volume term having interpretation in terms of cosmological constant would have been a possible cure but would have broken conformal invariance bringing in an *ad hoc* dimensional coupling.

Decades later the proposal for a twistor lift of TGD led to the identification of fundamental action as an analog of Kähler action for 6-D twistor spaces having X^4 as base space and S^2 as fiber [L99]. The induction of the twistor structure from that for the 6+6-D product of twistor spaces of M^4 and CP_2 (these spaces are the only 4-spaces allowing twistor space with Kähler structure [A63] so that TGD is unique) to the 6-surface forces a dimensional reduction reducing 6-D Kähler action to a sum of 4-D Kähler action and volume them. The counterpart of the cosmological constant emerges dynamically. Λ depends on the p-adic length scale characterizing space-time surfaces and approaches to zero in long length scales [L99].

The ontology of standard quantum theory in which 3-D $t = constant$ slice of space-time contains the quantum states, does not fit nicely with TGD framework. Space-time surfaces in 1-1 correspondence with 3-surfaces are more natural objects to consider. This conforms also with the notion of holography implied by GCI: actually SH is highly suggestive and means that 2-D data at partonic 2-surfaces and string world sheets determined the X^4 as a preferred extremal. In particular, various anomalies suggest that the arrow of time need not be fixed.

Eventually this led to zero energy ontology (ZEO) [L91] in which quantum states are essentially superpositions of preferred extremals inside causal diamond (CD): space-time surfaces have ends at the boundaries of CD and these pairs of 3-surfaces or equivalently the 4-surfaces are the basic objects. CDs form a hierarchy: there are CDs with CDs and CDs can also intersect. They would form an analog of atlas of coordinate charts. Each CD would serve as a correlate for a conscious entity so that the charts can be said to be conscious.

ZEO leads to a quantum measurement theory and allows avoiding the basic problems of the standard quantum measurement theory. Zero energy states correspond to state pairs at opposite boundaries of CD or equivalently, superpositions of deterministic time evolutions. In state function reduction (SFR) as a superposition of classical deterministic time evolutions is replaced with a new one.

"Big" and "small" state function reduction - BSFR and SSFR - are the basic notions. In SSFRs as analogs of "weak" measurements following a unitary time evolution, the size of CD increases in statistical sense. The members of the state pairs associated with the passive boundary of CD do not change during SSFRs: this gives rise to the analog of Zeno effect. The active boundary and the states at it change. Active boundary also shifts farther from the passive one. BSFRs correspond to ordinary state function reductions and in BSFRs the arrow of time changes. One could speak of a death of a conscious entity in universal sense and reincarnation with an opposite arrow of time. For instance, the findings of Minev *et al* [L77] provide support for the time reversal [L77].

18.1.1 How to construct the TGD counterpart of unitary S-matrix?

The concrete construction of scattering amplitudes remained a challenge from very beginning. During years I have proposed several proposals and many important aspects of the problem are understood but simple rules are still lacking.

1. The time evolutions assignable to SSFRs should be describable by a unitary S-matrix or its analog.

2. The counterpart of S-matrix should have the huge super-symplectic algebra (SSA) and Kac-Moody algebras related to isometries of H as symmetries. These symmetries, extended further to Yangian symmetries and quantum groups with both algebra and co-algebra structure, are expected to be a key element in the construction of the counterpart of S-matrix. In particular, product and co-product in the super-symplectic algebra define excellent candidates for vertices. What has been missing was a concrete guiding principle.

3. Feynman (or twistor) diagrammatics should generalize. Point-like particles are replaced with 3-surfaces and topologically incoming and outgoing many-particle states correspond to disjoint unions of 3-surfaces at the boundaries of CD. The first guess is that the vertices

correspond to 3-surfaces at which 4-D lines of the analog of Feynman diagram meet. SH and $M^8 - H$ duality [L85] however suggest that the lines of the diagrams should correspond to 3-D light-like orbits of partonic 2-surfaces defining boundaries between space-time regions with Euclidian and Minkowskian signature of the induced metric. Also string world sheets connecting them and also serving as carriers of information in SH should be considered. The 1-D light-like intersections of strings world sheets with partonic orbits would define carriers of fermion number.

4. The identification of fermionic anti-commutation relations was a longstanding challenge. It turned out that the induction of second quantized free fermion fields from H to X^4 fixes the anti-commutations of the induced spinor fields and allows to calculate fermionic propagators. Therefore quantum algebra would give what is needed to calculate scattering amplitudes: the interaction vertices assignable to partonic 2-surfaces and fermionic propagators would result from the induction procedure. 8-D fermions have however 7-D delta functions as anti-commutators and normal ordering of fermions can produce divergences already at the level of the MDA.

   The problem disappears if the MDA is made bilocal [L115]: in this article a more detailed discussion is given and leads to a rather detailed picture about MDA.

5. $M^8 - H$ duality [L85, L49, L50, L51] allows to concretize this picture. One can regard X^4 either as a surface in the complexified M^8 or in H. $M^8 - H$ duality maps space-time surfaces from M^8 to H. Space-time surfaces in the complexified M^8 correspond to algebraic 4-surfaces determined by real polynomials with real (rational if one requires p-adicization) coefficients. Also rational and even analytic functions can be considered, in which case polynomials could be seen as approximations. The roots of the real polynomial dictate the space-time surfaces as quaternionic/associative 4-surfaces in complexified octonionic M^8. Holography becomes discrete.

   The algebraic equations defining space-time surfaces also have special solutions, in particular 6-spheres. These analogs of 6-branes have as M^4 projections in both M^8 and $H = M^4 \times CP_2 t = r_n$ hyperplanes, where r_n corresponds to a root of a real polynomial defining X^4 in complexified M^8 The interpretation of these hyper-planes is in TGD inspired consciousness is as "very special moments in the life of self".

   The solutions of the analog of Dirac equation in M^8 as algebraic equation [L116] are localized to 3-D light-like surfaces and mapped to light-like 3-surfaces in H identifiable as orbits of partonic 2-surfaces. Partonic 2-surfaces serving as vertices of topological analogs of Feynman diagrams would reside at the above described $t = r_n$ hyperplanes of $H = M^4 \times CP_2$. Scattering amplitudes would have partonic 2-surfaces as vertices and their 3-D light-like orbits as lines. The intersections of string world sheets with the partonic orbits would be 1-D lines and could be interpreted as fermion lines so that also the point particle description would be part of the picture.

   CDs inside CD would define the regions inside which particle reactions occur and this suggests a fractal hierarchy of CDs within CDs as a counterpart for the hierarchy of radiative corrections.

What is still missing is the general principle allowing a bird's eye of view about the counterpart of S-matrix. Wheeler was the first to introduce the notion of unitary S-matrix, which generalizes probability conservation to an infinite number of conditions. Could one challenge the unitary principle and consider something else instead of it?

1. Unitary time evolution is natural in non-relativistic quantum mechanics but is already problematic in quantum field theory (QFT), in particular in twistor Grassmannian approach [B28]. The idea about the reduction of physics to Kähler geometry inspires the question whether Kähler geometry of WCW could provide a general principle for the construction of the scattering amplitudes and perhaps even an explicit formulas for them.

   Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurements. Could the Kähler metric of state space code the counterpart of S-matrix and even unitary S-matrix? Also the Kähler metric satisfies conditions analogous to unitarity conditions.

An amazingly simple argument demonstrates that one could construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric the analog of a unitary S-matrix by assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied.

(a) If the probabilities are identified as the real parts of complex analogs $p^c_{i,j} = g_{i,\bar{j}}g^{\bar{j},i}$ of probabilities, it is enough to require $Re(p^c_{i,j}) \geq 0$. The complex analogs of $ip^c_{i,j})$ would define the analog of Teichmueller matrix [A35, A55, A43] (`https://en.wikipedia.org/wiki/Teichm\unhbox\voidb@x\bgroup\let\unhbox\voidb@x\setbox\@tempboxa\hbox{u\global\mathchardef\accent@spacefactor\spacefactor}\let\begingroup\endgroup\relax\let\ignorespaces\relax\accent127u\egroup\spacefactor\accent@spacefactorller_space`) for which imaginary parts of matrix elements are non-negative. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By SH, the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

(b) By positing the condition that $g_{i,\bar{j}}$ and $g^{\bar{j},i}$ have opposite phases, one can assign to Kähler metric a unitary S-matrix but this does not seem to be necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique. These solutions would be special case of Teichmueller solutions: Teichmueller matrix would be purely imaginary. The condition looks too restrictive. For instance, for torus, this would correspond to a metric conformally equivalent with a flat metric.

2. This inspires the idea that quantum physics could be geometrized by the same way as Einstein geometrized gravitation. Take a flat Hilbert space bundle (in the case of TGD) and replace its flat Kähler metric both base space and fiber with a non-flat Kähler metric. The replacement of flat metric with a curved one would lead from a non-interacting quantum theory to an interacting one. Quantum theory would be gravitation at the level of this Hilbert bundle! This replacement is completely universal.

   In the TGD framework the world of classical worlds (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at X^4 and induced from second quantized spinors of the embedding space. Scattering amplitudes would be determined by the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in ZEO and satisfying Teichmueller condition guaranteeing non-negative probabilities.

   WCW geometry is also characterized by zero modes corresponding to non-complex coordinates for WCW giving no contribution to WCW metric. This is self-evident from SH. The zero modes would be in 1-1 correspondence with Teichmueller parameters and WCW Kähler metrics.

   Equivalence Principle (EP) generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give a possible interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic QFT [K105].

There is also challenge of constructing the Kähler metric and associated spinor structure for the spinor bundle of WCW. This would mean a specification of the analogs of Feynman rules so that instead of two problems one would have only one problem.

1. WCW gamma matrices can be identified as superpositions of fermionic oscillator operators associated with quark spinors [L92]. One can consider two approaches to the quantization

of these spinors: one studies induced spinor fields obeying MDE and quantizes this or one generalizes the induction of spinors from H to the induction of second quantized spinor fields in H: this would mean simply projecting the spinor fields to X^4. The latter option is extremely simple. It seems possible to avoid divergence problems if the anti-commutators are assigned to different 3-surfaces at different boundaries of CD. This would allow the identification of the Dirac propagator. As a matter of fact, the two approaches are equivalent.

2. WCW gamma matrices would allow the identification as super generators of SSA identified as contractions of gamma matrices SSA with Killing vectors. Quantum states would be created by bosonic and fermionic SSA generators.

3. I have proposed a further supersymmetrization of both H coordinates and spinors by replacing them with expansions in powers of local composites of oscillator operators for quarks and antiquarks [L92]. This however requires Kronecker delta type anti-commutators natural for cognitive representations defining unique discretization of X^4: this allows to avoid normal ordering divergences. Induction of the H spinor fields would lead to 8-D delta function type divergences. This suggests that local composites are not quite local but states consisting of quarks and antiquarks at opposite throats of wormhole contacts identifiable as partonic 2-surfaces. One would obtain leptons as 3-quark states with quarks at the same partonic 2-surface but not at the same point anymore as in the proposal of [L92].

4. The matrix elements of the Kähler metric of WCW Hilbert bundle correspond to scattering amplitudes analogous to Feynman diagrams. What are the Feynman rules? Partonic two surfaces and their orbits correspond to vertices and propagators topologically. The TGD counterpart for $F\overline{F}B$ vertex would correspond to a bosonic wormhole contact with a fermion and antifermion at opposite wormhole throats and representing SCA generator which decomposes to two partonic 2-surfaces carrying fermions at opposite throats representing fermionic SCA generators. This allows avoiding of normal ordering divergences.

   The vertex would correspond to a product or co-product, which can be said to be time reversals of each other. The structure constants of SCA extended to quantum algebra would fix the vertices and thus the analogs of Feynman diagrams completely. Their number is presumably finite for a X^4 with fixed 3-surfaces at its ends and summation over Feynman diagrams would correspond to integration in WCW.

Before discussing them current proposal in detail, the complementary way to overview TGD as either WCW geometry or as number theory are discussed below. Readers might skip these sections at their first reading and choose to read the section discussing the basic idea in more detail.

In the sequel the basic idea about representation of scattering amplitudes as elements of Kähler metric satisfying what I call "Teichmueller condition", is discussed in TGD framework.

The detailed formulation allows a formulation of conditions for the cancellation of normal ordering divergences and also other divergences. The induction of the second quantized free spinor field from H to space-time surface fixes the propagators at the space-time level. If the creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts, divergences are avoided. ZEO suggests an alternative but not exclusive option that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

The fact that the Dirac propagators for massive particles in the TGD sense reduce in a good approximation to massless propagators when the propagation takes place along light-like distances, allows to considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

Of course, one must not forget that this proposal is only an interesting thought game. It is quite possible that zero energy ontology allows to define a natural way a unitary S-matrix or a more general isometric map between the states spaces associated with the extensions of rationals with different algebraic dimensions assignable naturally to to space-time regions inside causal diamonds. The huge symmetries of WCW generalized to Yangian symmetries could lead to a unique S-matrix and number theoretic conditions pose extremely powerful constraints. In [L162], a proposal along these lines was developed 3 years after writing this.

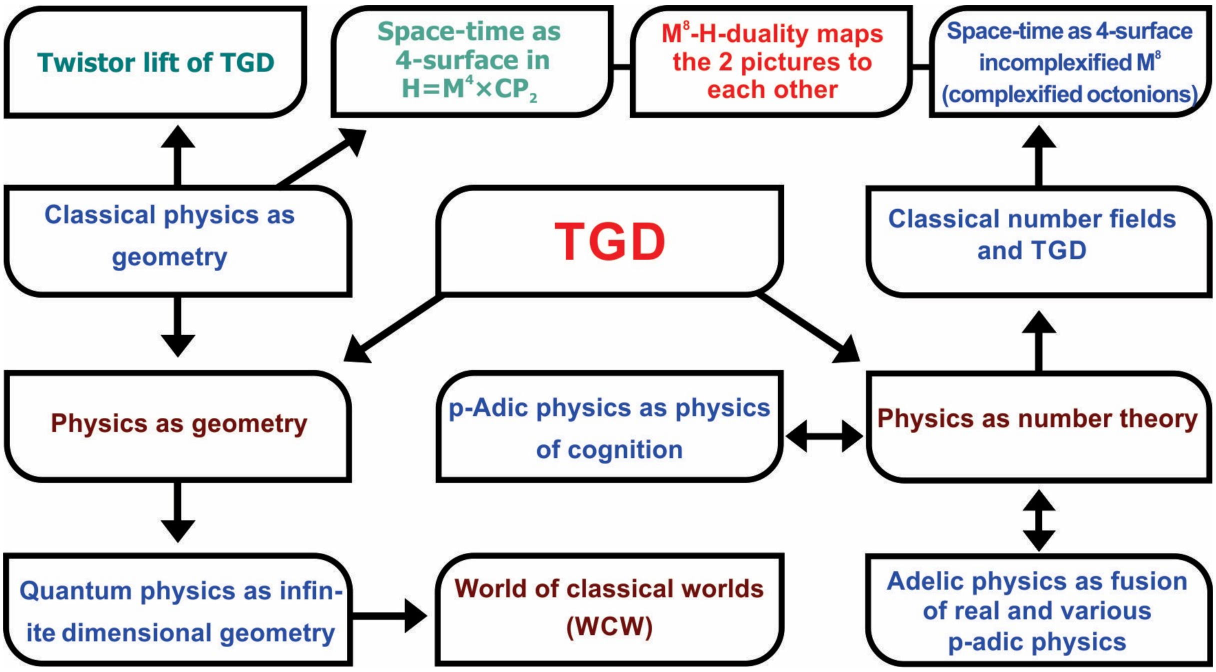

Figure 18.1: TGD is based on two complementary visions: physcs as geometry and physics as number theory.

18.2 Physics as geometry

One can end up with TGD in two ways (see **Fig. 18.2**). Either as a solution of energy problem of GRT realizing Einstein's dream about geometrization of classical physicsor as a generalization of hadronic string model or of superstring theory [B43]. In case of hadronic string model the generalization of string to 3-surface would allow to get rid of spontaneous compactification and the landscape catastrophe implied by it.

At fundamental level TGD could be seen as a hybrid of GRT and SRT: the notion of force does not disappear and can be defined as rate for an exchange of conserved quantity which can be Poincare or color charge. This connection with Newtonian limit is more clear than in GRT, where the conservation laws are lost.

18.2.1 Classical physics as sub-manifold geometry

The new elements are many-sheeted space-time topologically non-trivial in all scales, and topological field quantization implying that physical systems have field identity, field body, in particular magnetic body (MB) central in applications [L6, L5] (see **Fig. 18.3**).

Induction procedure

One ends up to a geometrization of gravitational field and gauge fields of the standard model as induced fields. Induction means induction of bundle structure is in question. Parallel translation at X^4 is carried out by using spinor connection of H and distances are measured using the metric

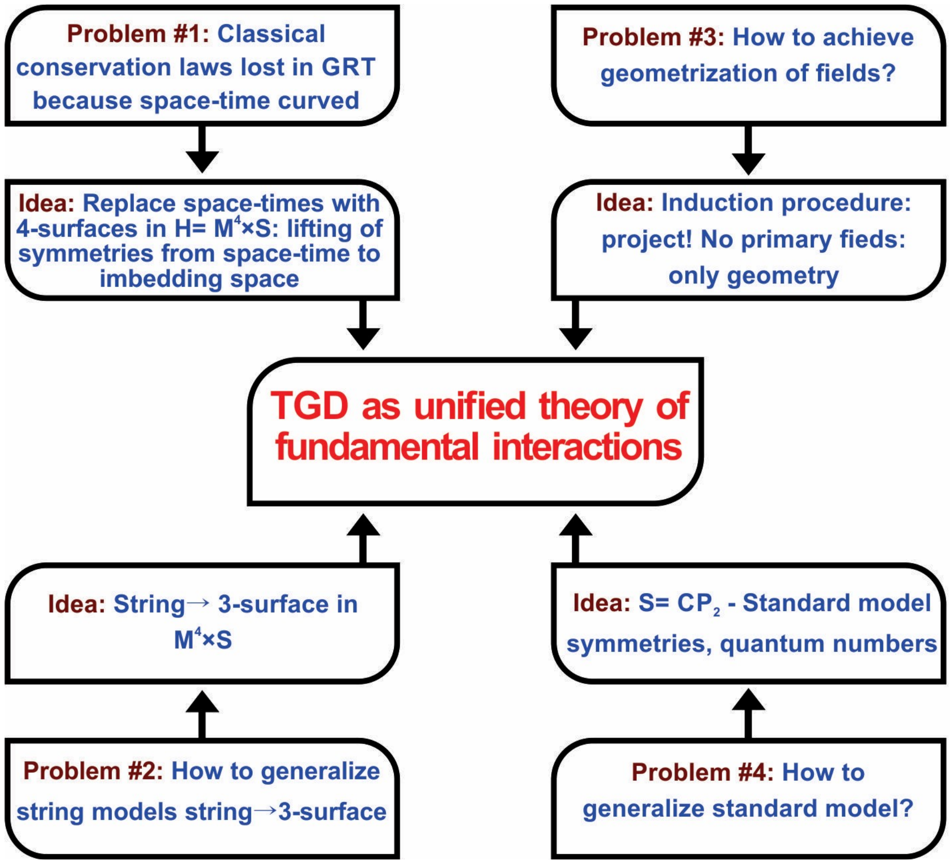

Figure 18.2: The problems leading to TGD as their solution.

of H. The components of induced gauge potentials and metric are projections to X^4. Color gauge potentials are identified as projections of Killing vector fields of CP_2 and one can define for them gauge algebra structure. The components of the induced color field are proportional to $H_A J$, where H_A is the Hamiltonian of color isometry and J induced Kähler form. For details see [L7] or the material at my homepage.

The induction of spinor structure allows to avoid the problems related to the definition of spinor structure for general 4-geometry encountered in GRT. For the induced spinor structure induction means projection of gamma matrices to X^4. The definition of gamma matrices is modified when classical action defining the space-time dynamics contains besides volume term also Kähler action with the projection of CP_2 Kähler form defining the analog of Maxwell field. Modified gamma matrices are contractions $T^{\alpha k}\gamma_k$ of the embedding space gamma matrices γ_k with canonical momentum currents $T^{\alpha k}$ associated with the action: this is required by the hermiticity of the modified Dirac action and means existence of infinite number of super currents labelled by the modes of the modified Dirac action.

Spacetime is topologically complex

Locally the theory is extremely simple: by GCI there are only 4 field-like variables corresponding to a suitable identification of embedding space coordinates as space-time coordinates. The possibility to choose the coordinates in this manner means enormous simplification since the problems caused by GCI in GRT disappear. It is however obvious that 4 field-like variables does not conform with standard model and GRT. This simplicity is compensated by topological complexity in all scales implied by the many-sheeted space-time. The QFT-GRT limit explained in introduction gives the space-time of gauge theories and GRT.

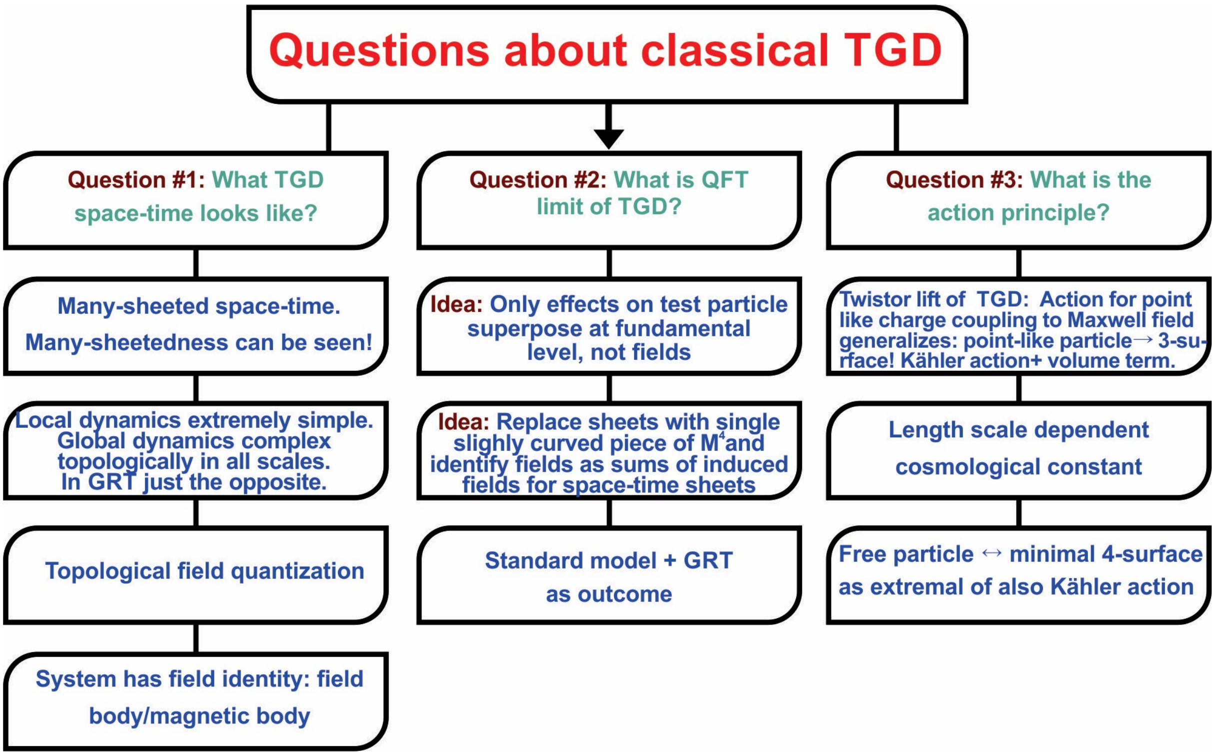

Figure 18.3: Questions about classical TGD.

Geometrically the QFT limit for space-time surfaces having 4-D M^4 projection is obtained by replacing the sheets of many-sheeted space-time with slightly curved region of M^4 and identifying gauge potentials and gravitational field (deviation of the metric from M^4 metric) as superpositions of induced fields at various space-time sheets. Einstein's equations hold true as a remnant of the Poincare invariance.

The presence of space-time regions with M^4 projection of dimension $D < 4$ must be described at QFT limit as particle- or string-like entities. Particle-like entities correspond to CP_2 type extremals having Euclidian signature of induced metric and light-like M^4 projection. 3-D light-like surfaces serve as boundaries between them and Minkowskian space-time regions: the identification is as partonic orbits carrying fermion number serving as building bricks of elementary particles [L58].

The topology of partonic 2-surface is characterized by its genus (number of handless attached to sphere) and is propose to explain family replication for fermions. Also for bosons 3 families are predicted. The existence of 3 light fermion families is understood in terms of the fact that only 3 lowest genera have global Z_2 as conformal symmetry making possible bound state of 2 handles. For the higher genera handles would behave like particles and mass spectrum would be continuum.

Cosmic strings are fundamental objects of this kind and appear as two different species. Those carrying monopole flux mean deviation from Maxwell's theory. They are unstable against perturbations making their M^4 projection 4-D and transforming them to magnetic flux tubes playing a key role in TGD inspired cosmology.

Twistor lift

One could end up with the twistor lift of TGD from problems of twistor Grassmannian approach originally due to Penrose [B61] and developed to a powerful computational tool in $\mathcal{N} = 4$ SYM [B24, B18, B31, B14, B25].

Twistor lift of TGD [L40, L106, L107] generalizes the ordinary twistor approach [L73, L74] (see **Fig. 18.4**). The 4-D masslessness implying problems in twistor approach is replaced with 8-D masslessness so that masses can be non-vanishing in 4-D sense.

The basic recipe is simple: replaced fields with surfaces. Twistors as field configurations are replaced with 6-D surfaces in the 12-D product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having the structure of S^2 bundle and analogous to twistor space $T(X^4)$. Bundle structure requires dimensional reduction. The induction of twistor structure allows to avoid the problems with the non-existence of twistor structure for arbitrary 4-geometry encountered in GRT.

The pleasant surprise is that twistor space has Kähler structure only for M^4 and CP_2 [A63]: this had been discovered already when started to develop TGD! Since Kähler structure is necessary for the twistor lift of TGD, TGD is unique. One outcome is length scale dependent cosmological constant Λ assignable to any system - even hadron - taking a central role in the theory. At long length scales Λ approaches zero and this solves the basic problem associated with it. At this limit action reduces to Kähler action, which for a long time was the proposal for the variational principle.

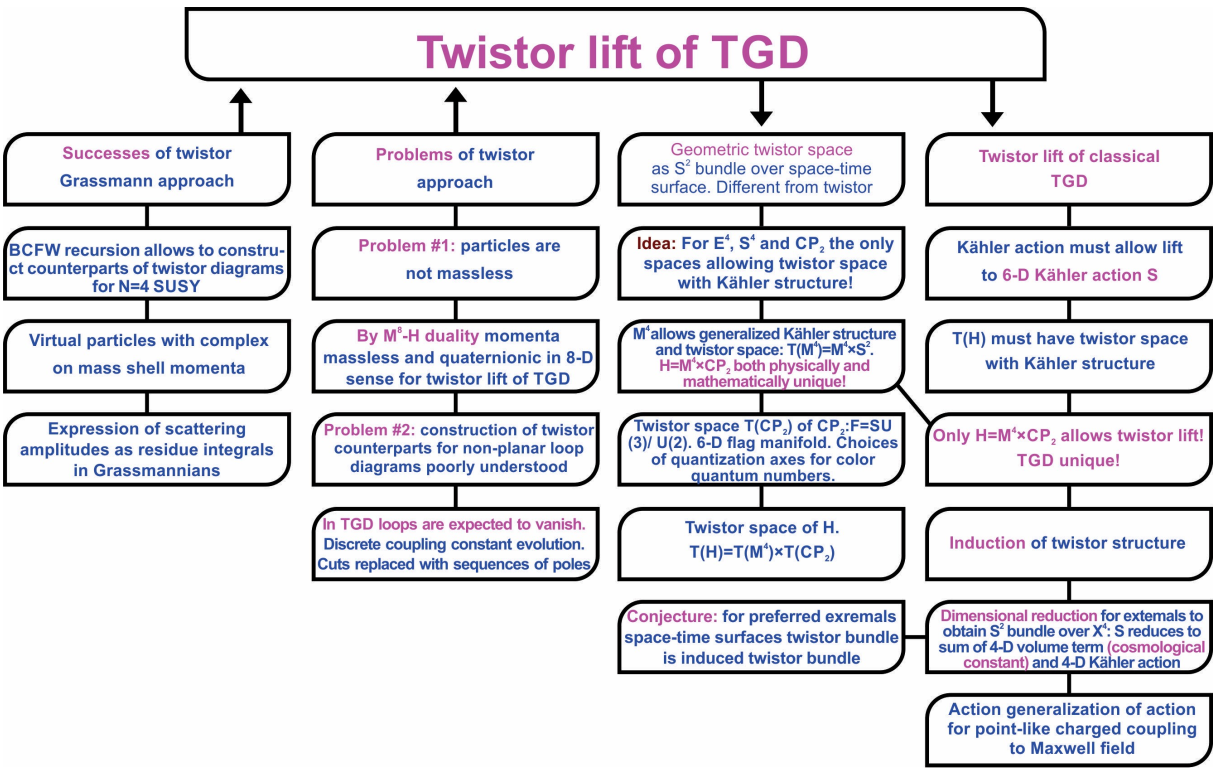

Figure 18.4: Twistor lift

18.2.2 Quantum physics as WCW geometry

WCW as an analog of Wheeler's superspace

Quantum TGD replaces Wheeler's superspace of 3-geometries with the "World of Classical Worlds" (WCW) as the space of 3-surfaces (see **Fig. 18.5**). The holography forced by general coordinate invariance (GCI) implies their 1-1 correspondence with space-time surfaces identified as preferred extremals (PEs) of the basic variational principle analogous to Bohr orbits. Classical physics becomes an exact part of quantum physics [L9, L8]. Einstein's geometrization of classical physics extends to that of quantum physics.

The geometry of infinite-D WCW (see **Fig. 18.5**) and physics is highly unique from its mere existence requiring maximal group of isometries: a result proved by Freed for loop spaces [A44]. The group of WCW isometries is identified as the group of symplectic (contact) transformations of $\delta M^3_+ \times CP_2$ having the light-like radial coordinate in the role of complex variable z in conformal field theories

Remark: The geometric properties of boundary of 4-D light-cone are unique by its metric 2-dimensionality. In particular, the ordinary 2-D conformal symmetries involving local scaling of the radial light-like coordinate give rise to isometries).

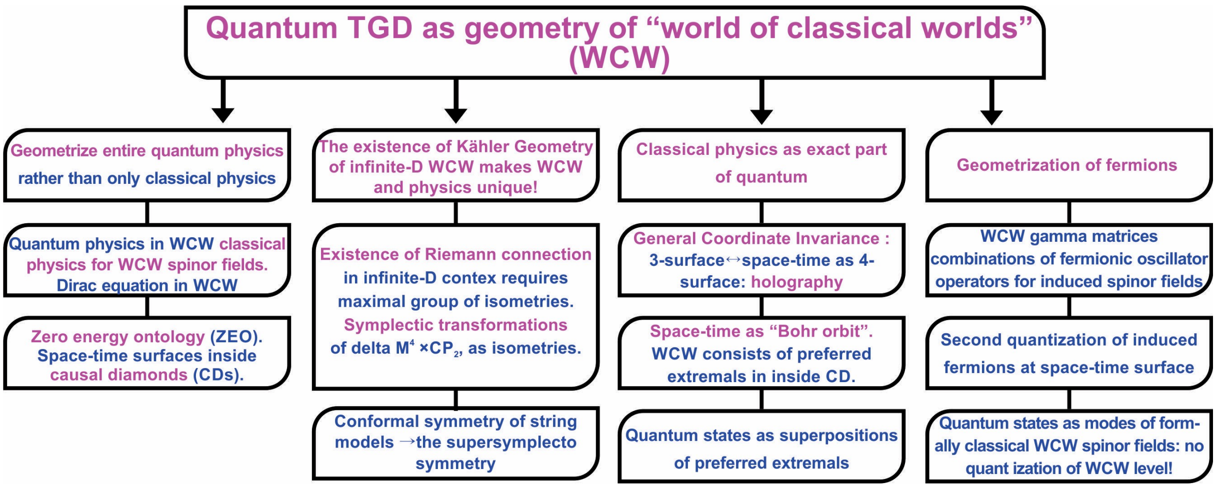

Figure 18.5: Geometrization of quantum physics in terms of WCW

The assumption that space-time surfaces as preferred extremals (PEs) are fundamental entities leads to zero energy ontology (ZEO) in which quantum superpositions of pairs (X^3_1, X^3_2) of 3-surfaces at opposite boundaries of causal diamond (CD) and connected by PE represent quantum states [L127]. This leads to a solution of the basic problem of quantum measurement theory due to the conflict between the determinism of field equations and non-determinism of state function reduction (SFR) and quantum measurement theory extends to a theory of consciousness bringing observer a part of the physisal system.

Quantum states are identified as modes of classical WCW spinor fields so that apart from quantum jump the theory is formally classical. WCW spinor structure involves complexified gamma matrices expressible as superpositions of second quantized oscillator operators of the induced spinor fields at space-time so that a geometrization of fermionic statistics is achieved [L25, L108, L115]. The simplest formulation assumes only quark spinors and would predict that lepton are local composites of 3 quarks.

18.2.3 Super-symplectic group as isometries of WCW

The work of Freed related to the geometrization of loop spaces [A44] demonstrated that the Kähler metric allows awell-defined Riemann connection onlyif it has a maximal group of isometries. This fixes the metric completely. The natural conjecture isthat this is true also in 3-D case and that the group consists of symplectic (contact) transformations at $\delta M^4_\pm \times CP_2$. Here $\delta M^4_\pm$ is future/past directed lightcone boundary containing the "upper"/"lower" boundary of a causal diamond of M^4.

WCW allows as infinitesimal isometries huge super-symplectic algebra (SSA) [K57, K29] acting on space-like 3-surfaces at the ends of space-time surfaces inside causal diamond (CD) and also generalization of Kac-Moody and conformal symmetries acting on the 3-D light-like orbits of partonic 2-surfaces (partonic super-conformal algebra (PSCA)). These symmetry algebras have a fractal structure containing a hierarchy of sub-algebras isomorphic to the full algebra. Even ordinary conformal algebras with non-negative conformal weights have similar fractal structure as also Yangian. In fact, quantum algebras are formulated in terms of these half algebras.

The proposal is that physical states are annihilated by a sub-algebra SSA$_n$ of SSA (with non-negative conformal weights), $n = 1, 2, ...,$ with conformal weights coming as n-multiples of those for SSA and thus isomorphic to the entire SSA, and by the commutator $[SSA_n, SSA]$. What remains seems to be a finite-D Kac-Moody algebra as an effective "coset" algebra obtained. Note that the resulting analog of a normal sub-group could actually be a quantum group.s There is a direct analogy with the decomposition of the Galois group Gal to a product of sub-group and normal subgroup H. If the normal subgroup H acts trivially on the representation of Gal reduces to that of the group Gal/H. Now one works at Lie algebra level: Gal is replaced with SSA and H with its sub-algebra with conformal weights multiples of those for SSA. These two hierarchies of subgroups could correspond to each other and to the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs) [K126, K44]. These conditions would guarantee preferred extremal property of the space-time surface and holography or even its strong form.

Holography from GCI

Gravitational holography has been one of the dominating themes in recent day theoretical physics. It was originally proposed by Susskind [B55], and formulated by Maldacena as AdS/CFT correspondence [B52]. One application is by Preskill *et al* to quantum error correcting codes [B33].

By holography implied by GCI the basic variational problem can be seen either as boundary value problem with 3-surfaces at opposite boundaries of CD or as initial value problem caused by PE property. Ordinary 3-D holography is thus forced by general coordinate invariance (GCI) and loosely states that the data at 3-surface at either boundary of CD allows to determine $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

Holography poses additional strong conditions on X^4.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SSA_n with radial conformal weights coming as n-multiples of those for the entire algebra SSA and its commutator $[SSA_n, SSSA]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations. An open question is whether this hierarchy corresponds to the hierarchy of EQs with n identifiable as dimension of the extension.

2. Second conjecture is that PEs are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor

structure from the product of twistor spaces of M^4 and CP_2 to 6-D S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [A63] so that TGD is unique.

Strong form of holography

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated

1. either with light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian,

2. or the space-like 3-surfaces at the ends of CD to determine X^4 as PE (in case that it exists),

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken the strong form of $M^8 - H$ duality [L101] mapping $X^4 \subset M^8$ to $X4 \subset H = M^4 \times CP_2$ that it allows to map only certain 2-D sub-manifolds $X^2 \subset X^4 \subset M^8$: SH allows to determine $X^4 \subset H$ from this 2-D data.

Further generalizations

This picture about WCW is not general enough.

1. $M^8 - H$ duality [L101] suggests that the notion of WCW applies also M^8 picture. The parameters of polynomials defining $X^4 \subset M^8$ are assumed to be rational. The points of M^8 counterpart of WCW have the rational coefficients of these polynomials as coordinates so that WCW should be discrete in real topology. This should be the case also for H counterpart of WCW. Could one see real and p-adic variants of WCW as completions of this discrete WCW.

2. Adelic physics inspires the question whether p-adic and adelic variants of WCW make sense or is it enough to have number theoretically universal cognitive representations to define unique discretized variants of X^4 and correspondingly discretized WCW.

3. For TGD variant of SUSY [L105, L104] super coordinates for H correspond to hermitian local composites of quark oscillator operators. For super-quarks they correspond to local components with fixed quark number. Leptons can be understand as local composites of quarks - super field components [L115]. SUSY replaces modes of super-field with super-surfaces so that the components of super-field correspond to sets of disjoint 4-surfaces. This is true also for the points of super WCW.

18.3 Physics as number theory

Number theoretical vision is second thread of TGD. It decomposes to 3 threads corresponding to various p-adic physics [L10] fusing to adelic physics [L58], classical number fields [L11], and infinite primes [L12] (not discussed in the sequel).

18.3.1 p-Adic and adelic physics and extensions of rationals (EQs)

p-Adic number fields would serve as correlates of cognition and imagination (see **Fig. 18.6**) . Space-time is replaced with a book like structure having both real and various p-adic space-time sheets as pages. The outcome is adelic physics as fusion of various p-adic physics [L54, L58] (see `http://tinyurl.com/ycbhse5c`). The EQ induces extensions of p-adic numbers fields and of adele giving rise to a hierarchy of physics having interpretation in terms of evolution induced by the increase of the complexity of the EQ.

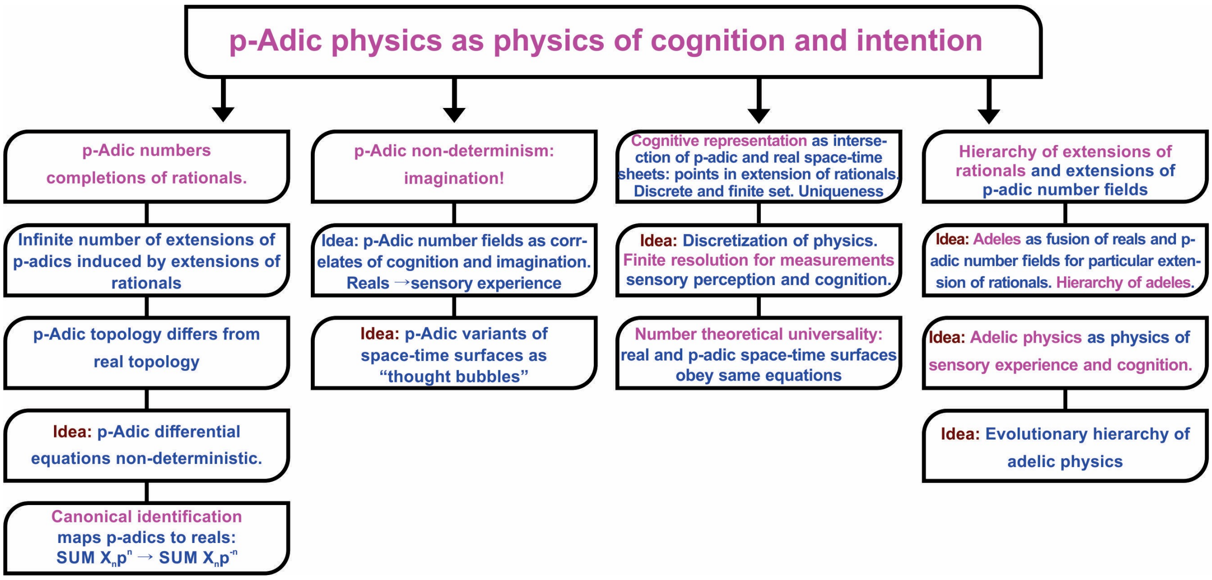

Figure 18.6: p-Adic physics as physics of cognition and imagination.

Adelic physics leads also the hierarchy of Planck constants $h_{eff}/h_0 = n$ with n identified as dimension of EQ labelling phases of ordinary matter behaving like dark mater, and making possible quantum coherence in arbitrarily long time scales essential for understanding living matter.

EQs are characterized by discriminant D assignable to a polynomial giving rise to the extension (for second order polynomials D has expression familiar from school days). Now polynomials with rationals (equivalently integer) valued coefficients are interesting. The primes dividing the discriminant are known as ramified primes and they have a property that for p-adic variant of polynomial degenerate roots appear in $O(p) = 0$ approximation [L103]. The interpretation could be in terms of quantum criticality and physically preferred p-adic primes are identified as ramified primes of extension [L111].

Remark: One can also consider polynomials with algebraic coefficients. The notion of Galois group make sense also for real coefficients.

The hierarchy of EQs labelling levels of dark matter hierarchy and of hierarchy of adelic physics follows from $M^8 - H$ duality allowing to identify $X^4 \subset M^8$ as a projection of $X^4_c \subset M^8_c$ - identified as complexified octonions O_c - and satisfying algebraic equations associated with a polynomial of degree n.

Real and p-adic physics are strongly correlated and mass calculations represent the most important application of p-adic physics [K65]. Elementary particles seem to correspond to p-adic primes near powers 2^k (there are also indicatons for powers of 3). Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of period doubling in which prime $p = 2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p = 3$ (period tripling as approach to chaos) [?, ?]. Many elementary particles and also hadron physics and electroweak physics seem to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2 and the challenge is to understand this [L41].

18.3.2 Classical number fields

Second aspect of number theoretical vision are classical number fields: reals, complex numbers, quaternions and octonions and their complexifications by a commuting imaginary unit i (see **Fig. 18.7**).

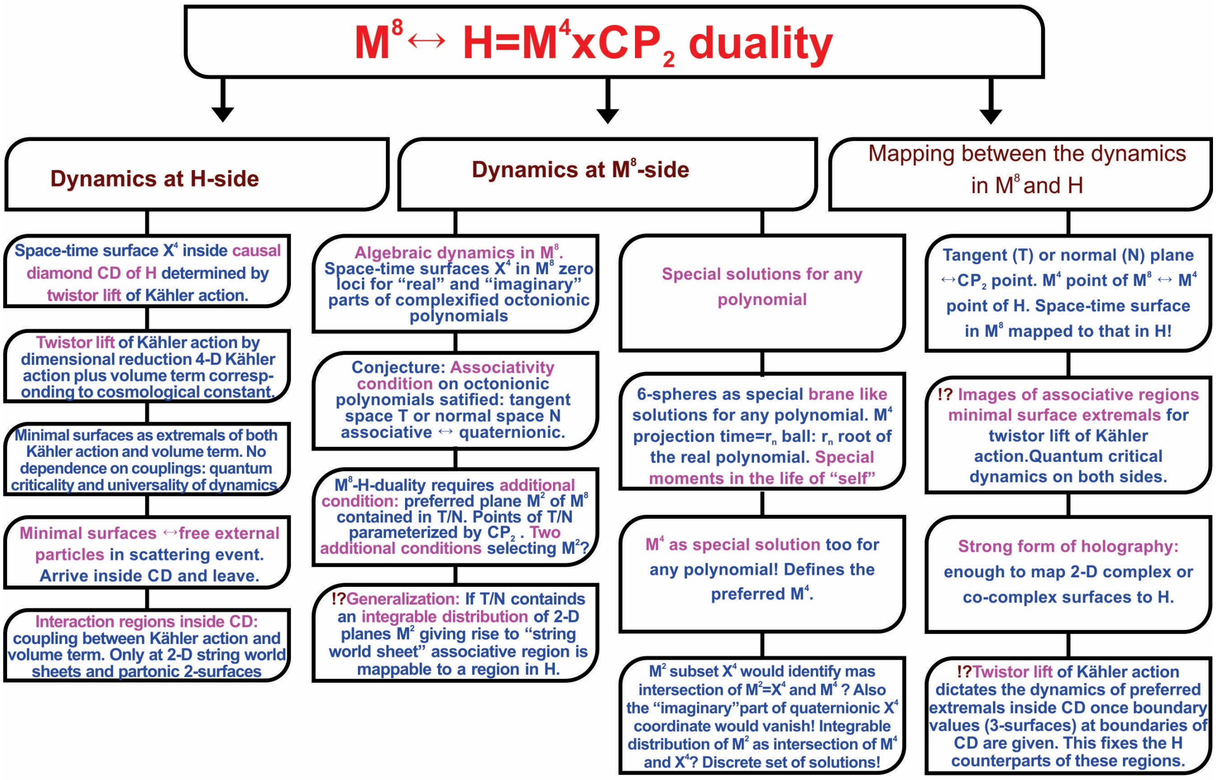

Figure 18.7: $M^8 - H$ duality

Space-time as 4-surface in $M_c^8 = O_c$

One can regard real space-time surface $X^4 \subset M^8$ as a M^8–-projection of $X_c^4 \subset M_c^8 = O_c$. M_c^4 is identified as complexified quaternions H_c [L101, L111]. The dynamics is purely algebraic and therefore local.

1. The basic condition is associativity of $X^4 \subset M^8$ in the sense that either the tangent space or normal space is associative - that is quaternionic. This would be realized if X_c^4 as a root for the quaternion-valued "real" or "imaginary part" for the O_c algebraic continuation of real analytic function $P(x)$ in octonionic sense. Number theoretical universality requires that the Taylor coefficients are rational numbers and that only polynomials are considered.

   The 4-surfaces with associative normal space could correspond to elementary particle like entities with Euclidian signature (CP_2 type extremals) and those with associative tangent space to their interaction regions with Minkowskian signature. These two kinds space-time surfaces could meet along these 6-branes suggesting that interaction vertices are located at these branes.

2. The conditions allow also exceptional solutions for any polynomial for which both "real" and "imaginary" parts of the octonionic polynomial vanish. Brane-like solutions correspond to 6-spheres S^6 having $t = r_n$ 3-ball B^3 of light-cone as M^4 projection: here r_n is a root of the real polynomial with rational coefficients and can be also complex - one reason for complexification by commuting imaginary unit i. For scattering amplitudes the topological vertices as 2-surfaces would be located at the intersections of X_c^4 with 6-brane. Also Minkowski space M^4 is a universal solution appearing for any polynomial and would provide a universal reference space-time surface.

3. Polynomials with rational coefficients define EQs and these extensions form a hierarchy realized at the level of physics as evolutionary hierarchy. Given extension induces extensions of p-adic number fields and adeles and one obtains a hierarchy of adelic physics. The dimension n of extension allows interpretation in terms of effective Planck constant $h_{eff} = n \times h_0$. The phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ behave like dark matter and galactic dark matter could correspond to classical energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes. It is not completely clear whether number galactic dark matter must have $h_{eff} > h$. Dark energy in would correspond to the volume part of the energy of the flux tubes.

   There are good arguments in favor of the identification $h = 6h_0$ [?] "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining X^4 as a covering space locally. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

   The ramified primes of extension in turn are identified as preferrred p-adic primes. The moduli for the time differences $|t_r - t_s|$ have identification as p-adic time scales assignable to ramified primes [L111]. For ramified primes the p-adic variants of polynomials have degenerate zeros in $O(p) = 0$ approximation having interpretation in terms of quantum criticality central in TGD inspired biology.

4. During the preparation of this article I made a trivial but overall important observation. Standard Minkowski signature emerges as a prediction if conjugation in O_c corresponds to the conjugation with respect to commuting imaginary unit i rather than octonionic imaginary units as though earlier. If X^4 corresponds to the projection $O_c \rightarrow M^8 \rightarrow M^4$ with real time coordinate and imaginary spatial coordinates the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

How to realize $M^8 - H$ duality?

$M^8 - H$ duality (see **Fig. 18.7**) allows to $X^4 \subset M^8$ to $X^4 \subset H$ so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D preferred 2-surfaces defining holography making possible $M^8 - H$ duality and possibly appearing as singularities in H. The dynamics of minimal surfaces, which are also extremals of Kähler action, reduces for known extremals to purely algebraic conditions analogous to holomorphy conditions in string models and thus involving only gradients of coordinates. This condition should hold generally and should induce the required huge reduction of degrees of freedom proposed to be realized also in terms of the vanishing of super-symplectic Noether charges already mentioned [L108].

Twistor lift allows several variants of this basic duality [L106, L107]. M_H^8 duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of EQs defining an evolutionary hierarchy. This forms the basics for the number theoretical vision about TGD.

As already noticed, $X^4 \subset M^8$ would satisfy an infinite number of additional conditions stating vanishing of Noether charges for a sub-algebra $SSA_n \subset SSA$ of super-symplectic algebra SSA actings as isometries of WCW.

$M^8 - H$ duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions [L101].

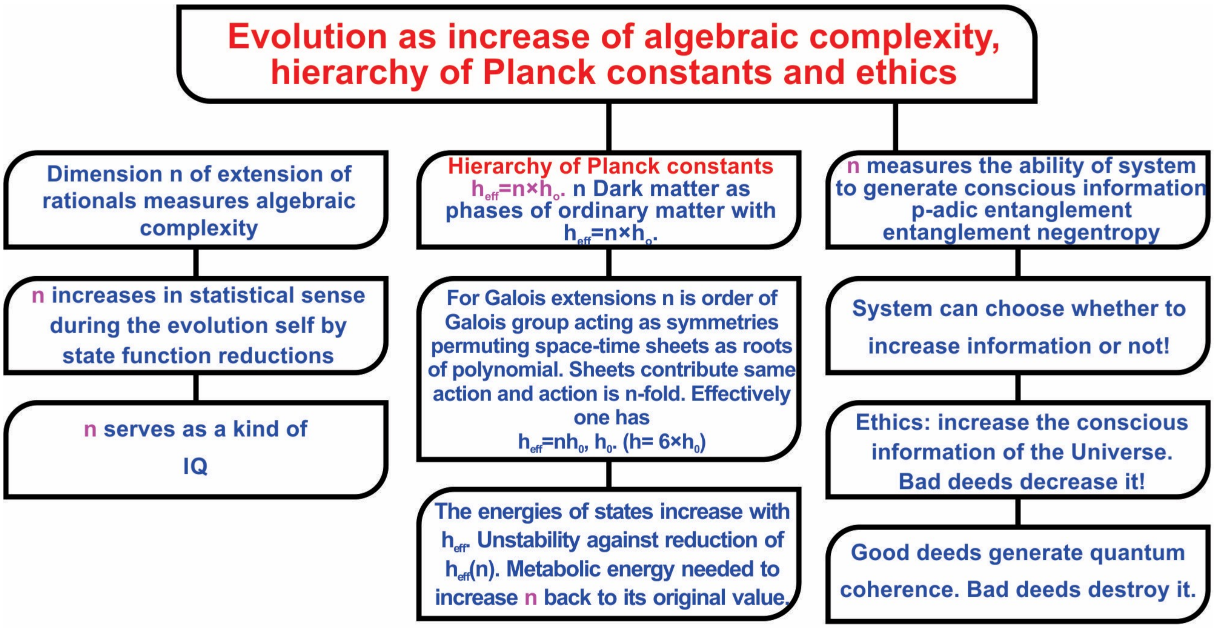

Figure 18.8: Number theoretic view about evolution

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of M^8-H duality and means that tangent - or normal space is associative/quaternionic.

2. Each tangent space of X^4 at x must contain a preferred $M^2_c(x) \subset M^4_c$ such that $M^2_c(x)$ define an integrable distribution and therefore complexified string world sheet in M^4_c. This gives similar distribution for their orthogonal complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense. $E^2_c(x)$ would correspond to partonic 2-surface. This condition generalizes for X^4 with quaternionic normal space.

One can imagine two realizations for this condition.

Option I: Global option states that the distributions $M^2_c(x)$ and $E^2_c(x)$ define a slicing of X^4_c.

Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce $X^4 \subset H$. This would be the minimal option.

It seems that only **Option II** can be realized.

1. The basic observation is that X^2_c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M^2_c would be the simplest solution but also more general complex sub-manifolds $X^2_c \subset M^4_c$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. SH makes possible $M^8 - H$ duality assuming that associativity conditions hold true only at 2-surfaces including partonic 2-surfaces or string world sheets or perhaps both. Thus one can give up the conjecture that the polynomial ansatz implies the additional condition globally.

   SH indeed states that PEs are determined by data at 2-D surfaces of X^4. Even if the conditions defining X_c^2 have only a discrete set of solutions, SH at the level of H could allow to deduce the PEs from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. The existence of $M^2(x)$ would be required only at the 2-D surfaces.

3. There is however a delicacy involved: the X^2 might be only metrically 2-D but not topologically. The partonic orbits are 3-D light-like surfaces with metric dimension $D = 2$. The 4-metric degenerates to 2-D metric at them. Therefore their pre-images would be natural candidates for the singularities at which the dimension of the quaternionic tangent or normal space reduces to 2 [L102] . If this happens, SH would not be quite so strong as expected. The study of fermionic variant of $M^8 - H$ correspondence indeed leads to this conclusion.

One can generalize the condition selecting X_c^2 so that it selects 1-D surface inside X_c^2. By assuming that R_c-valued "real" or "imaginary" part of complex part of P at this 2-surface vanishes. One obtains preferred M_c^1 or E_c^1 containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as a complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

What about $M^8 - H$ duality in the fermionic sector?

During the preparation of this article I became aware of the fact that the realization M^8-H duality in the fermionic sector has remained poorly understood. This led to a considerable integration of the ideas about $M^8 - H$ duality also in the bosonic sector and the existing phenomenological picture follows now from $M^8 - H$ duality. There are powerful mathematical guidelines available.

1. Octonionic spinors

By supersymmetry, octonionicity should have also a fermionic counterpart.

1. The interpretation of M_c^8 as complexified octonions suggests that one should use complexified octonionic spinors in M_c^8. This is also suggested by SO(1,7) triality unique for dimension $d = 8$ and stating that the dimensions of vector representation, spinor representation and its conjugate are same and equal to $D = 8$. I have already earlier considered the possibility to interpret M^8 spinors as octonionic [L26]. Both octonionic gamma matrices and spinors have interpretation as octonions and gamma matrices satisfy the usual anti-commutation rules. The product for gamma matrices and spinors is replaced with the non-associative octonionic product.

2. Octonionic spinors allow only one M^8-chirality, which conforms with the assumption of TGD inspired SUSY that only quarks are fundamental fermions and leptons are their local composites [L105, L104].

3. The decomposition of $X^2 \subset X^4 \subset M^8$ corresponding to $R \subset C \subset Q \subset O$ should have an analog for the O_c spinors as a tensor product decomposition. The special feature of dimension $D = 8$ is that the dimensions of spinor spaces associated with these factors are indeed 1, 2, 4, and 8 and correspond to dimensions for the surfaces!

   One can define for the octonionic spinors associative/co-associative sub-spaces as quaternionic/co-quaternionic spinors by posing chirality conditions. For $X^4 \subset M_c^8$ one could define the analogs of projection operators $P_\pm = (1 \pm \gamma_5)/2$ as projection operators to either factor of the spinor space as tensor product of spinor space associated with the tangent and normal spaces of X^4: the analog of γ_5 would correspond to tangent or normal space depending on whether tangent or normal space is associative. For the spinors with definite chirality there would be no entanglement between the tensor factors. The condition would generalize the chirality condition for massless M^4 spinors to a condition holding for the local M^4 appearing as tangent/normal space of X^4.

4. The chirality condition makes sense also for $X^2 \subset X^4$ identified as a complex/co-complex surface of X^4. Now γ_5 is replaced with γ_3 and states that the spinor has well-defined spin in the direction of axis defined by the decomposition of X^2 tangent space to $M^1 \times E^1$ with M^1 defining real octonion axis and selecting rest frame. Interpretation in terms of quantum measurement theory is suggestive.

What about the sigma matrices associated with the octonionic gamma matrices? The surprise is that the commutators of M^4 sigma matrices and those of E^4 sigma matrices close to the same $SO(3)$ algebra allowing interpretation as representation for quaternionic automorphisms. Lorentz boosts are represented trivially, which conforms with the fact that octonion structure fixes unique rest system. Analogous result holds in E^4 degrees of freedom. Besides this one has unit matrix assignable to the generalized spinor structure of CP_2 so that also electroweak $U(1)$ factor is obtained.

One can understand this result by noticing that octonionic spinors correspond to 2 copies of a tensor products of the spinor doublets associated with spin and weak isospin. One has $2 \otimes 2 = 3 \oplus 1$ so that one must have $1 \oplus 3 \oplus 1 \oplus 3$. The octonionic spinors indeed decompose like $1 + 1 + 3 + \overline{3}$ under $SU(3)$ representing automorphisms of the octonions. $SO(3)$ could be interpreted as $SO(3) \subset SU(3)$. $SU(3)$ would be represented as tangent space rotations.

2. Dirac equation as partial differential equation must be replaced by an algebraic equation

Algebraization of the dynamics should be supersymmetric. The modified Dirac equation in H is linear partial differential equation and should correspond to a linear algebraic equation in M^8.

1. The key observation is that for the ordinary Dirac equation the momentum space variant of Dirac equation for momentum eigenstates is algebraic! Could the interpretation for $M^8 - H$ duality as an analog of momentum-position duality of wave mechanics considered already earlier make sense! This could also have something to do with the dual descriptions of twistorial scattering amplitudes in terms of either twistor and momentum twistors. Already the earlier work excludes the interpretation of the octonionic coordinate o as 8-momentum. Rather, $P(o)$ has this interpretation and o corresponds to the embedding space coordinates.

2. The first guess for the counterpart of the modified Dirac equation at the level of $X^4 \subset M^8$ is $P\Psi = 0$, where Ψ is octonionic spinor and the octonionic polynomial P defining X^4 can be seen as a generalization of momentum space Dirac operator with octonion units representing gamma matrices. If associativity/co-associativity holds true, the equation becomes quaternionic/co-quaternionic and reduces to the 4-D analog of massless Dirac equation and of modified Dirac equation in H. Associativity holds true if also Ψ satisfies associativity/co-associativity condition as proposed above.

3. What about the square of the Dirac operator? There are 3 conjugations involved: quaternionic conjugation assumed in the earlier work, conjugation with respect to i, and their combination. The analog of octonionic norm squared defined as the product $o_c o_c^*$ with conjugation with respect to i only, gives Minkowskian metric $m_{kl} o^k \overline{o}^l$ as its real part. The imaginary part of the norm squared is vanishing for the projection $O_c \rightarrow M^8 \rightarrow M^4$ so that time coordinate is real and spatial coordinates imaginary. Therefore Dirac equation allows solutions only for the M^4 projection X^4 and M^4 ($M8$) signature of the metric can be said to be an outcome of quaternionicity (octonionicity) alone in accordance with the duality between metric and algebraic pictures.

   Both $P^\dagger P$ and PP should annihilate Ψ. $P^\dagger P\Psi = 0$ gives $m_{kl} P^k \overline{P}^l = 0$ as the analog of vanishing mass squared in M^4 signature in both associative and co-associative cases. $PP\Psi = 0$ reduces to $P\Psi = 0$ by masslessness condition. One could perhaps interpret the projection $X^4_c \rightarrow M^8 \rightarrow M^4$ in terms of Uncertainty Principle.

There is a $U(1)$ symmetry involved: instead of the plane M^8 one can choose any plane obtained by a rotation $exp(i\phi)$ from it. Could it realize quark number conservation in the M^8 picture?

For $P = o$ having only $o = 0$ as root $Po = 0$ reduces to $o^\dagger o = 0$ and o takes the role of momentum, which is however vanishing. 6-D brane like solutions S^6 having $t = r_n$ balls $B^3 \subset CD_4$ as M^4 projections one has $P = 0$ so that the Dirac equation trivializes and does not pose conditions on Ψ. o would have interpretation as space-time coordinates and $P(o)$ as position dependent momentum components P^k.

The variation of P at the mass shell of M^8_c (to be precise) could be interpreted in terms of the width of the wave packet associated with a particle. Since the light-like curve at partonic 2-surface for fermion at X^3_L is not a geodesic, mass squared in M^4 sense is not vanishing. Could one understand mass squared and the decay width of the particle geometrically? Note that mass squared is predicted also by p-adic thermodynamics [K65].

4. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. $M^8 - H$ duality [L101] suggests that this boundary is mapped to $X^3_L \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X^3_L gives a very powerful constraint on SH and $M^8 - H$ duality.

5. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. $M^8 - H$ duality [L101] suggests that this boundary is mapped to $X^3_L \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X^3_L gives a very powerful constraint on SH and $M^8 - H$ duality.

6. The variant Dirac equation would hold true also at 2-surfaces $X^2 \subset X^4$ and should commute with the corresponding chirality condition. Now $D^\dagger D\Psi = 0$ defines a 2-D variant of masslessness condition with 2-momentum components represented by those of P. 2-D masslessness locates the spinor to a 1-D curve X^1_L. Its H-image would naturally contain the boundary of the string word sheet at X^3_L assumed to carry fermion quantum numbers and also the boundary of string world sheet at the light-like boundary of CD_4. The interior of the string world sheet in H would not carry an induced spinor field.

7. The general solution for both 4-D and 2-D cases can be written as $\Psi = P\Psi_0$, Ψ_0 a constant spinor - this is in a complete analogy with the solution of modified Dirac equation in H. P depends on position: the WKB approximation using plane waves with position dependent momentum seems to be nearer to reality than one might expect.

3. The phenomenological picture at H-level follows from the M^8-picture

Remarkably, the partly phenomenological picture developed at the level of H is reproduced at the level of M^8. Whether the induced spinor fields in the interior of X^4 are present or not, has been a long standing question since they do not seem to have any role in the physical picture. The proposed picture answers this question.

Consider now the explicit realization of $M^8 - H$-duality for fermions.

1. SH and the expected analogy with the bosonic variant of $M^8 - H$ duality lead to the first guess. The spinor modes in $X^4 \subset M^8$ restricted to X^2 can be mapped by $M^8 - H$-duality to those at their images $X^2 \subset H$, and define boundary conditions allowing to deduce the solution of the modified Dirac equation at $X^4 \subset H$. X^2 would correspond to string world sheets having boundaries X^1_L at X^3_L.

The guess is not quite correct. Algebraic Dirac equation requires that the solutions are restricted to the 3-D and 1-D mass shells $P_kP^k = 0$ in M^8. This should remain true also in H and X^3_L and their 1-D intersections X^1_L with string world sheets remain. Fermions would live at boundaries. This is just the picture proposed for the TGD counterparts of the twistor amplitudes and corresponds to that used in the twistor Grassmann approach!

For 2-D case constant octonionic spinors Ψ_0 and gamma matrix algebra are equivalent with the ordinary Weyl spinors and gamma matrix algebra and can be mapped as such to H. This gives one additional reason for why SH must be involved.

2. At the level of H the first guess is that the modified Dirac equation $D\Psi = 0$ is true for D based on the modified gamma matrices associated with both volume action and Kähler action. This would select preferred solutions of modified Dirac equation and conform with the vanishing of super-symplectic Noether charges for SSA_n for the spinor modes. The guess is not quite correct. The restriction of the induced spinors to X^3_L requires that Chern-Simons action at X^3_L defines the modified Drac action.

3. The question has been whether the 2-D modified Dirac action emerges as a singular part of 4-D modified Dirac action assignable to singular 2-surface or can one assign an independent 2-D Dirac action assignable to 2-surfaces selected by some other criterion. For singular surfaces $M^8 - H$ duality fails since tangent space would reduce to 2-D space so that only their images can appear in SH at the level of H.

   This supports the view that singular surfaces are actually 3-D mass shells M^8 mapped to X^3_L for which 4-D tangent space is 2-D by the vanishing of $\sqrt{g_4}$ and light-likeness. String world sheets would correspond to non-singular $X^2 \subset M^8$ mapped to H and defining data for SH and their boundaries $X^1_L \subset X^3_L$ and $X^1_L \subset CD_4$ would define fermionic variant of SH.

What about the modified Dirac operator D in H?

1. For X^3_L modified Dirac equation $D\Psi = 0$ based on 4-D action S containing volume and Kähler term is problematic since the induced metric fails to have inverse at X^3_L. The only possible action is Chern-Simons action S_{CS} used in topological quantum field theories and now defined as sum of C-S terms for Kähler actions in M^4 and CP_2 degrees of freedom. The presence of M^4 part of Kähler form of M^8 is forced by the twistor lift, and would give rise to small CP breaking effects explaining matter antimatter asymmetry [L105, L104]. S_{C-S} could emerge as a limit of 4-D action.

   The modified Dirac operator D_{C-S} uses modified gamma matrices identified as contractions $\Gamma^\alpha_{CS} = T^{\alpha k}\gamma_k$, where $T^{\alpha k} = \partial L_{CS}/\partial(\partial_\alpha h^k)$ are canonical momentum currents for S_{C-S} defined by a standard formula.

2. CP_2 part would give conserved Noether currents for color in and M^4 part Poincare quantum numbers: the apparently small CP breaking term would give masses for quarks and leptons! The bosonic Noether current $J_{B,A}$ for Killing vector j^k_A would be proportional to $J^\alpha_{B,A} = T^\alpha_k j_A k$ and given by $J_{B,A} = \epsilon^{\alpha\beta\gamma}[J_{\beta\gamma}A_k + A_\beta J_{\gamma k}]j^k_A$.

   Fermionic Noether current would be $J_{F,A} = \overline{\Psi}J^\alpha\Psi$ 3-D Riemann spaces allow coordinates in which the metric tensor is a direct sum of 1-D and 2-D contributions and are analogous to expectation values of bosonic Noether currents. One can also identify also finite number of Noether super currents by replacing $\overline{\Psi}$ or Ψ by its modes.

3. In the case of X^3_L the 1-D part light-like part would vanish. If also induced Kähler form is non-vanishing only in 2-D degrees of freedom, the Noether charge densities J^t reduce to $J^t = JA_k j^k_A$, $J = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma}$ defining magnetic flux. The modified Dirac operator would reduce to $D = JA_k\gamma^k D_t$ and 3-D solutions would be covariantly constant spinors along the light-like geodesics parameterized by the points 2-D cross section. One could say that the number of solutions is finite and corresponds to covariantly constant modes continued from X^1_L to X^3_L. This picture is just what the twistor Grassmannian approach led to [L73, L74].

18.4 Could Kähler metric of state space replace S-matrix?

In the sequel a more detailed view about the reduction of S-matrix to a non-flat Kähler geometry of Hilbert space consisting of WCW spinor fields is considered. The proposal is novel in the sense that the state space would codes interactions to its geometry just like space-time geometry codes gravitational interaction in general relativity.

18.4.1 About WCW spinor fields

Induction of second quantized spinor fields from H

There are two approaches to the quantization of induced spinors at space-time surfaces, and these approaches are equivalent.

1. Induction means that gamma matrices are determined by Kähler action as analogs for projections of embedding space gamma matrices and space-time spinor field χ is simply the restriction of H spinor field Ψ. For a given action determining X^4, supersymmetry allows the identification of the modified Dirac operator D and finding of the modes of the induced H spinor field as solutions of the modified Dirac equation (MDE) $D\chi = 0$. Second quantization would replace their coefficients with oscillator operators. However, it is not clear what the anti-commutation relations for the oscillator operators are.

2. One can generalize the classical induction of spinors Ψ to an induction of second quantized spinor fields in H as a restriction of the second quantized Ψ in H to the X^4. One must however get rid of normal ordering diverges due the fact that the anti-commutators for coinciding points give 7-D delta functions. One gets rid of them, if the Ψ and $\overline{\Psi}$ are assigned to disjoin space-time regions. This leads to bi-local modified Dirac action (MDA), implying automatically the classical field equations for the action determining D.

   What does $D\Psi = 0$ really mean when Ψ is quantum field? One can develop the restrictions of the c-valued modes of Ψ in terms of modes of χ satisfying $D\chi = 0$, and obtain an expression for Ψ at X^4 in terms of these modes each satisfying MDE. The operator valued coefficients of Ψ modes contributing to a given mode of χ would define the corresponding oscillator value fermionic oscillator operators at X^4.

   Also the generalizations of the variants of MDE restricted to sub-manifolds of X^4 make sense and are needed. The beauty is that there is no need to introduce spinor fields at lower-D surfaces as independent dynamical degrees of freedom. For instance, one only a variant of a modified Dirac action defined by Cherns-Simons analog of Kähler action makes sense at light-like partonic orbits so that one has an analog of a topological quantum field theory (TQFT).

How to avoid normal ordering divergences from fermionic oscillator operators?

The normal ordering divergences due to the anti-commutators of fermionic fields at the same point are really serious since induce spinor fields of 8-D $H = M^4 \times CP_2$ so that normal ordering singularities are proportional to 7-D delta function $\delta^7(0)$. They are encountered already for the ordinary MDA giving rise to bosonic SCA charges as Noether charges, which also are plagued by these divergences. Normal ordering for the oscillators in the Noether charges associated with MDA would allow to get rid of the divergences but is a mere trick. The proposal considered in [L115] is to make MDA bi-local at the space-time level.

Consider the general constraints on bi-locality coming from the cancellation of the normal ordering divergences.

1. Consider first 4-D variant of MDA. The most general option for MDA is that there is an integral over the entire X^4 for both Ψ and $\overline{\Psi}$ separately sothat one has 2 4-D integrations. One obtains potential normal ordering divergences proportional to $\delta^7(0)d^8x$. If one has two space-time sheets which in the generic case intersect transversally at discrete set of points, one obtains a vanishing result. However, the self-pairing of a given space-time sheet gives a divergence as a 4-D volume integral of $\delta^3(0)$. The definition of the self-pairing as a limit of separate space-time sheets approaching each other to get rid of the divergences looks like a trick.

   This suggests that the pairing can occur only between disjoint space-time regions, most naturally space-time sheets. For instance, parallel space-time sheets with overlapping M^4 projections. Allowing pairing only between disjoint regions eliminates also the divergences associated with the bosonic Noether charges deduced from MDA and involving 3+4-D integral instead of 3-D integral.

What could be the precise definition for these disjoint regions? M^8-H duality suggests that they correspond to different roots of the octonionic polynomial defined by real polynomials. When 2 roots coincide, one obtains a term of type $\delta^7(0)d^7x$ giving a finite result. What if the number of coinciding roots is higher than 2? This case will be discussed later in number theoretic context.

What about space-like regions, in particular the wormhole contacts expected to be small deformations of a warped embedding of CP_2 having light-like M^4 projection but having same Kähler metric and Kähler form as CP_2 [K9]? There is no pairing with a parallel space-time sheet now. It seems that the pairing must be between different wormhole contacts. This pairing could be essential for the understanding of string like entities as paired wormhole contacts providing a model for elementary particles.

2. For the bilinear MDA, the variation of the 4+4-D modified Dirac action with respect to $\overline{\Psi}$ and Ψ yields both the modified Dirac equation $D\Psi = 0$ plus the field equations for the preferred extremal. This gives the modes of the induced spinor field. In the standard picture the hermiticity condition for the Dirac action yields the same outcome and has interpretation as a supersymmetry between classical and fermionic degrees of freedom.

3. Both the phenomenological picture developed during years and $M^8 - H$ duality strongly suggest that spinors can be restricted also to lower-D surfaces. For the lower-D variants of MDA the normal ordering divergences appear already for tranversal intersections. For instance, for 3-D variant of MDA one has $\delta^7(0)d^7x$ type divergences. The only possible manner to avoid them is to require that paired regions are disjoint. For the 3-D Chern-Simons-Kähler action associated with the light-like partonic orbits the paired space-time sheets are very naturally the opposite wormhole throats so that fermions and antifermions would reside at opposite wormhole throats.

   Physical picture also suggests the assignment of actions to 2-D string world sheets and 1-D light-boundaries defining their intersections with partonic orbits.

4. Also 6-D brane-like solutions having the topology of S^6 and $t = r_n$ hyper-plane as intersection with M^4 are of physical interest. Different 4-D space-time surfaces could be glued together along 3-surfaces or 2-D partonic 2-surfaces at S^6. Arguments similar to those already discussed exclude the pairing of various objects with these 6-branes as also their self-pairing.

   Also M^4 and CP_2 define special solutions to the algebraic equations in M^8. MDA reduces to ordinary massless Dirac equation in M^4. In the case of CP_2 one has a massless Dirac equation in CP_2 and only the right-handed neutrino ν_R is possible as a solution. If only quarks are allowed, this solution is excluded. What happens for the deformations of CP_2? Could it be that quarks cannot reside inside wormhole contacts as 4-D entities? Or could one allow solutions of $D\Psi = 0$ as analytic functions of CP_2 coordinates finite in the region in which they are defined - wormhole contact does not span the entire CP_2?

Cognitive representations provide additional insights to the problem of normal ordering divergences, and it could be even argued that they are the only possible manner to define scattering amplitudes as a sequences of improving approximation natural in the approach based on hyper-finite factors of type II_1 (HFFs).

1. For a given extension of rationals determined by the polynomial defining the space-time region in M^8, the space-time surfaces inside CD are replaced with their discretizations consisting of points of M^8 in the extension considered. This surface and cognitive representation are mapped to H by $M^8 - H$ correspondence [L85]. For cognitive representations one can perform discretization by replacing the integrals defining SCA generators with discrete sums over points of the cognitive representation. This replacement is very natural since in the p-adic context the counterpart of the Riemann integral does not exist.

2. The Galois group of extension serves as a symmetry group and one can form analogs of group algebra elements - wave functions in discrete Galois group - acting on the cognitive representation and giving rise to discrete representation of quantum states. This state space has as its dimension the dimension n of the Galois group which for Galois extensions coincides

with the dimension of extension [L42, L121]. This group algebra-like structure can be given Kähler metric and also spinor structure and this spinor structure could discretize the spinor structure of WCW if gamma matrices are identified as fermionic oscillator operators.

3. Also now one can avoid divergences if the paired space-time regions, say space-time sheets, in MDA are disjoint. It can however happen that n separate points at the orbit of the Galois group approach each other and coincide: this would correspond to the touching of space-time sheets meaning coinciding roots of the octonionic polynomial. In this situation a subgroup of the Galois group would leave the intersection point invariant.

   The possible normal ordering divergence comes from different pairs of the m points, which coincide. In 4-D case, the situation corresponds to transversal space-time sheets so that the divergence vanishes. For lower-dimensional surfaces, say partonic orbits, the intersections do not occur in the generic situation but if they occur, the divergence is multiplied by a sum over the values of wave function at coinciding branches and vanishes if the representation is *non-singlet*. It would thus seem that the non-singlet character of Galois representations must be posed as an additional condition.

4. This cancellation mechanism works even without discretization since the notions of Galois group and its representations make sense for arbitrary polynomial surfaces without a restriction to rational or algebraic polynomial coefficients so that the cancellation occurs for non-singlet representations when the space-time sheets intersect.

Are fermions 4-D in H but 3-D in M^8?

M^8-H duality suggests the restriction of the induced spinor fields to light-like 3-surfaces having 2-D partonic surfaces as ends. M^8-H duality reduces space-time surfaces in M^8 to algebraic surfaces defined by polynomials of real variable. The coefficients can be complex. Concerning p-adicization real rationals defines the most attractive option. This leads to a picture in which a hierarchy of extensions of rationals defines evolutionary and cognitive hierarchies. The extensions provide cognitive representations as unique discretizations of the X^4 with embedding space coordinates in extension of rationals and the one can formulate quantum TGD in finite measurement resolution at least using these representations.

The fermionic variant of M^8-H duality [L116] leads to the conclusion that spinor modes in M^8 are restricted at 3-D light-like surfaces obeying an algebraic equations analogs to the momentum space variant of massless Dirac equation. Are H fermions also always restricted to the 3-D light-like orbits of the partonic 2-surfaces at which the signature of the induced metric changes?

On the other hand, the picture deduced at the level of H from the cancellation of the normal ordering divergences allows 4-D fermions, and also implies field equations for X^4 itself. Can one say that free fermions can reside in 4-D space-time but reside only at the 3-D mass shell in momentum space. M^8-H duality would be analogous to the duality between space-time and momentum space descriptions of particles.

Even more, string world sheets have light-like boundaries at the parton orbits. Also fermions in H would be naturally located at string boundaries and behave like point-like particles. One would obtain a picture resembling that provided by twistor Grassmannian approach. Also the cancellation of normal ordering divergences supports this picture and leads to a detailed form of bi-linear modified Dirac action. Also strong form of holography (SH) stating that 2-surfaces carry all information needed to construct the X^4 supports this view. This is actually the same as the phenomenological picture that has been applied.

M^8-H duality predicts also "very special moments in the life self" to have as correlates 6-branes with M^4 time defining in M^8 octonionic real axis (unique rest system) having as values roots of the polynomial defining the space-time surfaces. These surfaces should contain the partonic 2-surfaces defining the reaction vertices. If there is a non-determinism associated with these surfaces it should preserve classical charges and also SSA charge.

Is the proposed counterpart of QFT supersymmetry only an approximate symmetry?

The proposal for the cancellation of the normal ordering divergences allows overviewing leptons as three quark composites with 3 quarks at the same wormhole throat. This option is strongly suggested by the conceptual economy since quarks are enough for WCW spinor structure.

An interesting question is whether TGD allows a counterpart of QFT supersymmetry (SUSY). This was proposed in [L92]. The idea was that both embedding space coordinates and spinors can be expanded as polynomials in the local compositesof quark and antiquark oscillator operators - rather than anticommuting hermitian theta parameters leading to problems with fermion number conservation - with a well-defined quark number.

The proposal was that leptons are purely local 3-quark-composite analogous to a superpartner of quark: note however that quark superspinor would have quark number one so that precise spartner interpretation fails. This option and only its slightly local variant is possible only for the TGD view about color as angular momentum rather than spin-like quantum number.

This proposal was based on discrete cognitive representations as unique discretizations of the X^4 and on the crucial assumption that fermionic oscillator operators obey Kronecker delta type anticommutations rather than the 8-D anticommutations giving $\delta^7(0)$ anti-commutator singularities for the induced second quantized quark field in H. Can the notion of super-field based on local composites of quarks and antiquarks with a definite fermion number avoid normal ordering divergences for the induced anticommutation relations? One can of course think of a normal ordering of monomials but one expects problems with vertices.

This suggests that the super coordinates of H and superspinors can be only approximate notions. Superfield components would correspond to states with a fixed quark number but quarks and antiquarks would reside at opposite wormhole throats rather than forming exactly local composites. Since the throat is expected to have CP_2 size, these states would be for all practical purposes strictly local composites.

18.4.2 Kähler metric as the analog of S-matrix

Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurement. Could the Kähler metric define unitary S-matrix? Under simple additional conditions this is true!

The analogs of unitarity conditions

The following little arguments show that given Kähler metric defines an analog of unitary S-matrix giving rise positive transition probabilities, and under additional conditions also a unitary S-matrix between states with quantum numbers labeling basis of complex vectors or of complexified gamma matrices. This defines an S-matrix like entity and under some additional conditions even an unitary S-matrix.

1. The defining conditions for unitary S-matrix and Kähler metric are very similar. S and $S^\dagger$ would correspond to the covariant metric $g_{m\overline{n}}$ and contravariant metric $g_{\overline{m}n}$. Unitary for S-matrix corresponds to the conditions

$$S_{mr}S^\dagger_{rn} = S_{mr}S_{nr} = \delta_{m,n} \quad .$$

(there is summation over repeated indices). The rows of S-matrix are orthonormalized. The definition of the contravariant metriccorresponds the conditions

$$g_{m\overline{r}}g^{\overline{r}n} = \delta_{m,n} \quad .$$

The complex rows of metric tensor and contravariant metric are orthonormalized also now and rows are orthonormal

2. For S-matrix the probabilities are given by $p_{mn} = S_{mn}S^\dagger_{nm} = S_{mn}S_{mn}*$ and are real and non-negative and their sum is equal to one. Also for the Kähler metric the complex analogs of probabilities defined by

$$p^c_{mn} = g_{m\overline{r}}g^{\overline{r}n}$$

sum up to unity. Hence the real parts $Re(p^c_{mn})$ of p^c_{mn} sum up to unity whereas the imaginary parts sum up to zero.

3. p^c_{mn} are not however automatically real and non-negative and it is not clear how to interpret complex or even real but negative probabilities physically. One can however pose the positivity of the real parts of p^c_{mn} as an additional condition on the phase factors $U_{m\overline{n}} = exp(\Phi_{m\overline{n}})$ and $V_{m\overline{n}} = exp(\Psi_{m\overline{n}})$ associated with $g_{m\overline{n}} = R_{mn}U_{m\overline{n}}$ and $g^{\overline{n}m} = S_{nm}V_{\overline{n}m}$. The condition for positivity is

$$U_{m\overline{n}}V_{\overline{n}m} = cos(\Phi_{\overline{n}m} - \Psi_{\overline{n}m} \geq 0$$

and is rather mild requiring the angle difference to be in the range $(-\pi/2, \pi/2)$. This is true of the angles are in the range $(\pi/4, \pi/4)$. The condition $Re(p^c_{mn}) \geq 0$ is equivalent with the condition $Im(ip^c_{mn}) \geq 0$, and characterizes the coefficients of Teichmueller matrices [A35, A55, A43] [K26]: the meaning of this condition will be discussed below.

4. Under what conditions p^c_{mn} reduce to non-negative real numbers? One can express the probabilities as $p_{mn} = g_{m\overline{n}} \times cof(g_{m\overline{n}})/det(g)$. Note that $Z = det(g)$ is constant depending only on the point of the Kähler manifold. One can express g_{mn} as $g_{m\overline{n}} = A_{mn}U_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ as $cof(g_{m\overline{n}}) = B_{mn}V_{m\overline{n}}$. The reality condition implies

$$U_{m\overline{n}} = \overline{V_{m\overline{n}}} \quad .$$

The phases of $g_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ are opposite.

This gives additional conditions. Kähler metric involves $N_{tot} = 2N^2$ real parameters There are $(N^2-N)/2$ elements in say upper diagonal and by hermiticity they are complex conjugates of the lower diagonal. This is the number N_{cond} of conditions coming from the reality. There is also one additional condition due to the fact that the probabilities do not depend on the normalization of g. The total number of real parameters is

$$N_{param} = N_{tot} - N_{cond} - 1 = N(N-1) - 1 \quad .$$

For instance, for $N \in \{2, 3, 4\}$ one has $N_{param} = \in \{1, 5, 11\}$. Unitary matrix allows $N_{unit} = N^2$ real parameters and the ratio $N_{param}/N_{unit} = (N(N-1)-1)/N^2$ approaches unity for large values of N. Note that a unitary matrix with real diagonals has $N^2 - N$ parameters so that the number of parameters is the same as for a hermitian metric with unit determinant.

5. Could one transform the metric defining non-negative probabilities to a unitary matrix by a suitable scaling? One can indeed define a matrix S as a matrix $S_{mn} = \sqrt{A_{mn}B_{mn}/Z}U_{mn}$. One has $S_{mn}S_{mn}* = A_{mn}B_{mn}/Z$ given also by the product of $g_{m\overline{n}}g^{\overline{n}m}$ so that the probabilities are the same. The unitarity conditions reduce to $g_{m\overline{r}}g^{\overline{r}n} = \delta^n_m$.

   In infinite dimensional case problems might be produced by the appearance of the square root of determinant expected to be infinite. However, also the cofactors are expected to diverge, and one can express them as partial derivatives of the metric determinant with respect to the corresponding element of the metric. This is expected to give a finite value for the elements of the contravariant metric. Note that the ratios of the probabilities do not depend on the metric determinant.

Can one distinguish between the descriptions based on Kähler metric and S-matrix?

For the Teichmueller option the proposed analog for S-matrix involves imaginary part. Does it have some physically observable consequences?

Could one imagine a physical situation allowing to test whether the S-matrix description or its TGD variant is nearer to truth? One can indeed imagine an analog of a Markov process characterized by a matrix p of transition probabilities p_{mn} at a given step. For a two-step process the transition matrix would be p^2_{mn}.

In the TGD context one would have $p_{mn} = Re(p^c_{mn})$. What happens in a two-step process? Should one use use p^2_{mn} or $Re((p^c)^2)_{mn} = Re((p^c)2)_{mn} - Im(p^c)^2_{mn}$? If both options are possible, what could distinguish physically between them?

Could the correct interpretation be that p_{mn}^2 describes the process when the outcome is measured in both steps, and $Re((p^c)^2)_{mn}$ the process in which only initial and final states are measured? This picture would generalize to n-step processes and predict a deviations from the ordinary Markov process and perhaps allow to compare the predictions of the TGD view and standard view and deduce $Im(p^c)$.

S-matrix and its Hermitian conjugate correspond in standard physics to situations related by CPT symmetry defined as the product of charge conjugation C, spatial reflection P and time reversal T. The transition probabilities would remain invariant in this transformation although transition amplitudes are replaced with their complex conjugates.

What happens to CPT in TGD framework? In TGD framework CPT induces a hermitian conjugation $g_{m\overline{n}} \rightarrow g_{\overline{n}m} =$

18.5 The role of fermions

In this section the role of fermions (quarks as it seems) is discussed in more detail. In particular, the conditions on the scattering amplitudes from the cancellation of normal ordering divergences and co-associative octonionic spinors at the level of M^8 are discussed. Also the formulation of scattering amplitudes the level of M^8 is briefly considered.

18.5.1 Some observations about Feynman propagator for fundamental quark field

In the sequel the divergence cancellation mechanism and the properties of Dirac propagator are discussed in detail. The surprise is that the massive propagators with CP_2 mass scale reduce essentially to massless propagators for light-like separations. This allows understanding of why quarks can give rise to light elementary particles.

The second quantized free quark field Ψ in H defines fundamental fermions appearing as a building brick of elementary particles. The Feynman propagator for Ψ appears in the analogs of Feynman diagrams. Apart from the right handed neutrino (present only as a 3 quark composite at partonic 2-surface if only quarks are involved) the modes of Ψ are extremely massive. Elementary particles are light. How can one understand this?

In p-adic thermodynamics the generation of small mass was assumed to involve a generation of a negative, "tachyonic", ground state conformal weight encountered also in string models. $M^8 - H$ correspondence allows a more sophisticated description based on the choice of $M^4 \subset M^8$ mapped to $M^4 \subset H$. By 8-D Lorentz invariance the 4-D mass squared of ground state massless in 8-D sense, depends on the choice of $M^4 \subset H$, and with a proper re-choice of M^4 the particle having large M^4 mass becomes massless.

The action of the generators of super-conformal algebra creates states with a well-defined conformal weight, which are massless for a proper choice of $M^4 \subset M^8$. In p-adic thermodynamics the choice of $M^4 \subset M^8$ would correspond to a generation of negative ground state conformal weight.

The states can however mix slightly with states having higher value of conformal weight, and since one cannot choose M^4 separately for these states, a small mass is generated and described by p-adic thermodynamics. The classical space-time correlate for the almost masslessness is minimal surface property, which provides a non-linear geometrization for massless fields as surfaces. The non-linearity at the classical level leads to a generation of small mass in 4-D sense for which p-adic thermodynamics provides a model.

The propagators for the fundamental quarks in H correspond to CP_2 mass scale. Can this be consistent with the proposed picture? The following simple observations about the properties of predicted fermion propagator and anticommutator for the induced spinor fields lead to a result, which was a surprise to me. The propagators and anti-commutators of massive quarks at light cone boundary are in excellent approximation massless for light-like distances. This makes it possible to understand why elementary fermions are light.

This mechanism does not work in QFT defined in M^4 since inverse propagator is $\gamma^k p_k + m$ so that M^4 chiralities mix for massive states. In TGD picture H-chirality is fixed by 8-D masslessnes and the product of M^4 and CP_2 chiralities for spinors equals to the H chirality. The inverse progator is proportional to the operator $p^k\gamma + D_{CP_2}$, where D_{CP2} is CP_2 part of Dirac operator.

General form of the Dirac propagator in H

Second quantized quark field Ψ restricted to the space-time surface determines the Feynman propagator fundamental quark. The propagator can be expressed as a sum of left- and right-handed propagators as

$$S_F = S_{F,L} + S_{F,R} = D_L G_{F,L} + D_R G_{F,R} \ .$$

Here D_L and D_R are the left- and right-handed parts of a massless (in 8-D sense) Dirac operator D in H involving couplings to CP_2 spinor connection depending on CP_2 chirality in accordance with electroweak parity breaking. $G_{F,L}$ *resp.* $G_{F,R}$ is the propagator for a massless (in 8-D sense) scalar Laplacian in H coupling to the spinor connection assignable to left *resp.* right handed modes. G_F can be expressed by generalizing the formula from 4-D case

$$G_{F,I} \sum_n \int d^4p \frac{1}{p^2 - M_{n,I}^2} exp(ip\cdot(m_1 - m_2))\Phi_{n,I}^*(s_1)\Phi_{n,I}(s_2) \ .$$

Here one has $I \in \{L,R\}$ and the mass spectra are different for these modes. Here m_i denote points of M^4 and s_i points of CP_2. n, I, $I \in \{L,R\}$, labels the modes $\Phi_{n,I}$ of a scalar field in CP_2 associated with right and left handed modes having mass squared $M_{n,R}$. Since H-chirality is fixed to be quark chirality, there is a correlation between M^4 - and CP_2 chiralities. Apart from ν_R all modes are massive (ν_R is need not be present as a fundamental fermion) and the mass M_n, which is of order CP_2 mass about 10^{-4} Planck masses, is determined by the CP_2 length scale and depends on CP_2 chirality.

$G_{F,I}$ reduces to a superposition over massive propagators with mass $M_{n,I}$:

$$G_{F,I} = \sum_n G_F(m_1 - m_2|M_n)\Phi_{n,I}^*(s_1)\Phi_{n,I}(s_2)P_I \ .$$

Here P_I, $I \in \{L,R\}$ is a projector to the left/right handed spinors. One can express $S_{F,I}$ as a sum of the free M^4 part and interaction term proportional to the left - or right-handed part of CP_2 spinor connection:

$$S_{F,I} = D(M^4)G_{F,I} + A_I G_{F,I} \ .$$

A_I, $I \inf\{L,R\}$ acts either on s_1 or s_2 but the outcome should be the same. The first term gives sum over terms proportional to massive free Dirac propagator in M^4 allowing to get a good idea about the behavior of the propagator.

About the behavior of the quark propagator

The quark propagator reduces to left- and right-handed contributions corresponding to various mass values $M_{n,I}$. To get view about the behaviour of the quark propagator it is useful to study the behavior of $G_F(x,y|M)$ for a given mass as well as the behaviors of free and interacting parts of S_F its free part

From the explicit expression of $G_F(m_1 - m_2|M_n)$ one can deduce the behavior of the corresponding contribution to the Feynman propagator. Only ν_R could give a massless contribution to the progator. Explicit formula for G_F can be found from Wikipedia [A5] (`https://en.wikipedia.org/wiki/Propagator#Feynman_propagator`):

$$G_F(x,y|m) = \left\{ \begin{array}{ll} -\frac{1}{4\pi}\delta(s) + \frac{m}{8\pi\sqrt{s}}H_1^{(1)}(m\sqrt{s}), & s \geq 0 \\ -\frac{im}{4\pi^2\sqrt{-s}}K_1^{(1)}(m\sqrt{-s}), & s \leq 0 \ . \end{array} \right.$$

Here $H_1^{(1)}(x)$ is Hankel function of first kind and $K_1^{(1)}$ is modified Bessel function [A2](`https://en.wikipedia.org/wiki/Bessel_function`). Note that for massless case the Hankel term vanishes.

Consider first Hankel function.

1. Hankel function $H_\alpha^{(1)}(x)$ [A2, A5] obeys the defining formula

$$H_\alpha^{(1)}(x) = \frac{J_{-\alpha}(x) - exp(i\alpha\pi i)J_\alpha(x)}{isin(\alpha\pi)} \quad .$$

For integer values of α one has $J_{-n}(x) = (-1)^n J_n(x)$ so that $\alpha = n$ case gives formally $0/0$ and the limit must be obtained using Hospital's rule.

2. Hankel function $H_1^{(1)}(x)$ can be expressed as sum of Bessel functions of first and second kind

$$H_1^{(1)}(x) = J_1(x) + iY_1(x) \quad .$$

J_1 vanishes at origin whereas Y_1 diverges like $1/x$ at origin.

3. The behaviors of Bessel functions and their variants near origin and asymptotically are easy to understand by utilizing Schrödinger equation inside a cylinder as a physical analogy. The asymptotic behaviour of Hankel function for large values of x is

$$H_\alpha^{(1)}(x) = \frac{2}{\pi x} exp(i(x - 3\pi/4)) \quad ,$$

4. The asymptotic behavior of Hankel function implies that the massive Feynman propagator an oscillatory behavior as a function of $m\sqrt{s}$. Modulus decreases like $1/\sqrt{m\sqrt{s}}$. The asymptotic behavior for the real and imaginary parts corresponds to that for Bessel functions of first kind (J_1) and second kind (Y_1). At origin $H_\alpha^{(1)}(x)$ diverges like $Y_1(x) \sim \frac{(x/2)^{-n}}{\pi}$ near origin. For large values of x $K_1(x)$ decreases exponentially like $exp(-x)\frac{\sqrt{\pi}}{2x}$. At origin $K_1(x)$ diverges.

5. In the recent case the quark propagator would oscillate extremely rapidly leaving only the $\delta(s)$ part so that the propagator behaves like massless propagator!

   The localization of quarks to the partonic surfaces with a size scale of CP_2 radius implies that that the oscillation does not lead to a vanishing of the Hankel contribution to the scattering amplitudes. For induced spinor fields in the interior of space-time surfaces destructive interference is however expected to occur so that behavior is like that for a massless particle. This should explain why the observed particles are light although the fundamental fermions are extremely massive. The classical propagation would be essentially along light-like rays.

   The long range correlations between quarks would come from the $\delta(s)$ part of the propagator, and would not depend on quark mass so that it would effectively behave like a massless particle. Also the action of Dirac operator on $G_F(x,y)$ in M^4 degrees of freedom is that of a massless Dirac propagator coupling to induced gauge potentials. The quarks inside hadrons and also elementary particles associated with the wormhole throats of flux tubes could be understood as quarks at different partonic 2-surfaces at the boundary of CD having light-like distance in an excellent approximation.

6. The above argument is for the Feynman propagator but should generalize also for anticommutator. The anticommutator for Dirac operator D in M^4 can be expressed as $D\Delta(x,y)$, where D is a scalar field propagator.

$$\Delta(x,y|m) \propto \left\{ \begin{array}{ll} \frac{m}{8\pi\sqrt{s}} H_1^{(1)}(m\sqrt{s}), & s \geq 0 \\ -\frac{m}{\sqrt{-s}} K_1^{(1)}(m\sqrt{-s}), & s \leq 0 \quad . \end{array} \right.$$

   Apart from possible proportionality constants the behavior is very similar to that for Feynman propagator except that the crucial $\delta(s)$ term making possible effectively massless propagation is absent. At light-cone boundary however $\sqrt{s}$ is zero along light rays, and this gives long range correlations between fermions at different partonic 2-surfaces intersected by light rays from the origin. Hence one could have a non-vanishing Hermitian inner product for 3-D states at boundaries of CD.

   Rather remarkably, these results provide a justification for twistor-diagrams identified as polygons consisting of light-like segments.

Possible normal ordering divergences

Concerning the cancellation of normal ordering divergences the singularities of the propagators G_F are crucial. The bi-linearity of the modified Dirac action forcing anticommuting quark and antiquark oscillator operators at different throats of wormhole contacts but this need not guarantee the absence of the divergence since the free quark propagator in M^4 contains mass independent $\delta(s)$ part plus the divergent part from Hankel function behaving like $1/\sqrt{sm}$. For the massless propagator assignable to ν_R the propagator would reduce to M^4 propagator and only the $\delta(s)$ would contribute.

$s = 0$ condition tells that the distance between fermion and anti-fermion is light-like and is possible to satisfy at the light-like boundary of CD. Paired quark and antiquark at the wormhole throats must reside at the same light-like radial ray from the tip of cd (cd corresponds to causal diamond in M^4). Since partonic surfaces are 2-D this condition selects discrete pairs of points at the pair of the partonic surfaces. The integration over the position of the end of the propagator line over paired partonic 2-surfaces should smooth out the divergences and yield a finite result. This would be crucial for having an inner product for states at the boundary of the light-cone.

This applies also to the point pairs at opposite throats of wormhole contact. Time-ordered product vanishing for $t_1 = t_2$ so that the points must have different values of t and this is possible. The two 2-D integrations are expected to smooth out the singularities and eliminate divergences also now.

18.6 Conclusions

TGD predicts revolution in quantum theory based on three new principles.

1. ZEO solving the basic paradox of quantum measurement theory. Ordinary ("big") state function reduction involves time reversal forcing a generalization of thermodynamics and leading to a theory of quantum self-organization and self-organized quantum criticality (homeostasis in living matter).

2. Phases of ordinary matter labelled by effective Planck constant $h_{eff} = nh_0$ identified as dark matter and explaining the coherence of living matter in terms of dark matter at magnetic body serving as a master, and predicting quantum coherence in all scales at the level of magnetic bodies. $h_{eff}/h_0 = n$ has interpretation as the dimension for an extension of rationals and is a measure of algebraic complexity. Evolution corresponds to the increase of n.

   Extensions of rationals are associated with adelic physics providing description of sensory experience in terms of real physics and of cognition in terms of p-adic physics. Central notion is cognition representation providing unique discretization of X^4 in terms of points with embedding space coordinates in the extension of rationals considered $M^8 - H$ duality realizes the hierarchy of rational extensions and assigns them to polynomials defining space-time regions at the level of M^8 and mapped to minimal surfaces in H by $M8 - H$ duality.

3. The replacement of the unitary S-matrix with the Kähler metric of the Kähler space defined by WCW spinor fields satisfying the analog of unitarity and predicting positive definite transition probabilities defining matrix in Teichmueller space. Einstein's geometrization of classical physics extends to the level of state space, Equivalence Principle generalizes, and interactions are coded by the geometry of the state space rather than by an *ad hoc* unitary matrix. Kähler geometry for the spinor bundle of WCW has Riemann connection only for a maximal group of isometries identified as super-symplectic transformations (SS). This makes the theory unique and leads to explicit analogs of Feynman rules and to a proof that theory is free of divergences.

In this work the third principle, which is new, is formulated and some of its consequences are discussed. The detailed formulation allows understanding of how normal ordering divergences and other divergences cancel. The key idea is to induce the second quantized free spinor field from H to space-time surface. This determines the propagators at the space-time level. The condition that creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts is enough. An alternative but not exclusive option suggested by ZEO is that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

A further observation is that the Dirac propagators for particles reduce in a good approximation to massless propagators when the propagation takes place along light-like distances: this provides a considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

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Chapter 19

Breakthrough in understanding of $M^8 - H$ duality

19.1 Introduction

$M^8 - H$ duality [L85, L81, L82, L116] has become a cornerstone of quantum TGD but several aspects of this duality are still poorly understood.

19.1.1 Development of the idea about $M^8 - H$ duality

A brief summary about the development of the idea is in order.

1. The original version of $M^8 - H$ duality assumed that space-time surfaces in M^8 can be identified as associative or co-associative surfaces. If the surface has associative tangent/normal space and contains a complex co-complex surface, it can be mapped to a 4-surface in $M^4 \times CP_2$.

2. Later emerged the idea that octonionic analyticity realized in terms of a real polynomials P algebraically continued to polynomials of complexified octonion might realize the dream [L49, L50, L51]. The original idea was that the vanishing condition for the real/imaginary part of P in quaternion sense could give rise to co-assocative/associative sense.

   $M^8 - H$ duality concretizes number theoretic vision [L54, L53] summarized as adelic physics fusing ordinary real number based physics for the correlates of sensory experience and various p-adic physics ($p = 2, 3, ...$) as physics for the correlates of cognition. The polynomials of real variable restricted to be rational valued defines an extension or rationals via the roots of the polynomials and one obtains an evolutionary hierachy associated with these extensions increasing in algebraic complexity. These extensions induce extensions of p-adic numbers and the points of space-time surface in M^8 with coordinates in the extension of rationals define cognitive representations as unique discretizations of the space-time surface.

3. The realization of the general coordinate invariance in TGD framework [K57, K29, K96, L124] [L120] motivated the idea that strong form of holography (SH) in H could allow realizing $M^8 - H$ duality by assuming associativity/co-associativity conditions only at 2-D string world sheet and partonic 2-surfaces and possibly also at their light-like 3-orbits at which the signature of the induced metric changes from Minkowskian to Euclidian.

19.1.2 Critical re-examination of the notion

In this article $M^8 - H$ duality is reconsidered critically.

1. The healthy cold shower was the learning that quaternion (associative) sub-spaces of quaternionic spaces are geodesic manifolds [A36]. The distributions of quaternionic normal spaces are however always integrable. Hence, co-associativity remains the only interesting option.

Also the existence of co-commutative sub-manifolds of space-time surface demanding the existence of a 2-D integrable sub-distribution of subspaces is possible. This learning experience motivated a critical examination of the $M^8 - H$ duality hypothesis.

2. The basic objection is that for the conjectured associative option, one must assign to each state of motion of a particle its own octonionic structure since the time axis would correspond to the octonionic real axis. It was however clear from the beginning that there is an infinite number of different 4-D planes O_c in which the number theoretical complex valued octonion inner product reduces to real - the number theoretic counterpart for Riemann metric. In the co-associative case this is the only option. Also the Minkowski signature for the real projection turns out to be the only physically acceptable option. The mistake was to assume that Euclidian regions are co-associative and Minkowskian regions associative: both must be co-associative.

3. The concrete calculation of the octonion polynomial was the most recent step - carried already earlier [L49, L50, L51] but without realizing the implications of the extremely simple outcome. The imaginary part of the polynomial is proportional to the imaginary part of octonion itself. It turned out that the roots $P = 0$ of the octonion polynomial P are 12-D complex surfaces in O_c rather than being discrete set of points defined as zeros $X = 0, Y = 0$ of two complex functions of 2 complex arguments. The analogs of branes are in question. Already earlier 6-D real branes assignable to the roots of the real polynomial P at the light-like boundary of 8-D light-cone were discovered: also their complex continuations are 12-D [L81, L91].

4. P has quaternionic de-composition $P = Re_Q(P) + I_4 Im_Q(P)$ to real and imaginary parts in a quaternionic sense. The naive expectation was that the condition $X = 0$ implies that the resulting surface is a 4-D complex surface X^4_c with a 4-D real projection X^4_r, which could be co-associative.

   The expectation was wrong! The equations $X = 0$ and $Y = 0$ involve the same(!) complex argument o_c^2 as a complex analog for the Lorentz invariant distance squared from the tip of the light-cone. This implies a cold shower. Without any additional conditions, $X = 0$ conditions have as solutions 7-D complex mass shells H^7_c determined by the roots of P. The explanation comes from the symmetries of the octonionic polynomial.

   There are solutions $X = 0$ and $Y = 0$ only if the two polynomials considered have a common a_c^2 as a root! Also now the solutions are complex mass shells H^7_c.

5. How could one obtain 4-D surfaces X^4_c as sub-manifolds of H^7_c? One should pose a condition eliminating 4 complex coordinates: after that a projection to M^4 would produce a real 4-surface X^4 .

   A co-associative X^4_c is obtained by acting with a local SU_3 transformation g to a co-associative plane $M^4 \subset M^8_c$. If the image point $g(p)$ is invariant under $U(2)$, the transformation corresponds to a local CP_2 element and the map defines $M^8 - H$ duality even if the co-associativity in geometric sense were not satisfied.

   The co-associativity of the plane M^4 is preserved in the map because G_2 acts as an automorphism group of the octonions. If this map also preserves the value of 4-D complex mass squared, one can require that the intersections of X^4_c with H^7_c correspond to 3-D complex mass shells. One obtains holography with mass shells defined by the roots of P giving boundary data. The condition H images are analogous to Bohr orbits, corresponds to number theoretic holography.

It this, still speculative, picture is correct, it would fulfil the original dream about solving classical TGD exactly in terms of roots for real/imaginary parts of octonionic polynomials in M^8 and by mapping the resulting space-time surfaces to H by $M^8 - H$ duality. In particular, strong form of holography (SH) would not be needed at the level of H, and would be replaced with a dramatically stronger number theoretic holography.

Octonionic Dirac equation, which is purely algebraic equation and the counterpart for ordinary Dirac equation in momentum space, serves as a second source of information.

1. The first implication is that O_c has interpretation as an analog of momentum space for quarks: this has profound implications concerning the interpretation. The space-time surface in M^8 would be analog of Fermi ball. The octonionic Dirac equation reduces to the mass shell condition $m^2 = r_n$, where r_n is a root of the polynomial P defining the 4-surface but only in the co-associative case.

2. Cognitive representations are defined by points of M^8 with coordinates having values in the extensions of rational defined by P and allowing an interpretation as 4-momenta of quarks. In the generic case the cognitive representations are finite. If the points of M^8 correspond to quark momenta, momentum conservation is therefore expected to make the scattering trivial.

   However, a dramatic implication of the reduction of the co-associativity conditions to the vanishing of ordinary polynomials Y is that by the Lorentz invariance of roots of P, the 3-D mass shells ofd M^4 have an infinite number of points in a cognitive representation defined by points with coordinates having values in the extensions of rational defined by P and allowing an interpretation as 4-momenta. This is what makes interesting scattering amplitudes for massive quarks possible.

3. What is the situation for the images of M^4 points under the effective local CP_2 element defined by local $SU(3)$ element g preserving the mass squared and mapping H^3 to $g(H^3)$? If g is expressible in terms of rational functions with rational coefficients, algebraic points are mapped to algebraic points. This is true also in the interior of M^4.

   This would mean a kind of cognitive explosion for massive quark momenta. Without the symmetry one might have only forward scattering in the interior of X^4_r. Note that massless quarks can however arrive at the boundary of CD which also allows cognitive representation with an infinite number of points.

4. In the number theoretic approach, kinematics becomes a highly non-trivial part of the scattering. The physically allowed momenta would naturally correspond to algebraic integers in the extension E of rationals defined by P. Momentum conservation and on-mass-shell conditions together with the condition that momenta are algebraic integers in E are rather strong. The construction of Pythagorean squared generalize to the case of quaternions provides a general solutions to the conditions: the solutions to the conditions are combinations of momenta which correspond to squares of quaternions having algebraic integers as components.

5. The original proposal was that local $G_{2,c}$ element $g(x)$ defines a vanishing holomorphic gauge field and its restriction to string world sheet or partonic 2-surface defines conserved current. $M^8 - H$ duality however requires that local $SU(3)$ element with the property that image point is invariant under $U(2)$ is required by $M^8 - H$ duality defines $X^4 \subset M^8$.

   In any case, these properties suggest a Yangian symmetry assignable to string world sheets and partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The generators of the Yangian algebra have a representation as Hamiltonians which are in involution. They define conserved charges at the orbits for a Hamiltonian evolution defined by any combination of these the Hamiltonians. ZEO suggests a concrete representation of this algebra in terms of quark and antiquark oscillator operators. This algebra extends also to super-algebra. The co-product of the associated Yangian would give rise to zero energy states defining as such the scattering amplitudes.

19.1.3 Octonionic Dirac equation

The octonionic Dirac equation allows a second perspective on associativity. Everything is algebraic at the level of M^8 and therefore also the octonionic Dirac equation should be algebraic. The octonionic Dirac equation is an analog of the momentum space variant of the ordinary Dirac equation and also this forces the interpretation of M^8 as momentum space.

Fermions are massless in the 8-D sense and massive in 4-D sense. This suggests that octonionic Dirac equation reduces to a mass shell condition for massive particle with $q \cdot q = m^2 = r_n$, where $q \cdot q$ is octonionic norm squared for quaternion q defined by the expression of momentum p as $p = I_4 q$, where I_4 is octonion unit orthogonal to q. r_n represents mass shell as a root of P.

For the co-associative option the co-associative octonion p representing the momentum is given in terms of quaternion q as $p = I_4 q$. One obtains $p \cdot p = qq = m^2 = r_n$ at the mass shell defined as a root of P. Note that for M^4 subspace the space-like components of $p\,p$ are proportional to i and the time-like component is real. All signatures of the number theoretic metric are possible.

For associative option one would obtain $qq = m^2$, which cannot be satisfied: q reduces to a complex number $zx + Iy$ and one has analog of equation $z^2 = z^2 - y^2 + 2Ixy = m_n^2$, which cannot be true. Hence co-associativity is forced by the octonionic Dirac equation.

Before continuing, I must apologize for the still fuzzy organization of the material related to $M^8 - H$ duality. The understanding of its details has been a long and tedious process, which still continues, and there are unavoidably inaccuracies and even logical inconsistencies caused by the presence of archeological layers present.

19.2 The situation before the cold shower

The view about $M^8 - H$ duality before the cold shower - leading to what I dare to call a break-through - helps to gain idea about the phenomenological side of $M^8 - H$ duality. Most of the phenomenology survives the transition to a more precise picture. This section is however not absolutely necessary for what follows it.

19.2.1 Can one deduce the partonic picture from $M^8 - H$ duality?

The M^8 counterparts for partons and their light like orbits in H can be understood in terms of octonionic Dirac equation in M^8 as an analog for the algebraic variant of ordinary Dirac equation at the level of momentum space [L116, L115] but what about the identification of partonic 2-surfaces as interaction vertices at which several partonic orbits meet? Can one deduce the phenomenological view about elementary particles as pairs of wormhole contacts connected by magnetic flux tubes from $M^8 - H$ duality? There is also the question whether partonic orbits correspond to their own sub-CDs as the fact that their rest systems correspond to different octonionic real axes suggests.

There are also some questions which have become obsolote. For instance: qhy should the partonic vertices reside at $t = r_n$ branes? This became obsolste with the realization that M^8 is analogous to momentum space so that the identification as real octonionic coordinate corresponds now to a component of 8-momentum identifiable as energy. Furthermore, the assumption the associativity of the 4-surface in M^8 had to be replaced with-co-associtivity and octonionic real coordinate does not have interpretation as time coordinate is associative surface

$M^8 - H$ duality indeed conforms with the phenomenological picture about scattering diagrams in terms of partonic orbits [L124, L123] [L123, L124] [L124], and leads to the view about elementary particles as pairs of Euclidian wormhole contacts associated with flux tubes carrying monopole flux.

19.2.2 What happens to the "very special moments in the life of self"?

The original title was "What happens at the "very special moments in the life of self?" but it turned out that "at" must be replaced with "to". The answer to the new question would be "They disappear from the glossary".

The notion of "very special moments in the life of self" (VPM) [L81, L91] makes sense if M^8 has interpretation as an 8-D space-time. M^4 projections of VPMs were originally identified as hyperplanes $t = r_n$, where t is time coordinate and r_n is a root of the real polynomial defining octonionic polynomial as its algebraic continuation.

The interpretation of M^8 as cotangent space of H was considered from the beginning but would suggest the interpretation of M^8 as the analog of momentum space. It is now clear that this interpretation is probably correct and that $M^8 - H$ duality generalizes the momentum-position duality of wave mechanics. Therefore one should speak of $E = r_n$ plane and simply forget the misleading term VMP. VPMs would correspond to constant values of the M^8 energy assignable to M^4 time coordinate.

The identification of space-time surface as co-associative surface with quaternionic normal space containing integrable distribution of 2-D commutative planes essential for $M^8 - H$ duality is also in conflict with the original interpretation. Also the modification of $M^8 - H$ duality in M^4 degrees of freedom forced by Uncertainty Principle [L141] has led to the conclusion that VMPs need not have a well-defined images in H.

19.2.3 What does SH mean and its it really needed?

SH has been assumed hitherto but what is its precise meaning?

1. Hitherto, SH at the level of H is believed to be needed: it assumes that partonic 2-surfaces and/or string world sheets serve as causal determinants determining X^4 via boundary conditions.

   (a) The normal or tangent space of X^4 at partonic 2-surfaces and possibly also at string world sheets has been assumed to be associative that is quaternionic. This condition should be true at the entire X^4.

   (b) Tangent or normal space has been assumed to contain preferred M^2 which could be replaced by an integrable distribution of $M^2(x) \subset M^4$. At string world sheets only the tangent space can be associative. At partonic 2-surfaces also normal space could be associative. This condition would be true only at string world sheets and partonic 2-surfaces so that only these can be mapped to H by $M^8 - H$ duality and continued to space-time surfaces as preferred extremals satisfying SH.

   The current work demonstrates that although SH could be used at the level of SH, this is not necessary. Co-associativity together with co-commutativity for string world sheets allows the mapping of the real space-time surfaces in M^8 to H implying exact solvability of the classical TGD.

19.2.4 Questions related to partonic 2-surfaces

There are several questions related to partonic 2-surfaces.

Q1: What are the M^8 pre-images of partons and their light-like partonic orbits in H?

It will be found that the octonionic Dirac equation in M^8 implies that octo-spinors are located to 3-D light-like surfaces Y^3_r - actually light-cone boundary and its 3-D analogs at which number theoretic norm squared is real and vanishes - or to the intersections of X^3_r with the 6-D roots of P in which case Dirac equation trivializes and massive states are allowed. They are mapped to H by $M^8 - H$ duality.

Remark: One can ask whether the same is true in H in the sense that modified Dirac action would be localized to 3-D light-like orbits and 3-D ends of the space-time surfaces at the light-like boundaries of CD having space-like induced metric. Modified Dirac action would be defined by Cherm-Simons term and would force the classical field equations for the bosonic Chern-Simons term. If the interior part of the modified Dirac action is absent, the bosonic action is needed to define the space-time surfaces as extremals. They would be minimal surfaces and universal by their holomorphy and would not depend on coupling parameters so that very general actions can have them as preferred extremals. This issue remains still open.

The naïve - and as it turned out, wrong - guess was that the images of the light-like surfaces should be light-like surfaces in H at the boundaries of Minkowskian and Euclidian regions (wormhole contacts). In the light-like case Y^3_r corresponds to the light-cone boundary so that this would be the case. X^3_r however turns out to correspond to a hyperboloid in M^4 as an analog of a mass shell and is not identifiable as a partonic orbit.

It turned out that the complex surface X^4_c allows real sections in the sense that the number theoretic complex valued metric defined as a complex continuation of Minkowski norm is real at 4-D surfaces: call them Z^4_r. They are bounded by a 3-D region at Z^3_r at which the value of norm squared vanishes. This surface is an excellent candidate for the pre-image of the light-like orbit of partonic 2-surface serving as a topological vertex. One has therefore strings worlds sheets, partonic 2-surfaces and their light-like orbits and they would connect the "mass shells" at X^4_r. All ingredients for SH would be present.

The intersections of Z^3_r with X^3_r identifiable as the section of X^4_r $a = constant$ hyperboloid would give rise to partonic 2-surfaces appearing as topological reaction vertices.

The assumption that the 4-D tangent space at these light-like 3-surfaces is co-associative, would give an additional condition determining the image of this surface in H, so that the boundary conditions for SH would become stronger. One would have boundary conditions at light-like partonic orbits. Note that string world sheets are assumed to have light-like boundaries at partonic orbits.

Q2: Why should partonic 2-surfaces appear as throats of wormhole contact in H? Wormhole contacts do not appear in M^8.

1. In M^8 light-like orbits are places where the Minkowskian signature changes to Euclidian. Does $M^8 - H$ duality map the images of these coinciding roots for Euclidian and Minkowskian branches to different throats of the wormhole contact in H so that the intersection would disappear?

2. This is indeed the case. The intersection of Euclidian and Minkowskian branches defines a single 3-surface but the tangent and normal spaces of branches are different. Therefore their H images under $M^8 - H$ duality for the partonic 2-surface are different since normal spaces correspond to different CP_2 coordinates. These images would correspond to the two throats of wormhole contact so that the H-image by SH is 2-sheeted. One would have wormhole contacts in H whereas in M^8 the wormhole contact would reduce to a single partonic 2-surface.

3. The wormhole contact in H can have only Euclidian signature of the induced metric. The reason is that the M^4 projections of the partonic surfaces in H are identical so that the points with same M^4 coordinates have different CP_2 coordinates and their distance is space-like.

Q3: In H picture the interpretation of space-time surfaces as analogs of Feynman graphs assumes that several partonic orbits intersect at partonic 2-surfaces. This assumption could be of course wrong.This raises questions.

What the pre-images of partonic 2-surfaces are in M^8? Why should several partonic orbits meet at a given partonic 2-surface? Is this needed at all?

The space-time surface X^4_r associated intersects the surface X^6_r associated with different particle - say with different value of mass along 2-D surface. Could this surface be identified as partonic 2-surface X^2_r? This occurs symmetrically so that one has a pair of 2-surfaces X^2_r. What does this mean? Could these surface map to the throats of wormhole contact in H?

Why several partonic surfaces would co-incide in topological reaction vertex at the level of H? At this moment is is not clear whether this is forced by M^8 picture.

Octonionic Dirac equation implies that M^8 has interpretation as analog of momentum space so that interaction vertices are replaced by multilocal vertices representing momenta and propagators become local being in this sense analogous to vertices of QFT. One could of course argue that without the gluing along ends there would be no interactions since the interactions in X^6_r for two 3-surfaces consist in the generic case of a discrete set of points. One could also ask whether the surfaces Y^3_r associated with the space-time surfaces X^4_r associated with incoming particles must intersect along partonic 2-surface rather than at discrete set of points.

The meeting along ends need not be true at the level of M^8 since the momentum space interpretation would imply that momenta do not differ much so that particles should have identical masses: for this to make sense one should assume that the exchanged virtual particles are massless. One other hand, if momenta are light-like for Y^3_r, this might be the case.

Q4: Why two wormhole contacts and monopole flux tubes connecting them at the level of H? Why monopole flux?

1. The tangent spaces of the light-like orbits have different light-like direction. Intuitively, this corresponds to different directions of light-like momenta. Momentum conservation requires

more than one partonic orbit changing its direction meeting at partonic 2-surface. By light-likeness, the minimum is 2 incoming and two outgoing lines giving a 4-vertex. This allows the basic vertices involving Ψ and $\overline{Psi}$ at opposite throats of wormhole contacts. Also a higher number of partonic orbits is possible.

2. A two-sheeted closed monopole flux tube having wormhole contacts as its "ends" is suggested by elementary particle phenomenology. Since M^8 homology is trivial, there is no monopole field in M^8. If M^8-H duality is continuous it maps homologically trivial partonic 2-surfaces to homologically trivial 2-surfaces in H. This allows the wormhole throats in H to have opposite homology charges. Since the throats cannot correspond to boundaries there must be second wormhole contact and closed flux tube.

3. What does the monopole flux for a partonic 2-surface mean at the level of M^8? The distribution of quaternionic 4-D tangent/normal planes containing preferred M^2 and associated with partonic 2-surface in M^8 would define a homologically on-trivial 2-surface in CP_2. The situation is analogous to a distribution of tangent planes or equivalently normal vectors in S^2.

Q4: What is the precise form of M^8-H duality: does it apply only to partonic 2-surfaces and string world sheets or to the entire space-time surfaces?

M^8-H duality is possible if the X^4 in M^8 contains also integrable distribution of complex tangent or normal 2-planes at which 4-D tangent space is quaternionic/associative. String world sheets and partonic 2-surfaces define these distributions.

The minimum condition allowed by SH in H is that string world sheets and there is a finite number of partonic 2-surfaces and string world sheets. In this case only these 2-surfaces can be mapped to H and SH assigns to them a 4-D space-time surface. The original hypothesis was that these surfaces define global orthogonal slicings of the X^4 so that M^8-H duality could be applied to the entire X^4. This condition is probably too strong.

19.3 Challenging M^8-H duality

M^8-H duality involves several alternative options and in the following arguments possibly leading to a unique choice are discuses.

1. Are both associativity and co-associativity possible or is only either of these options allowed? Is it also possible to pose the condition guaranteeing the existence of 2-D complex sub-manifolds identifiable as string world sheets necessary to map the entire space-time surface from M^8 to H? In other words, is the strong form of holography (SH) needed in M^8 and/or H or is it needed at all?

2. The assignment of the space-time surface at the level of M^8 to the roots of real or imaginary part (in quaternionic sense) of octonionic polynomial P defined as an algebraic continuation of real polynomial is an extremely powerful hypothesis in adelic physics [L53, L54] and would mean a revolution in biology and consciousness theory.

   Does P fix the space-time surface with the properties needed to realize M^8-H duality or is something more needed? Does the polynomial fix the space-time surface uniquely - one would have extremely strong number theoretic holography - so that one would have number theoretic holography with coefficients of a real polynomial determining the space-time surface?

3. M^8-H duality involves mapping of $M^4\subset M^8$ to $M^4\subset H$. Hitherto it has been assumed that this map is direct identification. The form of map should however depend on the interpretation of M^8. In octonionic Dirac equation M^8 coordinates are in the role of momenta [L116]. This suggests the interpretation of M^8 as an analog of 8-D momentum space. If this interpretation is correct, Uncertainty Principles demands that the map $M^4\subset M^8\rightarrow M^4\subset H$ is analogous to inversion mapping large momenta to small distances.

4. Twistor lift of TGD [K116] is an essential part of the TGD picture. Ttwistors and momentum twistors provide dual approaches to twistor Grassmann amplitudes. Octonionic Dirac

equation suggests that M^8 and H are in a similar dual relation. Could $M^8 - H$ duality allow a generalization of twistorial duality to TGD framework?

19.3.1 Explicit form of the octonionic polynomial

What does the identification of the octonionic polynomial P as an octonionic continuation of a polynomial with real or complexified coefficients imply? In the following I regard M^8_c as O^8_c and consider products for complexified octonions.

Remark: In adelic vision the coefficients of P must be rationals (or at most algebraic numbers in some extension of rationals).

One interesting situation corresponds to the real subspace of O_c spanned by $\{I_0, iI_k\}, = 1, ..7$, with a number theoretic metric signature $(1, -1, -1..., -1)$ of M^8 which is complex valued except at in various reals subspaces.This subspace is associative. The original proposal was that Minkowskian space-time regions as projections to this signature are associative whereas Euclidian regions are co-associative. It however turned out that associative space-time surfaces are physically uninteresting.

The canonical choice $(iI_0, I_1, I_2, iI_3, I_4, iI_5, I_6, iI_7)$ defining the complexification of the tangent space represents a co-associative sub-space realizing Minkowski signature. It turns out that both Minkowskian and Euclidian space-time regions must be co-associative .

Surprises

The explicit calculation of the octonionic polynomial yielded a chilling result. If one poses (co-)associativity conditions as vanishing of the imaginary or real part in quaterionic sense: $Im_Q(P) = 0$ or $Re_Q(P) = 0$, the outcome is that the space-time surface is just M^4 or E^4. Second chilling result is that quaternionic sub-manifolds are geodesic sub-manifolds. This led to the question how to modify the (co-)associativity hypothesis.

The vision has been that space-time surfaces can be identified as roots for the imaginary (co-associative) part $Im_Q(O)$ or real part $Im_Q(O)$ of octonionic polynomial using the standard decomposition $(1, e_1, e_2, e_3)$.

1. The naïve counting of dimensions suggests that one obtains 4-D surfaces. The surprise was that also 6-D brane like entities located at the boundary of M^8 light-cone and with topology of 6-sphere S^6 are possible. They correspond to the roots of a real polynomial $P(o)$ for the choice $(1, iI_1, ..., iI_7)$. The roots correspond to the values of the real octonion coordinate interpreted as values of linear M^4 time in the proposal considered. Also for the canonical proposal one obtains a similar result. In O_c they correspond to 12-D complex surfaces X^6_c satisfying the same condition conditions $x_0^2 + r^2 = 0$ and $P(x_0) = 0$.

2. There was also another surprise. As already described, the general form for the octonionic polynomial $P(o)$ induced from a real polynomial is extremely simple and reduces to $X(t^2, r^2)I_0 + iY(t^2, r^2)Im(o)$. There are only two complex variables t and r^2 involved and the solutions of $P = 0$ are 12-D complex surfaces X^6_c in O_c. Also the special solutions have the same dimension.

3. In the case of co-associativity 8 conditions are needed for $Re_Q(P) = 0$: note that $X = 0$ is required. The naive expectation is that this gives a complex manifold X^4_c with 4-D real projection X^4_r as an excellent candidate for a co-associative surface.

   The expectation turned out to be wrong: in absence of any additional conditions the solutions are complex 7-dimensional mass shells! This is due to the symmetries of the octonionic polynomials as algebraic continuation of a real polynomial.

4. The solution of the problem is to change the interpretation completely. One must assign to the 7-D complex mass shell H^7_c a 3-D complex mass shell H^3_c.

   One can do this by assuming space-time surface is surface intersecting the7-D mass shell obtained as a deformation of $M^4_c \subset M^8_c$ by acting with local $SU(3)$ gauge transformation and requiring that the image point is invariant under $U(2)$. If the 4-D complex mass squared remains invariant in this transformation, X^4_c intersects H^7_c.

With these assumptions, a local CP_2 element defines X^4_c and X^4_r is obtained as its real projection in M^4. This definition assigns to each point of M^4 a point of CP_2 so that M^8-H duality is well-defined.

One obtains holography in which the fixing of 3-D mass shells fixes the 4-surface and also assigns causal diamond with the pair of mass shells with opposite energies. If the space-time surface is analog of Bohr orbit, also its preimage under $M^8 - H$ duality should be such and P would determine 4-surface highly uniquely [L146] and one would have number theoretic holography.

General form of P and of the solutions to $P = 0$, $Re_Q(P) = 0$, and $Im_Q(P) = 0$

It is convenient to introduce complex coordinates for O_c since the formulas obtained allow projections to various real sections of O_c.

1. To see what happens, one can calculate o_c^2. Denote o_c by $o_c = tI_0 + \overline{o_c}$ and the norm squared of $\overline{o}$ by r^2, where $r^2 = \sum o_k^2$ where o_k are the complex coordinates of octonion. Number theoretic norm squared for o_c is $t^2 + r^2$ and reduces to a real number in the real sections of O_c. For instance, in the section $(I_1, iI3, iI_5, iI_7)$ the norm squared is $-x_1^2 + x_3^2 + x_5^2 + x_7^2$ and defines Minkowskian norm squared.

   For o^2 one has:

   $$o^2 = t^2 - r^2 + 2t\overline{o} \equiv X_2 + \overline{Y}_2 \ .$$

   For o^3 one obtains

   $$o^3 = tX_2 - \overline{o}\cdot\overline{Y}_2 + t\overline{Y}_2 + X_2\overline{o} \ .$$

   Clearly, $Im_Q(o^n)$ has always the same direction as $Im_Q(o)$. Hence one can write in the general case

   $$o^n = X + Y\overline{o} \ . \tag{19.3.1}$$

   This trivial result was obtained years ago but its full implications became evident only while preparing the current article. The point is that the solutions to associativity/co-associativity conditions by putting $Re(Q(P) = 0$ or $Im_Q(P) = 0$ are trivial: just M^4 or E^4. What goes wrong with basic assumptions, will be discussed later.

   Remark: In M^8 sub-space one has imaginary $\overline{o}$ is proportional to the commuting imaginary unit.

2. It is easy to deduce a recursion formula for the coefficients for X and Y for n:th power of o_c. Denote by t the coordinate associated with the real octonion unit (not time coordinate). One obtains

   $$\begin{array}{l} o_c^n = X_nI_0 + Y_n\overline{o} \ , \\ X_n = tX_{n-1} - rY_{n-1} \ , \\ Y_n = tY_{n-1} + rX_{n-1} \ . \end{array} \tag{19.3.2}$$

   In the co-associative case one has $t = 0$ or possibly constant $t = T$ (note that in the recent interpretation t does not have interpretation as time coordinate). The reason is that the choice of octonionic coordinates is unique apart from translation along the real axis from the condition that the coefficients of P remain complex numbers in powers of the new variable.

3. The simplest option correspond to $t=0$. One can criticize this option since the quaternionicity of normal space should not be affected if t is constant different from zero. In any case, for $t=0$ the recursion formula gives for the polynomial $P(o_c)$ the expression

$$P(o_c)=\sum(-1)^n r^{2n}(p_{2n-1}I_0+p_{2n}\overline{o}) \ . \tag{19.3.3}$$

Denoting the even and of odd parts of P by P_{even} and P_{odd}, the roots $r_{k,odd}$ of $X=Re(P(o_c))$ are roots P_{odd} and roots $r_{k,even}$ of $Y=Im(P(o_c))$ are roots of P_{even}. Co-associativity gives roots of X and the roots of P as simultaneous roots of P_{odd} and P_{even}. The interpretation of roots is as in general complex mass squared values.

In the general case, the recursion relation would give the solution

$$\begin{aligned}\left(\begin{array}{c} X_n \\ Y_n \end{array}\right) = A^n\left(\begin{array}{c} t \\ r \end{array}\right) \\ A=\left(\begin{array}{cc} t & -r \\ r & t \end{array}\right)\end{aligned} \tag{19.3.4}$$

One can diagonalize the matrix appearing in the iteration by solving the eigenvalues $\lambda_\pm = t\pm ir$ and eigenvectors $X_\pm=(\pm i,1)$ and by expressing $(X_1,Y_1)=(t,r)$ in terms of the eigenvectors as $(t,r)=((it+r)X_+ +(r-it)X_-)/2$. This gives

$$\left(\begin{array}{c} X_n \\ Y_n \end{array}\right)=\frac{1}{2}\left(\begin{array}{c} (t+ir)^n i-(t-ir)^n i \\ (t+ir)^n+(t-ir)^n \end{array}\right) \tag{19.3.5}$$

This gives

$$\begin{aligned} P(o_c) &= P_1I_0+P_2\overline{o} \ , \\ P_1(r) &= \sum X_np_nr^{2n} \ , \\ P_2(r) &= \sum Y_np_nr^{2n} \ . \end{aligned} \tag{19.3.6}$$

For the restriction to M_c^4, r^2 reduces to complex 4-D mass squared given by the root r_n. In general case r^2 corresponds to complex 8-D mass squared. All possible signatures are obtained by assuming M_c^8 coordinates to be either real or imaginary (the number theoretical norm squared is real with this restriction).

How does one obtain 4-D space-time surfaces?

Contrary to the naive expections, the solutions of the vanishing conditions for the $Re_Q(P)$ ($Im_Q(P)$) (real (imaginary) part in quaternionic sense) are 7-D complex mass shells $r^2=r_{n,1}$ as roots of $P_1(r)=0$ or $r^2=r_{n,2}$ of $P_2(r)=0$ rather than 4-D complex surfaces (for a detailed discussion see [K20]) A solution of both conditions requires that P_1 and P_2 have a common root but the solution remains a 7-D complex mass shell! This was one of the many cold showers during the development of the ideas about M^8-H duality! It seems that the adopted interpretation is somehow badly wrong. Here zero energy ontology (ZEO) and holography come to the rescue.

1. Could the roots of P_1 or P_2 define only complex mass shells of the 4-D complex momentum space identifiable as M^4_c? ZEO inspires the question whether a proper interpretation of mass shells could be as pre-images of boundaries of cd:s (intersections of future and past directed light-cones) as pairs of mass shells with opposite energies. If this is the case, the challenge would be to understand how X^4_c is determined if P does not determine it.

   Here holography, considered already earlier, suggests itself: the complex 3-D mass shells belonging to X^4_c would only define the 3-D boundary conditions for holography and the real mass shells would be mapped to the boundaries of cds. This holography can be restricted to X^4_R. Bohr orbit property at the level of H suggests that the polynomial P defines the 4-surface more or less uniquely.

2. Let us take the holographic interpretation as a starting point. In order to obtain an X^4_c mass shell from a complex 7-D light-cone, 4 complex degrees of freedom must be eliminated. M^8-H duality requires that X^4_c allows M^4_c coordinates.

   Note that if one has $X^4_c = M^4_c$, the solution is trivial since the normal space is the same for all points and the H image under M^8-H duality has constant $CP_2 = SU(3)/U(2)$ coordinates. X^4_c should have interpretation as a non-trivial deformation of M^4_c in M^8.

3. By M^8-H duality, the normal spaces should be labelled by $CP_2 = SU(3)/U(2)$ coordinates. M^8-H duality suggests that the image $g(p)$ of a momentum $p \in M^4_c$ is determined essentially by a point $s(p)$ of the coset space $SU(3)/U(2)$. This is achieved if M^4_c is deformed by a local $SU(3)$ transformation $p \to g(p)$ in such a way that each image point is invariant under $U(2)$ and the mass value remains the same: $g(p)^2 = p^2$ so that the point represents a root of P_1 or P_2.

   Remark: I have earlier considered the possibility of G_2 and even $G_{2,c}$ local gauge transformation. It however seems that that local $SU(3)$ transformation is the only possibility since G_2 and $G_{2,c}$ would not respect M^8-H duality. One can also argue that only real $SU(3)$ maps the real and imaginary parts of the normal space in the same manner: this is indeed an essential element of M^8-H duality.

4. This option defines automatically M^8-H duality and also defines causal diamonds as images of mass shells $m^2 = r_n$. The real mass shells in H correspond to the real parts of r_n. The local $SU(3)$ transformation g would have interpretation as an analog of a color gauge field. Since the H image depends on g, it does not correspond physically to a local gauge transformation but is more akin to an element of Kac-Moody algebra or Yangian algebra which is in well-defined half-algebra of Kac-Moody with non-negative conformal weights.

The following summarizes the still somewhat puzzling situation as it is now.

1. The most elegant interpretation achieved hitherto is that the polynomial P defines only the mass shells so that mass quantization would reduce to number theory. Amusingly, I started to think about particle physics with a short lived idea that the d'Alembert equation for a scalar field could somehow give the mass spectrum of elementary particles so that the issue comes full circle!

2. Holography assigns to the complex mass shells complex 4-surfaces for which M^8-H duality is well-defined even if these surfaces would fail to be 4-D co-associative. These surfaces are expected to be highly non-unique unless holography makes them unique. The Bohr orbit property of their images in H indeed suggests this apart from a finite non-determinism [L146]. Bohr orbit property could therefore mean extremely powerful number theoretical duality for which the roots of the polynomial determine the space-time surface almost uniquely. $SU(3)$ as color symmetry emerges at the level of M^8. By M^8-H duality, the mass shells are mapped to the boundaries of CDs in H.

3. Do we really know that X^4_r co-associative and has distribution of 2-D commuting subspaces of normal space making possible M^8-H duality? The intuitive expectation is that the

answer is affirmative [A36]. In any case, $M^8 - H$ duality is well-defined even without this condition.

4. The special solutions to $P = 0$, discovered already earlier, are restricted to the boundary of CD_8 and correspond to the values of energy (rather than mass or mass squared) coming as roots of the real polynomial P. These mass values are mapped by inversion to "very special moments in the life of self" (a misleading term) at the level of H as special values of light-cone proper time rather than linear Minkowski time as in the earlier interpretation [L81]. The new picture is Lorenz invariant.

Octonionic Dirac equation as analog of momentum space variant of ordinary Dirac equation forces the interpretation of M^8 as an analog of momentum space and Uncertainty Principle forces to modify the map $M^4 \subset M^8 \to M^4 \subset H$ from identification to inversion. The equations for $Re_Q(P) = 0$ reduce to simultaneous roots of the real polynomials defined by the odd and even parts of P having interpretation as complex values of mass squared mapped to light-cone proper time constant surfaces in H. This leads to the idea that the formulation of scattering amplitudes at M^8 levels provides the counterpart of momentum space description of scattering whereas the formulation at the level of H provides the counterpart of space-time description.

This picture combined with zero energy ontology (ZEO) leads also to a view about quantum TGD at the level of M^8. Local $SU(3)$ element has properties suggesting a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by P. The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

19.3.2 The input from octonionic Dirac equation

The octonionic Dirac equation allows a second perspective on associativity. Everything is algebraic at the level of M^8 and therefore also the octonionic Dirac equation should be algebraic. The octonionic Dirac equation is an analog of the momentum space variant of ordinary Dirac equation and also this forces the interpretation of M^8 as momentum space.

Fermions are massless in the 8-D sense and massive in 4-D sense. This suggests that octonionic Dirac equation reduces to a mass shell condition for massive particle with $q \cdot q = m^2 = r_n$, where $q \cdot q$ is octonionic norm squared for quaternion q defined by the expression of momentum p as $p = I_4 q$, where I_4 is octonion unit orthogonal to q. r_n represents mass shell as a root of P.

For the co-associative option the co-associative octonion p representing the momentum is given in terms of quaternion q as $p = I_4 q$. One obtains $p \cdot p = q\overline{q} = m^2 = r_n$ at the mass shell defined as a root of P. Note that for M^4 subspace the space-like components of $p\,p$ are proportional to i and the time-like component is real. All signatures of the number theoretic metric are possible.

For associative option one would obtain $qq = m^2$, which cannot be satisfied: q reduces to a complex number $zx + Iy$ and one has analog of equation $z^2 = z^2 - y^2 + 2Ixy = m_n^2$, which cannot be true. Hence co-associativity is forced by the octonionic Dirac equation.

One of the big surprises was that the cognitive representations for both light-like boundary and X^4_r are not generic meaning that they would consist of a finite set of points but infinite due to the Lorentz symmetry: a kind of cognitive explosion would happen by the Lorentz symmetry. The natural assumption is that for a suitable momentum unit, physical momenta satisfying mass shell conditions are algebraic integers in the extension of rationals defined by P. Periodic boundary conditions in turn suggest that for the physical states the total momenta are ordinary integers and this leads to Galois confinement as a universal mechanism for the formation of bound states.

Hamilton-Jacobi structure and Kähler structure of $M^4 \subset H$ and their counterparts in $M^4 \subset M^8$

The Kähler structure of $M^4 \subset H$, forced by the twistor lift of TGD, has deep physical implications and seems to be necessary. It implies that for Dirac equation in H, modes are eigenstates of only the longitudinal momentum and in the 2 transversal degrees of freedom one has essentially harmonic oscillator states [L141, L136], that is Gaussians determined by the 2 longitudinal momentum components. For real longitudinal momentum the exponents of Gaussians are purely imaginary or purely real.

The longitudinal momentum space $M^2 \subset M^4$ and its orthogonal complement E^2 is in a preferred role in gauge theories, string models, and TGD. The localization of this decomposition leads to the notion of Hamilton-Jacobi (HJ) structure of M^4 and the natural question is how this relates to Kähler structures of M^4. At the level of H spinors fields only the Kähler structure corresponding to constant decomposition $M^2 \oplus E^2$ seems to make sense and this raises the question how the H-J structure and Kähler structure relate. TGD suggests the existence of two geometric structure in M^4: HJ structure and Kähler structure. It has remained unclear whether HJ structure and Kähler structure with covariantly constant self-dual Kähler form are equivalent notions or whether there several H-J structures accompaning the Kähler structure.

In the following I argue that H-J structures correspond to different choices of symplectic coordinates for M^4 and that the properties of $X^4 \subset H$ determined bt M^-H duality make it natural to to choose particular symplectic coordinates for M^4.

Consider first what H-J structure and Kähler structure could mean in H.

1. The H-J structure of $M^4 \subset H$ would correspond to an integrable distribution of 2-D Minkowskian sub-spaces of M^4 defining a distribution of string world sheets $X^2(x)$ and orthogonal distribution of partonic 2-surfaces $Y^2(x)$. Could this decomposition correspond to self-dual covariantly Kähler form in M^4?

   What do we mean with covariant constancy now? Does it mean a separate covariant constancy for the choices of $M^2(x)$ and $Y^2(x)$ or only of their sum, which in Minkowski coordinates could correspond to a constant electric and magnetic fields orthogonal to each other?

2. The non-constant choice of $M^2(x)$ ($E^2(x)$) cannot be covariantly constant. One can write $J(M^4) = J(M^2(x)) \oplus J(E^2(x)$ corresponding to decomposition to electric and magnetic parts. Constancy of $J(M^2(x)$ would require that the gradient of $J(M^2(x)$ is compensated by the gradient of an antisymmetric tensor with square equal to the projector to $M^2(x)$. Same condition holds true for $J(E^2(x))$. The gradient of the antisymmetric tensor would be parallel to itself implying that the tensor is constant.

3. H-J structure can only correspond to a transformation acting on J but leaving $J_{kl}dm^k dm^l$ invariant. One should find analogs of local gauge transformations leaving J invariant. In the case of CP_2, these correspond to symplectic transformations and now one has a generalization of the notion. The M^4 analog of the symplectic group would parameterize various decompositions of $J(M^4)$.

   Physically the symplectic transformations define local choices of 2-D space $E^2(x)$ of transversal polarization directions and longitudinal momentum space M^2 emerging in the construction of extremals of Kähler action.

4. For the simplest Kähler form for $M^4 \subset H$, this decomposition in Minkowski coordinates would be constant: orthogonal constant electric and magnetic fields. This Kähler form extends to its number theoretical analog in M^8. The local $SU(3)$ element g would deform M^4 to $g(M^4)$ and define an element of local CP_2 defining $M^8 - H$ duality. g should correspond to a symplectic transformation of M^4.

Consider next the number theoretic counterparts of H-J- and Kähler structures of $M^4 \subset H$ in $M^4 \subset M^8$.

1. In M^4 coordinates H-J structure would correspond to a constant $M^2 \times E^2$ decomposition. In M^4 coordinates Kähler structure would correspond to constant E and B orthogonal to each other. Symplectic transformations give various representations of this structure as H-J structures.

2. The number theoretic analog of H-J structure makes sense also for $X^4 \subset M^8$ as obtained from the distribution of quaternionic normal spaces containing 2-D commutative sub-space at each point by multiplying then by local unit $I_4(x)$ orthogonal to the quaternionic units $\{1, I_1 = I_2 = I_3\}$ with respect to octonionic inner product. There is a hierarchy of CDs and the choices of these structures would be naturally parameterized by G_2.

   This would give rise to a number theoretically defined slicing of $X^4_c \subset M^8_c$ by complexified string world sheets X^2_c and partonic 2-surfaces Y^2_c orthogonal with respect to the octonionic inner product for complexified octonions.

3. In $M^8 - H$ duality defined by $g(p) \subset SU(3)$ assigns a point of CP_2 to a given point of M^4. $g(p)$ maps the number theoretic H-J to H-J in $M^4 \subset M^8$. The space-time surface itself - that is $g(p)$ - defines these symplectic coordinates and the local $SU(3)$ element g would naturally define this symplectic transformation.

4. For $X^4 \subset M^8$ g reduces to a constant color rotation satisfying the condition that the image point is $U(2)$ invariant. Unit element is the most natural option. This would mean that g is constant at the mass and energy shells corresponding to the roots of P and the mass shell is a mass shell of M^4 rather than some deformed mass shell associated with images under $g(p)$.

   This alone does not yet guarantee that the 4-D tangent space corresponds to M^4. The additional physically very natural condition on g is that the 4-D momentum space at these mass shells is the same. $M^8 - H$ duality maps these mass shells to the boundaries of these cd:s in M^4 (CD= $cd \times CP_2$). This conforms with the identification of zero energy states as pairs of 3-D states at the boundaries of CD.

This generalizes the original intuitive but wrong interpretation of the roots r_n of P as "very special moments in the life of self" [L81].

1. Since the roots correspond to mass squared values, they are mapped to the boundaries of cd with size $L = \hbar_{eff}/m$ by $M^8 - H$ duality in M^4 degrees of freedom. During the sequence of SSFRs the passive boundary of CD remains does not shift only changes in size, and states at it remain unaffected. Active boundary is shifted due to scaling of cd.

   The hyperplane at which upper and lower half-cones of CD meet, is shifted to the direction of geometric future. This defines a geometric correlate for the flow of experienced time.

2. A natural proposal is that the moments for SSFRs have as geometric correlates the roots of P defined as intersections of geodesic lines with the direction of 4-momentum p from the tip of CD to its opposite boundary (here one can also consider the possibility that the geodesic lines start from the center of cd). Also energy shells as roots $E = r_n$ of P are predicted. They decompose to a set of mass shells $m_{n.,k}$ with the same $E = r_n$: similar interpretation applies to them.

3. What makes these moments very special is that the mass and energy shells correspond to surfaces in M^4 defining the Lorentz quantum numbers. SSFRs correspond to quantum measurements in this basis and are not possible without this condition. At $X^4 \subset M^8$ the mass squared would remain constant but the local momentum frame would vary. This is analogous to the conservation of momentum squared in general relativistic kinematics of point particle involving however the loss of momentum conservation.

4. These conditions, together with the assumption that g is a rational function with real coefficients, strongly suggest what I have referred to as preferred extremal property, Bohr orbitology, strong form of holography, and number theoretical holography.

In principle, by a suitable choice of M^4 one can make the momentum of the system light-like: the light-like 8-momentum would be parallel to M^4. I have asked whether this could be behind the fact that elementary particles are in a good approximation massless and whether the small mass of elementary particles is due to the presence of states with different mass squares in the zero state allowed by Lorentz invariance.

The recent understanding of the nature of right-handed neutrinos based on M^4 Kähler structure [L136] makes this mechanism un-necessary but poses the question about the mechanism choosing some particular M^4. The conditions that $g(p)$ leaves mass shells and their 4-D tangent spaces invariant provides this kind of mechanism. Holography would be forced by the condition that the 4-D tangent space is same for all mass shels representing inverse images for very special moments of time.

What about string world sheets and partonic 2-surfaces?

One can apply the above arguments also to the identification of 2-D string world sheets and partonic 2-surfaces.

1. One has two kinds of solutions: M^2 and 3-D surfaces of X^4 as analogs of 6-brane. The interpretation for 3-D *resp.* 2-D branes as a light-like 3-surface associated with the octonionic Dirac equation representing mass shell condition *resp.* string world sheet is attractive.

2. M^2 would be replaced with an integrable distribution of $M^2(x)$ in local tangent space $M^4(x)$. The space for the choices of $M^2(x)$ would be S^3 corresponding to the selection of a preferred quaternion imaginary unit equal to the choices of preferred octonion imaginary unit.

   The choices of the preferred complex subspace $M^2(x)$ at a given point would be characterized by its normal vector and parameterized by sphere S^2: the interpretation as a quantization axis of angular momentum is suggestive. One would have space $S^3 \times S^2$. Also now the integrability conditions $de_A = 0$ would hold true.

3. String world sheets could be regarded as analogs of superstrings connecting 3-D brane like entities defined by the light-like partonic orbits. The partonic 2-surfaces at the ends of light-like orbits defining also vertices could correspond to the 3-surfaces at which quaternionic 4-surfaces intersect 6-branes.

19.3.3 Is (co-)associativity possible?

The number theoretic vision relying on the assumption that space-time surfaces are 8-D complex 4-surfaces in o_c^8 determined as algebraic surfaces for octonionic continuations of real polynomials, which for adelic physics would have coefficients which are rational or belong to an extension of rationals. The projections to subspaces Re^8 of o_c^8 defined as space for which given coordinate is purely real or imaginary so that complexified octonionic norm is real would give rise to real 4-D space-time surfaces. $M^8 - H$ duality would map these surfaces to geometric objects in $M^4 \times CP_2$. This vision involves several poorly understood aspects and it is good to start by analyzing them.

Challenging the notions of associativity and co-associativity

Consider first the notions of associativity *resp.* co-associativity equivalent with quaternionicity *resp.* co-quaternionicity. The original hope was that both options are possible for surfaces of real sub-spaces of O_c ("real" means here that complexified octonionic metric is real).

1. The original idea was that the associativity of the tangent space or normal space of a real space-time surface X^4 reduces the classical physics at the level of M^8 to associativity. Associativity/co-associativity of the space-time surface states that at each point of the tangent-/normal space of the real space-time surface in O is quaternionic. The notion generalizes also to $X_c^4 \subset O_c^8$. (Co-)associativity makes sense also for the real subspaces space of O with Minkowskian signature.

2. It has been however unclear whether (co-)associativity is possible. The cold shower came as I learned that associativity allows only for geodesic sub-manifolds of quaternionic spaces about which octonions provide an example [A36]. The good news was that the distribution of co-associative tangent spaces always defines an integrable distribution in the sense that one can find sub-manifold for which the associative normal space at a given point has tangent space as an orthogonal complement. Should the number theoretic dynamics rely on co-associativity rather than associativity?

3. Minkowskian space-time regions have been assumed to be associative and to correspond to the projection to the standard choice for basis as $\{1, iI_1, iI_2, iI_3\}$. The octonionic units $\{1, I_1, I_2, I_3\}$ define quaternionic units and associative subspace and their products with unit I_4 define the orthogonal co-associative subspace as $\{I_4, I_5 = I4I_1, I_6 = I_4I_2, I_7 = I_4I_3\}$. This result forces either to weaken the notion of associativity or to consider alternative identifications of Minkowskian regions, which can be co-associative: fortunately, there exists a large number of candidates.

The article [A36] indeed kills the idea about the associativity of the space-time surface. The article starts from a rather disappointing observation that associative sub-manifolds are geodesic sub-manifolds and therefore trivial. Co-associative quaternion sub-manifolds are however possible. With a motivation coming from this observation, the article discusses what the author calls RC quaternionic sub-manifolds of quaternion manifolds. For a quaternion manifold the tangent space allows a realization of quaternionic units as antisymmetric tensors. These manifolds are constant curvature spaces and typically homogeneous spaces.

1. Quaternion sub-manifold allows a 4-D integrable distribution of quaternion units. The normal complement of this distribution is expressible in terms of the second fundamental form and the condition that it is trivial implies that the second fundamental form is vanishing so that one has a geodesic submanifold. Quaternionic sub-manifolds are thus too trivial to be interesting. As a diametric opposite, one can also define totally real submanifolds for which the normal space contains a distribution of quaternion units. In this case the distribution is always integrable. This case is much more interesting from the TGD point of view.

2. Author introduces the notion of CR quaternion sub-manifold $N \subset M$, where M is quaternion manifold with constant sectional curvatures. N has quaternion distribution D in its tangent spaces if the action of quaternion units takes D to itself. $D^{\perp}$ is the co-quaternionic orthogonal complement D in the normal space N. D would take also $D^{\perp}$ to itself. $D^{\perp}$ can be expressed in terms of the components of the second fundamental form and vanishes for quaternion sub-manifolds.

3. Author deduces results about CR quaternion sub-manifolds, which are very interesting from the TGD point of view.

   (a) Sub-manifold is CR quaternion sub-manifold only if the curvature tensor of R_M of the embedding space satisfies $R_M(D, D, D^{\perp}, D\ \ ) = 0$. The condition is trivial if the quaternion manifold is flat. In the case of octonions this would be the case.

   (b) D is integrable only if the second fundamental form restricted to it vanishes meaning that one has a geodesic manifold. Totally real distribution $D^{\perp}$ is always integrable to a co-associative surface.

   (c) If $D^{\perp}$ integrates to a minimal surface then N itself is a minimal surface.

Could one consider RC quaternion sub-manifolds in TGD framework? Both octonions and their complexifixation can be regarded as quaternionic spaces. Consider the real case.

1. If the entire D is quaternionic then N is a geodesic sub-manifold. This would leave only E^4 and its Minkowskian variants with various signatures. One could have however 4-D totally real (co-associative) space-time surfaces. Simple arguments will show that the intersections of the conjectured quaternionic and co-quaternionic 4-surfaces have 2- and 3-D intersections with 6-branes.

   Should one replace associative space-time surfaces with CR sub-manifolds with $d \leq 3$ integrable distribution D whereas the co-quaternionic surfaces would be completely real having 4-D integrable $D^{\perp}$? Could one have 4-D co-associative surfaces for which $D^{\perp}$ integrates to $n \geq 1$-dimensional minimal surface (geodesic line) and the X^4 itself is a minimal surface?

   Partially associative CR manifold do not allow M^8H duality. Only co-associative surfaces allow it and also their signature must be Minkowskian: the original idea [L85, L49, L50, L51] about Euclidian (Minkowskian) signature for co-associative (associative) surfaces was wrong.

2. The integrable 2-D sub-distributions D defining a distribution of normal planes could define foliations of the X^4 by 2-D surfaces. What springs to mind is foliations by string world sheets and partonic 2 surfaces orthogonal to them and light-like 3-surfaces and strings transversal to them. This expectation is realized.

How to identify the Minkowskian sub-space of O_c?

There are several identifications of subspaces of O_c with Minkowskian signature. What is the correct choice has been far from obvious. Here symmetries come in rescue.

1. Any subspace of O^c with 3 (1) imaginary coordinates and 1 (3) real coordinates has Minkowskian signature in octonionic norm algebraically continued to O_c (complex valued continuation of real octonion norm instead of real valued Hilbert space norm for O_c). Minkowskian regions should have local tangent space basis consisting of octonion units which in the canonical case would be $\{I_1, iI_3, iI_5, iI_7\}$, where i is commutative imaginary unit. This particular basis is co-associative having whereas its complement $\{iI_0, I_2, I_4, I_6\}$ is associative and has also Minkowskian signature.

2. The size of the isometry group of the subspace of M^8_c depends on whether the tangent basis contains real octonion unit 1 or not. The isometry group for the basis containing I_0 is $SO(3)$ acting as automorphisms of quaternions and $SO(k, 3-k)$ when $3-k$ units are proportional to i. The reason is that G_2 (and its complexification $G_{2,c}$) and its subgroups do not affect I_0. For the tangent spaces built from 4 imaginary units I_k and iI_l the isometry group is $SO(k, 4-k) \subset G_{2,c}$.

   The choice therefore allows larger isometry groups and also co-associativity is possible for a suitable choice of the basis. The choice $\{I_1, iI_3, iI_5, iI_7\}$ is a representative example, which will be called canonical basis. For these options the isometry group is the desired $SO(1,3)$ as an algebraic continuation of $SO(4) \subset G_2$ acting in $\{I_1, I_3, I_5, I_7\}$, to $SO(1,3) \subset G_{2,c}$.

   Also Minkowskian signature - for instance for the original canonical choice $\{I_0, iI_1, iI_2, iI_3\}$ - can have only $SO(k, 3-k)$ as isometries. This is the basic objection against the original choice $\{I_0, iI_1, iI_2, iI_3\}$. This identification would force the realization of $SO(1,3)$ as a subgroup of $SO(1,7)$. Different states of motion for a particle require different octonion structure with different direction of the octonion real axis in M^8. The introduction of the notion of moduli space for octonion structures does not look elegant. For the option $\{I_1, iI_3, iI_5, iI_7\}$ only a single octonion structure is needed and $G_{2,c}$ contains $SO(1,3)$.

   Note that also the signatures $(4,0)$, $(0,4)$ and $(2,2)$ are possible and the challenge is to understand why only the signature (1,3) is realized physically.

Co-associative option is definitely the only physical alternative. The original proposal for the interpretation of the Minkowski space in terms of an associative real sub-space of M^4 had a serious problem. Since time axis was identified as octonionic real axis, one had to assign different octonion structure to particles with non-parallel moment: $SO(1,7)$ would relate these structures: how to glue the space-time surfaces with different octonion structures together was the problem. This problem disappears now. One can simply assign to particles with different state of motion real space-time surface defined related to each other by a transformation in $SO(1,3) \subset G_{2,c}$.

The condition that $M^8 - H$ duality makes sense

The condition that $M^8 - H$ duality makes sense poses strong conditions on the choice of the real sub-space of M^8_c.

1. The condition that tangent space of O_c has a complexified basis allowing a decomposition to representations of $SU(3) \subset G_2$ is essential for the map to $M^8 \to H$ although it is not enough. The standard representation of this kind has basis $\{\pm iI_0 + I_1\}$ behaving like $SU(3)$ singlets $\{I_2 + \epsilon iI_3, I_4 + \epsilon iI_5, \epsilon I_6 \pm iI_7\}$ behaves like SU(3) triplet 3 for $\epsilon = 1$ and its conjugate $\overline{3}$ for $\epsilon = -1$. $G_{2,c}$ provides new choices of the tangent space basis consistent with this choice. $SU(3)$ leaves the direction I_1 unaffected but more general transformations act as Lorentz

transformation changing its direction but not leaving the M^4 plane. Even more general $G_{2,c}$ transformations changing M^4 itself are in principle possible.

Interestingly, for the canonical choice the co-associative choice has $SO(1,3)$ as isometry group whereas the complementary choice failing to be associative correspond to a smaller isometry group $SO(3)$. The choice with M^4 signature and co-associativity would provide the highest symmetries. For the real projections with signature $(2,2)$ neither consistent with color structure, neither full associativity nor co-associativity is possible.

2. The second essential prerequisite of $M^8 - H$ duality is that the tangent space is not only (co-)associative but contains also (co-)complex - and thus (co-)commutative - plane. A more general assumption would be that a co-associative space-time surface contains an integrable distribution of planes $M^2(x)$, which could as a special case reduce to M^2.

   The proposal has been that this integrable distribution of $M^2(x)$ could correspond to string sheets and possibly also integrable orthogonal distribution of their co-complex orthogonal complements as tangent spaces of partonic 2-surfaces defining a slicings of the space-time surface. It is now clear that this dream cannot be realized since the space-time surface cannot be even associative unless it is just E^4 or its Minkowskian variants.

3. As already noticed, any distribution of the associative normal spaces integrates to a co-associative space-time surface. Could the normal spaces also contain an integrable distribution of co-complex planes defined by octonionic real unit 1 and real unit $I_k(x)$, most naturally I_1 in the canonical example? This would give co-commutative string world sheet. Commutativity would be realized at the 2-D level and associativity at space-time level. The signature of this plane could be Minkowskian or Euclidian. For the canonical example $\{I_1, iI_3, iI_5, iI_7\}$ the 2-D complex plane in quaternionic sense would correspond to $(a \times 1, +n_2I_2 + n_4I_6 + n_6I_6$, where the unit vector n_i has real components and one has $a = 1$ or $a = i$ is forced by the complexification as in the canonical example.

   Since the distribution of normal planes integrates to a 4-surface, one expects that its sub-distribution consting of commutative planes integrates to 2-D surface inside space-time surface and defines the counterpart of string worlds sheet. Also its normal complement could integrate to a counterpart of partonic 2-surface and a slicing of space-time surface by these surfaces would be obtained.

4. The simplest option is that the commutative space does not depend on position at X^4. This means a choice of a fixed octonionic imaginary unit, most naturally I_1 for the canonical option. This would make $SU(3)$ and its sub-group $U(2)$ independent of position. In this case the identification of the point of $CP_2 = SU(3)/U(2)$ labelling the normal space at a given point is unique.

   For a position dependent choice $SU(3)(x)$ it is not clear how to make the specification of $U(2)(x)$ unique: it would seem that one must specify a unique element of $G_2(x)$ relating $SU(3)(x)$ to a choice at special point x_0 and defining the conjugation of both $SU(3)(x)$ and $U(2)(x)$. Otherwise one can have problems. This would also mean a unique choice for the direction of time axis in O and fixing of $SO(1,3)$ as a subgroup of $G_{2,c}$. Also this distribution of associative normal spaces is integrable. Physically this option is attractive but an open question is whether it is consistent with the identification of space-time surfaces as roots $Re_Q(P) = 0$ of P.

Co-associativity from octonion analyticity or/and from G_2 holography?

Candidates for co-associative space-time surfaces X^4_r are defined as restrictions X^4_r for the roots X^4_c of the octonionic polynomials such that the O_c coordinates in the complement of a real co-associative sub-space of O_c vanish or are constant. Could the surfaces X^4_r or even X^4_c be co-associative?

1. X^4_r is analogous to the image of real or imaginary axis under a holomorphic map and defines a curve in complex plane preserving angles. The tangent vectors of X^4_r and X^4_c involve gradients of all coordinates of O_c and are expressible in terms of all octonionic unit vectors. It is not

obvious that their products would belong to the normal space of X_r^4 a strong condition would be that this is the case for X_c^4.

2. Could octonion analyticity in the proposed sense guarantee this? The products of octonion units also in the tangent space of the image would be orthogonal to the tangent space. Ordinary complex functions preserve angles, in particular, the angle between x- and y-axis is preserved since the images of coordinate curves are orthogonal. Octonion analyticity would preserve the orthogonality between tangent space vectors and their products.

3. This idea could be killed if one could apply the same approach to associative case but this is not possible! The point is that when the real tangent space of O_c contains the real octonion unit, the candidate for the 4-D space-time surface is a complex surface X_c^2. The number theoretic metric is real only for 2-D X_r^2 so that one obtains string theory with co-associativity replaced with co-commutativity and $M^4 \times CP_2$ with $M^2 \times S^2$. One could of course ask whether this option could be regarded as a "sub-theory" of the full theory.

My luck was that I did not realize the meaning of the difference between the two cases first and realized that one can imagine an alternative approach.

1. G_2 as an automorphism group of octonions preserves co-associativity. Could the image of a co-associative sub-space of O_c defined by an octonion analytic map be regarded as an image under a local G_2 gauge transformation. $SU(3) \subset G_2$ is an especially interesting subgroup since it could have a physical interpretation as a color gauge group. This would also give a direct connection with $M^8 - H$ duality since $SU(3)$ corresponds to the gauge group of the color gauge field in H.

2. One can counter-argue that an analog of pure gauge field configuration is in question at the level of M^8. But is a pure gauge configuration for $G_{2,c}$ a pure gauge configuration for G_2? The point is that the $G_{2,c}$ connection $g^{-1}\partial_\mu g$ trivial for $G_{2,c}$ contains by non-linearity cross terms from $g_2g, c = g_{2,1} + ig_{2,2}$, which are of type $Re = X[g_{2,1}, g_{2,1}] - X[g_{2,2}, g_{2,2}] = 0$ and $Im = iZ[g_{2,1}, g_{2,2}] = 0$. If one puts $g_{2,2}$ contributions to zero, one obtains $Re = X[g_{2,1}, g_{2,1}]$, which does not vanish so that $SU(3)$ gauge field is non-trivial.

3. X_r^4 could be also obtained as a map of the co-associative M^4 plane by a local $G_{2,c}$ element. It will turn out that $G_{2,c}$ could give rise to the speculated Yangian symmetry [L45] at string world sheets analogous to Kac-Moody symmetry and gauge symmetry and crucial for the construction of scattering amplitudes in $M8$.

4. The decomposition of the co-associative real plane of O_c should contain a preferred complex plane for $M^8 - H$ duality to make sense. $G_{2,c}$ transformation should trivially preserve this property so that SH would not be necessary at H side anymore.

There is a strong motivation to guess that the two options are equivalent so that $G_{2,c}$ holography would be equivalent with octonion analyticity. The original dream was that octonion analyticity would realize both associative and co-associative dynamics but was exaggeration!

Does one obtain partonic 2-surfaces and strings at boundaries of ΔCD_8?

It is interesting to look for the dimensions of the intersections of the light-like branes at the boundary of CD_8 giving rise to the boundary of CD_4 in M^4 to see whether it gives justification for the existing phenomenological picture involving light-like orbits of partonic 2-surfaces connected by string world sheets.

1. Complex light-cone boundary has dimension $D = 14$. $P = 0$ as an additional condition at δCD_8 gives 2 complex conditions and defines a 10-D surface having 5-D real projections.

2. The condition $Im_Q(P) = 0$ gives 8 conditions and gives a 2-D complex surface with 1-D real projection. The condition $Re_Q(P) = 0$ gives 3 complex conditions since $X = 0$ is already satisfied and the solution is a 4-D surface having 2-D real projection. Could the interpretation be in terms of the intersection of the orbit of a light-like partonic surface with the boundary of CD_8?

3. Associativity is however not a working option. If only co-associative Minkowskian surfaces allowing mapping to H without SH are present then only 4-D space-time surfaces with Minkowskian signature, only partonic 2-surfaces and their light-like orbits would emerge from co-associativity.

   This option would not allow string world sheets for which there is a strong intuitive support. What could a co-complex 2-surface of a co-associative manifold mean? In the co-associative case the products of octonion imaginary units are in the normal space of space-time surface. Could co-complex surface $X_c^2 \subset X_c^4$ be defined by an integrable co-complex sub-distribution of co-associative distribution. The 4-D distribution of normal planes is always integrable.

   Could the 2-D sub-distributions of co-associative distribution integrate trivially and define slicings by string world sheets or partonic 2-surfaces. Could the distribution of string distributions and its orthogonal complement be both integrable and provide orthogonal slicings by string world sheets and partonic 2-surfaces? String world sheets with Minkowskian signature should intersect the partonic orbits with Euclidian signature along light-like lines. This brings in mind the orthogonal grid of flow lines defined by the $Re(f) = 0$ and $Im(f) = 0$ lines of an analytic function in plane.

4. In this picture the partonic 2-surfaces associated with light-like 3-surface would be physically unique and could serve as boundary values for the distributions of partonic 2-surfaces. But what about string world sheets connecting them? Why would some string world sheets be exceptional? String world sheets would have a light-like curve as an intersection with the partonic orbit but this is not enough.

   Could the physically special string world sheets connect two partonic surfaces? Could the string associated with a generic string world sheet be like a flow line in a hydrodynamic flow past an obstacle - the partonic 2-surface? The string as a flowline would go around the obstacle along either side but there would be one line which ends up to the object.

Interactions would correspond geometrically to the intersections of co-associative space-time surfaces X_r^4 associated with particles and corresponding to different real sub-spaces of O_c related by Lorentz boost in $SO(1,3) \subset G_{2,c}$. In the generic case the intersection would be discrete. In the case that X and Y have a common root the real surfaces $X_r^4 \subset X_r^6$ associated with quarks and depending on their state of motion would reside inside the same 6-D surface X_r^6 and have a 2-D surface X_r^2 as intersection. Could this surface be interpreted as a partonic 2-surface? One must however bear in mind that partonic 2-surfaces as topological vertices are assumed to be non-generic in the sense that the light-like partonic orbits meet at them. At the level of H, the intersections would be partonic 2-surfaces X^2 at which the four 3-D partonic orbits would meet along their ends. Does this hold true at the level of M^8? Or can it hold true even at the level H?

The simplest situation corresponds to 4 external quarks. There are 6 different intersections. Not all of them are realized since a given quark can belong only to a single intersection. One must have two disjoint pairs -say 12 and 34. Most naturally positive *resp.* negative energy quarks form a pair. These pairs are located in different half-cones. The intersections would give two partonic 2-surfaces and this situation would be generic. This suggests a modification of the description of particle reaction in M^8 . $M^8 - H$ duality suggests a similar description in H.

What could be the counterparts of wormhole contacts at the level of M^8?

The experience with H, in particular the presence of extremals with Euclidian signature of the induced metric and identified as building bricks of elementary particles, suggest that also the light-like 3-surfaces in M_c^8 could have a continuation with an Euclidian signature of the number theoretic metric with norm having real values only for the projections to planes allowing real coordinates.

The earlier picture has been that the wormhole contacts as CP_2 type extremals correspond to co-associative regions and their exteriors to associative regions. If one wants $M^8 - H$ duality in strong form and thus without need for SH, one should assume that both these regions are co-associative.

1. The simplest option is that the real Minkowskian time coordinate becomes imaginary. Instead of the canonical (I_1, iI_3, iI_5, iI_7) the basis would be (iI_1, iI_3, iI_5, iI_7) having Euclidian

signature and SO(4) as isometry group. The signature would naturally change at light-like 3-surface the time coordinate along light-like curves becomes zero - proper time for photon vanishes - and can ransforms continuously from real to imaginary.

2. Wormhole contacts in H behave like pairs of magnetic monopoles with monopole charges at throats. If one does not allow point-like singularity, the monopole flux must go to a parallel Minkowskian space-time sheet through the opposite wormhole throat. Wormhole contact with effective magnetic charge would correspond in M^8_c to a distribution of normal 4-planes at the partonic 2-surfaces analogous to the radial magnetic field of monopole at a sphere surrounding it. To avoid singularity of the distribution, there must be another light-like 3-surface M^8 such that its partonic throat has a topologically similar distribution of normal planes.

In the case of X^3_c dimension does not allow co-quaternion structure: could they allow 4-D co-associative sub-manifolds? It will be found that this option is not included since co-associative tangent space distributions in a quaternion manifold (now O) are always integrable.

19.3.4 Octonionic Dirac equation and co-associativity

Also the role of associativity concerning octonionic Dirac equation in M^8 must be understood. It is found that co-associativity allows very elegant formulation and suggests the identification of the points appearing as the ends of quark propagator lines in H as points of boundary of CD representing light-like momenta of quarks. Partonic vertices would involve sub-CDs and momentum conservation would have purely geometric meaning bringing strongly in mind twistor Grassmannian approach [B27, B25, ?]. I have discussed the twistor lift of TGD replacing twistors as fields with surfaces in twistor space having induced twistor structure in [K116, L55, L72] [L98, L99].

Octonionic Dirac equation

The following arguments lead to the understanding of co-associativity in the case of octonion spinors. The constant spinor basis includes all spinors but the gamma matrices appearing in the octonionic Dirac equation correspond to co-associative octonion units.

1. At the level of O_c the idea about massless Dirac equation as partial differential equation does not make sense. Dirac equation must be algebraic and the obvious idea is that it corresponds to the on mass shell condition for a mode of ordinary Dirac equation with well-define momentum: $p^k\gamma_k\Psi = 0$ satisfying $p^kp_k = 0$. This suggests that octonionic polynomial P defines the counterpart of $p^k\gamma_k$ so that gamma matrices γ_k would be represented as octonion components. Does this make sense?

2. Can one construct octonionic counterparts of gamma matrices? The imaginary octonion units I_k indeed define the analogs of gamma matrices as $\gamma_k \equiv iI_k$ satisfying the conditions $\{\gamma_k, \gamma_l\} = 2\delta_{kl}$ defining Euclidian gamma matrices. The problem is that one has $I_0 I_l k + I_k I_0 = 2I_k$. One manner to solve the problem would be to consider tensor products $I_0\sigma_3$ and $I_k\sigma_2$ where σ_3 and $sigma_2$ are Pauli's sigma matrices with anti-commutation relations $\{\sigma_i, \sigma_j\} = \delta_{i,j}$. Note that I_k do not allow a matrix representation.

   Co-associativity condition suggests an alternative solution. The restriction of momenta to be co-associative and therefore vanishing component p^0 as octonion, would selects a sub-space spanned by say the canonical choice $\{I_2, iI3, iI_5, iI_7\}$ satisfying the anticommutation relations of Minkowskian gamma matrices. Octonion units do not allow a matrix representation because they are not associative. The products for a co-associative subset of octonion units are however associative $(a(bc) = (ab)c$ so that they can be mapped to standard gamma matrices in Minkowski space. Co-associativity would allow the representation of 4-D gamma matrices as a maximal associative subset of octonion units.

3. What about octonionic spinors. The modes of the ordinary Dirac equation with a well-defined momentum are obtained by applying the Dirac operator to an orthogonal basis of constant spinors u_i to give $\Psi = p^k\gamma_k u_i$. Now the counterparts of constant spinors u_i would naturally

be octonion units $\{I_0, I_k\}$: this would give the needed number 8 of real spinor components as one has for quark spinors.

Dirac equation reduces to light-likeness conditions $p^k p_k = 0$ and p_k must be chosen to be real - if pk are complex, the real and imaginary parts of momentum are parallel. One would obtain an entire 3-D mass shell of solution and a single mode of Dirac equation would correspond to a point of this mass shell.

Remark: Octonionic Dirac equation is associative since one has a product of form $(p_k\gamma_k)^2 u_i$ and octonion products of type $x^2 y$ are associative.

4. p^k would correspond to the restriction of $P(o_c)$ to M^4 as sub-space of octonions. Since co-associativity implies $P(o_c) = Y(o_c)o_c$ restricted to counterpart of M^4 (say subspace spanned by $\{I_2, iI3, iI_5, iI_7\}$), Dirac equation reduces to the condition $o^k o_k = 0$ in M^4 defining a light-cone of M^4. This light-cone is mapped to a curved light-like 3-surface X^3 in o_c as $o_c \to P(o_c) = Y o_c$. $M^8 - H$ duality maps points of space-time surface on M^8 H and therefore the light-cone of M^4 corresponds to either light-like boundary of CD. It seems that the image of X^3 in H has M^4 projection to the light-like boundary of CD.

   Co-associative space-time surfaces have 3-D intersections X^3 with the surface $P = 0$: the conjecture is that X^3 corresponds to a light-like orbit of partonic 2-surfaces in H at which the induced metric signature changes. At X^3 one has besides $X = 0$ also $Y = 0$ so that octonionic Dirac equation $P(o_c)\Psi = P^k I_k \Psi = Y p^k I_k \Psi = 0$ is trivially satisfied for all momenta $p^k = o^k$ defined by the M^4 projections of points of X^3 and one would have $P^k = Y p^k = 0$ so that the identification of P^k as 4-momentum would not allow to assign non-vanishing momenta to X^3. The direction of p^k is constrained only by the condition of belonging to X^3 and the momentum would be in general time-like since X^3 is inside future light-cone.

   $Y = 0$ condition conforms with the proposal that X^3 defines a boundary of Minkowskian and Euclidian region: Euclidian mass shell condition for real P^k requires $P^k = 0$. The general complex solution to $P^2 = 0$ condition is $P = P_1 + iP_2$ with $P_1^2 = P_2^2$.

A single mode of Dirac equation with a well-defined value of p^k as the analog of 4-momentum would correspond to a selection of single time-like point at X^3 or light-like point at the light-like boundary of CD. X^3 intersects light-cone boundary as part of boundary of 7-D light-cone. The picture about scattering amplitudes - consistent with the view about cognitive representations as a unique discretization of space-time surface - is that quarks are located at discrete points of partonic 2-surfaces representing the ends of fermionic propagator lines in H and that one can assign to them light-like momenta.

Challenging the form of $M^8 - H$ duality for the map $M^4 \subset M^8$ to $M^4 \subset H$

The assumption that the map $M^4 \subset M^8$ to $M^4 \subset H$ in $M^8 - H$ duality is a simple identification map has not been challenged hitherto.

1. Octonionic Dirac equation forces the identification of M^8 as analog of 8-D momentum space and the earlier simple identification is in conflict with Uncertainty Principle. Inversion allowed by conformal invariance is highly suggestive: what comes first in mind is a map $m^k \to \hbar_{eff} m^k / m^k m_k$.

   At the light-cone boundary the map is ill-defined. Here on must take as coordinate the linear time coordinate m^0 or equivalently radial coordinate $r_M = m^0$. In this case the map would be of fporm $t \to \hbar_{eff}/m^0$: m^0 has interpretation as energy of massless particle.

   The map would give a surprisingly precise mathematical realization for the intuitive arguments assigning to mass a length scale by Uncertainty Principle.

2. Additional constraints on $M^8 - H$ duality in M^4 degrees of freedom comes from the following argument. The two half-cones of CD contain space-time surfaces in M^8 as roots of polynomials $P_1(o)$ and $P_2(2T - o)$ which need not be identical. The simplest solution is $P_2(o) = P_1(2T - o)$: the space-time surfaces at half-cones would be mirror images of each other. This gives $P_1(T, Im_R(o)) = P_1(T - Im_R(o))$ Since P_1 depends on $t^2 - \overline{o}^2$ only, the condition is identically satisfied for both options.

There are two options for the identification of the coordinate t.

Option a): t is identified as octonionic real coordinate o_R identified and also time coordinate as in the original option. In the recent option octonion o_R would correspond to the Euclidian analog of time coordinate. The breaking of symmetry from $SO(4)$ to $SO(3)$ would distinguish t as a Newtonian time.

At the level of M^8, The M^4 projection of CD_8 is a union of future and past directed light-cones with a common tip rather than CD_4. Both incoming and outgoing momenta have the same origin automatically. This identification is the natural one at the level of M^8.

Option b): t is identified as a Minkowski time coordinate associated with the imaginary unit I_1 in the canonical decomposition $\{I_1, iI_3, iI_5, iI_7\}$. The half-cone at $o = 0$ would be shifted to $O = (0, 2T, 0...0)$ and reverted. M^4 projection would give CD_4 so that this option is consistent with ZEO. This option is natural at the level of Hbut not at the level of M^8.

If **Option a)** is realized at the level of M^8 and **Option b)** at the level of H, as seems natural, a time translation $m^0 \rightarrow m^0 + 2T$ of the past directed light-cone in $M^4 \subset H$ is required in order to to give upper half-cone of CD_4.

3. The map of the momenta to embedding space points does not prevent the interpretation of the points of M^8 as momenta also at the level of H since this information is not lost. One cannot identify p^k as such as four-momentum neither at the level of M^8 nor H as suggested by the naïve identification of the Cartesian factors M^4 for M^8 and H. This problem is circumvented by a conjugation in M^8_c changing the sign of 3-momentum. The light-like momenta along the light-cone boundary are non-physical but transform to light-like momenta arriving into light-cone as the physical intuition requires.

   Therefore the map would have in the interior of light-cone roughly the above form but there is still a question about the precise form of the map. Does one perform inversion for the M^4 projection or does one take M^4 projection for the inversion of complex octonion. The inversion of M^4 projection seems to be the more plausible option. Denoting by $P(o_c)$ the real M^4 projection of X^4 point one therefore has:

$$P(o_c) \rightarrow \hbar_{eff} \frac{\overline{P(o_c)}}{P(o_c) \cdot P(o_c)} \quad . \tag{19.3.7}$$

   Note that the conjugation changes the direction of 3-momentum.

   At the light-cone boundary the inversion is ill-defined but Uncertainty Principle comes in rescue, and one can invert the M^4 time coordinate:

$$Re(m^0) = t \rightarrow \hbar_{eff} \frac{1}{t} \quad . \tag{19.3.8}$$

A couple of remarks are in order.

1. The presence of $\hbar_{eff}$ instead of $\hbar$ is required by the vision about dark matter. The value of $\hbar_{eff}/h_0$ is given by the dimension of extension of rationals identifiable as the degree of P.

2. The image points $\overline{p}^k$ in H would naturally correspond to the ends of the propagator lines in the space-time representation of scattering amplitudes.

The information about momenta is not lost in the map. What could be the interpretation of the momenta $\overline{p}^k$ at the level of H?

1. Super-symplectic generators at the partonic vertices in H do not involve momenta as labels. The modes of the embedding space spinor field assignable to the ground states of super-symplectic representations at the boundaries of CD have 4-momentum and color as labels. The identification of $\overline{p}^k$ as this momentum label would provide a connection with the classical picture about scattering events.

   At the partonic 2-surfaces appearing as vertices, one would have a sum over the ground states (spinor harmonics). This would give integral over momenta but M^8-H duality and number theoretic discretization would select a finite subset and the momentum integral would reduce to a discrete sum. The number of M^8 points with coordinates in a given extension of rationals is indeed finite.

2. $M^4 \subset M^8$ could be interpreted as the space of 4-momenta labeling the spinor harmonics of M^8. Same would apply at the level of H: spinor harmonics would correspond to the ground states of super-symplectic representations.

3. The interpretation of the points of M^4_c as complex 4-momenta inspires the question whether the interpretation of the imaginary part of the momentum squared in terms of decay decay width so that M^8 picture would code even information about the dynamics of the particles.

19.4 How to achieve periodic dynamics at the level of $M^4 \times CP_2$?

Assuming M^8-H duality, how could one achieve typical periodic dynamics at the level of H - at least effectively?

It seems that one cannot have an "easy" solution to the problem?

1. Irreducible polynomials which are products of monomials corresponding to roots r_n which are in good approximation evenly spaced $r_n = r_0+nr_1\Delta r_n$ would give "very special moments in the life of self" as values of M^4 time which are evenly spaced [L85, L81]. This could give rise to an effective periodicity but it would be at the level of M^8, not H, where it is required.

2. Is it enough that the periodic functions are *only* associated with the spinor harmonics of H involved with the construction of scattering amplitudes in H [L123]? For the modified Dirac equation [K127] the periodic behavior is possible. Note also that the induced spinors defining ground states of super-symplectic representations are restrictions of second quantized spinors of H proportional to plane waves in M^4. These solutions do not guarantee quantum classical correspondence.

19.4.1 The unique aspects of Neper number and number theoretical universality of Fourier analysis

Could one assume more general functions than polynomials at the level of H? Discrete Fourier basis is certainly an excellent candidate in this respect but does it allow number theoretical universality?

1. Discrete Fourier analysis involves in the Euclidian geometry periodic functions $exp(2\pi x)$, n integer and in hyperbolic geometry exponential functions $exp(kx)$.

   Roots of unity $exp(i2\pi/n)$ allow to generalize Fourier analysis. The p-adic variants of $exp(ix)$ exist for rational values of $x = k2\pi/n$ for $n = K$ if $exp(i2\pi/K)$ belongs to the extension of rationals. $x = k = 2pi/n$ does not exist as a p-adic number but $exp(x) = exp(i2\pi/n)$ can exist as phase replacing x as coordinate in extension of p-adics. One can therefore define Fourier basis $\{exp(inx)|n \in Z\}$ which exist at discrete set of rational points $x = k/n$

   Neper number e is also p-adically exceptional in that e^p exists as a p-adic number for all primes p. One has a hierarchy of finite-D extensions of p-adic numbers spanned by the roots $e^{1/n}$. Finiteness of cognition might allow them. Hyperbolic functions $exp(nx)$, $n = 1, 2...$ would have values in extension of p-adic number field containing $exp(1/N)$ in a discrete set of points $\{x = k/N|k \in Z\}$.

2. (Complex) rationality guarantees number theoretical universality and is natural since CP_2 geometry is complex. This would correspond to the replacement $x \rightarrow exp(ix)$ or $x \rightarrow exp(x)$ for powers x^n. The change of the signature by replacing real coordinate x with ix would automatically induce this change.

3. Exponential functions are in a preferred position also group theoretically. Exponential map maps $g \rightarrow exp(itg)$ the points of Lie algebra to the points of the Lie group so that the tangent space of the Lie algebra defines local coordinates for the Lie group. One can say that tangent space is mapped to space itself. M^4 defines an Abelian group and the exponential map would mean replacing of the M^4 coordinates with their exponential, which are p-adically more natural. Ordinary Minkowski coordinates have both signs so that they would correspond to the Lie algebra level.

4. CP_2 is a coset space and its points are obtained as selected points of $SU(3)$ using exponentiation of a commutative subalgebra t in the decomposition $g = h + t + \bar{t}$ in the Lie-algebra of $SU(3)$. One could interpret the CP_2 points as exponentials and the emergence of exponential basis as a basis satisfying number theoretical universality.

19.4.2 Are CP_2 coordinates as functions of M^4 coordinates expressible as Fourier expansion

Exponential basis is not natural at the level of M^8. Exponential functions belong to dynamics, not algebraic geometry, and the level H represents dynamics.

It is the dependence of CP_2 coordinates on M^4 coordinates, where the periodicity is needed. The map of the tangent spaces of $X^4 \subset M^8$ to points of CP_2 is slightly local since it depends on the first derivatives crucial for dynamics. Could this bring in dynamics and exponential functions at the level of H?

These observations inspire the working hypothesis that CP_2 points as functions of M^4 coordinates are expressible as polynomials of hyperbolic and trigonometric exponentials of M^4 coordinates.

Consider now the situation in more detail.

1. The basis for roots of e would be characterized by integer K in $e^{1/K}$. This brings in a new parameter characterizing the extension of rationals inducing finite extensions of p-adic numbers. K is analogous to the dimension of extension of rationals: the p-adic extension has dimension $d = Kp$ depending on the p-adic prime explicitly.

2. If CD size T is given, $e^{-T/K}$ defines temporal and spatial resolution in H. K or possibly Kp could naturally correspond to the gravitational Planck constant [L60] [K11] [E1] $K = n_{gr} = \hbar_{gr}/h_0$.

3. In [L125] many-sheetedness with respect to CP_2 was proposed to correspond to flux tubebundles in M^4 forming quantum coherent structures. A given CP_2 point corresponds to several M^4 points with the same tangent space and their number would correspond to the number of the flux tubes in the bundle.

   Does the number of these points relate to K or Kp? p-Adic extension would have finite dimension $d = Kp$. Could $d = Kp$ be analogous to a degree of polynomial defining the dimension of extension of rationals? Could this be true in p-adic length scale resolution $O(p^2) = 0$ The number of points would be Kp and very large. For electron one has $p = M_{127} = 2^{127} - 1$.

4. The dimension n_A Abelian extension associated with EQ would naturally satisfy $n_A = K$ since the trigonometric and hyperbolic exponentials are obtained from each other by replacing a real coordinate with an imaginary one.

5. There would be two effective Planck constants. $h_{eff} = nh_0$ would be defined by the degree n of the polynomial P defining $X^4 \subset M^8$. $\hbar_{gr} = n_{gr}h_0$ would define infra-red cutoff in M^4 as the size scale of CD in $H = M^4 \times CP_2$. n *resp.* $n_{gr} = Kp$ would characterize many-sheetedness in M^4 *resp.* CP_2 degrees of freedom.

19.4.3 Connection with cognitive measurements as analogs of particle reactions

There is an interesting connection to the notion of cognitive measurement [L125, L126, L135].

1. The dimension n of the extension of rationals as the degree of the polynomial $P = P_{n_1} \circ P_{n_2} \circ ...$ is the product of degrees of degrees n_i: $n = \prod_i n_i$ and one has a hierarchy of Galois groups G_i associated with $P_{n_i} \circ ....$ G_{i+1} is a normal subgroup of G_i so that the coset space $H_i = G_i/G_{i+1}$ is a group of order n_i. The groups H_i are simple and do not have this kind of decomposition: simple finite groups appearing as building bricks of finite groups are classified. Simple groups are primes for finite groups.

2. The wave function in group algebra $L(G)$ of Galois group G of P has a representation as an entangled state in the product of simple group algebras $L(H_i)$. Since the Galois groups act on the space-time surfaces in M^8 they do so also in H. One obtains wave functions in the space of space-time surfaces. G has decomposition to a product (not Cartesian in general) of simple groups. In the same manner, $L(G)$ has a representation of entangled states assignable to $L(H_i)$ [L125, L135].

This picture leads to a model of analysis as a cognitive process identified as a cascade of "small state function reductions" (SSFRs) analogous to "weak" measurements.

1. Cognitive measurement would reduce the entanglement between $L(H_1)$ and $L(H_2)$, the between $L(H_2)$ and $L(H_3)$ and so on. The outcome would be an unentangled product of wave functions in $L(H_i)$ in the product $L(H_1) \times L(H_2) \times ....$ This cascade of cognitive measurements has an interpretation as a quantum correlate for analysis as factorization of a Galois group to its prime factors. Similar interpretation applies in M^4 degrees of freedom.

2. This decomposition could correspond to a replacement of P with a product $\prod_i P_i$ of polynomials with degrees $n = n_1 n_2 ...$, which is irreducible and defines a union of separate surfaces without any correlations. This process is indeed analogous to analysis.

3. The analysis cannot occur for simple Galois groups associated with extensions having no decomposition to simpler extensions. They could be regarded as correlates for irreducible primal ideas. In Eastern philosophies the notion of state empty of thoughts could correspondto these cognitive states in which SSFRs cannot occur.

4. An analogous process should make sense also in the gravitational sector and would mean the splitting of $K = n_A$ appearing as a factor $n_{gr} = Kp$ to prime factors so that the sizes of CDs involved with the resulting structure would be reduced. This process would reduce to a simultaneous measurement cascade in hyperbolic and trigonometric Abelian extensions. The IR cutoffs having interpretation as coherence lengths would decrease in the process as expected. Nature would be performing ordinary prime factorization in the gravitational degrees of freedom.

Cognitive process would also have a geometric description.

1. For the algebraic EQs, the geometric description would be as a decay of n-sheeted 4-surface with respect to M^4 to a union of n_i-sheeted 4-surfaces by SSFRs. This would take place for flux tubes mediating all kinds of interactions.

   In gravitational degrees of freedom, that is for trascendental EQs, the states with $n_{gr} = Kp$ having bundles of Kp flux tubes would deca to flux tubes bundles of $n_{gr,i} = K_i p$, where K_i is a prime dividing K. The quantity $log(K)$ would be conserved in the process and is analogous to the corresponding conserved quantity in arithmetic quantum field theories (QFTs) and relates to the notion of infinite prime inspired by TGD [K105].

2. This picture leads to ask whether one could speak of cognitive analogs of particle reactions representing interactions of "thought bubbles" i.e. space-time surfaces as correlates of cognition. The incoming and outgoing states would correspond to a Cartesian product of simple

subgroups: $G = \prod_i^{\times} H_i$. In this composition the order of factors does not matter and the situation is analogous to a many particle system without interactions. The non-commutativity in general case leads to ask whether quantum groups might provide a natural description of the situation.

3. Interestingly, Equivalence Principle is consistent with the splitting of gravitational flux tube structures to smaller ones since gravitational binding energies given by Bohr model in $1/r$ gravitational potential do not depend on the value of $\hbar_{gr}$ if given by Nottale formula $\hbar_{gr} = GMm/v_0$ [L144]. The interpretation would be in terms of spontaneous quantum decoherence taking place as a decay of gravitational flux tube bundles as the distance from the source increases.

19.4.4 Still some questions about $M^8 - H$ duality

There are still on questions to be answered.

1. The map $p^k \rightarrow m^k = \hbar_{eff} p^k/p \cdot p$ defining $M^8 - H$ duality is consistent with Uncertainty Principle but this is not quite enough. Momenta in M^8 should correspond to plane waves in H.

   Should one demand that the momentum eigenstate as a point of cognitive representation associated with $X^4 \subset M^8$ carrying quark number should correspond to a plane wave with momentum at the level of $H = M^4 \times CP_2$? This does not make sense since $X^4 \subset CD$ contains a large number of momenta assignable to fundamental fermions and one does not know which of them to select.

2. One can however weaken the condition by assigning to CD a 4-momentum, call it P. Could one identify P as

   (a) the total momentum assignable to either half-cone of CD

   (b) or the sum of the total momenta assignable to the half-cones?

The first option does not seem to be realistic. The problem with the latter option is that the sum of total momenta is assumed to vanish in ZEO. One would have automatically zero momentum planewave. What goes wrong?

1. Momentum conservation for a single CD is an ad hoc assumption in conflict with Uncertainty Principle, and does not follow from Poincare invariance. However, the sum of momenta vanishes for non-vanishing planewave when defined in the entire M^4 as in QFT, not for planewaves inside finite CDs. Number theoretic discretization allows vanishing in finite volumes but this involves finite measurement resolution.

2. Zero energy states represent scattering amplitudes and at the limit of infinite size for the large CD zero energy state is proportional to momentum conserving delta function just as S-matrix elements are in QFT. If the planewave is restricted within a large CD defining the measurement volume of observer, four-momentum is conserved in resolution defined by the large CD in accordance with Uncertainty Principle.

3. Note that the momenta of fundamental fermions inside half-cones of CD in H should be determined at the level of H by the state of a super-symplectic representation as a sum of the momenta of fundamental fermions assignable to discrete images of momenta in $X^4 \subset H$.

$M^8 - H$-duality as a generalized Fourier transform

This picture provides an interpretation for $M^8 - H$ duality as a generalization of Fourier transform.

1. The map would be essentially Fourier transform mapping momenta of zero energy as points of $X^4 \subset CD \subset M^8$ to plane waves in H with position interpreted as position of CD in H. CD and the superposition of space-time surfaces inside it would generalize the ordinary Fourier

transform . A wave function localized to a point would be replaced with a superposition of space-time surfaces inside the CD having interpretation as a perceptive field of a conscious entity.

2. $M^8 - H$ duality would realize momentum-position duality of wave mechanics. In QFT this duality is lost since space-time coordinates become parameters and quantum fields replace position and momentum as fundamental observables. Momentum-position duality would have much deeper content than believed since its realization in TGD would bring number theory to physics.

How to describe interactions of CDs?

Any quantum coherent system corresponds to a CD. How can one describe the interactions of CDs? The overlap of CDs is a natural candidate for the interaction region.

1. CD represents the perceptive field of a conscious entity and CDs form a kind of conscious atlas for M^8 and H. CDs can have CDs within CDs and CDs can also intersect. CDs can have shared sub-CDs identifiable as shared mental images.

2. The intuitive guess is that the interactions occur only when the CDs intersect. A milder assumption is that interactions are observed only when CDs intersect.

3. How to describe the interactions between overlapping CDs? The fact the quark fields are induced from second quantized spinor fields in in H *resp.* M^8 solves this problem. At the level of H, the propagators between the points of space-time surfaces belonging to different CDs are well defined and the systems associated with overlapping CDs have well-defined quark interactions in the intersection region. At the level of M^8 the momenta as discrete quark carrying points in the intersection of CDs can interact.

Zero energy states as scattering amplitudes and subjective time evolution as sequence of SSFRs

This is not yet the whole story. Zero energy states code for the ordinary time evolution in the QFT sense described by the S-matrix. What about subjective time evolution defined by a sequence of "small" state function reductions (SSFRs) as analogs of "weak" measurements followed now and then by BSFRs? How does the subjective time evolution fit with the QFT picture in which single particle zero energy states are planewaves associated with a fixed CD.

1. The size of CD increases at least in statistical sense during the sequence of SSFRs. This increase cannot correspond to M^4 time translation in the sense of QFTs. Single unitary step followed by SSFR can be identified as a scaling of CD leaving the passive boundary of the CD invariant. One can assume a formation of an intermediate state which is quantum superposition over different size scales of CD: SSFR means localization selecting single size for CD. The subjective time evolution would correspond to a sequence of scalings of CD.

2. The view about subjective time evolution conforms with the picture of string models in which the Lorentz invariant scaling generator L_0 takes the role of Hamiltonian identifiable in terms of mass squared operator allowing to overcome the problems with Poincare invariance. This view about subjective time evolution also conforms with super-symplectic and Kac-Moody symmetries of TGD.

   One could perhaps say that the Minkowski time T as distance between the tips of CDs corresponds to exponentiated scaling: $T = exp(L_0 t)$. If t has constant ticks, the ticks of T increase exponentially.

The precise dynamics of the unitary time evolutions preceding SSFRs has remained open.

1. The intuitive picture that the scalings of CDs gradually reveal the entire 4-surface determined by polynomial P in M^8: the roots of P as "very special moments in the life of self" would correspond to the values of time coordinate for which SSFRs occur as one new root emerges.

These moments as roots of the polynomial defining the space-time surface would correspond to scalings of the size of both half-cones for which the space-time surfaces are mirror images. Only the upper half-cone would be dynamical in the sense that mental images as sub-CDs appear at "geometric now" and drift to the geometric future.

2. The scaling for the size of CD does *not* affect the momenta associated with fermions at the points of cognitive representation in $X^4 \subset M^8$ so that the scaling is not a genuine scaling of M^4 coordinates which does not commute with momenta. Also the fact that L_0 for super symplectic representations corresponds to mass squared operator means that it commutes with Poincare algebra so that M^4 scaling cannot be in question.

3. The Hamiltonian defining the time evolution preceding SSFR could correspond to an exponentiation of the sum of the generators L_0 for super-symplectic and super-Kac Moody representations and the parameter t in exponential corresponds to the scaling of CD assignable to the replaced of root r_n with root r_{n+1} as value of M^4 linear time (or energy in M^8). L_0 has a natural representation at light cone boundaries of CD as scalings of light-like radial coordinate.

4. Does the unitary evolution create a superposition over all over all scalings of CD and does SSFR measure the scale parameter and select just a single CD?

   Or does the time evolution correspond to scaling? Is it perhaps determined by the increase of CD from the size determined by the root r_n as "geometric now" to the root r_{n+1} so that one would have a complete analogy with Hamiltonian evolution? The scaling would be the ratio r_{n+1}/r_n which is an algebraic number.

   Hamiltonian time evolution is certainly the simplest option and predicts a fixed arrow of time during SSFR sequence. L_0 identifiable essentially as a mass squared operator acts like conjugate for the logarithm of the logarithm of light-cone proper time for a given half-cone.

   One can assume that L_0 as the sum of generators associated with upper and lower half-cones if the fixed state at the lower half-cone is eigenstate of L_0.

How does this picture relate to p-adic thermodynamics in which thermodynamics isdetermined by partition function which would in real sector be regarded as a vacuum expectation value of an exponential $exp(iL_0t)$ of a Hamiltonian for imaginary time $t = i\beta$ $\beta = 1/T$ defined by temperature. L_0 is proportional to mass squared operator.

1. In p-adic thermodynamics temperature T is dimensionless parameter and $\beta = 1/T$ is integer valued. The partition function as exponential $exp(-H/T)$ is replaced with $p^{\beta L_0)}$, $\beta = n$, which has the desired behavior if L_0 has integer spectrum. The exponential form $e^{L_0/T_R)}$, $\beta_R = nlog(p)$ equivalent in the real sector does not make sense p-adically since the p-adic exponential function has p-adic norm 1 if it exists p-adically.

2. The time evolution operator $exp(-iL_0t)$ for SSFRs (t would be the scaling parameter) makes sense for the extensions of p-adic numbers if the phase factors for eigenstates are roots of unity belonging to the extension. $t = 2\pi k/n$ since L_0 has integer spectrum. SSFRs would define a clock. The scaling $exp(t) = exp(2\pi k/n)$ is however not consistent with the scaling by r_{n-1}/r_n.

   Both the temperature and scaling parameter for time evolution by SSFRs would be quantized by number theoretical universality. p-Adic thermodynamics could have its origins in the subjective time evolution by SSFRs.

3. In the standard thermodynamics it is possible to unify temperature and time by introducing a complex time variable $\tau = t + i\beta$, where $\beta = 1/T$ is inverse temperature. For the space-time surface in complexified M^8, M^4 time is complex and the real projection defines the 4-surface mapped to H. Could thermodynamics correspond to the imaginary part of the time coordinate?

   Could one unify thermodynamics and quantum theory as I have indeed proposed: this proposal states that quantum TGD can be seen as a "complex square root" of thermodynamics. The exponentials $U = exp(\tau L_0/2)$ would define this complex square root and thermo-dynamical partition function would be given by $UU^\dagger = exp(-\beta L_0)$.

19.5 Can one construct scattering amplitudes also at the level of M^8?

$M^8 - H$ duality suggests that the construction is possible both at the level of H and M^8. These pictures would be based on differential geometry on one hand and algebraic geometry and number theory on the other hand. The challenge is to understand their relationship.

19.5.1 Intuitive picture

H picture is phenomenological but rather detailed and M^8 picture should be its pre-image under $M^8 - H$ duality. The following general questions can be raised.

1. Can one construct the counterparts of the scattering amplitudes also at the level of M^8?
2. Can one use $M^8 - H$ duality to map scattering diagrams in M^8 to the level of H?

Consider first the notions of CD and sub-CD.

1. The intuitive picture is that at the level of H that one must surround partonic vertices with sub-CDs, and assign the external light-like momenta with the ends of propagator lines from the boundaries of CD and other sub-CDs. The incoming momenta $\overline{p}^k$ would be assigned to the boundary of sub-CD.
2. What about the situation in M^8? Sub-CDs must have different origin in the general case since the momentum spectrum would be shifted. Therefore the sub-CDs have the same tip - either upper or lower tip, and have as their boundary part of either boundary of CD. A hierarchy of CDs associated with the same upper or lower tip is suggestive and the finite maximal size of CD in H gives IR cutoff and the finite maximal size of CD in M^8 gives UV cutoff.
3. Momentum conservation at the vertices in M^8 could decompose the diagram to sub-diagrams for which the momentum conservation is satisfied. On the basis of QFT experience, one expects that there are some minimal diagrams from which one can construct the diagram: in the TGD framework this diagram would describe 4-quark scattering. The condition that the momenta belong to the extension of rationals gives extremely strong constraints and it is not clear that one obtains any solutions to the conditions unless one poses some conditions on the polynomials assigned with the two boundaries of CD.

   The two half-cones (HCs) of CD contain space-time surfaces in M^8 as roots of polynomials $P_1(o)$ and $P_2(2T-o)$ which need not be identical. The simplest solution is $P_2(o) = P_1(2T-o)$: the space-time surfaces at HCs would be mirror images of each other. This gives $P_1(T, Im_R(o)) = P_1(T - Im_R(o))$ Since P_1 depends on $t^2 - r^2$ only, the condition is identically satisfied for both options.

   There are two options for the identification of the coordinate t.

   Option (a): t is identified as octonionic real coordinate o_R identified and also time coordinate as in the original option. In the recent option octonion o_R would correspond to the Euclidian analog of time coordinate. The breaking of symmetry from $SO(4)$ to $SO(3)$ would distinguish t as a Newtonian time. The M^4 projection of CD_8 gives a union of future and past directed light-cones with a common tip rather than CD_4 in M^4 at the level of M^8 . Both incoming and outgoing momenta have the same origin automatically. This identification seems to be the natural one at the level of M^8.

   Option (b): t is identified as a Minkowski time coordinate associated with the imaginary unit I_1 in the canonical decomposition $\{I_1, iI_3, iI_5, iI_7\}$. The HC at $o = 0$ would be shifted to $O = (0, 2T, 0...0)$ and reverted. M^4 projection would give CD_4 so that this option is consistent with ZEO. This option is natural at the level of Hbut not at the level of M^8.

   If Option (a) is realized at the level of M^8 and Option b) at the level of H, as seems natural, a time translation of the past directed light-cone by T in $M^4 \subset H$ is required to give CD_4. The momentum spectra of the two HCs differ only by sign and at least a scattering diagram in which all points are involved is possible. In fact all the pairs of subsets with opposite momenta are allowed. These however correspond to a trivial scattering. The decomposition to say 4-vertices with common points involving momentum space propagator suggests a decomposition into sub-CDs. The smaller the sub-CDs at the tips of the CD, the smaller the momenta are and the better is the IR resolution.

4. The proposal has been that one has a hierarchy of discrete size scales for the CDs. Momentum conservation gives a constraint on the positions of quarks at the ends of propagator lines in M^8 mapped to a constraint for their images in H: the sum of image points in H is however not vanishing since inversion is not a linear map.

5. QFT intuition would suggest that at the level of M^8 the scattering diagrams decompose to sub-diagrams for which momentum conservation is separately satisfied. If two such sub-diagrams A and B have common momenta, they correspond to internal lines of the diagram involving local propagator D_p, whose non-local counterpart at the level of H connects the image point to corresponding point of all copies of B.

   The usual integral over the endpoint of the propagator line $D(x, y)$ at space-time level should correspond to a sum in which the H image of B is shifted in M^4. Introduction of a large number of copies of H image of the sub-diagram looks however extremely ugly and challenges the idea of starting from the QFT picture.

   What comes in mind is that all momenta allowed by cognitive representation and summing up to zero define the scattering amplitude as a kind of super-vertex and that Yanigian approach allows this construction.

19.5.2 How do the algebraic geometry in M^8 and the sub-manifold geometry in H relate?

Space-time surfaces in H have also Euclidian regions - in particular wormhole contacts - with induced metric having Euclidian signature due to the large CP_2 contribution to the induced metric. They are separated from Minkowskian regions by a light-like 3-surfaces identifiable as partonic orbits at which the induced metric becomes degenerate.

1. The possible M^8 counterparts of these regions are expected to have Euclidian signature of the number theoretic metric defined by complexified octonion inner product, which must be real in these regions so that the coordinates for the canonical basis $\{I_1, iI_3, iI_5, iI_7\}$ are either imaginary or real. This allows several signatures.

2. The first guess is that the energy p^0 assignable to I_1 becomes imaginary. This gives tachyonic p^2. The second guess is that all components of 3-momentum $\{iI_3, iI_5, iI_7\}$ become imaginary meaning that the length of 3-momentum becomes imaginary.

3. One cannot exclude the other signatures, for instance the situation in which 1 or 2 components of the 3-momentum become imaginary. Hence the transition could occur in 3 steps as $(1, -1, -1-, 1) \to (1, 1, -1, -1) \to (1, 1, 1, -1) \to (1, 1, 1, 1)$. The values of $p^2 \equiv Re(p_c^2)$ would be non-negative and also their images in $M^4 \subset H$ would be inside future light-cone. This could relate to the fact that all these signatures are possible inthe twistor Grassmannian approach.

4. These regions belong to the complex mass shell $p_c^2 = r_n = m_0^2 = r_n$ appearing as a root to the co-associativity condition $X = 0$. This gives the conditions

$$\begin{aligned} Re(p_c) \cdot Im(p_c^2) &= Im(r_n) \ , \\ Re(p_c^2) \equiv p^2 &= Im(p_c^2) + m_n^2 \ , \\ m_n^2 &\equiv Re(r_n) \geq 0 \ . \end{aligned} \tag{19.5.1}$$

Consider first the case $(1,1,1,1)$.

1. The components of p_c are either real or imaginary. Using the canonical basis $\{I_1, iI_3, iI_5, iI_7\}$ the components of p_c are real in the Minkowskian region and imaginary in the totally time-like Euclidian region. One has for the totally time-like momentum $p = (p_0, iIm(p_3))$ in the canonical basis.

   This would give

$$Re(p_c^2) \equiv p^2 = p_0^2 = -Im(p_3)^2 + m_n^2 \quad . \tag{19.5.2}$$

   The number theoretic metric is Euclidian and totally time-like but one has $p^2 \geq 0$ in the range $[m_0^2, 0]$. This region is a natural counterpart for an Euclidian space-time region in H. The region $p^2 \geq m_0^2$ has Minkowskian signature and counterpart for Minkowskian regions in H. The region $0 \leq p^2 < m_0^2$ is a natural candidate for an Euclidian region in M^4.

   Remark: A possible objection is that Euclidian regions in O_c are totally time-like and totally space-like in H.

2. The image of these regions under the map $Re(p^k) \rightarrow M^k$ under inversion plus octonionic conjugation defined as $p^k \rightarrow \hbar_{eff}\overline{p^k}/p^2$ (to be discussed in more detail in the sequel) consists of points M^k in the future light-cone of $M^4 \subset H$. The image of the real Euclidian region of O_c with $p^2 \in [0, m_0^2)$ is mapped to the region $M^k M_k < \hbar_{eff}^2/m_0^2$ of $M^4 \subset H$.

3. The contribution of CP_2 metric to the induced metric is space-like so that it can become Euclidian. This would naturally occur in the image of a totally time-like Euclidian region and this region would correspond to small scales $M^k M_k < \hbar_{eff}^2/m_0^2$. The change of the signature should take place at the orbits of partonic 2-surfaces and the argument does not say anything about this. The boundary of between the two regions corresponds to momenta $p = (p_0, 0)$ which is is a time-like line perhaps identifiable as the analog of the light-like geodesic defining the M^4 projection of CP_2 type extremal, which is an idealized solution to actual field equations.

This transition does not explain the M^8 counterpart of the 3-D light-like partonic orbit to which the light-light geodesic thickens in the real situation?

The above argument works also for the other signatures of co-associative real sub-spaces and provides additional insights. Besides the Minkowskian signature, 3 different situations with signatures $(1,1,1,1)$, $(1,-1,1,1)$, and $(1,-1-1,1)$ with non-space-like momentum squared are possible.

The following formulas list the signatures, the expressions of real momentum squared, and dimension D of the transition transition $Im(p_c^2) = 0$ as generalization of partonic orbit and the possible identification of the transition region.

$$\begin{array}{lll}
\textbf{Signature} & p^2, & D \\
(+,-,-,+): & (p^0)^2 - (p^1)^2 - (p^2)^2 = -Im(p^3)^2 + m_n^2 & 3 \ , \\
\textbf{Identification} & \text{partonic orbit} & . \\
 & & \\
\textbf{Signature} & p^2 & D \\
(+,-,+,+): & (p^0)^2 - (p^1)^2 = -Im(p^2)^2 - Im(p^3)^2 + m_n^2 \ , & 2 \ , \\
\textbf{Identification} & \text{string world sheet} & . \\
 & & \\
\textbf{Signature} & p^2 & D \\
(+,+,+,+): & (p^0)^2 = -Im(p^1)^2 - Im(p^2)^2 - Im(p^3)^2 + m_n^2 \ , & 1 \ . \\
\textbf{Identification} & \text{string boundary} & .
\end{array} \tag{19.5.3}$$

Since the map of the co-associative normal space to CP_2 does not depend on the signature, $M^8 - H$ duality is well defined for all these signatures. One can ask whether a single transition creates partonic orbit, two transitions a string world sheet and 3 transitions ends of string world sheet inside partonic orbit or even outside it.

19.5.3 Quantization of octonionic spinors

There are questions related to the quantization of octonionic spinors.

1. Co-associative gamma matrices identified as octonion units are associative with respect to their octonionic product so that matrix representation is possible. Do second quantized octonionic spinors in M^8 make sense? Is it enough to second quantize them in M^4 as induced octonionic spinors? Are the anti-commutators of oscillator operators Kronecker deltas or delta functions in which case divergence difficulties might be encountered? This is not needed since the momentum space propagators can be identified as those for E^8_c restricted to X^4_r as a subspace with real octonion norm.

   The propagators are just massless Dirac propagators for the choice of M^4 for which light-like M^8 momentum reduces to M^4 momentum. Could one formulate the scattering amplitudes using only massless inverse propagators as in the twistor Grassmannian approach?This does not seem to be the case.

2. Could the counterpart of quark propagator as inverse propagator in M^8 as the idea about defining momentum space integrals as residue integrals would suggest? This would allow on-mass-shell propagation like in twistor diagrams and would conform with the idea that inversion relates M^8 and H descriptions. This is suggested by the fact that no integration over intermediate virtual momenta appears in the graphs defined by the algebraic points of the pre-images of the partonic 2-surfaces X^2_r .

How to identify external quarks? Note that bosons would consist of correlated quark-antiquark pairs with the propagator obtained as a convolution of quark propagators. The correlation would be present for the external states and possibly also for the states in the diagram and produced by topologically.

1. The polynomial P and the $P = 0$ surface with 6-D real projection X^6_r is not affected by octonion automorphisms. Quarks with different states of motion would correspond to the same P but to different choices of M^4 as co-associative subspace for M^8_c. P could be seen as defining a class of scattering diagrams. P determines the vertices.

2. The space-time surface associated with a quark carrying given 4-momentum should be obtainable by a Lorenz transformation in $SO(3,1) \subset G_{2,c}$ to give it light-like M^4 so that complexified octonionic automorphisms would generate 3-surfaces representing particles. If $M^4 \subset M^8$ and thus the CD associated with the quark is chosen suitably, the quark is massless. Any incoming particle would be massless in this frame.

   Lorentz invariance however requires a common Lorentz frame provided by the CD. The momentum of a quark in CD would be obtained by $G_{2,c}$ transformation. In the frame of CD the external quark momenta arriving to the interior of CD at vertices associated with $X^3_r \cap Y^3_r$ are time-like. Momentum conservation would hold in this frame. The difference between massive constituent quarks and massless current quarks could be understood as reflecting M^8 picture.

To sum up, the resulting picture is similar to that at the level of H these diagrammatic structures would be mapped to H by momentum inversion. Quantum classical correspondence would be very detailed providing both configuration space and momentum space pictures.

19.5.4 Does $M^8 - H$ duality relate momentum space and space-time representations of scattering amplitudes?

It would seem that the construction of the scattering amplitudes is possible also at the level of M^8 [L123]. M^8 picture would provide momentum representation of scattering diagrams whereas H picture would provide the space-time representation.

Consider first a possible generalization of QFT picture involving propagators and vertices.

1. At first it seems that it is not possible to talk about propagation at the level of momentum space: in positive energy ontology nothing propagates in momentum space if the propagator

is a free propagator D_p! In ZEO this is not quite so. One can regard annihilation operators as creation operators for the fermionic vacuum associated with the opposite HC of CD (or sub-CD): one has momentum space propagation from p to $-p$! The expressions of bosonic charges would be indeed bi-local with annihilation and creation operators associated with the mirror paired points in the two HCs of CD forming pairs. The momentum space propagator D_p would actually result from the pairing of creation creation operators with the opposite values of p and the notation $D(p,-p)$ would be more appropriate.

2. In QFT interaction vertices are local in space-time but non-local in momentum space. The n-vertex conserves the total momentum. Therefore one should just select points of M^8 and they are indeed selected by cognitive representation and assign scattering amplitude to this set of points. To each point one could assign momentum space propagator of quark in M^8_c but it would not represent propagation! The vertex would be a multilocal entity defined by the vertices defining the masses involved at light cone boundary and mass shells.

   The challenge would be to identify these vertices as poly-local entities. In the QFT picture there would be a set of n-vertices with some momenta common. What could this mean now? One would have subset sets of momenta summing up to zero as vertices. If two subsets have a common momentum this would correspond to a propagator line connecting them. Should one decompose the points of cognitive representation so that it represents momentum space variant of Feynman graph? How unique this decomposition is and do this kind of decompositions exist unless one poses the condition that the total momenta associated with opposite boundaries sum up to zero as done in ZEO. A given n-vertex in the decomposition means the presence of sub-CDs for which the external momenta sum up to zero. This poses very tight constraints on the cognitive representation, and one can wonder they can be satisfied if the cognitive representation is finite as it is in the generic case.

3. Note that for given a polynomial P allowing only points in cognitive representation, one would *not* have momentum space integrations as in QFT: they could however come from integrations over the polynomial coefficients and would correspond to integration of WCW. In adelic picture one allows only rational coefficients for the polynomials. This strongly suggests that the twistor Grasmmannian picture [B25, **?**, B65, B15] in which residue integral in the momentum space gives as residues inverse quark propagators at the poles. M^8 picture would represent the end result of this integration and only on mass shell quarks would be involved. One could even challenge the picture based on propagators and vertices and start from Yangian algebra based on the generalization of local symmetries to multilocal symmetries [A25, A82] [B22] [L45].

4. In the case of H restriction of the second quantized free quark field of H to space-time surface defines the propagators. In the recent case one would have a second quantized octonionic spinor field in M^8. The allowed modes of H spinor field are just the co-associative modes for fixed selection of M^4 analogous to momentum space spinors and restricted to Y^3_r. One could speak of wave functions at Y^3_r, which is very natural since they correspond to mass shells.

   The induced spinor field would have massless part corresponding to wave functions at the M^4 light-cone boundary and part corresponding to X^3 at which the modes would have definite mass. $P = 0$ would select a discrete set of masses. Could second quantization have the standard meaning in terms of anti-commutation relations posed on a free M^8 spinor field. In the case of M^8_c one avoids normal ordering problems since there is no Dirac action. The anti-commutators however have singularities of type 7-D delta function. The anti-commutators of oscillator operators at the same point are the problem. If only a single quark oscillator operator at a given point of M^8 is allowed since there is no local action in coordinate space with the interaction part producing the usual troubles.

5. Could one perform a second quantization for E^8 spinor field using free Dirac action? Could one restrict the expansion of the spinor field to co-associative space-time surfaces giving oscillator operators at the points of cognitive representation with the additional restriction to the pre-image of given partonic 2-surface, whose identification was already considered. Scattering amplitudes would involve n-vertices consisting of momenta summing up to zero and connected to opposite incoming momenta at the opposite sides of the HCs with the

same tip in M^8. Scattering amplitude would decompose to sub-diagrams defining a cluster decomposition, and would correspond to sub-CDs. The simplest option is that there are no internal propagator lines. The vanishing of the total momenta poses stringent conditions on the points of cognitive representation.

Normal ordering divergences can however produce problems for this option in the case of bosonic charges bilear in oscillator operators. At the level of H the solution came from a bilocal modified Dirac action leading to bilocal expressions for conserved charges. Now Yangian symmetry suggests a different approach: local vertices in momentum space can involve only commuting oscillator operators.

Indeed, in ZEO one can regard annihilation operators as creation operators for the fermionic vacuum associated with the opposite HC of CD (or sub-CD). The expressions of bosonic charges would be indeed bi-local with annihilation and creation operators associated with the mirror paired points in the two HCs of CD forming pairs. As already noticed, also the momentum space propagator $D_p = D(p, -p)$ would be also a bi-local object.

6. This is not enough yet. If there is only a single quark at given momentum, genuine particle creation is not possible and the particle reactions are only re-arrangements of quarks but already allowing formation of bosons as bound states of quarks and antiquarks. Genuine particle creation demands local composites of several quarks at the same point p having interpretation as a state with collinear momenta summing up to p and able to decay to states with the total momentum p. This suggests the analog of SUSY proposed in [L92]. Also Yangian approach is highly suggestive.

   To sum up, momentum conservation together with the assumption of finite cognitive representations is the basic obstacle requiring new thinking.

19.5.5 Is the decomposition to propagators and vertices needed?

One can challenge the QFT inspired picture.

1. As already noticed, the relationship $P_1(t) = P(2T - t)$ makes it possible to satisfy this condition at least for the entire set of momenta. This does not yet allow non-trivial interactions without posing additional conditions on the momentum spectrum. This does not look nice. One can ask whether there is a kind of natural selection leading to polynomials defining space-time surfaces allowing cognitive representations with vertex decompositions and polynomials $P(t)$ and $P_r(t)$ without this symmetry? This idea looks ugly. Or could evolution start from simplest surfaces allowing 4 vertices and lead to an engineering of more complex scattering diagrams from these?

2. The map of momentum space propagators regarded as completely local objects in M^8 to H propagators is second ugly feature. The beauty and simplicity of the original picture would be lost by introducing copies of sub-diagrams mapped to the various translations in H.

3. The Noether charges of the Dirac action in H fail to give rise to 4-fermion vertex operator. The theory would be naturally just free field theory if one assumes cognitive representations.

The first heretic question is whether the propagators are really needed at the level of momentum space. This seems to be the case.

1. In ZEO the propagators pair creation and operators with opposite 4-momenta assignable to the opposite HCs of CD having conjugate fermionic vacua (Dirac sea of negative energy fermions and Dirac sea of positive energy fermions) so that momentum space propagators $D(p, -p)$ are non-local objects. The propagators would connect positive and negative energy fermions at the opposite HCs and this should be essential in the formulation of scattering amplitudes. They cannot be avoided.

2. The propagators would result from the contractions of fermion oscillator operators giving a 7-D delta function at origin in continuum theory. This catastrophe is avoided in the

number theoretic picture. Since one allows only points with M^8 coordinates in an extension of rationals, one can assume Kronecker delta type anti-commutators. Besides cognitive representations, this would reflect the profound difference between momentum space and space-time.

This would also mean that the earlier picture about the TGD analog of SUSY based on local composites of oscillator operators [L92] makes sense at the level of M^8. The composites could be however local only for oscillator operators associated with the HC of CD. With the same restriction they could be local also in the$H$ picture.

What about vertices? Could Yangian algebra give directly the scattering amplitudes? This would simplify dramatically the $M^8 - H$ duality for transition amplitudes. For this option the $P_1(t) = P(2T - t)$ option required by continuity would be ideal.

1. Without vertices the theory would be a free field theory. The propagators would connect opposite momenta in opposite HCs of CD. Vertices are necessary and they should be associated with sub-CDs. Unless sub-CDs can have different numbers of positive and negative energy quarks at the opposite HCs, the total quark number is the same in the initial and final states if quarks and antiquarks associated with bosons as bound states of fermion and antiquark are counted. This option would require minimally 4-quark vertex having 2 fermions of opposite energies at the two hemi-spheres of the CD. A more general option looks more plausible. One obtains non-trivial scattering amplitudes by contracting fermions assigned to the boundary P (F) past (future) HC of CD to the past (future) boundary P_{sub} (F_{sub}) of a sub-CD. Sub-CD and CD must have an opposite arrow of time to get the signs of energies correctly.

   Sub-CDs would thus make particle creation and non-trivial scattering possible. There could be an arbitrary number of sub-CDs and they should be assignable to the pre-images of the partonic 2-surfaces X^2_r if the earlier picture is correct. The precise identification of the partonic 2-surfaces is still unclear as also the question whether light-like orbits of partonic 2-surfaces meet along their ends in the vertices.

2. As in the case of H, one could assign the analogs of n-vertices at pre-images of partonic 2-surfaces at X^2_r representing the momenta of massive modes of the octonionic Dirac equation and belonging to the cognitive representations. The idea is to use generators of super-Yangian algebra to be discussed later which are both bosonic and fermionic. The simplest construction would assign these generators to the vertices as points in cognitive representation.

   An important point is that Yangian symmetry would be a local symmetry at the level of momentum space and correspond to non-local symmetry at the level of space-time rather than vice versa as usually. The conserved currents would be local composites of quark oscillator operators with same momentum just as they are in QFTs at space-time level representing parallelly propagating quarks and antiquarks.

   The simplest but not necessary assumption is that they are linear and bilinear in oscillator operators associated with the same point of M^8 and thus carrying 8-momenta assignable to the modes of E^8 spinor field and restricted to the co-associative 4-surface. Their number of local composites is finite and corresponds to the number 8 of different states of 8-spinors of given chirality.

   Also a higher number of quarks is possible, and this was indeed suggested in [L92]. The proposal was that instance leptons would correspond to local composites of 3 quarks. The TGD based view about color allows this. These states would be analogous to the monomials of theta parameters in the expansion of super-field. The H picture allows milder assumptions: leptonic quarks reside at partonic 2-surface at different points but this is not necessary.

3. Instead of super-symplectic generators one has $G_{2,c}$ as the complexified automorphism group. Also the Galois group of the extension acts as an automorphism group and is proposed to have a central role in quantum TGD with applications to quantum biology [L42, L121]. As found, $G_{2,c}$ acts as an analog of gauge or Kac-Moody group. Yangian has analogous structure but the analogs of conformal weights are non-negative.

4. The identification of the analogs of the poly-local vertex operators as produces of charges generators associated with FHC anbd PHC is the basic challenge. They should consist of quark creation operators (annihilation operators being associated as creation operators at the opposite HC) and be generators of infinitesimal symmetries which in number theoretic physics would correspond instead of isometries of WCW to the octonionic automorphism group G_2 complexified to $G_{2,c}$ containing also the generators of $SO(4) \subset G_2$ and thus also those of Lorentz group $SO(1,3) \subset G_{2,c}$.

   The construction Noether charges of E^8 second quantized spinor field at momentum space representation gives bilinear expressions in creation and annihilation operators associated with opposite 3-momenta and would have a single fermion in a given HC. This is not enough: there should be at least 4 fermions.

What strongly suggests itself are Yangian algebras [A25] [L45] having poly-local generators and considered already earlier and appearing in the twistor Grassmannian approach [B25, **?**]. The sums of various quantum numbers would vanish for the vertex operators. These algebras are quantum algebras and the construction of n-vertices could involve co-algebra operation. What is new as compared to Lie algebras is that Yangian algebras are quantum algebras having co-algebra structure allowing to construct n-local generators representing scattering amplitudes. It might be possible replace oscillator operators with the quantum group counterparts.

19.5.6 Does the condition that momenta belong to cognitive representations make scattering amplitudes trivial?

Yangian symmetry is associated with 2-D integrable QFTs which tend to be physically rather uninteresting. The scattering is in the forward direction and only phase shifts are induced. There is no particle creation. If the relationship $P_1(t) = P(2T - t)$ is applied the momentum spectra for FHC and PHC differ only by the sign. If all momenta are involved and the cognitive representations are finite, the situation would be the same! Also the existence of cluster compositions involving summations of subsets of momenta to zero is implausible. Something seems to go wrong!

The basic reason for the problem is the assumption that the momenta belong to cognitive representations assumed to be finite as they indeed are in the generic case. But are they finite in the recent situation involving symmetries?

1. The assumption that all possible momenta allowed by cognitive representation are involved, allows only forward scattering unless there are several subsets of momenta associated with either HC such that the momenta sum-up to the same total momentum. This would allow the change of the particle number. The subsets S_i with same total momentum p_{tot} in the final state could save as final states of subsets S_j with the same total momentum p in the initial state. What could be the number theoretical origin of this degeneracy?

2. In the generic case the cognitive representation contains only a finite set of points (Fermat theorem, in which one considers rational roots of $x^n + y^n = z^n$, $n > 2$ is a basic example of this) . There are however special cases in which this is not true. In particular, M^4 and its geodesic sub-manifolds provide a good example: all points in the extension of rationals are allowed in M^4 coordinates (note that there are preferred coordinates in the number theoretic context).

   The recent situation is indeed highly symmetric due to the Lorentz invariance of space-time surfaces as roots reducing the equations to ordinary algebraic equations for a single complex variable. $X = 0$ condition gives as a result $a_c^2 = constant$ complex hyperboloid with a real mass hyperboloid as a real projection. $a_c^2 = r_n$ is in the extension of rationals as a root of n:th order polynomial. One has the condition $Re(m^2)^2 - Im(m^2) = Re(r_n)$ giving X_r^4 a slicing by real mass hyperboloids. If $Im(m)$ and the spatial part of $Re(m)$ belongs to the extension, one has for real time coordinate $t = \sqrt{r_M^2 + Im(m^2) + r_n}$. If $r_M^2 + Im(m)^2 + r_n$ is a square in the extension also t belongs to the extension. Cognitive representation would contain an infinite number of points and the it would be possible to have non-trivial cluster decompositions. Scattering amplitude would be a sum over different choices of the momenta of the external particles satisfying momentum conservation condition.

As found, the intersection of X_r^4 and X_r^6 is either empty or X_r^4 belongs to X_r^6, Cognitive representations would have an infinite number of points also now by the previous argument. Partonic 2-surfaces at X_r^3 would be replaced with 3-D surfaces in X_r^4 in this situation and would contain a large number of roots. The partonic 2-surfaces would be still present and correspond to the intersections of incoming space-time surfaces of quarks inside X_r^6. These surfaces would also contain the vertices.

3. Could number theoretic evolution gradually select space-time surfaces for which the number theoretic dynamics involving massive quarks is possible? First would be generic polynomials for which X_r^3 would be empty and only massless quarks arriving at the light-cone boundary would be possible. After that surfaces allowing non-empty X_r^3 and massive quarks would appear. There is a strong resemblance with the view about cosmological evolution starting from massless phases and proceeding as a sequence of symmetry breakings causing particle massivation. Now the massivation would not be caused by Higgs like fields but have purely number theoretic interpretation and conform with the p-adic mass calculations [K65].

   Also a cognitive explosion would occur since these space-time surfaces would be cognitively superior after the emergence of massive quarks. If this picture has something to do with reality, the space-time surfaces contributing to the scattering amplitudes would be very special and interactions could be seen as a kind of number theoretical resonance phenomenon.

4. Even is not enough to obtain genuine particle reaction instead of re-arrangements: one must have also local composites of collinear quarks at the same momentum p identifiable as the sum of parallel momenta discussed in [L92]. This kind of situation is also encountered for on-mass-shell vertices in twistor Grassmannian approach. The local composites could decay to local composites with a smaller number of quarks but respecting momentum conservation. Here the representations of Yangian algebra would come in rescue.

19.5.7 Momentum conservation and on-mass-shell conditions for cognitive representations

Momentum conservation and on-mass shell-conditions are very powerful for cognitive representations, which in the generic case are finite. At mass shells the cognitive representations consist of momenta in the extension of rationals satisfying the condition $p^2 = Re(r_n)$, r_n a complex root of X, which is polynomial of degree n in p^2 defined by the odd part of P. If $\sqrt{Re(r_n)}$ does not belong to the extension defined by P, it can be extended to contain also $\sqrt{Re(r_n)}$.

For Pythagorean triangles in the field of rationals, mass shell condition gives for the momentum components in extension an equation analogous to the equation $k^2 + l^2 = m^2$, which can be most easily solved by noticing that the equation has rotation group $SO(2)$ consisting of rational rotation matrices as symmetries. The solutions are of form $(k = r^2 - s^2, l = 2rs, m = r^2 + s^2)$. By $SO(2)$ invariance, one can choose the coordinate frame so that one has $(k, l) = (r^2 + s^2, 0)$. By applying to this root a rational rotation with $cos(\phi) = (r^2 - s^2)/(r^2 + s^2), sin(\phi) = 2rs/(r^2 + s^2)$ to obtain the general solution $(k = r^2 - s^2, l = 2rs, n = r^2 + s^2)$. The expressions for k and l can be permuted, which means replacing ϕ with $\phi - pi/2$. For a more general case $k^2 + l^2 = n$ one can replace n with $\sqrt{n}$ so that one has an extension of rationals.

For the hyperbolic variants of Pythagorean triangles, one has $k^2 - l^2 = m^2$ or equivalently $l^2 + m^2 = k^2$ giving a Pythagorean triangle. The solution is $k = r^2 + s^2, l = r^2 - s^2, m^2 = 2rs$. The expressions for l and ma can be permuted. Rotation is replaced with 2-D Lorentz boost $cosh(\eta) = (r^2 + s^2)/(r^2 - s^2)$ and $sinh(\eta) = 2rs/(r^2 - s^2)$ with rational matrix elements.

Consider now the 4-D case.

1. The algebra behind the solution depends in no manner on the number field considered and makes sense even for the non-commutative case if m and n commute. Hence one can apply the Pythagorean recipe also in 4-D case to the extension of rationals defined by P by adding to it $\sqrt{r_n}$.

2. Assume that a Lorentz frame can be chosen to be the rest frame in which one has $p = (E = \sqrt{Rer_n}, 0)$ (this might not be possible always). As in the Pythagorean case, there must be a consistency condition. Now it would be of form $E = \sqrt{r_n} = p_0^2 - p_1^2 - p_2^2 - p_3^2$ in the

extension defined by $\sqrt{r_n}$. It is not clear whether this condition can be solved for all choices of momentum components in the extension or assuming that algebraic integers of extension are in question. One can also consider an option in which one has algebraic integer divided by some integer N. p-Adic considerations would suggest that prime powers $N = p^k$ might be interesting.

The solutions $\sqrt{r_n} = p_1^2 - p_2^2$ represent a special case. The general solution is obtained by making Lorenz transformation with a matrix with elements in the discrete subgroup of Lorentz group with matrix elements in the extension of rationals.

3. The solutions would define a discretization of the mass shell (3-D hyperbolic space) defined as the orbit of the infinite discrete subgroup of $SO(1,3)$ considered - perhaps the subgroup of $SL(2,C)$ with matrix elements identified as algebraic integers.

If the entire subgroup of SL(2,C) with matrix elements in the extension of rationals is realized, the situation would correspond effectively to a continuous momentum spectrum for infinite cognitive representations. The quantization of momenta is however physically a more realistic option.

1. An interesting situation corresponds to momenta with the same time component, in which case the group would be a discrete subgroup of $SO(3)$. The finite discrete symmetry subgroups act as symmetries of Platonic solids and polygons forming the ADE hierarchy associated to the inclusions of hyperfinite factors of type II_1 and proposed to provide description of finite measurement resolution in TGD framework.

2. The scattering would be analogous to diffraction and only to the directions specified by the vertices of the Platonic solid. Platonic solids, in particular, icosahedron appear also in TGD inspired quantum biology [L19, L117], and also in Nature. Could their origin be traced to $M^8 - H$ duality mapping the Platonic momentum solids to H by inversion?

A more general situation would correspond to the restriction to a discrete non-compact sub-group $\Gamma \subset SL(2,C)$ with matrix elements in the extension of rationals. $Sl(2,C)$ has a representation as Möbius transformations of upper half-plane H^2 of complex plane acting as conformal transformations whereas the action in H^3 is as isometries. The Möbius transformation acting as isometries of H^2 corresponds to $SL(2,Z)$ having also various interesting subgroups, in particular congruence subgroups.

1. Subgroups Γ of the modular group $SL(2,Z)$ define tessellations (analogs of orindary lattices in a curved space) of both H^2 and H^4. The fundamental domain [A7] (`https://cutt.ly/ahBrtT5`) of the tessellation defined by $\Gamma \subset SL(2,C)$ contains exactly one point at from each orbit of Γ. The fundamental domain is analogous to lattice cell for an Euclidian 3-D lattice.

   Γ must be small enough since the orbits would be otherwise dense just like rationals are a dense sub-set of reals. In the case of rationals this leaves into consideration tje modular subgroup $SL(2,Z)$ or its subgroups. In the recent situation an extension of the modular group allowing matrix elements to be algebraic integers of the extension is natural. Physically this would correspond to the quantization of momentum components as algebraic integers. The tessellation in M^8 and its image in H would correspond to reciprocal lattice and lattice in condensed matter physics.

2. So called uniform honeycombs [A14, A9, A23] (see `https://cutt.ly/xhBwTph`, `https://cutt.ly/lhBwPRc`, and `https://cutt.ly/OhBwUOO`) in H^3 assignable to $SL(2,Z)$ can be regarded as polygons in 4-D space and H^3 takes the roles of sphere S^2 for platonic solids for which the tessellation defined by faces is finite.

   The four regular compact honeycombs in H^3 for which the faces and vertex figures (the faces meeting the vertex) are finite are of special interest physically. In the Schönflies notation characterizing polytopes (tessellations are infinite variants of them) they are labelled by (p,q,r), where p is the number of vertices of face, q is the number of faces meeting at vertex, and s is the number of cells meeting at edge.

The regular compact honeycombs are listed by (5,3,4), (4,3,5), (3,5,3), (5,3,5). For Platonic solids (5,3) characterizes dodecahedron, (4,3) cube, and (3,5) for icosahedron so that these Platonic solids serve as basic building bricks of these tessellations. Rather remarkably, icosahedral symmetries central in the TGD based model of genetic code [L19, L117], characterize cells for 3 uniform honeycombs.

Consider now the momentum conservation conditions explicitly assuming momenta to be algebraic integers. It is natural to restrict the momenta to algebraic integers in the extension of rationals defined by the polynomial P. This allows linearization of the constraints from momentum conservation quite generally.

Pythagorean case allows to guess what happens in 4-D case.

1. One can start from momentum conservation in the Pythagorean case having interpretation in terms of complex integers $p = (r+is)^2 = r^2 - s^2 + 2irs$. The momenta in the complex plane are squares of complex integers $z = r+is$ obtained by map $z \rightarrow w = z^2$ and complex integers. One picks up in the w-plane integer momenta for the incoming and outgoing states satisfying the conservation conditions $\sum_i P_{out,i} = \sum_k P_{in,k}$: what is nice is that the conditions are linear in w-plane. After this one checks whether the inverse images $\sqrt{P_{out,i}}$ and $\sqrt{P_{in,i}}$ are also complex integers.

2. To get some idea about constraints, one can check what CM system for a 2-particle system means (it is not obvious whether it is always possible to find a CM system: one could have massive particles which cannot form a rest system). One must have opposite spatial momenta for $P_1 = (r_1+is_1)^2$ and $P_2 = (r_2+is_2)^2$. This gives $r_{s1} = r_2s_2$. The products r_is_i correspond to different compositions of the same integer N to factors. The values of $r_i^2+si^2$ are different.

3. In hyperbolic case one obtains the same conditions since the roles of $r^2 - s^2$ and $r^2 + s^2$ in the conditions are changed so that $r^2 - s^2$ corresponds now to mass mass mass and differs for different decomposition of N to factors. The linearization of the conservation conditions generalizes also to the algebraic extensions of rationals with integers replaced by algebraic integers.

The generalization to the 4-D case is possible in terms of octonions.

1. Replace complex numbers by quaternions $q = q_0+\overline{q}$. The square of quaternion is $q^2 = q_0^2-\overline{q}\cdot\overline{q}+2iq_0\overline{q}$. Allowed momenta for given mass correspond to points in q^2-plane. Conservation conditions in the q^2 plane are linear and satisfied by quaternionic integers, which are squares. So that in the q^2 plane the allowed momenta form an integer lattice and the identification as a square selects a subset of this lattice. This generalizes also to the algebraic integers in the extension of rationals.

2. What about the co-associative case corresponding to the canonical basis $\{I_1, iI_3, iI_5, iI_7\}$? Momenta would be as co-associative octonion o but o^2 is a quaternion in the plane defined by $\{I_0, iI_2, iI_4, iI_6\}$. o representable in terms of a complexified quaternion $q = q_0 + i\overline{q}$ as $o = I_4q$ and the in general complex values norm squared is give by $o\overline{o}$ with conjugation of octonionic imaginary units but not i: this gives Minkowskian norm squared. This reduces the situation to the quaternionic case.

3. In this case the CM system for two-particle case corresponds to the conditions $q_{1,0}\overline{q}_1 = q_{2,0}\overline{q}_2$ implying that q_1 and q_2 have opposite directions and $q_{1,0}|\overline{q}_1| = q_{2,0}|\overline{q}_2|$. The ratio of the lengths of the momenta is integer. Now the squares $q_{i,0}|\overline{q}_i|^2$, $i = 1,2$ are factorizations of the same integer N. Masses are in general different.

4. The situation generalizes also to complexified quaternions - the interpretation of the imaginary part of momentum might be in terms of a decay width - and even to general octonions since associativity is not involved with the conditions.

19.5.8 Further objections

The view about scattering amplitudes has developed rather painfully by objections creating little shocks. The representation of scattering amplitudes is based on quark oscillator operator algebra. This raises two further objections.

The non-vanishing contractions of the oscillator operators are necessary for obtaining non-trivial scattering amplitudes but is this condition possible to satisfy.

1. One of the basic deviations of TGD from quantum field theories (QFTs) is the hypothesis that all elementary particles, in particular bosons, can be described as bound states of fermions, perhaps only quarks. In TGD framework the exchange of boson in QFT would mean an emission of a virtual quark pair and its subsequent absorption. In ZEO in its basic form this seems to be impossible.

2. If scattering corresponds to algebra morphism mapping products to products of co-products - the number of quarks in say future HC is higher than in the past HC as required. But how to obtain non-vanishing scattering amplitudes? There should be non-vanishing counterparts of propagators between points of FHC but this is not possible if only creation operators are present in a given HC as ZEO requires. All particle reactions would be re-arrangements of quarks and antiquarks to elementary fermions and bosons (OZI rule of the hadronic string model: `https://en.wikipedia.org/wiki/OZI_rule`). The emission of virtual or real bosons requires the creation of quark antiquark pairs and seems to be in conflict with the OZI rule.

3. It would be natural to assign to quarks and bosons constructed as their bound states non-trivial inner product in a given HC of CD. Is this possible if the counterparts of annihilation operators act as creation operators in the opposite HC? Can one assign inner product to a given boundary of CD by assuming that hermitian conjugates of quark oscillator operators act in the dual Hilbert space of the quark Fock space? Could this dual Hilbert space relate to the Drinfeld's double?

How could one avoid the OZI rule?

1. Is it enough to also allow annihilation operators in given HC? Bosonic $G_{2,c}$ generators could involve them. The decay of boson to quark pair would still correspond to re-arrangement but one would have inner product for states at given HC. The creation of bosons would still be a problem. Needless to say, this option is not attractive.

2. A more plausible solution for this problem is suggested by the phenomenological picture in which quarks at the level of H are assigned with partonic 2-surfaces and their orbits, string world sheets, and their boundaries at the orbits of partonic 2-surfaces. By the discussion in the beginning of this section, these surfaces could correspond at the level of M^8 to space-time regions of complexified space-time surface with real number theoretic metric having signature (+,+,-,-), (+,+,+,-), (+,+,+,+) having 2,3, or 4 time-like dimensions. They would allow non-negative values of mass squared and would be separated from the region of Minkowskian signature by a transition region space-time region with dimension $D \in \{3, 2, 1\}$ mapped to CP$_2$.

   In these regions one would have 1, 2, or 3 additional energy like momentum components $p_i = E_i$. E_i. Could the change of sign for E_i transform creation operator to annihilation operator as would look natural. This would give bosonic states with a non-vanishing norm and also genuine boson creation. What forces to take this rather radical proposal seriously that it conforms with the phenomenological picture.

   In this region one could have a non-trivial causal diamond CD with signature (+,+,-,-), (+,+,+,-). For the signature (+,+,+,+) CD reduces to a point with a vanishing four-momentum and would correspond to CP_2 type extremals (wormhole contacts). Elementary fermions and bosons would consist of quarks in regions with signature (+,+,-,-) and (+,+,+,-). It would seem that the freedom to select signature in twistorial amplitude is not mere luxury but has very deep physical content.

One can invent a further objection. Suppose that the above proposal makes sense and allows to assign propagators to a given HC. Does Yangian co-product allow a construction of zero energy states giving rise to scattering amplitudes, which typically have a larger number of particles in the future HC (FHC) than in past HC (PHC) and represent a genuine creation of quark pairs?

1. One can add to the PHC quarks and bosons one-by-one by forming the product super $G(2,c)$ generators assignable to the added particles. To the FHC one would add the product of co-products of these super $G(2,c)$ generators (co-product of product is product of co-products as an algebra morphism).

2. By the basic formula of co-product each addition would correspond to a superposition of two states in FHC. The first state would be the particle itself having suffered a forward scattering. Second state would involve 2 generators of super $G_{2,c}$ at different momenta summing up to that for the initial state, and represent a scattering $q \to q+b$ for a quark in the initial state and scattering $b \to 2b$, $b \to 2b$, or $b \to 2q$ for a boson in the initial state.

   Number theoretic momentum conservation assuming momenta to be algebraic integers should allow processes in which quark oscillator operators are contracted between the states in FHC and PHC or between quarks in the FHC.

3. Now comes the objection. Suppose that the state in PC consists of fundamental quarks. Also the FC containing the product of co-products of quarks must contain these quarks with the same momenta. But momentum conservation does not allow anything else in FC! The stability of quarks is a desirable property in QFTs but something goes wrong! How to solve the problem?

   Also now phenomenological picture comes to the rescue and tells that elementary particles - as opposed to fundamental fermions - are composites of fundamental fermions assignable to flux tubes like structures involving 2 wormhole contacts. In particular, quarks as elementary particles would involve quark at either throat of the first wormhole contact and quark-antiquark pair associated with the second wormhole contact. The state would correspond to a quantum superposition of different multilocal momentum configurations defining multi-local states at M^8 level. The momentum conservation constraint could be satisfied without trivializing the scattering amplitudes since the contractions could occur between different components of the superposition - this would be essential.

   Note also that at H level there can be several quarks at a given wormhole throat defining a multilocal state in M^8: one could have a superposition of these states with different momenta and again different components of the wave function could contract. By Uncertainty Principle the almost locality in H would correspond to strong non-locality in M^8. This could be seen as an approximate variant of the TGD variant of H variant of SUSY considered in [L92].

   Could the TGD variant of SUSY proposed in [L92] but realized at the level of momentum space help to circumvent the objection? Suppose that the SUSY multiplet in M^8 can be created by a local algebraic product possessing a co-product delocalizing the local product of oscillator operators at point p in PC and therefore represents the decay of the local composite to factors with momenta at p_1 and $p - p_1$ in FC. This would not help to circumvent the objection. Non-locality and wave functions in momentum space is needed.

19.6 Symmetries in M^8 picture

19.6.1 Standard model symmetries

Can one understand standard model symmetries in M^8 picture?

1. $SU(3) \subset G_2$ would respect a given choice of time axis as preferred co-associative set of imaginary units ($I_2 \subset \{I_2, iI_3, iIb, iI_7\}$ for the canonical choice). The labels would therefore correspond to the group $SU(3)$. $SU(3)_c$ would be analogous to the local color gauge group in

the sense that the element of local $SU(3)_c$ would generate a complecofied space-time surface from the flat and real M^4. The real part of pure $SU(3)_c$ gauge potential would not however reduce to pure $SU(3)$ gauge potential. Could the vertex factors be simply generators of $SU(3)$ or $SU(3)_c$?

2. What about electroweak quantum numbers in M^8 picture? Octonionic spinors have spin and isospin as quantum numbers and can be mapped to H spinors. Bosons would be bound states of quarks and antiquarks at both sides.

   How could electroweak interactions emerge at the level of M^8? At the level of H an analogous problem is met: spinor connection gives only electroweak spinor connection but color symmetries are isometries and become manifest via color partial waves. Classical color gauge potentials can be identified as projections of color isometry generators to the space-time surface.

   Could electroweak gauge symmetries at the level of M^8 be assigned with the subgroup $U(2) \subset SU(3)$ of $CP_2 = SU(3)/U(2)$ indeed playing the role of gauge group? There is a large number of space-time surfaces mapped to the same surface in H and related by a local $U(2)$ transformation. If this transformation acted on the octonionic spinor basis, it would be a gauge transformation but this is not the case: constant octonion basis serves as a gauge fixing. Also the space-time surface in M^8 changes but preserves its "algebraic shape".

19.6.2 How the Yangian symmetry could emerge in TGD?

Yangian symmetry [A25, A82] appears in completely 2-D systems. The article [B36] (`https://arxiv.org/pdf/1606.02947.pdf`) gives a representation which is easy to understand by a physicist like me whereas the Wikipedia article remains completely incomprehensible to me.

Yangian symmetry is associated with 2-D QFTs which tend to be physically rather uninteresting. The scattering is in forward direction and only phase shifts are induced. There is no particle creation. Yangian symmetry appears in 4-D super gauge theories [B22] and in the twistor approach to scattering amplitudes [B23, B29, B25, ?]. I have tried to understand the role of Yangian symmetry in TGD [L45].

Yangian symmetry from octonionic automorphisms

An attractive idea is that the Yangian algebra having co-algebra structure could allow to construct poly-local conserved charges and that these could define vertex operators in M^8.

1. Yangian symmetry appears in 2-D systems only. In TGD framework strings world sheets could be these systems as co-commutative 2-surfaces of co-associative space-time surface.

2. What is required is that there exists a conserved current which can be also regarded as a flat connection. In TGD the flat connection would a connection for $G_{2,c}$ or its subgroup associated with the map taking standard co-associative sub-space of O_c for which the number theoretic norm squared is real and has Minkowski signature (M^4 defined by the canonical choice $\{I_2, iI_3, iI_5, iI_7\}$.

   The recent picture about the solution of co-associativity conditions fixes the subgroup of G_2 to $SU(3)$. X^4 corresponds to element g of the local $SU(3)$ acting on preferred $M^4 \subset M^8_c$ with the additional condition that the 4-surface $X^4 \subset M^8$ is invariant under $U(2) \subset SU(3)$ so that each point of X^4 corresponds to a CP_2 point. At the mas shells as roots of a polynomial P, g reduces to unity and the 4-D tangent space is parallel to the preferred M^4 on which g acts.

   One can induce this flat connection to string world sheet and holomorphy of g at this surface would guarantee the conservation of the current given by $j_{0)} = g^{-1}dg$.

3. Under these conditions the integral of the time component of current along a space-like curve at string world sheets with varying end point is well-defined and the current

$$j_{1)}(x) = \epsilon\mu\nu j_{0),\nu}(x) - \frac{1}{2}[j^{\mu}_{0)}(x,t), \int^x j^0_{0)}(t,y)dy]$$

is conserved. This is called the current at first level. Note that the currents have values in the Lie algebra considered. It is essential that the integration volume is 1-D and its boundary is characterized by a value of single coordinate x.

4. One can continue the construction by replacing j_0 with j_1 in the above formula and one obtains an infinite hierarchy of conserved currents $j_{n)}$ defined by the formula

$$j_{n+1)}(x) = \epsilon\mu\nu j_{n),\nu}(x) - \frac{1}{2}[j^{\mu}_{n)}(x,t), \int^{x} j^{0}_{n)}(t,y)dy] \qquad (19.6.1)$$

   The corresponding conserved charges Q_n define the generators of Yangian algebra.

5. 2-D metric appears in the formulas. In the TGD framework one does not have Riemann metric - only the number theoretic metric which is real only at real space-time surfaces already discussed. Is the (effective) 2-dimensionality and holomorphy enough to avoid the possible problems? Holomorphy makes sense also number theoretically and implies that the metric disappears from the formulas for currents. Also current conservation reduces to the statement of that current is equivalent to complex differential form.

6. Conserved charges would however require a 1-D integral and number theory does not favor this. The solution of the problem comes from the observation that one can construct a slicing of string world sheet to time-like curves as Hamiltonian orbits with Hamiltonian belonging to the Yangian algebra and defined by the conserved current by standard formula $j^{\alpha} = J^{\alpha\beta}\partial_{\beta}H$ in terms of Kähler form defined by the 2-D Kähler metric of string world sheet. This generalizes to Minkowskian signature and also makes sense for partonic 2-surfaces. Hamiltonians become the classical conserved charges constant along the Hamiltonian orbit. This gives an infinite hierarchy of conserved Hamiltonian charges in involution. Hamiltonian can be any combination of the Hamiltonians in the hierarchy and labelled by a non-negative integer and the label of $G_{2,c}$ generator. This is just what integrability implied by Yangian algebra means. Co-associativity and co-commutativity would be the deeper number theoretic principles implying the Yangian symmetry.

7. Could one formulate this argument in dimension $D = 4$? Could one consider instead of local current the integral of conserved currents over 2-D surfaces labelled by single coordinate x for a given value of t? If the space-time surface in M^8 (analog of Fermi sphere) allows a slicing by orthogonal strings sheets and partonic 2-surfaces, one might consider the fluxes of the currents $g^{-1}dg$ over the 2-D partonic 2-surfaces labelled by string coordinates (t, x) as effectively 2-D currents, whose integrals over x would give the conserved charge. Induced metric should disappear from the expressions so that fluxes of holomorphic differential forms over partonic 2-surface at (t, x) should be in question. Whether this works is not clear.

One should interpret the above picture at the level of momentum space instead of ordinary space-time. The roles of momentum space and space-time are changed. At this point, one can proceed by making questions.

1. One should find a representation for the algebra of the Hamiltonians associated with $g(x)$ defining the space-time surface. The charges are associated with the slicings of string world sheets or partonic 2-surfaces by the orbits of Hamiltonian dynamics defined by a combination of conserved currents so that current conservation becomes charge conservation. These charges are labelled by the coordinate x characterizing the slices defined by the Hamiltonian orbits and from these one can construct a non-local basis discrete basis using Fourier transform.

2. What the quantization of these classical charges - perhaps using fermionic oscillator operators in ZEO picture for which the local commutators vanish - could mean (only the anti-commutators of creation operators associated with the opposite half-cones of CD with opposite momenta are non-vanishing)? Do the Yangian charges involve only creation operators of either type with the same 8-momentum as locality at M^8 level suggests? Locality is

natural l since these Yangian charges are analogous to charges constructed from local currents at space-time level.

3. Could the Yangian currents give rise to poly-local charges assignable to the set of vertices in a cognitive representation and labelled by momenta? Could the level n somehow correspond to the number n of the vertices and could the co-product Δ generate the charges? What does the tensor product appearing in the co-product really mean: do the sector correspond to different total quark numbers for the generators? Is it a purely local operation in M^8 producing higher monomials of creation operators with the same momentum label or is superposition over Hamiltonian slices by Fourier transform possibly involved ?

How to construct quantum charges

One should construct quantum charges. In the TGD framework the quantization of $g(x)$ is not an attractive idea. Could one represent the charges associated with g it in terms of quark oscillator operators induced from the second quantized E^8 spinors so that propagators would emerge in the second quantization? Analogs of Kac Moody representations but with a non-negative spectrum of conformal weights would be in question. Also super-symplectic algebra would have this property making the formulation of the analogs of gauge conditions possible, and realizing finite measurement resolution in terms of hierarchy of inclusions of hyper-finite factors of type II_1 [K126, K44]. The Yangian algebra for $G_{2,c}$ or its subgroup could be the counterpart for these symmetries at the level of H.

The following proposal for the construction for the charges and super-charges of Yangian algebra in terms of quark oscillator operators is the first attempt.

1. One knows the Lie-algebra part of Yangian from the Poisson brackets of Hamiltonians associated with string world sheet slicing and possibly also for a similar slicing for partonic 2-surfaces. One should construct a representation in terms of quark ocillator operators in ZEO framework for both Lie-algebra generators and their super-counterparts. Also co-product should be needed.

2. The oscillator operators of E^8 spinor field located at the points of X^4 are available. The charges must be local and describe states with non-linear quarks and antiquarks.

   One must construct conserved charges as currents associated with the Hamiltonian orbits. Bosonic currents are bilinear in quark and antiquark oscillator operators and their super counterparts linear in quark or antiquark oscillator operators.

3. Since the system is 2-D one can formally assume in Euclidian signature (partonic 2-surface) Kähler metric $g^{z\overline{z}}$ and Kähler form $J^{z\overline{z}} = igz\overline{z}$, which is antisymmetric and real in real coordinates ($J^{kl} = -J^{lk}$) knowing that they actually disappear from the formulas. One can also define gamma matrices $\Gamma_\alpha = \gamma_k \partial_\alpha p^k$ as projections of embedding space gamma matrices to the string world sheet. In the case of string world sheet one can introduce light-like coordinates (u,v) as analogous of complex coordinates and the only non-vanishing component of the metric is g^{uv}.

4. The claim is that the time components J_n^u the bosonic currents

$$J_n^\alpha = b_p^\dagger \overline{v}(p)\Gamma^\alpha H_n u(p) a^\dagger \tag{19.6.2}$$

   at the Hamiltonian curves with time coordinate t define conserved charges ($\alpha \in \{u,v\}$ at the string world sheet).

   Remark: v_p corresponds to momentum $-p$ for the corresponding plane wave in the Fourier expansion of quark field but the physical momentum is p and the point of M^8 that this state corresponds.

   Therefore one should have

$$\frac{J^u_n}{du} = 0 \qquad (19.6.3)$$

One can check by a direct calculation what additional conditions are possibly required by this condition.

5. The first point is that H_n is constant if $v = constant$ coordinate line is a Hamiltonian orbit. Also oscillator operators creating fermions and antifermions are constant. The derivative of $u(p)$ is

$$\frac{du(p)}{du} = \frac{\partial u(p)}{\partial_{p^k}}\frac{dp^k}{du} \ .$$

. u_p is expressible as $u_p = Du_a$, where D is a massless Dirac operator in M^8 and u_a is a constant 8-D quark spinor with fixed chirality. D is sum of M^4- and E^4 parts and M^4 part is given by $D(M^4) = \gamma^k p_k$ so that one has $dp^k/dt = \gamma_r dp^r/dt$.

This gives

$$\frac{d(\Gamma^u H_n u(p))}{du} = g^{uv}\gamma_k\partial_v p^k\frac{du(p)}{du} = g^{uv}\partial_u p\cdot\partial_v p \ .$$

If the tangent curves of u and v are orthogonal in the induced metric and $v = 0$ constant lines are Hamiltonian orbits the bosonic charges are conserved.

One can perform a similar calculation for $d\frac{d\overline{v}(p)}{du}$ and the result is vanishing.

One must also have $dg^{uv}/du = 0$. This should reduce to the covariant constancy of g^{uv}. If the square root of the metric determinant for string world sheet is included it cancels g^{uv}.

6. From the bosonic charges one construct corresponding fermionic super charges by replacing the fermionic or anti-quark oscillator operator part with a constant spinor.

The simplest option is that partonic 2-surfaces contain these operators at points of cognitive representation. One can ask whether co-product could forces local operators having a higher quark number. What is clear that this number is limited to the number $n = 0$ of spin degrees of $n = 8$.

1. The commutators of bosonic and fermionic charges are fermionic charges and co-product would in this case be a superposition of tensor products of bosonic and fermionic charges, whose commutator gives bosonic charge. Now however the bosonic and fermionic charges commute in the same half-cone of CD. Does this mean that the tensor product in question must be tensor product for the upper and lower half-cones of CD?

   For instance, in the fermionic case one would obtain superposition over pairs of fermions at say lower half-cone and bosons at the upper half-cone. The momenta would be opposite meaning that a local bosonic generator would have total momentum $2p$ at point p and fermionic generator at opposite cone would have momentum $-p$. The commutator would have momentum p as required. In this manner one could create bosons in either half-cone.

2. One can also assign to the bosonic generators a co-product as a pair of bosonic generators in opposite half-cones commuting to the bosonic generator. Assume that bosonic generator is at lower half-cone. Co-product must have a local composite of 4 oscillator operators in the lower half-cone and composite of 2 oscillator operators in the upper half-cone. Their anti-commutator contracts two pairs and leaves an operator of desired form. It therefore seems.

   Statistics allows only generators with a finite number of oscillator operators corresponding to 8 spin indices, which suggests an interpretation in terms of the proposed SUSY [L92]. The roots of P are many-sheeted coverings of M^4 and this means that there are several 8-momenta with the same M^4 projection. This degree of freedom corresponds to Galois degrees of freedom.

3. Only momenta in cognitive representation are allowed and momentum is conserved. The products of generators appearing in the sum defining the co-product of a given generator T, which is a local composite of quarks, would commute or anti-commute to T, and their momenta would sum-up to the momentum associated with T. The co-product would be poly-local and receive contributions from the points of the cognitive representation. Also other quantum numbers are conserved.

About the physical picture behind Yangian and definition of co-product

The physical picture behind the definition of Yangian in the TGD framework differs from that adopted by Drinfeld, who has proposed - besides a general definition of the notion of quantum algebra - also a definition of Yangian. In the Appendix Drinfeld's definition is discussed in detail: this discussion appears almost as such in [L45].

1. Drinfeld proposes a definition in terms of a representation in terms of generators of a free algebra to which one poses relations [B57]. Yangian can be seen as an analog of Kac-Moody algebra but with generators labelled by integer $n \geq 0$ as an analog of non-negative conformal weight. Also super-symplectic algebra has this property and its Yangianization is highly suggestive. The generators of Yangian as algebra are elements J_n^A, $n \geq 0$, with $n = 0$ and $n = 1$. Elements J_0^A define the Lie algebra and elements J_1^A transform like Lie-algebra elements so that commutators at this level are fixed.

   Remark: I have normally used generator as synonym for the element of Lie algebra: I hope that this does not cause confusion

   The challenge is to construct higher level generators J_n^A. Their commutators with JA^0 with J^An are fixed and also the higher level commutators can be guessed from the additivity of n and the transformation properties of generators J_n^A. The commutators are very similar to those for Kac-Moody algebra. In the TGD picture the representation as Hamiltonians fixes these commutation relations as being induced by a Poisson bracket. The Lie-algebra part of Yangian can be therefore expressed explicitly.

2. The challenge is to understand the co-product Δ. The first thing to notice is that Δ is a Lie algebra homomorphism so that one has $\Delta(XY) = \Delta(X)\Delta(Y)$ plus formulas expressing linearity. The intuitive picture is that Δ adds a tensor factor and is a kind of time reversal of the product conserving total charges and the total value of the weight n. Already this gives a good overall view about the general structure of the co-commutation relations.

   The multiplication of generators by the unit element Id of algebra gives the generator itself so that $\Delta(J_A)$ should involve part $Id \otimes J^A \oplus J^A \otimes Id$. Generators are indeed additive in the ordinary tensor product for Lie-algebra generators - for instance, rotation generators are sums of those for the two systems. However, one speaks of interaction energy: could the notion of "interaction quantum numbers" make sense quite generally. Could this notion provide some insights to proton spin puzzle [**?**] meaning that quark spins do not seem to contribute considerably to proton spin? A possible TGD based explanation is in terms of angular momentum associated with the color magnetic flux tubes [K71], and the formulation of this notion at M^8 level could rely on the notion of "interaction angular momentum".

   The time reversal rule applied to $[J_A^m, J_B^n] \propto f_{ABC} J_C^{m+n}$ suggests that $\Delta(T_A^n)$ contains a term proportional to $f_{CBA} J_C^m \otimes J_B^{n-m}$. This would suggest that co-product as a time reversal involves also in the case of J_A^0 the term $k_1 f_{CBA} J_C^0 \otimes J_B^0$, where k_1 as an analog of interaction energy.

   Drinfeld's proposal does not involve this term in accordance with Drinfeld's intuition that co-product represents a deformation of Lie-algebra proportional to a parameter denoted by $\hbar$, which need not (and cannot!) actually correspond to $\hbar$. This view could be also defended by the fact that J_0^A do not create physical states but only measures the quantum numbers generated by J_A^n, $n > 0$. TGD suggests interpretation as the analog of the interaction energy.

3. In Drinfeld's proposal, the Lie-algebra commutator is taken to be $[J_A^0, J_B^0] = k f_{ABC} J_C^0$, $k = 1$. Usually one thinks that generators have the dimension of $\hbar$ so that dimensional

consistency requires $k = \hbar$. It seems that Drinfeld puts $\hbar = 1$ and the $\hbar$ appearing in the co-product has nothing to do with the actual $\hbar$.

The conservation of dimension applied to the co-product would give $k_1 = 1/\hbar$! What could be the interpretation? The scattering amplitudes in QFTs are expanded in powers of gauge coupling strengths $\alpha = g^2/4\pi\hbar$. In ZEO co-product would be essential for obtaining non-trivial scattering amplitudes and the expansion in terms of $1/\hbar$ would emerge automatically from the corrections involving co-products - in path integral formalism this expansion emerges from propagors

This view would also conform with the vision that Mother Nature loves her theoreticians. The increase of $h_{eff}/h_0 = n$ as dimension of extension of rationals would be Mother Nature's way to make perturbation theory convergent [K43]. The increase of the degree of P defining the space-time surface increases the algebraic complexity of the space-time surface but reduces the value of α as a compensation.

4. Drinfeld gives the definition of Yangian in terms of relations for the generating elements with weight $n = 0$ and $n = 1$. From these one can construct the generators by applying Δ repeatedly. Explicit commutation relations are easier to understand by a physicist like me, and I do not know whether the really nasty looking representation relations - Drinfeld himself calls "horrible" [B36] - are the only manner to define the algebra. In the TGD framework the definition based on the idea about co-product as a strict time reversal of product would mean deviation in the $n = 0$ sector giving rise to an interaction term having natural interpretation as analog of interaction energy.

5. Drinfeld proposes also what is known as Drinfeld's double [A83] (see `http://tinyurl.com/y7tpshkp`) as a fusion of two Hopf algebras and allowing to see product and co-product as duals of each other. The algebra involves slight breaking of associativity characterized by Drinfeld's associator. ZEO suggests [K59] that the members of Drinfeld's double correspond to algebra and co-algebra located at the opposite half-cones and there are two different options. Time reversal occurring in "big" state functions reductions (BSFRs) would transform the members to each other and change the roles of algebra and co-algebra (fusion would become decay).

In the TGD framework there is also an additional degree of freedom related to the momenta in cognitive representation, which could be regarded also as a label of generators. The idea that commutators and co-commutators respect conservation of momentum allows the fixing of the general form of Δ. Co-product of a generator at momentum p ina given half-cone would be in the opposite half-cone and involve sum over all momentum pairs of generators at p_1 and p_2 with the constraint $p_1 + p_2 + p = 0$.

Summation does not make sense for momenta in the entire extension of rationals. The situation changes if the momenta are algebraic integers for the extension of rationals considered: quarks would be particles in a number theoretic box. In the generic case, very few terms - if any - would appear in the sum but for space-time surfaces as roots of octonionic polynomials this is not the case. The co-products would as such define the basic building bricks of the scattering amplitudes obtained as vacuum expectation reducing the pairs of fermions in opposite half-cones to propagators.

19.7 Appendix: Some mathematical background about Yangians

In the following necessary mathematical background about Yangians are summarized.

19.7.1 Yang-Baxter equation (YBE)

YBE has been used for more than four decades in integrable models of statistical mechanics of condensed matter physics and of 2-D quantum field theories (QFTs) [A82]. It appears also in topological quantum field theories (TQFTs) used to classify braids and knots [B22] (see http://tinyurl.com/mcvvcqp) and in conformal field theories and models for anyons. Yangian symmetry appears also in the twistor Grassmann approach to scattering amplitudes [B23, B29] and thus involves YBE. At the same time new invariants for links were discovered and a new braid-type relation was found. YBEs emerged also in 2-D conformal field theories.

Yang-Baxter equation (YBE) has a long history described in the excellent introduction to YBE by Jimbo [B57] (see http://tinyurl.com/l4z6zyr, where one can also find a list of references). YBE was first discovered by McGuire (1964) and 3 years later by Yang in a quantum mechanical many-body problem involving a delta function potential $\sum_{i<j}\delta(x_i-x_j)$. Using Bethe's Ansatz for building wave functions they found that the scattering matrix factorized that it could be constructed using as a building brick 2-particle scattering matrix - R-matrix. YBE emerged for the R-matrix as a consistency condition for factorization. Baxter discovered in 1972 a solution of the eight vertex model in terms of YBE. Zamolodchikov pointed out that the algebraic mechanism behind factorization of 2-D QFTs is the same as in condensed matter models.

1978-1979 Faddeev, Sklyanin, and Takhtajan proposed a quantum inverse scattering method as a unification of classical and quantum integrable models. Eventually the work with YBE led to the discovery of the notion of quantum group by Drinfeld. Quantum group can be regarded as a deformation $U_q(g)$ of the universal enveloping algebra $U(g)$ of Lie algebra. Drinfeld also introduced the universal R-matrix, which does not depend on the representation of algebra used.

R-matrix satisfying YBE is now the common aspect of all quantum algebras. I am not a specialist in YBE and can only list the basic points of Jimbo's article. The interested reader can look for details and references in the article of Jimbo.

In 2-D quantum field theories R-matrix $R(u)$ depends on one parameter u identifiable as hyperbolic angle characterizing the velocity of the particle. $R(u)$ characterizes the interaction experienced by two particles having delta function potential passing each other (see the figure of http://tinyurl.com/kyw6xu6). In 2-D quantum field theories and in models for basic gate in topological quantum computation the R-matrix is unitary. R-matrix can be regarded as an endomorphism mapping $V_1\otimes V_2$ to $V_2\otimes V_1$ representing permutation of the particles.

YBE

R-matrix satisfies Yang-Baxter equation (YBE)

$$R_{23}(u)R_{13}(u+v)R_{12}(v) = R_{12}(v)R_{13}(u+v)R_{23}(u) \tag{19.7.1}$$

having interpretation as associativity condition for quantum algebras.

At the limit $u,v\rightarrow\infty$ one obtains R-matrix characterizing braiding operation of braid strands. Replacement of permutation of the strands with braiding operation replaces permutation group for n strands with its covering group. YBE states that the braided variants of identical permutations (23)(13)(12) and (12)(13)(23) are identical.

The equations represent n^6 equations for n^4 unknowns and are highly over-determined so that solving YBE is a difficult challenge. Equations have symmetries, which are obvious on the basis of the topological interpretation. Scaling and automorphism induced by linear transformations of V act as symmetries, and the exchange of tensor factors in $V\otimes V$ and transposition are symmetries as also shift of all indices by a constant amount (using modulo N arithmetics).

One can pose to the R-matrix some boundary condition. For $V\otimes V$ the condition states that $R(0)$ is proportional to the permutation matrix P for the factors.

General results about YBE

The following lists general results about YBE.

1. Belavin and Drinfeld proved that the solutions of YBE can be continued to meromorphic functions in the complex plane with poles forming an Abelian group. R-matrices can be classified to rational, trigonometric, and elliptic R-matrices existing only for $sl(n)$. Rational and trigonometric solutions have a pole at origin and elliptic solutions have a lattice of poles. In [B57] (see http://tinyurl.com/l4z6zyr) simplest examples about R-matrices for $V_1=V_2=C^2$ are discussed, one of each type.

2. In [B57] it is described how the notions of R-matrix can be generalized to apply to a collection of vector spaces, which need not be identical. The interpretation is as commutation relations of abstract algebra with co-product Δ - say quantum algebra or Yangian algebra. YBE guarantees the associativity of the algebra.

3. One can define quasi-classical R-matrices as R-matrices depending on Planck constant like parameter $\hbar$ (which need have anythingto do with Planck constant) such that small values of u one has $R = constant \times (I + \hbar r(u) + O(\hbar^2))$. $r(u)$ is called classical r-matrix and satisfies CYBE conditions

$$[r_{12}(u), r_{13}(u+v)] + [r_{12}(u), r_{23}(v)] + [r_{13}(u+v), r_{23}(v)] = 0$$

obtained by linearizing YBE. $r(u)$ defines a deformation of Lie-algebra respecting Jacobi-identities. There are also non-quasi-classical solutions. The universal solution for r-matrix is formulated in terms of Lie-algebra so that the representation spaces V_i can be any representation spaces of the Lie-algebra.

4. Drinfeld constructed quantum algebras $U_q(g)$ as quantized universal enveloping algebras $U_q(g)$ of a Lie algebra g. One starts from a classical r-matrix r and Lie algebra g. The idea is to perform a "quantization" of the Lie-algebra as a deformation of the universal enveloping algebra $U_q(g)$ of $U(g)$ by r. Drinfeld introduces a universal R-matrix independent of the representation used. This construction will not be discussed here since it does not seem to be as interesting as Yangian: in this case co-product Δ does not seem to have a natural interpretation as a description of interaction. The quantum groups are characterized by parameter $q \in C$. For a generic value the representation theory of q-groups does not differ from the ordinary one. For roots of unity situation changes due to degeneracy caused by the fact $q^N = 1$ for some N.

5. The article of Jimbo discusses also a fusion procedure initiated by Kulish, Restetikhin, and Sklyanin allowing to construct new R-matrices from existing one. Fusion generalizes the method used to construct group representation as powers of fundamental representation. Fusion procedure constructs the R-matrix in $W \otimes V^2$, where one has $W = W_1 \otimes W_2 \subset V \otimes V^1$. Picking W is analogous to picking a subspace of tensor product representation $V \otimes V^1$.

19.7.2 Yangian

Yangian algebra $Y(g(u))$ is associative Hopf algebra (see `http://tinyurl.com/qfl8dwu`) that is bi-algebra consisting of associative algebra characterized by product μ: $A \otimes A \to A$ with unit element 1 satisfying $\mu(1, a) = a$ and co-associative co-algebra consisting of co-product $\Delta A \in A \otimes A$ and co-unit $\epsilon : A \to C$ satisfying $\epsilon \circ \Delta(a) = a$. Product and co-product are "time reversals" of each other. Besides this one has antipode S as algebra anti-homomorphism $S(ab) = S(b)S(a)$. YBE has interpretation as an associativity condition for co-algebra $(\Delta \otimes 1) \circ \Delta = (1 \otimes \Delta) \circ \Delta$. Also ϵ satisfies associativity condition $(\epsilon \otimes 1) \circ \Delta = (1 \otimes \epsilon) \circ \Delta$.

There are many alternative formulations for Yangian and twisted Yangian listed in the slides of Vidas Regelskis at `http://tinyurl.com/ms9q8u4`. Drinfeld has given two formulations and there is FRT formulation of Faddeev, Restetikhin and Takhtajan.

Drinfeld's formulation [B57] (see `http://tinyurl.com/qfl8dwu`) involves the notions of Lie bi-algebra and Manin triple, which corresponds to the triplet formed by half-loop algebras with positive and negative conformal weights, and full loop algebra. There is isomorphism mapping the generating elements of positive weight and negative weight loop algebra to the elements of loop algebra with conformal weights 0 and 1. The integer label n for positive half loop algebra corresponds in the formulation based on Manin triple to conformal weight. The alternative interpretation for $n+1$ would be as the number of factors in the tensor power of algebra and would in TGD framework correspond to the number of partonic 2-surfaces. In this interpretation the isomorphism becomes confusing.

In any case, one has two interpretations for $n + 1 \geq 1$: either as parton number or as occupation number for harmonic oscillator having interpretation as bosonic occupation number in quantum field theories. The relationship between Fock space description and classical description for n-particle states has remained somewhat mysterious and one can wonder whether these two interpretations improve the understanding of classical correspondence (QCC).

Witten's formulation of Yangian

The following summarizes my understanding about Witten's formulation of Yangian for $\mathcal{N}=4$ SUSY [B22], which does not mention explicitly the connection with half loop algebras and loop algebra and considers only the generators of Yangian and the relations between them. This formulation gives the explicit form of Δ and looks natural, when n corresponds to parton number. Also Witten's formulation for Super Yangian will be discussed.

However, it must be emphasized that Witten's approach is not general enough for the purposes of TGD. Witten uses the identification $\Delta(J_1^A) = f^A_{BC}J_0^B\times J_0^C$ instead of the general expression $\Delta(J_1^A)=J_1^A\otimes 1+1\times J_1^A+f^A_{BC}J_0^B\times J_0^C$ needed in TGD strongly suggested by the dual roles of the super-symplectic conformal algebra and super-conformal algebra associated with the light-like partonic orbits realizing generalized EP. There is also a nice analogy with the conformal symmetry and its dual twistor Grassmann approach.

The elements of Yangian algebra are labelled by non-negative integers so that there is a close analogy with the algebra spanned by the generators of Virasoro algebra with non-negative conformal weight. The Yangian symmetry algebra is defined by the following relations for the generators labeled by integers $n=0$ and $n=1$. The first half of these relations discussed in very clear manner in [B22] follows uniquely from the fact that adjoint representation of the Lie algebra is in question

$$\left[J^A,J^B\right]=f^{AB}_CJ^C \quad , \quad \left[J^A,J^{(1)B}\right]=f^{AB}_CJ^{(1)C} \quad . \tag{19.7.2}$$

Besides this Serre relations are satisfied. These have more complex form and read as

$$\begin{aligned}
&\left[J^{(1)A},\left[J^{(1)B},J^C\right]\right]+\left[J^{(1)B},\left[J^{(1)C},\ J^A\right]\right]+\left[J^{(1)C},\left[J^{(1)A},\ J^B\right]\right]\\
&\qquad\qquad =\frac{1}{24}\ f^{ADK}f^{BEL}f^{CFM}f_{KLM}\{J_D,J_E,J_F\} \quad ,\\
&\left[\left[J^{(1)A},J^{(1)B}\right],\ \left[J^C,J^{(1)D}\right]\right]+\left[\left[J^{(1)C},J^{(1)D}\right],\ \left[J^A,J^{(1)B}\right]\right]\\
&\qquad\qquad =\frac{1}{24}(f^{AGL}f^{BEM}f^{CD}_K\\
&\qquad\qquad +f^{CGL}f^{DEM}f^{AB}_K)f^{KFN}f_{LMN}\{J_G,J_E,J_F\} \quad .
\end{aligned} \tag{19.7.3}$$

The indices of the Lie algebra generators are raised by invariant, non-degenerate metric tensor g_{AB} or g^{AB}. $\{A,B,C\}$ denotes the symmetrized product of three generators.

The right hand side often has coefficient $\hbar^2$ instead of $1/24$. $\hbar$ need not have anything to do with Planck constant and as noticed in the main text has dimension of $1/\hbar$. The Serre relations give constraints on the commutation relations of $J^{(1)A}$. For $J^{(1)A}=J^A$ the first Serre relation reduces to Jacobi identity and second to the antisymmetry of the Lie bracket. The right hand side involved completely symmetrized trilinears $\{J_D,J_E,J_F\}$ making sense in the universal covering of the Lie algebra defined by J^A.

Repeated commutators allow to generate the entire algebra, whose elements are labeled by a non-negative integer n. The generators obtained in this manner are n-local operators arising in $(n-1)$-commutator of $J^{(1)}$: s. For $SU(2)$ the Serre relations are trivial. For other cases the first Serre relation implies the second one so the relations are redundant. Why Witten includes it is for the purpose of demonstrating the conditions for the existence of Yangians associated with discrete one-dimensional lattices (Yangians exist also for continuum one-dimensional index).

Under certain consistency conditions, a discrete one-dimensional lattice provides a representation for the Yangian algebra. One assumes that each lattice point allows a representation R of J^A so that one has $J^A=\sum_i J^A_i$ acting on the infinite tensor power of the representation considered. The expressions for the generators J^{1A} in Witten's approach are given as

$$J^{(1)A} = f^A_{BC}\sum_{i<j} J_i^B J_j^C \ . \tag{19.7.4}$$

This formula gives the generators in the case of conformal algebra. This representation exists if the adjoint representation of G appears only one in the decomposition of $R\otimes R$. This is the case for SU(N) if R is the fundamental representation or is the representation of by k^{th} rank completely antisymmetric tensors.

This discussion does not apply as such to $\mathcal{N}=4$ case the number of lattice points is finite and corresponds to the number of external particles so that cyclic boundary conditions are needed guarantee that the number of lattice points reduces effectively to a finite number. Note that the Yangian in color degrees of freedom does not exist for SU(N) SYM.

As noticed, Yangian algebra is a Hopf algebra and therefore allows co-product. The co-product Δ is given by

$$\begin{aligned}\Delta(J^A) &= J^A\otimes 1 + 1\otimes J^A \ ,\\ \Delta(J^{(1)A}) &= J^{(1)A}\otimes 1 + 1\otimes J^{(1)A} + f^A_{BC}J^B\otimes J^C\end{aligned} \tag{19.7.5}$$

Δ allows to imbed Lie algebra into the tensor product in a non-trivial manner and the non-triviality comes from the addition of the dual generator to the trivial co-product. In the case that the single spin representation of $J^{(1)A}$ is trivial, the co-product gives just the expression of the dual generator using the ordinary generators as a non-local generator. This is assumed in the recent case and also for the generators of the conformal Yangian.

Super-Yangian

Also the Yangian extensions of Lie super-algebras make sense. From the point of physics especially interesting Lie super-algebras are $SU(m|m)$ and $U(m|m)$. The reason is that $PSU(2,2|4)$ (P refers to "projective") acting as super-conformal symmetries of $\mathcal{N}=4$ SYM and this super group is a real form of $PSU(4|4)$. The main point of interest is whether this algebra allows Yangian representation and Witten demonstrated that this is indeed the case [B22].

These algebras are Z_2 graded and decompose to bosonic and fermionic parts which in general correspond to n- and m-dimensional representations of $U(n)$. The representation associated with the fermionic part dictates the commutation relations between bosonic and fermionic generators. The anti-commutator of fermionic generators can involve besides the unit operator also bosonic generators if the symmetrized tensor product in question contains adjoint representation. This is the case if fermions are in the fundamental representation and its conjugate. For $SU(3)$ the symmetrized tensor product of adjoint representations contains adjoint (the completely symmetric structure constants d_{abc}) and this might have some relevance for the super $SU(3)$ symmetry.

The elements of these algebras in the matrix representation (no Grassmann parameters involved) can be written in the following form

$$x = \left(\begin{array}{cc} a & b\\ c & d\end{array}\right) \ .$$

a and d representing the bosonic part of the algebra are $n\times n$ matrices and $m\times m$ matrices corresponding to the dimensions of bosonic and fermionic representations. b and c are fermionic matrices are $n\times m$ and $m\times n$ matrices, whose anti-commutator is the direct sum of $n\times n$ and $n\times n$ matrices. For $n=m$ bosonic generators transform like Lie algebra generators of $SU(n)\times SU(n)$ whereas fermionic generators transform like $n\otimes\overline{n}\oplus\overline{n}\otimes n$ under $SU(n)\times SU(n)$. Supertrace is defined as $Str(x) = Tr(a) - Tr(b)$. The vanishing of Str defines $SU(n|m)$. For $n\neq m$ the super trace condition removes the identity matrix and $PU(n|m)$ and $SU(n|m)$ are the same. This does not happen for $n=m$: this is an important delicacy since this case corresponds to $\mathcal{N}=4$ SYM. If any two matrices differing by an additive scalar are identified (projective scaling as a new physical effect) one obtains $PSU(n|n)$ and this is what one is interested in.

Witten shows that the condition that adjoint is contained only once in the tensor product $R\otimes\overline{R}$ holds true for the physically interesting representations of $PSU(2,2|4)$ so that the generalization of the bilinear formula can be used to define the generators of $J^{(1)A}$ of super Yangian of $PU(2,2|4)$. The defining formula for the generators of the Super Yangian reads as

$$\begin{aligned} J_C^{(1)} &= g_{CC'}J^{(1)C'} = g_{CC'}f^{C'}_{AB}\sum_{i<j} J_i^A J_j^B \\ &= g_{CC'}f^{C'}_{AB}g^{AA'}g^{BB'}\sum_{i<j} J^i_{A'}J^j_{B'} \quad . \end{aligned} \tag{19.7.6}$$

Here $g_{AB} = Str(J_AJ_B)$ is the metric defined by super trace and distinguishes between $PSU(4|4)$ and $PSU(2,2|4)$. In this formula both generators and super generators appear.

19.8 Conclusions

$M^8 - H$ duality plays a crucial role in quantum TGD and this motivated a critical study of the basic assumptions involved.

19.8.1 Co-associativity is the only viable option

The notion of associativity of the tangent or normal space as a number theoretical counterpart of a variational principle. This is not enough in order to have $M^8 - H$ duality. The first guess was that the tangent space is associative and contains a commutative 2-D sub-manifold to guarantee $M^8 - H$ duality.

1. The cold shower came as I learned that 4-D associative sub-manifolds of quaternion spaces are geodesic manifolds and thus trivial. Co-associativity is however possible since any distribution of associative normal spaces integrates to a sub-manifold. Typically these sub-manifolds are minimal surfaces, which conforms with the physical intuitions. Therefore the surface X^4_r given by holography should be co-associative. By the same argument space-time surface contains string world sheets and partonic 2-surfaces as co-complex surfaces.

2. $X = Re_Q(o) = 0$ and $Y = Im_Q(P) = 0$ allow M^4 and its complement as associative/co-associative subspaces of O_c. The roots $P = 0$ for the complexified octonionic polynomials satisfy two conditions $X = 0$ and $Y = 0$.

   Surprisingly, universal solutions are obtained as brane-like entities X^6_c with real dimension 12, having real projection X^6_r ("real" means that the number theoretic complex valued octonion norm squared is real valued).

   Equally surprisingly, the non-universal solutions to the conditions to $X = 0$ correspond complex mass shells with real dimension 6 rather than 8. The solutions to $X = Y = 0$ correspond to common roots of the two polynomials involved and are also 6-D complex mass shells.

   The reason for the completely unexpected behavior is that the equations $X = 0$ and $Y = 0$ are reduced by Lorentz invariance to equations for the ordinary roots of polynomials for the complexified mass squared type variable. The intersection is empty unless X and Y have a common root and X^4_r belongs to X^6_r for a common root.

How to associate to the polynomial P a real 4-surface satisfying the conditions making $M8 - H$-duality?

1. P would fix complex mass shells in terms of its roots but not the 4-surfaces, contrary to the original expectations. The fact that the 3-D mass shells belong to the same M^4 and also their tangent spaces are parallel to M^4 together with rationality conditions for local $SU(3)$ element suggests number theoretical holography.

2. The key observation is that G_2 as the automorphism group of octonions respects the co-associativity of the 4-D real sub-basis of octonions. Therefore a local G_2 gauge transformation applied to a 4-D co-associative sub-space $M^c \subset O_c$ gives a co-associative four-surface as a real projection. Also octonion analyticity allows G_2 gauge transformation. If X^4 is the image M^4 by a local $SU(3)$ element such that it also remains invariant under $SU(2)$ at each point, one obtains automatically $M^8 - H$ duality.

   The image of X^4 under $M^8 - H$ duality depends on g so that gauge invariance is not in question. The plausible interpretation in case of $SU(3)$ is in terms of Kac-Moody - or even Yangian symmetry. Note that at QFT limit the gauge potentials defined at H level as projections of Killing vector fields of $SU(3)$ are replaced by their sums over parallel space-time sheets to give gauge fields as the space-time sheets are approximated with a single region of Minkowski space.

The study of octonionic Dirac equation shows that the solutions correspond to momenta at mass shells $m^2 = r_n$ obtained as roots of the polynomial P and that co-associativity is an essential for the octonionic Dirac equation. This conforms with the reduction of everything to algebraic conditions at the level of M^8.

19.8.2 Construction of the momentum space counter parts of scattering amplitudes in M^8

The construction of scattering amplitudes in M^8 was the main topic of this article. ZEO and the interpretation of M^8 as a momentum space analogous to the interior of the Fermi sphere give powerful constraints on the scattering amplitudes. 0

1. The fact that $SU(3)$ gauge transformation with boundary conditions defined by the mass shells as roots of polynomial P defines space-time surface and the corresponding gauge field vanishes plus the fact that at string world sheets the gauge potential defines a conserved current by holomorphy strongly suggest Yangian symmetry differing from Kac-Moody symmetry in that the analogs of conformal weights are non-negative. This leads to a proposal for how vertex operators can be constructed in terms of co-product using fermionic oscillator operators but with Kronecker delta anti-commutators since the cognitive representation is discrete.

2. The main objection is that the scattering amplitudes are trivial if quark momenta belong to cognitive representations, which are finite in the generic case. This would be the case also in 2-D integrable theories. The objection can be circumvented. First, the huge symmetries imply that cognitive representations can contain a very large - even an infinite - number of points. At partonic 2-surface this number could reduce to finite. Equally importantly, local composites of quark oscillation operators with collinear quark momenta are possible and would be realized in terms of representations of Yangian algebra for $G_{2,c}$ serving as the counterpart for super-symplectic and Kac-Moody algebras at the level of H.

3. ZEO leads to a concrete proposal for the construction of zero energy states - equivalently scattering amplitudes - by using a representation of Yangian algebra realized in terms of positive and negative energy quarks in opposite half-cones. Co-product plays a key role in the construction. Also the proposed local composites of quarks proposed in [L92] make sense.

4. Momentum conservation conditions and mass shell conditions combined with the requirement that the momenta are algebraic integers in the extension of rationals determined by the polynomial P look rather difficult to solve. These conditions however linearize in the sense that one can express the allowed momenta as squares of integer quaternions.

Also the construction of scattering amplitudes in M^8 is considered. ZEO and the interpretation of M^8 as a momentum space analogous to the interior of the Fermi sphere give powerful constraints on the scattering amplitudes. The fact that $G_{2,c}$ gauge transformation defines space-time surface and the corresponding gauge field vanishes plus the fact that at string world sheets the gauge potential defines a conserved current by holomorphy strongly suggest Yangian symmetry differering from Kac-Moody symmetry in that the analogs of conformal weights are non-negative. This leads to a proposal for how vertex operators can be constructed in terms of co-product using fermionic oscillator operators but with Kronecker delta anticommutators since the cognitive representation is discrete.

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Chapter 20

New findings related to the number theoretical view of TGD

20.1 Introduction

TGD could be seen as a holy trinity of three visions about quantum physics based on physics as geometry, physics as number theory, and physics as topology.

Quite recently I gave a talk on TGD and TGD inspired theory of consciousness and was asked about the motivations for the number theoretic vision. My response went roughly as follows.

1. The attempt to find a mathematical description for the physical correlates of cognition could have led to the vision of quantum TGD as a number theory. What are the possibly geometric/number theoretic/topological correlates of thought bubbles?

   A bold guess could have been p-adic numbers, $p = 2, 3, 5, 7, ...$ provide natural mathematical correlates for cognition. Rationals, algebraic extensions of rationals, and the extensions of p-adic number fields induced by them are natural candidates as also complex numbers, quaternions, and octonions. Also finite number fields emerged quite recently as natural ingredients of the number theoretic vision [K106, K107, K105] [L157].

   As a matter of fact, I ended up to a proposal that p-adic physics provides the correlates of cognition via a different route, by p-adic mass calculations based on p-adic thermodynamics, which turned out to have surprisingly high predictive power due to the number theoretic existence conditions [K65].

2. Sensory experience corresponds to real number based physics. There is a strong correlation between cognition and sensory experience, but it is not perfect. Sensations arouse thoughts, but cognition is also able to dream and imagine.

3. Cognition includes mathematical thought. The concretization of mathematical thinking as computation requires discretization. This suggests that discretization should correspond to what one might call a cognitive representation transforming thoughts to sensory percepts and it should have a number theoretic representation.

4. Mathematical thinking is able to imagine spaces with an arbitrary dimension, while the dimension of the perceptual world is fixed and is the dimension of three-space. How does cognition achieve this?

5. Cognition has evolved. Why and how can this be the case?

6. If the correlates of cognition are part of reality, then cognition must be optimally efficient. How?

This leads to the following questions and answers.

1. Could p-adic spacetime surfaces represent thought bubbles, equivalent to real 4-surfaces? They are a number-theoretic concept, they also involve a different topology than the sense-world, and p-adic space-time surfaces would be examples of algebraic geometry.

2. How is cognition able to imagine? p-Adic differential equations are non-deterministic: integration constants, which by definition have vanishing derivatives are only piecewise constants. Could this make imagination possible [K79]??

3. How the strong correlation between cognition and sensory experience could be realized? All p-adic number fields and their extensions must be allowed. Consider first the simplet book involving only reals and p-adic number fields. p-Adic number fields Q_p, $p = 2, 3, 5, ...$ can be combined into a book, an adele [L54, L53]. Different number fields as extensions of rationals represent the pages of this book. Real numbers correspond to sensory experience and various p-adic number fields to cognition. The back of the book corresponds to rational numbers that are common to all chapters.

   Every algebraic extension of rationals defines extensions of p-adic number fields. The p-adic pages of the algebraically extended book are algebraic extensions of various p-adic number fields. One obtains an infinite library with books labeled by algebraic extensions of rationals.

   Now the back of the book consists of algebraic numbers for the extension generated by the roots of a polynomial with integer coefficients. The back of the book gives a cognitive representation, a number theoretic discretization of the 4-surface that is unique for a given extension. The bigger the extension, the more accurate the discretization. Cognitive evolution would correspond to a refinement of cognitive representation induced by the increase of the order of the polynomial defining the extension.

4. How is cognition able to imagine higher-dimensional mathematical objects that do not exist at the level of sensory experience? algebraic extensions for p-adic numbers can have an arbitrarily high dimension if the corresponding polynomial has high enough degree. One can have p-adic 4-surfaces for which the associated algebraic dimension is arbitrarily high! p-Adic cognition is liberated from the chains of matter!

5. Why is evolution related to cognition? One gets an infinite number of books labeled algebraic extensions, a whole library. Does the evolution of cognition present a hierarchy? The bigger the algebraic extension, the better the approximation to real numbers and thus to sensory experience.

6. Can p-adic cognition be maximally effective? Here p-adic thermodynamics suggests the answer. p-Adic mass calculations assign to each elementary particle a p-adic prime. For instance, for electrons it is Mersenne prime $p = M_{127} = 2^{127} - 1 \simeq 10^{38}$. p-Adic mass squared value is expansion powers of p and its real counterpart is power series in negative powers of p. This series converges extremely rapidly for large primes such as $p \simeq 10^{38}$ and two lowest orders give a practically exact answer so all errors would be due to the assumptions of the model rather than due to computations.

How to realize number theoretic physics?

1. Number theoretic discretization does not resonate with the idea of general coordinate invariance. For $H = M^4 \times CP_2$ allows linear Minkowski coordinates but CP_2 coordinates are not linear although also now complex coordinates consistent with the isometries of $SU(3)$ are natural.

   What about M^8 or possibly its complexification suggested by twistorial considerations and also by the fact that classical TGD predicts that Euclidian space-time regions give an imaginary contribution to the conserved four momenta. M^8 allows highly unique linear Minkowski coordinates and the idea that M^8_c corresponds to complexified octonions is very natural. The automorphism group G_2 of octonions poses additional conditions.

2. This leads to the idea that number theoretic physics is realized at the level of M^8_c and that it is dual to the geometric physics realized at the level of H and that these physics are related

by M^8-H duality mapping 4-D surfaces in M^8 to H. TGD can be regarded as a wave mechanics for point-like particles replaced with 3-D surfaces in H, which, by the failure of complete determinism for holography, must be replaced by analogs of Bohr orbits. Wave mechanics is characterized by momentum-position duality, which naturally generalizes to M^8-H duality [L109, L110, L154, L157].

3. The physics in M^8_c should be purely algebraic as is also the ordinary physics at the level of momentum space for free fields. This physics should make sense also in all p-adic number fields. This suggests that polynomials with integer coefficients, in particular their roots, together with number theoretic holography based on associativity, partially characterize the 4-surfaces in M^8, which would make sense also as their p-adic variants.

   It is not clear whether the p-adicization is needed at the level of H: it might be enough to have it only at the level of M^8 so that only the p-adic variants of M^8 would be needed.

The geometric vision of TGD is rather well-understood (see for instance [L141]), but one need not think long to realize that there is still a lot of fog in the number theoretic vision (see for instance [K106, K107, K105] and [L109, L110, L128, L154, L157]).

1. There are uncertainties related to the interpretation of the 4-surfaces in M^8 what the analogy with space-time surface in $H = M^4 \times CP_2$ time evolution of 3-surface in H could mean physically?

2. The detailed realization of M^8-H duality [L109, L110] involves uncertainties: in particular, how the complexification of M^8 to M^8_c can be consisted with the reality of $M^4 \subset H$.

3. The formulation of the number theoretic holography with dynamics based on associativity involves open questions. The polynomial P determining the 4-surface in M^8 doesn't fix the 3-surfaces at mass shells corresponding to its roots. Quantum classical correspondence suggests the coding of fermionic momenta to the geometric properties of 3-D surfaces: how could this be achieved?

4. How unique is the choice of 3-D surfaces at the mass shells $H^3_m \subset M^4 \subset M^8$ and whether a strong form of holography as almost $2 \rightarrow 4$ holography could be realized and make this choice highly unique.

5. The understanding of 3-geometries is essential for the understanding of the holography in both M^8 and H. The mathematical understanding of 3-geometries is at a surprisingly high level: the prime 3-manifolds can be constructed using 8 building bricks. Do these building bricks, model geometries, have counterparts as prefered extremals of action in the TGD framework.

   The known extremals $X^4 \subset H$ satisfying holography should be analogues of Bohr orbits [L146]. They are proposed to satisfy a 4-D generalization of 2-D holomorphy and apart from lower-D singularities would be the same for any general coordinate invariant action based on induced geometry and spinor structure. They would be minimal surfaces both in H and M^8 except at singularities at which the details of the action principle would matter [L170]. This suggests that the preferred extremals could have maximal isometries and provide topological invariants as also do the model geometries in the classification of 3-geometries.

20.2 What does one mean with M^8 physics?

In TGD, the point-like particle is replaced by a 3-surface $X^3 \subset H = M^4 \times CP_2$, and the holography required by the general coordinate invariance requires the replacment of the 3-surfaces with the analogues of Bohr trajectories passing through them. The Bohr trajectories are not completely deterministic as already the case of hydrogen atoms demonstrates. The "World of Classical Worlds" (WCW) is thus the space of generalized Bohr orbits as the counterpart of the superspace of Wheeler which originally inspired the notion of WCW [L169, L170, L166, L167]).

In wave mechanics, the duality between the descriptions using momentum and position space applies in wave mechanics but does not generalize to field theory. The M^8-H duality [L109, L110] can be seen as a generalization of this duality. M^8 is the momentum space counterpart and $H = M^4 \times CP_2$ is the position space counterpart in this duality.

20.2.1 Physical interpretation of the 4-surfaces of the space M^8 and their singularities

The physical interpretation of 4-surfaces in the complexifixation of the momentum space M^8 is far from straightforward. There are many reasons for the complexification.

1. Complexified octonionicity requires that M^8, or equivalently E^8, is complexified: one has $M^8_c = E^8_c$ giving as its subspaces various real subspaces with various signatures of the number theoretical. The metric obtained from the Minkowski norm $\delta_{kl} z^k z^l$, where z^k are 8 complex coordinates. M^4 with signature $(1, -1, -1, -1)$ is in a special physical role and one can of course ask, whether also other signatures might be important.

2. If complex roots are allowed for polynomials P determining together with associativity the holography, complexification must be allowed. Virtual momenta could therefore be complex, but Galois confinement says that the total momenta of physical states are real and have integer components in the momentum scale determined by the size of the causal diamond (CD). Physical intuition suggests that the imaginary parts of the momenta code for the decay width of the particle. This is natural if the imaginary part is associated with the energy in the rest system.

3. The conserved momenta given by Noether's theorem at the level of H have real parts assignable to Minkowskian space-time regions. The fact that $\sqrt{g_4}$ appears in the integral defining a conserved quantity differs in Minkowski and Euclidean regions by an imaginary unit suggests that the contributions to momenta from the Euclidian regions are imaginary. The momenta from the Minkowskian space-time regions can be transferred to the light-like boundaries between Minkowskian and Euclidian regions identified as light-like partonic orbits. Quantum-classical correspondence requires that the classical total momenta, like all conserved quantities, correspond to the total momenta of the fermion state.

   Euclidian regions most naturally correspond to CP_2 type extremals as preferred extremals. They can be regarded as singularities resulting in the blow up of tip-like cusp singularities (see `https://rb.gy/0p30o` and `https://rb.gy/fd4dz`) in M^8 . This would suggest that the real parts of momenta are associated with the Minkowskian regions of space-time surfaces and imaginary parts to the Euclidian regions. This applies also to other conserved quantities.

20.2.2 Number theoretic holography

Number theoretic holography has two forms.

1. The weak $3 \to 4$ form corresponds to the ordinary holography $Y^3 \subset M^8 \to Y^4 \subset M^8$, which is by $M^8 - H$ duality equivalent of the holography for $X^3 \subset H \to X^4 \subset H$ for space-time surfaces. The proposed interpretation of Y^3 is as a fundamental region of H^3/Γ.

2. For the strong $2 \to 4$ form of the holography Y^3 is determined by a 2-D data defined by the boundary of the fundamental region of H^3/Γ. The proposal to be considered is that the boundary of the fundamental region of H^3/Γ can be identified as 2-D hyperbolic space H^2/Γ.

Consider next the weak form of the holography.

1. The 4-surface $Y^4 \subset M^8_c$ is determined from number-theoretic dynamics and is an associative surface, i.e. its normal space is associative and therefore quaternionic.

2. There are also commutative 2-D surfaces that most naturally correspond to string world sheets, and for them commutativity of tangent space (as analog of associativity) as subspace of normal space of Y^2 defines holography. Holographic data corresponds now to strings connecting wormhole contacts assignable to Euclidian singularities inside $Y^3 \subset H^3_m$. One

can also consider the possibility that partonic 2-surfaces correspond to co-commutative 2-surfaces. The situation is not completely clear here.

3. One must also identify the 3-surfaces $Y^3 \subset H^3_m$ defining the holography. Holography is subject to very strong conditions and I have proposed that these surfaces are hyperbolic 3-manifolds X^3 obtained as coset spaces H^3/Γ, where G is suitably chosen discrete but infinite subgroup of $SL(2,C)$ acting as Lorentz transformations in H^3. The spaces H^3/Γ are fundamental domains of H^3 tessellations.

   $Y^3 = H^3/\Gamma$ is counterpart for the unit cells of a lattice in E^3, which effectively has this topology and geometry due to boundary conditions stating that G leaves various "field configurations" invariant. The situation is the same as in the case of ordinary condensed matter, where periodic boundary conditions for a cube as a unit cell make it effectively a 3-torus.

   Also the crystal-like structures consisting of a finite number of copies of the fundamental domain of H^3/Γ glued together are possible choices for Y^3. They would be analogous to the unit cells of the lattices of Euclidian space E^3 or finite crystals formed from them. Therefore the analog of solid state physics would be realized at the fundamental level.

   One can also consider closed 3-manifolds $Y^3 = H^3/\Gamma$ obtained by gluing two copies of the fundamental region with different S^3 coordinates connected together along their 2-D boundaries. The gluing could be performed by a cylinder of $S^3 \subset E^4 \subset M^4 \times E^4$ connecting the boundaries.

4. The quantum state at H^3_m consists of several Galois singlets assignable to 3-surfaces Y^3_i. The total momenta for X^3_i would be real and have integer valued components for the momentum unit defined by the size scale of CD involved. This condition is analogous to the periodic boundary conditions.

5. Quantum classical correspondence requires that the many-fermion state on H^3_m, characterized partially by momenta, which are in the algebraic extension of rationals associated with the polynomial P, determines $Y^3_i \subset H^3_m$. For a given Y^3_i, the accompanying fermions correspond to the points of H^3_m. The classical momentum of the state given by Noether theorem in H would the sum of the fermionic momenta.

   An attractive idea is that at least a subset of the fermionic momenta corresponds to cusp singularities (see `https://rb.gy/0p30o` and `https://rb.gy/fd4dz`), which can be visualized geometrically as vertices of an algebraic surface at which the direction of normal space is ill-defined.

   The cusps correspond to parabolic subgroups $P \subset G \subset SL(2,C)$ (`https://rb.gy/b5t55`), where the $\Gamma \subset SL(2,C)$ defines the fundamental domain H^3/Γ of the tessellation. Parabolic subgroups P are automorphic to the subgroup of translations of upper half-plane H generated by $SL(2,C)$ matrix $(1a;0,1)$, a a real algebraic number. This particular P acts as Möbius transformations in H representing hyperbolic space H^2. The cusp singularities would encode at least a subset of fermionic momenta of the state into the hyperbolic geometry of Y^3_i. Each fermion would correspond to its own parabolic generator in the subgroup G.

   In the TGD view of hadron physics [L161], the fermions associated with the cusps could be identified as analogs of valence quarks. They would be associated with singularities identifiable as light-like 3-D partonic orbits serving as boundaries of 4-D CP_2 type extremals with Euclidian signature of the induced metric.

   Also fermionic momenta, which have algebraic integers as components but do not correspond to cusps, can be considered. These would be naturally associated with strings predicted to connect cusps at the throats of different wormhole contacts. The blow-up would be now 2-sphere relating to cusp singularity like line charge to point charge. It is not clear whether the sea partons could correspond to these string-like singularities. In any case, hyperbolic 3-manifolds have string-like singularities connecting cusps.

6. If Y^3_i corresponds to a Galois singlet, then its total 4-momentum is real and integer-valued and should be mapped to a discrete plane wave in the finite lattice defined by the crystal like structure formed by the copies $X^3(Y^3)$ in H^3_a given by inversion. Each Galois singlet Y^3_i

would define such a plane wave and one can imagine a hierarchy of such structures just as in the case of condensed matter with crystals of different sizes. The analogy with condensed matter physics suggests that Γ is a lattice. This follows also from the condition that H^3/Γ has a finite volume.

7. This picture would suggest that also $X^3(Y_i^3)$ is hyperbolic manifold of its fundamental region and perhaps isometric with Y^3. This would mean a geometric realization of Lorentz invariance analogous to the dual of conformal invariance encountered in the twistorialization.

20.2.3 Quantum classical correspondence for momenta

Quantum classical correspondence for momenta and also other conserved charges poses very strong conditions.

1. Noether charges for the classical action define momenta and other conserved charges. The classical contribution is a c-number. In addition, quantum contributions from fermions are included. They correspond to the momenta related to the second quantized spinor modes of H and from the orbital degrees of freedom associated with the "world of classical worlds" (WCW). The fermionic contributions are related to the ground states of the super symplectic representation characterized in terms of spinor modes for H spinor fields [K29, K96] [L157].

2. Are the classical contributions separate and do they add up to the total momentum? The fact that classical contributions are separately conserved, does not support this view.

3. Quantum classical correspondence would mean that the classical total momentum is the sum of the fermionic momenta determined by the multi-fermion state. This would hold quite generally for Cartan algebra of observables. For example, in the case of hadrons, the dominant classical contribution could correspond to the gluon sea, that is to multi-gluon state with gluons expressible in terms of quark-antiquark pairs. This picture is consistent with QCD and is therefore perhaps a more realistic guess.

4. Wormhole contact has Euclidean induce metric and the related classical conserved momentum is naturally imaginary. Could the sum of the imaginary parts of complex fermionic momenta of fermions for a wormhole throat correspond to the classical imaginary momentum assignable with the wormhole contact? Could the imaginary part of the fermionic momentum be assigned with the end of the euclidean string inside CP_2 type extremals, while the real momentum would be assigned with an end of a Minkowskian string?

5. Quantum-classical correspondence would be realized if the fermionic conserved four-momenta on the H side corresponded to M^8 points at hyperbolic 3-surfaces H^3_m. Their inversions in the M^8-H duality would be points of M^4_c of the spacetime surface $H^3_m \subset M^4_c \times CP_2$. It would seem that one must map only the real parts of the momenta at H^3_m to H^3_a.

   It would also seem that H^3_m must be associated with the M^4 projection of M^4_c. Whether the variant of H^3_m for complex valued m^2 makes even sense is not obvious.

20.2.4 The analog of time evolution in M^8 as a coupling constant evolution conserving dual quantum numbers

The proposal that 4-D surfaces appear at the level of M^8 suggests that it makes sense to talk about dynamics also in M^8 and the 4-D analogies of space-time surfaces make sense. This does not fit the usual classical intuition.

The twistor picture for conformal invariant field theories predicts that conformal invariance has a dual counterpart. This would mean that 4-momenta and other Poincare charges in H have dual counterparts in M^8. In TGD, the dual counterparts would be obtained by inversion from the defining the M^8-H duality and mapped to points of the space-time surface at the mass shells H^3_a in H. They would be analogs of the representation of lattice momenta as points of the heavenly sphere in crystal diffraction.

1. In zero energy ontology (ZEO), the time evolution at the level of H by "small" state function reductions (SSFRs), which are analogous to the so called weak measurements introduced by quantum opticians, would correspond to time evolution in terms of scalings rather than time translations. They would scale the size of the causal diamond (CD) and leave the passive boundary of CD invariant. These analogs of time evolutions would be generated by the scaling generator L_0. This would naturally also apply to M^8. This time evolution would be induced by scalings of the mass-scale, which need not be identical.

2. Could the "energy evolution", by identifying the square of the mass as the counterpart of time, correspond to the development related to the renormalization group? M^8-H duality would map the renormalization group evolution from M^8 to time evolution in H. This is quite a strong prediction.

3. Mass squared values for the fundamental fermions would not define particle masses but mass scales. 4-momenta for physical particles would correspond to total momenta for many fermion states, which obey Galois confinement, which requires that the momentum components are integers, when the mass unit is defined by the size scale of CD.

4. What would be the interpretation of the mass shells M^4_c determined by the roots of the polynomial P in the coupling constant evolution? Could the related hyperbolic 3-manifolds correspond to fixed points for the coupling constant evolution? With these mass values, something special would happen. Could H^3_a correspond to critical moments of light-cone proper time a when the SSFRs occur and a new unitary time evolution begins and ends with the next SSFR, as I have suggested?

5. What about the M^8 side? Could one talk about conserved quantities with respect to the evolution determined by scalings? Could these dual charges, dual momenta, and. also the charges related to E^4 isometries, be invariants of the renormalization group evolution?

   I have proposed that the $SO(4)$ symmetry of of hadrons in old-fashioned hadron physics involving notions like conserved vector current (CVC) and partially conserved axial current (PCAC) could correspond to the color symmetry of higher energy hadron physics by M^8-H duality in which the natural conserved charges on M^8 side are associated with the product of isometry groups of M^4 and E^4 and perhaps even with $SO(1,7)\times T^8$ or G_2 as automorphism group of octonions. At H side one would have a product of Poincare group and color group. Also holonomy groups are involved. At least $SO(4)$ symmetry could define invariants of the coupling constant evolution in M^8.

Consider now a more detailed interpretation of 4-surfaces $Y^4 \subset M^8$ in terms of a generalized coupling constant evolution.

1. The changes $m_i^2 \rightarrow m_{i+1}^2$ for the roots of P would define a discrete evolution in both M^8 and H. Discrete coupling constant evolution affects the mass resolution and brings in or deletes details and therefore would induce changes for the representations of the states. The 4-surfaces in M^8 would represent renormalization group flows. The failure of a complete determinism is expected and could be interpreted in terms of phase transitions occurring at critical masses.

2. A given mass shell m_i^2 determined by a root of P would define a discrete mass scale for the evolution having perhaps an interpretation as a fixed point or a critical point of the coupling constant evolution. It would be natural to assume that the evolution induced by the change of resolution conserves other total quantum numbers than 4-momentum.

3. What about the conservation of 4-momentum? At $m^2 = m_{i+1}^2$ the value of mass squared for fundamental fermions defining the mass scale changes. The structure of the state must change in $m_i^2 \rightarrow m_{i+1}^2$ if 4-momentum conservation is assumed.

   The normalization group evolution for the mass m^2 of the physical state, is typically logarithmic in QFTs, and must be distinguished from the discrete evolution for the mass scale m_i^2. Hence the change of m^2 in $m_i^2 \rightarrow m_{i+1}^2$ is expected to be small. This could be realized if n corresponds to a (possibly normal) subgroup of the Galois group of P perhaps spanned by the roots $m_k^2 \leq m_i^2$.

Could the phase transition $m_i^2 \rightarrow m_{i+1}^2$ change the rest energy of the state? Does the change require an energy feed between the CD and its environment as ordinary phase transitions require? This is not the case if CD is interpreted as a perceptive field rather than a physical system.

4. Does it make sense to talk about the conservation of dual momentum $X^k = \sum_i X_i^k$, $X_i^k = Re(\hbar_{eff} p^k/p_{k,i} p_i^k) = Re(\hbar_{eff} p^k/m_i^2)$? The conservation of momentum p^k does not imply the conservation of dual momentum since it is proportional to $1/m_i^2$: X^k would scale as $1/m_i^2$. The size of the CD is assumed to increase in statistical sense during the sequence of small state function reductions (SSFRs). The increase of the size of the CD would gradually make the mass shells inside it visible.

   $M^4 \subset H$ center of mass position X^k therefore changes $m_i^2 \rightarrow mi + 1^2$ unless h_{eff} is not scaled to compensate the change $m_i^2 \rightarrow mi + 1^2$ in the formula for X_i^k. The integer n in $h_{eff} = nh_0$ is assumed to correspond to the order of the Galois group of P. It could also correspond to the order of a subgroup of the Galois group of P defined by the roots m_k^2, $k \leq i$. If so, h_{eff} would increase in evolution and one can even imagine a situation in which $Re(h_{eff}/m_i^2)$ remains constant.

20.3 $M^8 - H$ duality

The proposed $M^8 - H$ duality maps 4-surfaces $Y^4 \subset M_+^4 \subset M^8 = M^4 \times E^4$ to space-time surfaces $X^4 \subset M_+^4 \subset M^4 \subset M^4 \times CP_2$.

20.3.1 $M^8 - H$ duality as inversion

$M^8 - H$ duality relates also the hyperbolic spaces $H_m^3 \subset M_+^4 \subset M^8 = M^4 \times E^4$ and $H_a^3 \subset M_+^4 \subset M^4 \subset M^4 \times CP_2$. The hyperbolic space $H_m^3 \subset M_+^4 \subset M^8$ corresponds to the mass shell for which mass squared value m^2 is a root of a polynomial P. The hyperbolic space $H_a^3 \subset M^4 \subset= M^4 \subset M^4 \times CP_2$ corresponds to light-cone proper time constant surface $t^2 - r^2 = a^2$.

1. The root of P is in general a complex algebraic number. The first guess is that $M^8 - H$ duality is defined by inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^l p_l$. Or briefly, $p \rightarrow \hbar_{eff} p/m^2$. The light-cone proper time $a = h_{eff}/m$ characterized the hyperboloid $H_a^3 \subset M^4$. $H_m^3 \rightarrow H_a^3$ is consistent with the Uncertainty Principle. In this case the image would be complex. This creates interpretational problems. There is no need for the complexification of CP_2, which also suggests that the image un H_a^3 must be real.

2. One can consider the possibility that only the real projection of the complex variant of H_m^3 to $H_{Re(m)}^3$ is involved. The image of the real part $Re(p^k)$ in H_a^3 obtained by inversion would be real but would not code information about the imaginary part $Im(p^k)$.

3. One could however take the real part of a complex inversion to get a point in H_a^3. $Re(\hbar_{eff} p^k/p^2$ would code information about the imaginary value of m^2.

Inversion fails at the light-cone boundary with $m = 0$. In this case, the inversion must be defined as the inversion of the energy of the massless state: $p^k \rightarrow \hbar_{eff} Re(p^k/E^2)$.

20.3.2 The technical problems posed M^8-H duality the complexification of M^8

Complexification of M^8 is highly desirable in the number-theoretic vision. But how to deal with the fact that fermion momenta, for which with components are algebraic integers in the algebraic extension determined by a polynomial P, are in general complex?

1. Without additional conditions, the mass shells in $M^4_c \subset M^8_c$ for complex m^2 as a root of P are 6-D. There are 2 conditions coming from the conditions fixing the value of $Re(m^2) = Re(p^2) - Im(p^2)$ and $Im(m^2) = 2Re(p) \cdot Im(p)$. If one only energy is complex, the dimension of the mass shell is 3. This looks natural. The preferred time axis would be determined by the rest system for massive states. A possible interpretation for the imaginary part is as decay width in the rest system.

2. The complexified mass shells of complexified $M^4 \subset H$ must be considered. Does this make sense? Since the CP_2 point labelling tangent space of Y^4 does not depend on complexification there is no need to consider complexification of CP_2. Therefore the natural conclusion is that also the $M^4 \subset H$ images should be real.

The inversion $p^k \to p^k/m_r p_r p_s$ is the simplest realization for $M^8 - H$ duality and would naturally fit into a generalization of 2-D conformal invariance to 4-D context. $h_{eff} = nh_0$ hierarchy comes along in a natural way. The polynomial P determines the algebraic extension and the value of h_{eff}. The size of the CD would scale like h_{eff} on the H side. There would be no scaling on the M^8 side.

1. The first thing to notice is that one could understand classically complex momenta. On the H side, Euclidean regions could give an imaginary contribution to the classical momentum.

2. The complex inversion $p^k \to p^k/m_r p_r p_s$ maps complex H^3_m to complex H^3_a. What would be the interpretation of the complex M^4_c coordinates? The same problem is also encountered in twistorization. One can ask whether a complex time coordinate corresponds to, for example, the inverse temperature?

   However, since no complexification is needed for CP_2, it seems that the only natural option is that the $M^4 \subset H$ image is real.

3. One can consider 3 options guaranteeing the reality.

   (a) Only the real parts of the complex $M^4 \subset M^8$ momenta are mapped to H. The information about the imaginary parts would be lost.

   (b) The complex algebraic integer valued momenta are allowed and the real part $Re(p^k \to p^k/m_r p_r p_s)$ of the complex inversion defines the image points in H. The $M^4 \subset H$ complexification would not be needed for this option but the information about the imaginary part of the momenta would not be lost.

   (c) By Galois confinement, the physical multiparticle states consist of momenta with integer value components using the momentum unit assignable to CD at the M^8 level of space with mass shells. These would define the 3-D data for the holography, which determines the 4-surface $Y^4 \subset M^8$ through the associativity of the normals space of Y^4. Only the real, integer-valued momenta of Galois confined states would be mapped from the mass shells of $M8$ to their images in H.

   The information about the fermion composition of the many-particle states would be lost completely. Therefore this option does not look realistic.

   The realization of this view might be possible however. 4-momenta determine the 3-surfaces Y^3 with real mass shells H^3 as data for associative holography. Momenta could correspond to point-like singularities on $Y^3 \subset H^3$ and these should be assigned as CP_2 type extremals at H side as blow-ups of the singularities.

To conclude, the option $p^k \to Re(p^k \to p^k/m_r p_r p_s)$ seems to be the physically realistic option.

20.3.3 Singularities and $M^8 - H$ duality

Consider next the description of the cusp singularities (see `https://rb.gy/0p30o` and `https://rb.gy/fd4dz`) in $M^8 - H$ duality. The condition that information is not lost, requires that the map is given by $p^k \to Re(p^k/m_{rs} p^r p^s)$.

1. The cusps in M^8-H duality would be mapped to a 3-D surface of CP_2. It would correspond to the 3-D section CP_2 of the extremum, which corresponds to a wormhole contact associated with fermions at its throats.

   At H^3_m there is a temptation to assign to the cusp singularity, identified as a blow-up, the 3-D sphere $S^3 \subset E^4$ of the normal space E^4 defined by the mass shell condition. The simplest option is that this sphere is mapped to $U(2)$ invariant sphere $S^3 \subset CP_2$ for which the radius would be fixed by the mass squared value.

   The metric of H^3/Γ is singular at the cusp. The elimination of the singularity requires that one must allow a hole Z^3 around the cusp. The boundary of X^3 can have any genus. The size scale of the hole should be determined by the mass squared value.

   This view conforms with the physical picture of the CP_2 type extremal as an orbit of an Euclidian wormhole contact connecting two Minkowskian space-time sheets. S^3 would be replaced with $S^3 \setminus Z^3$ mapped to CP_2, where it corresponds to a wormhole throat having arbitrary genus.

   This picture would suggest that a given cusp singularity can correspond only to a single wormhole throat. This is not in conflict with the recent view of what elementary particles having wormhole contacts as composites should be. Composite, involving 2 wormhole contacts (required by the conservation of the monopole flux forming a loop involving two space-time sheets) and therefore 2 wormhole throats, can have spin varying from 0 to 2 which conforms with the popular wisdom that elementary particle spins vary in this range.

2. In the case of string-like objects $Y^2 \times R \subset H^3_m$, that is $S^2 \times R$ and their higher genus counterparts $H^2/\Gamma \times R$, the counterpart of the blow-up would be $Y^2 \subset S^3 \subset E^4$. Y^2 would be mapped to $X^2 \subset CP_2$ such that the radius assignable to S^2 or the size scale assigned to H^2/Γ would correspond to the mass squared.

3. Fermion trajectories at the partonic orbits would be light-like curves defining boundaries of string world sheets. CP_2 extremal would be associated with a fermionic cusp by holography and M^8-H duality.

   Fundamental fermion as an analog of valence quark [L161] could correspond to a cusp at the boundary of the string world sheet. Cusps would be related to the boundary of X^3_a composed of partonic 2-surfaces.

4. In principle, fermion momenta in the interior of $Y^3 \subset H^3_m$ are also possible. The picture given by hadron physics would indicate that the interior contribution corresponds to the sea partons. They can also be associated with string world sheets and correspond to virtual bosons appearing as fermion-antifermion pairs. These singularities would be string-like objects of the form $X^2 \times R$ and $X^2 \subset CP_2$ would replace the sphere of CP_2. One could say that fermions are delocalized at string.

20.3.4 Realization of the Uncertainty Principle

Inversion alone is not enough to realize Uncertainty Principle (UP), which requires that M^8-H duality is analogous to the Fourier transform. However, with the help of H^3 tessellations, it is possible to understand how the UP is realized in a finite measurement resolution.

The invariance of points of H^3/Γ under the subgroup $G \subset SL(2,C)$ is analogous to the periodic boundary conditions that replace the cubic unit cell of a crystal lattice with a torus. Now the tessellation of H^3, which quantizes the momenta, would be replaced by H^3/Γ. The momentum lattice having the fundamental region for H^3_m/G as unit cell would be mapped by inversion to a position lattice having the fundamental region H^3_a/G as a unit cell. A point in H^3_m would correspond to an analog of plane wave as a superposition of all positions of $X^3(Y^3)$ in a part of the tessellation in H^3_a: a wave function in finite crystal. One would have a superposition of 3 surfaces X^3_a corresponding to different lattice points multiplied by the phase factor. For a multi-fermion state the cusp singularities (see `https://rb.gy/0p30o` and `https://rb.gy/fd4dz`) assigned to the momenta of fermions would characterize H^3/G so that the information about ("valence") fermion state would be code geometrically. Similar coding would be realized also for the string-like entities $H^2/\Gamma \times R$ at H^3/Γ. What is new and surprising, and also challenges the interpretation, is that the genus of H^2/Γ would code for the momenta of many-fermion states. Does the number of fermion-antifermion pairs correlate with the genus which in turn is proposed to label fermion families? There would be one fermion-antifermion pair per single handle. This would conform with the quantum classical correspondence. The proposed explanation [K26] for the number of observed fermion families would be in terms of hyper-ellipticity meaning that Z_2 acts as a conformal symmetry for all genera smaller than 3. Genus two would correspond to a formation of a bound state of two handles. Could this mean a formation of a graviton-like bound state of 2 fermion pairs and that higher spin states are not possible as bound states of handle. If fermions correspond to cusp singularities surrounded by holes, this picture might make sense: fermion antifermion pair would correspond to two holes connected by a handle.

The M^8-H duality maps the surface $Y^4 \subset M^8$ to the space-time surface $X^4 \subset H$. The point of $M^4 \subset H$ is obtained as the real part of the inversion of the point of the M^4 projection of the surface Y^4.

There would be a direct analogy to the physics of condensed matter.

A hyperbolic 3-manifold would correspond to a fundamental domain of a tessellation. It would be the equivalent of a unit cell both in position space and momentum space. These unit cells would correspond to each other at the H^3 level by M^8-H duality. Both would involve discretization. By finite momentum and position resolution UP would be reduced to the interior of the finite tessellation analogous to finite crystal. Quantum-classical correspondence and inversion are consistent with the realization of the UP related to Bohr's orbitology. Momenta in H^3_m would be mapped to equivalents of plane waves, i.e. superpositions of positions of the fundamental region in the tessellation. This picture generalizes to the multi-fermion states. Each fermion momentum defines a cusp and fermionic statistics makes it possible to avoid several cusps at the same points. Fermions for which other quantum numbers, such as spin differ, can however have the same momentum. They should correspond to the same cusp. How can this make sense? Could S^3 be involved somehow. Could they correspond to different holes in S^3 whose sizes and locations correlate with the other quantum numbers somehow? I have considered this problem earlier in the twistor picture where spin corresponds to a geometric degree of freedom in twistor space, which has identification at the level of M^8. The space of causal diamonds (CDs) as a kind of spine of WCW is discussed in [L169]. Lorentz transformations also occur at the level of CDs. The moduli of CD correspond to cm degrees of freedom in H. The finite volume of CD allows states for which Poincare quantum numbers are not exactly opposite for the boundaries of CD. Therefore the values of the total Poincare quantum numbers can be assigned to the CD. Only at the limit of infinitely large CDs does the zero energy property become exact. Therefore the CD wave function carries genuine information. At the p-adic level, translations and Lorentz transformations have the same effect as transformations of a compact group. Translations or Lorentz transformations of order $O(p^n)$ do not increase the p-adic norm of a point.

20.4 Holography

4-D general coordinate invariance forces holography at the level of $H = M^4 \times CP_2$ and one can regard space-time surfaces as analogues of Bohr orbits determined almost uniquely by 3-D surfaces. Quantum TGD is therefore very much like wave mechanics with point-like particles replaced with 3-surfaces in turn replaced with 4-D Bohr orbits. In fact, a wave-mechanical toy model for TGD would replace electron wave functions in atoms with wave functions in the space of its Bohr orbits.

M^8 is analogous to the momentum space in wave mechanics and the 4-surfaces in M^8 obey number theoretical holography based on associativity.

20.4.1 What does one mean with holography?

Consider now a more precise definition of holography.

1. The standard form of holography as $3 \rightarrow 4$ assigning to a 3-surface at the boundary of causal diamond (CD) an almost unique 4-D surface is the weakest form of holography. The non-uniqueness of the holography forces zero energy ontology (ZEO) in which analogues of Bohr orbits are basic geometrical objects.

2. $2 \rightarrow 4$ holography is the strongest form of holography. I have called it strong holography (SH). The 2-D partonic surfaces and possibly also the string world sheets would encode the data about the 4-surface and also the data about quantum holography. The strong form of holography could be realized as super symplectic and super-Kac Moody invariance and super-conformal invariance being minimally broken. Only the scaling generator L_0 would not annihilate the states. This condition is however too strong.

3. For the weaker form of SH super-symplectic and conformal symmetries are broken such that the algebras A_n (there are several of them), whose conformal weights are n-multiples of the conformal weights of the entire algebra A, and $[A_n, A]$ annihilate the physical states [L157, L87]. This requires half-algebra with non-negative conformal weights.

   The breaking hierarchy labelled by the values of n makes sense also for the ordinary conformal invariance but to my best albeit non-professional knowledge is not considered as a physical option. Hierarchies corresponding to the inclusion hierarchies of rational extensions and HFFs are obtained.

Both holographies set very strong conditions for the 3-surfaces appearing as holographic data.

Role of polynomials

At the level of M^8 physics is algebraic as it is also for the momentum space in the case of free field theory and reduces to algebraic conditions like mass shell condition and orthogonality of polarization vector and momentum. Polynomials P having integer coefficients determined the physics.

1. P as such does not fix the 4-surface nor even the 3-surface defining the data for number theoretic holography.

2. The polynomial P must have integer coefficients to guarantee number theoretical universality in the sense that they make sense also in p-adic number fields. If the coefficients are smaller than the degree of P, also finite fields become natural mathematical structures in TGD so that all number fields are involved. The roots of P give rise to the mass shells in M_c^8 with mass squared values defined by the roots of P. The roots define an extension of rationals.

3. Polynomials are also characterized by ramified primes as the divisors of the discriminant of the polynomial determined by the product of the differences of its roots [L170]. They are not a property of the algebraic extensions. They depend on P and the exponent of the classical action is proposed to correspond to the discriminant D. Ramified primes are identified as p-adic primes playing a central role in p-adic mass calculations [K65].

The role of fermions

Quantum classical correspondence requires that the 3-surfaces Y^3 at the mass shells are determined by the quantum numbers of fermions associated with the quantum states. What assumptions could provide this additional data and how could this data be coded to the geometry of Y^3?

The data in question correspond to fermion momenta, spins and electroweak quantum numbers. Color does not appear as spin-like quantum numbers but corresponds to color partial waves in CP_2. Consider next how momenta are coded to the properties of 3-surfaces.

1. I have proposed that the 3-surfaces Y^3 in H_m^3 could correspond to the fundamental domains of tessellations of H^3. The unit cell of ordinary crystal in E^3 serves as an analog for the fundamental domain of a tessellation in H^3. The disjoint components of Y^3 would naturally

correspond to surfaces Y_i^3 at H_m^3 and would correspond to fundamental domains of analogs of finite crystals formed by gluing them together.

The points of $E^4 \subset M^4 \times E^4$ correspond to 3-sphere with radius determined by mass m and for given Y_i^3 the values of E^4 coordinates would be constant. A stronger condition would be that the values are the same for all Y_i^3. At the cusp points the point would be replaced by S^3, which could touch two disjoint sheets with different values of S^3 coordinates. Since the metric becomes singular at cusp, a natural proposal is that a small hole is drilled around the point and to S^3 and they are glued along their boundaries. The scale of the hole would be determined by the mass.

2. TGD predicts as basic objects also string-like objects $X^2 \times R \subset H$ and their deformations to magnetic flux tubes. By $M^8 - H$ duality they are expected to be present also in M^8 in particular at the hyperboloids H_m^3.

   There are two kinds of string-like objects depending on whether their CP_2 projection is homologically trivial or not. In the latter case the string carries monopole flux.

Quantum classical correspondence suggests that the momenta of fermions as points of $H_m^3 \subset M^8$ are coded into the geometry of Y^3 as singularities. $M^8 - H$ duality based on inversion in turn maps the momenta to singular points of H_a^3.

1. Singularities would be naturally cusps as analogs of tips of algebraic surfaces allowing all normal spaces of Y^4 at the singularity: M^8 duality would assign a 3-D subset of CP_2 to the tip.

2. Is it possible to have singularities, where the throats of the opposite wormhole throats touch? Or could the wormhole throats of the incoming partons fuse to single throat? This could occur in the topological counterpart of 3-vertex describing pair annihilation to a single particle. The singularities emerging in this way could relate to the description of the creation of fermion-antifermion pairs and would also define defects essential for exotic differential structures occurring only in dimension $D = 4$ [L152].

Can one code also spin to geometry or should it be regarded as a fermionic quantum number?

1. At the level of H one would have a product of twistor spaces $T(M^4)$ and $T(CP_2)$: these twistor spaces are unique in the sense that they have Kähler structure. This makes H a unique choice for the embedding space.

   Twistorialization replaces space-time surface with 6-D surface $X^6 \subset T(M^4) \times T(CP_2)$ having S^2-bundle structure as possessed also by $T(M^4)$ and $T(CP_2)$. Spinor description of spin and electroweak isospin is replaced by a wave function in twistor spheres S^2.

   The embedding corresponds to dimensional reduction producing X^6 as S^2 bundle. The twistor spheres associated with M^4 and CP_2 must be identified by the embedding of $X^6 \subset T(M^4) \times T(CP_2)$.

   The identification of the twistor spheres forces spin doublets to correlate almost completely with electroweak spin doublets apart from the directions of the two spins. This picture allows only spin- and electroweak spin doublest. Does this force a complete correlation between the values of spin and electroweak to be identical or do the details of the identification for the the embedding of $X^6 \subset T(M^4) \times T(CP_2)$ allow to regard spin and electroweak spin as independent?

   The identification of the two twistor spheres is not unique. The spin rotations and possibly also electroweak spin rotations (, which are not isometries) changes the identification of the two twistor spheres. This would make spin and electroweak spin independent quantum numbers.

   One can argue that only the relative rotation of the two spheres matters. Could this mean that electroweak spin axes can be thought of being completely fixed. Electroweak quantization axes are indeed completely fixed physically.

2. Something similar could happen at the level of M^8. Now one must consider twistor spaces of M^4 and E^4 and similar embedding of a 6-D surface $X^6 \subset M^8$ as twistor space with S^2 bundle structure.

   In M^8 one would have an algebraic description of spin and electroweak spin instead of a wave function at S^2. A direction of S^2 would define a quantization axis and the diametrically opposite points of S^2 associated with it would provide a geometric correlate for the spin and electroweak spin values of fermion. The relative rotations of the twistor spheres of M^4 and E^4 associated with their identification are also now possible so that the two quantum numbers can be regarded as independent but with the electroweak quantization axes fixed.

In the twistorial picture one would have $5 \rightarrow 6$ weak holography or even $4 \rightarrow 6$ strong almost unique holography.

20.4.2 What kind of 3-geometries are expected in the TGD framework?

To get a wider perspective, it is good to have an overall view of the Geometrization conjecture of Thurston `https://rb.gy/9x3pm` proven by Perelman by studying Ricci flows. Geometrization theorem implies Poincare conjecture and so called spherical space conjecture.

The inspiration comes from the classification of 2-D manifolds expressed by uniformization theorem (`https://rb.gy/ts8va`). There are only 3 closed simply connected Riemann manifolds: sphere, disk, and hyperbolic plane. These are constant curvature spaces with corresponding Lie groups of isometries. One can obtain connected closed 2-manifolds with a nontrivial fundamental group by identifying the points related by a discrete subgroup of isometries.

In the case of the hyperbolic plane the isometry group is infinite and gives rise to a nontrivial fundamental group. For the hyperbolic plane, one obtains 2-manifolds with nonvanishing genus allowing a negative constant curvature. Constant curvature can be normalized to be -1, 1 or 0 in various cases. For non-vanishing curvature, the area serves as a topological invariant. For torus this is not the case.

The following provides the summary of my non-professional understanding of the 3-D case. The TGD inspired comments rely on what I know from the universal preferred extremals of practically any variational principle which is general coordinate invariant and can be constructed from the induced geometric quantities. They are always minimal surfaces outside 3- or lower-dimensional singularities at which the field equations depend on the action. The known extremals are discussed in [K9, K15, L146].

1. Thurston's conjecture `https://rb.gy/9x3pm` states that every oriented and closed irreducible (prime) 3-manifold can be cut along tori, so that the interior of each of the resulting manifolds has a geometric structure with a finite volume which becomes a topological invariant in geometric topology. For instance, knots give rise to a 3-manifold in this way.

   An important difference is that the closed 3-manifold decomposes to a union of different types of 3-manifolds rather than only of single type as in the 2-D case.

2. The notion of model geometry is essential. There exists a diffeomorphism to X/Γ for some model geometry such that Γ is a discrete subgroup of a Lie group of isometries acting in G. There are 8 types of model geometries.

3. Irreducible 3-manifolds appear as building bricks of 3-manifolds using connected sum. There are 8 types of model geometries for closed prime 3-manifolds, which by definition do not allow a connected sum decomposition. These geometries are E^3, S^3, H^3, $S^2 \times R$, $H^2 \times R$, $SL(2,"R")$, Nil, and $Solve$.

4. The model geometries allow a constant curvature metric. The finite volume of the manifold becomes a topological characteristic if one has constant curvature equal to ± 1.

5. All types except one, $S^2 \times R$, which corresponds to a string-like objects in TGD, allow a 3-D Lie group as subgroup of isometries (Bianchi group).

6. All model geometries except hyperbolic manifold (`https://rb.gy/snpft`) and Solv manifold are Seifert fiber spaces (`https://rb.gy/uxszk`), which are fibered by S^1 fiber. Hyperbolic manifolds are atoroidal but have an infinite fundamental group since Γ must be infinite from the finite volume property. Atoroidality means that there is no embedding of torus which would not be parallel to the boundary of the hyperbolic manifold.

   The finite volume property of H^3/Γ also requires that Γ is a lattice: this implies a deep analogy with condensed matter physics. The group elements in the TGD frame-work be $SL(2,C)$ matrices with elements which are algebraic integers in an extension of rationals defined by the polynomial P defining 4-surface in M^8. Note that also momentum components are predicted to be algebraic integers using a unit defined by the scale of the causal diamond (CD).

   TGD leads to the proposal [K102] that the H^3 lattices could appear in cosmological scales and explain "God's fingers" [E3] discovered by Halton Arp. They are astrophysical objects appearong along a line and having quantized redshifts.

7. One can form the spaces of the orbits for a discrete subgroup $\Gamma \subset G$ to obtain 3-manifolds with non-trivial fundamental group or orbifolds as in the case of S^3 and $S^2 \times R$. For hyperbolic 3-manifolds, the fundamental group is infinite and generated by elements of parabolic subgroups of G (`https://rb.gy/b5t55`). Cusp point and cusp neighborhood (`https://rb.gy/fd4dz`) are related to the infinite part of the fundamental group. Since parabolic subgroups $P \subset \Gamma$ are infinite groups, the fundamental group of the hyperbolic manifold is infinite.

8. One can decompose an irreducible 3-manifold to pieces, which are either Seifert manifolds or atoroidal. All 8 model geometries except hyperbolic geometries and so called Solv manifolds are Seifert manifolds.

   Solv manifolds are fiber spaces over a circle with 2-D plane with Minkowski signature as a fiber. In TGD solv manifolds could correspond to the so-called massless extremals [K9, K15, K82] serving representing classical radiation fields having only Fourier components with wave vectors in a single direction: laser beam is a good analog for them. They are not embeddable to H^3.

   In Ricci flows the hyperbolic pieces expand whereas other pieces contract so that asymptotically the manifold becomes hyperbolic. In fact, the collapse occurs in some caes in a finite time as found already by Richard S. Hamilton. The flow "kills" the positive curvature geometries S^3 and $S^2 \times R$ in the connected sum. What is left at large times is "thick-thin" decomposition. The "thick" piece is a hyperbolic geometry whereas the "thin" piece is a so-called graph manifold.

Hyperbolic manifolds and Seifert fiber spaces

Hyperbolic space and Seifert fiber space (`https://rb.gy/uxszk`) are in a central role in the TGD framework and therefore deserve short discussion. The following just gives the basic definitions and brief TGD inspired comments.

1. Hyperbolic manifolds

A hyperbolic n-manifold (`https://rb.gy/2esup`) is a complete Riemannian n-manifold of constant sectional curvature. Every complete, connected, simply-connected manifold of constant negative curvature -1 is isometric to the real hyperbolic space H^n. As a result, the universal cover of any closed manifold M of constant negative curvature -1.

Every hyperbolic manifold (`https://rb.gy/snpft`) can be written as H^n/Γ, where Γ is a torsion-free discrete group of isometries on H^n. That is, Γ is a discrete subgroup of $SO^+_{1,n}$. The manifold has a finite volume if and only if Γ is a lattice.

Its "thick–thin" decomposition has a "thin" part consisting of tubular neighborhoods of closed geodesics and ends which are the product of a Euclidean $(n-1)$-manifold and the closed half-ray. The manifold is of finite volume if and only if its "thick" part is compact.

In the TGD framework, the lattice structure is natural and would mean that the elements of the matrices of Γ *are algebraic extensions in the extension of rational defined by the polynomial* P *determining* Y^4*. The tubular neighborhoods of the "thin" part would correspond to string-like objects (tubular neighborhoods) as geodesics whereas the ends would correspond to cusp singularities inducing blow-up as 3-surface* $S^3 \subset E^3$*.*
At the level of H *the tubular neighborhoods correspond to a string-like object and their ends to* CP_2 *type extremals serving as building bricks of elementary particles. Hadronic strings would represent examples of string-like objects and all elementary particles would involve them as monopole flux tubes connecting wormhole contacts.*

For $n > 2$ the hyperbolic structure on a finite volume hyperbolic n-manifold is unique by Mostow rigidity theorem and so geometric invariants are in fact topological invariants. One of these geometric invariants used as a topological invariant is the hyperbolic volume of a knot or link complement, which can allow us to distinguish two knots from each other by studying the geometry of their respective manifolds.

The identification of geometric invariants as topological invariants conforms with the TGD vision about "holy trinity" geometry-number theory-topology. Number theory would leak in through the identification of Γ *as a lattice determined by the polynomial* P*.*

2. Seifert fiber spaces

A Seifert manifold `https://rb.gy/uxszk` is a closed 3-manifold together with a decomposition into a disjoint union of circles (called fibers) such that each fiber has a tubular neighborhood that forms a standard fibered torus.

A standard fibered torus corresponding to a pair of coprime integers (a, b) with $a > 0$ is the surface bundle of the automorphism of a disk given by rotation by an angle of $2\pi b/a$ (with the natural fibering by circles). If $a = 1$ the middle fiber is called ordinary, while if $a > 1$ the middle fiber is called exceptional. A compact Seifert fiber space has only a finite number of exceptional fibers.

The set of fibers forms a 2-dimensional orbifold, denoted by B and called the base — also called the orbit surface — of the fibration. It has an underlying 2-dimensional surface B_0, but may have some special orbifold points corresponding to the exceptional fibers.

The definition of Seifert fibration can be generalized in several ways. The Seifert manifold is often allowed to have a boundary (also fibered by circles, so it is a union of tori). When studying non-orientable manifolds, it is sometimes useful to allow fibers to have neighborhoods that look like the surface bundle of a reflection (rather than a rotation) of a disk, so that some fibers have neighborhoods looking like fibered Klein bottles, in which case there may be one-parameter families of exceptional curves. In both of these cases, the base B of the fibration usually has a non-empty boundary. 6 of the 8 basic geometries of Thurston are Seifert fiber spaces.

In the TGD framework, the Seifert fiber spaces would correspond to string-like objects, which appear as several variants.

The eight simply connected 3-geometries appearing in the Thurston's conjecture from the TGD point of view

This section contains as almost verbatim the description of the 8 Thurston geometries provided by the Wikipedia article `https://rb.gy/9x3pm`. There is a good reason for this: I am not a professional and do not understand the technical details. There is a good reason for not giving a mere Wikipedia link: I have added comments relating to the TGD based identification of these model geometries as 3-surfaces.

It turns out that the geometries could correspond to fundamental regions of H^3, to energy $E = constant$ (M^4 time $t = constant$ in H) surfaces $D^3 \subset M^4_+ \subset M^8$, to string-like objects $X^2 \times R$ allowing Seifert fiber space structure, or to masless extremals with structure $M^2 \times E^2$ with M^2 and E^2 corresponding to the orthogonal planes defined by light-like momentum and polarization vector.

First some definitions:

1. A model geometry is a simply connected smooth manifold X together with a transitive action of a Lie group G on X having compact stabilizers (the isotropy group of a point is compact).

2. A model geometry is called maximal if G is maximal among groups acting smoothly and transitively on X with compact stabilizers.This condition can be aso included in the definition of a model geometry.

3. A geometric structure on a manifold M is a diffeomorphism from M to X/Γ for some model geometry X, where Γ is a discrete subgroup of G acting freely on X; this is a special case of a complete (G, X)-structure. If a given manifold admits a geometric structure, then it admits a structure, whose model is maximal.

   One can say that the spaces X provide the raw material from which one obtains various 3-geometries by identifications using a discrete subgroup of G.

A 3-dimensional model geometry X is relevant for the geometrization conjecture if it is maximal and if there is at least one compact manifold with a geometric structure modelled on X. Thurston classified the 8 model geometries satisfying these conditions; they are listed below and are sometimes called Thurston geometries. (There are also uncountably many model geometries without compact quotients.)

There is a connection with the Bianchi groups, which are the 3-dimensional Lie groups. Most Thurston geometries can be realized as a left invariant metric on a Bianchi group. However, $S^2 \times R$ does not allow Bianchi geometry; Euclidean space corresponds to two different Bianchi groups; and there are an uncountable number of solvable non-unimodular Bianchi groups, most of which give model geometries having no compact representatives.

1. Spherical geometry S^3

The point stabilizer is $O(3, R)$, and the group G is the 6-dimensional Lie group $O(4, R)$, with 2 components. The corresponding manifolds are exactly the closed 3-manifolds with a finite fundamental group. Examples include the 3-sphere, the Poincaré homology sphere, and lens spaces. This geometry can be modeled as a left invariant metric on the Bianchi group of type IX. Manifolds with this geometry are all compact, orientable, and have the structure of a Seifert fiber space (often in several ways). The complete list of such manifolds is given in the article on spherical 3-manifolds. Under Ricci flow, manifolds with this geometry collapse to a point in finite time.

> *In the TGD framework S^3 geometry could be associated with cusp singularities (see `https://rb.gy/0p30o` and `https://rb.gy/fd4dz`) of hyperbolic 3-manifold and represent the blow-up of a the cusp to S^3 which can be regarded as sphere in $E^4 \subset M^8 = M^4 \times E^4$. This is mapped to a 3-sphere of CP_2 in $M^8 - H$-duality.*

2. Euclidean geometry E^3

The point stabilizer is $O(3, R)$, and the group G is the 6-dimensional Lie group $R^3 \times O(3, R)$, with 2 components. Examples are the 3-torus, and more generally the mapping torus of a finite-order automorphism of the 2-torus; see torus bundle. There are exactly 10 finite closed 3-manifolds with this geometry, 6 orientable and 4 non-orientable. This geometry can be modeled as a left invariant metric on the Bianchi groups of type I or VII_0.

Finite volume manifolds with this geometry are all compact, and have the structure of a Seifert fiber space (sometimes in two ways). The complete list of such manifolds is given in the article on Seifert fiber spaces. Under Ricci flow, manifolds with Euclidean geometry remain invariant.

> *In M^8 one has two kinds of roots of polynomials. For the first option they correspond mass square values defining mass shells H^3 . For the second option applying to the light-cone boundary as mass shell, energy E replaces mass and roots correspond to discrete energies. E = constant surface corresponds to E^3 as 3-balls inside the light-cone.*

3. Hyperbolic geometry H^3

The point stabilizer is $O(3,R)$, and the group G is the 6-dimensional Lie group $O^+(1,3,R)$, with 2 components. There are enormous numbers of examples of these, and their classification is not completely understood. The example with the smallest volume is the Weeks manifold. Other examples are given by the Seifert–Weber space, or "sufficiently complicated" Dehn surgeries on links, or most Haken manifolds.

The geometrization conjecture implies that a closed 3-manifold is hyperbolic if and only if it is irreducible, atoroidal, and has an infinite fundamental group. This geometry can be modeled as a left invariant metric on the Bianchi group of type V or $\mathrm{VII}_{h\neq 0}$. Under Ricci flow, manifolds with hyperbolic geometry expand.

In TGD H^3 has an interpretation as a mass shell in $M^4 \subset M^8$ determined by the roots of the polynomial P or as a light-cone proper time constant hyperboloid in M^4.

This geometry does not allow Seifert fiber space structure unlike most other geometries.

4. The geometry of $S^2 \times R$

The point stabilizer is $O(2,R) \times Z/2Z$, and the group G is $O(3,R) \times R \times Z/2Z$, with 4 components. The four finite volume manifolds with this geometry are: $S^2 \times S^1$, the mapping torus of the antipode map of S^2, the connected sum of two copies of 3-dimensional projective space, and the product of S^1 with two-dimensional projective space.

The first two are mapping tori of the identity map and antipode map of the 2-sphere, and are the only examples of 3-manifolds that are prime but not irreducible. The third is the only example of a non-trivial connected sum with a geometric structure. This is the only model geometry that cannot be realized as a left invariant metric on a 3-dimensional Lie group.

Finite volume manifolds with this geometry are all compact and have the structure of a Seifert fiber space (often in several ways). Under normalized Ricci flow manifolds with this geometry converge to a 1-dimensional manifold.

In the TGD framework, these surfaces could correspond to the simplest string-like objects for which S^2 corresponds to a geodesic sphere (homologically trivial or non-trivial) with a finite length connecting fundamental regions of H^3 or finite tessellations formed by them. S^2, which would correspond to a 2-D surface in CP_2 would be the base and string the fiber. One might argue that S^2 is more natural as fiber.

5. The geometry of $H^2 \times R$

The point stabilizer is $O(2,R) \times Z/2Z$, and the group G is $O^+(1,2,R) \times R \times Z/2Z$, with 4 components. Examples include the product of a hyperbolic surface with a circle, or more generally the mapping torus of an isometry of a hyperbolic surface.

Finite volume manifolds with this geometry have the structure of a Seifert fiber space if they are orientable. (If they are not orientable the natural fibration by circles is not necessarily a Seifert fibration: the problem is that some fibers may "reverse orientation"; in other words their neighborhoods look like fibered solid Klein bottles rather than solid tori.) The classification of such (oriented) manifolds is given in the article on Seifert fiber spaces. This geometry can be modeled as a left invariant metric on the Bianchi group of type III. Under normalized Ricci flow manifolds with this geometry converge to a 2-dimensional manifold.

In the TGD context, these geometries would correspond to closed string-like objects for which the CP_2 projection is a 2-surface with genus $g > 0$. Seifert fiber space property corresponds to closed strings.

6. The geometry of the universal cover of $SL(2,"R")$

The universal cover of $SL(2,R)$ is denoted $\widetilde{\mathrm{SL}}(2,\mathbf{R})$. It fibers over H^2, and the space is sometimes called "Twisted $H^2 \times R$". The group G has 2 components. Its identity component has the structure $(\mathbf{R} \times \widetilde{\mathrm{SL}}_2(\mathbf{R}))/\mathbf{Z}$. The point stabilizer is $O(2,R)$.

Examples of these manifolds include: the manifold of unit vectors of the tangent bundle of a hyperbolic surface, and more generally the Brieskorn homology spheres (excepting the 3-sphere and the Poincare dodecahedral space). This geometry can be modeled as a left invariant metric on the Bianchi group of type VIII or III. Finite volume manifolds with this geometry are orientable and have the structure of a Seifert fiber space. The classification of such manifolds is given in the article on Seifert fiber spaces. Under normalized Ricci flow manifolds with this geometry converge to a 2-dimensional manifold.

Also now the interpretation as a closed string-like entity is possible in TGD.

7. Nil geometry

This fibers over E^2, and so is sometimes known as "Twisted $E2 \times R$". It is the geometry of the Heisenberg group. The point stabilizer is $O(2,R)$. The group G has 2 components, and is a semidirect product of the 3-dimensional Heisenberg group by the group $O(2,R)$ of isometries of a circle. Compact manifolds with this geometry include the mapping torus of a Dehn twist of a 2-torus, or the quotient of the Heisenberg group by the "integral Heisenberg group". This geometry can be modeled as a left invariant metric on the Bianchi group of type II.

Finite volume manifolds with this geometry are compact and orientable and have the structure of a Seifert fiber space. The classification of such manifolds is given in the article on Seifert fiber spaces. Under normalized Ricci flow, compact manifolds with this geometry converge to R^2 with the flat metric. *In TGD also this geometry might be assigned with a closed string-like object as all Seifert fiber spaces.*

8. Sol geometry

This geometry (also called Solv geometry) fibers over the line with fiber the plane, and is the geometry of the identity component of the group G. The point stabilizer is the dihedral group of order 8. The group G has 8 components, and is the group of maps from 2-dimensional Minkowski space to itself that are either isometries or multiply the metric by - 1. The identity component has a normal subgroup R^2 with quotient R, where R acts on R^2 with 2 (real) eigenspaces, with distinct real eigenvalues of product 1.

This is the Bianchi group of type VI_0 and the geometry can be modeled as a left invariant metric on this group. All finite volume manifolds with solv geometry are compact. The compact manifolds with solv geometry are either the mapping torus of an Anosov map of the 2-torus (such a map is an automorphism of the 2-torus given by an invertible 2 by 2 matrix whose eigenvalues are real and distinct, such as

$$\left(\begin{array}{cc} 2 & 1 \\ 1 & 1 \end{array}\right)) \ ,$$

or quotients of these by groups of order at most 8. The eigenvalues of the automorphism of the torus generate an order of a real quadratic field, and the solv manifolds can be classified in terms of the units and ideal classes of this order. Under normalized Ricci flow compact manifolds with this geometry converge (rather slowly) to R^1.

Unlike in the case of Seifert fiber spaces, a plane or disk appears as a fiber. Could one consider the possibility whether boundary conditions guaranteeing conservation laws could allow string-like objects for which the cross section is disk rather than a closed 2-surface? The appearance of isometries of 2-D Minkowski space suggests that the disk X^2 must have Minkowski signature so that the embedding to H^3 would not be possible.
Could one assign this structure to massless extremals [K9, K82], which in the TGD framework define the representations for classical radiation fields, which involve the decomposition $M^4 = M^2 \times D^2$. The circle $S^1 \subset D^2$ defining its boundary would define the base space. Boundaries would be light-like and might allow to solve the boundary conditions. It is not clear how to obtain the counterparts of massless extremals at the M^8 level.

20.4.3 $3 \rightarrow 4$ form of holography

One can consider two forms of holography. The first, weak, form corresponds to the ordinary $3 \rightarrow$ holography in which 3-D boundaries provide the data defining the 4-surface. The second, strong form, corresponds to $2 \rightarrow 4$ holography in which conformal boundaries provide the data defining the 4-surface. In this section the $3 \rightarrow 4$ form of the holography is considered.

Fundamental domains of hyperbolic tessellations as data for $3 \rightarrow 4$ holography

Good candidates for the surfaces Y_i^3 are fundamental domains assignable to hyperbolic 3-manifolds H^3/Γ_i represented as surfaces in $H^3 \subset M^4 \subset M^8$ (or its complexification). In the case of string-like objects, the fundamental domains would correspond to the analogs of fundamental domains for $S^2/\Gamma \times R$ and $H^2/\Gamma \times R$. The treatment of this case is a rather straightforward modification of the first case so that the discussion is restricted to H^3/Γ_i.

The surfaces Y_i^3 would correspond topologically to many-particle states of free particles. Holography would induce topological interactions in the interior of Y^4 and $X^4(Y^4)$. The momenta (positions) of the fermions analogous to valence quarks correspond to the cusp singularities.

For fundamental fermions momenta would have components, which are algebraic integers. Galois confinement states that the momenta for many-fermion states are ordinary integers. This poses a condition for H^3/Γ and it would be interesting to understand what the condition means.

The degrees of freedom orthogonal to H^3 correspond to a complexified sphere S^3 of E^4, whose radius squared corresponds to the square of the complex mass squared.

1. Hyperbolicity is a generic property of 3-manifolds and probably preserved in small enough deformations. In other words, deformations of hyperbolic 3-manifold X_i^3 probably allow a hyperbolic metric although the induced metric for the deformation is not in general hyperbolic.

   Deformation of the hyperbolic manifold (`https://rb.gy/snpft`) could take place in its evolution defining Y_i^4 and $X^4(Y_i^4)$ and could lead to, for example, to singularities such as the touching of different surfaces and interaction vertices at which partonic 2-surfaces meet.

2. There is an interesting connection to the geometrization conjecture of Thurston (`https://rb.gy/9x3pm`), especially with the work of the Russian mathematician Grigori Perelman, who studied 3-D Ricci flows (`https://rb.gy/n6qlv`) for metrics and proved that, apart from scaling, they lead to hyperbolic geometries.

   Interestingly, hyperbolic manifolds decompose into "thin" and "thick" pieces and the "thin" piece corresponds to cusp neighborhoods (`https://rb.gy/fd4dz`). This decomposition brings in mind the notions of valence partons and sea partons with sea partons, in particular gluons assignable to the interior of Y_i^3 and giving the dominant contribution to the hadron mass.

Consider now what one can assume about Y_i^3.

1. The simplest assumption is that the S^3 coordinates are constant for Y_i^3 identified as the fundamental domain of a tessellation defined by H^3/Γ. It would represent a piece of H^3.

   Could one consider the allowance of S^3 deformations H_d^3 of H^3 in the direction of S^3, which are invariant under G so that the space H_d^3/G would exist. They would define what mathematicians would call a model of hyperbolic geometry.

2. Can one allow for a given Y_i^3 a multiple covering of H^3 by copies of Y_i^3 with different constant values of S^3 coordinates? Could this state correspond topologically to a many-sheeted covering naturally associated with the polynomial P?

   An interesting possibility is that Galois symmetry implies the existence of several copies of Y_i^3 with different S^3 coordinates as the orbit of the Galois group or its sub-group. Z_2 would be the simplest Galois group and give two sheets.

3-D data for $3 \rightarrow 4$ holography with 3-surfaces as hyperbolic 3-manifolds

It is good to start with questions.

1. Could the 3-surfaces X^3 associated with the mass shells $H_m^3 \subset M^8$ appearing as holographic data be fundamental domains (analogs of unit cell for crystals) of the tessellation H^3/Γ? Could a fermionic many-particle state for an algebraic extension determined by a given polynomial P assign a singularity to the fundamental domain and fix it?

   The TGD view of hadron physics provides some clues. Gluon sea consists of gluons identifiable as fermion-antifermion pairs and fermions and antifermions. Here is the data for the given hyperbolic 3-manifold of singularities. The valence fermions could reside at throats and virtual sea gluons could be associated with strings $Y^2 \times R$ inside flux tubes and would give to the classical string tension?

Hyperbolic 3-manifolds also have string-like singularities connecting the cusp singularities. In the physical picture of TGD, these would correspond to strings connecting wormhole throats of different wormhole contacts which in turn would correspond to blow-ups of cusps.

2. Is the situation the same in M^8 and H? Could 4-surfaces assignable to the X_i^3 be minimal surfaces in both H and M^8 having a generalization of holomorphic structure to dimension 4? It would be possible to map X_i^3 to each other by inversion. Note that M^8-H correspondence would map the $M^4 \subset M^8$ projections of the points of Y^4 by inversion to H also in the interior of 4-surface.

   Could this realize the dual conformal invariance proposed by the twistors, which would therefore be behind the analogy of Langlands duality and $M^8 - H$ duality?

20.4.4 Strong form of the hyperbolic holography

Holography roughly means an assignment of, not necessarily a unique 4-surface, to a set of 3-surfaces at the mass shells defined by roots of the polynomial P. The 4-surface is analogous to Bohr orbit.

A stronger form of the holography would be approximate $2 \rightarrow 4$ holography suggesting that the 3-surfaces allow $2 \rightarrow 3$ holography, which need not be completely deterministic. To understand what is involved one must have an idea about what kind of 3-surfaces could be involved.

1. Irreducible closed 3-surfaces Y_i^3 at H_m^3 consist of regions of 8 different types. Could these regions correspond to model geometries or at least have the symmetries of model geometries?

   This conjecture is natural if the 3-surfaces $Y_i^3 \subset H_m^3 \subset M^4 \subset M^8$ belong to (possibly complex) mass shells of M^4. In this case, the composites of fundamental regions of hyperbolic manifolds (`https://rb.gy/snpft`) as analogs of finite crystals would be natural.

   The interiors of these regions would correspond to the "thick" part of the 3-manifold whereas the cusp singularities and string singularities as boundaries of string world sheets would correspond to the "thin" part. The blow-ups of cusp singularities would give rise to 3-D regions of CP_2.

2. Also monopole flux tubes connecting hyperbolic regions to form a network should be involved. Here the natural model geometries would be of type $S^2 \times R$ or $H^2 \times R$ with the ends of R at the two hyperbolic regions. By replacing H^2 with H^2/Γ, one would obtain higher flux tubes with a cross section having a higher genus.

The natural idea is that hyperbolic holography gives rise to $2 \rightarrow 3$ holography. In the case of H^3/Γ, the holography would assign H^3/Γ its fundamental region Y^3 to H^2/Γ.

In the case of H^2/Γ, applying for string-like objects, holography would assign $Y^2 \times R$ to a union of circles H^1/Γ defining its boundary. The rule would be simple: Y^n/Γ is a union of fundamental regions Y_i^n having H^{n-1}/Γ as boundary.

Hyperbolic holography from H^2/G to the fundamental domain of H^3/Γ

The representation of M^4 momenta in terms of bispinors is possible only for massless particles. This raises the question whether one must assume a strong form of holography in which 2-D surfaces at the boundaries of H_m^3 dictate the 4-D surface almost completely. The hyperbolic 2-manifold H^2/G should define the boundary for Y_i^3 identifiable as a fundamental domain Y^3 of a hyperbolic 3-manifold H^3/Γ.

1. This would conform with the proposed realization of super-symplectic invariance and Kac-Moody type symmetries for light-like partonic orbits meaning that the interior degrees of freedom associated with the 3-surfaces X_i^3 and light-like orbits of partonic 2-surfaces are eliminated with a suitable gauge choice formulated in terms of a generalization the Virasoro and Kac-Moody conditions [L154, L157].

2. Physically this would mean that the fermion momenta at cusp point are light-like. This would conform with the view that fermions move along light-like curves inside the light-like partonic orbit.

3. If hyperbolic holography makes sense, the above formulation for H^2 would generalize to the case of H^3. Cusp neighborhood U/P as a projection $U \rightarrow H^2/G$ has a counterpart for H^3/Γ and the fundamental domain for H^2/G extends to a fundamental domain for H^3/Γ. For H^2/G as boundary it would correspond to the condition $p_3 > 0$ for the momentum component in the chosen direction.

4. The cusp singularity is analogous to a cusp of an algebraic surface. This suggests that near the cusp point of H^3/Γ the metric behaves like the induced metric of 3-D cusp in 4-D space. Near the cusp one has $t = k\sqrt{\rho}$ where t and ρ are vertical coordinate and transversal coordinates of the cusp in 4-D space. The radial component of the induced metric orthogonal to tip direction should behave like $g_{\rho\rho} = 1 + k^2/4\rho$ and the radial distance from the tip would diverge logarithmically. One cold say that this point is missing so that the hyperbolic manifold is compact but not closed since it has boundaries. The singularity of the metric is a good motivation for cutting off a small ball around the singularity in M^4 and a small ball from S^3 and for gluing the two together along boundaries. At the level of H this would correspond to wormhole throat.

20.4.5 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \rightarrow f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex

Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. Is this deformation of H^3 in imaginary time direction equivalent with a region of H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} \ . \qquad (20.4.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \rightarrow Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \rightarrow 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \rightarrow SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

1. This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

2. A physically motivated proposal is that the real parts of $SU(3)_c$ gauge potential and color gauge field can be lifted to H and the lifts are equal to the classical gauge potentials and color gauge field proposed in H. Color gauge potentials in H are proportional to the isometry generators of the color gauge field and the components of the color gauge field are proportional to the products of color Hamiltonians with the induced Kähler form.

3. The color gauge field $Re(G)$ obeys the formula $Re(G) = dRe(A) + [Re(A), Re(A)] = [Re(A), Re(A)]$ and does not vanish since the contribution of $[Im(A), Im(A)]$ cancelling the real part is absent. The lift of $A_R = g^{-1}dg$ to H is determined by g using M^4 coordinates for Y^4 . The

M^4 coordinates $p^k(M^8)$ having interpretation as momenta are mapped to the coordinates m^k of H by the inversion

$$I : m^k = \hbar_{eff} Re(\frac{p^k}{p^2}) \ , \ p^2 \equiv p^k p_k \ ,$$

where p^k is complex momentum. $Re(A)_H$ is obtained by the action of the Jacobian

$$dI^k_l = \frac{\partial p^k}{\partial m^l} \ ,$$

as

$$A_H = dI \cdot Re(A_{M^8}) \ .$$

dI^k_l can be calculated as the inverse of the Jacobian $\partial m^k/\partial Re(p)^l$. Note that $Im(p^k)$ is expressible in terms of $Re(p^k)$.

For $Im(p^k) = 0$ the Jacobian for I reduces to that for $m^k = \hbar_{eff}\frac{p^k}{p^2}$ and one has

$$\frac{\partial m^k}{\partial p^l} = \frac{\hbar_{eff}}{p^2}(\delta^k_l - \frac{p^k p_l}{p^2}) \ .$$

This becomes singular for $m^2 = 0$. The nonvanishing of $Im(p^k)$ however saves from the singularity.

4. The $M^8 - H$ duality obeys a different formula at the light-cone boundaries associated with the causal diamond: now one has $p^0 = \hbar_{eff}/m^0$. This formula should be applied for $m^2 = 0$ if this case is encountered. Note that number theoretic evolution for masses and classical color gauge fields is directly coded by the mass squared values and holography.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.

2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.

4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.

5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same. The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of $M^8 - H$ duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4 \times CP_2$?

1. The selection of $SU(3) \subset G_2$ for ordinary $M^8 - H$ duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3 \subset M^8_c$ and its image $H^3 \subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?

2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3) \subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.

3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of $M^8 - H$ duality as something more fundamental than the ordinary $M^8 - H$ duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3 \times CP_2$ assignable to mass shells would make sense physically. In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1) \times U(1)$. Color gauge fields would be

non-vanishing at the mass shells but outside the mass shells one would have nonvanishing G_2 gauge fields.

There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1 \oplus 3 \oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$: this requires that the decomposition contains $3 \oplus \overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8 \oplus 3 \oplus \overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3) \times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x) \times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X_i^3.

20.5 Singularities, quantum classical correspondence, and hyperbolic holography

The point-like fermions and their 1-D trajectories appear as singularities of the minimal surfaces [L146]. Strings that connect fermions located at their ends, and string world sheets in the interior of X^4 appear also as singularities. Also partonic 2-surfaces separating Minkowskian and Euclidian regions should correspond to singularities of X_i^3 and their light-like radii.

There would therefore be singularities in dimensions $D = 0, 1, 2, 3$. These singularities should relate to the fundamental domains $Y_i^3 \subset H_m^3 \subset M^8$ and holography would suggest that they correspond to the singularities of 3-D hyperbolic manifolds (`https://rb.gy/snpft`).

20.5.1 Cusp singularities and fermionic point singularities

The singularities should be associated with hyperbolic manifolds Y_i^3 identified as fundamental domains of coset spaces H^3/Γ , that is, as effective geometries H^3/Γ defined by the boundary conditions for various "fields". In the same way as, for example, a torus geometry appears in condensed matter physics for a unit cell of lattice.

Cusp singularity (`https://rb.gy/fd4dz`) is a natural candidate for a point-like singularity and geometrically corresponds to a cusp. For abstract Riemann geometry, the cusp property would correspond to a singularity of the metric for a cusp (tip) and mean that the radial component of the metric diverges at the tip.

Consider first the basic concepts and ideas in the case of 2-D hyperbolic space H^2 and corresponding hyperbolic manifold H^2/G case.

1. Riemann surface can be regarded as a coset space H^2/G, which is represented as a fundamental region for a tessellation of H^2.

2. Cusp singularities of H^2/G correspond to parabolic subgroups P (`https://rb.gy/b5t55`) generated by a parabolic element for $G \subset SL(2, C)$. Parabolic subgroup P is isomorphic to a discrete group of translations along, say, the real axis as a boundary of the upper half-plane and is noncompact. It is represented as Möbius transformations induced by the matrices $(1, n : 0, n)$. P can be regarded as a subgroup generated by a Lorentz boost in a fixed direction.

The cusp singularity results from the identification of points related by the elements of P, which form a non-compact group. Let U denote the set with $Im(z) > 1$ which corresponds to the set $p^3 > 0$ in momentum space. U and $P(U)$ are disjoint. The cusp neighborhood (`https://rb.gy/fd4dz`) can be identified as the set U/P which is the projection of U to H/G.

3. In the simplest situation, one has $G \subset SL(2,Z) \subset SL(2,R) \subset SL(2,C))$, where $S(2,R)$ leaves the real axis invariant. Z could be replaced by an algebraic extension for rationals of algebraic integers in this extension.

$SL(2,C)$ and therefore also $SL(2,R)$ acts in M^4 as Lorentz transformations.

1. A given M^4 momentum has the representation $p^k = \overline{\Psi}\sigma^k\Psi$, $\Psi = (z_1, z_2)$. The representation is unique apart from a complex scaling of z_i so that $z = z_1/z_2$ can be taken as a complex coordinate for the plane and $SL(2,C)$ acts as Möbius transformation. $SL(2,R)$ leaves the real axis invariant.

   The automorphism of sigma matrices induced by $SL(2,C)$ transformation in turn induces Lorentz transformation in momentum space.

2. Under what conditions can bi-spinors correspond to M^4 coordinates? Bi-spinor can be assumed to be of the form $(z_1, z_2) = (z, 1)$. From the formula $p^k = \overline{\Psi}\sigma^k\Psi$, $\Psi = (z_1, z_2) = (z, 1)$ one can deduce an expression of the condition $Im(z_1/z_2) = Im(z_1) > 1$ in terms of p^k. The condition implies that the z-component of momentum satisfies $p^z = z\overline{z} - 1 > 0$.

   The description of M^4 momenta in terms of bi-spinors and H^2 identified as upper half-plane, denoted by H, is possible only for massless particles.

What happens at cusp singularity

What happens at the cusp singularity?

1. The normal space of the singularity is completely ill-defined as the direction of the electric field of a point-like charge. If so, CP_2 would always be a companion to the cusp. CP_2 would be a blow-up of the cusp points of X_i^3 as a hyperbolic manifold (`https://rb.gy/snpft`). One would have $X_i^3 \subset H^3$ and the cusp points would correspond to a 3-D sub-manifold of CP_2 defined by the normal spaces at the cusp singularity.

2. In the interior of the space-time surface the 3-D submanifold of CP_2 would extend to CP_2 type extremal with a light-like M^4 projection or its deformation. Several cusp singularities (see `https://rb.gy/0p30o` and `https://rb.gy/fd4dz`) could be associated with a single CP_2 type extremal representing wormhole contact. This corresponds to the view that wormhole throats can carry more than one fermion although the recent model assumes only a single fermion.

3. CP_2 type extremal defines a wormhole contact connecting two Minkowskian space-time sheets in $H = M^4 \times CP_2$. This would mean that the 3-D submanifold of CP_2 as a blow up is deformed to CP_2 type extremal with 2 throats at opposite sheets: at them the Euclidian induced metric transforms to Minkowskian signature.

   The conservation of monopole flux indeed forces the presence of two Minkowskian space-time sheets in the picture based on H. If the throat as a boundary of the 3-D region of $X^4 \subset H$ involves an incoming radial monopole flux, there must be another throat CP_2, where this flux runs to the other space-time sheet.

4. How could the throats connecting the two Minkowskian space-time sheets emerge in the M^8 picture? Should one allow several copies of Y_i^3 with the same H^3 projection but different constant S^3 coordinates and with a common cusp point. The blow-up would give several copies of 3-D regions of CP_2, and in holography they would define wormhole contact with 2 or even more throats.

   The simplest view is that quarks are the only fundamental fermions and leptons correspond to wormhole contacts carrying three antiquarks. They could have three throats associated with the same CP_2 type extremal but this is not the only possibility.

The singularities associated with string-like objects

For string-like objects, the fundamental domains would correspond to the analogs of fundamental domains for $S^2/\Gamma \times R$ and $H^2/\Gamma \times R$. For $S^2 \times R$ the spaces S^2/Γ, Γ a finite non-trivial subgroup of $SO(3)$ are orbifolds: the faces of Platonic solids are basic examples. For P^2/Γ one obtains $g > 0$ 2-manifolds with constant curvature metric with negative curvature.

The physical interpretation would be that $S^2 \times R$ and $H^2/\Gamma \times R$ are glued along their ends S^2 or P^2/Γ to partonic 2-surfaces associated with wormhole contacts.

For string-like objects, the fundamental domains would correspond to the analogs of fundamental domains for $S^2/\Gamma \times R$ and $H^2/\Gamma \times R$. For $S^2 \times R$ the spaces S^2/Γ are orbifolds if Γ is finite non-trivial subgroup of $SO(3)$: the triangular, quadrilateral, and pentagonal faces of Platonic solids are key examples. From these one can build finite lattices at S^2. For P^2/Γ one obtains $g > 0$ 2-manifolds with constant curvature metric with a negative curvature.

The physical interpretation would be that $S^2 \times R$ and $H^2/\Gamma \times R$ are glued along their ends S^2 or P^2/Γ to partonic 2-surfaces associated with wormhole contacts.

What could be the quantal counterpart for the geometric holography? This has been a long standing open question. Suppose that the strong form of holography is realized.

1. In [L161], I considered quantal holography as a counterpart of geometric holography discussed in this article. This led to a suggestion that valence quarks at the wormhole throats could pair with pairs of dark quark and antiquark at strings associated with magnetic flux tubes in the interior of the hadronic 3-surfaces. Could these strings correspond to string-like singularities assignable to geodesic lines inside fundamental regions of $H^3/Gamma$?

2. The flux tubes were proposed to have an effective Planck constant $h_{eff} > h$. The correspondence between valence quarks and dark quarks was proposed to be holographic. The spin and electroweak quantum numbers of dark antiquark would be opposite to those of valence quark and dark quark would have quantum numbers valence quark. There would be entanglement in color degrees of freedom for valence quark and dark antiquark to form color single: this would screen the color of valence quark and transfer it to the magnetic body. The holography in this way would allow a convergent perturbation theory. Nature would be theoretician friendly: a phase transition increasing h_{eff}, transferring color to dark quarks, and reducing color coupling strength to $\alpha_s = 2^2/4\pi\hbar_{eff}$ would occur.

   Whether the dark quark-antiquark pairs as analog for gluon pairs as explanation for hadron mass could explain most of hadron mass remained open: if the classical conserved quantities are identical with the quantum contribution from fermions for Cartan algebra, this could be the case. Whether they could correspond to sea gluons remains also an open question.

3. Quantal holography allowing to obtain a convergent perturbation theory might be realized quite generally, also for leptons which correspond to color partial waves in CP_2 neutralized by super symplectic generator [K65, L72] [L87].

It should be noticed that leptonic dark holography would be very natural if leptons consist of 3 antiquarks [L129]. This option would explain matter-antimatter asymmetry in a new way. Antimatter would be identifiable leptons. For the simplest option, the 3 antiquarks would be associated with a single single wormhole throat. The generalized Kähler structure assignable to M^4 in twistor lift [L147, L148] allows a CP violation, which could favor the condensation of quarks to baryons and antiquarks to leptons.

There are however objections against this idea. The considerations of this article inspire the question whether a single wormhole throat can carry only a single quark assignable to the cusp singularity, as suggested already earlier. Two wormhole contacts would be required. This is required also by the fact that stable wormhole contact must carry a monopole flux and monopole flux flux loops must be closed. Uncertainty Principle would suggest that the flux tube must have length of order lepton Compton length. Can this be consistent with the point-like nature of leptons? These arguments favor the option in which leptons and quarks as opposite H chiralities of H-spinors are the fundamental fermions.

Other kinds of point-like singularities and analogy with Fermi surface

Point-like singularities as cusps would naturally correspond to fundamental fermions at the light-like orbits of partonic 2-surfaces.

1. The 2-D boundaries of the fundamental region Y_i^3 associated with H^3/Γ would be analogues for 2-D pieces of the Fermi surface corresponding to atomic energy levels as energy bands.

   In condensed matter physics, the energy shells can deform and the components of the Fermi surface can touch. These singularities are central to topological physics.

2. At M^8 level the 2-D boundaries of Y_i^3 are analogues of energy bands. The evolution defined by the number theoretic holography, identifiable as a coupling constant evolution at the level of M^8, induces deformations of Y_i^3. One expects that this kind of touching singularities take place.

   At the level of H this would correspond to simple touching of the outer boundaries of the physical objects. In particular, these touchings could take place at the partonic 2-surfaces identified as vertices at which severa partonic orbits meet as the partonic surfaces as their ends are glued to single surface just like the ends of lines of a vertex of Feynman diagram are glued together along their ends.

   Could the meeting of fermion and antifermion cusp singularity in this way relate to an annihilation to a boson regarded as a fermion antifermion pair?

3. One can of course challenge the assumption that all fermions correspond to cusps, which correspond to parabolic subgroups of $G \subset SL(2,C)$ (`https://rb.gy/b5t55`). The proposal that all momenta, whose components are algebraic integers for the extension defined by P, are possible. What could be the interpretation of fermions which do not correspond to cusp.

   What the addition of a fermion to a particular allowed momentum could mean? Could it mean that its momentum defines a parabolic subgroup of G? Or is it true only for the "thin" part of Y_i^3 perhaps representing analogs of valence quarks.

   Or could the non-singular momenta correspond to the momenta for the analogues of sea partons, in particular analogs of sea gluons as fermion-antifermion pairs so that their total momentum would dominate in the total momentum of hadron. These would correspond to the "thick" part of Y_i^3. Could these interior momenta correspond to states delocalized at the string world sheets in the interior of monopole flux tubes and also states delocalized in the interiors of the flux tubes. Are these fermions present too?

   The presence of these states should be coded by the geometry of the hyperbolic manifold H^3/Γ (`https://rb.gy/snpft`) and Y_i^3 as its fundamental domain. Somehow the group $G \subset SL(2,C)$ should be responsible for this coding.

20.5.2 About the superconformal symmetries for the light-like orbits of partonic 2-surfaces

Are the cusp singularities (see `https://rb.gy/0p30o` and `https://rb.gy/fd4dz`) giving rise to CP_2 type extremals and the fermion momenta inside string world sheets and flux tubes associated with Y_i^3 sufficient to fix the 3-surfaces Y_i^3 in turn fixing number-theoretic holography?

1. The total energy for the classical action associated with these two kinds of fermions should correspond to the "sea" (thick part) and "valence fermions" assigned to the cusps (thin part).

2. Supersymplectic invariance and generalized conformal and Kac-Moody invariance assignable to light-like partonic orbits allows a large number of alternatives for the light-like surfaces [L141, L157, L154]. If supersymplectic and Kac-Moody symmetries act as gauge symmetries, the surfaces related by these symmetries are physically equivalent.

   The proposal is that these symmetries are partially broken and there is a hierarchy of breakings labelled by subalgebras $A_n \subset A$ of these algebras. The vanishing conditions for classical and quantal charges for A_n and $[A_n, A]$ serve as gauge conditions and also select the partonic 3-surfaces.

Interpretation of the partially broken gauge symmetries giving rise to dynamical symmetries is in terms of number theoretical measurement resolution and inclusion of hyperfinite factors of type II_1. These hierarchies relate to the hierarchies of extensions of rationals defined by the polynomials P defining the space-time surfaces apart from the effect of fermions.

If the preferred extremal property means generalization of holomorphy from 2-D case to 4-D case, one can conclude that the preferred extremals differ only at the singularities of space-time surfaces such as partonic orbits where the entire action comes into play. The regions outside the singularities would be universal: the minimal surface property would realize the 4-D generalization of the holomorphy.

3. Could different choices of the classical action, which determine the expressions of the classical and fermionic (quantal) Noether charges in terms of the modified Dirac action, correspond to different gauge choices selecting singular surfaces, in particular the CP_2 type extremals differently?

   The standard view would suggest that the change of the parameters of the action at the level of H corresponds to coupling constant evolution, which in the TGD framework is discrete and in terms of p-adic length scales. On the other hand, the existence of dual M^4 conformal invariance suggests that the coupling constant evolution at the level of M^8 is realized as "energy" evolution by using associativity as a dynamical principle. Can these two views be consistent?

   Note that the discriminant of the polynomial P is proposed to correspond to the exponent of action [L147, L149, L157, L154, L151]. The discriminant should change if the action changes. Does this mean that the change of the (effective) action in the discrete coupling constant evolution changes the polynomial?

20.6 Birational maps as morphisms of cognitive structures

https://en.wikipedia.org/wiki/Birational$_{g}eometry$ and their inverses are defined in terms of rational functions. They are very special in the sense that they map algebraic numbers in a given extension E of rationals to E itself.

1. In the TGD framework, the algebraic extensions E are defined by rational polynomials P at the level of M^8_c identifiable as complexified octonions. E defines a unique discretization for the number theoretically preferred coordinates of M^8_c by the condition that the M^8 coordinates have values in E: I call these discretizations cognitive representations. They make sense also in the extensions of p-adic number fields induced by E serving as correlates of cognition in TGD inspired theory of conscious experience. Birational maps respect the extension E associated with the cognitive representations and map cognitive representations to cognitive representation of same kind. They are clearly analogous to morphisms in category theory.

2. M^8-H duality [L109, L110, L168, L173] is a number theoretic analogue of momentum-position duality. M^8_c serves as the analog of momentum space and $H=M^4\times CP_2$ as the analog of position space. M^8-H duality maps the 4-surface defined in M^8_c by number theoretic holography based on 3-D data to a 4-D space-time surface in H.

3. Should M^8-H duality respect the algebraic extension? If so, it would map the cognitive representation defined by points belonging to 4-D surface $Y^4\subset M^8$ with the values of preferred coordinates in E to points of $M^4\subset H$ with coordinate values in E. One could say that M^8-H duality respects the number theoretical character of cognitive representations. The precise meaning of this intuition is however far from clear.

There are also questions related to the choice of preferred coordinates in which the cognitive representation is defined.

1. Number theoretic constraints fix the preferred coordinates at M^8 side rather uniquely and this induces a preferred choice also on $M^4\subset H$. For hyperbolic spaces (mass shells) a cognitive explosion happens and a natural question whether cognitive explosion happens also

for the light-like curves assignable to the partonic orbits. If the light-like curve is geodesic, the explosion indeed occurs. For more general light-like curves this is not the case always: could these more general light-like curves be related by a birational map to light-like geodesics?

2. At the H side one can also imagine besides standard Minkowski coordinates also other physically preferred choices of coordinates: are they also theoretically preferred? The notion of Hamilton-Jacobi structure [L164] suggests that in the case of M^4 Hamilton-Jacobi coordinates are very natural for the holomorphic realization of holography. If these are allowed, a natural condition would be that the Hamilton-Jacobi coordinates are related to each other by birational maps mapping the point of E to points of E so that cognitive representations are mapped to cognitive representations.

20.6.1 M^8-H duality, holography as holomorphy, Hamilton-Jacobi structures, and birational maps as cognitive morphisms

In the sequel the questions raised in the introduction are considered. The basic notions are M^8-H duality [L109, L110, L168, L173], holography as a generalized holomorphy [L160, L171], Hamilton-Jacobi structures [L164], and birational maps as cognitive morphisms.

About more precise definitions of the basic concepts

Consider first more precise definitions of various notions involved.

1. What are the preferred coordinates of M^8_c in which the cognitive representation is constructed? M^8_c has a number theoretic interpretation in terms of complexified octonions and physical interpretation as 8-D momentum space. Linear Minkowski coordinates are number-theoretically preferred since octonionic multiplication and other arithmetic operations have a very simple form in these coordinates. Also the number theoretic automorphisms respect the arithmetic operations. The allowed automorphisms correspond to the group G_2 which is a subgroup of $SO(1,7)$. Physically Minkowski space coordinates are preferred coordinates in the momentum space and also in $M^4 \subset H$.

2. How the algebraic extension of rationals, call it E, is determined? The proposal is that rational polynomials characterize partially the 3-D data for number theoretic holography [L168]. The roots of a rational polynomial P define an algebraic extension of rationals, call it E. A stronger, physically motivated, condition on P is that its coefficients are integers smaller than the degree of P.

   The roots of P define mass shells $H^3_c \subset M^4_c \subset M^8_c$, which in turn assign to the mass shells a 4-D surface Y^4 of M^8_c going through the mass shells by associative holography requiring that the normal space of Y^4 is associative, that is quaternionic. It has been be assumed that the roots are complex although also the condition that the roots are real can be considered. The imaginary unit i associated with the roots commutes with the octonionic imaginary units.

3. How the cognitive representation is defined? The points of $Y^4 \subset M^8_c$ with M^4 coordinates in E define a unique discretization of Y^4, called a cognitive representation, making sense also in the extensions of p-adic number fields induced by E. In general, the number of algebraic points in the interior of Y^4 is discrete and even finite but at the mass shells H^3 a cognitive explosion takes place. All points of H^3 with coordinates in E are algebraic.

   The algebraic points with coordinates, which are algebraic *integers* are physically and cognitively in very special role in number theoretic physics and make sense also as points of various p-adic number fields making possible number theoretical universality. The points of H^3 have interpretation as momenta and for physical states the total momentum as sum of momenta at mass shells defined by the roots of P has components which are integers, called Galois confinement [L147, L148], would define fundamental mechanism for the formation of bound states.

4. M^8-H duality maps the points of $H^3_c \subset M^4_c \subset M^8_c$ to points of $H^3 \subset M^4 \subset M^4 \times CP_2 = H$ by a map, which is essentially an inversion: this form is motivated by Uncertainty Principle:

for the most recent formulation of the duality see [L173]. This map is a birational map and takes points of E points of E. Also the points of cognitive representation belonging to the interior of $Y^4 \subset M^8_c$ are mapped to the interior of $X^4 \subset M^8_c$. One can ask whether the discrete set of points of cognitive representations in the interiors are of special physical interest, say having interpretation as interaction vertices.

Questions to be pondered

There are many questions to be considered.

1. Also partonic orbits in $X^4 \subset H$ define 3-D holographic data in H. What are these partonic orbits? The simplest partonic orbits have light-like M^4 projection but also more general light-like H projection can be considered (note the analogy with a 2-D rigid body rotating along a light-like geodesic of H). A general light-like geodesic of H is a combination of time-like geodesic of M^4 and space-like geodesic of CP_2.

   A point of the light-like partonic orbit correspond at the level of M^8 to the 3-D blowup of a point of M^8 at which the quaternionic normal space parametrized by CP_2 point is not unique so that the normal spaces for a 3-D section of CP_2, whose union along (probably light-like) geodesic is CP_2 with two holes corresponding to the ends of the partonic orbit. This singularity is highly analogous to the singularity of the electric field of a point charge. Partonic orbits define part of the 3-D holographic data.

2. Could one associate cognitive representations also to the partonic orbits? Could also partonic orbits allow a cognitive explosion? The simplest way to guarantee light-likeness for the H projection is as a light-like geodesic and this indeed allows an infinite number of algebraic points in Minkowski coordinates. Same applies to more general light-like orbits. One would have at least 1-D explosion of the cognitive representation.

3. What can one say about the CP_2 and M^4 projections of the partonic 2-surface? Could also these projections to X^2 and Y^2 allow an infinite number of points with coordinates in E or do these kinds of points have some special physical meaning, say as vertices for particle reactions? Concerning cognitive representation, the blow-up would mean that the point has an infinite but discrete set of quaternionic normal spaces at the level of M^8. Since the partonic surface can have an arbitrary complex sub-manifold as CP_2, there is indeed information to be cognitively represented.

The most general cognitively preferred coordinate choices for space-time surfaces and H?

In the case of M^8_c, number theoretical considerations fix the preferred coordinates highly uniquely. In the case of H the situation is not so obvious and one cannot exclude alternative coordinate choices related by a birational map.

A possible motivation comes from the following argument.

1. String world sheets are candidates for singularities analogous to partonic orbits. At a given point of the string world sheet a blow up to a 2-D complex sub-manifold of CP_2 would occur. This would mean that the normal spaces of the point in M^8_c form this sub-manifold. Cosmic strings are the simplest objcts of this kind. Monopole flux tubes are deformations of the cosmic strings and allow also an interpretation in terms of maps from M^4 to CP_2.

   If string world sheets define part of the data needed to define holography, one could argue that it makes sense to assign cognitive explosion to the string world sheet.

2. Cognitive explosion takes place if the string world sheets are 2-D geodesic submanifolds of H. Planes $M^2 \subset M^4$ represent the simplest example. A more complex example is obtained by taking a space-like geodesic in H and rotating it along a time-like geodesic of H. One can also take a light-like geodesic in H and rotate it along a light-like geodesic in dual light-like

direction (ruler surface would be in question). In which case the gluing of the string world sheet along the boundary to the partonic orbit could be possible.

One might perhaps think of building string world sheets by gluing these kinds of ultrasimple regions along their boundaries so that one would have edges. An interpretation as a discretization would be appropriate. One might even go further and argue that the cognitive explosion explains why we are able to think of these kinds of regions in terms of simple formulas. One might argue that number theoretic physics realizes exactly what is usually regarded as approximation. One can however wonder whether life is so simple.

This argument encourages to consider a more complex option allowing more general string world sheets.

1. In the case of M^4 projection, the notion of the Hamilton-Jacobi structure [L164], generalizing the notion of ordinary complex structure, is highly interesting in this respect. It involves a generalization of complex coordinates involving local decompositions $M^4(x) = M^2(x) \times E^2(x)$ of the 4-D tangent space of M^4. The integrable distribution of $E^2(x)$ corresponds to complex coordinates $(w, \overline{w}$ integrating to a partonic 2-surface whereas the integrable distribution of $M^2(x)$ to light-like coordinate pairs (u, v) integrating to a string world sheet in M^4.

   Cognitive representation mean that the discretized values of the Hamilton-Jacobi coordinates $(u, v, w, \overline{w})$ are in E. Hamilton-Jacobi structure generalizes also to the level of $X^4 \subset H$ and now Y^2 can also correspond to CP_2 projection as in the case of cosmic strings and magnetic flux tubes. Note that in TGD one can use a subset of H coordinates as coordinates of X^4.

2. The simplest assumption is that the 1-D parton orbit corresponds to a light-like geodesic but could one map light-like geodesics to more general light-like curves by a birational map? Hamilton-Jacobi structure gives rise to a pair of curved (u, v) of light-like coordinates: could it relate to the standard flat light-like coordinates of M^2 by a birational map? Could a birational map relate standard complex coordinates of E^2 to the pair $(w, \overline{w})$? Could one also consider more general birational maps of $M^4 \to M^4$? If so, the Hamilton-Jacobi structures would be related by maps respecting algebraic extensions and cognitive representations. This would give a very powerful constraint on the Hamilton-Jacobi structures.

In the case of CP_2, projective coordinates are group-theoretically highly unique and determined apart from color rotations. Could one require that the CP_2 projection Y^2 associated with the partonic 2-surface and cosmic string or magnetic flux tube involves cognitive explosion. Are the allowed M^4 and CP_2 projections related by birational maps? Note that color rotations are birational maps.

These considerations suggest the following speculative view.

1. $M^8 - H$ duality, when restricted to 3-D holographic data at both sides, is analogous to a birational map expressible in terms of rational functions and respects the number theoretical character of cognitive representations.

2. Cognitive explosion occurs for the holographic data (this is very natural from the information theoretic perspective): this includes also string world sheets. Hamilton-Jacobi structures in the same cognitive class, partially characterized by the extension E of rationals, are related by a birational map.

3. $M^8 - H$ duality maps the quaternionic normal spaces to points of CP_2 and is an example of a birational map in M^4 degrees of freedom. It is not however easy to guess how the number theoretic holography is realized explicitly and how the 4-surfaces in M^8 are mapped to holomorphic 4-surfaces in H.

4. An interesting additional aspect relates to the non-determinism of partonic orbits due to the non-determinism of the light-likeness condition deriving from the fact that the action is Chern-Simons-Kähler action. The deformation of the partonic orbit induces the deformation of time derivatives of H coordinates at the boundary of $\delta M^4_+ \times CP_2$ to guarantee that boundary conditions at the orbit are realized. This suggests a strong form of holography [L171].

Already the 3-surfaces at $\delta M^4_+ \times CP_2$ or partonic orbits would be enough as holographic data. This in turn suggests that the analog of birational cognitive correspondence between the holographic data at $\delta M^4_+ \times CP_2$ and at partonic orbits.

20.6.2 Appendix: Some facts about birational geometry

Birational geometry has as its morphisms birational maps: both the map and its inverse are expressible in terms of rational functions. The coefficients of polynomials appearing in rational functions are in the TGD framework rational. They map rationals to rationals and also numbers of given extension E of rationals to themselves (one can assign to each space-time region an extension defined by a polynomial).

Therefore birational maps map cognitive representations, defined as discretizations of the space-time surface such that the points have physically/number theoretically preferred coordinates in E, to cognitive representations. They therefore respect cognitive representations and are morphisms of cognition. They are also number-theoretically universal, making sense for all p-adic number fields and their extensions induced by E. This makes birational maps extremely interesting from the TGD point of view.

The following lists basic facts about birational geometry as I have understood them on the basis of Wikipedia articles about birational geometry and Enriques-Kodaira classification. I have added physics inspired associations with TGD.

Birational geometries are one central approach to algebraic geometry.

1. They provide classification of complex varieties to equivalence classes related by birational maps. The classification complex curves (real dimension 2) reduces to the classification of projective curves of projective space CP_n determined as zeros of a homogeneous polynomial. Complex surfaces (real dimension 4) are of obvious interest in TGD: now however the notion of complex structure is generalized and one has Hamilton-Jacobi structure.

2. In TGD, a generalization of complex surfaces of complex dimension 2 in the embedding space $H = M^4 \times CP_2$ of complex dimension 4 is considered. What is new is the presence of the Minkowski signature requiring a combination of hypercomplex and complex structures to the Hamilton-Jacobi structure. Note however the space-time surfaces also have counterparts in the Euclidean signature $E^4 \times CP_2$: whether this has a physical interpretation, remains an open question. Second representation is provided as 4-surfaces in the space M^8_c of complexified octonions and an attractive idea is that $M^8 - H$ duality corresponds to a birational mapping of cognitive representations to cognitive representations.

3. Every algebraic variety is birationally equivalent with a sub-variety of CP_n so that their classification reduces to the classification of projective varieties of CP_n defined in terms of homogeneous polynomials. $n = 2$ (4 real dimensions) is of special relevance from the TGD point of view. A variety is said to be rational if it is birationally equivalent to some projective variety: for instance CP_2 is rational.

4. A concrete example of birational equivalence is provided by stereographic projections of quadric hypersurfaces in n+1-D linear space. Let p be a point of quadric. The stereographic projection sends a point q of the quadric to the line going through p and q, that is a point of CP_n in the complex case. One can select one point on the line as its representative. Another exammple is provided by Möbius transformations representing Lorentz group as transformations of complex plane.

The notion of a minimal model is important.

1. The basic observation is that it is possible to eliminate or add singularities by using birational maps of the space in which the surface is defined to some other spaces, which can have a higher dimension. The zeros of a birational map can be used to eliminate singularities of the algebraic surface of dimension n by blowups replacing the singularity with CP_n. Poles in turn create singularities. Peaks and self-intersections are examples of singularities.

   The idea is to apply birational maps to find a birationally equivalent surface representation, which has no singularities. There is a very counter-intuitive formal description for this. For instance, complex curves of CP_2 have intersections since their sum of their real dimensions is 4. The same applies to 4-surfaces in H. My understanding is as follows: the blowup for CP_2 makes it possible to get rid of an intersection with intersection number 1. One can formally say that the blow up by gluing a CP_1 defines a curve which has negative intersection number -1.

2. In the TGD framework, wormhole contacts have the same metric and Kähler structure as CP_2 and light-like M^4 projection (or even H projection). They appear as blowups of singularities of 4-surfaces along a light-like curve of M^8. The union of the quaternionic/associative normal spaces along the curve is not a line of CP_2 but CP_2 itself with two holes corresponding to the ends of the light-like curve. The 3-D normal spaces at the points of the light-like curve are not unique and form a local slicing of CP_2 by 3-D surfaces. This is a Minkowskian analog of a blow-up for a point and also an analog of cut of analytic function.

The Italian school of algebraic geometry has developed a rather detailed classification of these surfaces. The main result is that every surface X is birational either to a product $\mathbb{P}^1 \times$ $Cforsomecurve\mathrm{C}ortoaminimalsurface\mathrm{Y}.Preferredextremalsareindeedminimalsurfacessothatspace-$ $timesurfacesmightdefine\ minimalmodels.Theabsenceofsingularities(typicallypeaksorself-intersections)cha$

There are several birational invariants listed in the Wikipedia article. Many of them are rather technical in nature. The canonical bundle K_X for a variety of complex dimension n corresponds to n:th exterior power of complex cotangent bundle that is holomorphic n-forms. For space-time surfaces one would have $n = 2$ and holomorphic 2-forms.

1. Plurigenera corresponds to the dimensions for the vector space of global sections $H_0(X, K_X^d)$ for smooth projective varieties and are birational invariants. The global sections define global coordinates, which define birational maps to a projective space of this dimension.

2. Kodaira dimension measures the complexity of the variety and characterizes how fast the plurigenera increase. It has values $-\infty, 0, 1, ..n$ and has 4 values for space-time surfaces. The value $-\infty$ corresponds to the simplest situation and for $n = 2$ characterizes CP_2 which is rational and has vanishing plurigenera.

3. The dimensions for the spaces of global sections of the tensor powers of complex cotangent bundle (holomorphic 1-forms) define birational invariants. In particular, holomorphic forms of type $(p, 0)$ are birational invariants unlike the more general forms having type (p, q). Betti numbers are not in general birational invariants.

4. Fundamental group is birational invariant as is obvious from the blowup construction. Other homotopy groups are not birational invariants.

5. Gromow-Witten invariants are birational invariants. They are defined for pseudo-holomorphic curves (real dimension 2) in a symplectic manifold X. These invariants give the number of curves with a fixed genus and 2-homology class going through n marked points. Gromow-Witten invariants have also an interpretation as symplectic invariants characterizing the symplectic manifold X.

   In TGD, the application would be to partonic 2-surfaces of given genus g and homology charge (Kähler magnetic charge) representatable as holomorphic surfaces in $X = CP_2$ containing n marked points of CP_2 identifiable as the loci of fermions at the partonic 2-surface. This number would be of genuine interest in the calculation of scattering amplitudes.

What birational classification could mean in the TGD framework?

1. Holomorphic ansatz gives the space-time surfaces as Bohr orbits. Birational maps give new solutions from a given solution. It would be natural to organize the Bohr orbits to birational equivalence classes, which might be called cognitive equivalence classes. This should induce similar organization at the level of M_c^8.

2. An interesting possibility is that for certain space-time surfaces CP_2 coordinates can be expressed in terms of preferred M^4 coordinates using birational functions and vice versa. Cognitive representation in M^4 coordinates would be mapped to a cognitive representation in CP_2 coordinates.

3. The interpretation of $M^8 - H$ duality as a generalization of momentum position duality suggests information theoretic interpretation and the possibility that it could be seen as a cognitive/birational correspondence. This is indeed the case M^4 when one considers linear M^4 coordinates at both sides.

4. An intriguing question is whether the pair of hypercomplex and complex coordinates associated with the Hamilton-Jacobi structure could be regarded as cognitively acceptable coordinates. If Hamilton-Jacobi coordinates are cognitively acceptable, they should relate to linear M^4 coordinates by a birational correspondence so that M^8-H duality in its basic form could be replaced with its composition with a coordinate transformation from the linear M^4 coordinates to particular Hamilton-Jacobi coordinates. The color rotations in CP_2 in turn define birational correspondences between different choices of Eguchi-Hanson coordinates.

   If this picture makes sense, one could say that the entire holomorphic space-time surfaces, rather than only their intersections with mass shells H^3 and partonic orbits, correspond to cognitive explosions. This interpretation might make sense since holomorphy has a huge potential for generating information: it would make TGD exactly solvable.

Chapter 21

Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole

21.1 Introduction

I have had a very interesting discussions with Baba Ilya Iyo Azza about von Neumann algebras [A70]. I have a background of physicist and have suffered a lot of frustration in trying to understand hyperfinite factors of type II_1 (HFFs, `https://cutt.ly/OX8uP32`) by trying to read mathematicians' articles.

I cannot understand without a physical interpretation and associations to my own big vision TGD. Again I stared at the basic definitions, ideas and concepts trying to build a physical interpretation. This is not my first attempt to understand the possible role of HFFs in TGD: I have written already earlier of the possible role of von Neumann algebras in the TGD framework [K126, K44]. In the sequel I try to summarize what I have possibly understood with my meager technical background.

In the first section I will redescribe the basic notions and ideas related to von Neumann algebras as I see them now, in particular HFFs, which seem to be especially relevant for TGD because of their "hyperfiniteness" property implying that they are effectively finite-D matrix algebras.

There are also more general factors of type II_1, in particular those related to the notion of free probability (`hhttps://cutt.ly/SX2ftyx`), which is a notion related to a theory of non-commutative random variables. The free group generated by a finite number of generators is basic notion and the group algebras associated with free groups are factors of type II_1. The isomorphism problem asks whether these algebras are isomorphic for different numbers of generators. These algebras are not hyperfinite and from the physics point of view this is not a good news.

21.1.1 Basic notions of HFFs from TGD perspective

In this section I will describe my recent, still rather primitive physicist's understanding of HFFs. Factor M and its commutant M' are central notions in the theory of von Neumann algebras. An important question, not discussed earlier, concerns the physical counterparts of M and M'. I will not discuss technical details: I have made at least a noble attempt to do this earlier [K126, K44].

1. In the TGD framework, one can distinguish between quantum degrees of freedom and classical ones, and classical physics can be said to be an exact part of quantum physics.

2. The formulation of physics as Kähler geometry of the "world of classical worlds" (WCW) is briefly summarized in the Appendix. The formulation involves hierarchies A_n of 3 kinds of algebras; supersymplectic algebras SSA_n acting on $\delta M^4_+ \times CP_2$ and assumed to induce isometries of WCW, affine algebras Aff_n associated with isometries and holonomies of $H = M^4 \times CP_2$ acting on light-like partonic orbits, and isometries I_n of the light-cone boundary δM^4_+.

At the H-side, quantum degrees of freedom are assignable to A_n, which would correspond to M.

In zero energy ontology (ZEO) [K130] states are quantum superpositions of preferred extremals. Preferred extremals depend on zero modes, which are symplectic invariants and do not appear in the line element of WCW. Zero modes serve as classical variables, which commute with super symplectic transformations and could correspond to M' for SSA_n at H-side. Similar identification of analogs of zero modes should be possible for Aff_n and I_n.

3. In the number theoretic sector at the M^8-side, braided group algebras would correspond to quantum degrees of freedom, that is M. M' would correspond to some number theoretic invariants of polynomials P determining the space-time surface in H by M^8-H duality [L109, L110]. The set of roots of P and ramified primes dividing the discriminant of P are such invariants.

21.1.2 Bird's eye view of HFFs in TGD

A rough bird's eye view of HFFs is discussed with an emphasis on their physical interpretation. There are two visions of TGD: the number theoretic view [L54, L53] and the geometric view [K57, K29, K127, K96] and M^8-H duality relates these views [L141, L109, L110].

1. At the M^8 side, the p-adic representations of braided group algebras of Galois groups associated with hierarchies of extensions of rationals define natural candidates for the inclusion hierarchies of HFFs.

   Braid groups represent basically permutations of tensor factors and the same applies to the braided Galois groups with S_n restricted to the Galois group.

   A good guess is that braid strands correspond to the roots of a polynomial labelling mass shells H^3 in $M^4 \subset M^8$.

2. The 3-D mass shells define a 4-surface in M^8 by holography based on associativity, which makes possible holography.

   The condition that the normal space N of the 4-surface X^4 in M^8 is associative and contains a 2-D commutative sub-space X^2, guarantees both holography and M^8-H duality mapping this 4-surface $X^4 \subset M^8$ to a space-time surface $Y^4 \subset H$.

   The 2-D commutative space $X^2 \subset N$ can be regarded as a normal space of the 6-D counterpart of twistor space $T(M^4)$. $T(M^4)$ is mapped by M^8-H duality to a point of the twistor space $T(CP_2) = SU(3)/U(1)\times U(1)$ of CP_2. This map is assumed to define the twistor space $T(Y^4) \subset T(M^4) \times T(CP_2)$ as a preferred extremal [L147, L148].

3. The physical picture strongly suggests that also string world sheets and partonic 2-surfaces in $Y^4 \subset H$ are needed. They are are assumed to correspond to singularities for the map to H. A natural conjecture is that the 2-D subspace $X^2 \subset N$ is mapped to a 2-D subspace $Y^2 \subset T$ of the tangent space T of X^4 by a multiplication with a preferred octonionic imaginary unit in T.

   How could this preferred octonionic unit be determined?

   (a) Complexified octonionic units in the tangent space of M^8_c decomposes under $SU(3) \subset G_2$, having interpretation as color group, to representations $1_1 \oplus 1_2 \oplus 3 \oplus \overline{3}$. 1_1 and 1_2 correspond to the real unit I_0 and imaginary unit I_1 and 3 and $\overline{3}$ correspond to color triplets analogous to quarks and antiquarks.

   (b) Complexified quaternionic sub-space defining N corresponds to color singlets I_0, I_1, and quarks I_2, I_3 with $(Y=-1/3, I_3=1/2)$ and $Y=2/3, I_3=0)$. The complement T corresponds to quark I_4 $(Y=-1/3, I_3=1/2)$ and 3 antiquarks (I_5, I_6, I_7). The octonionic multiplication of the units of quaternionic subspace by quark I_4 gives T as the orthogonal complement of the quaternionic sub-space N.

(c) This multiplication would assign to $X^2 \subset N$ 2-D subspace of T and also its orthogonal complement Y^2 in T. If the distributions of X^2 and Y^2 are integrable, they define the slicing of X^4 by partonic 2-surfaces and string world sheets. The tangent spaces for them would correspond to the local choice of I_0, I_1 and I_2, I_3. X^2 and Y^2 at different points would differ by a local SU(3) transformation. In fact, the 4-surface in M^8 would correspond to a complex color gauge transformation [L109, L110].

This choice could correspond to what I have called Hamilton-Jacobi (H-J) structure [K9] in X^4 defining a slicing of X^4 defined by an integrable distribution of pairs of orthonormal 2-surfaces analogous to the choice of massless wave vector and orthogonal polarization plane depending on the point of X^4 or equivalently on the point of M^4 as its projection. H-J structure would also define the analog of Kähler structure in M^4 strongly suggested by twistor lift.

The original proposal was that the H-J structure is associated with M^4, and one cannot completely exclude the possibility that the projection of the proposed slicing to M^4 defines H-J. The idea about single H-J structure is not physical. Dynamical H-J structure does not conform with the idea that M^4 is completely non-dynamical. However, if the H-J structure is determined by the choice $X^2 \subset N$ and defines H-J structure in X^4, this objection can be circumvented.

At the H side there are 3 algebras.

1. The subalgebras SSA_n of super-symplectic algebra (SSA) are assumed to induce isometries of WCW. Since SSA and also other algebras have non-negative conformal weights, it has a hierarchy of subalgebras SSA_n with conformal weights coming as n-multiples of those for SSA.

2. There are also affine algebras Aff associated with H isometries acting on light-like orbits of partonic 2-surfaces and having similar hierarchy of Aff_n. Both isometries and holonomies of H are involved.

3. Light-cone boundary allows infinite dimensional isometry group I consisting of generalized conformal transformation combined with a local scaling allowing similar hierarchy I_n.

One should understand how the number theoretic and geometric hierarchies relate to each other and a good guess is that braided group algebras act on braids assignable to SSA_n with n interpreted as the number of braid strands and thus the degree n of P.

Also the interpretational problems related to quantum measurement theory and probability interpretation are discussed from the TGD point of view, in which zero energy ontology (ZEO) allows us to solve the basic problem of quantum measurement theory.

21.1.3 $M^8 - H$ duality and HFFS

$M^8 - H$ duality [L109, L110] suggests that the hierarchies of extensions of rationals at the number theoretic side and hierarchies of HFFs at the geometric side are closely related.

The key idea is that the braided Galois groups at M^8-side interact on algebras $A_n \in \{SSA_n, Aff_n, I_n\}$ at H level as number theoretic braid groups permuting the tensor factors assignable to the braid strands, which correspond to the roots of the polynomial P.

The basic notions associated with a polynomial P with rational coefficients having degree n are its n roots, ramified primes as factors of the discriminant defined by the difference of its roots, and Galois group plus a set of Galois invariants such as symmetric polynomials of roots. The Galois group is the same for a very large number of polynomials P. The question concerns the counterparts of these notions at the level of H?

An educated guess is that the n roots of P label the strands of an n-braid in H assignable to A_n, ramified primes correspond to physically preferred p-adic primes in the adelic structure formed by various p-adic representations $A_{n,p}$ of the algebras A_n and the Galois group algebra associated with the polynomial P with degree n.

This picture suggests a generalization of arithmetics to quantum arithmetics based on the replacement of + and × with ⊕ and ⊗ and replacement of numbers with representations of groups or algebras [L153]. This implies a generalization of adele by replacing p-adic numbers with the p-adic quantum counterparts of algebras A_n.

The mysterious McKay correspondence [A78] has inspired several articles during years [L46, L96, L95, L153] but it is fair to say that I do not really understand it. Hence I could not avoid the temptation to attack this mystery also in this article.

21.1.4 Infinite primes

The notion of infinite primes [K105, K69] is one of the ideas inspired by TGD, which has waited for a long time for its application. Their construction is analogous to a quantization of supersymmetric arithmetic quantum field theory.

1. The analog of Dirac sea X is defined by the product of finite primes and one "kicks" from sea a subset of primes defining a square free integer n_F to get the sum $X/n_F + n_F$. One can also add bosons to X/n_F *resp.* n_F multiplying it with integer n_{B_1} *resp.* n_{B_2}, which is divisible only by primes dividing Z/n_F *resp.* n_F.

2. This construction generalizes and one can form polynomials of X to get infinite primes analogous to bound states. One can consider instead of $P(X)$ a polynomial $P(X,Y)$, where Y is the product of all primes at the first level thus involving the product of all infinite primes already constructed, and repeat the procedure. One can repeat the procedure indefinitely and the formal interpretation is as a repeated quantization. The interpretation could be in terms of many-sheeted space-time or abstraction process involving formation of logical statements about statements about ...

3. The polynomials Q could also be interpreted as ordinary polynomials. If $Q(X) = P(X)$, where $P(X)$ is the polynomial defining a 4-surface in M^8, the space-time surface X^4 in H would correspond to infinite prime. This would give a "quantization" of P defining the space-time surface.

   The polynomial P defining 4-surface in H would fix various quantum algebras associated with it. The polynomials $P(X_1, X_2, ...X_n)$ could be interpreted as $n-1$-parameter families defining surfaces in the "world of classical worlds" (WCW) [L141] (for the development of the notion see [K57, K29, K127, K96]).

4. X is analogous to adele and infinite primes could be perhaps seen as a generalization of the notion of adele. One could assign p-adic variants of various HFFs to the primes defining the adele and + and × could be replaced with ⊕ and ⊗. The physical interpretation of ramified primes of P is highly interesting.

In the last section, I try to guess how the fusion of these building blocks by using the ideas introduced in the previous sections could give rise to what might be called quantum TGD. It must be made clear that the twistor lift of TGD [L147, L148] is not considered in this work.

21.2 Basic notions related to hyperfinite factors of type II_1 from TGD point of view

In this section, the basic notions of hyperfinite factors (HFFs) as a physicists from the TGD point of view will be discussed. I have considered HFFs earlier several times [K126, K44] and will not discuss here the technical details of various notions.

21.2.1 Basic concepts related to von Neumann algebras

John von Neumann proposed that the algebras, which now carry his name are central for quantum theory [A70]. Von Neumann algebra decomposes to a direct integral of factors appearing and there are 3 types of factors corresponding to types I, II, and III.

Inclusion/embedding as a basic aspect of physics

Inclusion (https://cutt.ly/NX8eWwa, https://cutt.ly/cX8eUuf, https://cutt.ly/4X8ePn6)) is a central notion in the theory of factors. Inclusion/embedding involving induction of various geometric structures is a key element of classical and quantum TGD.

One starts from the algebra $B(H)$ of bounded operators in Hilbert space. This algebra has naturally hermitian conjugation $*$ as an antiunitary operation and therefore one talks of C_* algebras. von Neumann algebra is a subalgebra of $B(H)$. Already here an analog of inclusion is involved (https://cutt.ly/3XkPO2s). There are also inclusions between von Neumann algebras, in particular HFFs.

What could the inclusion of von Neumann algebra to $B(H)$ as subalgebra mean physically? In the TGD framework, one can identify several analogies.

1. Space-time is a 4-surface in $H = M^4 \times CP_2$: analog of inclusion reducing degrees of freedom.

2. Space-time is not only an extremal of an action [K9] [L146] but a preferred extremal (PE), which satisfies holography so that it is almost uniquely defined by a 3-surface. This guarantees general coordinate invariance at the level of H without path integral. I talk about preferred extremals (PEs) analogous to Bohr orbits. Space-time surface as PE is a 4-D minimal surface with singularities [L146]: there is an analogy with a soap film spanned by frames. This implies a small failure of determinism localizable at the analogs of frames so that holography is not completely unique.

   Holography means that very few extremals are physically possible. This Bohr orbit property conforms with the Uncertainty Principle. Also HFFs correspond to small sub-spaces of $B(H)$. Quantum classical correspondence suggests that this analogy is not accidental.

The notion of commutant and its physical interpretation in the TGD framework

The notion of the commutant M' of M, which also defines HFF, is also essential. What could be the physical interpretation of M'? TGD suggests 3 important hierarchies of HFFs as algebras A_n. A_n could correspond to super-symplectic algebras SSA_n acting at $\delta M^4_{\pm} \times CP_2$; to an affine algebras Aff_n acting at the light-like partonic orbits; or to an isometry algebra I_n acting at δM^4_+. All these HFF candidates have commutants and would have interpretation in terms of quantum-classical correspondence.

One can consider SSA as an example.

1. In TGD, one has indeed an excellent candidate for the commutant. Supersymplectic symmetry algebra (SSA) of $\delta M^4_+ \times CP_2$ (δM^4_+ denotes the boundary of a future directed light-cone) is proposed to act as isometries of the "world of classical worlds" (WCW) consisting of space-time surfaces as PEs (very, very roughly).

   Symplectic symmetries would be generated by Hamiltonians, which are products of Hamiltonians associated with δM^4_+ (metrically sphere S^2) and CP_2. Symplectic symmetries are conjectured to act as isometries of WCW and gamma matrices of WCW extend symplectic symmetries to super-symplectic ones.

   Hamiltonians and their super-counterparts generate the super-symplectic algebra (SSA) and quantum states are created by using them. SSA is a candidate for HFF. Call it M. What about M?

2. The symplectic symmetries leave invariant the induced Kähler forms of CP_2 and contact form of δM^4_+ (assignable to the analog of Kähler structure in M^4).

3. The wave functions in WCW depending of magnetic fluxes defined by these Kähler forms over 2-surfaces are physically observables which commute SSA and with M. These fluxes are in a central role in the classical view about TGD and define what might perhaps be regarded as a dual description necessary to interpret quantum measurements.

   Could M' correspond or at least include the WCW wave functions (actually the scalar parts multiplying WCW spinor fields with WCW spinor for a given 4-surface a fermionic Fock state) depending on these fluxes only? I have previously talked of these degrees of freedom as zero modes commuting with quantum degrees of freedom and of quantum classical correspondence.

4. There is $M-M'$ correspondence also for number theoretic degrees of freedom, which naturally appear from the number theoretic M^8 description mapped to H-description. Polynomials P associated with a given Galois group are analogous to symplectic degrees of freedom with given fluxes as symplectic invariants. Galois groups and Galois invariants are "classical" invariants at the M^8 side and should have counterparts on the H side. For instance, the degree n of polynomial P could correspond to the number of braid stran

More algebraic notions

There are further algebraic notions involved. The article of John Baez (`https://cutt.ly/VXlQyqD`) describes these notions nicely.

1. The condition $M'' = M$ is a defining algebraic condition for von Neumann algebras. What does this mean? Or what could its failure mean? Could M'' be larger than M? It would seem that this condition is achieved by replacing M with M''.

   $M'' = M$ codes algebraically the notion of weak continuity, which is motivated by the idea that functions of operators obtained by replacing classical observable by its quantum counterpart are also observables. This requires the notion of continuity. Every sequence of operators must approach an operator belonging to the von Neumann algebra and this can be required in a weak sense, that is for matrix elements of the operators.

   Does $M'' = M$ mean that the classical descriptions and quantum descriptions are somehow equivalent? At first, this looks nonsensical but when one notices that the scalar parts of WCW spinor fields correspond to wave functions in the zero mode of WCW which do not appear in the line element of WCW, this idea starts to look more sensible. In quantum measurements the outcome is indeed expressed in terms of classical variables. Zero modes and quantum fluctuating modes would provide dual descriptions of physics.

2. There is also the notion of hermitian conjugation defined by an antiunitary operator J: $a^\dagger = JAJ$. This operator is absolutely essential in quantum theory and in the TGD framework it is geometrized in terms of the Kähler form of WCW. The idea is that entire quantum theory, rather than only gravitation or gravitation and gauge interactions should be geometrized. Left multiplication by JaJ corresponds to right multiplication by a.

Connes tensor product and category theoretic notions

Connes tensor product (Connes fusion) [A28] appears in the construction of the hierarchy of inclusions of HFFs. For instance, matrix multiplication has an interpretation as Connes tensor product reduct tensor product of matrices to a matrix product. The number of degrees of freedom is reduced. The tensor product $A \otimes_R B$ depends on the coefficient ring R acting as right multiplication in A and left multiplication in B. If the dimension of R increases, the dimension of A (B) as a left/right R module is reduced. For instance, A as an A-module is 1-dimensional.

Also category theory related algebraic notions appear. I still do not have an intuitive grasp about category theory. In any case, one would have a so-called 2-category (`https://cutt.ly/3XkPO2s`). M and N correspond to 0-morphisms (objects). One can multiply arguments of functions in $L^2(M)$ and $L^2(N)$ by M or N.

Bimodule (`https://cutt.ly/EX885WA`) is a key notion. For instance the set of $R_{m,n}$ of $m\times n$ matrices is a bimodule, which is a left (right) module with respect to $m \times m$ ($n \times n$) matrices. One can replace matrices with algebras. The bimodule $_MM_M$ *resp.* $_NN_N$ is analogous to $m \times m$ *resp.* $n \times n$ matrices. They correspond to 1-morphisms, which behave like units. The bimodule $_MN_N$ *resp.* $_NN_M$ is analogous to $m \times n$ *resp.* $n \times m$ matrices. These two bimodules correspond to a generating 1-morphisms mapping N to M *resp.* M to N. Bimodule map corresponds to 2-morphisms. Connes tensor product defines what category theorists call a tensor functory.

The notions of factor and trace

The notion of factor as a building block of more complex structures is central and analogous to the notion of simple group or prime. Factor is a von Neumann algebra, which is simple in the sense that it has a trivial center consisting of multiples of unit operators. The algebra is direct sum or integral over different factors.

The notion of trace is fundamental and highly counter intuitive. For the factors of type I, it is just the ordinary trace and the trace $Tr(I)$ of the unit operator is equal to the dimension n of the Hilbert space. This notion is natural when direct sum is the key notion. For the other factors, the situation is different.

Factors can be classified into three types: $I, II, andIII$.

1. For factors of type I associated with three bosons, the trace equals n in the n-D case and ∞ in the infinite-D case.

2. A highly non-intuitive and non-trivial axiom relating to HFFs as hyperfinite factors of type II_1 is that the trace of the unit operator satisfies $Tr(Id) = 1$: for factors of type II (see the article of Popa at `https://cutt.ly/KX8y0Fs`). This definition is natural in the sense that being a subsystem means being a tensor factor rather than subspace.

   The intuitive idea is that the density matrix for an infinite-D system identified as a unit operator gives as its trace total probability equal to one. These factors emerge naturally for free fermions. "Hyperfinite" expresses the fact that the approximation of a factor with its finite-D cutoff is an excellent approximation.

   HFFs are extremely flexible and can look like arbitrarily high-dimensional factor I_n. For instance, one can extract any matrix algebra $M^n(C)$ as a tensor factor so that one has $M = M^n(C) \otimes M^{1/n}$ by the multiplicativity of dimensions in the tensor product. Should one interpret this by saying that measurement can separate from a factor an $n-D$ Hilbert space and that $M^{1/n}$ is something that remains inaccessible to the measurements considered? If one introduces the notion of measurement resolution in this manner, the description of measurement could be based on factors of tyoe I_n.

3. The factors of type II_∞ are tensor products of infinite-D factors of type I and HFFs and could describe free bosons and fermions.

4. In quantum field theory (QFT), factors of type III appear and in this case the notion of trace becomes useless. These factors are pathological and in QFT they lead to divergence difficulties. The physical reason is the idea about point-like particles, which is too strong an idealization.

   In the TGD framework, the generalization of a point-like particle to 3-surface saves from these difficulties and leads to factors of type I and HFFs. In TGD, finite measurement resolution is realized in terms of a unique number theoretic discretization, which further simplifies the situation in the TGD framework.

21.2.2 Standard construction for the hierarchy of HFFs

Consider now the standard construction leading to a hierarchy of HFFs and their inclusions.

1. One starts from an inclusion $M \subset N$ of HFFs. I will later consider what these algebras could be in the TGD framework.

2. One introduces the spaces $L^2(M)$ *resp.* $L^2(N)$ of square integrable functions in M *resp.* N.

   From the physics point of view, bringing in "L^2" is something extremely non-trivial. Space is replaced with wave functions in space: this corresponds to what is done in wave mechanics, that is quantization! One quantizes in M, particles as points of M are replaced by wave functions in M, one might say.

3. At the next step one introduces the projection operator e as a projection from $L^2(N)$ to $L^2(M)$: this is like projecting wave functions in N to wave functions in M. I wish I could really understand the physical meaning of this. The induction procedure for second quantized spinor fields in H to the space-time surface by restriction is completely analogous to this procedure.

After that one generates a HFF as an algebra generated by e and $L^2(N)$: call it $\langle L^2(N), e\rangle$. One has now 3 HFFs and their inclusions: $M_0 \equiv M$, $M_1 \equiv N$, and $\langle L^2(N), e\rangle \equiv M_2$.

An interesting question is whether this process could generalize to the level of induced spinor fields?

4. Even this is not enough! One constructs $L^2(M_2) \equiv M_3$ including M_2. One can continue this indefinitely. Physically this means a repeated quantization.

   One could ask whether one could build a hierarchy M_0, $L^2(M_0)$,..., $L^2(L^2...(M_0))..)$: why is this not done?

   The hierarchy of projectors e_i to M_i defines what is called Temperley-Lieb algebra [A93] involving quantum phase $q = exp(i\pi/n)$ as a parameter. This algebra resembles that of S_∞ but differs from it in that one has projectors instead of group elements. For the braid group $e_i^2 = 1$ is replaced with a sum of terms proportional to e_i and unit matrix: mixture of projector and permutation is in question.

5. There is a fascinating connection in TGD and theory of consciousness. The construction of what I call infinite primes [K105, K69] is structurally like a repeated second quantization of a supersymmetric arithmetic quantum field theory involving fermions and bosons labelled by the primes of a given level I conjectured that it corresponds physically to quantum theory in the many-sheeted space-time.

   Also an interpretation in terms of a hierarchy of statements about statements about .... bringing in mind hierarchy of logics comes to mind. Cognition involves generation of reflective levels and this could have the quantization in the proposed sense as a quantum physical correlate.

21.2.3 Classification of inclusions of HFFs using extended ADE diagrams

Extended ADE Dynkin diagrams for ADE Lie groups, which correspond to finite subgroups of $SU(2)$ by McKay correspondence [A78, A77, A64], discussed from the TGD point of view in [L153], characterize inclusions of HFFs.

For a subset of ADE groups not containing E_7 and D_{2n+1}, there are inclusions, which correspond to Dynkin diagrams corresponding quantum groups. What is interesting that E_6 (tetrahedron) and E_8 (icosahedron/dodecahedron) appear in the TGD based model of bioharmony and genetic code but not E_7 (cube and octahedron) [L133].

1. Why finite subgroups of $SU(2)$ (or $SU(2)_q$) should characterize the inclusions in the tunnel hierarchies with the same value of the quantum dimension $M_{n+1} : M_n$ of quantum group?

   In the TGD interpretation M_{n+1} reduces to a tensor product of M_n and quantum group, when M_n represents reduced measurement resolution and quantum group the added degrees of freedom. Quantum groups would represent the reduced degrees of freedom. This has a number theoretical counterpart in terms of finite measurement resolution obtained when an extension of ... of rationals is reduced to a smaller extension. The braided relative Galois group would represent the new degrees of freedom.

2. One can algebraically identify HFF as a "tunnel" obtained by iterated standard construction as an infinite tensor power of $GL(2, c)$ or $GL(n, C)$. The analog of the McKay graph for the irreps of a closed subgroup of $GL(2, C)$ defines an invariant characterizing the fusion rules involved with the reduction of the Connes tensor products. This invariant reduces to the McKay graph for the tensor products of the canonical 2-D representation with the irreps of a *finite* rather than only closed subgroups of $SU(2)$. This must take place also for $GL(n, C)$. Why?

   The reduction of degrees of freedom implied by the Connes tensor product seems to imply a discretization at the level of $SU(2)$ and replace closed subgroups of $SU(2)$ with finite subgroups. This conforms with the similarity of the representation theories of discrete and closed groups. In the case of quantum group representations only a finite number of ordinary finite-D group representations survive.

All this conforms with the TGD view about the equivalence of number-theoretic discretization and inclusions as descriptions of finite measurement resolution.

In the TGD framework, $SU(2)$ could correspond to a covering group of quaternionic automorphisms and number theoretic discretization (cognitive representations) would naturally lead to discrete and finite subgroups of $SU(2)$.

21.3 TGD and hyperfinite factors of type II_1: a bird's eye of view

In this section, a tentative identification of hyperfinite factors of type II_1 (HFFS) in the TGD framework [K126, K44] is discussed. Also some general related to the interpretation of HFFs and their possible resolution in the TGD framework are considered.

21.3.1 Identification of HFFs in the TGD framework

Inclusion hierarchies of extensions of rationals and of HFFs

I have enjoyed discussions with Baba Ilya Iyo Azza about von Neumann algebras. Hyperfinite factors of type II_1 (HFF) (`https://cutt.ly/lXp6MDB`) are the most interesting von Neumann algebras from the TGD point of view. One of the conjectures motivated by TGD based physics, is that the inclusion sequences of extensions of rationals defined by compositions of polynomials define inclusion sequences of hyperfinite factors. It seems that this conjecture might hold true!

Already von Neumann demonstrated that group algebras of groups G satisfying certain additional constraints give rise to von Neuman algebras. For finite groups they correspond to factors of type I in finite-D Hilbert spaces.

The group G must have an infinite number of elements and satisfy some additional conditions to give a HFF. First of all, all its conjugacy classes must have an infinite number of elements. Secondly, G must be amenable. This condition is not anymore algebraic. Braid groups define HFFs.

To see what is involved, let us start from the group algebra of a finite group G. It gives a finite-D Hilbert space, factor of type I.

1. Consider next the braid groups B_n, which are coverings of S_n. One can check from Wikipedia that the relations for the braid group B_n are obtained as a covering group of S_n by giving up the condition that the permutations σ_i of nearby elements e_i, e_{i+1} are idempotent. Could the corresponding braid group algebra define HFF?

   It is. The number of conjugacy classes $g_i\sigma_i g_i^{-1}$, $g_i == \sigma_{i+1}$ is infinite. If one poses the additional condition $\sigma_i^2 = U \times 1$, U a root of unity, the number is finite. Amenability is too technical a property for me but from Wikipedia one learns that all group algebras, also those of the braid group, are hyperfinite factors of type II_1 (HFFs).

2. Any finite group is a subgroup G of some S_n. Could one obtain the braid group of G and corresponding group algebra as a sub-algebra of group algebra of B_n, which is HFF. This looks plausible.

3. Could the inclusion for HFFs correspond to an inclusion for braid variants of corresponding finite group algebras? Or should some additional conditions be satisfied? What the conditions could be?

Here the number theoretic view of TGD could comes to the rescue.

1. In the TGD framework, I am primarily interested in Galois groups. The vision/conjecture is that the inclusion hierarchies of extensions of rationals correspond to the inclusion hierarchies for hyperfinite factors. The hierarchies of extensions of rationals defined by the hierarchies

of composite polynomials $P_n \circ ... \circ P_1$ have Galois groups, which define a hierarchy of relative Galois groups such that the Galois group G_k is a normal subgroup of G_{k+1}. One can say that the Galois group G is a semidirect product of the relative Galois groups.

2. One can decompose any finite subgroup to a maximal number of normal subgroups, which are simple and therefore do not have a further decomposition. They are primes in the category of groups.

3. Could the prime HFFs correspond to the braid group algebras of simple finite groups acting as Galois groups? Therefore prime groups would map to prime HFFs and the inclusion hierarchies of Galois groups induced by composite polynomials would define inclusion hierarchies of HFFs just as speculated.

   One would have a deep connection between number theory and HFFs.

How could HFFs emerge in TGD?

What could HFFs correspond to in the TGD framework? Consider first the situation at the level of M^8.

1. Braid group $B(G)$ of group (say Galois group as subgroup of S_n) and its group algebra would correspond to $B(G)$ and $L^2(B(G)\ )$. Braided Galois group and its group algebra could correspond to $B(G)$ and $L^2(B(G))$.

   The inclusion of Galois group algebra of extension to its extension could naturally define a Connes tensor product. The additional degrees of freedom brought in by extension of extension would be below measurement resolution.

2. Composite polynomials $P_n \circ .... \circ P_1$ are used instead of a product of polynomials naturally characterizing free n-particle states. Composition would describe interaction physically: the degree is the product of degrees of factors for a composite polynomial and sum for the product of polynomials.

   The multiplication rule for the dimensions holds also for the tensor product so that functional composition could be also seen as a number theoretic correlate for the formation of interacting many particle states.

3. Compositeness implies correlations and formation of bound states so that the number of degrees of freedom is reduced. The interpretation as bound state entanglement is suggestive. This hierarchical entanglement could be assigned with directed attention in the TGD inspired theory of consciousness [L121].

   An alternative interpretation is in terms of braids of braids of ... of braids with braid strands at a given level characterized by the roots of P_i. These interpretations could be actually consistent with each other.

4. Composite polynomials define hierarchies of Galois groups such that the included Galois group is a normal subgroup. This kind of hierarchy could define an increasing sequence of inclusions of braided Galois groups.

Consider the situation at the H level.

1. At the level of H, elements of the algebras $A \in \{SSA, Aff, I\}$a associated with super-symplectic symmetries acting at δM^4_+, affine isometries acting at light-like partonic orbites, isometries of δM^4_+, are labelled by conformal weights coming as non-negative integers. Also algebraic integers can be considered but for physical states conformal confinement requires integer valued conformal weights.

2. One can construct a hierarchy of representations of A such that subalgebras A_n with conformal weights $h \geq 0$ coming as multiples of n and the commutator $[A_n, A]$ annihilate the physical states. These representations are analogous to quantum groups and one can say that A_n defines a finite measurement resolution in A. A_{nk}, $k \geq 1$ is included by A_n for and one has a reversed sequence of inclusions.

One can construct inclusion hierarchies defined by the sequences $1 \div n_1 \div n_2 \div ....$ "$n_{-1} = 1$" corresponds to SSA. The factor spaces $A_{n_k}/A_{n_{k+1}}$ are analogs of quantum group-like objects associated with Jones inclusions and the interpretation is in terms of finite measurement resolution defined by $A_{n_{k+1}}$.

The factor spaces A/A_{n_k} define inclusion hierarchies with an increasing measurement resolution.

21.3.2 Could the notion of free probability be relevant in TGD?

In discussions with Baba Ilya Iyo Azza, I learned about the notion of free probability (`https://cutt.ly/LCY51sy`) assignable to von Neumann algebras. This algebra is II_1 factor. Originally, the notion was discovered by Voiolescu [A91] in order to attack some operator algebra problems, in particular free group factor isomorphism problem and Voiolescu demonstrated there is an infinity of von Neumann free group factors, which can be isomorphic. One can ask whether the free probability could have physical applications. In particular, whether the HFFs emerging naturally in TGD are consistent with this notion.

I try first to describe the notion of free probability as I understand it, on basis of what I learned in the discussions.

1. Free probability theory and classical probability theory differ because the latter is commutative and the former is highly noncommutative, and the notion of independence differs for them.

   In the classical theory, the expectations of the variables $X, Y, ...$ are commutative whereas in free probability theory they become observables represented by operators, which in general are non-commutative. The expectations for independent variables satisfy $E(XY) = E(X)E(Y)$ and more generally $E(X^mY^n) = E(X^m)E(Y^n)$. The expectations for powers are called moments.

   The free probability theory generalizes independent variables to free, in general non-commutative, operators $a, b, ...$ of von Neumann algebra M. The mean value $E(X)$ is replaced with a vacuum expectation value $\tau(a)$ of a, as physicists would call it. The expectations define what mathematicians call a normal state. Here τ, defining the vacuum expectation, denotes a linear functional in M.

   The random variable a can act on the argument of square integrable functions $F(m)$ defined in non-commutative von Neumann algebra M defining a commutative algebra $L^2(M)$. The action of a is non-commutative right or left multiplication of the argument of $F(m)$. One can speak of non-commutative probability space.

   Could group algebras and braid group algebra represent free algebras? Unfornatunately not. It is known that HFF probability is not consistent with free algebra property.

2. In the classical theory of independent random variables, one has $E(XY) = E(X)E(Y)$ and it is possible to express all expectations of monomials of $X_1, X_2, ...$ of polynomials of variables $X_1, X_2, ..$ in terms of moments $E(X_i^n)$.

   For free probability theory an analogous situation prevails although the formulas are not identical. Consider factor M of type II_1, which in the case ff free algebras cannot be hyperfinite. A linear functional $\tau(a)$ corresponds to vacuum expectation value, using the language of physicists. One has $\tau(1) = 1$. This corresponds to the condition $Tr(Id) = 1$. One has pointless space in the sense that the projector to a ray of Hilbert state defined by M has a vanishing trace. This corresponds to a finite measurement resolution requiring that the trace of the projector characterizing quantum measurement is a nonvanishing number.

   $\tau(ab) = \tau(a)\tau(b)$ is true for the generators of the free algebra and states that there is no correlation between a and b. This is however not true in general.

   [Note that an analogous condition holds true for the correlators of free quantum field fields at the level of momentum space: the n-point correlation functions reduce to products of momentum space propagators.]

   For instance, one would have

$$\tau(abab) = \tau(a^2)\tau(b)^2 + \tau(a)^2\tau(b^2) - 2\tau(a)^2\tau(b)^2$$

instead of $E(XYXY) = E(X^2)E(Y^2)$ for classical independent variables. Also now however only powers of a and b appear in the formula. This reduction of the expectations to the momenta $\tau(a^n)$ would hold quite generally.

3. A more precise definition is as follows (`https://cutt.ly/LCY51sy`). Unital subalgebras $A_1, ..., A_m$ are said to be freely independent if the expectation of the product $a_1...a_n$ is zero whenever each a_j has zero expectation, lies in an A_k, no *adjacent* a_j's come from the same subalgebra A_k, and n is nonzero. Random variables are freely independent if they generate freely independent unital subalgebras.

4. The lattice of non-crossing partitions (`https://cutt.ly/jCY6jNe`) for a finite set ordered cyclically, distinguishes free probability theory from lattice of all partition in the theory if independent random variables. Two partitions ab and xy are non-crossing if their elements do not correspond to order $axby$. The subsets of a non-crossing partition consist of elements, which are adjacent in this ordering and form connected subsets with k_i elements in which a cyclic subgroup $Z_{k_i} \subset Z_n$ acts. The expression of an element of S_n as a product of elements of cyclic subgroups Z_{n_k} of S_n corresponds to this kind of partition.

   Interestingly, in the construction of the non-planar parts of the twistor amplitudes similar cyclic ordering plays an important role. The problem of the twistor constructed are non-planar amplitudes which do not allow cyclic ordering. Could it be that the non-planar parts of the amplitudes do not have counterparts in a deeper theory utilizing HFFs? If so, free probability could code a very profound aspect of quantum theory.

Free random variables could correspond to the generators of von Neumann algebra of type II_1. My un-educated guess was that also HFFs realize free probability. I was wrong: thanks for Baba Ilya Iyo Azza for noticing this. It however seems that something highly reminiscent of free probability emerges for the algebras involved with TGD.

1. In the TGD framework, the generators typically generate an algebra of observables having interpretation as algebra of symmetries, such as affine algebra or super symplectic algebra.

2. ab would correspond to the product of say affine algebra generators a and b labelled by quantum numbers which are additive in the product. $\tau(a)$ would vanish as a vacuum expectation value of a generator with non-vanishing quantum numbers so that for generators one should have $\tau(ab) = \tau(a)\tau(b) = 0$.

   ab is expressible in terms of commutator and anticommutator as the sum $[a, b]/2 + \{a, b\}/2$. Both terms vanish if the quantum numbers of ab are non-vanishing. Only when the quantum numbers of a and b are opposite, the vanishing need not take place. If a and its Hermitian conjugate $a^\dagger$ with opposite quantum numbers belong to the set of generators of the free algebra, one has $\tau(aa^\dagger) > 0$ and is different from $\tau(a)\tau(a^\dagger) = 0$.

   Therefore the hermitian conjugates of generators cannot belong to the generators of the algebra creating the physical states. This algebra is highly reminiscent of free algebra since all vacuum expectations for the products vanish.

3. For affine and conformal algebras this condition corresponds to the requirement that physical states are created using only the generators with non-negative conformal weight $n \geq 0$ analogous to the algebra of creation operators. Also the generators of this algebra, whose number is finite, satisfy this condition. One could speak of half-algebra.

4. In TGD half-algebras appear for a different reason. The TGD Universe is fractal in several senses of the word. Also the algebra A of observables is fractal in the sense that it contains an infinite hierarchy of sub-algebras A_n for which the conformal weights are n-multiples of those for A. The finite measurement resolution is realized by the conditions that A_n and $[A_n, A]$ annihilate physical states and that also the corresponding classical Noether charges vanish, which gives strong conditions on space-time surfaces. These sub-algebras define hierarchies of measurement resolutions related to inclusions of HFFs.

If the generators of super symplectic algebra and extensions of affine algebras indeed define free algebras, the rules of free probability theory could bring in dramatic computational simplifications if the scattering amplitudes correspond to expectations for the polynomials of the free-algebra generators.

5. In ZEO, zero energy states are generated by this kind of half-algebra and its hermitian conjugate as superpositions of state pairs assigned to the opposite boundaries of causal diamond (CD= $cd \times CP_2$, where cd is the intersection of future and past directed light-ones).

   The members of the state pair are created by the half-algebra *resp.* its hermitian conjugate and are assigned with opposite boundaries of CD (intersection of light-ones with opposite time directions). The corresponding vacua are analogous to Dirac sea of negative energy fermions and its hermitian conjugate consisting of positive energy fermions. The zero energy states are analogous to pairs formed by Dirac's bras and kets. This allows to code the scattering matrix elements [L147, L148] as zero energy states.

21.3.3 Some objections against HFFs

One cannot avoid philosophical considerations related to the notion of probability and to the interpretations of quantum measurement theory (`https://cutt.ly/YXxSLS1`).

Standard measurement theory and HFFs

The standard interpretations of quantum measurement theory are known to lead to problems in the case of HFFs.

1. An important aspect related to the probabilistic interpretation is that physical states are characterized by a density matrix so that quantum theory reduces to a purely statistical theory. Therefore the phenomenon of interference central in the wave mechanics does not have a direct description.

   Another problem is that for HFFs, pure states do not exist as so-called normal states, which are such that it is possible to assign a density operator to them. This is easy to understand intuitively since by the $Tr(Id) = 1$ property of the unit matrix, there is no minimal projection. Selection of a ray would correspond to an infinite precision and delta function type density operator. The axiom of choices in mathematics is quite a precise analogy.

   One can of course argue that even if pure states as normal states are possible, in practice the studied system is entangled with the environment and that this forces the description in terms of a density matrix even when pure states are realized at the fundamental level.

2. In the purely statistical approach, the notion of quantum measurement must be formulated in terms of what occurs for the density matrix in quantum measurement. The expectation value of any observable A for the new density matrix generated in the measurement of observable O with a discrete spectrum must be a weighted sum for the expectations for the eigenstates of the observable with weights given by the state function reduction probabilities.

   Problems are however encountered when the spectrum contains discrete parts. In the TGD framework, the number theoretic discretization would make it possible to avoid these problems.

Should density matrix be replaced with a more quantal object?

These problems force us to ask whether there could be something deeply wrong with the notion of density matrix? The TGD inspired view of HFFs [K126, K44] suggests a generalization of the state as a density matrix to a "complex square root" of the density matrix. At the level of WCW as vacuum functional, it would be proportional to exponent of a real valued Kähler function of WCW identified as Kähler action for the space-time region as a preferred extremal and a phase factor defined by the analog of of action exponential. Zero energy state would be proportional to an exponent of Kähler function of WCW identified as Kähler action for space-time surface as a preferred extrema.

Problems with the interpretations of quantum theory

HFFs based probability concept has also problems with the interpretations of quantum theory, which actually strongly suggest that something is badly wrong with the standard ontology.

1. In TGD, this requires a generalization of quantum measurement theory [L91] [K130] based on zero energy ontology (ZEO) and Negentropy Maximization Principle (NMP) [K70] [L43], which is consistent with the second law [L135]. What is essential is that physics is extended to what I call adelic physics [L54, L53] to describe also the correlates of cognition. This brings in a measure for conscious information based on a p-adic generalization of Shannon entropy.

2. ZEO [K130] is forced by an almost exact holography in turn implied by general coordinate invariance for space-time as 4-surface. States in ZEO are superpositions of classical time evolutions and and is replaced by a new one in a state function reduction (SFR) [L91, L149]. The determinism of the unitary time evolution is consistent with the non-determinism of SFR. The basic problem of quantum measurement theory disappears since there are two times and two causalities. Causality of field equations and geometric time of physicists can be assigned to the classical time evolutions. The causality of free will and flow of experienced time can be assigned to a sequence of SFRs. The findings of Minev et al [L77] provide support for ZEO [L77].

   Quantum measurement as a reduction of entanglement can in principle occur for any entangled system pair if NMP favors it. There is no need to assume mysterious decoherence as a separate postulate. By NMP, entanglement negentropy can also increase by the formation of entangled states. Since entanglement negentropy is the sum of positive p-adic contribution and negative contribution from real entanglement and is positive, the increase of negentropy is consistent with the increase of real entanglement entropy.

   However, since classical determinism is slightly broken [L146] (there is analogy with the non-uniqueness of the minimal surfaces spanned by frames), the holography is not quite exact. This has important implications for the understanding of the space-time correlates of cognition and intentionality in the TGD framework.

The notion of finite measurement resolution and probabilistic interpretation

One can also ask whether something could go wrong with the quantum measurement theory itself. This notion of quantum measurement does not take into account the fact that the measurement resolution is finite.

The notion of finite measurement resolution realized in terms of inclusion, replacing Hilbert space ray with the included factor and reducing state space to quantum group like object, could allow us to overcome the problems due to the absence of minimal projectors for HFFs implying that the notion of Hilbert space ray does not make sense.

Quantum group like object would represent the degrees of freedom modulo finite measurement resolution described by the included factor. The quantum group representations form a finite subset of corresponding group representations and the state function reductions could occur to quantum group representations and the standard quantum measurement theory for factors of type I would generalize.

Connes tensor product and finite measurement resolution

In the TGD framework Connes tensor product could provide a description of finite measurement resolution in terms of inclusion.

1. In the TGD framework, inclusion of HFFs are interpreted in terms of measurement resolution. The included factor $M \subset N$ would represent the degrees of freedom below measurement resolution. N as M module would mean that M degrees of freedom are absorbed to the coefficient ring and are not visible in the physical states. Complex numbers as a coefficient ring of the Hilbert space are effectively replaced with M. In the number theoretic description of the measurement resolution, the extension of extension is replaced with the extension. The

quantum group, N as M, quantum group with quantum dimension $N : M$ would characterize the observable degrees of freedom.

This fits with the hierarchy of SSA_n:s. SSA_{n+1} would take the role of M and SSA_n that of N. This conforms with the physical intuition. Since n corresponds to conformal weight, the large values of n would naturally correspond to degrees of freedom below UV cutoff.

Could also IR cutoff have a description in the super symplectic hierarchy of SSA_n:s. It should correspond to a minimal value for conformal weight. The finite size of CD defining a momentum unit gives a natural IR cutoff. The proposal is that the total momentum assignable to the either half-cone of CD defines by $M^8 - H$ duality the size scale L as $L = h_{eff}/M$ [L109, L110].

2. For the hierarchies of extensions of rationals the upper levels of the extension hierarchy would not be observed. The larger the value of $n = h_{eff}/h_0$, n a dimension of extension of rationals associated with polynomial P defining the space-time region by $M^8 - H$ duality, the longer the quantum coherence scale.

   In this case large values for the dimension of extension would correspond to IR cutoff. Therefore UV and IR cutoffs would correspond to number theoretic and geometric cutoffs. This conforms with the view that $M^8 - H$ duality as an analog of Langlands duality is between number theoretic and geometric descriptions.

3. Duality suggests that also UV cutoff should have a number theoretic description. In the number theoretic situation, Galois confinement for these levels might imply that they are indeed unobservable, just like color-confined quarks. In fact, the hypothesis $n = h_{eff}/h_0$, n a dimension of extension of rationals associated with polynomial P defining the space-time region by $M^8 - H$ duality, for the effective Planck constant leads to estimate for ordinary Planck constant as $h = n_0 h_0$ where n_0 corresponds to the order of permutation group S_7.

   Could the interpretation be that these degrees of freedom are Galois confined and unobservable in the scales at which measurements are performed. Smaller values of h_{eff} would appear only in length scales much below the electroweak scale and at the limit of CP_2 scale?

How finite measurement resolution could be realized using inclusions of HFFs?

The basic ideas are that finite measurement resolution corresponds to inclusions of HFFs on one hand, and to number theoretic discretizations defined by extensions of rationals. In both cases one has inclusion hierarchies.

One can consider realizations at the level of WCW (geometry) and at the level of number theory in terms of adelic structures assignable to the extensions of rationals. Space-time surfaces can be discretized and this induces discretization of WCW. Even more, WCW should be in some natural manner effectively discrete.

In [K57, K29, K96] the construction WCW Kähler metric is considered and the mere existence of the K "ahler metric is expected to require infinite-D isometry group and imply constant curvature property. The Kähler function K is defined in terms of action consisting of the Kähler action and volume part for a preferred extremal (PE). There are however zero modes present and the metric depends on the zero modes. Twistor lift fixes the choices of H uniquely [L147, L148].

How to define WCW functional integral and how to discretize it? I have proposed that the Gaussian approximation to WCW integration is exact and allows to define a discretization in terms of the maxima (maybe also other extrema) of Kähler function. The proposal is that the exponential of Kähler function should correspond to a number theoretic invariant, perhaps the discriminant of the polynomial P defining PE by $M^8 - H$ duality.

Consider first the standard realization of the restriction $P : N \to M$ reducing the measurement resolution.

1. The definition of a unitary S-matrix for HFFs is non-trivial. Usually one considers only density matrices expressible in terms of projection operators P to subspaces of HFF.

   I have earlier proposed the notion of a complex square root of the density matrix as a generalization of the density matrix. In a direct sum representation of S over projections, in which S-matrix is diagonal, and the projection operators would be multiplied by phase factors. This definition looks sensible at the level of WCW but perhaps as a generalization of the density matrix rather than the S-matrix.

The exponent of Kähler function could have a modulus multiplied by a phase factor. Also an additional state dependent phase factor can be considered. The mathematical existence of the WCW integral fixes the modulus essentially uniquely to an exponent of Kähler function K multiplied by the metric volume element. K could also have an imaginary part.

2. The projected S-matrix PSP is unitary if the projection operator P must commute with S. S-matrix is realized at the level of HFFs so that the matrix representation does not make sense in a strict sense since the notion of ray is not sensical.

3. Projection $N \rightarrow M$ respects unitarity only if P commutes with S and $S^{\dagger}$. The S-matrix does not have matrix elements between M and N. This is a very tough condition.

How the finite measurement resolution could be realized in the TGD framework?

1. In WCW spin degrees of freedom plus algebras A_n. Number theoretic degrees of freedom are discrete and correspond to various p-adic degrees of freedom. Continuous WCW is associated with the real part of the adelic structure. Its number theoretic parts correspond to the p-adic degrees of freedom, which are discrete.

2. Discretization could be a natural and necessary part of the definition of WCW. Could discrete WCW degrees of freedom be identified in terms of symplectic and number theoretic invariants? They would represent for WCW spinor fields scalar degrees scalar degrees of freedom separable from spin degrees of freedom representable in terms of algebras A_n. These two kinds of degrees of freedom correspond to M and M' if the proposed general picture is correct.

Measurement resolution would be realized in terms of braid group algebras and algebras A_n defining the measurement resolution. What does this mean at the level of WCW?

1. Bosonic generators of SSA_n and possible other algebras A_n define tangent space basis for WCW. The gauge conditions stating that A_n and $[A_n, A]$ annihilate WCW spinor fields define a finite measurement resolution selecting only a subset of tangent space-generators and their super counterparts.

2. Consider first ideal measurement resolution in a function space. There is a complete basis of scalar functions Φ_m in a given space. The sum $\overline{\Phi}_m(x)\Phi_m(y) = \delta(x,y)$ would hold true for an infinite measurement resolution.

In a finite measurement resolution one uses only a finite subset of the scalar function basis, and completeness relation becomes non-local and is smoothed out: $\delta(x,y) \rightarrow D(x,y)$, which is non-vanishing for different point pairs x, y.

3. The condition of finite measurement resolution should define a partition of WCW to disjoint sets. In real topology, the condition $|x-y|^2$ would define a natural measurement resolution but would not define a partition.

In p-adic topology, the situation is different: the p-adic distance function $d(x-y)$ has values p^{-n} and the sets $d(x-y) < d$ are either disjoint or identical. One would have the desired partition. Therefore it seems that p-adicization is essential and the p-adic variants of WCW, or rather regions of WCW, obtained by discretization could allow partitions corresponding to various p-adic number fields forming the adele. Different p-adic representations of algebras A_n would define measurement resolutions.

There is a connection with spin glasses where spin energy landscape consisting of free energy minima allows ultrametric topology: p-adic topologies are indeed ultrametric. The TGD view of spin glasses is discussed in [L139]. One expects the decomposition of WCW to different p-adic topologies with ramified primes of polynomial P defining the p-adic sectors to which a given space-time surface can belong.

4. The consistency condition is that the transition probabilities $P(m \to n)$ between the states satisfying the gauge conditions representing finite measurement resolution, predicted by S-matrix or its TGD counterpat, should be constant should be constant in the subsets of WCW for which the completeness relation gives a non-vanishing $D(x,y)$ for the point pairs (x,y).

5. Does WCW have hierarchies of partitions such that the constancy of $P(m \to n)$ holds true within each partition?

   Do these partitions correspond to hierarchies of inclusions of HFFs defining increasing resolution? $M^8 - H$ duality does not allow all kinds of hierarchies. The hierarchies should be induced by the hierarchies of extensions of rationals. As the measurement precision increases, the partition involves an increasing number of sets and at the limit of ideal measurement resolution, the partition consists of algebraic points of WCW and of space-time surfaces.

6. $P = Q$ condition implying that space-time surfaces correspond to infinite prime, could appear as a consistency condition for allowed hierarchies. Preferred extremals and preferred polynomials would correspond to each other. Note that $P = Q$ conditions fixes the scaling of P.

In the TGD framework, one can challenge the idea, originally due to Wheeler, that transition probabilities are given by a unitary S-matrix.

1. The TGD based proposal is that in spin degrees of freedom, that is for many-fermion states for a given space-time surface, the counterpart of S-matrix could be be given by the analog of Kähler metric in the fermionic Hilbert space [L124]. This would mean a geometrization of quantum theory, at least in fermionic degrees of freedom.

   The transition probabilities would be given by $P(m \to n) = K_{\overline{m}n}K^{\overline{n}m}$ and the properties of Kähler metric K give analogs of unitary conditions and probability conservation plus some prediction distinguishing the proposal from the standard view.

2. In the infinite-D situation, the existence of Hilbert space Kähler metric in the fermionic sector is an extremely powerful condition and one expects that the Kähler metric is a unique constant curvature metric allowing a maximal group of isometries. This, together with p-adization, would help to satisfy the constancy conditions for $P(m \to n)$ inside the sets for which $D(x,y)$ is non-vanishing. In fact, one expects that since super-generators are proportional to isometry generators contracted with WCW gamma matrices the metric in the fermionic degrees of freedom is induced by Kähler metric in the basis of isometry generators.

3. This condition could allow a generalization to include the states obtained by application of the bosonic generations of A_n the to ground state. This would mean that in bosonic degrees of freedom Kähler metric of WCW in the isometry basis defines the transition probabilities. Tangent vectors of WCW correspond to the isometry generators. An arbitrary number of isometry generators is involved in the definition of the state. However, the Kähler metric of WCW induces a Kähler metric in the algebra generated by the isometry generators, which is analogous to the algebra of tensors.

21.4 $M^8 - H$ duality and HFFs

$M^8 - H$ duality [L109, L110] gives strong constraints on the interpretation of HFFs at the number theoretic M^8 side and the geometric H side of the duality. One must also understand the relation between $M^8 - H$ duality and $M - M'$ duality, identifiable as quantum-classical correspondence (QCC).

Although McKay correspondence [A78, A77, A64, A54, A53] is not quite at the core of $M^8 - H$ duality, it is difficult to avoid its discussion. I have considered McKay correspondence also before [L46, L95, L96, L153].

21.4.1 Number theoretical level: M^8 picture

Braided Galois group algebras

For n-braids the permutation group has extension to a braid group B_n defining an infinite covering of S_n for which permutation corresponds to a geometric operation exchanging the two strands of a braid. There are also hierarchies of finite coverings.

S_n is replaced with the Galois group which is a subgroup of S_n and the property of being a subgroup of S_n allows to identify a braided Galois group as a braided Galois subgroup of braided S_n. In the same way one can identify the braided Galois group algebra defining HFF as a subalgebra of HFF associated with braid group algebra defined by S_n. One can ask whether the property of being a number theoretic braid could be interpreted as a kind of symmetry breaking to S_n to the Galois group of P.

M^8-H duality [L109, L110] suggests that the roots correspond to braid strands of geometric braids in H. If so, the braided Galois group would be both topological and number theoretic: topology, natural at the level of H, and number theory, natural at the level of M^8, would meet by M^8-H duality.

This picture looks nice but one can make critical questions.

1. Can the n roots really correspond to n braid strands at the level of H? The n roots correspond to, in general complex, algebraic numbers associated with the extension of rationals. The real projections correspond to mass shells with different mass values mapped to light-cone proper time surfaces in H by M^8-H duality. Therefore the action of the Galois group changes mass squared values and does not commute with Lorentz transformations. This suggests a violation of causality.

   Should one restrict the Galois group to the isotropy group of a given root? This would mean number theoretic symmetry breaking and could relate to massivation. This restriction would however trivialize the braid.

2. Zero energy ontology (ZEO) could come to the rescue here. In fact, ZEO implies space-time surfaces are the basic objects rather than 3-surfaces so that quantum states are superpositions of space-time surfaces as preferred extremals (PEs). This is forced by the slight violation of determinism of field equations implying also a slight violation of ideal holography.

   Space-time surfaces are minimal surfaces [L146] analogous to soap films spanned by frames and there can be a slight violation of the strict determinism localized to frames as already 2-D case suggests. This could be also seen as violation of classical causality. At the level of consciousness theory it would be a classical correlate for the non-determinism of intentional free will.

   In particular, time-like braids for which the braiding is time-like and corresponds to a dynamical dance pattern, make sense. For these braids one can in principle select the mass squared value mapped to a value of light-cone proper time a to belong to the braid. The values of a need not be the same.

Also Galois confinement, which is a key aspect of the number theoretic vision, is involved.

1. Galois confinement states that physical states transform trivially under the Galois group of extension. This condition for physical states follows as a consequence of periodic boundary conditions for causal diamond (CD), which takes the role of box for a particle in a box.

   A weaker condition would be that singlet property holds only for the isotropy group of a given root of the polynomial P characterizing the space-time region and corresponding to mass squared value and at the level of H to a value of the light-cone proper time a.

2. In M^8, the momenta of particles are points at the mass shells of $M^4 \subset M^8$ identifiable as hyperbolic spaces $H^3 \subset M^4$ defined with mass squared values defined as the roots of P. The momenta correspond to algebraic integers (the momentum unit is defined by CD) for the extension defined by P, and in general they are complex. The interpretation is as virtual particles which form physical particles as composites. The physical states must have total momenta, which are ordinary integers. This gives the simplest form of Galois confinement.

3. Commutativity with the Lorentz group would favor the isotropy group instead of the full Galois group. One must be however very cautious since in zero energy ontology (ZEO) physical states correspond to a superposition of space-time surfaces and time-like braids are natural. There is a small violation of strict determinism at the level of preferred extremas. The labelling of braid strands based on the images of roots as mass squared values at level of H is quite natural and is not in conflict with causality.

The Galois group for a polynomial $P_n \circ ... \circ P_1$ has a decomposition to normal subgroups GA_i acting as Galois groups for the i:th sub-extension.

1. The number of roots is a product of the numbers of roots for P_i. Therefore the natural identification is that number theoretic braid groups allow a natural interpretation in terms of braids of braids ... of braids.

2. This hierarchy defines an inclusion hierarchy for the braided HFFs assignable to the polynomials $P_k \circ ... \circ P_1$, $k = 1, ..., n$. It is not quite clear to me whether these inclusions reduce to Jones inclusions and whether one can characterize the inclusions in the sequence by the same invariants as in the case of Jones inclusions.

3. In this picture the Connes tensor product would correspond to formation of composite polynomials $P \circ Q$. The reduction in the number of degrees of freedom from that for the ordinary tensor product of braided Galois group algebras would be due to interactions described in terms of polynomial decomposition. Various braids in the hierarchy could correspond to braids at different sheets of the many-sheeted space-time.

4. Any normal subgroup Gal_i of Galois group Gal defining a sequence of inclusions of normal sub-groups Gal_i can be trivially represented. By normal subgroup property, the elements of Gal can be represented as semidirect products of elements of the factor groups $G_i = Gal_i/Gal_{i-1}$. Any representation of Gal can be decomposed to a direct sum of tensor products of representations of G_i.

   From this decomposition it is clear that any group G_i in the decomposition can be trivially represented so that one obtains a rich structure of representation in which some G_i:s are trivially represented.

A possible interpretation is that in TGD, rational polynomials give discrete cognitive representations as approximations for physics. Cognitive representations are in the intersection of p-adicities and reality defined by the intersection of reals and extension of p-adics defined by the algebraic extension of the polynomial P defining a given space-time surface. Continuum theory would represent real numbers as a factor of the adele.

One can ask whether the various zeta functions consistent with the integer spectrum for the conformal weights and possibly also with conformal confinement, appear at the continuum limit and provide representations for the space-time surfaces at this limit? In this framework, it would be natural for the roots of zeta to be algebraic numbers [K97]. Also in the case of ζ, the virtual momenta of fermions would be algebraic integers for virtual fermions and integers for the physical states. This makes sense if the notions of Galois group and Galois confinement are sensible for ζ.

As noticed, the notion of ζ generalizes. The so-called global L-functions (`https://cutt.ly/3VNPYmp`) are formally similar to ζ and the extended Riemann Hypothesis could be true for them. The physical motivation for RH would be that it would allow fermion with any conformal weight to appear in a state which is conformal singlet. Algebraic integers for a finite extension of rationals replace integers in the ordinary ζ and one has an entire hierarchy of L-functions. Could one think that the global L-functions could define preferred extremals at the continuum limit?

How could the degrees of prime polynomials associated with simple Galois groups and ramified primes relate to the symmetry algebras acting in H?

The goal is to relate various parameters characterizing polynomials P for which braided Galois group algebras define HFFs to the parameters labelling the symmetry algebras defining hierarchies of HFFs at the level H. There are good reasons to believe that polynomial composition defines inclusion of HFFs and that this inclusion induces the inclusions for the symmetry algebras A_n at the level of H.

One can identify simple Galois groups as prime groups having no normal subgroups. The polynomial P associated with a simple Galois group cannot have no non-trivial functional decomposition $P_n \circ ... \circ P_1$ if one stays in the field of rationals (say). This leads to the notion of prime polynomials. Note that this notion of primeness does not correspond to the irreducibility stating that polynomials with coefficients in a given number field do not allow decomposition to lower degree polynomials.

A polynomial P is also partially characterized by ramified primes and discriminant defines a Galois invariant for the polynomial as also the symmetric polynomials formed from the roots.

How do these two notions of primeness relate to the p-adic prime decomposition of adelic structures defined by the algebras A_n, which act at the level of H and decomposed adelically to a tensor product of all $A_{n,p}$:s?

Simple Galois groups correspond to prime polynomials. This notion looks fundamental concerning the understanding of the situation at the level of H.

1. Polynomials can be factorized into composites of prime polynomials [A34, A74] (`https://cutt.ly/HXAKDzT` and `https://cutt.ly/5XAKCe2`). A polynomial, which does not have a functional composition to lower degree polynomials, is called a prime polynomial. It is not possible to assign to prime polynomials prime degrees except in special cases. Simple Galois groups with no normal subgroups must correspond to prime polynomials.

2. For a non-prime polynomial, the number N of the factors P_i, their degrees n_i are fixed and only their order can vary so that n_i and $n = \prod n_i$ is an invariant of a prime polynomial and of simple Galois group [A34, A74]. Note that this composition need not exist for monic polynomials even if the Galois group is not simple so that polynomial primes in the monic sense need not correspond to simple Galois groups.

3. The number of the roots of P_i is given by its order n_i, and since Galois group and its braided variant permute the roots as subgroup of S_{n_i}, it is natural to assume that the roots define an n_i-braid. The composite polynomial would define braid of braids of ... of braids. At the level of H the braid strands would correspond to flux tubes and braiding would have a geometric interpretation.

4. The integer n characterizing the algebra A_n acting in H would naturally correspond to the degree of n of P and the decomposition of P to polynomial primes would naturally correspond to an inclusion hierarchy A_{n_i} $A_{n_1} \subset A_{n_1 n_2} \subset ... \subset A_n$ with improving resolution allowing to see braids and braids of braids.

   The corresponding factor spaces realizing the notion of finite measurement resolution, would be analogous to quantum groups obtained when some number of the highest levels in the hierarchy of braids in the braid of braids of ... braids are neglected and the entire algebra is replaced with a quantum group-like structure. This means cutting off some number of the highest levels in the tree-like hierarchy. The trunk is described by a quantum group-like object.

5. This hierarchy corresponds to the hierarchy of Galois groups as normal subgroups assignable to braids in the decomposition and the hierarchy of corresponding braided Galois group algebras defining inclusions of HFFs. Galois group algebras would act as braid groups inc corresponding algebras A_n. Therefore number theoretic and geometric views would fuse together.

6. Connes tensor product is a central notion in the theory of HFFs and it could be naturally associated with the inclusions of brided Galois group algebras. The counterpart for the quantum group as factor space N/M of the factors would correspond to the inclusion $Gal_{i-1} \subset Gal_i$ as a normal subgroup. The inclusion defines group $G_i = Gal_i/Gl_{i-1}$. Also its braided variant is defined. The factor space of braided group algebras would be the counterpart of the quantum group G_i.

   Note that these quantum group-like objects could be much more general than the quantum groups defined by subgroups of $SU(2)$ appearing in Jones inclusions.

What about the interpretation of the ramified primes, which are Galois invariants as also the root spectrum (but not the roots themselves) and depends on the polynomial.

In accordance with the proposed physical interpretation of the ramified primes as preferred p-adic primes labelling particles in p-adic thermodynamics, ramified primes p_i would define preferred p-adic primes for the p-adic variants of the algebras A_n in the adelic generalization of A_n as tensor product of p-adic representations of $A_{n,p}$ of A_n. A_{n,p_i} would be physically and also mathematically special.

Both the degree n as the number of braids of P and the ramified primes of P would dictate the physically especially relevant algebras A_{n,p_i}. For instance, un-ramified primes could be such that corresponding p-adic degrees of freedom are not excited.

21.4.2 Geometric level: H picture

The hierarchies of algebras SSA_n, Aff_n and I_n

The algebras $A_n \in \{SSA_n, Aff_n, I_n\}$ for $n = p$ acting at the level of WCW seem to have special properties since the values of the conformal weights for the factor algebras defined by the conditions that A_n and $[A_n, A]$ annihilate physical states, allow the structure of finite field $G(p)$ or even its extension $G(p,k)$ for conformal weights in extension of rationals. The representations would be finite-D. Also the values $n = p^k$ seem special and the finite field representations of SSA_p could be extended to p-adic representations.

This raises the question, whether one could regard n as a p-adic number? The interpretation of n as the number of braid strands assignable to roots of the polynomial P with degree n defining the space-time surface, looks more approriate since it allows braid group algebra of P to act in SSA_n, This identification does not favor this interpretation.

A more plausible interpretation is that the p-adic primes, identifiable as ramified primes of P, characterize the p-adic representations of SSA_n. This also conforms with the interpretation of preferred p-adic primes characterizing elementary particles as ramified primes.

The polynomials with prime degree could be however physically special. The algebras SSA_p, with p defining the degree of polynomial p allow finite field representations, which extend to p-adic representations and one can ask whether the prime decomposition of n could allow some kind of inclusion hierarchy of representations.

This would also give a possible content for the p-adic length scale hypothesis $p \simeq 2^k$, k prime, or its generalization involving primes near powers of prime $q = 2, 3, 5, ...$ A more general form of p-adic length scale hypothesis would be $p \simeq q^n$, n the degree of P.

Commutants for algebras A_n and braid group algebras

For the super $A \in \{SSA, Aff, I\}$, the inclusion An_{nk} to SSA_n should define a Connes tensor product. One would obtain inclusion hierarchies labelled by divisibility hierarchies $n_1 \div n_2 \div ....$. For braid group algebras one obtains similar hierarchies realized in terms of composite polynomials.

What about the already mentioned "classical" degrees of freedom associated with the fluxes of the induced Kähler form? They should be included to M' at the level of H. The hierarchies of flux tubes within ... within flux tubes correspond to the hierarchies assignable to M' at the level of H.

The number theoretic degrees of freedom identifiable as invariants of Galois groups should be included to M' at the number theoretical level. The hierarchies of roots assignable to composite polynomials $P_n \circ ... \circ P_1$ with roots assigned to the strands of time like braid strands could correspond to these hierarchies at the level of M^8.

21.4.3 Wild speculations about McKay correspondence

McKay correspondence is loosely related to the HFFs in TGD framework [L46, L96, L95, L153] and I cannot avoid the temptation to try to understand it in TGD framework.

1. The origin of the McKay graphs for inclusions is intuitively understood. Representations of finite subgroups of $SU(2)$ are assignable to 2-D factors. These representations could correspond to closed subgroups of quaternionic $SU(2)$ on the basis of the reduction to

$M_2(C) \otimes M_2(C) \otimes .....$ A reduction of degrees of freedom happens for HFFs since they are subalgebras of $B(H)$ and this could reduce the closed subgroup to a finite subgroup.

Also the interpretation N as tensor product of M and quantum group $SU(2)$ suggests the same since quantum groups have a finite number of irreps, when q equal is a root of unity. The analog of McKay graph coding fusion rules for the quantum group tensor products would reduce to McKay graphs.

2. Why would the McKay graphs for finite subgroups of $U(2)$ correspond to those for affine or ordinary Lie algebras? Could these Lie-algebras emerge from the inclusions. This is a mystery, at least to me.

3. In the TGD framework one can ask why there should be Weyl group of extended ADE Dynkin diagram assignable to SSA_n? SSA_n defines a representation of SSA with SSA_n and $[SSA_n, SSA]$ acting trivially. Could this representation correspond to an affine or ordinary ADE algebra? Similar question makes sense for all algebras $A_n \in \{SSA_n, Aff_n, I_n\}$. A_nn would define a cutoff of the SSA so that all generators with conformal weight larger than n would be represented trivially.

   Note that for $n = p$, the conformal weights of A_n would define a finite field and if algebraic integers also its extension. This case could correspond to polynomials defining cyclic extension of order p with roots coming as roots of unity.

4. The Weyl groups assignable to the "factor algebra" of SSA_n defined by the gauge conditions for A_n and $[A_n, A]$ and proposed to reduce to ADE type affine or ordinary Lie algebra should relate to Galois groups for polynomial P with degree n as number of braid strands.

   (a) Could the braid strands correspond to the roots of ADE algebra so that roots in the number theoretic sense would correspond to the roots in the group theoretic sense? This would conform with Langlands correspondence [K61, A46, A45] discussed from the TGD perspective in [K61] [L14, L34].

   (b) Could the Weyl groups allow identification as subgroups of corresponding Galois groups?

   Note that simple Galois groups correspond to so-called prime polynomials [A34, A74] allowing no decomposition to polynomials of lower degree so that the preferred values of n would correspond to prime polynomials.

5. Affine electroweak and color algebras an their M^4 counterparts would be special since they wuuld not emerge a dynamical symmetries of SSA_n but define algebras Aff_n and I_n related to the light-like partonic orbits. They would also correspond to symmetries both at the level of M^8 and H.

This inspires the following questions, which of course look very naive from the point of view of a professional mathematician. My only excuse is the strong conviction that the proposed picture is on the right track. I might be wrong.

1. The Jones inclusion of HFFs [A56, A94, A95] involves an extended or ordinary ADE Dynkin diagram assignable also to finite subgroups of $SU(2)$ by McKay correspondence [A78].

   Could the Weyl group of an extended ADE diagram really correspond to an affine algebra or quantum group assignable to A_n? If so, one would have dynamical symmetries and should relate to the "factor" space SSA/SSA_n in which SSA_n defines a measurement resolution.

2. HFF can be regarded algebraically as an infinite tensor power of $M_2(C)$. Does the representation as a 2×2 matrix imply the emergence of representations of a closed subgroup of $SU(2)$ or its quantum counterpart. Could the reduction of degrees of freedom due to the finite measurement resolution imply that the closed subgroup reduces to a finite subgroup?

3. The algebraic decomposition of HFF to an infinite tensor power of $M^2(C)$ would suggest that the including factor N with dimension 1 is equal to $M^{d_q} \otimes M^{1/d_q}$, where d_q is the quantum dimension characterizing eith M or N. Could these two objects correspond to an ADE type affine algebra and quantum group with inverse quantum dimensions? Or could either of them correspond to ADE type affine algebra or quantum group?

4. Could one think that the analog of McKay graph for the quantum group-like object assignable to affine group by a finite measurement resolution reduces to the McKay graph for a finite subgroup of $SU(2)$ because only a finite number of representations survives?

5. Could the finite subgroups of $SU(2)$ correspond to finite subgroups for the covering group of quaternion automorphisms acting naturally in M^8? Could these finite subgroups correspond to finite subgroups of the rotation group $SU(2)$ at H side?

Could only the n_C (dimension of Cartan algebra) roots appearing in the Dynkin diagram be represented as roots of a polynomial P in extension of rationals or its quantum variant? This option fails since the Dynkin diagram does not allow a symmetry group identifiable as the Galois group. The so called Steinberg symmetry groups (`https://cutt.ly/GXMb8Si`) act as automorphisms of Dynkin diagrams of ADE type groups and seem quite too small and fail to be transitive as action of the Galois group of an irreducible polynomial is.

$M^8 - H$ duality inspires the question whether a subgroup of Galois group could act as the Weyl group of ADE type affine or ordinary Lie algebra at H side.

1. The Galois group acts as a braid group and permutes the roots of P represented as braid strands. Weyl group permutes the roots of Lie algebra

   The crazy question is whether the roots of P and roots of the ADE type Lie-algebra could correspond to each other. Could the roots of P in $N \to 1$-correspondence with the non-vanishing roots of the representation of Lie algebra or of its affine counterpart containing an additional root corresponding to the central extension?

   If the roots appearing in the Dynkin diagram correspond to a subset of roots of polynomial P, the Weyl group could correspond to a minimal subgroup of the Galois group generated by reflections and generating all non-vanishing roots of the Lie algebra.

2. The action of the Weyl group should give all roots for the representation of G. Could the Weyl group, which is generated by reflections, correspond to a minimal subgroup of Gal giving all roots as roots of P when applied to the McKay graph?

   The obvious objection is that the order of the Weyl group increases rapidly with the order of the Cartan group so that also the Gal and also the order of corresponding polynomials P would increase very rapidly. Gal is a subgroup of S_n having order $n!$ for a polynomial of degree n so that the degree of P need not be large and this is what matters.

   If the m braid strands labelled by the m roots correspond to the roots of the affine algebra, it would be natural to assign affine algebra generators to these roots with the braid strands. The condition $n = Nm$ implies that m divides n. For $Gal = S_n$ with order $n!$ this condition is very mild. $Gal = Z_p$ fixes the Lie algebra to A_p.

   The root space of the dynamical symmetry group would have dimension m, which is a factor of n. For Lie algebras A_n and D_{2n} (with $n \geq 4$) appear besides E_6 and E_8. For affine Lie algebras $\hat{A}_n$ or $hatD_n$ (with $n \geq 3$) and $\hat{E}_6$, $\hat{E}_7$ and $\hat{E}_8$ appear. For large values of n, there are two alternatives for even values of n.

3. One can also consider quantum arithmetics based on $\oplus$ and $\otimes$ and replace P with its quantum counterpart and solve it in the space of irreps of the finite subgroup G of $U(2)$ defining a quantum analog for an extension of rationals. The roots of the quantum variant of P would be direct sums of irreps of G.

   These quantum roots define nodes of a diagram. This diagram should include as nodes the roots of the Dynkin diagram defined by positive roots, whose number is the dimension n_C of Cartan algebra.

   Could the missing edges correspond to the edges of the Mac-Kay graph in the tensor product with a 2-D representation of $SU(2)$ restricted to a subgroup? The action of 2-D representation would generate the (extended) Dynkin diagram ADE type.

One can look this option in more detail.

1. Assume that adjoint representation Adj of an affine or ordinary ADE Lie group L emerges in the tensor product $M^2(C)\otimes...\otimes M^2(C)$ allowing embedding of $SU(2)$ as diagonal embedding. One can imbed the finite subgroup $G \subset SU(2)$ as a diagonal group $G \times G \times ... \times G$ to $M^2(C) \otimes ... \otimes M^2(C)$.

   Also a given representation of G can be embedded as a direct sum of the copies of the representation, each acting in one factor of $M^2(C) \otimes ... \otimes M^2(C)$. The $2-D$ canonical representation of $G \subset SU(2)$ has a natural action in the $G\times G\times...\times G$ to $M^2(C)\otimes...\otimes M^2(C)$ and would generate a McKay graph.

   One can also embed G to L as $G \subset SU(2) \subset L$. Adj can be decomposed to irreps G. Therefore the tensor product action of various irreps of G, in particular the canonical 2-D representation, in Adj is well-defined. The tensor action of the 2-D canonical representation of G gives a McKay graph such that the nodes have weights telling how many times a given irrep appears in the decomposition of Adj to irreps of G. The weighted sum of the dimensions of irreps of G is equal to the dimension of Adj.

2. This construction is possible for any Lie group and some consistency conditions should be satisfied. That McKay graph is the same as the generalized Dynkin diagram would be such a consistency condition and leave only simply laced Lie groups.

3. What can one say about the weights of the weighted McKay graph? Could the weights be the number of the images of the positive root under the action of the Weyl group W of L.

   The McKay graph would correspond only to the n_C (dimension of the Cartan algebra) positive roots appearing in the Dynkin diagram of Adj. How to continue the Dynkin dynkin to a root diagram of Adj?

4. Could the n_C roots in the Dynkin diagram correspond to the roots of a polynomial P in a quantum extension of rationals with roots as irreps of G appearing in the McKay graph. The multiple of a given root would correspond to its orbit under W. The action of W as reflections in the quantum extension of rationals, spanned by the roots of Adj, as vectors with integer components would generate all roots of Adj as quantum algebraic integers in the quantum extension of rationals.

5. As proposed, one could interpret the Dynkin diagram as a subdiagram of the root diagram of Adj and identify its nodes as roots of Gal for a suitable polynomial P. The Weyl group could be the minimal transitive subgroup of Gal.

6. The Galois group of extension of ... of rationals is a semidirect of Galois groups which can be chosen to be simple so that the polynomials considered are prime polynomials unless one poses additional restrictions. What does this restriction mean for the ADE type Weyl group of assignable to the exrtension

21.5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K57, K96].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

21.5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [L55] [L141, L147, L148] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$ of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A63] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.

2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.

3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

21.5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.

2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L157]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

   The super symplectic algebra A has an infinite hierarchy of sub-algebras [L157] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

   This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.

2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L157] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by M^8-H duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.

2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L157] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P) = n(braid) = n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.

3. The actions related to different values of $n(P) = n(braid) = n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.

4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L151] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i = 2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r = 1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

   A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.

5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p \simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K71, K72]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L151], and the forward scattering amplitude corresponds to the propagator.

   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L151], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L151].

3. p-Adic length scale hypothesis [L158] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

   Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L157] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K57, K29]. As isometries they would naturally permute the maxima with each other.

21.6 About the TGD based notions of mass, of twistors and hyperbolic counterpart of Fermi torus

The notion of mass in the TGD framework is discussed from the perspective of $M^8 - H$ duality [L109, L110, L158, L148].

1. In TGD, space-time regions are characterized by polynomials P with rational coefficients [L109, L110]. Galois confinement defines a universal mechanism for the formation of bound states. Momenta for virtual fermions have components, which are algebraic integers in an extension of rationals defined by a polynomial P characterizing a space-time region. For the physical many fermion states, the total momentum as the sum of fermion momenta has components, which are integers using the unit defined by the size of the causal diamond (CD) [L91, L138, L149].

2. This defines a universal number theoretical mechanism for the formation of bound states as Galois singlets. The condition is very strong but for rational coefficients it can be satisfied since the sum of all roots is always a rational number as the coefficient of the first order term.

3. Galois confinement implies that the sum of the mass squared values, which are in general complex algebraic numbers in E, is also an integer. Since the mass squared values correspond to conformal weights as also in string models, one has conformal confinement: states are conformal singlets. This condition replaces the masslessness condition of gauge theories [L158].

Also the TGD based notion of twistor space is considered at concrete geometric level.

1. Twistor lift of TGD means that space-time surfaces X^4 is $H = M^4 \times CP_2$ are replaced with 6-surfaces in the twistor space with induced twistor structure of $T(H) = T(M^4) \times T(CP_2)$ identified as twistor space $T(X^4)$. This proposal requires that T(H) has Kähler structure and this selects $M^4 \times CP_2$ as a unique candidate [A63] so that TGD is unique.

2. One ends up to a more precise understanding of the fiber of the twistor space of CP_2 as a space of "light-like" geodesics emanating from a given point. Also a more precise view of the induced twistor spaces for preferred extremals with varying dimensions of M^4 and CP_2 projections emerges. Also the identification of the twistor space of the space-time surface as the space of light-like geodesics itself is considered.

3. Twistor lift leads to a concrete proposal for the construction of scattering amplitudes. Scattering can be seen as a mere re-organization of the physical many-fermion states as Galois singlets to new Galois singlets. There are no primary gauge fields and both fermions and bosons are bound states of fundamental fermions. 4-fermion vertices are not needed so that there are no divergences.

4. There is however a technical problem: fermion and antifermion numbers are separately conserved in the simplest picture, in which momenta in $M^4 \subset M^8$ are mapped to geodesics of $M^4 \subset H$.The led to a proposal for the modification of $M^8 - H$ duality [L109, L110]. The modification would map the 4-momenta to geodesics of X^4. Since X^4 allows both Minkowskian and Euclidean regions, one can have geodesics, whose M^4 projection turns backwards in time. The emission of a boson as a fermion-antifermion pair would correspond to a fermion turning backwards in time. A more precise formulation of the modification shows that it indeed works

The third topic of this article is the hyperbolic generalization of the Fermi torus to hyperbolic 3-manifold H^3/Γ. Here $H^3 = SO(1,3)/SO(3)$ identifiable the mass shell $M^4 \subset M^8$ or its $M^8 - H$ dual in $H = M^4 \times CP_2$. Γ denotes an infinite subgroup of $SO(1,3)$ acting completely discontinuously in H^3. For virtual fermions also complexified mass shells are required and the question is whether the generalization of H^3/Γ, defining besides hyperbolic 3-manifold also tessellation of H^3 analogous to a cubic lattice of E^3.

21.6.1 Conformal confinement

The notion of mass distinguishes TGD from QFT. As in string models, mass squared corresponds to a conformal weight in TGD. However, in the TGD framework tachyonic states are not a curse but an essential part of the physical picture and conformal confinement, generalizing masslessness condition, states that the sum of conformal weights for physical states vanishes. This view conforms with the fact that Euclidean space-time regions are unavoidable at the level of H. Positive *resp.* negative *resp.* vanishing conformal weights can be assigned with Minkowskian *resp.* Euclidean space-time regions *resp.* light-like boundaries associated with them.

Mass squared as conformal weight, conformal confinement and its breaking

At the level of M^8, the momentum components for momenta as points of $H_c^3 \subset M_c^4 \subset M_c^8$ are (in general complex) algebraic integers in an extension of rationals defined by the polynomial P defining the space-time region. For physical states the momentum components for the sum of the momenta are ordinary integers when the momentum unit is defined by the size scales of causal diamond (CD). This scale corresponds to a p-adic length scale for p-adic prime, which is a ramified prime of the extension of rationals defined by the polynomial P.

For virtual many-fermion states the mass squared is an algebraic integer but an ordinary integer for the physical states [L158]. The question is whether the mass squared for the physical states can be negative so that one would have tachyons. The p-adic mass calculations require the presence of tachyonic mass squared values and the proposal is conformal confinement in the sense that the sum of mass squared values for the particles present in state and identifiable as conformal weights sum up to zero. Conformal confinement would generalize the masslessness condition of gauge field theories.

The observed mass squared values would correspond to the Minkowskian non-tachyonic parts of the mass squared values assignable to states, which in general are entangled states formed from tachyonic and non-tachyonic states. p-Adic thermodynamics would describe the entanglement in terms of the density matrix and observed mass squared would be thermal average. p-Adic thermodynamics leads to a breaking of the generalized conformal invariance and explains why different values of the Virasoro scaling generator L_0 are involved. Since complex mass squared values with a negative real part are allowed as roots of polynomials, the condition is highly non-trivial.

Association of mass squared values to space-time regions

$M^8 - H$ duality [L109, L110] would make it natural to assign tachyonic masses with CP_2 type extremals and with the Euclidean regions of the space-time surface. Time-like masses would be assigned with time-like space-time regions. In [L155] it was found that, contrary to the beliefs held hitherto, it is possible to satisfy boundary conditions for the action action consisting of the Kähler action, volume term and Chern-Simons term, at boundaries (genuine or between Minkowskian and Euclidean space-time regions) if they are light-like surfaces satisfying also $detg_4 = 0$. Masslessness, at least in the classical sense, would be naturally associated with light-like boundaries (genuine or between Minkowskian and Euclidean regions).

Riemann zeta, quantum criticality, and conformal confinement

The assumption that the space-time surface corresponds to rational polynomials in TGD is not necessary. One can also consider real analytic functions f [L148]. The condition that momenta of physical states have integer valued momentum components implies integer valued conformal weights poses extremely strong conditions on this kind of functions since the sum of the real parts of the roots of f must be an integer as a conformal weight identified as the sum of in general complex virtual mass squared values.

There are strong indications Riemann zeta (`https://cutt.ly/iVTV1kqs`) has a deep role in physics, in particular in the physics of critical systems. TGD Universe is quantum critical. What quantum criticality would mean at the space-time level is discussed in [L155]. This raises the question whether Riemann zeta could have a deep role in TGD.

First some background relating to the number theoretic view of TGD.

1. In TGD, space-time regions are characterized by polynomials P with rational coefficients [L109, L110]. Galois confinement defines a universal mechanism for the formation of bound states. Momenta for virtual fermions have components, which are algebraic integers in an extension of rationals defined by a polynomial P characterizing space-time region. For the physical many fermion states, the total momentum as the sum of fermion momenta has components, which are integers using the unit defined by the size of the causal diamond (CD).

   This defines a universal number theoretical mechanism for the formation of bound states. The condition is very strong but for rational coefficients it can be satisfied since the sum of all roots is always a rational number as the coefficient of the first order term.

2. Galois confinement implies that the sum of the mass squared values, which are in general complex algebraic numbers in E, is also an integer. Since the mass squared values correspond to conformal weights as also in string models, one can have conformal confinement: states would be conformal singlets. This condition replaces the masslessness condition of gauge theories [L158].

Riemann zeta [A49] (`https://cutt.ly/oVNSltD`)is not a polynomial but has infinite number of roots. How could one end up with Riemann zeta in TGD? One can also consider the replacement of the rational polynomials with analytic functions with rational coefficients or even more general functions [L148].

1. For real analytic functions roots come as pairs but building many-fermion states for which the sum of roots would be a real integer, is very difficult and in general impossible.

2. Riemann zeta and the hierarchy of its generalizations to extensions of rationals (Dedekind zeta functions, and L-functions in general) is however a complete exception! If the roots are at the critical line as the generalization of Riemann Hypothesis (RH) assumes, the sum of the root and its conjugate is equal to 1 and it is easy to construct many fermion states as 2N fermion states, such that they have integer value conformal weight.

   Since zeta has also trivial zeros for even negative integers interpretable in terms of tachyonic states, also conformal confinement with vanishing net conformal weight for physical states is possible. The trivial zeros would be associated with Euclidean space-time regions and non-trivial ones to Minkowskian ones.

   One can wonder whether one could see Riemann zeta as an analog of a polynomial such that the roots as zeros are algebraic numbers. This is however not necessary. Could zeta and its analogies allow it to build a very large number of Galois singlets and they would form a hierarchy corresponding to extensions of rationals. Could they represent a kind of second abstraction level after rational polynomials?

A possible interpretation is that in TGD, rational polynomials give discrete cognitive representations as approximations for physics. Cognitive representations are in the intersection of p-adicities and reality defined by the intersection of reals and extension of p-adics defined by the algebraic extension of the polynomial P defining a given space-time surface. Continuum theory would represent real numbers as a factor of the adele.

One can ask whether the various zeta functions consistent with the integer spectrum for the conformal weights and possibly also with conformal confinement, appear at the continuum limit and provide representations for the space-time surfaces at this limit? In this framework, it would be natural for the roots of zeta to be algebraic numbers [K97]. Also in the case of ζ, the virtual momenta of fermions would be algebraic integers for virtual fermions and integers for the physical states. This makes sense if the notions of Galois group and Galois confinement are sensible for ζ.

As noticed, the notion of ζ generalizes. The so-called global L-functions (`https://cutt.ly/3VNPYmp`) are formally similar to ζ and the extended Riemann Hypothesis (RH) could be true for them. The physical motivation for RH would be that it would allow a fermion with any conformal weight to appear in a state which is conformal singlet. Algebraic integers for a finite extension of rationals replace integers in the ordinary ζ and one has an entire hierarchy of L-functions. Could one think that the global L-functions could define preferred extremals at the continuum limit?

21.6.2 About the notion of twistor space

For the twistor lift of TGD, twistor space $T(X^4)$ of the space-time surface X^4 is identified an S^2 bundle over X^4 obtained by the induction of the twistor bundle $T(H) = T(M^4) \times T(CP_2)$. The definition of the $T(X^4)$ as 6-surface in $T(H)$ identifies the twistor spheres of $T(M^4)$ and $T(CP_2)$ and identifies it as a twistor sphere of $T(X^4)$.

The notion of twistor space for different different types of preferred extremals

I have not previously considered the notion of the induced twistor space for the different types of preferred extremals. Here some technical complications emerge.

1. Since the points of the twistor spaces $T(M^4)$ and $T(CP_2)$ are in 1-1 correspondence, one can use either $T(M^4)$ or $T(CP_2)$ so that the projection to M^4 or CP_2 would serve as the base space of $T(X^4)$. One could use either CP_2 coordinates or M^4 coordinates as space-time coordinates if the dimension of the projection is 4 to either of these spaces. In the generic case, both dimensions are 4 but one must be very cautious with genericity arguments, which turned out to fail at the level of M^8 [L109, L110].

2. There are exceptional situations in which genericity fails at the level of H. String-like objects of the form $X^2 \times Y^2 \subset M^4 \subset CP_2$ is one example of this. In this case, X^6 would not define 1-1 correspondence between $T(M^4)$ or $T(CP_2)$.

   Could one use partial projections to M^2 and S^2 in this case? Could $T(X^4)$ be divided locally into a Cartesian product of 3-D M^4 part projecting to $M^2 \subset M^4$ and of 3-D CP_2 part projected to $Y^2 \subset CP_2$?

3. One can also consider the possibility of defining the twistor space $T(M^2 \times S^2)$. Its fiber at a given point would consist of light-like geodesics of $M^2 \times S^2$. The fiber consists of direction vectors of light-like geodesics. S^2 projection would correspond to a geodesic circle $S^1 \subset S^2$ going through a given point of S^2 and its points are parametrized by azimuthal angle Φ. Hyperbolic tangent $tanh(\eta)$ with range $[-1, 1]$ would characterize the direction of a time like geodesic in M^2. At the limit of $\eta \rightarrow \pm\infty$ the S^2 contribution to the S^2 tangent vector to length squared of the tangent vector vanishes so that all angles in the range $(0, 2\pi)$ correspond to the same point. Therefore the fiber space has a topology of S^2.

   There are also other special situations such as $M^1 \times S^3$, $M^3 \times S^1$ for which one must introduce specific twistor space and which can be treated in the same way.

To deal with these special cases in which the dimensions of both M^4 and CP_2 are not equal to 4, one must allow also 6-surfaces X^6 which can have dimension of M^4 and CP_2 projections which are different from the canonical value 4. For CP_2 type extremals the dimension of CP_2

projection would be 6 and the dimension of M^4 projection would be 1. For cosmic strings the dimensions of M^4 projection and CP_2 projection would be 2.

The concrete definition of the twistor space of H as the space of light-like geodesics

During the writing of this article I realized that the twistor space of H defined geometrically as a bundle, which has as H as base space and fiber as the space of light-like geodesic starting from a given point of H, need not be equal to $T(M^4)\times T(CP_2)$, where $T(CP_2)$ is identified as $SU(3)/U(1)\times U(1)$ characterizing the choices of color quantization axes. Is this really the case?

1. The definition of $T(CP_2)$ as the space of light-like geodesics from a given point of CP_2 is not possible. One could also define the fiber space of $T(CP_2)$ geometrically as the space of geodesics emating from origin at $r=0$ in the Eguchi-Hanson coordinates [L1] and connecting it to the homologically non-trivial geodesic sphere S^2_G $r=\infty$. This relation is symmetric.

   In fact, all geodesics from $r=0$ end up to S^2. This is due to the compactness and symmetries of CP_2. In the same way, the geodesics from the North Pole of S^2 end up to the South Pole. If only the endpoint of the geodesic of CP_2 matters, one can always regard it as a point S^2_G.

   The two homologically non-trivial geodesic spheres associated with distinct points of CP_2 always intersect at a single point, which means that their twistor fibers contain a common geodesic line of this kind. Also the twistor spheres of $T(M^4)$ associated with distinct points of M^4 with a light-like distance intersect at a common point identifiable as a light-like geodesic connecting them.

2. Geometrically, a light-like geodesic of H is defined by a 3-D momentum vector in M^4 and 3-D color momentum along CP_2 geodesic. The scale of the 8-D tangent vector does not matter and the 8-D light-likeness condition holds true. This leaves 4 parameters so that $T(H)$ identified in this way is 12-dimensional.

   The M^4 momenta corresponds to a mass shell H^3. Only the momentum direction matters so that also in the M^4 sector the fiber reduces to S^2. If this argument is correct, the space of light-like geodesics at point of H has the topology of $S^2\times S^2$ and $T(H)$ would reduce to $T(M^4)\times T(CP_2)$ as indeed looks natural.

The twistor space of the space-time surface

The twistor lift of TGD allows to identify the twistor space of the space-time surface X^4 as the base space of the S^2 bundle induced from the 12-D twistor space $T(8)=T(M^4)\times CP_(2)$ to the 6-surface $X^6\subset T(H)$ by a local dimensional reduction to $X^4\times S^2$ occurring for the preferred extremals of 6-D Kähler action existing only in case of $H=M^4\times CP_2$.

Could the geometric definition of $T(X^4)$ as the space of light-like geodesics make sense in the Minkowskian regions of X^4?

1. By their definition, stating that the length of the tangent vector of the geodesic is conserved, the geodesic equations conserve the value of the velocity squared so that light-likeness can be forced via the initial values. This allows the assignment of a twistor sphere to a given point of a Minkowskian space-time region. Whether this assignment can be made global is not at all trivial and the difficulties related to the definition of twistor space in general relativity probably reflects this problem. If this is the case, then the direct geometric definition might not make sense unless the very special properties of the PEs come to rescue.

2. The twistor lift of TGD is proposed to modify the definition of the twistor space so that one can assign twistor structure to the space-time surface by inducing the twistor structure of H just as one can assign spinor structure with the space-time surface by inducing the spinor structure of H.

   Could the generalized holomorphic structure, implying that PEs are extremals of both volume and of 4-D Kähler action, make possible the existence of light-like geodesics and even allow to assign to a given point of the space-time surface sphere parametrizing light-like geodesics?

3. The light-like 3-surfaces X^3 representing partonic orbits carry fermionic lines as light-like geodesics and are therefore especially interesting. They are metrically 2-D and boundary conditions for the field equations force the vanishing of the determinant $det(g_4)$ of the induced metric at them so that the dimension of the tangent space is effectively reduced. Light-like 3-surfaces allow a generalization of isometries such that conformal symmetries accompanied by scaling of the light-like radial coordinate depending on transversal complex coordinates is isometry.

   It seems that to a given point of the space-like intersection, only a single light-like geodesic can be assigned so that the twistor space at a given point would consist of a single light-like geodesic. This would be caused by the light-likeness of X^3.

The geometric definition of the twistor space for CP_2

In the case of the Euclidean regions, the notion of a light-like geodesic does not make sense. The closed geodesics and the presence of pairs of points analogous to North pole-South pole pairs, where diverging geodesics meet, would be required. This condition is very strong and the minimal requirement is that the space has a positive curvature so that the geodesics do not diverge. Also symmetries seem to be necessary. Clearly, something new is required.

1. The addition of Kähler coupling term equal to an odd multiple of the induced Kähler gauge potential A to the spinor connection is an essential element in the definition of a generalized spinor structure of CP_2.

2. Should one replace the light-like geodesics with orbits of Kähler charged particles for which CP_2 has been replaced with $p - q_K A$. For the counterparts of light-like geodesics $p - q_K A$ would vanish and the analog of mass squared would vanish but one would have a line. For a geodesic p would be constant.

   Is it possible to have $A = constant$ along a closed geodesic? In the case of sphere, the Kähler gauge potential in the spherical coordinates is $(A_\theta = A_\phi = kcos(\theta)$ and is constant along the geodesics going through South and North Poles. Something like this could happen in the case of CP_2 but it seems that a special pair of homological non-trivial spheres S^2 invariant under $U(2) \subset SU(3)$ is selected. One might perhaps speak of symmetry breaking.

   To obtain entire S^2 of light-like geodesics in this sense, the geodesics must emanate from a coordinate singularity, the origin of Eguchi-Hanson coordinates at $r = 0$, where the values of the coordinates (θ, ϕ, ψ) correspond to the same point. The space for the light-like geodesics must be 2-D rather than 3-D. This must be forced by the $p - A = 0$ condition. For the homologically trivial geodesic sphere $r = \infty$, Ψ coordinate is redundant so that the conserved value of A_ψ must vanish for the light-like geodesics and the associated velocities cannot have component in the direction of Ψ.

3. Note that this definition could apply also in Minkowskian regions of space-time surface.

The description of particle reactions without vertices

In standard field theory, particles are point-like and particle reactions are described using vertices assignable to non-linear interaction terms in the action.

1. In the TGD framework, particles are replaced with 3-surfaces and elementary particles are assigned to partonic 2-surface whose orbits correspond to light-like 3-surfaces identifiables as the boundary regions between Minkowskian and Euclidean space-time regions and modelled as wormhole contacts between two space-time sheets with a Minkowskian signature. Vertices are replaced with topological vertices at which incoming partonic 2-surfaces, whose orbits are light-like 3-surfaces, meet at partonic 2-surfaces.

2. In TGD, all particles are composites of fundamental fermions assignable to the wormhole throats identified as partonic orbits. In particular, bosons consist of fermions and antifermions assignable to the throats of wormholes. Since wormhole contact contains homologically trivial 2-surface of CP_2, there is a monopole flux throwing out of the throat and one must have at least two wormhole contacts so that one obtains a closed monopole flux flowing between the sheets and forming a closed flux tube.

3. The light-like orbits of the partonic 2-surfaces contain fermionic lines defined at the ends of string world sheets connecting different partonic orbits. In QFT description, this would require a 4-fermion vertex as a fundamental vertex involving dimensional coupling constant and leading to a non-renormalizable QFT. Therefore there can be no vertices at the level of fermion lines.

In the number theoretic vision based on Galois confinement [L147, L148], the interactions correspond at the level of M^8 to re-arrangements of virtual fermions, having virtual momentum components in the extension of rationals defined by P, to new combinations required to be Galois singlets and therefore having momentum components, which are ordinary integers. Note that P fixes by holography the 4-surface in M^8 in turn defining the space-time surface in H by $M^8 - H$ duality based on associativity.

There is however a problem. If the particle reactions are mere re-arrangements of fundamental fermions and antifermions, moving along light-like geodesic lines in fixed time direction, the total numbers of fermions and antifermions are separately conserved. How can one overcome this problem without introducing the disastrous 4-fermion vertex?

Consider FFB vertex describing boson emission by fermion as a concrete example.

1. B is described as a pair of partonic surfaces containing at least one fermion-antifermion pair, which must be created in the vertex. Incoming particles for the topolocal FFB 3-vertex correspond to partonic orbits for incoming F and outgoing F, each containing one fermion line and possibly a pair of fermion and antifermion.

2. The idea is that boson emission as a pair creation could be described geometrically as a turning of fermion backwards in time. This forces us to reconsider the definition of $M^8 - H$ duality. The simplest view of $M^8 - H$ duality is that momenta of $M^4 \subset M^8$ are mapped to the geodesic lines of M^4. Tachyonic momenta in $M^4 \subset M^8$ would be mapped to space-like geodesics in H emanating from the center of CD which is a sub-CD of a larger CD in general. It seems that this definition does not allow us to understand boson emission by fermion in the way proposed in [L148].

3. This led to a proposal that the images of momenta could be geodesics of the space-time surface X^4, rather than H. Since X^4 allows also Euclidean regions and the interiors of the deformed CP_2 type extremals are Euclidean, one ends up with the idea that the geodesics lines of X^4 can have M^4 projections, which turn backwards in the time direction [L109, L110, L141].

   This would allow us to interpret the emission of a boson as a fermion-antifermion pair as the turning of a fermionic line backwards in time. Fermions lines would be identified as the boundaries of string world sheets. Sub-manifold gravitation would play a key role in the elimination of 4-fermion vertex and thus of QFT type divergences.

4. But is it possible to have a light-like geodesic arriving at the partonic 2-surface and continuing as a light-like geodesic in the Euclidean wormhole contact and returning back? The problem is that in Euclidean regions, ordinary light-like geodesics degenerate to points. The generalization of the light-like geodesics satisfying $p = qA$ implying $(p-qA)^2 = 0$ is possible. At the space-time level, these conditions could be true quite generally and give as a special case light-like geodesics with $p^2 = 0$ in the Minkowskian regions.

21.6.3 About the analogies of Fermi torus and Fermi surface in H^3

Fermi torus (cube with opposite faces identified) emerges as a coset space of E^3/T^3, which defines a lattice in the group E^3. Here T^3 is a discrete translation group T^3 corresponding to periodic boundary conditions in a lattice.

In a realistic situation, Fermi torus is replaced with a much more complex object having Fermi surface as boundary with non-trivial topology. Could one find an elegant description of the situation?

Hyperbolic manifolds as analogies for Fermi torus?

The hyperbolic manifold assignable to a tessellation of H^3 defines a natural relativistic generalization of Fermi torus and Fermi surface as its boundary. To understand why this is the case, consider first the notion of cognitive representation.

1. Momenta for the cognitive representations [L157] define a unique discretization of 4-surface in M^4 and, by M^8-H duality, for the space-time surfaces in H and are realized at mass shells $H^3 \subset M^4 \subset M^8$ defined as roots of polynomials P. Momentum components are assumed to be algebraic integers in the extension of rationals defined by P and are in general complex.

   If the Minkowskian norm instead of its continuation to a Hermitian norm is used, the mass squared is in general complex. One could also use Hermitian inner product but Minkowskian complex bilinear form is the only number-theoretically acceptable possibility. Tachyonicity would mean in this case that the real part of mass squared, invariant under $SO(1,3)$ and even its complexification $SO_c(1,3)$, is negative.

2. The active points of the cognitive representation contain fermion. Complexification of H^3 occurs if one allows algebraic integers. Galois confinement [L157, L153] states that physical states correspond to points of H^3 with integer valued momentum components in the scale defined by CD.

   Cognitive representations are in general finite inside regions of 4-surface of M^8 but at H^3 they explode and involve all algebraic numbers consistent with H^3 and belonging to the extension of rationals defined by P. If the components of momenta are algebraic integers, Galois confinement allows only states with momenta with integer components favored by periodic boundary conditions.

Could hyperbolic manifolds as coset spaces $SO(1,3)/\Gamma$, where Γ is an infinite discrete subgroup $SO(1,3)$, which acts completely discontinuously from left or right, replace the Fermi torus? Discrete translations in E^3 would thus be replaced with an infinite discrete subgroup Γ. For a given P, the matrix coefficients for the elements of the matrix belonging to Γ would belong to an extension of rationals defined by P.

1. The division of $SO(1,3)$ by a discrete subgroup Γ gives rise to a hyperbolic manifold with a finite volume. Hyperbolic space is an infinite covering of the hyperbolic manifold as a fundamental region of tessellation. There is an infinite number of the counterparts of Fermi torus [L133]. The invariance respect to Γ would define the counterpart for the periodic boundary conditions.

   Note that one can start from $SO(1,3)/\Gamma$ and divide by $SO(3)$ since Γ and $SO(3)$ act from right and left and therefore commute so that hyperbolic manifold is $SO(3)\setminus SO(1,3)/\Gamma$.

2. There is a deep connection between the topology and geometry of the Fermi manifold as a hyperbolic manifold. Hyperbolic volume is a topological invariant, which would become a basic concept of relativistic topological physics (`https://cutt.ly/RVsdNl3`).

   The hyperbolic volume of the knot complement serves as a knot invariant for knots in S^3. Could this have physical interpretation in the TGD framework, where knots and links, assignable to flux tubes and strings at the level of H, are central. Could one regard the effective hyperbolic manifold in H^3 as a representation of a knot complement in S^3?

   Could these fundamental regions be physically preferred 3-surfaces at H^3 determining the holography and M^8-H duality in terms of associativity [L109, L110]. Boundary conditions at the boundary of the unit cell of the tessellation should give rise to effective identifications just as in the case of Fermi torus obtained from the cube in this way.

De Sitter manifolds as tachyonic analogies of Fermi torus do not exist

Can one define the analogy of Fermi torus for the real 4-momenta having negative, tachyonic mass squared? Mass shells with negative mass squared correspond to De-Sitter space $SO(1,3)/SO(1,2)$ having a Minkowskian signature. It does not have analogies of the tessellations of H^3 defined by discrete subgroups of $SO(1,3)$.

The reason is that there are no closed de-Sitter manifolds of finite size since no infinite group of isometries acts completely discontinuously on de Sitter space: therefore these is no group replacing the Γ in H^3/Γ. (`https://cutt.ly/XVsdLwY`).

Do complexified hyperbolic manifolds as analogies of Fermi torus exist?

The momenta for virtual fermions defined by the roots defining mass squared values can also be complex. Tachyon property and complexity of mass squared values are not of course not the same thing.

1. Complexification of H^3 would be involved and it is not clear what this could mean. For instance, does the notion of complexified hyperbolic manifold with complex mass squared make sense.

2. $SO(1,3)$ and its infinite discrete groups Γ act in the complexification. Do they also act completely discontinuously? p^2 remains invariant if $SO(1,3)$ acts in the same way on the real and imaginary parts of the momentum leaves invariant both imaginary and complex mass squared as well as the inner product between the real and imaginary parts of the momenta. So that the orbit is 5-dimensional. Same is true for the infinite discrete subgroup Γ so that the construction of the coset space could make sense. If Γ remains the same, the additional 2 dimensions can make the volume of the coset space infinite. Indeed, the constancy of $p_1 \cdot p_2$ eliminates one of the two infinitely large dimensions and leaves one.

   Could one allow a complexification of $SO(1,3)$, $SO(3)$ and $SO(1,3)_c/SO(3)_c$? Complexified $SO(1,3)$ and corresponding subgroups Γ satisfy $OO^T = 1$. Γ_c would be much larger and contain the real Γ as a subgroup. Could this give rise to a complexified hyperbolic manifold H^3_c with a finite volume?

3. A good guess is that the real part of the complexified bilinear form $p \cdot p$ determines what tachyonicity means. Since it is given by $Re(p)^2 - Im(p)^2$ and is invariant under $SO_c(1,3)$ as also $Re(p) \cdot Im(p)$, one can define the notions of time-likeness, light-likeness, and space-likeness using the sign of $Re(p)^2 - Im(p^2)$ as a criterion. Note that $Re(p)^2$ and $Im(p)^2$ are separately invariant under $SO(1,3)$.

   The physicist's naive guess is that the complexified analogies of infinite discrete and discontinuous groups and complexified hyperbolic manifolds as analogies of Fermi torus exist for $Re(P^2) - Im(p^2) > 0$ but not for $Re(P^2) - Im(p^2) < 0$ so that complexified dS manifolds do not exist.

4. The bilinear form in H^3_c would be complex valued and would not define a real valued Riemannian metric. As a manifold, complexified hyperbolic manifold is the same as the complex hyperbolic manifold with a hermitian metric (see `https://cutt.ly/qVsdS7Y` and `https://cutt.ly/kVsd3Q2`) but has different symmetries. The symmetry group of the complexified bilinear form of H^3_c is $SO_c(1,3)$ and the symmetry group of the Hermitian metric is $U(1,3)$ containing $SO(1,3)$ as a real subgroup. The infinite discrete subgroups Γ for $U(1,3)$ contain those for $SO(1,3)$. Since one has complex mass squared, one cannot replace the bilinear form with hermitian one. The complex H^3 is not a constant curvature space with curvature -1 whereas H^3_c could be such in a complexified sense.

21.7 The notion of generalized integer

This chapter was inspired by the article "Space Element Reduction Duplication (SERD) model produces photon-like information packets and light-like cosmological horizons" by Thomas L.

Wood, published in Metodologia IV B: Journal of International and Finnish Methodology, expresses the basic assumptions of the SERD approach very coherently and in a systematic way so that it easy to criticize them and compare with other views, in my case the TGD view.

My criticism, summarized below, is based on a different interpretations of the discreteness. In TGD framework would be assignable to cognitive representations based on p-adic numbers fields involving extensions of rationals rather than being a feature of space-time. The introduction of continuous number fields (reals, complex numbers, quaternions, octonions) besides p-adic number fields brings in real space-time as sensory representation and one ends up to a generalization of the standard model proving a number theoretic interpretation for its symmetries.

The approach of Wood looks is essentially topological: for instance, the information propagating in the hypergraph is assumed to be topological and characterize the graph. In TGD, discrete structures analogs define cognitive representations of the continuous sensory world and are basically number theoretic. The description of the sensory world involves both topology and geometry.

21.7.1 The first reactions to the abstract

The abstract gives a very concise summary of the approach and I have added below my reactions to it. The following commentary is my attempt to understand the basic ideas of SERD. I have also used the third section of the article to clarify my views. I must admit that I didn't quite get the two basic principles in the beginning of the third section. I have slightly re-organized the abstract and hope that I have not done any damage.

[TW] This document describes a correspondence between photons and propagating information packets (PIPs) that are emergent out of the Space Element Reduction Duplication (SERD) model introduced in a rudimentary form in [1, 2]. The SERD model is a discrete background independent microscopic space-time description.

[MP] The assumption of discreteness at the fundamental space-time level raises several challenges. 4-D space-time with Minkowskian signature should somehow emerge. The mere hypergraph might possess under additional assumptions a local dimension defined homologically/combinatorially but would vary. Note that in standard homology theory an embedding to some space is required and would give a metric. Now the distance and other geometric notions look problematic to me. One can also ask what kind of dynamics for hypergraphs could select the 4-D space-time? Should one have a variational principle of some kind?

The notion of symmetries is central in physics. Lorentz invariance or even Poincare invariance should emerge as approximate symmetries at least. Only discrete subgroups of these groups can emerge in the hypergraph approach. Lorentz invariance poses very, perhaps too, powerful constraints on the hypergraphs. The notion of discretized time is introduced. It should be Lorentz invariant and here the light-cone proper time a serves as an analog. a=constant sections would be analogs of hyperbolic 3-space H^3.

[TW] By observation of physically comparable behaviour emerging from this system, through analysis and computer simulation, we draw conclusions of what the form and dynamics of the true underlying space-time may be.

By treating elements of the system as fundamental observers, mathematical and empirical evidence is obtained of the existence of fully emergent light-like cosmological horizons, implying the existence of causally separated 'pocket universes'.

[MP] The emergence of the analogy with expanding cosmology presumably reflects the underlying dynamics implying the increase of the size of the hypergraph. The emergence of light-like causal horizons is natural if the dynamics involves maximal velocity of propagation for the signals. This is probably due to the locality of the basic dynamics involving only local changes of the hypergraph topology. Locality and classicality raise challenges if one wants to describe phenomena like quantum entanglement.

[TW] The SERD model is a hypergraph of connected hyperedges called Point Particles (PP) which represent the fundamental constituents of all matter and particles (and therefore observers) separated by strings of consecutive and fundamental elements or edges called Space Elements (SE).

[MP] I had to clarify myself what a hypergraph is. Hypergraph is a generalization of graphs. Also it contains the set of vertices/nodes. The notion of edge connecting a pair of vertices is however generalized to a hyperedge (PP) as a pair of subsets of vertices. PPs correspond to hyperedges as fundamental constituents of matter and formed by pairs of subsets of the set of nodes.

One could interpret this as a combinatorial counterpart for a length scale hierarchy of TGD in which a set can be approximated as a point. One might also interpret subsets of vertices as analogs of bound states of fundamental particles. In the TGD framework, many-sheeted space-time and various other hierarchies serve as its analogs.

Space elements (SEs) would bring in basic aspects of 3-space. It is said that they are infinitesimal or maximally small. SEs would be like edges (not hyperedges) of the hypergraph. Consecutive SEs in turn form interaction edges (IEs) connecting PPs. IEs store and transmit information relating to the structure space. What comes to mind is that functionally PPs are like neutrons and neuron groups and IEs are like axons.

[TW] All elements are separated by nodes called Information Gaps (IGs), that store propagating topological information of the hypergraph. Information gaps (IGs) are between PPs and SEs, between SEs and between PPs themselves.

[MP] What distinguishes the SERD model from physical theories, is that information takes the role of matter. Information is treated as some kind of substance. The basic objection is that conscious information is always about something, whereas matter just is.

IGs have the role of interfaces somewhat analogous to black-hole horizon assumed to store information in the holographic picture. One could see PPs as the nodes and IEs as the edges or SEs as the edges and IGs as the nodes. IGs could have synaptic contacts as analogs.

[TW] In time step (TS), SE can duplicate and reduce (disappear) while the PPs split and merge through discrete time. These processes create space or destroy it and increase or reduce the effective distance between PPs. Splitting generates an SE between the resulting PPs. These are known as the actions of the elements and create a highly dynamic multi-way system.

[MP] Time step (TS) is a further basic notion and corresponds to an elementary event as nearest neighbor interaction taking during the time chronon. The propagation rate for information is CS/TS and is analogous to maximal signal velocity. The counterpart of the space-time metric is thus brought in by the introduction of TS and CS.

SEs emerge or disappear so that the effective distances of the nearby points change: this would the counterpart for the dynamics of space-time metric in General Relativity. I understood that duplication and reduction effectively corresponds to the duplication or halving of the distance assignable to SE.

[TW] Elements have an 'awareness' of the information around them and communicate with their nearest neighbours through time.

[MP] The treatment of elements as fundamental observers is an interesting idea but can be criticized. Why not PPs? One could also argue that the SEs become conscious observers only under some additional assumptions. For instance, one can imagine that they represent matter and become fundamental conscious observers if fermions or fermion pairs can be assigned to them.

The abstract says nothing about quantum theory. To my view it is very difficult to imagine how quantum theory could emerge from an approach based on classical probability and some kind of quantum approach would be required to understand entanglement and state function reduction.

21.7.2 Fundamental discretization as a cognitive representation?

In the sequel TGD view of the discretization interpreted as cognitive representation is described. The surprise was the discovery of what I call generalized integers and rationals as a union of various p-adic number fields with different p-adic number fields glued together along numbers which belong to both p-adic number fields. I do not know whether mathematicians have played with this thought. This space has an ultrametric topology and could have application to the description of spin glass type systems [L139]. In TGD it could have application in the mathematical description of processes in which the p-adic prime associated with the particle changes.

Something is discrete but what it is?

Something is discrete at the fundamental level: is it space-time or only a discrete cognitive representation, a discretization of a continuous space-time? The essential assumption of SERD is that it is space-time, which is fundamentally discrete and realized as hypergraph. The basic problem is that it is not clear whether the notions of space-time dimensions, distance, angle, and curvature can emerge in a purely combinatorial approach in which only distance between nearby nodes is a metric notion. These notions also have a formal generalization to gauge theories.

The alternative approach would be based on the observation that cognition is discrete and finite. Cognition provides representations of the physical world. Could one assume that the physical world has continuous geometry and that only cognition is discrete?

Could the cognitive Universe consist of generalized integers?

Integers (and rationals) are the simplest discrete but infinite systems. Integers/rationals are usually assumed to have real topology. One can however imagine an infinite number of p-adic topologies, which are ultrametric and are defined by a p-adic norm having values coming as powers of prime p. p-Adic primes typically have an infinite expansion in powers of p and large powers of p have small p-adic norm in contrast to the real norm.

p-Adic integer/rational has expansion in powers of p and the inverse of the smallest power in the expansion determines the norm so that the notion of size is completely different for p-adic and real integers. Note that also the p-adic expansion of rationals involves an infinite number of powers of p but is periodic. p-Adic transcendentals do not have this property. Note also that p-adic integers modulo p define a finite field $G(p)$.

p-Adic integers are only weakly ordered. Only if two p-adic integers/ rationals have different p-adic norms, can one tell which is the larger one. One can however construct continuous maps from p-adics to reals to approximately preserve the norm. p-Adic norm is ultrametric and this property is essential in the thermodynamic models of spin glass energy landscape [L139].

One could, at least as the first guess, imagine that the Universe of cognition consists of integers/rationals or a finite subset of them and that one also allows integers/rationals, which are infinite as real integers but finite as p-adic integers for some prime p.

One can decompose generalized integers to subsets with different p-adic topologies.

1. Regions corresponding to two different p-adic topologies p_1 and p_2 have as an interface as the set integers, which have an expansion in powers of $n_{12} = p_1p_2$. Therefore the cognitive world decomposes into p-adic regions having interfaces, which consist of power series of $n_{12..k} = p_1^{k_1} ... \times p_k^{n_k}$. Ordinary integer n with a decomposition to primes belongs to the interface of the p-adic worlds corresponding to the prime factors.

   How does this decomposition relate to adeles [L54, L52], which can be regarded as a Cartesian product of p-adic number fields defining and of reals [L54, L52]? Adeles correspond to a Cartesian product but now one has a union so that these concepts seems to be different. I do not know whether mathematicians have encountered the notion of generalized integers and rationals.

2. Each p-adic region decomposes into shells, kinds of analogs of mass shells, consisting of p-adic integers with p-adic norm given by a power of p.

3. The distance between the points of the cognitive sub-landscape corresponding to p would be defined by the p-adic norm. The points with the same p-adic norm would have a distance defined as the p-adic norm of their difference. This distance is the same for several point pairs so that p-adic topology is much rougher than the real topology. For instance the p-adic norm of numbers $1, ..., p-1$ is the same.

4. One could define a distance between points associated with p-adic topologies p_1 and p_2 as the shortest distance between them identified as the sum of the distances to the interface between these regions.

In this framework, the analog of a hypergraph would be simply a subset of generalized integers decomposing to p-adic integers labeled by some subset of primes.

1. The simplest dynamical operation, having now an interpretation as a cognitive operation, would be addition or removal of a p-adic integer corresponding to some value of p-adic prime or several of them. The addition would have an interpretation or worsening or improving the cognitive representation for some prime p.

2. Arithmetic operations for the points inside a region corresponding to a given p are possible. Arithmetic operations of finite integers are basic elements of at least human cognition and their sum and product would correspond to "particle reactions" in which two points fuse together to form a sum or product. If infinite integers can be expressed as power series of integers n_1 and n_2, they can be regarded as p-adic integers for the factors of n_1 and n_2 and both sum and product make sense for common prime factors. Note that the operations are well-defined also for generalized rationals.

3. What happens in the arithmetic operations information theoretically? In the product operation, the outcome is in the interface region associated with n_1 and n_2 and the information about factors is not lost since a measurement revealing prime factors can be done repeatedly.

   The projection operator applied to a quantum superposition of integers would project to a subspace of integers, which are divisible by a given prime p. This operation could be repeated for different primes and eventually give the prime number decomposition for some integer n in the superposition.

   One strange fact about idiot savants described by Oliver Sacks (this is discussed from the TGD point of view in [K98]) is that they can decompose integers into prime factors and obviously see the emergence of the prime factors. Could this kind of cognitive measurement be in question?

   Sum does not in general belong to the interface region of either integer and information is lost since many number pairs give rise to the same sum. Therefore sum and product are information-theoretically very different operations.

Could there be a quantum physical realization for the arithmetic operations? Could they relate to our conscious arithmetic thinking?

1. Consider first the sum operation. Quantum numbers, such as momenta, represented as integers or even algebraic integers are conserved in the physical reaction vertices. The conserved quantum numbers for the final state for a fusion reaction are sums of integers so that these reactions have an arithmetic interpretation.

2. In the case of a product, the fusion reaction should give a product of integers n_1 and n_2 or a representation of it? One should have conserved multiplicative quantum numbers in the vertex.

   Phase factors as eigenvalues of unitary operators are such. They should form a multiplicative group as representation of integers or even rationals. Integer scalings define such a group. One can also consider eigenvalues $n^{i\phi}$, ϕ some fixed phase angle. The operator would therefore be a scaling represented unitarily by these phase factors.

   Initial state would be a product of eigenstates of the scaling operator with eigenphases $n_1^{i\phi}$ and $n_2^{i\phi}$ and the final state would be a single particle state with the eigenvalue $n_1^{i\phi}n_2^{i\phi} = (n_1n_2)^{i\phi}$. One can say that n_1 acts on n_2 by scaling or vice versa. Interestingly, at the fundamental level scalings replace time translations in the TGD framework (and also in superstring theory), and this is especially so for spin glass phase [L139].

   Interestingly, sum appears at the level of Lie algebras and product at the level of Lie groups.

In quantum groups also the reverse operations, co-product and co-sum, having pair creation as analog, are possible. For the co-sum the information increases for the product. These operations would be time reversals of each other. In the zero energy ontology (ZEO) of TGD time reversal occurs in "big" (ordinary) state function reductions (BSFRs) [L91, L149] [K130]. What comes to mind is that the idiot savants described by Sacks might perform a time reversal decomposing product to prime factors. The cognitive measurement would correspond to BSFR.

Note that ZEO also predicts "small" state function reductions (SSFRs), which do not change the arrow of time and give rise to the flow of consciousness whereas BSFR corresponds to a universal counterpart of death or of falling asleep. It is the TGD counterpart of repeated measurements in the Zeno effect and of weak measurements of quantum optics.

This cognitive world would in TGD correspond physically to the most general spin glass energy landscape having an ultrametric topology [L139].

The algebraic extensions of p-adic number fields are discrete

The proposed structure does not have any natural notion of dimension. We are however able to cognize higher dimensional spaces using formulas.

1. p-Adic number fields indeed allow infinite hierarchies of algebraic extensions obtained by adding to them roots of polynomials, which are algebraic numbers. These induce extensions of p-adic number fields as finite fields $G(p,k)$ having algebraic dimension, which is at most the dimension of the corresponding extensions of rationals.

2. It is natural to assume that cognitive representations are always finite. This suggests that the set of "populated" points of the cognitive space is discrete and even finite. Being "populated" could mean that a fermion, having an interpretation as a generator of Boolean algebra, is labelled by the algebraic number defining the point. In a more general formulation bringing in quaternions and octonions as number fields: algebraic complexified quaternions would define the momentum components of fermions.

   What has been said above, generalizes almost as such and one obtains a hierarchy of generalized integers as algebraic extensions of generalized integers at the lowest level. This could generalize the rational number based computationalism (Turing paradigm) to an entire hierarchy of cognitive computationalisms. The hierarchy of algebraic extensions suggests the same.

3. The algebraic complexity of generalized integers increases with the dimension of extension and in the TGD framework it corresponds to an evolutionary hierarchy. The dimension of extension defines what is identified in terms of an effective Planck constant.

But what about the real world?

A hierarchy of p-adicities and hierarchies of the algebraic extensions of p-adicities have been obtained. The 4-D world of sensory perceptions with its fundamental symmetries is however still missing. Could number theory come to rescue also here? This is indeed the case.

1. The fundamental continuous number fields consist of reals, complex numbers, quaternions and octonions with dimensions 1,2,4, 8 [L109, L110, L168]. Quaternions cannot as such correspond to 4-D space-time since the number theoretic purely algebraic norm defines the Euclidean metric.

2. This norm can be however algebraically continued to the complexification of quaternions obtained by adding a commuting imaginary unit i commuting with quaternionic and octonionic imaginary units. This algebraic norm squared does not involve complex conjugation as the Hilbert space norm and is in general complex but real for the subspaces corresponding to various metric signatures (a given component of quaternion are either real or imaginary). One obtains therefore Minkowski space and even more: its variants with various metric signatures.

3. One can imagine a generalization of the notion of generalized integer so that one would have hierarchies of generalized complex numbers, quaternions and octonions and their complexifications for various extensions of rationals.

   A possible problem relates to the p-adic variants of quaternions, octonions and complex numbers. Consider the inverse $z^{-1} = (x-iy)/(x^2+y^2)$ of p-adic complex numbers $z = x+iy$. The problem is that $x^2 + y^2$ can vanish since there is no notion of sign of the number. For $p \ mod \ 4 = 1$, $\sqrt{-1}$ is an ordinary p-adic number, albeit with an infinite pinary expansion so that for $y = \sqrt{-1}x$, one has this problem.

   Could the finiteness of cognition solve the problem? If only finite p-adic integers and rationals can define momentum components of fermions (finite cognitive and measurement resolution), the problem disappears.

   Could one give up the field property for the p-adic variants of classical number fields? Already the complexification by i forces to give up the field property but has physical meaning since it makes Minkowski signature possible.

This would give Minkowski space M^4 as a special case. This is however not enough. One wants curved 4-D space-times. The basic structure is complexified octonions.

1. One should obtain 4-D surfaces of M^8 generalizing empty Minkowski space M^4. Octonions fail to be associative and at the level of M_c^8 the natural proposal is that there is number theoretic dynamics based on associativity. The 4-D surfaces must be associative in some sense. The geometric vision predicts holography and this holography should have a number theoretic counterpart based on associativity.

2. The first guess is that the tangent space of 4-surface is associative and thus quaternionic. This gives only M^4 and is therefore trivial [L109, L110, L168].

   The requirement that the normal space of the 4-surface Y^4 in M_c^8 is associative/quaternionic however works. If one requires that the normal subspace contains also a commutative (complex) subspace, one ends up to M^8-H-duality ($H=M^4\times CP_2$ mapping the associative 4-D surfaces Y^4 of M_c^8 to space-time surfaces X^4 in H determined by holography forced by generalized coordinate invariance. The symmetries of H include Poincare symmetries and standard model symmetries.

3. At the level of M^8, associativity of the normal space allows also 6-D surfaces with 2-D commutative normal space and they can be interpreted in terms of analogs of 6-D twistor spaces of 4-D surfaces Y^4. They can be mapped to to the twistor space of H by M^8-H duality and define 6-D twistor spaces of space-time surfaces X^4 of H. What is beautiful is that the Kähler structure for the twistor space of H exists only for the choice $H=M^4\times CP_2$, which is also forced by the associative dynamics [A63]! TGD is unique!

4. The dynamics would rely on holography but how to get the algebraic extensions? The roots of a polynomial P with rational or even integer coefficients satisfying some additional conditions would define the needed extension of rationals. The roots would in the general case define complex mass shells H_c^3 as complex variants of hyperbolic 3-spaces H^3 in $M_c^4\subset M_c^8$ having interpretation as a momentum space. M^8-H duality serves as a generalization of momentum position duality. The 3-D surfaces as subsets of these H^3:s define the data of the associative holography and are contained by the 4-surface Y^4.

5. Cognitive representation would be defined as a unique number theoretic discretization of the 4-surface Y^4 of M_c^8 consisting of points, whose number theoretically preferred linear Minkowski coordinates are algebraic integers in an extension defining the 4-surface in question. This discretization induces discretization of the space-time surface via M^8-H duality. The cognitive representations are number-theoretically universal and belong to the intersections of realities and p-adicities.

6. The mass shells H_c^3 are very special since in the preferred Minkowski coordinates a cognitive explosion takes place. All algebraic rationals, in particular integers, are points of H_c^3. Algebraic integers are physically favored and define components of four-momenta. Galois confinement [L140] states that the total momenta have components which are ordinary integers when a suitable momentum unit is used.

21.8 Infinite primes as a basic mathematical building block

Infinite primes [K105, K58, K69] are one of the key ideas of TGD. Their precise physical interpretation and the role in the mathematical structure of TGD has however remained unclear.

3 new ideas are be discussed. Infinite primes could define a generalization of the notion of adele; quantum arithmetics could replace $+$ and $\times$ with $\oplus$ and $\otimes$ and ordinary primes with p-adic representations of say HFFs; the polynomial Q defining an infinite prime could be identified with the polynomial P defining the space-time surface: $P=Q$.

21.8.1 Construction of infinite primes

Consider first the construction of infinite primes [K105].

1. At the lowest level of hierachy, infinite primes (in real sense, p-adically they have unit norm) can be defined by polynomials of the product X of all primes as an analog of Dirac vacuum.

   The decomposition of the simplest infinite primes at the lowest level are of form $aX+b$, where the terms have no common prime divisors. More concretely $a=m_1/n_F$ $b=m_0n_F$, where n_F is square free integer analogous and the integer m_1 and n_F have no common prime divisors divisors. The divisors of m_2 are divisors of n_F and m_i has interpretation as n-boson state. Power p^k corresponds to k-boson state with momenta p. $n_F=\prod p_i$ has interpretation as many-fermion state satisfying Fermi-Dirac statistics.

   The decomposition of lowest level infinite primes to infinite and finite part has a physical analogy as kicking of fermions from Dirac sea to form the finite part of infinite prime. These states have interpretation as analogs of free states of supersymmetric arithmetic quantum field theory (QFT) There is a temptation to interpret the sum X/n_F+n_F as an analog of quantum superposition. Fermion number is well-defined if one assigns the number of factors of n_F to both n_F and X/n_F.

2. More general infinite primes correspond to polynomials $Q(X)=\sum_n q_nX^n$ required to define infinite integers which are not divisible by finite primes. Each summand q_nX^n must be a infinite integer. This requires that q_n is given by $q_n=m_{B,n}/\prod_{i_1}^n n_{F,i}$ of square free integers $n_{F,i}$ having no common divisors.

   The coefficients $m_{B,n}$ representing bosonic states have no common primes with $\prod n_{F,i}$ and there exists no prime dividing all coefficients $m_{B,n}$: there is no boson with momentum p present in all states in the sum.

   These states have a formal interpretation as bound states of arithmetic supersymmetric QFT. The degree k of Q determines the number of particles in the bound states.

   The products of infinite primes at given level are infinite primes with respect to the primes at the lower levels but infinite integers at their own level. Sums of infinite primes are not in general infinite primes. For instance the sum and difference of X/n_F+n_F and X/n_F-n_F are not infinite primes.

3. At the next step one can form the product of all finite primes and infinite primes constructed in this manner and repeat the process as an analog to second quantization. This procedure can be repeated indefinitely. This repeated quantization a hierarchy of infinite primes, which could correspond to the hierarchy of space-time sheets.

   At the n:th hierarchy level the polynomials are polynomials of n variables X_i. A possible interpretation would be that one has families of infinite primes at the first level labelled by n_1 parameters. If the polynomials $P(x)$ at the first level define space-time surfaces, the interpretation at the level of WCW could be that one has an $n-1$-D surface in WCW parametrized by $n-1$ parameters with rational values and defining a kind of sub-WCW. The WCW spinor fields would be restricted to this surface of WCW.

The Dirac vacuum X brings in mind adele, which is roughly a product of p-adic number fields. The primes of infinite prime could be interpreted as labels for p-adic number fields. Even more generally, they could serve as labels for p-adic representations of various algebras and one could even consider replacing the arithmetic operations with $\oplus$ and $\otimes$ to get the quantum variants of various number fields and of adeles.

The quantum counterparts of nfinite primes at the lowest and also at the higher levels of hierarchy could be seen as a generalization of adeles to quantum adeles.

21.8.2 Questions about infinite primes

One can ask several questions about infinite primes.

1. Could $\oplus$ and $\otimes$ replace $+$ and $-$ also for infinite primes. This would allow us to interpret the primes p as labels for algebras realized p-adically. This would give rise to quantal counterparts of infinite primes.

2. What could $+ \rightarrow \oplus$ for infinite primes mean physically? Could it make sense in adelic context? Infinite part has finite p-adic norms. The interpretation as direct sum conforms with the fermionic interpretation if the product of all finite primes is interpreted as Dirac sea. In this case, the finite and infinite parts of infinite prime would have the same fermion number.

3. Could adelization relate to the notion of infinite primes? Could one generalize quantum adeles based on $\oplus$ and $\otimes$ so that they would have parts with various degrees of infinity?

21.8.3 $P = Q$ hypothesis

One cannot avoid the idea that that polynomial, call it $Q(X)$, defining an infinite prime at the first level of the hierarchy, is nothing but the polynomial P defining a 4-surface in M^4 and therefore also a space-time surface. $P = Q$ would be a condition analogous to the variational principle defining preferred extremals (PEs) at the level of H.

There is however an objection.

1. $P = Q$ gives very powerful constraints on Q since it must define an infinite integer. The prime polynomials P are expected to be highly non-unique and an entire class of polynomials of fixed degree characterized by the Galois group as an invariant is in question. The same applies to polynomials Q as is easy to see: the only condition is that powers of $a_k X^k$ defining infinite integers have no common prime factors.

2. It seems that a composite polynomial $P_n \circ ... \circ P_1$ satisfying $P_i = Q_i$ cannot define an infinite prime or even infinite integer. Even infinite integer property requires very special conditions.

3. There is however no need to assume $P_i = Q_i$ conditions. It is enough to require that there exists a composite $P_n \circ ... \circ P_1$ of prime polynomials satisfying $P_n \circ ... \circ P_1 = Q$ defining an infinite prime.

   The physical interpretation would be that the interaction spoils the infinite prime property of the composites and they become analogs of off-mass-shell particles. Exactly this occurs for bound many-particle states of particles represented by P_i represented composite polynomials $P_1 \circ ... P_n$. The roots of the composite polynomials are indeed affected for the composite. Note that also products of Q_i are infinite primes and the interpretation is as a free many-particle state formed by bound states Q_i.

There is also a second objection against $P = Q$ property.

1. The proposed physical interpretation is that the ramified primes associated with $P = Q$ correspond to the p-adic primes characterizing particles. This would mean that the ramimied primes appearing in the infinite primes at the first level of the hierarchy should be physically special.

2. The first naive guess is that for the simplest infinite primes $Q(X) = (m_1/n_F)X + m_2 n_F$ at the first level, the finite part $m_2 n_F$ has an identification as the discriminant D of the polynomial $P(X)$ defining the space-time surface. This guess has no obvious generalization to higher degree polynomials $Q(X)$ and the following argument shows that it does not make sense.

   Since Q is a rational polynomial of degree 1 there is only a single rational root and discriminant defined by the differences of distinct roots is ill-defined that $Q = P$ condition would not allow the simplest infinite primes.

   Therefore one must give either of these conjectures and since $P = Q$ conjecture dictates the algebraic structure of the quantum theory for a given space-time surface, it is much more attractive.

The following argument gives $P = Q$. One can assign to polynomial P invariants as symmetric functions of the roots. They are invariants under permutation group S_n of roots containing Galois group and therefore also Galois invariants (for polynomials of second order correspond to sum and product of roots appearing as coefficients of the polynomial in the representation $x^2 + bx + cx$). The polynomial Q having as coefficients these invariants is the original polynomial. This interpretation gives $P = Q$.

21.9 Summary of the proposed big picture

In the previous sections the plausible looking building blocks of the bigger picture of the TGD were discussed. Here I try to summarize a guess for the big picture.

21.9.1 The relation between $M^8 - H$ and $M - M'$ dualities

The first question is whether $M^8 - H$ duality between number theoretical and geometric physics, very probably relating to Langlands duality, corresponds to a duality between M and its commutant M'. Physical intuition suggests that these dualities are independent. M' would more naturally correspond to classical description as dual to quantum description using M. One would assign classical and quantum views to both number theoretic (M^8) and geometric (H) descriptions.

1. At the geometric side M would be realized in terms of HFFs associated with SSA_n, Aff_n and I acting in H. At the number theoretic side, braided Galois group algebras would define the HFFs and have natural action in SSA_n, A_n and I .

2. The descriptions in terms of preferred extremals in H and of polynomials P defining 4-surfaces in M^8 would correspond to classical descriptions. $P = Q$ condition would define preferred polynomials and infinite primes.

3. At the geometric side, M' would correspond to scalar factors of WCW wave functions symplectic invariants identifiable as Kähler magnetic fluxes at both M^4 and CP_2 sectors. They are zero modes and therefore do not contribute to the WCW line element.

4. At the number theoretic side, the wave functions would depend on Galois invariants. Discriminant D, set of roots to which braid strands can be assigned to define n-braid, and ramified primes dividing it in the case of polynomials with rational/integer coefficients are Galois invariants analogous to Kähler fluxes. They code information about the spectrum of virtual mass squared values as roots of P. The strands of braid as Galois invariant correspond to (possibly) monopole flux tubes and one assign them quantized magnetic fluxes as integer valued symplectic invariants.

21.9.2 Basic mathematical building blocks

The basic mathematical building blocks of quantum aspects of TGD involve at least the following ones.

1. The generalization of arithmetics and even number theory by replacing sum and product by direct sum and tensor product for various algebras and associated representations is a mathematical notion expected to be important and a straightforward generalization of adeles and infinite primes to their quantum counterparts is highly suggestive.

2. Quantum version of adelic physics obtained by replacing ordinary arithmetic operations with direct sum and tensor product relates closely to the fusion of real and various p-adic physics at quantum level.

3. The hierarchy of infinite primes suggested by the many-sheeted space-time suggests a profound generalization of the notion of adelic physics. Infinite primes are defined by polynomials of several variables the basic equation in the general form would be $Q(X_1, ..., X_n) = P(X_1, ..., X_n)$.

21.9.3 Basic algebraic structures at number theoretic side

Number theoretic side involves several key notions that must have counterparts at the geometric side.

1. Number theoretic side involves Galois groups as counterparts of symplectic symmetries and can be regarded as number theoretic variants of permutation symmetries and lead to the notion of braided Galois group, whose group algebra defines HFF.

2. Galois groups can be decomposed to a hierarchy of normal subgroups, which are simple and therefore primes in group theoretic sense. Simple Galois groups correspond to polynomial primes with respect to functional composition, and one can assign to a given Galois group a set of polynomials with fixed degrees although the polynomials and their order of polynomials in composition are not unique.

3. There is a large class of polynomials giving rise to a given Galois group and they bring in additional degrees of freedom. The variation of the polynomial coefficients corresponding to the same Galois group is analogous to symplectic transformations leaving the induced Kähler form invariant.

   The roots of polynomials define analogs for the strands of n-braid, discriminant D, and ramified primes dividing the discriminant. They are central Galois invariants analogous to Kähler magnetic fluxes at the geometry side.

4. Ramified primes characterize polynomials P but are not fixed by the Galois group, are analogous to the zero modes at the level of H. Magnetic fluxes are their counterparts at the level of H. I have proposed the interpretation of ramified primes p as p-adic primes characterizing elementary particles in the model of particle masses based on p-adic thermodynamics. These primes are rather large: for instance, $M_{127} = 2^{127} - 1$ would characterize electrons. It would however seem that the prime k in SSA_k corresponds to the prime characterizing simple Galois group.

   Also affine algebras Aff_n assignable to the light-like partonic orbits and isometries of H are present and also they appear in p-adic mass calculations based on p-adic thermodynamics. Could the adelic hierarchy p-adic variants of algebras SSA, Aff and I have adelic factors labelled by ramified primes p form also an adelic structure with respect to $\oplus$ and $\otimes$?

21.9.4 Basic algebraic structures at the geometric side

The symmetry algebras at the level of H define the key quantal structures.

1. The symmetries at the geometric side involve hierarchies A_n of algebras $A_n \in SSA_n, A_n, I_n$ defining hierarchies of factor algebras. The condition that subalgebras A_n and $[A_n, A]$ annihilate physical states gives rise to hierarchies of algebras, which would correspond to those for Galois groups for multiple extensions of rationals. The braided Galois groups for polynomials of degree n n roots/braids would act naturally in A_n so that it would have number theoretic braiding.

2. The decomposition of the Galois group to simple normal subgroups would correspond to a functional composite of prime polynomials, which corresponds to the inclusion hierarchy of HFFs associated with A_n with n identified as the degree of polynomial.

   The polynomials $Q(X)$ defining infinite prime have decomposition to polynomial primes but the polynomial primes in the decomposition cannot define infinite primes.

   Kähler magnetic fluxes for CP_2 and M^4 Kähler forms are symplectic invariants and represent zero modes. At the number theoretic side the discriminant and root spectrum (mass squared spectrum) are classical Galois invariants. States as Galois singlets are Galois invariants at quantum level.

The key equation, not encountered before in the TGD framework, is $P = Q$ motivated by the notion of infinite prime. It would assign to polynomial P unique algebraic structures defining what might be called its quantization. Without this structure one should give up the notion of infinite prime and lose the notion of preferred P as analog of preferred extremal.

21.10 Appendix: The reduction of quantum TGD to WCW geometry and spinor structure

The first attempts to build quantum TGD were based on the standard method used to quantize quantum field theories. The path integral over all possible space-time surfaces connecting initial and final 3-surfaces for an action exponential using for instance Kähler action, would have given the scattering amplitudes.

21.10.1 The problems

The first problem is that the integrand is a phase factor $exp(iS)$, where S could be the Kähler action. Phase factor has modulus 1 and the integral does not converge even formally. One would need a real exponent to have any hopes of convergence. This problem can be circumvented in free quantum field theory by algebraic tricks.

The second problem is that all conceivable actions are extremely nonlinear and new kinds of divergences appear in each order of perturbation theory. This is essentially due to the locality of the action principle involving interaction vertices with arbitrarily high numbers of particles. Also ordinary QFTs meet the same problem and for renormalizable theories the addition of counterterms with suitably infinite coefficients can cancel the divergences without the addition of an infinite number of counter terms. It became clear that there are no hopes of getting rid of the divergences in TGD by addition of counterterms. The situation is the same in general relativity although heroic and ingenious attempts to calculate scattering amplitudes have been made.

Only $\mathcal{N}=4$ SUSY is a QFT that is hoped to be free of divergences without renormalization but here the problem is caused by the non-planar Feynman diagrams, to which the twistor approach does not apply.

21.10.2 3-D surfaces or 4-surfaces associated to them by holography replace point-like particles

The key idea of TGD is that point-like particles are replaced with 3-surfaces. This idea does not favour path integral approach.

1. In TGD, point-like particles are replaced with 3-surfaces. Local interaction vertices are smoothed out to non-local ones so that there should be no local divergences. Perhaps the path integral, derived originally as a representation of Schrödinger equation, is not only unnecessary but also a wrong way to compute anything in TGD. In superstring models, the replacement of a point-like particle with string indeed allows elimination of the local divergences.

   3-D surface should be the basic dynamical object. One should therefore have a functional integral over 3-surfaces, which is analogous to the Gaussian integral and converges.

2. This problem led to the idea of the "world of classical worlds" (WCW). 4-D General Coordinate Invariance implies that to a given 3-surface X^3 one must be able to assign a 4-surface $X^4(X^3)$ at which the 4-D general coordinate transformations act.

   Either 3-surfaces X^3 or almost unique 4-surfaces $X^4(X^3)$ are the fundamental objects so that holography holds true. At that time I did not talk about holography, which was introduced by Susskind much later, around 1995. Therefore the introduction of the path integral is not necessary.

   Later it became clear that the exact determinism of the classical dynamics can be lost, at least for Kähler action having huge spin glass degeneracy. Later 4-D Käehler action replaced in twistor lift of TGD by its sum with a volume term, and for this action the non-determinism is analogous to that for soap films spanned by frames, that is finite, and has physical interpretation.

21.10.3 WCW Kähler geometry as s geometrization of the entire quantum physics

This argument led to the vision about quantum TGD as WCW geometry, which generalizes Einstein's vision of geometrization of gravitational interaction to geometrization of all classical interactions and then to the geometrization of the entire quantum theory.

1. WCW is the space of all 3-surfaces or almost equivalently the space of 4-surfaces. Physical states correspond to WCW spinor fields.

2. WCW must have Kähler geometry since Kähler structure allows to geometrize the hermitian conjugation which is fundamental for quantum theory. Imaginary unit is represented geometrically by the Kähler form and the real unit by the Kähler metric. The tensor square Kähler form as an imaginary unit is equal to the negative of the real unit, that is the negative of the metric.

3. The construction of loop space geometries by Dan Freed [A44] led to a unique geometry of loop space. The mere existence of Riemann connection requires that the metric has maximal isometries and is unique apart from scaling. When basic objects are 3-D this condition is even more stringent. The Kähler geometry of WCW and thus physics could be unique from its mere mathematical existence!

   Why $H = M^4 \times CP_2$? The existence of the twistor lift fixes H uniquely since only M^4 (E^4) and CP_2 allow a twistor space with Kähler structure [A63]. The necessarily dimensionally reduced Kähler action at the twistor space level adds to the 4-D Kähler action a volume term removing the non-determinism and explaining cosmological constant and its smallness in long scales.

4. How is the Kähler geometry of WCW determined? The definition of the Kähler metric of WCW must assign to a 3-surface X^3 a more or less unique space-time surface $X^4(X^3)$ in order to have a general coordinate invariance. One must also have a connection with classical physics: classical physics must be an exact part of quantum physics and thus the definition of WCW Kähler geometry involves a classical action principle.

   The Kähler metric is defined by the Kähler function K. The idea is that K is the value of Kähler action S_K or of a more general action for a more or less unique space-time surface $X^4(X^3)$ containing a given 3-surface X^3.

5. It is convenient to speak of preferred extremal (PE) and there are several characterizations of what PE is. $M^8 - H$ duality gives the most concrete one. Twistor lift gives the second one and the gauge conditions associated with the WCW Dirac equation provide the third characterization.

21.10.4 Quantum physics as physics of free, classical spinor fields in WCW

How to develop quantum physics in WCW? The idea is that free, classical WCW spinor fields define all possible quantum states of the Universe and interactions reduce to topology. There would be no quantization at the level of WCW and the only genuinely quantal element of quantum theory would be state function reduction giving rise to conscious experience.

1. In order to have spinor fields in WCW, one must have the notion of spinor structure. Spinor structure is almost uniquely fixed by the metric and involves in an essential manner gamma matrices, which anticommute to metric.

2. The second quantization of H spinor fields assigns to the modes of H spinor fields fermionic oscillator operators. Why not build the conplexified gamma matrices of WCW (their hermitian conjugates) as linear combinations of the creation (annihilation) operators?! Second quantization for the *free* H spinor field, is completely unique and straightforward and avoids all problems of quantization in curved space-time.

One could interpret the second quantization of free fermions and fermionic statistics in terms of WCW geometry, which is something completely new.

3. WCW spinors (for given 4-surface as point of WCW) would be fermionic Fock states created using fermionic oscillator operators and depend on the space-time surface $X^4(X^3)$ as a 4-surface almost uniquely determined by 3-surface X^3.

   The fermionic Fock state basis can be interpreted as a representation of Boolean logic so that Boolean logic could be seen as a "square root" of Kähler geometry.

   The WCW spinor field would correspond to a superposition of preferred extremals X^4 with a WCW spinor assigned with each X^4.

21.10.5 Dirac equation for WCW spinor fields

Free Dirac equation is the key equation for classical spinor fields.

1. In string models it corresponds to the analogs of super-Virasoro and super-Kac-Moody conditions stating conformal invariance and Kac-Moody invariance analogous but not quite equivalent with gauge symmetry.

2. In TGD, these conditions as a counterpart of the WCW Dirac equation generalize. Super symplectic algebra associated with $\delta M^4_+ \times CP_2$ (δM^4_+ denotes light-cone boundary) SSA, the infinite-D algebras of conformal symmetries ($Conf$) and isometries (I) of δM^4_+ (unique to the 4-D Minkowski space), and the affine algebras Aff associated with the light-like orbits of partonic 2-surfaces would be the basic algebras.

3. To each of these algebras, one can assign a generalization of the gauge conditions of conformal field theories. What is new is that one obtains a hierarchy of gauge conditions. The algebra in question, call it A, and sub-algebra A_n, $n \geq 0$, with conformal weights coming as n-multiples of weights for A, and the commutator $[A_n, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish, which gives strong conditions on space-time surfaces and decomposes WCW to sectors characterized by n.

4. In superstring models one has only $n = 0$. In the number theoretic vision, the hierarchy of values of n would actually correspond to the hierarchy of extensions of rationals. If $M^8 - H$ duality holds true, n corresponds to the degree of polynomial P defining the space-time surface and polynomials P would decompose WCW to sectors.

21.10.6 $M^8 - H$ duality at the level of WCW

WCW emerges in the geometric view of quantum TGD. $M^8 - H$ duality should lso work for WCW. What is the number theoretic counterpart of WCW? What is the geometric counterpart of the discretization characteristic to the number theoretic approach?

In the number theoretic vision in which WCW is discretized by replacing space-time surfaces with their number theoretical discretizations determined by the points of $X^4 \subset M^8$ having the octonionic coordinates of M^8 in an extension of rationals and therefore making sense in all p-adic number fields? How could an effective discretization of the real WCW at the geometric H level, making computations easy in contrast to all expectations, take place?

1. The key observation is that any functional or path integral with integrand defined as exponent of action, can be *formally* calculated as an analog of Gaussian integral over the extrema of the action exponential $exp(S)$. The configuration space of fields would be effectively discretized. Unfortunately, this holds true only for the so called integrable quantum field theories and there are very few of them and they have huge symmetries. But could this happen for WCW integration thanks to the maximal symmetries of the WCW metric?

2. For the Kähler function K, its maxima (or maybe extrema) would define a natural effective discretization of the sector of WCW corresponding to a given polynomial P defining an extension of rationals.

The discretization of the WCW defined by polynomials P defining the space-time surfaces should be equivalent with the number theoretical discretization induced by the number theoretical discretization of the corresponding space-time surfaces. Various p-adic physics and corresponding discretizations should emerge naturally from the real physics in WCW.

3. The physical interpretation is clear. The TGD Universe is analogous to the spin glass phase [L139]. The discretized WCW corresponds to the energy landscape of spin glass having an ultrametric topology. Ultrametric topology of WCW means that discretized WCW decomposes to p-adic sectors labelled by polynomials P. The ramified primes of P label various p-adic topologies associated with P.

Chapter 22

Homology of "world of classical worlds" in relation to Floer homology and quantum homology

22.1 Introduction

One of the mathematical challenges of TGD is the construction of the homology of "world of classical worlds" (WCW). With my rather limited mathematical skills, I had regarded this challenge as a mission impossible. The popular article in Quanta Magazine with title "Mathematicians transcend the geometric theory of motion" (see `https://cutt.ly/vO4eb5V` however stimulated the attempts to think whether it might be possible to say something interesting about WWC homology.

The article told about a generalization of Floer homology by Abouzaid and Blumberg [A26] (`https://cutt.ly/ZPeOTSc`) published as 400 page article with the title "Arnold Conjecture and Morava K-theory". This theory transcends my mathematical skills but the article stimulated the idea WCW homology might be obtained by an appropriate generalization of the basic ideas of Floer homology (`https://cutt.ly/VO4dSPD`).

The construction of WCW homology as a generalization of Floer homology looks rather straightforward in the zero ontology (ZEO) based view about quantum TGD. The notions of ZEO and causal diamond (CD) [L91] [K130], the notion of preferred extremal (PE) [L141] [K9], and the intuitive connection between the failure of strict non-determinism and criticality pose strong conditions on the possible generalization of Floer homology.

WCW homology group could be defined in terms of the free group formed by preferred extremals $PE(X^3, Y^3)$ for which X^3 is a *stable* maximum of Kähler function K associated with the *passive* boundary of CD and Y^3 associated with the *active* boundary of CD is a more general critical point.

The stability of X^3 conforms with the TGD view about state function reductions (SFRs) [L91]. The sequence of "small" SFRs (SSFRs) at the active boundary of CD as a locus of Y^3 increases the size of CD and gradually leads to a PE connecting X^3 with stable 3-surface Y^3. Eventually "big" SFR (BSFR) occurs and changes the arrow of time and the roles of the boundaries of the CD changes. The sequence of SSFRs is analogous to a decay of unstable state to a stable final state.

The identification of PEs as minimal surfaces with lower-dimensional singularities as loci of instabilities implying non-determinism allows to assign to the set $PE(X^3, Y_i^3)$ numbers $n(X^3, Y_i^3 \to Y_j^3)$ as the number of instabilities of singularities leading from Y_i^3 to Y_j^3 and define the analog of criticality index (number of negative eigenvalues of Hessian of function at critical point) as number $n(X^3, Y_i^3) = \sum_j n(X^3, Y_i^3 \to Y_j^3)$. The differential d defining WCW homology is defined in terms of $n(X^3, Y_i^3 \to Y_j^3)$ for pairs Y_i^3, Y_j^3 such that $n(X^3, Y_j^3) - n(X^3, Y_i^3) = 1$ is satisfied. What is nice is that WCW homology would have direct relevance for the understanding of quantum criticality.

The proposal for the WCW homology also involves a generalization of the notion of quantum connectivity crucial for the definition of Gromow-Witten invariants. Two surfaces (say branes) can be said to intersect if there is a string world sheet connecting them generalizes. In ZEO quantum connectivity translates to the existence of a preferred extremal (PE), which by the weak form of holography is almost unique, such that it connects the 3-surfaces at the opposite boundaries of causal diamond (CD).

22.2 Some background

In this section some background, including Morse theory, Floer homology, its generalization by Abouzaid and Blumberg, and the basic ideas of TGD proposal, is discussed.

22.2.1 The basic ideas of Morse theory

Torus as a 2-D example helps to understand the idea of homology and Morse theory. Homologically non-trivial surfaces are surfaces without boundary but are not boundaries themselves. Entire torus represents the element of H^2, the 2 homologically non-trivial circles, and points indeed have vanishing boundaries without being boundaries. The basic homological operation d represents the operation of forming a boundary: the boundary of a boundary is empty and this corresponds to $d^2 = 0$. d reduces degree of homology by one unit: $H_n \rightarrow H_{n-1}$.

How to understand the homology of torus? Morse theory based on the notion of Morse function provides the tool.

1. Consider the embedding of torus to 3-space. The height-coordinate h defines a Morse function at torus and one can assign to it $h = constant$ level surfaces. It has 4 critical points: h_0, h_1, h_2, h_3 at which the topology of level suraace changes.

   h has maximum h_3 at the top of torus and minimum h_0 at the bottom of the torus. h_3 corresponds to the entire torus, element of homology group H_2 and h_0 to a point as element of H_0.

   h has saddle points h_1, h_2 at the top and bottom of the "hole" of the torus. The level surfaces $h = h_1$ and $h = h_2$ correspond to two touching circles: the topology of the intersection changes from a contractible circle to a union of oppositely oriented incontractible small circles representing elements of the homology group H_1. That they have opposite orientations states conservation of homology charge in the topological reaction in which the level circle splits to two: $0 = 1 - 1$.

   Outside the critical points the topology of the $h = constant$ level surface is a circle or two disjoint circles. The critical points of h clearly code part of the homology of torus. What however remains missing is the homology group element, which corresponds to the large circle around the torus. This element of H_1 would be obtained if the height function h were a horizontal coordinate.

2. One can deform the torus and also add handles to it to get 2-D topologies with a higher genus. Morse function also helps to understand the homology of higher-dimensional spaces for which visual intuition fails.

   This situation is finite-D and too simple to apply in the case of the space of orbits of a Hamiltonian system. Now the point of torus is replaced with a loop as a single orbit in phase space. The loop space is infinite-dimensional and the Morse theory does not generalize as such. In Floer homology one studies even the homology of infinite-dimensional spaces.

Homology involves also the d operation. d can be indeed visualized in terms of dynamics of a gradient flow. Assume that torus is in the gravitational potential of Earth proportional to h. Gravitation defines a downwards directed gradient force. One can speak of critical directions as directions in which the particle forced to stay at the torus can fall downwards when subjected to an infinitesimal push.

1. At the top $h = h_3$ of the torus there are 2 critical directions: either along a small or large incontractible circle of torus. This number corresponds to the dimension $d = 2$ of torus as

the element of the homology group H_2. At the bottom $h = h_0$ there are 0 critical directions and one has a point as an element of H_0. At the saddle points h_1, h_2 there is 1 critical direction and it corresponds to a nontrivial circle as an element of H_1. The number n of critical directions corresponds to the dimension for elements of the homology group H_n.

2. The particle at the top h_3 has 2 critical directions (criticality 2), and can fall to the saddle point h_2, having criticality 1, by moving along the small homologically non-trivial circle. Criticality decreases by 1 unit so that one has a map $H_2 \rightarrow H_1$. The particle can also move along the large circle to the bottom, in which case criticality decreases by 2 units.

   The particle at critical point h_2 moves to h_1 along a circle homologous to the large circle without a change in criticality and the particle at h_1 moves to h_0 also the small circle: the criticality changes by 1 unit so that one has a map $H_1 \rightarrow H_0$.

   Therefore the elements of the homology group correspond to critical points for the gradient flow defined by the gravitational field and the effect of the map d can be represented dynamically as a motion in the gravitational field reducing the criticality by one unit.

   The representability of homology elements as critical points of Morse function and the representation of d-operation in terms of gradient dynamics is extremely useful in higher dimensional spaces, where geometric intuition does not help much. In Floer homology this dynamics is applied as a tool.

22.2.2 The basic ideas of Floer homology

Consider first the motivations and ideas of Floer homology (`https://cutt.ly/lO6EMp6`). The original goal was to prove Arnold's conjecture. One considers a symplectic manifold with symplectic form ω. Arnold conjectured that the number of fixed points of a Hamiltonian symplectomorphism generated by an exponentiation of a Hamiltonian H, is bounded below by the number of critical points of a smooth function on M.

The goal is to generalize Morse theory.

1. Morse theory involves the height function h in a finite-D manifold M and the critical points of h correspond to elements of homology groups H_n. The number n of negative eigenvalues of Hessian of f at critical points defines the index of criticality f and one can associate with the critical point an element of the homology group H_n. $n = 0$ corresponds to maximum of f. Note that in infinite-D case, Morse theory need not work since n can be arbitrarily large and if the convention for criticality is changed so that $n = 0$ corresponds to minimum, a different theory is obtained.

2. In Morse homology, the n-simplices of the simplicial homology are replaced by critical points with criticality index n and the homology groups are replaced with the Abelian group defined by the critical points and graded by the criticality index n. The gradient flow lines connecting critical points with $\Delta n = 1$ allow to define an analog of the exterior derivative d: it is defined by the number of flow lines connecting critical points with $\Delta n = 1$.

22.2.3 Floer homology

The motivation for the symplectic Floer homology is the conjecture by Arnold related to the Hamiltonian systems. These systems are defined in phase space, whose points are pairs of position and momentum of the particle. This notion is extremely general in classical physics.

1. One considers compact symplectic manifolds M and symplectic action $S = \oint p_i dq_i$ and its critical points, which are loops. Note that symplectic action has interpretation as an area. The general case $S = \oint (p_i dq_i/dt - H)dt$ is not considered in the Floer homology.

   Remark: A more general question is whether there exist closed orbits, kind of islands of order, in the middle of oceans of chaos consisting of non-closed chaotic orbits. This is indeed the case: there is a fractal structure formed by islands of order in oceans of disorder. Hamiltonian chaos differs from dissipative chaos in that the fractal has the same dimension as the symplectic manifold since symplectic transformations preserve area and high 2n-dimensional volumes.

2. Arnold's conjecture was that the number of critical points of a given criticality index of a symplectomorphism has as an upper bound the number of critical points for a generic function. The inspiration behind the Floer homology is the intuition that a generalization of Morse theory to the loop space $L(M)$ allows us to understand the homology. The conjecture is that the closed orbits serve as minimal area representatives for the homology of $L(M)$. These closed orbits would be critical points of S defining the area closed by the curve.

The goal is to understand the homology of a finite-dimensional compact symplectic manifold M and Floer homology provides the needed tool. Floer homology for the infinite-D loop space $L(M)$ serves as a tool to achieve this goal and the proof of Arnold's conjecture follows as an outcome.

In symplectic Floer homology, one is interested in closed loops as orbits of a symplectic flow in a compact symplectic space M. One wants to identify them as critical points of an analog of Morse function in the loop space $L(M)$.

1. In the symplectic Floer homology, M is a finite-D symplectic manifold and one deduces information about it from the homology of loop space $L(M)$ by generalizing Morse homology to the homology of $L(M)$.

2. The counterpart of the Morse function is unique and defined by the symplectic action functional $S = \oint p_i dq_i$ in $L(M)$. Note that S depends only on M. S defines the counterpart of free action with a vanishing Hamiltonian H. For a general Hamiltonian one would have $S = \oint(p_i dq_i/dt - H)dt$. Note that closed orbits are possible if M is compact. For a generic H the dynamic becomes chaotic.

   Closed loops for free flows define the analogs of critical points of Morse function. For instance, for 2-torus the closed orbits correspond to loops with winding numbers n_1, n_2.

3. One must identify the counterpart for the gradient flow lines connecting the critical points with $\Delta n = 1$ in order to define d. Here one considers a deformation of the system by a time dependent Hamiltonian H and hopes that the predictions do not depend on the choices of H. This gives to orbits of the closed loops in the loop space giving rise to cylinders in M.

   These cylinders define pseudoholomorphic curves and define the counterparts of the gradient flows connecting critical points as closed loops in X. The differential d for the Floer homology is defined in terms of the numbers of these curves between critical points with the property that the criticality index increases by one unit.

4. The basic result is a proof for the Arnold conjecture and roughly states that for the ranks of homology groups of M are smaller than the Floer homology groups defined by arbitrary Hamilton.

Floer homology has a rich variety of applications discussed in the Wikipedia article (`https://cutt.ly/lO6EMp6`). One application relates to the Lagrangian manifolds of a symplectic manifold. Now the chain complex is generated by the intersection points of Lagrangian manifolds intersecting transversely.

A further application is associated with Yang- Mills theory. The action is the Chern-Simons action defining a topological quantum field theory. Its critical points are topologically non-trivial gauge connections with a trivial curvature form. Topological non-triviality means that the group defined by the parallel translations along closed curves is non-trivial. The counterpart of the gradient flow is defined by Yang-Mills action and the flow lines correspond to instantons approach at the ends of the counterpart of mapping cylinder trivial connections.

22.2.4 The generalization of Floer homology by Abouzaid and Blumberg

The work of mathematicians Abouzaid and Blumberg [A26] (`https://cutt.ly/ZPeOTSc`), which represents the generalization of Floer homology which, using popular terms, allows to "count holes" in the infinite-D space of loops.

The abstract of the article of Abouzaid and Blumberg is following.

> We prove that the rank of the cohomology of a closed symplectic manifold with coefficients in a field of characteristic p is smaller than the number of periodic orbits of any non-degenerate Hamiltonian flow.
> Following Floer, the proof relies on constructing a homology group associated to each such flow, and comparing it with the homology of the ambient symplectic manifold. The proof does not proceed by constructing a version of Floer's complex with characteristic p coefficients, but uses instead the canonical (stable) complex orientations of moduli spaces of Floer trajectories to construct a version of Floer homology with coefficients in Morava's K-theories, and can thus be seen as an implementation of Cohen, Jones, and Segal's vision for a Floer homotopy theory.
> The key feature of Morava K-theory that allows the construction to be carried out is the fact that the corresponding homology and cohomology groups of classifying spaces of finite groups satisfy Poincare duality.

I try to express what I understand as a physicist about this highly technical summary.

1. The main emphasis is in the homology of finite-D symplectic manifolds and the homology of the infinite-D loop space is only a tool to obtain this information.

2. The generalization of Arnold's conjecture is expressed in the first paragraph. For closed symplectic manifolds the cohomology groups of a closed symplectic manifold have rank smaller than the number of periodic orbits of *any* non-degenerate Hamiltonian flow.

   Therefore Hamiltonian flows give information about the cohomology and by Poincare duality also about homology of the symplectic manifold.

3. The coefficients of homology can be chosen in very many ways: rationals, integers, finite fields, p-adic number fields. Integers are however the natural ones in the situation in which one counts concrete objects. The homology has coefficients in finite field F_p, integers modulo prime p: for instance, the numbers of flow lines of gradient flow connecting the critical points of symplectic action are counted modulo p.

4. Time dependent Hamiltonians enter into the picture as perturbations of the symplectic action. One replaces the free symplectic action $S = \oint p_i dq_i/dt$ in loop space with $S = \oint (p_i dq_i/dt - H)dt$ playing a role analogous to that of Morse function. This is like adding an interaction term to free action. It is essential that the symplectic space is compact so that closed orbits as critical points of S are possible.

22.2.5 Gromow-Witten invariants

The proposed TGD based generalization of the notion of "being connected" by a flow line of gradient flow resonates with the definition of Gromow-Witten (G-W) invariant. G-W invariant emerges in enumerative geometry, which is essentially counting of particular kinds of points of enumerative geometry which is a branch of algebraic geometry.

G-W invariants (`http://tinyurl.com/y9b5vbcw`) are rational number valued topological invariants useful in algebraic and symplectic geometry. These quantum invariants give information about these geometries not provided by classical invariants. Despite being rational numbers in the general case G-W invariants in some sense give the number of string world sheets connecting given branes.

The definition of G-W invariant involves a non-locality, which is completely analogous to the non-locality in the proposed definition of WCW homology. In TGD, the string world sheet as connector of branes is replaced with PE as a connector of the boundaries of opposite boundaries of CD taking the role of brane.

Here is the definition of G-W invariants with some TGD induced coloring taken from [L49, K69].

1. One considers a collection of n surfaces ("branes") with even dimensions in some symplectic manifold X of dimension $D = 2k$ (say Kähler manifold) and pseudo-holomorphic curves ("string world sheets") X^2, which have the property that they connect these n surfaces in the sense that they intersect the "branes" in the marked points x_i, $i = 1, .., n$.

"Connect" does not reduce to intersection in a topologically stable sense since connecting is possible also for branes with dimension smaller than $D-2$. One allows all surfaces X^2 that intersect the n surfaces at marked points if they are pseudo-holomorphic even if the basic dimension rule is not satisfied. In the 4-dimensional case this does not seem to have implications since the partonic 2-surfaces automatically satisfy the dimension rule. The n branes intersect or touch in a quantum sense: there is no concrete intersection but intersection with the mediation of "string world sheet".

2. Pseudo-holomorphy means that the Jacobian df of the embedding map $f : X^2 \rightarrow X$ commutes with the symplectic structures j *resp.* J of X^2 *resp.* X: i.e. one has $df(jT) = Jdf(T)$ for any tangent vector T at given point of X^2. For $X^2 = X = C$ this gives Cauchy-Riemann conditions.

   In the symplectic case X^2 is characterized topologically by its genus g and homology class A as the surface of X. In algebraic geometry context the degree d of the polynomial defining X^2 replaces A. In TGD X^2 corresponds to a string world sheet having also a boundary. X^2 has also n marked points $x_1, ..., x_n$ corresponding to intersections with the n surfaces.

3. G-W invariant $GW_{g,n}^{X,A}$ gives the number of pseudo-holomorphic 2-surfaces X^2 connecting n given surfaces in X - each at single marked point. In TGD these surfaces would be partonic 2-surfaces and marked points would be carriers of sparticles.

22.3 About the generalization of Floer homology in the TGD framework

A generalization of homotopy and homology groups could help to understand WCW topology. One of the intuitive visions behind TGD has indeed been that, despite the explicit appearance of metric, TGD in a certain sense is a topological quantum theory. A mathematical motivation for this intuition comes from the fact that minimal surfaces provide representations for homological equivalence classes. Floer homology suggests concrete ideas, which might help to understand the homology of WCW.

22.3.1 Key ideas behind WCW homology

The encounter with Floer homology inspired the question whether one could say something interesting about WCW homology by an appropriate generalization of the concepts involved with it.

Preferred extremals (PEs) as counterparts of critical points

PEs are an obvious candidate for the counterparts of critical points. ZEO however implies some important delicacies crucial for WCW homology.

1. In the TGD Universe, space-time is a 4-surface in $H = M^4 \times CP_2$, in a loose sense an orbit of 3-surface. General Coordinate Invariance (GCI) requires that the dynamics associates to a given 3-surface a highly unique 4-surface at which the 4-D general coordinate transformations act. This 4-surface is a PE of the action principle determing space-time surfaces in H and analogous to Bohr orbit. GCI gives Bohr orbitology as an exact part of quantum theory and also holography.

   These PEs as 4-surfaces are analogous to the closed orbits in Hamiltonian systems about which Arnold speculated. In the TGD Universe, only these PEs would be realized and would make TGD an integrable theory. The theorem of Abouzaid and Blumberg allows to prove Arnold's conjecture in homologies based on cyclic groups Z_p. Maybe it could also have use also in the TGD framework.

2. WCW generalizes the loop space considered in Floer's approach. Very loosely, loop or string is replaced by a 3-D surface, which by holography induced is more or less equivalent with

4-surface. In TGD just these minimal representatives for homology as counterparts of closed orbits would matter.

3. Symplectic structure and Hamiltonian are central notions also in TGD. Symplectic (or rather, contact) transformations assignable to the product $\delta M^4_+ \times CP_2$ of the light-cone boundary and CP_2 act as the isometries of the infinite-D "world of classical worlds" (WCW) consisting of these PEs, or more or less equivalently, corresponding 3-surfaces. Hamiltonian flows as 1-parameter subgroups of isometries of WCW are symplectic flows in WCW with symplectic structure and also Kaehler structure.

4. The space-time surfaces are 4-D minimal surfaces in H with singularities analogous to frames of soap films. Minimal surfaces are known to define representatives for homological equivalence classes of surfaces. This has inspired the conjecture that TGD could be seen as a topological/homological quantum theory in the sense that space-time surfaces served as unique representatives or their homological classes.

5. There is also a completely new element involved. TGD can be seen also as number theoretic quantum theory. M^8-H duality can be seen as a duality of a geometric vision in which space-times are 4-surfaces in H an of a number theoretic vision in which one consideres 4-surfaces in octonionic complexified M^8 determined by polynomials with dynamics reducing to the condition that the normal space of 4-surface is associative (quaternionic). M^8 is analogous to momentum space so that a generalization of momentum-position duality of wave mechanics is in question.

The first sketch for WCW homology

A suitable generalization of Floer's theory might allow us to define WCW homology.

1. The PEs would correspond to the critical points of an analog of Morse function in the infinite-D context. In TGD the Kähler function K defining the Kahler geometry of WCW is the unique candidate for the analog of Morse function.

   The space-time surfaces for which the exponent $exp(-K)$ of the Kähler function is stationary (so that the vacuum functional is maximum) would define PEs. Also other space-time surfaces could be allowed and it seems that the continuity of WCW requires this. However the maxima or perhaps extrema would provide an excellent approximation and number theoretic vision would give an explicit realization for this approximation.

   It is however important to notice that the K for, in general non-unique, preferred external $PE(X^3, Y^3)$ can be maximum for X^3 and a more general critical point for Y^3. This option conforms with the ZEO view about SFRs in which the passive boundary of CD is stable and a sequence of SSFRs takes place at the active boundary and increases its size. The homology would be assigned to the criticality of the active boundary of CD.

   This would require a varying CD size, which should therefore be determined by PE and appear as a parameter in PE. By M^8-H duality the boundary of CD corresponds to the image of a mass shell H^3 in M^3. Perhaps this property at the active end of PE codes for the size scale of the CD. The size scale of CD, not necessarily the size, should correspond to the p-adic length scale L_p determined by the largest ramified prime of the polynomial coding for PE. Does this mean that L_p remains the same during the entire sequence of SSFRs or can it increase? The size could increase by factor $\sqrt{p}$ with change ibn L_p and for large p-adic primes such as $M_{127}=2^{127}-1$ this would mean very large scaling.

   Remark: Since WCW Kähler geometry has an infinite number of zero modes, which do not appear in the line element as coordinate differentials but only as parameters of the metric tensor, one expects an infinite number of maxima.

2. The PEs would correspond by M^8-H duality to roots of polynomials P in the complexified octonionic M^8 so that a connection with number theory emerges. M^8-H duality strongly strongly suggests that $exp(-K)$ is equal to the image of the discriminant D of P under canonical identification $I: \sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adic numbers to reals. The prime p would correspond to the largest ramified prime dividing D [L147, L148].

3. The number theoretic vision could apply only to the critical points of $exp(-K)$ with respect to both ends of PE and give rise to what I call a hierarchy of p-adic physics as correlates of cognition. Everything would be discrete and one could speak of a generalization of computationalism allowing also the hierarchy of extensions of rationals instead of only rationals as in Turing's approach. The real-number based physics would also include the non-maxima via a perturbation theory involving a functional integral around the maxima. Here Kähler geometry allows to get rid of ill-defined metric and Gaussian determinants.

G-W invariants and ZEO

Enumerative geometry is also a central element of adelic physics.

1. M^8-H duality involves the notion of cognitive representations consisting of special points of 4-surface, in particular, points of 3-D mass shell $H^3 \subset M^4_c \subset M^8_c$. The "active" points containing quark are identified as quark momenta. A generalization of momentum-position duality is in question.

2. The points of the cognitive representation, having interpretation as four-momenta [L109, L110, L147, L148], are identified are algebraic integers in the extensions defined by the real polynomial P with rational coefficients continued to a polynomial of a complexified octonion. P defines mass shells as its roots with $m^2 = r_n$ defining the spectrum of virtual mass squared values for quarks. The finite number of mass shells guarantees the absence of divergences due to momentum space integrations.

3. By the symmetries of H^3, the number of points in cognitive representations is especially high at the mass shells. Physical states correspond to Galois singlets (Galois confinement implying conformal confinement) for which the sum of quark momenta is an ordinary integer as one uses as unit the p-adic mass scale defined by the largest ramified prime associated with P.

4. The mass shells associated with a given polynomial P are connected by a 4-surface X^4 as a deformation of M^4_c, which defines M^8-H duality by assigning to $X^4 \subset M^8$ space-time surface in $H = M^4 \times CP_2$. This surface is a minimal surface with singularities analogous to frames of a soap film. M^8-H duality maps the points of cognitive representation to $X^4 \subset H$ [L146].

The TGD view about WCW homology could perhaps be regarded as a generalization of the quantum connectedness behind G-W invariants. The role of the string world-sheet as a quantum connector is taken by PE so that there is no need to introduce gradient dynamics separately. The quantum connection between X^3_1 and X^3_2 at the boundary A of CD exists if $X^3_1 = CPT(Y^3_1)$ is true for a PE having X^3_1 and Y^3_1 as ends. $\Delta n = \pm 1$ translates to an appearance or disappearance of minimal number of critical directions. The attribute "quantum" is well-deserved since the classical non-determinism serves as a space-time correlate for quantum jumps at WCW level [L146, L121, L147, L148].

22.3.2 A more concrete proposal for WCW homology as a generalization of the Floer homology

Consider first the notion of "world of classical worlds" (WCW).

1. In TGD, point-like particles are replaced by 3-surfaces. Zero energy ontology (ZEO) is assumed, which means that space-time surfaces X^4 as "orbits" of 3-surfaces are inside causal diamonds. These 4-surfaces are PEs of the action principle. For the exact holography, 3-surface at either boundary of CD would determine X^4 uniquely but determinism is expected to be slightly violated so that there are several PEs associated with a given X^3 at either boundary of CD. The failure of strict determinism is analogous to the failure of determinism for soap films with frames.

   Let PE have X^3 *resp.* Y^3 as its ends at the opposite boundaries A *resp.* B of CD.

2. WCW is identified as the space of PEs. One could regard WCW also as covering a space such that for a given X^3 at (say) A, the fiber contains the PEs having X^3 as the first end. WCW has symplectic and even a Kähler structure and symplectic transformations at the light-like boundaries of CD are conjectured to define isometries of WCW but not symmetries of S_K.

3. Kähler function K, serving as the analog of symplectic action, defines Kähler form and symplectic structure. K corresponds to 4-D Kähler action S_K plus volume term for a PE. This action is obtained as a dimensional reduction of 6-D Kähler action for the 6-D surface X^6 in the 6+6-D twistor space of $T(M^4) \times T(CP_2)$. X^6 carries induced twistor structure and has X^4 as base space and S^2 as fiber.

WCW homology based on minimal surfaces with singularities

The challenge is to identify the counterpart of gradient flow as a counterpart of quantum connectivity. This should not bring anything new to the existing picture. The following proposal is perhaps the simplest one and conforms with the physical intuition.

1. Morse theory and Floer homology would suggest that one should consider the Hessian of Kähler function $K(PE(X^3)$ of WCW as functional of preferred extremal $PE(X^3, Y^3)$. One could calculate the numbers n_+ resp. n_- of positive and negative eigenvalues of Hessian and identify n_- as the criticality index and number of unstable directions.

2. There are several problems. The identification of the analog of gradient flow seems very difficult. However, by the weak holography due the failure of strict determinism, for a given X^3, there are several 3-surfaces Y_i^3 at the opposite boundary of CD defining PEs. The meaning of criticality is far from obvious since instability for a given time direction looks like stability in the opposite time direction. This is a potential problem since in ZEO [L91] [K130] both arrows of time are possible. There should be a clear distinction between the ends of a CD.

3. By the failure of the strict determinism, the basic objects in ZEO are pairs (X_i^3, Y_j^3) connected by $PE(X_i^3, Y_j^3)$ identifiable as critical points of K with respect to variations of at least one end. The physical picture suggests that criticality is possible for both ends and that a maximum for the passive boundary of CD and criticality for the opposite active boundary of CD (where quantum fluctuations due to "small" state function reductions (SSFRs) are located) is possible. The instabilities associated with criticality at active end would correspond to a definite time direction. It is however difficult to proceed without a more concrete picture.

4. WCW homology could also involve a generalization of the notion of quantum connectivity crucial for the definition of Gromow-Witten invariants. The idea is that two surfaces (say branes) can be said to intersect when there is a string world sheet connecting them, generalizes.

   In ZEO this translates to the existence of a preferred extremal (PE), which by the weak form of holography is almost unique, such that it connects the boundaries of causal diamond (CD), which plays the role of brane.

The identification of PEs as minimal surfaces [L146] allows us to make this picture more concrete and gives a direct connection to quantum criticality as it would be realized in terms of classical non-determinism. One would not count critical directions but critical transitions assignable to singularities of minimal surfaces.

1. PEs are identified as minimal surfaces with singularities analogous to the frames of soap film. At the singularities the minimal surface property fails and the Kähler action and volume term couple together in field equations so that conservation laws are satisfied.

2. The singular surfaces have dimension $d < 4$ and and can be regarded as loci of instability leading to non-determinism. By suitably perturbing the singularities, one can generate new

preferred extremals $PE(X^3,Y_j^3)$ from $PE(X^3,Y_i^3)$. The maximum property of K with respect to the variations of X^3 would suggest that one cannot replace X^3 with a new maximum in this way.

3. For each Y_i^3, one can count the number of deformations of the singularities leading to $PE(X^3,Y_j^3)$ and call this number $n(X^3,Y_i^3 \to Y_j^3)$ as an the analog for the number of gradient lines between given critical points in Floer homology.

   One can define the analog of criticality index $n(X^3,Y_i^3)$ as $n(X^3,Y_i^3) = \sum_j n(X^3,Y_i^3 \to Y_j^3)$ as the analog of n_- of the negative eigenvalues of Hessian. One defines an Abelian group as the complex formed by $PE(X_i^3,Y_j^3)$. $n(X^3,Y_i^3)$ defines the grading for $PE(X^3,Y_j^3)$ as an element of this complex.

   The differential d for WCW homology can be defined in the same way as in Floer homology. Assume $n(X^3,Y_j^3) - n(X^3,Y_i^3) = 1$. Define the action of d as $d(PE(X^3,Y^i)) = \sum_j n(X^3,Y_i^3 \to Y_j^3)PE(X^3,Y^j)$.

The non-determinism of 6-D Kähler and 4-D action would be essential as also the asymmetry between the active and passive boundaries of CD crucial for TGD based quantum measurement theory. Nondeterminism is also essential for the non-triviality of scattering amplitudes since quantum non-determinism in WCW degrees of freedom has classical non-determinism as a space-time correlate [L146]. If the determinism were exact the homology groups H_n would correspond directly to the groups C_n and one would have a Cartesian product of spaces with the homology group $H_n = C_n$. Interesting questions relate to the interpretation of PE pairs with $\Delta n \neq 1$.

Could CPT allow the concretization of quantum connectedness

The quantum connectedness in some sense identifies the 3-surfaces connected by $PE(X_1^3,Y_1^3)$ such that X_1^3 and Y_1^3 are at opposite boundaries of $CD = cd \times CP_2$. If one could assign to Y_1^3 at B a 3-surface X_2^3 at A, quantum connectedness would become more concrete. There is no compelling reason to effectively for this but can ask whether PE could allow to achieve this formally.

1. This formal connection is achieved if there is a discrete symmetry mapping the boundaries A and B of CD to each other. This symmetry must involve time reflection T with respect to the center point of cd. If one requires that the symmetry is an exact symmetry of quantum theory, CPT remains the only candidate. C would act as charge conjugation, realized as a complex conjugation in CP_2.

   CPT maps the boundaries of CDs to each other and therefore also the positive and negative energy parts of zero energy states. The 3-surfaces $(X_1^3\ X_2^3)$ at a given boundary of CD are quantum connected if one has $X_2^3 = CPT(Y_1^3)$ for a PE connecting X_1^3 and Y_1^3.

2. Critical points of K must be mapped to critical points so that CPT should act as a symmetry of the variational principle. If M^4 has Kähler structure the self-dual covariantly constant Kähler form of M^4, strongly suggested by the twistor lift of TGD, must be invariant under CPT and this is indeed the case. The Kähler gauge potential would be also fixed apart from the decomposition $M^4 = M^2 \times E^2$ defined by electric and magnetic parts of $J(M^4)$.

Chapter i

Appendix

A-1 Introduction

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of embedding space and related spaces are discussed and the relationship of CP_2 to the standard model is summarized. The basic vision is simple: the geometry of the embedding space $H = M^4 \times CP_2$ geometrizes standard model symmetries and quantum numbers. The assumption that space-time surfaces are basic objects, brings in dynamics as dynamics of 3-D surfaces based on the induced geometry. Second quantization of free spinor fields of H induces quantization at the level of H, which means a dramatic simplification.

The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Also the relationship to string models is discussed briefly.

Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure analogous to adele [L54, L53]. In the recent view of quantum TGD [L157], both notions reduce to physics as number theory vision, which relies on $M^8 - H$ duality [L109, L110] and is complementary to the physics as geometry vision.

Zero energy ontology (ZEO) [L91] [K130] has become a central part of quantum TGD and leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory having quantum biology as an application. Also these aspects of TGD are briefly discussed.

A-2 Embedding space $M^4 \times CP_2$

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that embedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Embedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) [L91, L138] [K130] causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure [A63] so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2.1 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrate that it codes for the symmetries of standard models via its isometries and holonomies.

CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A51] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i \bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , & e^1 &= \tfrac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \tfrac{r\sigma_2}{\sqrt{F}} \ , & e^3 &= \tfrac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{array}{llllll} e^0 & = & \frac{dr}{F} \ , & e^1 & = & \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 & = & \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 & = & \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{array} \tag{A-2.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-2.12}$$

From this expression one finds that at coordinate infinity $r = \infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-2.13}$$

is given by

$$\begin{array}{llllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ . \end{array} \tag{A-2.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{llllll} R_{01} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-2.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -is_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \tag{A-2.17}$$

The condition states that J and g give representations of real unit and imaginary units related by the formula $i^2 = -1$.

Kähler form is expressible locally in terms of Kähler gauge potential

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

$dJ = ddB = 0$ gives the topological half of Maxwell equations (vanishing of magnetic charges and Faraday's induction law) and self-duality $*J = J$ reduces the remaining equations to $dJ = 0$. Hence the Kähler form can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling).

The magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2}dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta \wedge d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical (or symplectic or Darboux) coordinates in which the Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= -\frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A40]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential

$exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the embedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A89] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2.2 CP_2 geometry and Standard Model symmetries

Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B53] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-2.23}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \otimes \gamma_5$, $1 \otimes \gamma_5$ and $\gamma_5 \otimes 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4)$ having as its covering group $SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-2.24}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ , \end{array} \tag{A-2.25}$$

and

$$B = 2re^3 \ , \tag{A-2.26}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-2.27}$$

where one have defined

$$\begin{array}{lll} I_L^1 & = & \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 & = & \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{array} \tag{A-2.28}$$

A_{ch} is clearly left handed so that one can perform the identification of the gauge potential as

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-2.29}$$

where $W^\pm$ denotes the charged intermediate vector boson.

The covariantly constant curvature tensor is given by

$$\begin{array}{l} R_{01} = -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} = -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\ R_{03} = 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , \\ R_{12} = 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-2.30}$$

The charged part of the curvature tensor is left handed.

This is to be compared with the Weyl tensor, which defines a representation of quaternionic imaginary units.

$$
\begin{array}{l}
W_{03} = W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
W_{01} = W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
W_{02} = W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ .
\end{array} \tag{A-2.31}
$$

The charged part of the Weyl tensor is right-handed and that the relative sign of the two terms in the curvature tensor and Weyl tensor are opposite.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$
\begin{array}{rcl}
X & = & re^3 \ , \\
Y & = & \frac{e^3}{r} \ ,
\end{array} \tag{A-2.32}
$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$
\begin{array}{rcl}
\bar{\gamma} & = & aX + bY \ , \\
\bar{Z}^0 & = & cX + dY \ ,
\end{array} \tag{A-2.33}
$$

where the normalization condition

$$
ad - bc = 1 \ ,
$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$
\begin{array}{rcl}
A_{nc} & = & [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\
& + & [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ .
\end{array} \tag{A-2.34}
$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$
c \ = \ -d \ . \tag{A-2.35}
$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$
A_{nc} \ = \ \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.36}
$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$
\begin{array}{rcl}
Q_{em} & = & \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\
I_L^3 & = & \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ .
\end{array} \tag{A-2.37}
$$

The fields γ and Z^0 are defined via the relations

$$
\begin{array}{rcl}
\gamma & = & 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\
Z^0 & = & 4(a+b)\bar{Z}^0 = 4(X - Y) \ .
\end{array} \tag{A-2.38}
$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} , \tag{A-2.39}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of the Weinberg angle is a dynamical problem. The original approach was based on the assumption that it makes sense to talk about electroweak action defined at fundamental level and introduce a symmetry breaking by adding an additional term proportional to Kähler action. The recent view is that Kähler action plus volume term defines the fundamental action.

The Weinberg angle is completely fixed if one requires that the electroweak action contains no cross term of type γZ^0. This leads to a definite value for the Weinberg angle.

One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle. As a matter fact, color gauge action identifying color gauge field as proportional to $H^A J_{\alpha\beta}$ is proportional to Kähler action. A possible interpretation would be as a sum of electroweak and color gauge interactions.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) , \tag{A-2.40}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) , \end{aligned} \tag{A-2.41}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) . \tag{A-2.42}$$

Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} , \\ Z^0 &= 2R_{03} . \end{aligned} \tag{A-2.43}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) . \tag{A-2.44}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) , \end{aligned} \tag{A-2.45}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$
\begin{aligned}
X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\
K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ ,
\end{aligned}
\tag{A-2.46}
$$

This parameter can be calculated by substituting the values of quark and lepton charges and weak isospins.

In the general case the value of the coefficient K is given by

$$
K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9}\right] \ , \tag{A-2.47}
$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i (18+n_i^2))} \ . \tag{A-2.48}
$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-2.49}
$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is not far from the typical value 9/24 of GUTs at high energies [B12]. The experimental value at the scale length scale of the electron can be deduced from the ratio of W and Z boson masses as $sin^2\theta_W = 1-(m_W/m_Z)^2 \simeq .22290$. This ratio and also the weak boson masses depend on the length scale.

If one interprets the additional term proportional to J as color action, one could perhaps interpret the value of Weinberg angle as expressing a connection between strong and weak coupling constant evolution. The limit $f \rightarrow 0$ should correspond to an infinite value of color coupling strength and at this limit one would have $sin^2\theta_W = \frac{9}{28}$ for $f/g^2 \rightarrow 0$. This does not make sense since the Weinberg angle is in the standard model much smaller in QCD scale Λ corresponding roughly to pion mass scale. The Weinberg angle is in principle predicted by the p-adic coupling constant evolution fixed by the number theoretical vision of TGD.

One could however have a sum of electroweak action, correction terms changing the value of Weinberg angle, and color action and coupling constant evolution could be understood in terms of the coupling parameters involved.

Electroweak symmetry breaking

One of the hardest challenges in the development of the TGD based view of weak symmetry breaking was the fact that classical field equations allow space-time surfaces with finite but arbitrarily large size. For a fixed space-time surface, the induced gauge fields, including classical weak fields, are long ranged. On the other hand, the large mass for weak bosons would require a short correlation length. How can one understand this together with the fact that a photon has a long correlation length?

In zero energy ontology quantum states are superpositions of space-time surfaces as analogs of almost unique Bohr orbits of particles identified as 3-D surfaces. For some reason the superposition should be such that the quantum averages of weak gauge boson fields vanish below the weak scale whereas the quantum average of electromagnetic fields is non-vanishing.

This is indeed the case.

1. The supersymplectic symmetries form isometries of the world of classical worlds (WCW) and they act in CP_2 degrees of freedom as symplectic transformations leaving the CP_2 symplectic form J invariant and therefore also its contribution to the electromagnetic field since this part is the same for all space-time surfaces in the superposition of space-time surfaces as a representation of supersymplectic isometry group (as a special case a representation of color group).

2. In TGD, color and electroweak symmetries acting as holonomies are not independent and for the $SU(2)_L$ part of induced spinor connection the symplectic transformations induces $SU(2)_L \times U(1)_R$ gauge transformation. This suggests that the quantum expectations of the induced weak fields over the space-time surfaces vanish above the quantum coherence scale. The averages of W and of the left handed part of Z^0 should therefore vanish.

3. $\langle Z^0 \rangle$ should vanish. For $U(1)_R$ part of Z^0, the action of gauge transformation is trivial in gauge theory. Now however the space-time surface changes under symplectic transformations and this could make the average of the right-handed part of Z^0 vanishing. The vanishing of the average of the axial part of the Z^0 is suggested by the partially conserved axial current hypothesis.

One can formulate this picture quantitatively.

1. The electromagnetic field [L170] contains, besides the induced Kähler form, also the induced curvature form R_{12}, which couples vectorially. Conserved vector current hypothesis suggests that the average of R_{12} is non-vanishing. One can express the neutral part of the induced gauge field in terms of induced spinor curvature and Kähler form J as

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) = J + 2e^0 \wedge e^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) = 3J - 2e^0 \wedge e^3 \quad , \end{aligned} \tag{A-2.50}$$

2. The induced fields γ and Z^0 (photon and Z- boson) can be expressed as

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} = 2(J + 2e^0 \wedge e^3) && \text{(A-2.51)} \\ per. & && \text{(A-2.52)} \end{aligned}$$

The condition $\langle Z^0 \rangle = 0$ gives $2\langle e^0 \wedge e^3 \rangle = -2J$ and this in turn gives $\langle R_{12} \rangle = 4J$. The average over γ would be

$$\langle \gamma \rangle = (3 - 4sin^2\theta_W)J \ .$$

For $sin^2\theta_W = 3/4 \ langle\gamma\rangle$ would vanish.

The quantum averages of classical weak fields quite generally vanish. What about correlation functions?

1. One expects that the correlators of classical weak fields as color invariants, and perhaps even symplectic invariants, are non-vanishing below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as hadron.

2. The intuitive picture is that in longer length scales one has disjoint 3-surfaces with a size scale of Compton length. If the states associated with two disjoint 3-surfaces are separately color invariant there are no correlations in color degrees of freedom and correlators reduce to the products of expectations of classical weak fields and vanish. This could also hold when the 3-surfaces are connected by flux tube bonds.

   Below the Compton length weak bosons would thus behave as correlated massless fields. The Compton lengths of weak bosons are proportional to the value of effective Planck constant h_{eff} and in living systems the Compton lengths are proposed to be even of the order of cell size. This would explain the mysterious chiral selection in living systems requiring large parity violation.

3. What about the averages and correlators of color gauge fields? Classical color gauge fields are proportional to the products of Hamiltonians of color isometries induced Kähler form and the expectations of color Hamiltonians give vanishing average above Compton length and therefore vanishing average. Correlators are non-vanishing below the hadron scale. Gluons do not propagate in long scales for the same reason as weak bosons. This is implied by color confinement, which has also classical description in the sense that 3-surfaces have necessarily a finite size.

   A large value of h_{eff} allows colored states even in biological scales below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as dark hadron.

Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.

2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B17] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-2.53}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-2.54}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-2.55}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-3.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has embedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if embedding space at their image points. In the recent case the embedding of space-time surface to embedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the embedding space to the space-time surface (see `http://tgdtheory.fi/appfigures/induct.jpg`).

Induction procedure makes sense also for the spinor fields of embedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with embedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://tgdtheory.fi/appfigures/induct.jpg`.

A-3.2 Induced gauge fields for space-times for which CP$_2$ projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP$_2$ projection, only vacuum extremals and space-time surfaces for which CP$_2$ projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP$_2$ projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP$_2$ projection color rotations and weak symmetries commute.

A-3.3 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four embedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of embedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-3.4 Embedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

2. Spinor harmonics of embedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of embedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the embedding space spinor connection carries W gauge potentials one can say that the embedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the

spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-3.5 About induced gauge fields

In the following the induced gauge fields are studied for general space-time surface without assuming the preferred extremal property (Bohr orbit property). Therefore the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-3.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3 + 2p)\frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3 + p)\frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.2}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3 + 2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3 + p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-3.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned}
r &= \sqrt{\frac{X}{1-X}} \ , \\
X &= D\left[|\frac{k+u}{C}|\right]^{\epsilon} \ , \\
u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ ,
\end{aligned} \tag{A-3.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned}
J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\
Z^0 &= -\frac{6}{p} J \ .
\end{aligned} \tag{A-3.5}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2} Z^0 \ .
\end{aligned} \tag{A-3.6}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ ,
\end{aligned} \tag{A-3.7}$$

and is useful in the construction of vacuum embedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned}
\Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\
\Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ .
\end{aligned} \tag{A-3.8}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global embedding for, say a constant magnetic field. Although global embedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \tag{A-3.9}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-4 The relationship of TGD to QFT and string models

The recent view of the relationship of TGD to QFT and string models has developed slowly during years and it seems that in a certain sense TGD means a return to roots: instead of QFT like description involving path integral one would have wave mechanics for 3-surfaces.

A-4.1 TGD as a generalization of wave mechanism obtained by replacing point-like particles with 3-surfaces

The first vision of TGD was as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

The later work has revealed that TGD could be seen as a generalization of the wave mechanism based on the replacement of a point-like particle with 3-D surface. This is due to holography implied by general coordinate invariance. The definition of the metric of the "world of classical worlds" (WCW) must assign a unique or at least almost unique space-time surface to a given 3-surface. This 4-surface is analogous to Bohr orbit so that also Bohr orbitology becomes an exact part of quantum physics. The failure of strict determinism forces to replace 3-surfaces with 4-surfaces and this leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K57, K29, K96] [L141, L157].

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

A-4.2 Extension of superconformal invariance

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in a completely unique position as far as symmetries are considered.

In fact, this leads to a generalization of the Kac-Moody type symmetries of string models. $\delta M^4_+ \times CP_2$ allows huge supersymplectic symmetries for which the radial light-like coordinate of δM^4_+ plays the role of complex string coordinate in string models. These symmetries are assumed to act as isometries of WCW.

A-4.3 String-like objects and strings

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of the TGD Universe and the inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string-like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situations in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

A-4.4 TGD view of elementary particles

The TGD based view about elementary particles has two key aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidean signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

2. At the level of H Fermion number is carried by the modes of the induced spinor field. In space-time regions with Minkowski signature the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle orbit corresponds to a 4-D generalization of a world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidean signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at the same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having a hadronic string as a physical counterpart. Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.

3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particles along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K57, K96].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

A-5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [L55] [L141, L147, L148] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and

$T(CP_2)$ of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A63] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.

2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.

3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

A-5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.

2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L157]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

   The super symplectic algebra A has an infinite hierarchy of sub-algebras [L157] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

   This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.

2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L157] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by M^8-H duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.

2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L157] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P) = n(braid) = n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.

3. The actions related to different values of $n(P) = n(braid) = n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.

4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L151] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i = 2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r = 1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

   A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.

5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p \simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K71, K72]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L151], and the forward scattering amplitude corresponds to the propagator.

   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L151], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L151].

3. p-Adic length scale hypothesis [L158] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

   Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L157] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K57, K29]. As isometries they would naturally permute the maxima with each other.

A-6 Number theoretic vision of TGD

Physics as number theory vision is complementary to the physics as geometry vision and has developed gradually since 1993. Langlands program is the counterpart of this vision in mathematics [L154].

The notion of p-adic number fields emerged with the motivation coming from the observation that elementary particle mass scales and mass ratios could be understood in terms of the so-called p-adic length scale hypothesis [K76, K65, K26]. The fusion of the various p-adic physics leads to what I call adelic physics [L54, L53]. Later the hypothesis about hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter emerged [K33, K34, K35, K35].

Eventually this led to that the values of effective Planck constant could be identified as the dimension of an algebraic extension of rationals assignable to polynomials with rational coefficients. This led to the number theoretic vision in which so-called M^8-H duality [L109, L110] plays a key role. M^8 (actually a complexification of real M^8) is analogous to momentum space so that the duality generalizes momentum position duality for point-like particles. M^8 has an interpretation as complexified octonions.

The dynamics of 4-surfaces in M^8 is coded by polynomials with rational coefficients, whose roots define mass shells H^3 of $M^4 \subset M^8$. It has turned out that the polynomials satisfy stringent additional conditions and one can speak of number theoretic holography [L151, L154]. Also the ordinary 3$\rightarrow$ 4 holography is needed to assign 4-surfaces with these 3-D mass shells. The number theoretic dynamics is based on the condition that the normal space of the 4-surface in M^8 is associative (quaternionic) and contains a commutative complex sub-space. This makes it possible to assign to this surface space-time surface in $H = M^4 \times CP_2$.

At the level of H the space-time surfaces are by holography preferred extremals and are assumed to be determined by the twistor lift of TGD [L55] giving rise to an action which is sum of the Kähler action and volume term. The preferred extremals would be minimal surfaces analogous to soap films spanned by frames. Outside frames they would be simultaneous extremals of the Kähler action, which requires a generalization of the holomorphy characterizing string world sheets.

In the following only p-adic numbers and hierarchy of Planck constants will be discussed.

A-6.1 p-Adic numbers and TGD

p-Adic number fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A37]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k, \ x(k) = 0, ..., p-1 \ . \qquad \text{(A-6.1)}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) , \tag{A-6.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} . \tag{A-6.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D . \tag{A-6.5}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

2. Distances of points x and y inside single class are smaller than distances between different classes.

3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B39]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

1. Basic form of the canonical identification

There exists a natural continuous map $I : R_p \to R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \to x = \sum_{k<N} y_k p^{-k} , \\ y_k &\in \{0, 1, .., p-1\} . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1} \sum_{k=0,..} p^{k} \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

2. The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p|y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \tag{A-6.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p|y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-6.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

3. Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

4. Generalization of number concept and notion of embedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic embedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real embedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real embedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, “thought bubbles”.

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification violates general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic embedding space with chart maps to real embedding space and assuming preferred coordinates made possible by isometries of embedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-6.2 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the “impossible” quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale’s finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the embedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of embedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of embedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-6.3 $M^8 - H$ duality as it is towards the end of 2021

The view of $M^8 - H$ duality (see Appendix 8.5.6) has changed considerably towards the end 2021 [L141] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L141] of the M^8-H duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L141] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L109, L110]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$.

This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L81]. This notion does not make sense at the level M^8 anymore.

The modified $M^8 - H$ duality forces us to modify the original interpretation [L141]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L128] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L141]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

A-7 Zero energy ontology (ZEO)

ZEO is implied by the holography forced in the TGD framework by general coordinate invariance.

A-7.1 Basic motivations and ideas of ZEO

The following gives a brief summary of ZEO [L91] [K130].

1. In ZEO quantum states are not 3-dimensional but superpositions of 4-dimensional deterministic time evolutions connecting ordinary initial 3-dimensional states. By holography they are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution. One can say that in the TGD framework general coordinate invariance implies holography and the slight failure of its determinism in turn forces ZEO.

   Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced with a new superposition. Classical determinism of individual time evolution is not violated and this solves the basic paradox of quantum measurement theory. There are two kinds of quantum jumps: ordinary ("big") state function reductions (BSFRs) changing the arrow of time and "small" state function reductions (SSFRs) (weak measurements) preserving it and giving rise to the analog of Zeno effect [L91].

2. To avoid getting totally confused it is good to emphasize some aspects of ZEO.

   (a) ZEO does not mean that physical states in the usual 3-D sense as snapshots of time evolution would have zero energy state pairs defining zero energy states as initial and final states have same conserved quantities such as energy. Conservation implies that one can adopt the conventions that the values of conserved quantities are opposite for these states so that their sum vanishes: one can think that incoming and outgoing particles come from geometric past and future is the picture used in quantum field theories.

   (b) ZEO means two times: subjective time as sequence of quantum jumps and geometric time as space-time coordinate. These times are identifiable but are strongly correlated.

3. In BSFRs the arrow of time is changed and the time evolution in the final state occurs backwards with respect to the time of the external observer. BSFRs can occur in all scales since TGD predicts a hierarchy of effective Planck constants with arbitrarily large values. There is empirical support for BSFRs.

   (a) The findings of Minev et al [L77] in atomic scale can be explained by the same mechanism [L77]. In BSFR a final zero energy state as a superposition of classical deterministic time evolutions emerges and for an observer with a standard arrow of time looks like a superposition of deterministic smooth time evolutions leading to the final state. Interestingly, once this evolution has started, it cannot be stopped unless one changes the stimulus signal inducing the evolution in which case the process does not lead to anywhere: the interpretation would be that BSFR back to the initial state occurs!

(b) Libets' experiments about active aspects of consciousness [J2] can be understood. Subject person raises his finger and neural activity starts before the conscious decision to do so. In the physicalistic framework it is thought to lead to raising of the finger. The problem with the explanation is that the activity beginning .5 seconds earlier seems to be dissipation with a reversed arrow of time: from chaotic and disordered to ordered at around .15 seconds. ZEO explanation is that macroscopic quantum jump occurred and generated a signal proceeding backwards in time and generated neural activity and dissipated to randomness.

(c) Earthquakes involve a strange anomaly: they are preceded by ELF radiation. One would expect that they generate ELF radiation. The identification as BSFR would explain the anomaly [L80]. In biology the reversal of the arrow of time would occur routinely and be a central element of biological self-organization, in particular self-organized quantum criticality (see [L86, L177].

A-7.2 Some implications of ZEO

ZEO has profound implications for understanding self-organization and self-organized quantum criticality in terms of dissipation with non-standard arrow of time looking like generation of structures [L86, L177]. ZEO could also allow understanding of what planned actions - like realizing the experiment under consideration - could be.

1. Second law in the standard sense does not favor - perhaps even not allow - realization of planned actions. ZEO forces a generalization of thermodynamics: dissipation with a non-standard arrow of time for a subsystem would look like self-organization and planned action and its realization.

   Could most if not all planned action be like this - induced by BSFR in the geometric future and only apparently planned? There would be however the experience of planning and realizing induced by the signals from geometric future by a higher level in the hierarchy of conscious entities predicted by TGD! In long time scales we would be realizing our fates or wishes of higher level conscious entities rather than agents with completely free will.

2. The notion of magnetic body (MB) serving as a boss of ordinary matter would be central. MB carries dark matter as $h_{eff} = nh_0$ phases of ordinary matter with n serving as a measure for algebraic complexity of extension of rationals as its dimension and defining a kind of universal IQ. There is a hierarchy of these phases and MBs labelled by extension of rationals and the value of n.

   MBs would form a hierarchy of bosses - a realization for master slave hierarchy. Ordinary matter would be at the bottom and its coherent behavior would be induced from quantum coherence at higher levels. BSFR for higher level MB would give rise to what looks like planned actions and experienced as planned action at the lower levels of hierarchy. One could speak of planned actions inducing a cascade of planned actions in shorter time scales and eventually proceeding to atomic level.

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. `http://tgdtheory.fi/appfigures/reconect2.jpg`

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. `http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg`

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. `http://tgdtheory.fi/appfigures/cat.jpg`

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred embedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. http://tgdtheory.fi/appfigures/padictoreal.jpg

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. http://tgdtheory.fi/appfigures/sharing.jpg

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** http://tgdtheory.fi/appfigures/timemirror.jpg or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://tgdtheory.fi/appfigures/timemirror.jpg

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