The holography= holomorphy vision has led also to a dramatic progress in the understanding of Dirac equation in the TGD framework, in particular its variant involving M^4 Kähler form. This article summarizes the latest discoveries.

The first topic of this article is Hamilton-Jacobi $(H-J)$ structure, in particular the its hyper-complex aspects. The physical implications of M^4 Kähler structure are discussed and the classification of Hamilton-Jacobi structures at the level of $H=M^4\times CP_2$ and of space-time surfaces are discussed.

The main topic of the article are the solutions with non-trivial M^4 Kähler structure.

1. The mass squared spectrum of the M^4 Dirac operator turns out to be integer valued harmonic oscillator spectrum, being therefore "stringy". Mass squared has also negative tachyonic values.

   By 8-D masslessness, M^4 mass squared spectrum must be identical with CP_2 mass squared spectrum given by eigenvalues of $SU(3)$ Casimir operator with a spin term added. For the color partial waves, the Casimir operator is always integer valued whereas the spin term, which is proportional to the Kähler coupling q_K need not be integer valued. For CP_2, the ratio of Kähler couplings q_K of leptons and quarks is 3.

2. The condition that the M^4 and CP_2 mass squared values are identical has strong implications. For both $q_K(L,M^4)/q_K(q,M^4)=1$ and $q_K(L,M^4)/q_K(q,M^4)=3$, the M^4 - and CP_2 mass squared values can be identical if M^4 - and CP_2 chiralities are identical. For leptons this is a natural assumption. For quarks for which M^4 - and CP_2 chiralities would be opposite, and the spin terms for M^4- and CP_2 are necessarily different so that M^4- and CP_2 mass square cannot be identical.

3. There are several problems to be solved. How to obtain color triplet states for quarks and color singlets for leptons and weak screening? Is this possible at all for quarks? How to obtain massless states? Could the tachyonic mass spectrum of the M^4 Dirac operator help here? Is it possible to obtain massless quarks or are only many- quark states (baryons) for which the sums of M^4 and CP_2 mass squared values are identical.

   The possibility of M^4 tachyons leads to a proposal that, apart from the p-adic thermodynamic contribution to the mass squared, physical states are massless and that the addition of tachyonic fermion pairs to the states guarantees this. In the model proposed earlier, they would be pairs of left-handed and right-handed neutrinos, inducing a weak screening and guaranteeing that leptons/quarks correspond to color singlets/triplets. The different values of the spin terms for quarks and leptons however imply that this mechanism requires that the total number of quarks and antiquarks is a multiple of 3! In particular, massless bosons can be either lepton pairs or superposition of lepton pairs and baryon pairs. This condition also implies that the sum of spin terms for quarks is integer valued and the total M^4 and CP_2 contributions to the mass squared can be identical.

4. Also the modified Dirac equation for the induced spinors at the space-time surface in the special case $X^4=M^4$ is discussed for non-trivial Kähler structure. The solutions are holomorphic or antiholomorphic in the generalized sense. Holomorphic modes are possible also for the H Dirac operator and required by the induction of the spinor structure. Holomorphic massless modes are obtained only for the second M^4 chirality: this conforms with the intuitive view about massless fermions. This means a new kind of parity violation. The gauge theory analog of this phase would be a phase with a

vanishing value of the Higgs field.

The third topic are elliptic curves, which correspond to 2-D lattices consisting of parallelograms. These kinds of lattices are very interesting since the 2 periods could correspond to wavelengths. The periodicity of the function pair (f_1, f_2) with respect to the hypercomplex coordinate u provides the third period. There are however 4 momentum components: could the hypercomplex sector also allow a second period? Space-time surfaces are located inside causal diamond (CD) in zero energy ontology (ZEO): could the size scale of CD provide the other period? Also various realizations of the lattice structure are discussed.