

This article represents a critical re-examination of  $M^8-H$  duality, which is one of the cornerstones of Topological Geometro-dynamics (TGD). The original version of  $M^8-H$  duality assumed that space-time surfaces in  $M^8$  can be identified as associative or co-associative surfaces. If the surface has associative tangent or normal space and contains a complex or co-complex surface, it can be mapped to a 4-surface in  $H=M^4 \times CP_2$ .

Later emerged the idea that octonionic analyticity realized in terms of real polynomials  $P$  algebraically continued to polynomials of complexified octonion could fulfill the dream. The vanishing of the real part  $Re_Q(P)$  (imaginary part  $Im_Q(P)$ ) in the quaternionic sense would give rise to an associative (co-associative) space-time surface.

The realization of the general coordinate invariance motivated the notion of strong form of holography (SH) in  $H$  allowing realization of a weaker form of  $M^8-H$  duality by assuming that associativity/co-associativity conditions are needed only at 2-D string world sheet and partonic 2-surfaces and possibly also at their light-like 3-orbits.

The outcome of the re-examination was a positive surprise. Although no interesting associative space-time surfaces are possible, every distribution of normal associative planes (co-associativity) is integrable. Another positive surprise was that Minkowski signature is the only possible option. Equivalently, the image of  $M^4$  as real co-associative sub-space of  $O_c$  (complex valued octonion norm squared is real valued for them) by an element of local  $G_{2,c}$  or its subgroup  $SU(3,c)$  gives a real co-associative space-time surface. The conjecture is that the polynomials  $P$  determine these surfaces as roots of  $Re_Q(P)$ . These surfaces also possess co-complex 2-D sub-manifolds allowing the mapping to  $H$  to  $H$  by  $M^8-H$  duality as a whole. SH would not be needed and would be replaced with number theoretic holography determining space-time surface from its roots and selection of real subspace of  $O_c$  characterizing the state of motion of a particle.

Octonionic Dirac equation as analog of momentum space variant of ordinary Dirac equation forces the interpretation of  $M^8$  as an analog of momentum space and Uncertainty Principle forces to modify the map  $M^4 \subset M^8 \rightarrow M^4 \subset H$  from identification to inversion. The equations for  $Re_Q(P)=0$  reduce to simultaneous roots of the real polynomials defined by the odd and even parts of  $P$  having interpretation as complex values of mass squared mapped to light-cone proper time constant surfaces in  $H$ . This leads to the idea that the formulation of scattering amplitudes at  $M^8$  levels provides the counterpart of momentum space description of scattering whereas the formulation at the level of

$H$  provides the counterpart of space-time description

This picture combined with zero energy ontology leads also to a view about quantum TGD at the level of  $M^8$ . Local  $G_{2,c}$  element has properties suggesting a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by  $P$ . The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

The new view about  $M^8-H$  duality differs from the earlier one rather dramatically so that a summary of the differences is added to the end of paper.