In this chapter I try to formulate more precisely the recent TGD based view about fractional quantum Hall effect (FQHE). This view is much more realistic than the original rough scenario, which neglected the existing rather detailed understanding. The spectrum of ν, and the mechanism producing it is the same as in composite fermion approach. The new elements relate to the not so well-understood aspects of FQHE, namely charge fractionization, the emergence of braid statistics, and non-abelianity of braid statistics.

\begin{enumerate}

\item The starting point is composite fermion model so that the basic predictions are same. Now magnetic vortices correspond to (K\"ahler) magnetic flux tubes carrying unit of magnetic flux. The magnetic field inside flux tube would be created by delocalized electron at the boundary of the vortex. One can raise two questions.

Could the boundary of the macroscopic system carrying anyonic phase have identification as a macroscopic analog of partonic 2-surface serving as a boundary between Minkowskian and Euclidian regions of space-time sheet? If so, the space-time sheet assignable to the macroscopic system in question would have Euclidian signature, and would be analogous to blackhole or to a line of generalized Feynman diagram.

Could the boundary of the vortex be identifiable a light-like boundary separating Minkowskian magnetic flux tube from the Euclidian interior of the macroscopic system and be also analogous to wormhole throat? If so, both macroscopic objects and magnetic vortices would be rather exotic geometric objects not possible in general relativity framework.

\item Taking composite model as a starting point one obtains standard predictions for the filling fractions. One should also understand charge

fractionalization and fractional braiding statistics. Here the vacuum
degeneracy of K\"ahler action suggests the explanation. Vacuum degeneracy
implies that the correspondence between the normal component of the
canonical momentum current and normal derivatives of imbedding space
coordinates is 1- to-n. These kind of branchings result in
multi-furcations induced by variations of the system parameters and the
scaling of external magnetic field represents one such variation.

\item At the orbits of wormhole throats, which can have even macroscopic
M^4 projections, one has $1\rightarrow n_a$ correspondence and at the
space-like ends of the space-time surface at light-like boundaries of
causal diamond one has $1\rightarrow n_b$ correspondence. This implies that
at partonic 2-surfaces defined as the intersections of these two kinds of
3-surfaces one has $1\rightarrow n_a\times n_b$ correspondence. This
correspondence can be described by using a local singular n-fold covering
of the imbedding space. Unlike in the original approach, the covering
space is only a convenient auxiliary tool rather than fundamental notion.

\item The fractionalization of charge can be understood as follows. A
delocalization of electron charge to the n sheets of the multi-furcation
takes place and single sheet is analogous to a sheet of Riemann surface
of function $z^{1/n}$ and carries fractional charge $q=e/n$, $n=n_an_b$.
Fractionalization applies also to other quantum numbers. One can have also
many-electron stats of these states with several delocalized electrons: in
this case one obtains more general charge fractionalization: $q= \nu e$.

\item Also the fractional braid statistics can be understood. For
ordinary statistics rotations of M^4 rotate entire partonic 2-surfaces.
For braid statistics rotations of M^4 (and particle exchange) induce a
flow braid ends along partonic 2-surface. If the singular local
covering is analogous to the Riemann surface of $z^{1/n}$, the

braid rotation by $\Delta \Phi=2\pi$, where Φ corresponds to M^4 angle, leads to a second branch of multi-furcation and one can give up the usual quantization condition for angular momentum. For the natural angle coordinate Φ of the n-branched covering $\Delta \Phi=2/pi$ corresponds to $\Delta \Phi=n\times 2\pi$. If one identifies the sheets of multi-furcation and therefore uses Φ as angle coordinate, single valued angular momentum eigenstates become in general n-valued, angular momentum in braid statistics becomes fractional and one obtains fractional braid statistics for angular momentum.

\item How to understand the exceptional values $\nu=5/2,7/2$ of the filling fraction? The non- abelian braid group representations can be interpreted as higher-dimensional projective representations of permutation group: for ordinary statistics only Abelian representations are possible. It seems that the minimum number of braids is $n>2$ from the condition of non-abelianity of braid group representations. The condition that ordinary statistics is fermionic, gives $n>3$. The minimum value is $n=4$ consistent with the fractional charge $e/4$.

The model introduces Z_4 valued topological quantum number characterizing flux tubes. This also makes possible non-Abelian braid statistics. The interpretation of this quantum number as a Z_4 valued momentum characterizing the four delocalized states of the flux tube at the sheets of the 4-furcation suggests itself strongly. Topology would corresponds to that of 4-fold covering space of imbedding space serving as a convenient auxiliary tool. The more standard explanation is that $Z_4=Z_2\times Z_2$ such that Z_2:s correspond to the presence or absence of neutral Majorana fermion in the two Cooper pair like states formed by flux tubes.

What remains to be understood is the emergence of non-abelian gauge group realizing non-Abelian fractional statistics in gauge theory framework. Electroweak gauge group defined non-abelian braid group

in large
h_{eff} phase weak length above atomic length scale so that weak bosons
and even fermion behave as effectively massless particles below scaled up
weak scale. TGD also predicts the possibility of dynamical gauge groups
and maybe this kind of gauge group indeed emerges. Dynamical gauge groups
emerge also for stacks of N branes and the n sheets of multifurcation
are analogous to the N sheets in the stack for many-electron states.
\end{enumerate}