

In the TGD Universe space-time surfaces within causal diamonds (CDs) are fundamental objects.

1.  $M^8 - H$  duality means that one can interpret the space-time surfaces in two manners: either as an algebraic surface in complexified  $M^8$  or as minimal surfaces in  $H = M^4 \times CP_2$ .  $M^8 - H$  duality maps these surfaces to each other.
2. Minimal surface property holds true outside the frame spanning minimal surface as 4-D soap film and since also extremal of Kähler action is in question, the surface is analog of complex surface. The frame is fixed at the boundaries of the CD and dynamically generated in its interior. At frame the isometry currents of volume term and Kähler action have infinite divergences which however cancel so that conservation laws coded by field equations are true. The frames serve as seats of non-determinism.
3. At the level of  $M^8$  the frames correspond to singularities of the space-time surface. The quaternionic normal space is not unique at the points of a  $d$ -dimensional singularity and their union defines a surface of  $CP_2$  of dimension  $d_c = 4 - D < d$  defining in  $H$  a blow up of dimension  $d_c$ .

In this article, the inspiration provided by 2-D minimal surfaces is used to deepen the TGD view about space-time as a minimal surface and also about  $M^8 - H$  duality and TGD itself.

1. The properties of 2-D minimal surfaces encourage the inclusion of the phase with a vanishing cosmological constant  $\Lambda$  phase. This forces the extension of the category of real polynomials determining the space-time surface at the level of  $M^8$  to that of real analytic functions. The interpretation in the framework of consciousness theory would be as a kind of mathematical enlightenment, transcendence also in the mathematical sense.
2.  $\Lambda > 0$  phases associated with real polynomials as approximations of real analytic functions would correspond to a hierarchy of inclusions of hyperfinite-factors of type  $II_1$  realized as physical systems and giving rise to finite cognition based on finite-D extensions of rationals and corresponding extensions of p-adic number fields.
3. The construction of 2-D periodic minimal surfaces inspires a construction of minimal surfaces with a temporal periodicity. For  $\Lambda > 0$  this happens by gluing copies of minimal surface and its mirror image together and for  $\Lambda = 0$  by using a periodic frame.

A more general engineering construction using different basic pieces fitting together like legos gives rise to a model of logical thinking with thoughts as legos. This also allows an improved understanding of how  $M^8 - H$  duality manages to be consistent with the Uncertainty Principle (UP).

4. At the physical level, one gains a deeper understanding of the space-time correlates of particle massivation and of the TGD counterparts of twistor diagrams. Twistor lift predicts  $M^4$  Kähler action and its Chern-Simons implying CP breaking. This part is necessary in order to have particles with non-vanishing momentum in the  $\Lambda = 0$  phase.