

This chapter represents the most recent view about elementary particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters. In the following my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p -adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p -adic thermodynamics combined with p -adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S -matrix to what I call M -matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p -adic coupling constant evolution and p -adic length scale hypothesis from the first principles, the realization that the counterpart of Higgs mechanism involves generalized eigenvalues of the Kähler-Dirac operator: these are represent important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p -adic thermodynamics are not affected.

Since 2010 a further progress took place. These steps of progress relate closely to ZEO, bosonic emergence, the discovery of the weak form of electric-magnetic duality, the realization of the importance of twistors in TGD, and the discovery that the well-definedness of

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charge forces the modes of Kähler-Dirac operator to 2-D surfaces –
string
world sheets and possibly also partonic 2-surfaces. This allows to
assign
to elementary particle closed string with pieces at two parallel
space-time
sheets and accompanying a Kähler magnetic flux tube carrying
monopole
flux.

Twistor approach and the understanding of the solutions of
Kähler-Dirac
Dirac operator served as a midwife in the process giving rise to the
birth
of the idea that all fundamental fermions are massless and that
both
ordinary elementary particles and string like objects emerge from
them.
Even more, one can interpret virtual particles as being composed
of
these massless on mass shell particles assignable to wormhole
throats.
Four-momentum conservation poses extremely powerful constraints on
loop
integrals but does not make them manifestly finite as believed
first.
String picture is necessary for getting rid of logarithmic
divergences.

The weak form of electric-magnetic duality led to the realization
that
elementary particles correspond to bound states of two wormhole
throats
with opposite Kähler magnetic charges with second throat carrying
weak
isospin compensating that of the fermion state at second wormhole
throat.
Both fermions and bosons correspond to wormhole contacts: in the
case of
fermions topological condensation generates the second wormhole
throat.
This means that altogether four wormhole throats are involved with
both
fermions, gauge bosons, and gravitons (for gravitons this is
unavoidable in
any case). For p-adic thermodynamics the mathematical counterpart
of
string corresponds to a wormhole contact with size of order CP_2
size
with the role of its ends played by wormhole throats at which the
signature of the induced 4-metric changes. The key observation is
that for
massless states the throats of spin 1 particle must have opposite

three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N}=4$ SYM. At the level of effective space-time assigned to many-sheeted space-time this symmetry is broken. Besides this there is weak `{stringy}` contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets.

`{it 1. Physical states as representations of super-symplectic and Super Kac-Moody algebras}`

The basic constraint is that the super-conformal algebra involved must have five tensor factors. The precise identification of the Kac-Moody type algebra has however turned out to be a surprisingly difficult task. The latest view is as follows. Electroweak algebra $SU(2)_{ew} = SU(2)_L \times U(1)$ and symplectic isometries of light-cone boundary ($SU(2)_{rot} \times SU(3)_c$) give 2+2 factors and full supersymplectic algebra involving only covariantly constant right-handed neutrino mode would give 1 factor. This algebra could be associated with the 2-D surfaces X^2 defined by the intersections of light-like 3-surfaces with $\delta M^4_{\pm} \times CP_2$. These 2-surfaces have interpretation as partons.

For conformal algebra there are several candidates. For symplectic algebra radial light-like coordinate of light-cone boundary replaces complex coordinate. Light-cone boundary $S^2 \times R_+$ allows extended conformal symmetries which can be interpreted as conformal transformations of S^2 depending parametrically on the light-like coordinate of R_+ . There is infinite-D subgroup of conformal isometries with S^2 dependent radial scaling compensating for the conformal scaling in S^2 . Kähler-Dirac equation allows ordinary conformal symmetry very probably liftable to

imbedding space. The light-like orbits of partonic 2-surface are expected to allow super-conformal symmetries presumably assignable to quantum criticality and hierarchy of Planck constants. How these conformal symmetries integrate to what is expected to be 4-D analog of 2-D conformal symmetries remains to be understood.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams. The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces.

\vm{\it 2. Particle massivation}\vm

Particle massivation can be regarded as a generation of thermal mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The obvious objection is that Poincare invariance is lost. One could argue that one calculates just the vacuum expectation of conformal weight so that this is not case. If this is not assumed, one would have in positive energy ontology superposition of ordinary quantum states with different four-momenta and breaking of Poincare invariance since eigenstates of four-momentum are not in question. In Zero Energy Ontology this is not the case since all states have vanishing net quantum numbers and one has superposition of time evolutions with well-defined four-momenta. Lorentz invariance with respect to the either boundary of CD is achieved but there is small breaking of Poincare invariance characterized by the inverse of p-adic prime p characterizing the particle. For electron one has $1/p=1/M_{127}\sim$

10^{-38} .

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

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`\item` Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to \hbar so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0 !)). What basically matters is the number of tensor factors involved and five is the favored number.

`\item` There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

`\item` A natural identification of the non-integer contribution to the conformal weight is as stringy contributions to the vacuum conformal weight (strings are now `\blockquote{weak strings}`). TGD predicts Higgs particle and Higgs is necessary to give longitudinal polarizations for gauge bosons. The notion of Higgs vacuum expectation seems to be replaced by an analog of Higgs vacuum expectation which gives space-time correlate for the stringy mass

formula in case of fundamental fermions. Also gauge bosons usually regarded as exactly massless particles would naturally receive small mass from p-adic thermodynamics. The theoretical motivation for a small mass would be exact Yangian symmetry which broken at the QFT limit of the theory using GRT limit of many-sheeted space-time.

\item Hadron massivation requires the understanding of the CKM mixing of quarks reducing to different topological mixing of U and D type quarks. Number theoretic vision suggests that the mixing matrices are rational or algebraic and this together with other constraints gives strong constraints on both mixing and masses of the mixed quarks.

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p-Adic thermodynamics is what gives to this approach its predictive power.

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\item p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $\exp(-E/kT)$ is replaced with p^{L_0/T_p} , $1/T_p$ integer) and fermions correspond to $T_p=1$ whereas $T_p=1/n$, $n>1$, seems to be the only reasonable choice for gauge bosons.

\item p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5} \sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

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