

Cognitive representations are the basic topic of the third chapter related to  $M^8-H$  duality. Cognitive representations are identified as sets of points in extension of rationals for algebraic varieties with "active" points containing fermion. The representations are discussed at both  $M^8$ - and  $H$  level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces.

The notion is applied in various cases and the connection with  $M^8-H$  duality is rather loose.

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`\item` Extensions of rationals are essentially coders of information. There the possible analogy of extensions of rationals with genes deserves discussion. Extensions, which are not extensions of extensions would be analogous to genes. The notion of conserved gene as number theoretical analogy for Galois extensions as the Galois group of extension which is normal subgroup of Galois extension.

`\item` The possible physical meaning of the notion of perfectoid introduced by Peter Scholze is discussed in the framework of  $p$ -adic physics. Extensions of  $p$ -adic numbers involving roots of the prime defining the extension are involved and are not considered previously in TGD framework. There their possible physical meaning deserves discussion.

`\item` The construction of cognitive representation reduces to a well-known mathematical problem of finding the points of space-time surface with imbedding space coordinates in given extension of rationals. The work of Kim and Coates represents new ideas in this respect and there is a natural connection with TGD.

`\item` One expects that large cognitive representations are winners in the number theoretical fight for survival. Strong form of holography suggests that it is enough to consider cognitive representations restricted to string world sheets and partonic 2-surfaces. If the 2-surface possesses large group of symmetries acting in extension of rationals, one can have large cognitive representations as orbit of point in extension. Examples of highly symmetric 2-D surfaces are geodesic spheres assignable to partonic 2-surfaces and cosmic strings and elliptic curves assignable with string world sheets and cosmic strings.

`\item` Rationals and their extensions give rise to a unique discretizations of space-time surface (for instance) – cognitive representation – having interpretation in terms of finite measurement resolution. There are however many open questions. Should one allow only octonionic polynomials defined as algebraic continuations of real polynomials or should one allow also analytic functions and regard polynomials as approximations. Zeta functions

are especially interesting analytic functions and Defekind zetas characterize extensions of rationals and one can pose physically motivated questions about them.

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