$M^8 - H$ duality *viz.* Hubble law, and gravitational Planck constant *viz.* Allais effect and warping

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Abstract

In this article three developments in TGD are discussed. These developments emerged in a "wrong context".

The first discovery was the realization that M^8-H duality in zero energy ontology implies a fractal generalization of Hubble's law. It emerged in a work related to TGD inspired quantum biology, where gravitational Planck constant $h_{gr} = GMm/\beta_0$ (M and m are masses and β_0 is velocity parameter) introduced by Nottale plays a key role. This already also led to a partial understanding of Hubble tension: the value of β in cosmic scales is very near to 1 but its values differ slightly in long and short cosmic scales.

The second discovery emerged while developing a TGD based model for Allais effect, which has no explanation in the context of general relativity or Newtonian gravitation. The work led to the question concerning the interpretation of the velocity parameter β_0 appearing in the formula of the gravitational Planck constant h_{gr} introduced already by Nottale. The gravitational field bodies of the Earth and structures of cosmological size scale are characterized by $\beta_0 = v_0/c \simeq 1$ and cannot correspond to a velocity of matter for a matter flow. Solar system characterized by $\beta_0 \simeq 2^{-11}$, which cannot correspond to light-velocity in the general relativistic framework.

The solution of the problem came from a prediction of TGD, which emerged already 47 years ago during the first year of TGD. TGD predicts warped space-time surfaces which are flat like Minkowski space M^4 (no gravitation) and have vanishing gauge fields. They have Poincare symmetry but have reduced light-velocity due to the warping. Light-like geodesics of M^4 are replaced by light-like geodesics of $M^4 \times S^1 \subset H = M^4 \times CP_2$. Warped space-time surface can explain the Allais effect, in particular the dramatic reduction of the parabolic oscillator frequency in terms of reduced light velocity. The notion of warping generalizes: one can speak of warped Hamilton-Jacobi (H-J) structures for which the hypercomplex structure of M^4 and complex structure of CP_2 are mixed in the sense that M^4 H-J structure is replaced with that of a warped space-time surface.

The third development led to a more detailed understanding of how the CP$_2$ type extremals emerge from the H-H principle, which for ordinary H-J structure allows only Minkowskian space-time regions. The earlier view about CP_2 type extremals as blow-ups finds a nice description in terms of H-H principle and twisted H-J structure made possible by the warping of M^4 as tilting towards $M^4 \times S^1$, where S^1 is a geodesic circle of CP$_2$.

1 Introduction

In this article three separate developments in TGD are discussed. These developments emerged in a "wrong context" as side products and are discussed in separate articles [L21, L20]. Since these discoveries are very relevant for the general theoretical framework of TGD, it is natural to publish them in a separate article.

The first discovery was the realization that $M^8 - H$ duality in zero energy ontology implies a fractal generalization of Hubble's law. It emerged in a work [L21] related to TGD inspired quantum biology, where gravitational Planck constant $h_{gr} = GMm/\beta_0$ (M and m are masses and β_0 is velocity parameter) introduced by Nottale [E2] plays a key role [L21]. This already also led to a partial understanding of Hubble tension: the value of β in cosmic scales is very near to 1 but its values differ slightly in long and short cosmic scales.

The second discovery emerged while developing a TGD based model for Allais effect, which has no explanation in the context of general relativity or Newtonian gravitation. The work led to the question concerning the interpretation of the velocity parameter β_0 appearing in the formula of the gravitational Planck constant h_{gr} introduced already by Nottale [E2]. The gravitational field bodies of the Earth and structures of cosmological size scale are characterized by $\beta_0 = v_0/c \simeq 1$ and cannot correspond to a velocity of matter for a matter flow. Solar system characterized by $\beta_0 \simeq 2^{-11}$, which cannot correspond to light-velocity in the general relativistic framework.

The solution of the problem came from a prediction of TGD, which emerged already 47 years ago during the first year of TGD. TGD predicts warped space-time surfaces which are flat like Minkowski space M^4 (no gravitation) and have vanishing gauge fields. They have Poincare symmetry but have reduced light-velocity due to the warping. Light-like geodesics of M^4 are replaced by light-like geodesics of $M^4 \times S^1 \subset H = M^4 \times CP_2$. Warped space-time surface can explain the Allais effect, in particular the dramatic reduction of the parabolic oscillator frequency in terms of reduced light velocity. The notion of warping generalizes: one can speak of warped Hamilton-Jacobi (H-J) structures for which the hypercomplex structure of M^4 and complex structure of CP_2 are mixed in the sense that M^4 H-J structure is replaced with that of a warped space-time surface.

The third development led to a more detailed understanding of how the CP_2 type extremals emerge from the H-H principle, which for ordinary H-J structure [L10] allows only Minkowskian space-time regions. The earlier view about CP_2 type extremals as blow-ups finds a nice description in terms of H-H principle and twisted H-J structure made possible by the warping of M^4 as tilting towards $M^4 \times S^1$, where S^1 is a geodesic circle of CP_2.

1.1 $M^8 - H$ duality implies Hubble's law

An attempt to understand the highly suggestive connection of icosa tetrahedral tessellation [L8, L18] as a model of genetic code and icosahedral superclusters as hydrogen bonded clusters in water led to to completely unexpected developments suggesting deep connections between fundamental physics ($M^8 - H$ duality [L19] and the notions of gravitational Planck constant [E2] [L6] and electric Planck constant [L9] as implications of number theoretic vision), physics of water (hydrogen bonded water clusters), consciousness theory (field body as controller of biological body forming sensory representations of biological body), biology (ITT view of the genetic code) and cosmology (generalization of Hubble's law to all scales).

In particular, a prediction for the ordinary Planck Hubble constant implies a predictions for the mass density of the Universe and prediction is consistent with the observed mass density including contributions of dark matter and energy if the value of the velocity parameter $\beta_0 \leq 1$ appearing in the formula for the gravitational Planck constant is equal to $\beta_0 \simeq 1$ and the very large system containing observable Universe as quantum coherence region with Hubble radius L_H is a blackhole-like object with Schwartschild radius $r_s = 2\beta_0 L_H \simeq 2L_H$ for $\beta_0 \simeq 1$. Hubble tension means that the value of the Hubble length $L_H = 1/H_0$ in short scales is roughly 10 percent smaller than its value in long length scales. Since L_H is proportional to $1/\beta_0$ this can be understood if β_0 in the early universe (long scales) is roughly 10 per cent larger than in the long scales. This would imply that $\beta_0 \simeq .92$ in long length scales $\beta_0 \simeq 1$ in short scales. The predictions for the fraction of baryons would be about 5 percent in short length scales and 7.4 percent in long length scales.

1.2 Allais effect

The Allais effect [E1, E5] (see this and this) was first reported by Maurice Allais in 1954. It involves an abrupt change in the azimuth of a paraconical pendulum's oscillation plane during the solar eclipse, totaling up to 13.5 degrees.

1.2.1 Empirical findings

Consider first a brief summary of the findings of Allais and others [E5].

1. Paraconical pendulum consists of a rigid rod of ~ 1 meter and a metal ball. The bob, that is the weight at the bottom, has lense like shape. Paraconical pendulum differs from the conical pendulum in that the suspension point of the pendulum is not fixed but is a metal sphere able to roll without sliding in plane. Therefore it has 2 degrees of freedom rather than only one: both swinging and rotation around the vertical axis are possible.

2. In the absence of any other forces than the gravitation of Earth) paraconical pendulum can behave much like a conical or Foucault pendulum. The oscillation plane of the paraconical pendulum turned by 13,5 degrees during 14 minutes (see https://plus.maths.org/mathematical-mysteries-foucaults-pendulum-and-eclipse). It is difficult to see how the gravitational fields of the Sun and Moon could explain this behaviour by changing the effective value of the Earth's gravitational acceleration.

3. Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

4. Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5\times 10^{-4}$ [E1, E4] which happens to correspond to the constant $\beta_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant for the Sun. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.
5. There is also the finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical pendulum during solar eclipse [E6].
6. There is also evidence that the effect is present also before and after the full eclipse. The time scale is 1 hour. Allais emphasized that the effect is a dynamic, not static, phenomenon, connected to the variation of weight or inertia in the space swept by the pendulum during the eclipse. The 10% excessive bending of light is reported during some eclipses (the "residual arc") is also reported.

While many attempts to confirm it have met with varied or ambiguous results, several observations indicated anomalous behavior that cannot be easily explained by general relativity (GR) or standard Newtonian mechanics.

1.2.2 The TGD view of Allais effect briefly

The TGD view of Allais effect involves quantum physics based on TGD based quantum ontology. One can consider several different levels for how quantum physics appears in the description.

1. One can start from a harmonic oscillator model for the gravitational pendulum and perform a quantization using the gravitational Planck constants of the Sun or the Earth. The huge values of these Planck constants imply that small values of the harmonic oscillator quantum number are involved. The changes of this quantum number could explain the fluctuations at quantum criticality assignable to the transition to the eclipse. The effect would not be gravitational but quantum mechanical and due to the large value of $\hbar_{gr}$.
2. During a full eclipse, the screening of the solar gravitational field might explain the Allais effect. There is however evidence that the Allais effect appears also outside the regions of full eclipse and therefore in the scale of the Earth. This suggests that a description involving interference and diffraction effects besides screening is needed.

   In the TGD framework, models involving classical long range gravitational or Z^0/Kähler fields cannot be excluded. These models do not however look promising: the standard physics based expectation is that the effects are quite too small.
3. The description in terms of wave functions identified as spinor fields of the "world of classical worlds" (WCW) is more promising. The argument of the WCW spinor field would be the space-time surface as analog of Bohr orbit for a particle as 3-surface. One would have essentially wave mechanics in WCW. Instead of a Bohr orbit one would have a wave in the space of Bohr orbits (WCW).

   The Moon would act as an obstacle giving rise to quantum diffraction, which reduces to screening immediately behind the Moon. The diffraction would not be caused by the classical gravitational interaction but would be analogous to the diffraction of electrons in a double screen and a genuine quantum effect.
4. The observed reduction $r = \Delta f/f \simeq 2^{-11}$ of the oscillation frequency of the pendulum is several orders of magnitude larger than the prediction and happens to be near to the velocity parameter β_0 appearing in the solar gravitational Planck constant. r is also near to the electron proton mass ratio m_e/m_p. Which intepretation is correct?

   The pondering of this question led to a solution of a longstanding problem concerning the interpretation of the velocity parameter β_0 appearing in the Notale's hypothesis. Field equations allow as solutions warped space-time surfaces, which are flat just like Minkowski space but have reduced light velocity $c_\# < c$. $\beta_0 = c_\#$ identification is natural. Warping as a universal quantum critical phenomenon distinguishing between TGD and GRT, allows to identify a mechanism for the reduction of the oscillator frequency in the Allais effect: $\Delta f/f \sim m_e/mp$ would be the correct interpretation.

The cautious conclusion would be that the Allais effect does not tell so much about new gravitational physics than about the new quantum ontology predicting the notion of WCW realizing holography = holomorphy vision, the hierarchy of Planck constants, and ZEO.

1.3 CP_2 type extremals as blow-ups and birational cognition

For the ordinary H-J structure [L10], the H-H principle [L12, L15] allows only Minkowskian space-time regions. The earlier intuitive view about CP_2 type extremals was as blow-ups finds and finds an elegant description in terms of H-H principle and twisted H-J structure made possible by the warping of M^4 as tilting towards $M^4 \times S^1$, where S^1 is a geodesic circle of CP_2.

It is also found that birational maps $g : C^2 \times C^2$ allow to solve exactly the roots (c_1, c_2) for g in turn determining the space-time surfaces as surfaces $(f_1, f_2) = (c_1, c_2)$. Therefore they define easy cognitive representations for the functional number (f_1, f_2) as collections of space-time surfaces which correspond to collections of physical particles.

2 $M^8 - H$ duality implies generalized Hubble's law

The attempt to understand how the icosa tetrahedral tessellation (ITT) of H^3 [L8, L18] has a vertex figure the third shell of the icosahedral supercluster (ISC) [I1]. The problem was to understand how the complement of the vertex figure of ITT, which should be outside it, can correspond to the first and second shell of the ISC which are below the third shell.

The obvious guess is that the ITT realized at the field body of the ISC is related by inversion to ISC. $M^8 - H$ duality, as the TGD counterpart of the momentum position duality, involves inversion in $M^4 \subset M^8$, having interpretation as momentum space, mapping it to $M^4 \times CP_2$. Is $M^8 - H$ duality involved?

This question led to surprising developments suggesting deep connections between fundamental physics ($M^8 - H$ duality and the notions of gravitational and electric Planck constant as implications of number theoretic vision), physics of water (hydrogen bonded water clusters), consciousness theory (field body as controller of biological body forming sensory representations of biological body), biology (ITT view of the genetic code) and cosmology (generalization of Hubble's law to all scales).

2.1 $M^8 - H$ duality

In TGD, geometric and number theoretic visions of physics are complementary [L12, L15]: $M^8 - H$ duality [L19, L16] in which M^8 is analogous to 8-D momentum space associated with 8-D $H = M^4 \times CP_2$ is a formulation for this duality and makes Galois groups and their generalizations dynamic symmetries in the TGD framework. This complementarity is analogous to momentum position duality of quantum theory and implied by the replacement of a point-like particle with 3-surface, whose Bohr orbit defines space-time surface.

The points of $M^4 \subset M^8$ having interpretation as quaternions with number theoretic inner product identified as the real part of the square of the quaternion. The points p^k of $M^4 \subset M^8$ having interpretation as 4-momenta are mapped to points m^k of $M^4 \subset H = M^4 \times CP_2$ having interpretation as points of Minkowski space by inversion

$$p^k \rightarrow m^k = h_{eff}\frac{p^k}{p^2} \ .$$

Here h_{eff} is the effective Planck constant having a number theoretic interpretation. The proposal is that h_{eff} is a multiple of its minimal value h_0: $\hbar_{eff} = n\hbar_0$, $\hbar = n_0 h_0$. The assumption that CP_2 size scale $R(CP_2)$ is actually scaled up up Planck length $R(CP_2) = (\hbar/\hbar_0)l_P$, $l_P = \sqrt{\hbar G}$, gives the estimate $n_0 = (7!)^2$. This predicts also values $h_{eff} < h$ and there exists some evidence for them [D1] [L1].

There are at least two possible choices for h_{eff} if one assigns it to a field body mediating a particular type of interaction. The are at least gravitational [L7, L6] and electric [L9] Planck constants $\hbar_{gr}$ and $\hbar_{em}$. The gravitational Planck constant is given by

$$\hbar_{gr} = \frac{GMm}{\beta_0} = \frac{r_S m}{2\beta_0} \quad . \tag{2.1}$$

Here M and $m$are two masses, typically m is much smaller than M. This is something new a possible interpretation is in terms of a generalization of ordinary Lie symmetry algebras to multi-local symmetry algebras known as Yangians. The bilocal Planck constant could take the role of the ordinary Planck constant for Yangians [B1] [L2].

$\beta_0 = v_0/c \leq 1$ is a velocity parameter. Number theoretical arguments [L3] leads to ask whether β_0 is inverse integer valued but also rational values are possible. This would imply that Lorentz boosts generated by β_0 define a discrete subgroup of the Lorentz group. The formula for $\hbar_{gr}$ generalizes to massless particles by replacing the rest mass m with the energy E of the particle.

One can assign to $\hbar_{gr}$ gravitational Compton length

$$L_H = \Lambda_{gr} = \frac{r_S}{2\beta_0} \quad .$$

which by Equivalence Principle is independent of the small mass m.

A natural identification of the gravitational Compton length $\Lambda_{gr}(M) = GM/\beta_0(M) = r_S/2\beta_0(M)$ is as the size scale of gravitationally quantum coherent regions associated with a pair of systems with mass M and m. Note that by Equivalence Principle there is no dependence on mass m. The large mass M would correspond to the mass associated with a system with mass M and typically with a size scale considerably larger than its Schwartschild radius r_S.

A good guess is that the size of the CD associated with a gravitational field body of mass M corresponds to $\Lambda_{gr}(M) = r_s(M)/2\beta_0$ for a larger system with mass M creating the gravitational field in which m moves. This poses a condition on the parameter $\beta_0(M)$. For the Earth $\Lambda_{gr} = r_s/2 \simeq .5$ cm is the size scale of a snowflake. For the Sun with $\beta_0 \simeq 2^{-11}$ (, which happens to be electron-proton mass ratio) $\Lambda_{gr} \simeq R_E/2$ is considerably smaller than the solar radius. That the value is one half of the Earth's radius is probably not an accident.

The notion of the gravitational Planck constant generalizes to the electric case [L9] by the replacement $GMm \rightarrow Qqe^2$ where Q and q are interacting charges expressed using elementary charged e as unit.

2.2 $M^8 - H$ duality generalizes Hubble law

TGD predicts a Russian doll hierarchy defined by CDs. This leads to a generalization of the Hubble law for both gravitational and electric Planck constants. A natural proposal is that the gravitational Compton length $\Lambda_{gr} = GM(CD)/\beta_0$ is identifiable as the size scale $L(CD)$ of the CD. Here $M(CD)$ has an interpretation as mass of the system defining the gravitational field in which the system corresponds to quantum coherence regions as a space-time sheet. CD would contain the quantum coherence regions whose M^4 projection is light-cone proper time= constant hyperboloid H^3.

Since the half-cones of CD are analogous to empty cosmologies, another natural proposal is that the size scale $L(CD)$ of CD is identifiable as the Hubble radius $L_H(CD) = c/H_0(CD)$ of CD and that the Hubble law holds true inside the sub-cosmology associated with the CD and is apart from small corrections a consequence of the hyperbolic geometry of light-cone proper time = constant surface H^3.

Gravitational $M^8 - H$ duality predicts $m^k = \hbar_{gr} p^k/p^2$. If applied to particle with mass m appearing in $\hbar_{gr} = GMm/\beta_0$, this gives

$$m^k = \frac{r_S}{2}\frac{\beta^k}{\beta_0} \quad ,$$

where β^k is four-velocity of the particle. Four-velocity has unit length which fixes the value the light-cone proper time a as

$$a=\frac{r_S}{2\beta_0} \ .$$

The value of 3-dimensional M^4 distance r_3 is bounded by the condition

$$r_3\leq L_H=\frac{r_S}{2\beta_0} \ ,$$

stating that the perceptive field of the system is bounded by causal diamond with radius L_H.

If the particle is allowed to have mass $m_1\neq m$, where m defines $\hbar gr$, the outcome is different and summarized by the formulas

$$\begin{array}{l} m^k=\frac{r_S}{2}\frac{m}{m_1}\frac{\beta^k}{\beta_0} \ , \\ a=\frac{r_S}{2\beta_0}\frac{m}{m_1} \ , \\ r_3\leq\frac{r_S}{2\beta_0}\frac{m}{m_1} \ . \end{array}$$

Particles with different masses would correspond to different values of light-cone proper time and Hubble constant would be inversely proportional to the particle mass. This would suggest a violation of the Equivalence Principle so that this option looks implausible.

The formula

$$\beta_3=H_0(CD)r_3$$

looks like a generalization of the Hubble law with Hubble length $L_H(CD)=c/H_0(CD)$ given by the gravitational quantum coherence length

$$L_H=\Lambda_{gr}=\frac{r_S}{2\beta_0} \ .$$

For the Earth L_H would have the value of $\Lambda_{gr}=.5$ cm for $\beta_0=1$. For the Sun with $\beta_0\simeq 2^{-11}$, L_H would in a good approximation equal $R_E/2$, where R_E is the Earth radius.

Hubble's law generalizes also in the case of electric Planck constant $\hbar_{em}=Q_1q_2e^2/\beta_0$. Electric Compton length identifiable as Hubble length $L_{H,em}$ is

$$\Lambda_{em}=L_{H,em}=\frac{Q_1q_2e^2}{\beta_0 m}=\frac{4\pi\alpha_{em}}{\beta_0}Q_1q_2\times\lambda_C \ .$$

Q_1 and q_2 are charges using e as a unit. Now the Hubble length is proportional to Compton length.

In both electric and gravitational cases the predicted redshift is rather large and it is interesting to see whether this is a testable effect. The redshift reflects the fact that the Lorentz invariant light-cone proper time is the natural time coordinate for the CD. Its presence could be seen as a support for the notion of CD and the notion of many-sheeted space-time. In the gravitational case all particles would have the same time coordinate. In the electric case the time coordinate would be inversely proportional to the particle mass.

It is interesting to look the situation from the point of view of zero energy ontology (ZEO) [K4] [L4, L11]. The value of the light-cone proper time for cd$\subset$ CD=cd$\times$ CP_2 is quantized and equal to Λ_{em} *resp.* Λ_{gr} in the electric *resp.* gravitational case. ZEO predicts that CD is identifiable as a perceptive field of a conscious entity, self. The subjective time evolution corresponds to a sequence of "small" state function reductions (SSFRs) defining the TGD counterpart of the Zeno effect. The size of the cd increases in a statistical sense. The simplest assumption is that during this sequence the boundary of cd is only scaled (this induces a conformal transformation leaving the 3-D quantum state at the boundary invariant). The value of the geometric time, for instance identifiable as the temporal distance between the tips, of cd increases.

The problem is that the identification $a=L_H$ as subjective time means that subjective time would not increase. This is not consistent with the TGD view of memories. The interpretation of subjective memories [L13] in terms of classical non-determinism would assign them to definite values of a in the geometric past characterizing the loci of non-determinism. If only a single value of β_0 is assumed, this is not consistent with $a=L_H$. Could the moments associated with loci of memories correspond to different values of β_0? Smaller values of β_0 would correspond to earlier memories. The higher the evolutionary level, the larger the number of values of β_0 would be. The spectrum of β_0 would be fixed by the non-determinism of the space-time surface.

2.3 Ordinary Hubble law follows from $M^8 - H$ duality

One can test the generalization of Hubble's law on cosmological scales. The proposal implies that the ordinary Hubble radius $L_H = c/H_0 = 1.44 \times 10^9$ ly can be identified as gravitational Compton length:

$$L_H = \frac{GM(CD)}{\beta_0(CD)} = \frac{r_S(CD)}{2\beta_0(CD)} \quad .$$

where r_S is the Schwartschild radius associated with the mass inside the cosmic CD containing the quantum coherence regions with size $L(CD)$ as the application to the cases of the Earth and Sun forces to conclude. This gives $r_S(CD) = 2\beta_0(CD)L_H$.

It is important to notice that the mass M is the mass inside CD_1 for which the visible Universe corresponds to the quantum coherence region with radius L_H. From the examples of the Earth and Sun, M is considerably larger than the mass within radius L_H. The examples provided by the Earth and Sun raise the question whether there is a hierarchy of CD sizes corresponding to planets, stars, galaxies and whether the spectrum of β_0 reflects this hierarchy and serves as measure for the complexity of space-time surfaces and associated failure of strict classical determinism.

2.4 Estimate for the density of the Universe

The proposed picture predicts that the mass of the visible Universe inside CD using solar mass as a unit is

$$\frac{M(CD_1)}{M_{Sun}} = \frac{2\beta_0(CD)L_H(CD)}{r_S(Sun)} .$$

Here $r_S(Sun)$ equals 3 km. Assume that the radius of CD_1 is given by $L_H(CD_1) = xL_H(CD)$ so that one has $V(CD_1) = x^3V(CD)$.

This gives the estimate for

$$M(CD_1) \sim 2 \times 10^{22} \times x^{-3}\beta_0^4(CD)M_{Sun} \quad .$$

Here the mass $M(CD_1)$ corresponds to the mass within CD_1. $M_{Sun} \sim 1.88 \times 10^{57}m_p$, where m_p is proton mass. This predicts the average density

$$\rho \sim \beta_0^4 x^{-3} \times 12 \times 10^2 m_p/m^3 \quad .$$

The density of baryons is estimated to be $5.9 - 6$ protons per cubic meter (see this). The density ρ_B of ordinary (baryonic) matter is believed to be about $p = 1/20$ that is 5 percent of the total density: $\rho \sim \rho_B/p = 20\rho_B \simeq 120m_p/m^3$. This gives $\beta_0^4 x^{-3} \sim 1/10$.

p-Adic length scales are good candidates for the size scales of CDs and seem to correspond to octaves $p \simeq 2^{2k}$ so that minimal scaling relating the sizes of CD and CD_1 containing CD should correspond to $x = 2$. For $\beta_0 = 1$ the Universe would be a blackhole-like object with $L_H = r_s/2\beta_0 = r_s/2$. For $(p = 1/20, x = 2)$ would predict $\beta_0 \simeq .95$. $(\beta_0 = 1, x = 2)$ would predict $p \simeq 6.1$ per cent.

Hubble tension means that the Hubble length in short scales is 5-10 percent shorter than in long scales. This requires that in short scales β_0 is 5-10 per cent smaller than in long scales. By $\beta_0 \leq 1$ $\beta_0 = 1$ cannot be true in long scales $(\beta_0 = 1, x = 2)$ could be true in short scales (the rough estimate for $\hbar_{gr,E}$ gives $\beta_0 \simeq 1$) and $(\beta_0 = .95, x = 2)$ in long scales would predict difference 7.5 per cent $\Delta H_0/H_0$ and resolve the Hubble tension.

$\beta_0 = 1$ in short scales as opposed to $\beta_0 = .95$ in long length scales would require the scaling of baryon fraction from 5 percent in short scales to 6.1 percent in long scales. One would have $L_H = r_s/2$ and the Universe could be seen as a blackhole-like system for which the quantum coherence region would have radius $L_H = r_s/2$. This would give a p-adic fractal hierarchy of blackhole-like objects, which are quantum coherence regions of blackhole-like objects.

Why should the fraction of baryons be smaller in short scales than in long scales? A possible explanation is the transfer of baryons to dark baryons at monopole flux tubes, reducing the fraction of baryons in short scales (recent universe) from 6.1 percent to 5 percent. The cosmic evolution as an unavoidable increase of algebraic complexity would generate large h_{eff} phases and would also manifest as the formation of gravitational bound states such as galaxies, stars and planets.

2.5 Some implications of the generalized Hubble law

The generalized Hubble law gives for the 4-velocity $\beta_4^k = (m^k/H_0(CD))$. $\beta_4 = 1$ fixes the value light-cone proper time to $a = H_0(CD)$ and also the size of the CD. For 3-velocity β_3 Hubble law gives $\beta_3 = L_H r_3 \leq 1$, which implies

$$r_3 \leq L_H(CD) = \frac{r_S(CD)}{2\beta_0} = \frac{GM(CD_1)}{\beta_0} \quad .$$

The interpretation is in terms of a maximal value for the size of the CD coming from the mass $M(CD)$ and $\beta_0(CD)$. Note that the mass $M(CD_1)$ is for CD_1 for the object containing CD in its gravitational field.

In the case of the Sun with $\beta_0 \simeq 2^{-11}$, one has $L_H(Sun) \simeq 6,000$ km to be compared with the radius of the solar core about 139,000 km, which is roughly twice the value of $L_H(Sun)$. Note also that $L_H(Sun)$ is not far from the radius of Earth and also of the maximal radius of a solar spot: this plays a key role in the TGD based model of the Sun [L14].

In the case of Earth, L_H is of order .5 cm, much smaller than the size of Earth. From this one can conclude that the mass $M(CD)$ cannot be the mass assignable to the CD but to a larger system: for instance, the mass of the Sun or Earth. The maximal size of the CD is fixed by the gravitational field of a larger object.

$L_H(CD)$ as an analog of the event horizon defines an upper boundary of some object in the gravitational field of an object with mass M. One can consider two options.

1. L_H is a bound on the size of a space-time sheet interpreted as the size of quantum coherence regions.
2. L_H is a bound on the size of a CD defining the perceptive field of some object.

Consider now the interpretation as the size of the quantum coherence region.

1. Does L_H give an upper bound for the size of the Sun or of the quantum coherent objects inside the Sun and even outside it. Note that the rocky planets (Mercury, Venus, Earth and Mars) satisfy the condition $R \leq R_E$, whereas the outer planets fail to satisfy it. One can also consider the interpretation of space-time sheets as analogs of blackhole-like entities. The bound on M^4 radius r_3 would state that radiation cannot escape from this region.
2. The original model of Nottale for the planetary orbits as Bohr orbits assumes that the value of the velocity parameter β_0 for the outer planets is by factor 1/5 smaller than for the inner planets. This would increase the value of L_H as upper bound for their size to $5R_H$. For Jupiter, Saturn , Uranus and Neptune the values of R/R_E are 11.2, 9.5,4.0, and 3.88. For Jupiter and Saturn, which are gas planets, this proposal fails.
3. It is also possible that the value of β_0 is the same for the entire solar system: in this case only the principal quantum number n determining the Bohr radius for outer planets would be 5 times larger than for the first option. For this option Jupiter and Saturn should have an inner rocky core, which is not larger than Earth. This conforms with their character as gas planets.

The simplest interpretation is that the Sun and solar system decompose into space-time sheets with a size not exceeding $L_H(Sun)$. The notion of the many-sheeted space-time however allows us to ask whether there is a space-time sheet corresponding to the solar core topologically condensed at a larger connected space-time sheet identifiable as a quantum coherence region corresponding to the solar exterior or some other system, such as the field body of the Sun?

How to increase the value of L_H for the larger space-time sheet?

1. One could increase the mass M. In the case of the solar exterior, the increase of M would be small. Second option is that the value of $\beta_0 = 1/n$ is reduced. This would predict a spectrum of H_0 as integer multiples of a minimal value.

   Could the value of n label the onion-like layers of the field bodies with different values of the light-cone proper time a? Could they correspond to a hierarchy of values of $H_0(CD)$ coming as multiples of n? Note that some special values of n such as powers of 2 inspired by the p-adic length scale hypothesis can be considered.

2. Could the variation of n also resolve the Hubble tension: could the different values of the Hubble constant would correspond to different values of β_0. values of Hubble constant (10 per cent different) in long and short scales can be understood in terms of different values of β_0 but now one one must assume that $\beta_0 = 1/n$, with $n \sim 10$ or assume quantization as rational numbers.

3. There is evidence that the values of the cosmic recession velocities for distant astrophysical objects along lines of sight, originally discovered by Halton Arp, are quantized as multiples of n [E3, E7]. $\beta_0 = 1/n$ would predict subharmonics of the standard redshift rather than harmonics. Hyperbolic tessellations of H^3 could explain these mysterious 'God's fingers' as sequences of identical look stars or galaxies of hyperbolic tessellations along the line of sight [L5] [K3].

4. The notion of gravitational Planck constant makes sense only if one has $\hbar_{gr} \geq \hbar$. Could one assume that the upper layers of the solar surface have $h_{gr} = h$? This does not conform with the TGD based model for the surface layer of the Sun [L14], which assumes that the quantum coherence regions are of size of $L_H(Sun)$.

3 Allais effect as a key to the understanding of gravitational Planck constant in terms of warping

The Allais effect [E1, E5] (see this and this) was first reported by Maurice Allais in 1954. It involves an abrupt change in the azimuth of a paraconical pendulum's oscillation plane during the solar eclipse, totaling up to 13.5 degrees.

3.1 Empirical findings

Consider first a brief summary of the findings of Allais and others [E5].

1. Paraconical pendulum consists of a rigid rod of ~ 1 meter and a metal ball. The bob, that is the weight at the bottom, has lense like shape. Paraconical pendulum differs from the conical pendulum in that the suspension point of the pendulum is not fixed but is a metal sphere able to roll without sliding in plane. Therefore it has 2 degrees of freedom rather than only one: both swinging and rotation around the vertical axis are possible.

2. In the absence of any other forces than the gravitation of Earth) paraconical pendulum can behave much like a conical or Foucault pendulum. The oscillation plane of the paraconical pendulum turned by 13,5 degrees during 14 minutes (see). It is difficult to see how the gravitational fields of the Sun and Moon could explain this behaviour by changing the effective value of the Earth's gravitational acceleration.

3. Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

4. Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5 \times 10^{-4}$ [E1, E4] which happens to correspond to the constant $\beta_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant for the Sun. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

5. There is also the finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical pendulum during solar eclipse [E6].

6. There is also evidence that the effect is present also before and after the full eclipse. The time scale is 1 hour. Allais emphasized that the effect is a dynamic, not static, phenomenon, connected to the variation of weight or inertia in the space swept by the pendulum during the eclipse. The 10% excessive bending of light is reported during some eclipses (the "residual arc").

While many attempts to confirm it have met with varied or ambiguous results, several observations indicated anomalous behavior that cannot be easily explained by general relativity (GR) or standard Newtonian mechanics.

3.2 Anomalies and their brief explanations

Allais effect raises several problems which do not seem to have answers in the Newtonian and Einsteinian frameworks. The key observations are as follows.

1. Allais effect does not seem to involve any modification of classical gravitation in the sense that a modification of the classical gravitational force is not involved. This allows modification of gravitational potential by an addition of constant and in general relativity the addition of constant to the expression of the time component of the space-time metric.

2. The effect seems to be due to a horizontal force. The orbital plane is changed abruptly, which suggests a new kind of force. If is gravitational, it could be a force caused by the scattering of gravitons from the pendulum. The huge value of gravitational Planck constant implying long length scale quantum coherence and possibility of Bose-Einstein condensates together with the independence of the gravitational Compton length of graviton energy could make this effect large.

3. The fluctuations during the transition are large. This suggests quantum criticality. Classical field equations allow warped space-time surfaces, which are gravitational and gauge vacua and have a flat Minkowski metric with a reduced light-velocity $c_\# = \sqrt{1-R^2\omega^2}$.

   This leads to the notion of twisted (or warped) Hamilton-Jacobi structure [L21, L10] for which canonically embedded M^4 is tilted toward $M^4 \times S^1 \subset H$: this allows the generalization of warping to the case of general space-time surfaces as solutions of field equations obeying holography = holomorphy principle [L12, L15].

   A thin metal plate serves as an excellent analogy and is a critical system. Same is true for the deformations of canonically embedded M^4. This universal criticality reflecting itself as fluctuations of $c_\#$ could be behind very many forms of quantum criticality.

4. Gravitational Planck constant is expected to play a key role in the Allais effect. It is inversely proportional to a velocity parameter β_0. The identification of β_0 has remained a mystery. The identification $\beta_0 = c_\#$ is highly suggestive. This implies that one can talk about reflection, and refraction of waves at the boundaries of two regions with different values of β_0.

5. There are also diffractive effects and also these could be understood if gravitational waves or gravitations cause the transversal effects on the pendulum. Gravitational waves are indeed transversal. The gravitational Compton length $\Lambda_{gr} = GM/\beta_0$ is the same for all gravitons and would characterize the diffraction pattern. Irrespective of the energy of the gravitons (or of any particle). There are several alternative identifications of the large mass M and the value of β_0. Empirical findings suggest a wavelength of 44 m and this scale can be understood rather naturally.

6. By a dimensional argument, the force constant of the gravitational pendulum is proportional to $c_\# < c$. The fluctuations of $c_\# < c$ could induce the fluctuations of the pendulum's oscillation frequency. A possible quantum phase transition explaining the upper bound for $\Delta f/f \leq 2^{-11} \simeq m_e/m_p$ can identified.

3.2.1 Does gravitational pendulum behave as a harmonic oscillator at small quantum number limit?

In this article, the earlier model for the effect based on the replacement of the oscillator with its quantum counterpart with very large gravitational Planck constant is discussed. For $\hbar_{gr}$ the oscillator corresponds to a small oscillator quantum number limit, and this can give rise to large quantum fluctuations of the amplitude as transitions which change this quantum number so that the reason would not be classical gravitation but TGD based quantum theory allowing large quantum gravitational effects.

3.2.2 Are reflection, refraction and diffraction of gravitational waves responsible for the Allais effect?

There is evidence that the Allais effect does not involve screening of classical gravitational force. This raises the question whether reflection, refraction and diffraction type effects assignable to gravitational waves or gravitons are involved and explain the transversality of the effect.

Also diffractive effects are involved and conform with the long wavelengths implied by $\hbar_{gr}$. A rather promising model relies on quantum diffraction in the "world of classical worlds" (WCW) consisting of space-time surfaces obeying holography = holomorphy principle and having interpretation as Bohr orbits. Monopole flux tubes can be also interpreted as analogs for flowlines of an incompressible hydrodynamic flow past an obstacle. They can be regarded as quantum particles meaning analogy with quantum diffraction for Schrödinger equation.

3.2.3 The velocity parameter of the gravitational Planck constant as reduced light velocity induced by warping

The pondering of this question led as a by-product to a solution of a longstanding problem concerning the interpretation of the velocity parameter β_0 appearing in the Notale's hypothesis. Field equations allow as solutions warped space-time surfaces, which are flat just like Minkowski space but have reduced light velocity $c_\# = \sqrt{g_{tt}} = \sqrt{1-R^2\omega^2} < c$. The identification $\beta_0 = c_\#$ is natural. This motivates the notion of twisted (or warped) Hamilton-Jacobi structure allowing to generalize this phenomenon to non-vacuum extremals. Warping as a universal quantum critical phenomenon distinguishing between TGD and GRT, makes it possible to identify a mechanism for the fluctuations of the oscillator frequency in the Allais effect.

The warping only shifts the gravitational potential appearing in $g_{tt} = 1 - 2\Phi_{gr}$ but the classical gravitational force is unaffected. The reduction of the light velocity caused by the warping resembles that appearing for dielectrics and suggests that the shadow of the Moon involves the reduction $c \to c_\#$. The large value of $\hbar_{gr}$ and $c_\# < c$ suggest that the reflection, refraction and also diffraction of dark gravitons from the pendulum could cause the transversal effects in the transition zones.

The shadow of the Moon would be analogous to a dielectric. This would imply reflection and refraction of dark gravitational radiation from the Sun. Reflection at the surface of the Earth would induce transversal gravitational force amplified by the huge value of gravitational Planck constant and by the fact that the gravitational Compton length for gravitons does not depend on the energy of the dark graviton. The pendulum would become an ideal detector of gravitons. The 10% excessive bending of light is reported during some eclipses (the "residual arc") could be interpreted in terms of reflection for dark photons by the same mechanism.

3.2.4 The reason for huge size of the effect: dark gravitational radiation has always the same wavelength

Sun produces gravitational radiation in the energy range $(1-10^5)$ eV. The huge value of $\hbar_{gr}$ scales the wavelength range and makes possible long scale quantum coherence at the gravitational magnetic body amplifying the effect.

Gravitational Planck constant for a massless particle with energy E is GME/β_0. By Equivalence Principle, the expression for the wavelength of the graviton is $\lambda = \Lambda_{gr} = GM/\beta_0 = r_S/2\beta_0$ irrespective of graviton energy. All dark gravitons, in fact all dark particles, would have the same

Compton wavelength! This could explain why the Allais effect is so huge. The gravitational pendulum could become a detector ofgravitons.

For the Earth mass $M = M_E$ and for $\beta_{0,E} \simeq 1$ this gives 5 mm. The replacement $\beta_{0,E} \rightarrow \beta_{0,S} \simeq 2^{-11}$ assigned with the Sun would give $\lambda = 10$ m to be compared with 44 m suggested by the experiments. $\beta_0 = 2^{-13}$ would give a good fit. For $M = M_{Sun} = 3 \times 10^5 M_E$ one has $\lambda = \Lambda_{gr,S} = 3 \times 10^6$ m $\simeq R_E/2$, which is solar gravitational Compton length characterizing Sunspot size scale.

3.2.5 Reduction of the oscillator frequency

From the point of view of General Relativity, the maximal value for the reduction $r = \Delta f/f \simeq 2^{-11}$ of the oscillator frequency is huge. The identification of the fluctuations as being due to the fluctuations of $\beta_0 = c_\#$ is natural.

There are several intriguing co-incidences. r equals the velocity parameter β_0 appearing in the expression of the solar gravitational Planck constant and is also near to the electron-proton mass ratio m_e/mp. Also the velocity of the solar system with respect to the galactic center is of this order of magnitude? Which option is nearer to the truth?

$\Delta f/f \leq m_e/mp$ could be nearest to the correct interpretation for the maximal reduction of frequency. The mechanism could be the phase transition in which Rydberg atoms with very large size at the magnetic body decay to protons and electrons. The condition that $\hbar_{gr}(H) = \hbar_{gr}(p)$ guarantees that H atoms and protons can reside in the same monopole flux tubes: this condition holds true in biology also for base pairs of DNA. This would give $\beta_0(H)/\beta_0(p) = c_\#(H)/c_\#(p) = m_p/m_H$. The values of $Deltaf/f \leq m_e/mp$ could mean that only a part of the H atoms decay to a proton and electron.

The cautious conclusion would be that the Allais effect does not tell so much about classical gravitational physics than about the new quantum ontology predicting the notion of WCW realizing holography = holomorphy vision, the hierarchy of Planck constants, in particular huge values of gravitational Planck constant, and ZEO. The warping phenomenon distinguishing between General Relativity and TGD would be the central element.

The cautious conclusion would be that the Allais effect does not tell so much about new gravitational physics than about the new quantum ontology predicting the notion of WCW realizing holography = holomorphy vision, the hierarchy of Planck constants, and ZEO. The warping phenomenon distinguishing between General Relativity and TGD would be the central element.

3.3 Warping as an explanation for the reduction of the pendulum's oscillation frequency

The attempt to understand the reduction of the pendulum's oscillation frequency led to a solution of another long standing problem related to the understanding the physical meaning of the parameter β_0 appearing in the formula for the gravitational and electric Planck constants. The solution came from the notion of warping discovered already during the first year of TGD about 48 years ago and distinguishing between General Relativity and TGD. In the following I discuss two approaches. The first approach is based on the frequency reduction as an analog of reshift. The problem is that β_0 cannot be interpreted as a for the motion of matter. Second approach is based on the warped space-time surfaces inducing non-gravitational redshift which cannot be assigned to the motion of matter.

3.3.1 About the interpretation of the parameter β_0 and the reduction of the oscillator frequency in Allais effect

The problems related to the physical interpretation of the parameter β_0 led to a rather detailed understanding of the frequency reduction occurring in the Allais effect.

3.3.2 Problems related to the interpretation of the velocity parameter β_0

There are several longstanding questions related to the parameter β_0 appearing in the formula $\hbar_{gr} = GMm/\beta_0$ introduced originally by Nottale [E2].

1. Is the interpretation of β_0 as a velocity parameter necessary? The gravitational Compton length $\Lambda_{gr} = r_s/2\beta_0$ has no dependence on the small mass m, which conforms with the Equivalence Principle. Also the cyclotron frequencies at the monopole flux tubes of the gravitational field body are independent of m.

2. There are two preferred values for β_0: $\beta_0 \simeq 1$ assigned with the Earth's gravitational field body an and $\beta_0 \simeq 2^{-11}$ assigned with the field body of the Sun.

3. The velocity of the solar system with respect to the galaxy is of the same magnitude as $\beta_0 \simeq 2^{-11}$ assigned with the inner planets, which supports the interpretation as velocity. The interpretation of $\beta_0 = v_0/c \simeq 1$ assigned with the Earth as a velocity of a massive object does not however look sensible. The realization that $M^8 - H$ duality implies Hubble's law lead to the conclusion that the Hubble tension finds a solution if one has two slightly different values of β_0 near unity applying in short and long scales.

There might be a very simple solution to these interpretational problems, which I have failed to notice.

1. In the standard quantum theory two quantum lengths characterize a massive particle. The Compton length $\lambda_c = h/m$ and the de-Broglie wavelength $\lambda_{de-B} = h/m\beta_0$, where $\beta_0 = v_0/c$ is the velocity of particle using light velocity as a unit.

2. Could the gravitational Planck constant $\hbar_{gr}(S)$ assigned to Sun and also planets in the Bohr model for planetary orbits corresponds to de-Broglie wave length and could β_0 correspond to a velocity $220 - 230$ km/s giving $\beta \in [(.73, .77) \times 10^{-3}]$ of the solar system with respect to galactic center. The error is about 20 per cent. If the gravitational Planck constant assigned with the Earth would correspond to the gravitational Compton length Λ_{gr}, the problem with $\beta_0 = 1$ would disappear.

There are however objections against this proposal.

1. The problem is that the Bohr orbit quantization of the planetary system [?] does not make sense for this interpretation. The quantum input in the quantization is the quantization of angular momentum and it would say that L_z/m equals to a multiple of the gravitational de-Broglie wavelength. This does not make sense in the framework of standard QM. This suggests that β_0 cannot have an interpretation as physical velocity of a massive object. Could it correspond to an analog of light velocity? Neither can the value $\beta_0(E) \simeq 1$ for the Earth for cosmological scales be identified as a velocity for a massive object.

2. $M^8 - H$ duality for the gravitational Planck constant leads to a fractal generalization of Hubble's law suggesting that Hubble tension might relate to two slightly different values of $\beta_0 \simeq 1$ in short and long length scales differing by 5-6 percent [L21]. This interpretation is not consistent with the interpretation of Λ_{gr} for $\beta_0 = 1$ as gravitational Compton length.

   The problem disappears if one can interpret $v_0 \leq c$ as light velocity with $c_\# = \sqrt{g_{tt}}c \leq c$ along the space-time surface in the formula for the gravitational Compton length.

3. This interpretation would have non-trivial consequences. In the case of the Sun, the disappearance of the $1/\beta_0(S) \simeq 2^{11}$ from the formula h_{gr} reduces the gravitational Compton length and gives $\Lambda_{gr}(S) = 3 \times 10^5 \Lambda_{gr}(E)$ rather than $\Lambda_{gr}(S) \simeq 2^{11} \times 3 \times 10^5 \times \Lambda_{gr}(E)$. The energy $E = h_{gr}(S)f$ for a given frequency would be also reduced by $\beta_0(S) \simeq 2^{-11}$. And as noticed, the Bohr quantization of the planetary system would not make sense anymore.

4. It seems that the only solution to the problem is that β_0 is quite generally identifiable as reduced light velocity $c_\#$. The reduction of $c_\# = \sqrt{g_{tt}}$ to say $c_\# \simeq 2^{-11}$ would however require huge gravitational fields: this does not make sense in general relativistic framework.

3.3.3 Warping of the space-time surfaces as a solution of the problems

A possible solution of the problem comes from a basic distinction between TGD and General Relativity noticed already during the first year of TGD.

1. TGD allows solutions of field equations, which are gravitational vacua in the sense of GRT and also gauge theory vacua for induced gauge fields. The solutions however allow warping possible only for surfaces. A thin metal plate or a sheet of paper are good examples of a system unstable against warping and therefore critical systems.

2. TGD indeed allows minimal surface solutions with a 1-D CP_2 projection belonging to geodesic circle $S^1 \subset CP_2$ for which M^4 time coordinate in the rest system of the causal diamond CD is of form $m^0 = t-\Phi/\omega$. The induced metric of X^4 given by $ds^2 = (1-R^2\omega^2)-dz^2-dwd\overline{w}$ is flat and has a deformation of the Poinca group as isometries. The interpretation $c_\# = \sqrt{1-R^2\omega^2}$ as a reduced light velocity is natural: the path around a warped space-time surface is longer than along a non-warped one. There would be no gravitational force but the vacuum would be warped. This warping makes sense also for monopole flux tubes obtained as deformations of the Cartesian product $M^2 \subset Y^2 \subset M^4 \times CP_2$. M^2 would be completely analogous to a metal plate and could be warped.

3. The warping can occur also at the level of the embedding space $H = M^4 \times CP_2$ for the Hamilton-Jacobi structure [L10]. Now $M^2 \subset M^2$ and CP_2 degrees would mix. An analogy is provided by a cylinder surface for which the coordinates (z, Φ) are replaced with coordinates $z - k\phi, z + k\phi$ for which coordinate lines are dual helices.

   The hypercomplex coordinates $(u, v) \rightarrow (t - z, t + z)$ would be replaced with $(u = T - z, v = T + z)$ where T is defined as $T = t - \phi/\omega$. The canonical embedding of $M^2 \subset M^4$ with constant CP_2 coordinates would be tilted towards the direction of $S^1 \subset CP_2$. CP_2 complex coordinates would suffer a time dependent $U(1)$ rotation $\Phi \rightarrow \Phi - \omega t$, which is holomorphic transformation and gives rise to a twisted Hamilton-Jacobi structure.

4. Even more general twisted Hamilton-Jacobi structures can be imagined [L10]. The TGD based model for the honeybee dance [K2] led to the proposal that there are preferred extremals as sphere bundles, which assign to a given point of the space-time surface a geodesic sphere, whose position in CP_2 depends on 2 M^4 coordinates so that one speak of local $SU(3)$ rotation of the geodesic sphere depending on two M^4 coordinates. Could also these kinds of twistings define exotic Hamilton-Jacobi structures? Could also twistings depending on time coordinate and complex coordinate w define exotic exotic Hamilton-Jacobi structures?

5. The twisted Hamilton-Jacobi structures could be associated with monopole flux tubes serving as body parts of field bodies. This would give connection with $\hbar_{gr}$. Also space-time surfaces representable as graps $M^4 \times CP_2$ could have a twisted Hamilton-Jacobi structure and the Hubble tension [L21] could be understood if the Hamilton structures differ by a small twist in long and short cosmological scales.

   In the planetary system there are two options for the Bohr quantization. $\beta_0 \simeq 2^{-11}$ would be true for the inner planets. For outer planets there are two options. Either $\beta_0 \simeq 2^{-11}$ is true but the principal quantum number n comes as multiples of 5 or $\beta_0 = 2^{-11}/5$ is true and Earth corresponds to the principal quantum number $n = 1$ for outer planets or $n = 5$ for the inner planets. For the second option $c_\# = \beta_0$ would be different at the gravitational monopole flux tubes.

3.3.4 Warping of the space-time surfaces and dielectric constant

The flat warped space-time surfaces are characterized by the reduced light-velocity $\beta_0 = c_\#/c \leq 1$. There is a criticality with respect to the variations of $c_\#$ (instability of metal plates illustrates this). Also the twisted Hamilton-Jacobi structures would be characterized by $c_\#$.

The criticality of the warping could induce or accompany various kinds of quantum criticalities. In the case of the Allais effect, this kind of quantum criticality would explain the variation of the pendulum frequency cannot be explained in terms of gravitation.

Quite generally, one can write $f_\# = c_\#/\lambda = f/n$, where $n = c/c_\#$ is analogous to the refractive index appearing in electrodynamics in presence of matter. In Maxwellian electrodynamics, refractive index relates to the relative dielectric constant ϵ_r via the formula $n = c/c_\# = \sqrt{\epsilon_r}$. Could reflective index and dielectric properties have a geometric description in terms of the warping of the space-time surface? If so, the warping of the space-time surface could be seen directly via the reflection of light!

Refractive index depends on frequency. This can be understood in terms of quantum criticality implying the value of $c_\#$ associated with the massless extremal assignable to the photons depends on frequency. At resonance, at which ϵ_r diverges, the value $c_\#$ would in the ideal case vanish: there would be no propagation of signals. The standard interpretation would be in terms of absorption of the signal by atoms, which contribute to the resonance frequencies.

How the criticality of warping could manifest itself in critical systems?

1. For a harmonic oscillator, the frequency is given in terms of force constant and mass as $\omega = \sqrt{k/m}$. A reasonable dimensional guess is that the force constant k characterizing the electromagnetic force is proportional to $(c_\#/c)^2$. For instance, cyclotron frequency would be proportional to $c_\#$. More generally, the Coulomb force in a dielectric is scaled from its vacuum value by $1/\epsilon_r = (c_\#/c)^2$. Also capaticance of a capacitor would be proportional to $(c_\#/c)^2$. The variation of $c_\#$ at quantum criticality would make it possible to change the contribution of the electromagnetic force.

2. Gravitational masses have always the same sign so that the notion of dielectret does not make sense and $c_\#$ is not expected to play any role: this conforms with the character of warping. For instance, the gravitational force created by a constant mass density ρ corresponds to potential energy proportional to $Gm\rho r^2$, which is harmonic oscillator potential energy. The force constant $k \propto Gm\rho$ does not depend on $c_\#$.

3. If the system is in an equilibrium involving electromagnetic and gravitational forces, the variation of $c_\#$ appearing in the electromagnetic component of force could make possible the loss of equilibrium. The tuning of $c_\#$ could allow the field body to change the equilibrium point of a physical system and even destroy or create the equilibrium. In biology the generation of nerve pulse, the splitting of DNA double strand preceding transcription and replication could serve as examples of this.

There might also be a connection with the quantum criticality associated with the mild classical non-determinism assignable to the 3-D singularities of space-time surfaces. These singularities can be regarded as edges of the space-time surface and that the time evolution of 3-surface is analogous Brownian motion. What comes to mind is that the singular 3-surfaces could define interfaces of regions with different values of $c_\#$ as loci of quantum criticality. Could the gravitons propagating along monopole flux tubes experience refraction during the solar eclipse.

Besides dielectric constant, electrodynamics is characterized by conductivity. Could the analog of Brownian motion provide a classical space-time correlate for the finite conductivity?

3.3.5 A connection with the frequency reduction in Allais effect

There would be a connection with the model explaining the Allais effect [L20].

1. There is a surprisingly large reduction of the value of the oscillation frequency having upper bound $\Delta f/f \leq 2^{-11}$. This brings in mind $\beta_0(S)$ and the proposal was that the quantum critical transitions involve fluctuations reducing the oscillator frequency satisfying the formula $E = h_{gr}(E)f$: now the mass of the pendulum would be in the role of the small mass.

   The modification $\Delta c_\#/c_\# \simeq 2^{-11}$ would be needed. The gravitational fluctuations $\Delta c_\#/c_\#$ required to produce the effect would be quite too large as compared to the reduction of the value of c from its maximal value by $GM_S/AU = r_s(S)/2AU \sim 10^{-9}$ and $GM_E/R_E = r_s(E)/2R_E \sim 10^{-9}$.

2. The modification of the inherently quantum critical Hamilton-Jacobi structure makes a large change $\Delta c_\#/c_\# \simeq 2^{-11}$ possible. It could occur at the level of the ordinary space-time surface or at the level of the field body. In the case of a gravitational pendulum, the reduction of

oscillation frequency $\omega_1 \propto \sqrt{g/l} = \sqrt{GM/lR^2} = c_{\#,1}r_s/2R^2$ to $\omega_2 = c_{\#,2}r_s/R^2$ would be needed. That the velocity of the solar system with respect to the galactic center is near to $\beta_0 = c_\# \simeq 2^{-11}$ could follow from the warping in this framework. One could say that the solar system moves with a reduced light-velocity! One can wonder how general this phemenon is.

The physical mechanism causing this modification should be identified and explain the large value of $\Delta c_\#/c_\#$.

1. Warping is a critical phenomenon. Space-time warping as a fundamental quantum critical phenomenon could accompany and even induce many kinds of quantum critical phenomena, in particular Allais effect.

2. The model for the Allais effect proposes that diffraction-like effect for the gravitational flux tubes meaning a deviation of the monopole flux tubes, analogous to the deviation of flow lines of a hydrodynamic flow past solid object, could produce reduction of the effective gravitational flux. This would reduce the effective gravitational mass M_S experienced by the pendulum but the reduction is expected to be extremely small.

3. Could gravitational Planck constant and $c_\#$ of the gravitational field body change? $\Delta f/f \leq 2^{-11}$ is not far from the electron-proton mass ratio $m_e/m_p \simeq 1/1880$: the deviation is 9 per cent. If the field body contains hydrogen atoms, their ionization to protons and electrons transforming to ordinary electrons would reduce h_{gr} by the required amount.

   The hydrogen atoms should be Rydberg atoms with a very small binding energy and therefore with very large size: this is indeed possible at the field body. The dropped electrons should have smaller energy compensating for the energy needed for the energy needed for ionization. The transition could take place by tunnelling and therefore involve a pair of "big" state function reductions (BSFRs).

   This kind of phase transition should occur at quantum criticality assigned with the beginning of the solar eclipse? Why the turning of the monopole flux tubes meeting the Moon should induce a phase transition leading to the transformation of dark electrons to ordinary electrons? Are the electrons so near to ionization state the turning ionizes them?

3.4 The simplest model of Allais effect found hitherto

The proposed ideas allow us to imagine many concrete models. The following proposal is perhaps the simplest model that I have been able to imagine hitherto.

1. The criticality of the warped M^4 defining a twisted Hamilton-Jacobi structure [L21] could accompany the quantum criticality during the beginning and end of the eclipse. Suppose that the criticality sets on during the transition period. The value of $c_\#$ fluctuates at criticality and can have large values.

2. Suppose that the time component $g_{tt} = 1 - 2\Phi_{gr}$ of the induced metric (Φ_{gr} is gravitational potential) is reduced during the eclipse in the shadow of the Moon which is therefore analogous to a dielectric. For large values of $c_\#$, the gravitational contribution to g_{tt} is negligible and $c_\#$ makes itself physically visible. If the contributions of warping and gravitation of the Sun and Moon are additive, the gravitational force as a gradient of the gravitational potential is not affected. This would partially explain the absence of longitudinal effects.

3. The fluctuations of $c_\#$ would explain the fluctuations of the pendulum frequency proportional to $\sqrt{c_\#}$. The mere change $c \rightarrow c_\#$ in the transition zone would not be enough. The fluctuation must occur at least in the transition zone.

How to explain the transversality of the Allais effect? The following picture is the simplest one found hitherto.

1. Longitudinal effects are absent since the classical gravitational force is not affected. Gravitational waves are transversal. Could they explain the transversal effects, which are much larger than those caused by the ordinary gravitational torque? Could the large value of gravitational Planck constant $\hbar_{gr}$, implying quantum criticality, make the gravitational pendulum an extremely sensitive detector of gravitational waves?

2. Assume that the value of $c_{\#}$ is changed inside the shadow cone of the Moon. $c_{\#}$ could be constant inside the shadow cone but fluctuations only in the transition regions would occur. Snell's law implies that the reflection and refraction angles are largest during the transition phases. At the center point of the shadow cone the reflection occurs backwards and refraction forwards. Therefore $c_{\#}$ could fluctuate also inside the transition region but would have a smaller transversal effect, which would vanish at the middle point of the Moon's shadow. The situation at the middle point would be dominated by the ordinary gravitational force apart from a vertical momentum transfer from gravitons, which could affect the vertical gravitational force.

3. Due to the large value of $\hbar_{gr}$ and $c_{\#}$, reflection and refraction of the gravitational waves from the pendulum could become large effects during the transition. Dark gravitons with large $\hbar_{gr}$ have large energy $E = h_{gr}f$ and momentum $p = E/c_{\#}$. During the transition periods large reflection angles become possible by Snell's law and this causes large transfer of transversal momentum. This could explain the large change of the oscillation plane.

This model differs from the model based on the diffraction of the monopole flux tubes assumed to define analog of hydrodynamical flow since in this case the classical gravitational flux would be deflected and induce effective screening. This could occur in rather long length scales and there is evidence for a characteristic scale of 44 m.

4 Number theoretic aspects of holography = holomorphy principle

In the sequel, some number theoretic aspects of the holography = holomorphy (H-H) principle are considered.

4.1 Holography = holomorphy principle

First some background is needed.

1. The Holography = holomorphy (H-H) [L12, L15, L17] principle implies that space-time surfaces are determined as roots of the pairs of holomorphic functions $f = (f_1, f_2) : H = M^4 \times CP_2 \to C^2$. The function pairs $f = (f_1, f_2)$ define an algebra with respect to element-wise sum and product of functions f_i. If f_1 or f_2 is not varied, one obtains a function field.

2. The maps $g = (g_1, g_2) : C^2 \to C^2$ define symmetries via the composition $f \to g \circ f$ and give rise to hierarchies of space-time surfaces with exponentially increasing algebraic complexity in the case of polynomials.

   In particular, one obtains function field analogs of p-adic number fields and of adeles with powers of the p-adic prime p replaced with powers of polynomials of prime degree having coefficient polynomials with degree smaller than p. If the polynomial primeness is realized in the sense that the Galois group is not affected in deformations, there are also prime polynomials with non-prime degree having no decomposition to the functional composite of polynomials of lower degree.

   The holomorphic maps $g : C^2 \to C^2$ define infinite hierarchies of space-time surfaces. The interpretation is in terms of complexity hierarchies and cognitive hierarchies. In the general case the complexity increases exponentially.

3. Space-time surfaces are analogs of Bohr orbits for 3-surfaces replacing point-like particles and provide representations for the elements of function algebras and function fields. Space-time surfaces can also be interpreted as theorems: the slight failure of the classical determinism located at the singularities makes possible several analogs of theorems represented as Bohr orbits such that the 3-D holographic data at the boundary of CD provides the fixed premises of the theorem. The classical laws of physics would provide a physical representation for the axioms of mathematics.

4. The dynamics implied by the H-H principle is universal if the classical action is general coordinate invariant and constructible in terms of the induced geometry of the space-time surface. By holomorphy = holography (H-H))principle [L12, L15], the solutions of field equations are holographic minimal surfaces with 3-D singularities at which the minimal surface property and holography fail. At singularities the boundary conditions stating the conservation of classical isometry charges are satisfied and without additional symmetries the singularities, in particular their positions, depend on the classical action.

   This suggests that classical action serves in the role of effective action so that the parameters appearing in it can vary. In particular, they could depend on the extension E of rationals appearing in the coefficients of the pairs holomorphic functions $f = (f_1, f_2) : H = M^4 \times CP_2 \rightarrow C^2$.

5. WCW would define what might be called Platonia and by adding WCW spinor fields one obtains quantum Platonia and Boolean logic. By adding state function reductions, one obtains conscious Platonia, which remembers and learns more and more about itself. Number theoretic evolution as an increase of number theoretic complexity is unavoidable.

4.2 How do CP_2 type extremals emerge?

The proposed view seems to give only the Minkowskian regions of the space-time surface: also CP_2 type extremals with Euclidean signature are needed. The space-time surface is not affected if f_1 and f_2 are scaled by a non-vanishing analytic function. What happens if f_1 and f_2 are scaled with the same function which vanishes at some points? A good guess is that the allowance of this kind of scalings leads to the emergence of the CP_2 type regions with Euclidean induced metric and the geometry of CP_2.

1. The scaling invariance implies that in regions $z_3 \neq 0$, the natural interpretation of C^2 is as a projective space CP_2 obtained from C^3 by identifying the points (z_1, z_2, z_3) of C^3 differing by complex scaling and one can take $z_3 = 1$ are identified so that for instance $(\xi_1, \xi_2) = (z_1/z_3, z_2/z_3)$ serve as coordinates. The regions with $z_2 = 0$ and $(z_1, z_2) \neq (0, 0)$ correspond $(z_1, z_2) \neq (\infty, \infty)$ defining the homologically trivial geodesic sphere of CP_2 at infinity. By using $(z_1/z_2, z_3/z_2)$ or $(z_2/z_1, z_3/z_1)$ as coordinates one obtains the three coordinate patches of CP_2.

2. The points $(z_1, z_2, z_3) = (0, 0, 0)$ however remain still problematic and they are indeed of fundamental importance since the condition $(z_1 = 0, z_2 = 0)$ defines the space-time surface. At these points the ratios z_i/z_j are ill-defined and a blow-up takes place so that these kinds of points must be replaced with CP_2. Is this the mechanism for how CP_2 type extremals as Euclidian regions of the space-time surface emerges as a blow-up? I have indeed proposed the blow-up mechanism earlier.

3. The problem is that CP_2 type extremals have as an M^4 projection a light-like curve of M^4, which only in a special case is reduced to the standard embedding of CP_2 to H. This problem can be solved by replacing the notion of H-J structure [L10] with its twisted variant [L10, L20, L21]. Twisting means that the canonical embedding of CP_2 is replaced with a twisted embedding for which the geometry of CP_2 induced from H is not affected but a geodesic line of CP_2 is stretched to a light-like curve as a coordinate line for the Hamilton-Jacobi coordinates such that the dual of the light-coordinate is constant along it.

4. The twisting as tilting of M^4 to the direction of the geodesic circle $S^1 \subset CP_2$ has also an interpretation as warping of H-J structure. The induced metric of tilted M^4 is modified

but remains flat. A reduction $c \to c_\# < c$ of light-velocity however occurs. This leads to an elegant explanation of some aspects of the Allais effect impossible to understand in Newtonian and general relativistic views of gravitation [L20]. Warping is indeed possible only for space-time surfaces, not for abstract 4-D Riemann spaces.

5. Warping also explains the reduction of light-velocity in electrodynamics for non-vacuum systems. The notions of di-electric constant and refraction therefore reduce to the geometry of space-time surfaces. Warping is also a universal critical phenomenon allowing large numbers of almost vacuum extremals X^4 with energy density determined by the volume term in the classical action proportional to the cosmological constant Λ, which has an extremely small value in long length scales. Therefore it could therefore be behind very many quantum critical phenomena. This view corresponds to the original view based on the huage vacuum degeneracy of Kähler action interpreted as 4-D spin glass degeneracy. The introduction of small cosmological constant associated with thge volume action leaves only the approximate vacuum degeneracy appearing as warping.

The outcome would be that by allowing scaling factors of (f_1, f_2), which vanish at a discrete set of points and twisted H-J structure, one obtains also CP_2 type extremals as solutions of field equations.

4.3 Could classical action have interpretation as a number theoretic invariant?

The generalized Langlands duality [L12, L15, L17] and the universality of the classical dynamics suggest that the exponent of the classical action has interpretation as an effective action and can be regarded as a number theoretic invariant. This invariant would assign to the space-time surface a number, which is real, algebraic number or integer depending on how strong assumptions are made about the roots of $(f_1 - c_1, f_2 - c_2) = (0, 0)$.

The guess motivated by the H-H hypothesis is that this number is an analog of a discriminant D for a polynomial of a single variable expressible as a product of the non-vanishing root differences. In the recent case, the roots of $(f_1 - c_1, f_2 - c_2) = (0, 0)$ are 4-D regions of space-time surfaces. One should be able to pick points of the space-time surface as ordinary roots in order to define D using the standard formula.

There are several kinds of sub-manifolds of the space-time surface.

1. The 3-D singularities X^3 are physically in a preferred role since they define loci of non-determinism as memory seats. They also define generalized vertices.

2. The 3-D light-like partonic orbits Y^3 are interfaces of regions of the space-time surface with Minkowskian and Euclidean signature. The induced metric is effectively 2-D at them.

3. 2-D string worlds Z^2 sheets appear as 2-D self-intersections of space-time surfaces X^4 obtained at the limit of the intersections of X^4 when a small deformation X^4 becomes trivial. The intersections $Y^1 = Z^2 \cap Y^3$ of string world sheets and partonic 3-surfaces are identified as fermion lines.

4. 3-D light-like partonic orbits Y^3 and singular surfaces X^3 intersect along 2-D partonic 2-surfaces Y^2. The light-curves Y^1 defining the fermion lineas intersect these partonic 2-surfaces Y^2 at a discrete set of points for which the conditions $(f_1, f_2) = (0, 0)$ is true. By 2-dimensionality of Y^2 the roots are complex numbers. If f_i is polynomial, one can assign to the roots discriminant D_i. As a matter of fact, this is true even when f_i is not a polynomial.

5. One can assign to all intersections $Y^2_{i,j}$ of partonic orbits Y^3_i and singular 3-surfaces X^3_j discriminant D defined as the product $D(i, j) = D_1D_2(Y^2_{i,j})$ of the discriminants for f_1 and f_2 at the partonic 2-surface $Y^2_{ij} = X^3_i Y^3_i$. The roots as points of the partonic surface with coordinate z would be loci for fermions and the idea that fermions code for the classical dynamics is highly attractive.

If the complex coordinate z for Y^2 is unique, the roots have general coordinate invariant meaning. Whether this kind of generalized complex coordinate can be identified, is not clear. A weaker condition would be that only the discriminant is a general coordinate invariant, say conformal invariant. The Hamilton-Jacobi structure poses strong conditions on the coordinates of M^4.

6. The proposal has been that a power of the product $D = \prod_{i,j} D_{i,j}$ of the discriminants $D_{i,j}$ be identified as the exponent of the classical action for X^4. This would give strong constraints to the form of the action, in particular the parameters appearing in it so that the interpretation as an effective action would be appropriate. If one requires that the discriminants are rational numbers or in an extension E of rationals, the conditions are really strong: this kind of condition is not however necessary. The discriminant hypothesis would solve the theory in the sense that the calculations of the exponent of classical action defining the vacuum functional could be reduced to number theory.

7. How uniquely the value of D characterizes the space-time surface as a root of (f_1, f_2)? D depends on the choice of H-J structure but this conforms with the fact that also the space-time surface depends on this choice.

   Could the value of D and the fermionic loci as complex roots fix the space-time surface for a given Hamilton-Jacobi structure and the choice of CD? This would mean a strong form of holography and might be unrealistic. For instance, symmetry related space-time surfaces would have the same value of action and D. Also quantum criticality implies a degeneracy so that there could be a large number of space-time surfaces with the same action exponential. The fermionic loci at the singularities would allow to distinguish between symmetry-related space-time surfaces.

   One the other hand, if f_i are polynomials, there is a finite number $N = N_1+N_2$ of coefficients for both of them: fixing the values of $f = (f_1, f_2)$ at $N = N_1 + N_2$ points associated with the fermionic loci at the singularities could fix the coefficients.

8. The Gödelian dream would be that D is analogous to Gödel number fixing space-time surface and therefore the statement represented by the space-time surface.

4.4 The maps $g : C^2 \to C^2$ as symmetries of H-H princple

The maps $f = (f_1, f_2) : H \to C^2)$ define what might be called functional integers or rationals, depending on whether f_i are polynomials or rational functions. The coefficient field can be assumed to be an extension E of rationals. The functional numbers are fundamental and cognition made possible by the existence of symmetries $g : C^2 \to C^2$ and the slight failure of determinism should make possible the representation of these functional numbers in terms of space-time surfaces satisfying $(f_1, f_2) = (c_1, c_2)$. How to achieve this?

1. The value pairs $(f_1, f_2) = (c_1, c_2)$ define a representation of the functional number f as space-time surfaces identified as roots of $(f_1 - c_1, f_2 - c_2)$. A representation of the functional number as a many-particle system would be in question. Only a finite subset of values (c_1, c_2) can be realized in this way in practice.

2. If one is interested in representing only a subset $\{(c_1^{k)}, c_2^{k)})\}$ of the spectrum of f one can consider a map $g : H \to C$ defined as $F = \prod_k [(f_1 - c_1^{k)})^2 + (f_2 - c_2^{k)})^2]$ as a representation for the surfaces. Now the polynomials are reducible. c_i should belong to an extension of E. This kind of representation is however rather tricky.

3. For E-rational functions, the roots of $(f_1, f_2) = (0, 0)$ (and $(f_1 - c_1, f_2 - c_2) = 0$) can be solved by first solving from $f_2 = 0$ the the one of the 3 complex coordinates of H, say z_1 as an algebraic function $z_1(z_2, z_3, u)$ of the remaining two complex coordinate and one hypercomplex coordinate. After that one has equation $f_1(z_1(z_2, z_3, u)), z_2, z_3, u) = 0$ and for instance z_2 can be solved as algebraic function of $z_2(z_3, u)$. Note that the second light-like coordinate v dual to u is a dummy variable so that one obtains a 4-D algebraic surface.

4. For an E-rational map $g = (g_1, g_2) : C^2 \to C^2$ $g \circ (f_1, f_2) = (0,0)$ selects a set or parameters $(c_1^{k)}, c_2^{k)})$ as the roots of g in turn defining surfaces $(f_1, f_2) = (c_1^{k)}, c_2^{k)})$. A number theoretically motivated hypothesis is that the allowed values of $(c_1^{k)}, c_2^{k)})$ belong to the extension E of rationals so that one obtains hierarchies of discretizations of WCW.

Some general comments about the role of the maps g are in order.

1. This picture allows to understand why the hierarchies of the compositions of maps g applied to a space-time surface defined by $(f_1, f_2) = (0,0)$ which is prime in the sense that it does not allow a representation as a functional composite $f = g \circ h$ is so important. One can say that prime pairs f represent the substrate and the maps g represent the cognition for which the goal is to understand the substrate represented by f.

2. The degree d of rational function $R = P_1/P_2$, defined as the difference $d(P_1) - d(P_2)$ of the polynomials appearing in it, is analogous to polynomial degree and is multiplicative in the functional composition. Therefore rational functions with prime degree are special. One could define the generalization of functional p-adic numbers. Functional p-adic would be a sum of powers of rational function with prime degree multiplied by coefficients, which are rational functions of a lower degree.

3. Galois groups are an important part of the TGD view of cognition. One can also ask whether one can assign a Galois group to the roots of g identifiable as space-time surfaces. Google LLM informs that the notion of Galois group is extremely general and this makes sense. The Galois group would act as a flow permuting the roots. In the same way, one could assign a Galois group to the roots of $(f_1, f_2) = 0$ having identification as space-time regions. This interpretation was proposed in [L12, L15] as an intuitive notion.

4.5 Could birational maps $g: C^2 \to C^2$ play a special role?

The roots of $g: C^2 \to C^2$ define the corresponding spectrum $(f_1, f_2) = (c_1, c_2)$. One can argue that since the maps g relate to cognition, the maps $g: C^2 \to C^2$ for which roots can be solved analytically are cognitively very special and might be those, which appear first in the number theoretic evolution.

1. For birational maps $g = (g_1, g_2) : C^2 \to C^2$ also the inverses g^{-1} are rational maps. This allows us to solve the roots g analytically as $(c_1^{k)}, c_2^{k)}) = g^{-1}(0,0)$.

2. The birational maps $g: C^2 \to C^2$ are known as Cremona transformations (see this) and have a very rich structure. Their functional composition is possible but they do not form a group unlike in the case of CP_1 (Möbius transformations). Note that Möbius transformations as linear transformations act as symmetries of C^2.

3. Cremona transformations are generated by projective transformations of CP_2 and the so-called standard quadratic transformation $(z_1, z_2) \to (1/z_1, 1/z_2)$, which is singular at the origin. Projective transformations correspond to the group $SL(3, C)$ containing the isometry group $SU(3)$ of CP_2 as a subgroup. The implication of fundamental importance for TGD is that Cremona transforms a spectrum generating algebra rather than acting as a mere symmetry group.

4. The so called polynomial automorphisms are birational maps, which are everywhere well-defined isomorphisms of C^2 to itself. The Jung-van der Kulk theorem states that every polynomial automorphism is a composition of affine maps and elementary maps of form $(z_1, z_2) \to z_1, z_2) \to z_1, z_2 + P(z_1)$, where $P(z_1)$ is a composition of affine maps and polynomial. These transformations could be regarded as symmetries.

5. Every Cremona transformation can be factored into a sequence of point blow-ups followed by a finite sequence of point blow-downs. Geometrically this means blowing up a set of points where the map is undefined and then contracting the curves down to points to create the image surface.

6. At the blow-up points the Cremona transformation gives an indeterminate expression like $0/0$ and the value of the expression depends on the direction from which the point is approached. In the case of Cremona transformation a single point as image or pre-image is substituted by an entire projective in CP_1.

   A familiar analog is the Coulomb force of the point charge which has ill-defined direction at the charge. In the case of Coulomb force, the point is replaced by a sphere. By the argument already given, the blow-up means that C^2 is replaced by CP_2 by adding the sphere CP_1 at infinity.

7. A concrete example is provided by the standard Cremona transformation $(z_1, z_2) \rightarrow (z_2/z_1 z_2, z_2/z_1 z_2)$. Origin is the problem point. The approach to origin along the line $z_2 = m z_1$ gives in projective coordinates the point $(1/m, 1)$ which gives to CP_1 instead of a single point. Blow-up as the addition of CP_1 to infinity replacing C^2 withCP_2 is the solution of the problem.

   For g this would produce a sphere as an image point. There is no problem with the inverse image of $(0,0)$ since it does not belong to CP_1 at infinity so that $g^{-1}(0,0)$ is well-defined.

   For $g^{-1}(0,0)$, (c_1, c_2) would be ill-defined. For projective coordinates there would be an entire CP_1 of points (c_1, c_2) defining space-time surfaces. One can solve the problem by concluding that the coordinates for CP_2 become ill-defined in this situation and one must use another coordinate patch: 3 patches are enough. In a new coordinate patch $g^{-1}(0,0)$ is unique and obtains a single space-time surface.

   One could also argue that the inverse image is a union of the space-time surfaces labelled by the points of CP_1 and therefore 6-dimensional rather than 4-dimensional. Space-time surfaces can be regarded as intersections of 6-D $f_1 = 0$ and $f_2 = 0$ surfaces having a twistor space interpretat and the singularity could mean that the intersecting 6-surfaces are identical in this case In the case of Coulomb force, point is replaced by a sphere. The M-theory analog would be 4-dimensional 3-brane.

8. The degree of the rational map $g : C^2 \rightarrow C^2$ defines the analog for the degree of a polynomial. For the iterations of many families of maps this degree behaves like $d_n = 3d_{n-1} - 2d_{n-2}$. At the limit $n \rightarrow \infty$ the asymptotic behavior $d_{n+1} - d_n \rightarrow \Delta$ is consistent with this condition and means that complexity as the increase of the degree increases but not exponentially as in the general case. If genuine symmetries were in question, the complexity would not increase. The behavior of d_n can also be periodic and even chaotic.

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