Quantum Hall effect and Hierarchy of Planck Constants

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Contents

Abstract

In this chapter I try to formulate more precisely the recent TGD based view about fractional quantum Hall effect (FQHE). This view is much more realistic than the original rough scenario, which neglected the existing rather detailed understanding. The spectrum of ν, and the mechanism producing it is the same as in composite fermion approach. The new elements relate to the not so well-understood aspects of FQHE, namely charge fractionization, the emergence of braid statistics, and non-abelianity of braid statistics.

1. The starting point is composite fermion model so that the basic predictions are same. Now magnetic vortices correspond to (Kähler) magnetic flux tubes carrying unit of magnetic flux. The magnetic field inside flux tube would be created by delocalized electron at the boundary of the vortex. One can raise two questions.

   Could the boundary of the macroscopic system carrying anyonic phase have identification as a macroscopic analog of partonic 2-surface serving as a boundary between Minkowskian and Euclidian regions of space-time sheet? If so, the space-time sheet assignable to the macroscopic system in question would have Euclidian signature, and would be analogous to blackhole or to a line of generalized Feynman diagram.

   Could the boundary of the vortex be identifiable a light-like boundary separating Minkowskian magnetic flux tube from the Euclidian interior of the macroscopic system and be also analogous to wormhole throat? If so, both macroscopic objects and magnetic vortices would be rather exotic geometric objects not possible in general relativity framework.

2. Taking composite model as a starting point one obtains standard predictions for the filling fractions. One should also understand charge fractionalization and fractional braiding statistics. Here the vacuum degeneracy of Kähler action suggests the explanation. Vacuum degeneracy implies that the correspondence between the normal component of the canonical momentum current and normal derivatives of imbedding space coordinates is 1- to-n. These kind of branchings result in multi-furcations induced by variations of the system parameters and the scaling of external magnetic field represents one such variation.

3. At the orbits of wormhole throats, which can have even macroscopic M^4 projections, one has $1 \rightarrow n_a$ correspondence and at the space-like ends of the space-time surface at light-like boundaries of causal diamond one has $1 \rightarrow n_b$ correspondence. This implies that at partonic 2-surfaces defined as the intersections of these two kinds of 3-surfaces one has $1 \rightarrow n_a \times n_b$ correspondence. This correspondence can be described by using a local singular n-fold covering of the imbedding space. Unlike in the original approach, the covering space is only a convenient auxiliary tool rather than fundamental notion.

4. The fractionalization of charge can be understood as follows. A delocalization of electron charge to the n sheets of the multi-furcation takes place and single sheet is analogous to a sheet of Riemann surface of function $z^{1/n}$ and carries fractional charge $q = e/n$, $n = n_a n_b$. Fractionalization applies also to other quantum numbers. One can have also many-electron stats of these states with several delocalized electrons: in this case one obtains more general charge fractionalization: $q = \nu e$.

5. Also the fractional braid statistics can be understood. For ordinary statistics rotations of M^4 rotate entire partonic 2-surfaces. For braid statistics rotations of M^4 (and particle exchange) induce a flow braid ends along partonic 2-surface. If the singular local covering is analogous to the Riemann surface of $z^{1/n}$, the braid rotation by $\Delta\Phi = 2\pi$, where Φ corresponds to M^4 angle, leads to a second branch of multi-furcation and one can give up the usual quantization condition for angular momentum. For the natural angle coordinate Φ of the n-branched covering $\Delta\Phi = 2/pi$ corresponds to $\Delta\Phi = n \times 2\pi$. If one identifies the sheets of multi-furcation and therefore uses Φ as angle coordinate, single valued angular momentum eigenstates become in general n-valued, angular momentum in braid statistics becomes fractional and one obtains fractional braid statistics for angular momentum.

6. How to understand the exceptional values $\nu = 5/2, 7/2$ of the filling fraction? The non-abelian braid group representations can be interpreted as higher-dimensional projective representations of permutation group: for ordinary statistics only Abelian representations are possible. It seems that the minimum number of braids is $n > 2$ from the condition of non-abelianity of braid group representations. The condition that ordinary statistics is fermionic, gives $n > 3$. The minimum value is $n = 4$ consistent with the fractional charge $e/4$.

The model introduces Z_4 valued topological quantum number characterizing flux tubes. This also makes possible non-Abelian braid statistics. The interpretation of this quantum number as a Z_4 valued momentum characterizing the four delocalized states of the flux tube at the sheets of the 4-furcation suggests itself strongly. Topology would corresponds to that of 4-fold covering space of imbedding space serving as a convenient auxiliary tool. The more standard explanation is that $Z_4 = Z_2 \times Z_2$ such that Z_2:s correspond to the presence or absence of neutral Majorana fermion in the two Cooper pair like states formed by flux tubes.

What remains to be understood is the emergence of non-abelian gauge group realizing non-Abelian fractional statistics in gauge theory framework. Electroweak gauge group defined non-abelian braid group in large h_{eff} phase weak length above atomic length scale so that weak bosons and even fermion behave as effectively massless particles below scaled up weak scale. TGD also predicts the possibility of dynamical gauge groups and maybe this kind of gauge group indeed emerges. Dynamical gauge groups emerge also for stacks of N branes and the n sheets of multifurcation are analogous to the N sheets in the stack for many-electron states.

1 Introduction

Quantum Hall effect [D10, D13, D2] occurs in 2-dimensional systems, typically a slab carrying a longitudinal voltage V causing longitudinal current j. A magnetic field orthogonal to the slab generates a transversal current component j_T by Lorentz force. j_T is proportional to the voltage V along the slab and the dimensionless coefficient is known as transversal conductivity. Classically the coefficients is proportional ne/B, where n is 2-dimensional electron density and should have a continuous spectrum. The finding that came as surprise was that the change of the coefficient as a function of parameters like magnetic field strength and temperature occurred as discrete steps of same size. In integer quantum Hall effect the coefficient is quantized to $2\nu\alpha$, $\alpha = e^2/4\pi$, such that ν is integer.

Later came the finding that also smaller steps corresponding to the filling fraction $\nu = 1/3$ of the basic step were present and could be understood if the charge of electron would have been replaced with $\nu = 1/3$ of its ordinary value. Later also QH effect with wide large range of rational filling fractions of form $\nu = k/m$ was observed.

The observed fractions are not arbitrary but obey quite precise selection rules. This led to the notion of composite fermion (see `http://tinyurl.com/ycotdnyh`) as a bound state of electron and magnetic vortices carrying electron at their boundaries and FQHE reduces to QHE for these effective particles in an effective magnetic field than the original field and reduced by the binding of the magnetic vortices with the electron. Besides quasiparticles also corresponding holes contribute to FQHE.

What I see as a general problem of the composite model is the nature of the bound states. How both the number of vortices per electron and the number of electrons per vortex can be larger than one?

The composite model fails to explain only the observations for $\nu = 5/2$ and $\nu = 7/2$ (these values of ν belong to the spectrum but the these phases do not behave as predicted). The conjecture is that in these cases electron carrying magnetic vortices form Cooper pair like bound states. Non-Abelian braid statistics assigned with these phases would be essential for topological quantum computation but it has not been established convincingly yet [D3] (see `http://tinyurl.com/y8ahwh3x`).

Also the well-established charge fractionization should be understood. Whether fractional braid statistics is realized is still an open question. In these issues TGD might provide some new insights.

The phenomenology of FQHE is described in an extremely lucid manner in the Nobel lecture (see `http://tinyurl.com/y8mvdxpk`) of Horst L. Stormer [D7]. As a matter fact, I regret that I did not read it for years ago!

1.1 Abelian And Non-Abelian Anyons

The model explaining FQHE is based on pseudo particles known as anyons identifiable as magnetic vortices [A3] , [D10]. According to the general argument of [D15] anyons have a fractional charge νe. The braid statistics of anyon is believed to be fractional so that in the general case anyons are neither bosons nor fermions. Non-fractional statistics is absolutely essential for the vacuum degeneracy used to represent logical qubits.

In the case of Abelian anyons the gauge potential corresponds to the vector potential of the divergence free velocity field or equivalently of incompressible anyon current. For Abelian anyons the field theory defined by Chern-Simons action is free field theory and in well-defined sense trivial although it defines knot invariants. For non-Abelian anyons situation would be different. They would carry non-Abelian gauge charges possibly related to a symmetry breaking to a discrete subgroup H of gauge group [A3] each of them defining an incompressible hydrodynamical flow. According to [B3] the anyons associated with the filling fraction $\nu = 5/2$ are a good candidate for non-Abelian anyons and in this case the charge of electron is reduced to $Q = e/4$ rather than being $Q = \nu e$ [D4]. This finding favors non-Abelian models [D2].

Non-Abelian anyons [D10, D11] are always created in pairs since they carry a conserved topological charge. In the model of [B3] this charge should have values in 4-element group Z_4 so that it is conserved only modulo 4 so that charges +2 and -2 are equivalent as are also charges 3 and -1. The state of n anyon pairs created from vacuum can be shown to possess 2^{n-1}-dimensional vacuum degeneracy [D12]. When two anyons fuse the 2^{n-1}-dimensional state space decomposes to 2^{n-2}-dimensional tensor factors corresponding to anyon Cooper pairs with topological charges 2 and 0. The topological "spin" is ideal for representing logical qubits. Since free topological charges are not possible the notion of physical qubit does not make sense (note the analogy with quarks). The measurement of topological qubit reduces to a measurement of whether anyon Cooper pair has vanishing topological charge or not.

1.2 TGD Based View About Fqhe

In this chapter I try to formulate more precisely the recent TGD based view about fractional quantum Hall effect (FQHE) (see `http://tinyurl.com/y8mvdxpk`) . This view is much more realistic than the original rough scenario, which neglected the existing rather detailed understanding. The spectrum of ν, and the mechanism producing it is the same as in composite fermion approach. The new elements relate to the not so well-understood aspects of FQHE, namely charge fractionization, the emergence of braid statistics, and non-abelianity of braid statistics.

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The genuinely new element to the existing theory of FQHE are multi-furcations of partonic 2-surfaces and their second quantization. This notion leads to an explanation of the fractional charges, fractional braid statistics, and existence of Z_n valued topological quantum number in terms of many-sheeted space-time and mulfi-furcations of preferred extremals of Kähler action. One ends up also to a concrete geometric realization for the bound states of electron and flux quanta and geometric understanding of how n flux quanta "split" out from the magnetic field experienced by the electron. The rather radical "almost prediction" is that partonic 2-surfaces and their light-like orbits serving as boundaries between Euclidian and Minkowskian regions of space-time sheet would be realized even in macroscopic scales. Anyonic system would be in well-defined sense an elementary particle like object.

The first two sections of the chapter give brief summaries about FQHE and existing theories of FQHE. The third section represents a view about the effective hierarchy of Planck constants assignable to multi-furcations associated with Kähler action and the recent simplifications of this picture. The last section summarizes the TGD inspired model of FQHE, a model for flux tubes, a microscopic description for the 2-D surface representing the boundary of the anyonic system and with electrons attached to this surface. Here the TGD based view about elementary particles is in active role.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [?].

2 Fractional Quantum Hall Effect

2.1 Basic Facts About FQHE

2.2 A Simple Model For Fractional Quantum Hall Effect

Recall first the basic facts. Quantum Hall effect (QHE) [D10, D1, D13] is an essentially 2-dimensional phenomenon and occurs at the end of current carrying region for the current flowing transversally along the end of the wire in external magnetic field along the wire. For quantum Hall effect transversal Hall conductance characterizing the 2-dimensional current flow is dimensionless and quantized and given by

$$\sigma_{xy} = 2\nu\alpha_{em} \ ,$$

ν is so called filling factor telling the number of filled Landau levels in the magnetic field. In the case of integer quantum Hall effect (IQHE) ν is integer valued. For fractional quantum Hall effect (FQHE) ν is rational number.

The formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{2.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15...$, $2/3, 3/5, 4/7, 5/9, 6/11, 7/13...$, $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9...$, $1/5, 2/9, 3/13...$, $2/7, 3/11...$, $1/7....$ with odd denominator have been observed [D1]. Only fractions smaller than 1 are listed because the integer part of ν should not matter since it represents full Landau levels. Also $\nu = 1/2$, $\nu = 5/2$, $7/2$ states with even denominator have been observed. $\nu = 1/2$ can be understood easily in the existing theory. One might think that $\nu = 5/2 = 2 + 1/2$ and $\nu = 7/2 = 3 + 1/2$ would reduce to $\nu = 1/2$. This not however the case experimentally and these values of ν represent an unsolved problem of anyon physics.

The following gives a brief summary about the evolution of the understanding of FQHE.

1. Laughlin introduced his many-electron wave wave function predicting fractional quantum Hall effect for filling fractions $\nu = 1/m$ [D13]. The model of Laughlin [D13] cannot explain all observed filling fractions.

2. The best existing model proposed originally by Jain [D8] is based on the notion of composite fermion. These would result as bound states of electron and even number of magnetic flux quanta [D8]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

3. The description of the magnetic flux tubes led to the notion of anyon introduced by Wilzeck [D10]. Anyon has been compared to a vortex like excitation of a dense 2-D electron plasma formed by the current carriers. ν is inversely proportional to the magnetic flux and the fractional filling factor can be also understood in terms of fractional magnetic flux.

4. The starting point of the quantum field theoretical models is the effective 2-dimensionality of the system implying that the projective representations for the permutation group of n objects are representations of braid group allowing fractional statistics. This is due to the non-trivial first homotopy group of 2-dimensional manifold containing punctures. Quantum field

theoretical models allow to assign to the anyon-like states also magnetic charge, fractional spin, and fractional electric charge.

Topological quantum computation [K5, K3] , [B3], [C1] is one of the most fascinating applications of FQHE. It relies on the notion of braids with strands representing the orbits of of anyons. The unitary time evolution operator coding for topological computation is a representation of the element of the element of braid group represented by the time evolution of the braid. It is essential that the group involved is non-Abelian so that the system remembers the order of elementary braiding operations (exchange of neighboring strands). There is experimental evidence that $\nu = 5/2$ anyons possessing fractional charge $Q = e/4$ are non-Abelian [D2, D4].

Before continuing, it is good to represent both classical view about QHE effect and simple quantum explanation for IQHE effect.

1. Consider first the classical explanation. Electrons are assumed to drift in the orthogonal electric and magnetic fields with drift velocity $v = E \times B/B^2$ having magnitude $v = E/B$ It is easy to see that this solves Newtons equation of motion identically. Here the 2-D current transversal Hall current can be written as $j = e\rho v$, where ρ is 2-D electron density obtained by averaging in the direction of the electric field. This can be expressed as $j = e(N/S)(E/B)$, where one concludes that the Hall conductivity is given by

$$\sigma_{xy} = \frac{\rho}{B} = e^2\frac{N}{\Phi} \quad , \quad \Phi = eBS = e\int BdS \quad .$$

Using elementary flux quantum as a unit of magnetic flux, this says that Hall conductivity equals to the ratio of electrons per elementary flux quantum. To proceed further one must use quantization of electron's states in the magnetic field to concluded that N equals to integer multiple of $h\Phi$.

2. Consider next a quantum explanation. Choose the coordinates of the current currying slab so that x varies in the direction of Hall current and y in the direction of the main current. For IQHE the value of Hall conductivity is given by $\sigma = j_y/E_x = n_e ev/vB = n_e e/B = Ne^2/heBS = Ne^2/mh$, were m characterizes the value of magnetized flux and N is the total number of electrons in the current. In the Landau gauge $A_y = xB$ one can assume that energy eigenstates are momentum eigenstates in the direction of current and harmonic oscillator Gaussians in x-direction in which Hall current runs. This gives

$$\Psi \propto exp(iky)H_n(x + kl^2)exp(-\tfrac{(x+kl^2)^2}{2l^2}) \quad , \quad l^2 = \tfrac{\hbar}{eB} \quad . \tag{2.2}$$

Only the states for which the oscillator Gaussian differs considerably from zero inside slab are important so that the momentum eigenvalues are in good approximation in the range $0 \leq k \leq k_{max} = L_x/l^2$. Using $N = (L_y/2\pi)\int_0^{k_{max}} dk$ one obtains that the total number of momentum eigenstates associated with the given value of n is $N = eBdL_xL_y/h = n$. If ν Landau states are filled, the value of σ is $\sigma = \nu e^2/h$, where ν is the integer valued filling fraction.

The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires such a low temperature.

2.3 The Model Of FQHE Based On Composite Fermions

The model of FQHE based on omposite fermions (see `http://tinyurl.com/y9vwmjy5`) produces FQHE as integer QHE for effective particles - composite fermions. This phenomenological picture is described with enjoyable clarity in the Nobel lecture of Nobel lecture (see `http://tinyurl.com/y8mvdxpk`) of Horst L. Stormer [D7].

The empirical inspiration for the model is the observation that in strong enough magnetic fields electrons behave in an unexpected manner. For instance, they seem to respond only to effective magnetic field much weaker than the actual field. The difference in field strengths corresponds to an integer number of magnetic flux quanta multiplied by 2-D electron number density. It would seem that these flux quanta somehow separate from the external magnetic field and somehow combine with the electrons to form bound states, which become the basic dynamical units interacting with the external magnetic field. They of course have different mass.

From the experimental data one can conclude that the flux quanta behave like fermions. Most naturally they would carry a rotating electron current concentrated near their boundaries and serving as a source of a magnetic field concentrated around flux quantum or better to say, separating a magnetic field from incoming magnetic field outside the flux quantum. By the conservation of magnetic flux the external magnetic field is reduced correspondingly. The number of flux quanta per electron is integer valued. Since flux quanta behave like fermions, the number of the flux quanta per electron is even for fractional quantum Hall effect. For odd values of flux quanta one obtains composite bosons and something totally different.

The basic formula for the filling fraction is easy to deduce by using the assumption that FQHE is IQHE in effective magnetic field B_{eff} [D8] with even number $2p$ flux quanta subtracted. B_{eff} is given by

$$B_{eff} = B - 2pB_1 \ , \ B_1 = \rho\Phi_0 \ , \ \Phi_0 = h/e \ . \tag{2.3}$$

Here B is the external magnetic field, $2p$ the even number of flux quanta per electron, and Φ_0 the elementary flux quantum - twice the flux quantum in super-conductors because the charge carriers are now electrons rather than Cooper pairs.

The integer QHE for B_{eff} gives $\nu_{eff} = \rho\phi_0/B_{eff} = B_1/B_{eff} = n$ saying that n Landau levels are filled. This translates FQHE for B with ν given by completely analogous formula $\nu = B_1/B$. From $\nu_{eff} = 1/[(B/B_1) - 2p] = n = 1/(1/\nu) - 2p$ one obtains

$$\nu = \frac{\nu_{eff}}{1 + 2p\nu_{eff}} = \frac{n}{1 + 2pn} \ . \tag{2.4}$$

The formula is amazingly simple and consistent with all experimental findings hitherto. Note that at the limit $n \to \infty$ the formula gives filling fractions $1/2p$.

My understanding is that charge fractionalization is motivated by a paradox created by the flux quantum picture. Classically the number of electrons per flux quantum is higher than one since the generation of flux quantum requires at least one electron per flux quantum. How it is then possible that the number of flux quanta per electron given by ν is higher than one?

What this fractionalization actually means geometrically is not easy to visualize and might require new physics. The solution of the paradox might also require a de-localization of some kind. The number of electrons in the center and at the boundary of flux quantum is fractional. This could might be understood in terms of de-localization of electron wave functions at several flux quanta. In TGD framework electron is string like object defined by Kähler magnetic flux tube with wormhole contacts at its ends and could have rather long length: could it be that also electron charge is de-localized and shared between the two wormhole ends?

3 About Theories Of Quantum Hall Effect

The most elegant models of quantum Hall effect are in terms of anyons regarded as singularities due to the symmetry breaking of gauge group G down to a finite sub-group H, which can be also non-Abelian. Concerning the description of the dynamics of topological degrees of freedom topological quantum field theories based on Chern-Simons action are the most promising approach.

3.1 Quantum Hall Effect As A Spontaneous Symmetry Breaking Down To A Discrete Subgroup Of The Gauge Group

The system exhibiting quantum Hall effect is effectively 2-dimensional. Fractional statistics suggests that topological defects, anyons, allowing a description in terms of the representations of the homotopy group of $((R^2)^n - D)/S_n$. The gauge theory description would be in terms of spontaneous symmetry breaking of the gauge group G to a finite subgroup H by a Higgs mechanism [A3] , [D10]. This would make all gauge degrees of freedom massive and leave only topological degrees of freedom. What is unexpected that also non-Abelian topological degrees of freedom are in principle possible. Quantum Hall effect is Abelian or non-Abelian depending on whether the group H has this property.

In the symmetry breaking $G \rightarrow H$ the non-Abelian gauge fluxes defined as non-integrable phase factors $Pexp(i \oint A_\mu dx^\mu)$ around large circles (surrounding singularities (so that field approaches a pure gauge configuration) are elements of the first homotopy group of G/H, which is H in the case that H is discrete group and G is simple. An idealized manner to model the situation [D10] is to assume that the connection is pure gauge and defined by an H-valued function which is many-valued such that the values for different branches are related by a gauge transformation in H. In the general case a gauge transformation of a non-trivial gauge field by a multi-valued element of the gauge group would give rise to a similar situation.

One can characterize a given topological singularity magnetically by an element in conjugacy class C of H representing the transformation of H induced by a 2π rotation around singularity. The elements of C define states in given magnetic representation. Electrically the particles are characterized by an irreducible representations of the subgroup of $H_C \subset H$ which commutes with an arbitrarily chosen element of the conjugacy class C.

The action of $h(B)$ resulting on particle A when it makes a closed turn around B reduces in magnetic degrees of freedom to translation in conjugacy class combined with the action of element of H_C in electric degrees of freedom. Closed paths correspond to elements of the braid group $B_n(X^2)$ identifiable as the mapping class group of the punctured 2-surface X^2 and this means that symmetry breaking $G \rightarrow H$ defines a representation of the braid group. The construction of these representations is discussed in [D10] and leads naturally via the group algebra of H to the so called quantum double $D(H)$ of H, which is a quasi-triangular Hopf algebra allowing non-trivial representations of braid group.

Anyons could be singularities of gauge fields, perhaps even non-Abelian gauge fields, and the latter ones could be modelled by these representations. In particular, braid operations could be represented using anyons.

3.2 Witten-Chern-Simons Action And Topological Quantum FieldTheories

The Wess-Zumino-Witten action used to model 2-dimensional critical systems consists of a 2-dimensional conformally invariant term for the chiral field having values in group G combined with 2+1-dimensional term defined as the integral of Chern-Simons 3-form over a 3-space containing 2-D space as its boundary. This term is purely topological and identifiable as winding number for the map from 3-dimensional space to G. The coefficient of this term is integer k in suitable normalization. k gives the value of central extension of the Kac-Moody algebra defined by the theory.

One can couple the chiral field $g(x)$ to gauge potential defined for some subgroup of G_1 of G. If the G_1 coincides with G, the chiral field can be gauged away by a suitable gauge transformation and the theory becomes purely topological Witten-Chern-Simons theory. Pure gauge field configuration represented either as flat gauge fields with non-trivial holonomy over homotopically non-trivial paths or as multi-valued gauge group elements however remain and the remaining degrees of freedom correspond to the topological degrees of freedom.

Witten-Chern-Simons theories are labelled by a positive integer k giving the value of central extension of the Kac-Moody algebra defined by the theory. The connection with Wess-Zumino-Witten theory come from the fact that the highest weight states associated with the representations of the Kac-Moody algebra of WZW theory are in one-one correspondence with the representations R_i possible for Wilson loops in the topological quantum field theory.

In the Abelian case case 2+1-dimensional Chern-Simons action density is essentially the inner product $A \wedge dA$ of the vector potential and magnetic field known as helicity density and the theory in question is a free field theory. In the non-Abelian case the action is defined by the 3-form

$$\frac{k}{4\pi}Tr\left(A\wedge(dA+\frac{2}{3}A\wedge A)\right)$$

and contains also interaction term so that the field theory defined by the exponential of the interaction term is non-trivial.

In topological quantum field theory the usual n-point correlation functions defined by the functional integral are replaced by the functional averages for $Diff^3$ invariant quantities defined in terms of non-integrable phase factors defined by ordered exponentials over closed loops. One can consider arbitrary number of loops which can be knotted, linked, and braided. These quantities define both knot and 3-manifold invariants (the functional integral for zero link in particular). The perturbative calculation of the quantum averages leads directly to the Gaussian linking numbers and infinite number of perturbative link and not invariants.

The experience gained from topological quantum field theories defined by Chern-Simons action has led to a very elegant and surprisingly simple category theoretical approach to the topological quantum field theory [A1, A4] allowing to assign invariants to knots, links, braids, and tangles and also to 3-manifolds for which braids as morphisms are replaced with cobordisms. The so called modular Hopf algebras, in particular quantum groups $Sl(2)_q$ with q a root of unity, are in key role in this approach. Also the connection between links and 3-manifolds can be understood since closed, oriented, 3-manifolds can be constructed from each other by surgery based on links [K1].

Witten's article [A2] "Quantum Field Theory and the Jones Polynomial" is full of ingenious constructions, and for a physicist it is the easiest and certainly highly enjoyable manner to learn about knots and 3-manifolds. For these reasons a little bit more detailed sum up is perhaps in order.

1. Witten discusses first the quantization of Chern-Simons action at the weak coupling limit $k\rightarrow\infty$. First it is shown how the functional integration around flat connections defines a topological invariant for 3-manifolds in the case of a trivial Wilson loop. Next a canonical quantization is performed in the case $X^3=\Sigma^2\times R^1$: in the Coulomb gauge $A_3=0$ the action reduces to a sum of $n=dim(G)$ Abelian Chern-Simons actions with a non-linear constraint expressing the vanishing of the gauge field. The WCW consists thus of flat non-Abelian connections, which are characterized by their holonomy groups and allows Kähler manifold structure.

2. Perhaps the most elegant quantal element of the approach is the decomposition of the 3-manifold to two pieces glued together along 2-manifold implying the decomposition of the functional integral to a product of functional integrals over the pieces. This together with the basic properties of Hilbert of complex numbers (to which the partition functions defined by the functional integrals over the two pieces belong) allows almost a miracle like deduction of the basic results about the behavior of 3-manifold and link invariants under a connected sum, and leads to the crucial skein relations allowing to calculate the invariants by decomposing the link step by step to a union of unknotted, unlinked Wilson loops, which can be calculated exactly for $SU(N)$. The decomposition by skein relations gives rise to a partition function like representation of invariants and allows to understand the connection between knot theory and statistical physics [A5]. A direct relationship with conformal field theories and Wess-Zumino-Witten model emerges via Wilson loops associated with the highest weight representations for Kac Moody algebras.

3. A similar decomposition procedure applies also to the calculation of 3-manifold invariants using link surgery to transform 3-manifolds to each other, with 3-manifold invariants being defined as Wilson loops associated with the homology generators of these (solid) tori using representations R_i appearing as highest weight representations of the loop algebra of torus. Surgery operations are represented as mapping class group operations acting in the Hilbert space defined by the invariants for representations R_i for the original 3-manifold. The outcome is explicit formulas for the invariants of trivial knots and 3-manifold invariant of S^3 for $G=SU(N)$, in terms of which more complex invariants are expressible.

4. For $SU(N)$ the invariants are expressible as functions of the phase $q=exp(i2\pi/(k+N))$ associated with quantum groups [K1]. Note that for $SU(2)$ and $k=3$, the invariants are

expressible in terms of Golden Ratio. The central charge $k = 3$ is in a special position since it gives rise to $k+1 = 4$-vertex representing naturally 2-gate physically. Witten-Chern-Simons theories define universal unitary modular functors characterizing quantum computations [B4].

3.3 Chern-Simons Action For Anyons

In the case of quantum Hall effect the Chern-Simons action has been deduced from a model of electrons as a 2-dimensional incompressible fluid [D13]. Incompressibility requires that the electron current has a vanishing divergence, which makes it analogous to a magnetic field. The expressibility of the current as a curl of a vector potential b, and a detailed study of the interaction Lagrangian leads to the identification of an Abelian Chern-Simons for b as a low energy effective action. This action is Abelian, whereas the anyonic realization of quantum computation would suggest a non-Abelian Chern-Simons action.

Non-Abelian Chern-Simons action could result in the symmetry breaking of a non-Abelian gauge group G, most naturally electro-weak gauge group, to a non-Abelian discrete subgroup H [A3] so that states would be labelled by representations of H and anyons would be characterized magnetically H-valued non-Abelian magnetic fluxes each of them defining its own incompressible hydro-dynamical flow.

3.4 Topological Quantum Computation Using Braids And Anyons

By the general mathematical results braids are able to code all quantum logic operations [B1]. In particular, braids allow to realize any quantum circuit consisting of single particle gates acting on qubits and two particle gates acting on pairs of qubits. The coding of braid requires a classical computation which can be done in polynomial time. The coding requires that each dancer is able to remember its dancing history by coding it into its own state.

The general ideas are following.

1. The ground states of anyonic system characterize the logical qubits, One assumes non-Abelian anyons with Z_4 -valued topological charge so that a system of n anyon pairs created from vacuum allows 2^{n-1}-fold anyon degeneracy [D12]. The system is decomposed into blocks containing one anyonic Cooper pair with $Q_T \in \{2, 0\}$ and two anyons with such topological charges that the net topological charge vanishes. One can say that the states $(0, 1-1)$ and $(0, -1, +1))$ represent logical qubit 0 whereas the states $(2, -1, -1)$ and $(2, +1, +1)$ represent logical qubit 1. This would suggest 2^2-fold degeneracy but actually the degeneracy is 2-fold.

   Free physical qubits are not possible and at least four particles are indeed necessarily in order to represent logical qubit. The reason is that the conservation of Z^4 charge would not allow mixing of qubits 1 and 0, in particular the Hadamard 1-gate generating square root of qubit would break the conservation of topological charge. The square root of qubit can be generated only if 2 units of topological charge is transferred between anyon and anyon Cooper pair. Thus qubits can be represented as entangled states of anyon Cooper pair and anyon and the fourth anyon is needed to achieve vanishing total topological charge in the batch.

2. In the initial state of the system the anyonic Cooper pairs have $Q_T = 0$ and the two anyons have opposite topological charges inside each block. The initial state codes no information unlike in ordinary computation but the information is represented by the braid. Of course, also more general configurations are possible. Anyons are assumed to evolve like free particles except during swap operations and their time evolution is described by single particle Hamiltonians.

   Free particle approximation fails when the anyons are too near to each other as during braid operations. The space of logical qubits is realized as k-code defined by the 2^{n-1} ground states, which are stable against local single particle perturbations for $k = 3$ Witten-Chern-Simons action. In the more general case the stability against n-particle perturbations with $n < [k/2]$ is achieved but the gates would become $[k/2]$-particle gates (for $k = 5$ this would give 6-particle vertices).

3. Anyonic system provides a unitary modular functor as the S-matrix associated with the anyon system whose time evolution is fixed by the pre-existing braid structure. What this means that the S-matrices associated with the braids can be multiplied and thus a unitary representation for the group formed by braids results. The vacuum degeneracy of anyon system makes this representation non-trivial. By the NP complexity of braids it is possible to code any quantum logic operation by a particular braid [B5]. There exists a powerful approximation theorem allowing to achieve this coding classically in polynomial time [B1]. From the properties of the R-matrices inducing gate operations it is indeed clear that two gates can be realized. The Hadamard 1-gate could be realized as 2-gate in the system formed by anyon Cooper pair and anyon.

4. In [B3] the time evolution is regarded as a discrete sequence of modifications of single anyon Hamiltonians induced by swaps [B6]. If the modifications define a closed loop in the space of Hamiltonians the resulting unitary operators define a representation of braid group in a dense discrete sub-group of $U(2^n)$. The swap operation is 2-local operation acting like a 2-gate and induces quantum logical operation modifying also single particle Hamiltonians. What is important that this modification maps the space of the ground states to a new one and only if the modifications correspond to a closed loop the final state is in the same code space as the initial state. What time evolution does is to affect the topological charges of anyon Cooper pairs representing qubits inside the 4-anyon batches defined by the braids.

   In quantum field theory the analog but not equivalent of this description would be following. Quite generally, a given particle in the final state has suffered a unitary transformation, which is an ordered product consisting of two kinds of unitary operators. Unitary single particle operators $U_n = Pexp(i\int_{t_n}^{t_{n+1}} H_0 dt)$ are analogs of operators describing single qubit gate and play the role of anyon propagators during no-swap periods. Two-particle unitary operators $U_{swap} = Pexp(i\int H_{swap} dt)$ are analogous to four-particle interactions and describe the effect of braid operations inducing entanglement of states having opposite values of topological charge but conserving the net topological charge of the anyon pair. This entanglement is completely analogous to spin entanglement. In particular, the braid operation mixes different states of the anyon. The unitary time development operator generating entangled state of anyons and defined by the braid structure represents the operation performed by the quantum circuit and the quantum measurement in the final state selects a particular final state.

5. Formally the computation halts with a measurement of the topological charge of the left-most anyon Cooper pair when the outcome is just single bit. If decay occurs with sufficiently high probability it is concluded that the value of the computed bit is 0, otherwise 1.

4 Quantum Hall Effect, Charge Fractionalization, And Hierarchy Of Planck Constants

The proportionality $\sigma_{xy} \propto \alpha_{em} \propto 1/\hbar$ suggests an explanation of FQHE [D10, D1, D13] in terms of the hierarchy of Planck constants. The idea was that perhaps filling factors and magnetic fluxes are actually integer valued but the value of Planck constant defining the unit of magnetic flux is changed from its standard value - to its rational multiple in the most general case. This naive guess turned out be incorrect.

A careful study of what was known about FQHE much before 2005 (see for instance [D7]) - in particular understanding of the notion of composite fermion - would have demonstrated that FQHE is basically IQHE for composite fermions so that fractionization cannot be due to the integer values of Planck constant or of effective Planck constant. In fact, accepting that composite fermion description one has only to explain what really happens in charge fractionization and how braid statistics emerges. One should of course also have a concrete description for the bound states of electron and flux tubes.

In the picture using multi-sheeted covering of imbedding space as an auxiliary tool, the phase transition corresponds to the leakage of 3-surface from a given 8-D page to another one in the Big Book having local singular coverings of $CD \times CP_2$ as pages. This auxiliary tool is not absolutely necessary since multi-furcations of preferred extremals of Kähler action is the fundamental notion and one can see FQHE as a function of external magnetic field as a hierarchy of multi-furcations of preferred extremals. In the following this view is adopted since this minimizes the number un-necessary assumptions.

One particular assumption of this kind in the previous approach was that the singular coverings are products of those for CD and CP_2. The coverings has product structure in the sense that the number of sheets is product of two integers but this does not require that these integers could be assigned with singular coverings of CD and CP_2.

The proposed general principle governing the transition to large $\hbar$ phase states that Nature loves lazy theoreticians: if perturbation theory fails to converge, a phase transition increasing the effective value of Planck constant occurs and guarantees the convergence. The killer test for the hypothesis is to find whether higher order perturbative QED corrections in powers of α_{em} are reduced from those predicted by QED in QHE phase.

At the level of preferred extremals of Kähler action these phase transitions corresponds to multi-furcations and their presence is unavoidable due to the enormous vacuum degeneracy of Kähler action which makes also ordinary path integral quantization impossible and also implies 4-D spin glass degeneracy as a basic aspect of the dynamics.

In this section the most recent view about the relationship between dark matter hierarchy, effective hierarchy of Planck constants, and FQHE is discussed. Besides explanations for charge fractionization and fractional exchange statistics also a models for the magnetic flux quanta and the macroscopic 2-surface carrying the anyonic phase are proposed. All these models rely on the notion of many-sheeted space-time and the notion of multi-furcations for a preferred extremal of Kähler action implying also the effective hierarchy of Planck constants.

4.1 General Description Of The Anyonic Phase

It is appropriate to start with a general description of the anyonic phase in TGD framework. This involves two highly non-trivial new physics elements.

1. The first element corresponds to the description of electrons as pairs of Kähler magnetic flux tubes connecting two wormhole contacts (see **Fig.** `http://tgdtheory.fi/appfigures/wormholecontact.jpg` or **Fig. ??** in the appendix of this book) such that one obtains closed flux tube carrying monopole flux. This description applies to all elementary particles. The "upper" wormhole throat of the second end of this flux tube structure by definition contains electron's quantum numbers and they are assignable to the end of braid strand. This strand continues along the light-like end of the wormhole throat as well as along space-like braid strand assignable to the end of space-time at either end of causal diamond (CD).

   One can imagine a de-localization of electron's quantum numbers in the sense that the state superposition of flux tubes with electron's quantum numbers at either end. This might allow to understand the paradoxical aspects of FQHE in composite fermion description (number of flux quanta per electron large than one and number of electrons per flux quanta larger than one).

2. Second element corresponds to the assumption that wormhole contacts, which have induced metric with Euclidian signature can have M^4 projection which has macroscopic size. All macroscopic objects could correspond to macroscopic wormhole contacts and be analogous to black-holes.

3. Also the nanoscopic magnetic flux quanta with Minkowskian signature of metric and appearing in the composite fermion model of FQHE would have as their boundaries wormhole contacts, now with cylindrical M^4 projection.

4. The natural interpretation is that the generation of flux tubes changes the topology of the macroscopic boundary. It would describe the leakage of a Minkowskian region with magnetic field to Euclidian region occurring also in super-conductivity. Depending on the character of super-conductivity the penetration can take as flux tubes or as complex flux sheets. Flux quanta are long Minkowskian flux tubes connecting opposite sides of the boundary. Single flux tube boundary is a mesoscopic wormhole throat with tubular geometry - like a cave eaten by an worm in apple - and changes its topology by adding a handle.

5. This leads to the vision that macroscopic objects are obtained simply by somehow gluing elementary particles to the two throats of macroscopic wormhole contact along their second end. One can also imagine that Minkowskian flux tube like regions get branched and that there are Minkowskian islands connected by the flux tube Minkowskian flux quanta.

6. The flux tubes define space-like braids with effectively 1-D strands whereas the braids associated with electrons at the light-like orbit of the partonic 2-surface representing macroscopic boundary define time-like braids with literally 1-D braid strands. The space-like braids defined by magnetic flux tubes are in key role in TGD inspired quantum biology [K3].

4.1.1 Geometric description of the condensation of electrons to the anyonic 2-surface

There is a strong temptation to interpret the macroscopic 2-surface at which the anyonic phase resides as a partonic 2-surface or rather pair of parallel partonic 2-surfaces within distance of CP_2 size associated with macroscopic wormhole contact connecting two space-time sheets. The first space-time sheet would carry the external magnetic field with flux quanta subtracted and the other one the flux quanta.

The rather radical conceptual implication would be that the interior of this boundary surface - more precisely the space-time sheet corresponding to the interior of the entire macroscopic system - has Euclidian signature of metric, and is in several aspects analogous to blackhole interior and indeed proposed to replace blackhole in TGD Universe. In many-sheeted space-time this does not lead to any obvious problems and would say only that entire macroscopic system in this length scale behaves as a line of a generalized Feynman diagram.

Electrons can in some sense condense at this pair of space-time sheets. The simplest view would be that electronic flux tube pair attaches to this surface along its second wormhole contact. Another wormhole contact remains at the Minkowskian side. The two wormhole throats at the second end of electron attach to the macroscopic wormhole throats and the flux turns back through the macroscopic wormhole contact. This allows to have ordinary many-electron state - or rather, boundary state.

If one tries to add electrons as braid strands to the light-like orbit of the upper macroscopic partonic 2-surface, one obtains quite different state. This state has nothing to do with ordinary many-electron state but is more like super-conformal excitation of a primary state containing only single fermionic braid strand and its propagator as particle would be of form $1/p^N$, N large. In conformal theory conformal descendant of a primary field would be the analogy.

I have proposed that this kind of macroscopic and even astroscopic structures emerge naturally in TGD framework. so tht anyons could be important even in astrophysics.

4.1.2 Possible solution of the paradox

It has been already noticed that FQHE leads to what looks like a paradox - at least for an outsider to condensed matter physics like me. The number of flux tubes per electron is larger than 1 on one hand and the number of electrons per flux tube is larger than 1 on one hand.

The bi-locality of the electrons might solve the paradox. If the charge of free electron is delocalized to its both ends, electron can be said to reside at the both ends of its monopole flux tube.

Consider what the following two statements could mean. *Electron current generates the magnetic field inside flux quantum. Electron resides at the center of the flux quantum.*

1. Suppose first that electron is associated with either wormhole end of its monopole flux tube, call it E. If the electronic charge is always at the Minkowskian end of E, then two statements could be special cases of a more general statements E would connect second electron wormhole in the Minkowskian interior of the mesoscopic magnetic flux quantum - call it M - to electron wormhole fused to the boundary of M. The location of interior wormhole would be center of flux quantum or a point near to its boundary in the two cases respectively. It seems that the paradox remains unsolved in this picture.

2. Suppose that electron corresponds to a superposition of states for which charge is associated with either upper end of the flux tube perhaps having length of order Compton length. If

the electronic charge is de-localized and shared between ends of E, one cannot anymore say that the electron is either at the center or at the boundary. Paradox would disappear since quantum logic would not allow its formulation.

4.1.3 What happens to electron in external magnetic fields in FQHE?

What happens in external magnetic field when $2p$ flux quanta are formed? The first challenge is to construct a concrete model for what happens to electron as a geometric object in this process.

1. Assume that electron's "upper" space-time sheet by definition containing its quantum numbers suffers a $2p$-furcation. Each sheet of multi-furcation corresponds to flux quantum Φ_0. Electron near the center of flux quantum is de-localized at the sheets of the multi-furcation to "plane wave" like state. The corresponding conserved momentum defined modulo $2p$ defines a topological quantum number making in turn possible non-Abelian braid group representations. Intuitively it is clear that if one identifies electron's charge as that associated with single branch of the covering, charge fractionalization takes place.

2. Conservation of the magnetic flux requires that the lower sheet carries a reduced magnetic field $B_{eff} = B - \rho 2p\Phi_0$. Since electron experiences only this field one obtains IQHE in B_{eff} so that the basic formula for ν follows.

3. The flux of the magnetic flux quanta at the upper sheet must return back along the lower sheet and this leads to the replacement of B with B_{eff}. Also the fluxes assignable to electrons must return back to the lower sheet and this would take place at the boundaries of flux sheets representing second wormhole throat end of electron.

4.2 Basic Aspects Of FQHE

The following gives a brief summary about how one might understand basic aspects of FQHE in TGD framework

4.2.1 The identification of composite fermions

The basic aspects of FQHE can be understood in terms of composite fermions identified as bound states of electron and $2p$ magnetic flux tubes with magnetic field generated by electrons flowing around its boundary. The electrons are at the center of flux tube to minimize Coulomb repulsion. This picture is however somewhat problematic since it seem to be in conflict with $\nu < 1$ stating that the number of electrons per flux tube is smaller than 1. It has been already proposed that the TGD inspired identification of electron as a bi-local object consisting of two wormhole contacts attaching along its neutral wormhole contact to the cylindrical flux tube representing the magnetic flux quantum resolves the problem. Note that this requires that electron's geometric size is given by flux tube radius and can be large.

The formation of anyons - that is flux tubes would mean a topological transition changing the spherical (say) topology of the space-time sheet representing the macroscopic system- to sphere with handles with handle addition representing as drilling of wormhole connecting the opposite sides of the surface. In this process electrons de-localized at the spherical surfaces would be de-localized so that they would be de-localized also at the flux tube boundaries representing part of the macroscopic wormhole contact.

4.2.2 Charge fractionalization

Since the system is extremely non-linear, the increase of the external magnetic field is expected to lead to a series of multi-furcations meaning that the upper space-time sheet associated with electrons and attached to the upper anyonic 2-surface suffers a multi-furcation. The natural reason for the multi-furcation is that it allows to keep the local magnetic field strength at the flux quantum below critical value. Without multi-furcation this field strength would be proportional to $2p$. Once the first multi-furcation has taken place leading to the generation of the flux tubes, the subsequent multi-furcations only add the number of branches of the multi-furcation of the flux tube. Electron de-localizes to this n-branched structure and single branch carries fractional charge e/n. Also other quantum numbers are fractional.

One should demonstrate convincingly that the fractional charges identified in this sense correspond to measured fractional charges.

4.2.3 Fractional exchange statistics

Also the fractional braid statistics can be understood. For ordinary statistics rotations of M^4 rotate entire partonic 2-surfaces. For braid statistics rotations of M^4 (and particle exchange) induce a flow braid ends along partonic 2-surface. If the singular local covering is analogous to the Riemann surface of $z^{1/n}$, the rotation of 2π leads to a second branch of multi-furcation. For the natural angle coordinate of the n-branched covering its variation of $2/pi$ corresponds to a variation $n2\pi$ of M^4 angle coordinate, and a rotation by 2π in M^4 to a rotation of $2\pi/n$ in at space-time level and phase factor $exp(i2\pi)$ is mapped to $exp(i2\pi/n)$: one has fractional exchange statistics for angular momentum.

Quantum groups relate closely to the fractional statistics and the quantum phase $q = exp(i2\pi/n)$ characterizes the statistics. Quantum groups realize particles exchange as braiding and one can formulate statistics in terms of braid group representations. What is remarkable that also genuinely non-Abelian higher-dimensional braid group representations are possible and these representations are conjectured to be associated with the anomalously behaving filling fractions $\nu = 5/2, 7/2$ allowed also by the standard rules when the entire external magnetic field is transformed to flux quanta. Also the limit $n \to \infty$ gives $\nu = 1/2p$, given $\nu = 1/2$ for $p = 1$.

4.2.4 How non-Abelian gauge group is generated?

The emergence of Abelian braid statistics is explained in terms of the velocity field of electrons defining effectively Abelian gauge potential giving rise to Chern-Simons term defining a topological QFT. This requires that the electron flow is incompressible.

In the case of non-Abelian braid statistics a non-Abelian gauge group is needed to define Chern-Simons action. The challenge is to understand the physical origin of this gauge symmetry and to my best knowledge this problem is not well-understood.

In TGD framework Kähler action reduces to Abelian Chern-Simons terms for preferred extremals so that non-Abelian Chern-Simons term and corresponding gauge group should be generated dynamically. The study of the preferred extremals of Kähler action and solutions of Kähler-Dirac action indeed leads to a mechanism generating not only electro-weak gauge symmetries dynamically but also a larger gauge group [K6]. What might happen is follows. The core part of the dynamical gauge group would be $U(n)$ acting in the space of modes of the Kähler-Dirac operator. Its action commutes with electroweak and other quantum numbers. By taking the n^2 generators of $U(n)$ and the 4 generators of electroweak $U(2)$, and forming their tensor products, one would obtain $4n^2$ generators having interpretation as generators of $U(2n)$. The non-Abelian Chern-Simons term would be associated with $U(n)$.

The stack of N branes very near to each other gives rise to a dynamical gauge group $U(N)$ in M-theory context. This encourages to think that the n-furcation giving rise to n space-time branches gives rise to a dynamical gauge group $U(2)$ for $n = 4$: $SU(2)$ is the minimal requirement for non-trivial braid statistics.

4.2.5 Understanding the origin of braid statistics

Braid statistics requires a 2-dimensional system: plane with punctures in the simplest situation. The non-trivial homotopy allows non-trivial braid statistics since particle exchange as a homotopy need not be reducible to trivial one. The problem is that space is 3-dimensional. Isn't the idealization of 3-D system as 2-D system acceptable if it has so drastic implications as fractional statistics? Could braid statistics for anyons be a signature of something much deeper?

In TGD framework this would be the case. In many-sheeted space-time induced spinor fields are localized to 2-D string world sheets. The strings connecting partonic 2-surfaces have ends at partonic 2-surfaces, and one can perform braiding for the ends.

1. The first reason for the localization at string world sheets is the condition that em charge is well-defined for the spinor modes. This demands that the induced classical W boson fields vanish.

2. Also the condition that octonionic and ordinary spinor structures of imbedding space are equivalent inside the domains where induced spinor fields are non-vanishing can be satisfied if spinor modes are localized at string world sheets.

The Dirac action contains also a 1-D boundary term localized at the boundary of string world sheet at the orbits of partonic 2-surfaces and its bosonic counterpart which is length of this boundary. Field equation imply that fermion propagates with light-like momentum along piece of light-like geodesic. This is crucial for the generalization of twistorialization from 4-D to 8-D context [K8].

The 2-D character of string world sheets and of partonic 2-surfaces allows braid statistics. One can indeed modify the anti-commutation relations of fundamental fermions so that they realize braid statistics with quantum phase $q = exp(i2\pi/n)$ [K6].

4.2.6 Some problems of composite model

Composite model as at least the following not so well understood aspects.

1. The flux quanta must be assumed to behave like fermions. What gives them the fermionic statistics and maybe also fermion number?

2. How both the number of flux quanta per electron and the number of electrons per flux quantum can be larger than one?

3. How to understand charge fractionization. How general phenomenon the fractionization is?

Could TGD based model provide deeper justification for the com-posite model?

1. Composite model as starting point. Flux quanta now realized as magnetic flux tubes. An interesting possibility is that flux quanta correspond to mono- pole fluxes for which the transversal section of the flux tube is closed 2-surface rather than disk or annulus.

2. Interesting possibility is that the underlying 2-D system corresponds to a partonic 2-surface of macroscopic size at which electrons and accompanying flux tubes are attached. This kind of surfaces are proposed to appear even in astrophysical scales in TGD Universe and carry dark matter. This would give first a principle justification for braid statistics.

3. Could the braid statistics have justification in terms of hierachy of Planck constants? The proposal is that $h_{eff} = n \times h$ corresponds to a formation of effective n-sheeted covering of imbedding space. The original proposal was stronger. M^4 (CP_2) could be covered n_1 (n_2) times and one would have $n = n_1 \times n_2$. This means that ordinary rotations and color rotations in Cartan algebra induce only a phase correspond to $2\pi/n_1$ $(2\pi/n_2)$. This would bring in various kinds of fractionizations. This option seems however un-necessarily strong and the product formulas $n = n_1 n_2$ is somewhat questionable.

4. A delocalization of em charge to n sheets implies $1/n$ fractionization.

5. Non-abelian braid statistics is possible only for $n > 4$ and $n = 4$ would be minimal value of n.

6. What gives for the flux tube fermionic statistics? One possibility is based on the fact that a magnetic flux tube carrying Khler magnetic flux equal to Khler electric flux at its end is dyon with minimal magnetic charge and odd electric charge. By a well-known argument dyons obey fermionic statistics (`http://tinyurl.com/ybmld3hw`) [B2]. The objection is that in TGD physical fermions are obtained by adding "ur-fermions" at dyonic wormhole throats. Does this mean that fermions behave as bosons in scales longer than flux tube length and as fermions only at wormhole throats? This need not be the case since the two flux tube portions both would behave like fermion so that spin statistics would be correct since flux tubes are necessarily closed albeit in sense of many-sheeted space-time.

7. A solution of the problem consistent with basic TGD could be that the dyonic flux tube assignable to elementary particle defines only a classical space-time correlate for fermion.

8. An alternative explanation would be that the flux tube contains two parts located at parallel space-time sheets connected by wormhole contacts at the ends of the flux tubes so that a closed flux tube results (no Dirac magnetic monopoles in TGD). One would have two flux tube portions of this kind and statistics would be bosonic. The appearance of monopole fluxes as pairs in FQHE would conform with this picture about electron.

9. Another possibility is that covariantly constant right handed neutrino assignable to the flux tube gives rise to fermion number without contributing to four-momentum. The first geometric explanation would only define space-time correlate for spin 1/2. Note that this is consistent with the assumption that neutrino pairs neutralizes the weak isospin of electron. For larger h_{eff} the fractionization of weak isospin would be required.

5 Quantization Of Conductance In Neutral Matter As Evidence For Many-Sheeted Space-Time?

We are living really interesting times. Experimental sciences are producing with accelerating pace new discoveries challenging the existing theories and it is difficult to avoid the impression that a revolution is going on in physics and also in biology and neuroscience. It is a pity that colleagues do not seem to even realize what is going on. One example of fascinating experimental findings is described in an article published in Nature (`http://tinyurl.com/ybk5rkfl`) [D5].

The article reports quantization of conductance in neutral matter. In quantum Hall effect conductances is quantized in multiples of e^2/h. Now the however is in multiples of $1/h$. Looks strange! This is due to the fact that voltage is not present now: particles are neutral and electric field is replaced with the gradient of chemical potential and electric current with particle current. Hence elementary charge e is replaced with the unit for particle number which is just 1 rather than e. Hence the quantisation as multiples of $1/h$ but in complete analogy with Quantum Hall Effect (QHE).

What comes to my innocent in mind is that the effect is mathematically like QHE and that there is also fractional variant of it as in the case of QHE. In QHE magnetic field and cyclotron states at flux quanta of this field are in key role. But in the situation considered they are not present if we live in the standard model world.

What is the situation in TGD?

1. In many-sheeted space-time all classical electroweak fields are present as long range fields at given sheet. This has been one of they key interpretational problems of TGD from the beginning. In particular, Kähler electric and magnetic fields are always associated with non-vacuum extremals although ordinary electric field can vanish. Note that classical electro-weak fields affect the dynamics indirectly by forcing fermions to the string world sheets! They are clever power holders!

2. This has inspired the hypothesis that induced spinor fields describing fundamental fermions are localized at string world sheets at which only em fields are non-vanishing [K6]. This assumption guarantees that electromagnetic charge is well-defined quantum number for the modes of spinor field and thus also conserved. Classical Z^0 fields could be present below weak scale also at string world sheets. Weak scale is scaled up to macroscopic scale for large values of $h_{eff} = n \times h$ and this could explain the large parity breaking effects in living matter but also just the fact that fermionic fields are not where weak fields are, could explain the parity breaking effects.

3. At GRT-gauge theory limit the sheets of many-sheeted space-time are replaced with single one and interpreted as region of Minkowski space slightly curved and carrying gauge fields: now space-time is not regarded as a surface anymore. Only classical em field effectively present above weak scale since other electroweak gauge potentials associated with space-time sheets sum up to something which is zero on average at GRT limit.

These observations lead to ask whether the quantization of conductivity for neutral particles be a direct signature of many-sheeted space-time? Could the experiments probe physics at single sheet of many sheeted space-time? Could the needed magnetic and electric fields correspond to classical Z^0 fields, which can be present at string world sheets below weak scale now scaled up by h_{eff}/h.

If this approach is on the correct track then the thermodynamical description in terms of chemical potential cannot be fundamental (the gradient of the chemical potential replaces that of electric potential in this description). Leaving the realm of standard model, one could however wonder whether the thermodynamical description using chemical potentials (chemistry is by definition effective theory!) is really fundamental in quantum regime and whether it could reduce to something more fundamental which standard model can describe only phenomenologically.

1. I have considered two alternative models of cell membrane in zero energy ontology [K4] as a generalisation of thermodynamics as square root of thermodynamics with probability densities interpreted as square roots of thermodynamical weights which are exponentials of thermal energies. These models can be also combined. Both are characterized by a large value of $h_{eff} = h_{gr}$.

2. In the first model of the cell membrane Josephson energy determined by the voltage over the cell membrane is generalized by adding to it the difference of cyclotron energies at flux tubes at the two different sides of the membrane and the magnetic fields at flux tubes appear in the formula. This difference of cyclotron energies corresponds to chemical potential and affects the frequency associated with the Josephson current and corresponding energy proportional to h_{eff} and therefore above thermal energy.

3. For the second model classical Z^0 fields explaining the large parity breaking effects in living matter are assumed to be present. Chemical potential corresponds to the difference of Z^0 potential over the cell membrane. Could this phase be the phase in which "chemical" conductivity is quantized?

4. For the hybrid of the two models the theory of QHE would generalize by replacing em fields with combinations of em and Z^0 fields. This framework could be used to model also the observed quantization of neutral conductivity as an analog of QHE.

The most obvious objection that the quantum of conductivity for neutral particles is $1/h$ rather than g^2/h, where g is appropriate weak coupling strength does not bite. Experimentalists measure particle currents rather than Z^0 currents ($j = j_Z/g_Z$) and use gradient of chemical potential instead of Z^0 potentials $\mu = g_Z E_Z$). $j_Z = \sigma E_Z$ implies that the quantization of the conductance is in multiples of $1/h$.

6 Condensed matter simulation of 4-D quantum Hall effect from TGD point of view

I learned about an interesting experimental work related to the condensed matter simulation of physics in space-times with D=4 spatial dimensions meaning that one would have D=1+4=5-dimensional space-time. The simulation was discussed in popular article "*Leaving Flatland Quantum Hall Physics in 4D*" (see `http://tinyurl.com/ycoxr48s`).

What was simulated is 4-D quantum Hall effect (QHE). In M-theory D= 1+4 dimensional branes would have 4 spatial dimensions and also 4-D QH would be possible so that the simulation to study this speculative higher-D physics. To avoid misunderstandings it must be emphasized that it has not been demonstrated that 4:th spatial dimension exists as layman might think first.

A condensed matter simulation of a 4-D QHE possible in 1+4-dimensional space-time [D6] (see `http://tinyurl.com/y7nxd5k3`) is in question. Professors Immanuel Bloch (LMU/MPQ) and Oded Zilberberg (ETH Zürich) are the leaders of the team behind the work. Using ultracold atoms trapped in a periodically modulated two-dimensional superlattice potential, the scientists could observe a dynamical version of a novel type of QHE that is predicted to occur in four-dimensional systems.

The theory of the 4-D QHE is discussed in [D14] (see `http://tinyurl.com/y8nk5jp3`). This model assumes that spatial dimensions correspond to 4-sphere but also more general topologies are possible. In the simulation the topology was that of 4-torus.

4-D QH conductivity is proportional to a topological invariant known has second Chern number [D9]- gauge theorists talk about instanton number. This invariant is space-time integral of a quantity quadratic in gauge field so that the effect is non-linear.

2-D QH conductivity is proportional to the first Chern number which is essentially magnetic charge and non-vanishing if the second homology group is non-trivial (space has a non-contractible 2-D surface) and can be identified in the experiment considered as an analog of magnetic flux over torus but in momentum space rather than space-time. In the case of 2-D QHE in the real world the spatial topology is that of a 2-disk, which is compact only if boundary is included: one can define the first Chern class as Gauss-Bonnet invariant in this case. My interpretation is however that one considers Chern number in momentum space for the boundary of Fermi surface and that the effective monopole magnetic field corresponds to the area form of this surface: certainly this should be the case for the simulation.

6.1 The ideas of the simulation of 4-D QHE

The basic idea is that one tries to find an ordinary 1+3-D system having a dynamics mathematically equivalent to that of QHE in 4+1-D spacetime. Fig 1 of [D6] (see `http://tinyurl.com/y7nxd5k3`) illustrates the basic idea.

1. One wants to simulate the topology $(S^1 \times S^1) \times (S^1 \times S^1)$. 2-D QHE would take place at tori $S^1 \times S^1$. The basic observation is that the union $S^1 \times S^1 \cup S^1 \times S^1$ of two tori as 2-D surfaces in 3-space is Cartesian product $(S^1 \times S^1) \times (S^1 \times S^1)$ as far as degrees of freedom are counted. Therefore it might be possible to simulate physics of this system by using two 2-D tori plus suitable coupling between them. This idea is familiar from elementary quantum mechanism where the physics of N-particle system in 3-D space as physics of single particle system in 3N-D space.

   One cannot realize these tori as 2-D surfaces in 3-space. The problem is that magnetic field should be orthogonal to the torus. This would require monopole charge distribution along circle at the center of torus. This is not realizable at space-time level using the known physics. It can be however realized as effective magnetic field in momentum space at the boundary of Fermi surface, where one can define effective magnetic monopole field using the area form.

   I understand that the idea is to get effective torus topology in momentum space by using lattice like structure. The momenta differing by lattice momenta are equivalent: physically this means that wave lengths scale smaller than lattice constant are not detectable. This identification is standard manner to define torus topology. Even the lattice structure is realized in a rather exotic manner - as a photon lattice.

2. From the figure 1 one learns that for the first torus $S^1 \times S^1$ is obtained from a lattice-structure in z- and x-directions by the proposed identifications. The Fourier transform of the electric field E_z of 2-D QHE is in the z-direction and the transversal velocity component to Lorentz force is in x-direction. E_z is created by time varying real magnetic flux in x-direction of ordinary space-time by Faraday's law. Lorentz force in momentum space is caused by fictive circular monopole distribution in momentum space generating magnetic flux Φ_{xz}.

   The plane defined by the center circle of the second second torus is orthogonal to that of the first one. One has $(z, x) \rightarrow (w, y)$. x- and y-axis of the cyclinders are thus orthogonal as also induce orthogonal velocities v_x and v_y in 2-D QHE for these systems.

3. In order to get the analog of 4-D QHE one adds a coupling between the two systems modellable using real magnetic field B_{xw} orthogonal to the fictive magnetic flux Φ_{xz}. This implies additional Lorentz force F_w in the direction of E_w in momentum space. Φ_{yw} induces therefore an additional velocity component parallel to v_y and proportional to both Φ_{xz} and Φ_{yw}. This gives rise to additional 4-D QHE proportional to the second Chern number as the integral of the instanton density in momentum space, which is essentially the product of Φ_{xz} and Φ_{yw} so the second Chern number is product of first Chern numbers (I must admit

that I do not understand the details of the argument). This gives rise to QHE conductivity bi-linear in the effective magnetic fluxes and proportional to the second Chern number.

The actual realization of the situation involves quite refined condensed matter physics. The simulation of 2-D QH lattices is in terms of photon crystals creating 2-D periodic potentials to which a gas of ultracold boson atoms is added. As already confessed, I do not understand how the mathematical model for the situation leads to 4-D QHE. "*By implementing a 2D topological charge pump with ultracold bosonic atoms in an angled optical superlattice, we realize a dynamical version of the 4D integer quantum Hall effect*" does not tell much to a non-specialist. One can only admire the abstractness of the theory and skills of experimentalists.

6.2 TGD inspired comments about the simulation

The simulation raises several questions. Can one imagine 4 space-like dimensions or even 4+1 dimensions in TGD? Can one emerging a general simulation of imagined higher-D physics in terms of 4-D physics in TGD framework.

6.2.1 Are 4-D space-like regions possible in TGD?

In braneology of M-theory 4-D QHE is in principle possible and it might serve as a signature for the existence of fourth spatial dimension if branes really are there. There are however objections against large fourth space-like dimension.

1. Additional large spatial dimensions would have been probably detected if there are everywhere: for instance, additional conserved component of momentum is implied. This implies that the additional dimension must be small enough. One cannot however exclude regions of space-time, where the additional dimension is large.

2. The dimension 3 for hydrogen atom is very special. In fact, the $1/\hbar^2$ proportionality of the binding energies is crucial in TGD inspired biology, where Planck constant has spectrum: $\hbar_{eff}/\hbar = n$. At the level of chemistry one ends up with valence bond theory in which n characterizes the bonds [**?**].

   The binding energy spectrum changes dramatically in other dimensions. In particlar, in dimension $D = 4$ the dependence of binding energies on Planck constant is not a power law as it is in other dimensions [**?**] (see `http://tinyurl.com/yam7rbk6`). The energies of the hydrogen atom depend on $\hbar_{eff} = n \times h$ as $\hbar_{eff}^m$, $m = -2 < 0$. Hydrogen atoms in dimension D have Coulomb potential behaving as $1/r^{D-2}$ from Gauss law and the Schrödinger equation predicts for $D \neq 4$ that the energies satisfy $E_n \propto (h_{eff}/h)^m$, $m = 2 + 4/(D-4)$. For $D = 4$ the formula breaks since in this case the dependence on $\hbar$ is not given by power law. m is negative only for $D = 3$ and one has $m = -2$. There $D = 3$ would be unique dimension in allowing the hydrino-like states [**?**]. The temporary reduction of n makes possible bio-catalysis and life in the proposed scenario.

Are 4-D space-like regions possible in TGD?

1. In TGD space-times are 4-D surfaces in $H = M^4 \times CP_2$ picture. Space-time regions with Euclidian signature of metric (time is like fourth spatial coordinate) are predicted and could accompany any system as space-time sheet having same size as the system.

2. $M^8 - H$ duality is now a key piece of TGD and states that one can regard space-times as surfaces in either $H = M^4 \times CP_2$ or M^8 [**?**]see `http://tinyurl.com/yd43o2n2`). In M^8-picture space-time surfaces are zero loci for $RE(P)$ or $IM(P)$, where P is octonionic polynomial obtained as a continuation of real polynomial. In this picture one obtains also 1+4-D 1+5-D space-time surfaces as singular solutions but it is unclear whether they have any physical meaning since they do not have $M^4 \times CP_2$ counterpart. If the two descriptions are equivalent, 4-D QH effect is not possible.

6.2.2 Is 4-D QHE possible in TGD?

Is 4-D QHE possible in TGD? One can consider the question in two different pictures: $M^8 - M^4 \times CP_2$ duality [?] states that the descriptions of space-time surfaces as algebraic surfaces in M^8 on one hand, and as surfaces satisfying field equations in $H = M^4 \times CP_2$ are physically equivalent.

1. As noticed, space-time regions with Euclidian signature of metric are predicted but since one has only 4-D space rather than 1+4-D space-time, 4-D QHE is not possible.

   One could however consider the possibility that ZEO makes 4+1-D situation effectively possible. The size of CD increases in each "small" reduction identifiable as an analog of weak measurement since one can say that the active boundary of CD shifts farther away from the stationary passive boundary where the members of state pairs are unaffected [?] (see `http://tinyurl.com/ycxm2tpd`).

   The proper time parameter telling the distance between the tips of CD corresponds to clock time correlating with experienced time. Clock time is discrete since the increments are discrete for it but one can ask whether it could give rise to effective additional space-time coordinate and for space-like regions of space-time realized as surface inside CD this could make possible 4-D QHE. Perhaps a better manner to see this clock time is as the size scale of space-time surface which changes. One could also consider 4-D QHE in which time is replaced by a size scale.

2. In M^8-picture space-time surfaces are zero loci for real and imaginary parts $RE(P)$ or $IM(P)$ (in quaternionic sense using he decomposition of octonion to two quaternions) of octonionic polynomials P obtained as a continuation of real polynomials. Rather surprisingly, one obtains as singular solutions also 1+4-D and 1+5-D space-time surfaces but it is unclear whether they have any physical meaning since they do not have $M^4 \times CP_2$ counterpart. If the two descriptions are equivalent 4-D QH effect does not seem to be possible.

6.2.3 Other effects involving instanton number

One can of course imagine that there could be other effects involving 4-D instanton number (second Chern number). But can one have non-vanishing instanton number in TGD?

1. The induced color gauge field is proportional to induced Kähler gauge field and the counterpart of color action reduces to Kähler action. So that it seems to enough to consider the situation for the Kähler form (of CP_2) induced to space-time surface.

2. Instanton number is winding number for the map $X^4 \to CP_2$ and requires that the CP_2 projection of the space-time surface is 4-D. Therefore one can locally represent the instanton as a map $CP_2 \to M^4$. The asymptotic regions of M^4 and the boundary of CD are however exceptions. Call these regions just S. Here CP_2 coordinates are constant and M^4 coordinates are the appropriate coordinates near S. The map $M^4 \to CP_2$ can be however multiple-valued such that the branches co-indice in S.

Consider first Minkowskian signature for the induced metric, that is maps representable as graphs $M^4 \to CP_2$ (note that locally also the representation as map $CP_2 \to M^4$ are possible at points where the instanton density is non-vanishing).

1. One must can allow multiple-valued maps $M^4 \to CP_2$. One could see M^4- or CD coordinates as coordinates for CP_2, and CP_2 require at least 3 coordinate patches, which strongly suggests at least 3-fold covering and 3-valuedness except at singular regions in which some sheets coincide.

   The effective dynamical compactification of the space-time surface requires that the CP_2 coordinates are constant in S. All gauge field components therefore vanish at S. Instanton number is divergence of a topological current and reduces to a sum of surface integrals. The contribution from S vanishes.

   The topological current is proportional to Kähler gauge potential and since Kähler field is monopole field one must take into account the gauge discontinuities at coordinate patches coming from the gauge transformation associated with the transitions between patches. If one has instanton number n, there are $3n$ patches giving a non-vanising contribution and their sum could give a non-trivial instanton number.

2. There are good reasons to expect that the induced gauge fields have $n = 0$ in space-time regions with Minkowskian signature of the induced metric. At least this would be the case for the induced Kähler form. For the twistor lift of Kähler action reducing to a sum of Kähler action and volume term, preferred extremals representing a map of $M^4 \to CP_2$ or $CD \to CP_2$ with winding number n very probably do not exist [?] (see `http://tinyurl.com/yboog5sr`).

3. At QFT limit one consider only Minkowkian regions so that there would be no instantons in TGD Universe. Note that one would avoid the strong CP problem of QCD, which is due to instantons.

Consider next Euclidian signature of the induced metric.

1. For a non-vanishing value of n the representation as a map $CP_2 \to M^4$ is possible except at the intersections with S unless they are not discrete points. If the intersection with the boundary of CD discrete point or empty, one can have instanton number $n = 1$. One can represent CP_2 as a surface in $H = M^4 \times CP_2$ obtained by putting M^4 coordinates to constant. This solution is however not consistent with the assumption that space-time surfaces have ends at the opposite boundaries of CD.

   Elementary particles have wormhole contacts identifiable as deformed pieces of CP_2 as building bricks. CP_2 type extremal can be indeed deformed so that M^4 projection is a light-like geodesic. The resulting surface has two holes and they should reduce to points at the boundaries of CD. One can of course imagine also more holes. What could the instanton number of CP_2 with punctures be?

2. One could try to use 2-D analogy. Sphere CP_1 with punctures looks like a good analogy for CP_2 with punctures. The first Chern number for sphere with punctures is proportional to Gauss-Bonnet invariant expressible in terms of curvature scalar and corrections from the punctures. The first Chern number becomes proportional to $1 - n/2$, where n is the number of punctures. For two holes, one has vanishing Gauss-Bonnet invariant since one has topologically cylinder allowing flat metric.

   If an analogous formula holds also for CP_2, the second Chern number becomes fractional. CP_2 differs from sphere CP_1 in that it has 3 poles instead of 2. The removal of poles of CP_1 gives a vanishing first Chern number (cylinder). The removal of 3 poles from CP_2 should give vanishing second Chern number. Thus second Chern number would be proportional to $1 - n/3$.

   If CP_2 as surface in $H = M^4 \times CP_2$ allows n-fold coverings, they have instanton number n for the Abelian gauge field defined by the induced Kähler form. Is this possible? Could one have M^4 projection consisting of n light-like geodesics? One can argue that the sheets of n-fold covering defined by the light-like geodesics must be transformable continuously to each other so that the light-like geodesics must co-incide, and one can argue that one has 1-fold covering.

One can say that instanton number for Kähler form plays a fundamental role at the level of particle physics and has highly nontrivial physical implications and that they are directly seen in the scales of elementary particles if they have wormhole contacts as basic building bricks. This physics is however not seen at QFT limit of TGD.

6.2.4 Is the simulation of higher-dimensional physics/mathematic possible in TGD?

The idea of simulation of higher-D physics using 4-D physics is especially natural in TGD using N disjoint space-time surfaces. Time coordinate would be common to all N space-time surfaces, say proper time coordinate for either light-cone associated with CD so that the number of degrees of freedom would be $D = 3N + 1$. For light-line 3-D light-like partonic orbits defining the boundaries between Minkowskian and Euclidian regions the dimension would be $D = 2N + 1$ and for string world sheets it would be $D = N + 1$ so that multi-string states would allow the simulation of physics in any dimension $D \geq 2$.

At the level of imbedding space this would correspond to a simulation of physics for surfaces $H^N = (CD \times CP_2)^N$, such that time coordinate is same for all 3-D surfaces and one has effectively $(H_7)^N \times T = (E^3 \times CP_2)^N \times T$ where T denote time axis and E^3 to time= constant section. One can replace E^3 with the hyperbolic space H^3 and M^4 time t with the proper time a future or past directed light-cone.

I have proposed this possibility as a reaction to an objection against TGD. If space-time dimension is $D = 4$, how it is possible for a mathematician to imagine higher dimensions? Doesn't mathematical cognition of higher dimensions require a physical simulation of the higher-D dynamics? The proposed dynamics would indeed allow the physical simulation of the higher-D mathematics.

The simulation is trivial unless there is a non-trivial interaction between the separate space-time surfaces. This could be achieved by coupling them using flux tubes. If the surfaces are space-time sheets on top of each other with respect to CP_2 degrees of freedom, wormhole contacts define this interaction. What is interesting that homologically non-trivial wormhole contacts are basic building bricks of elementary particles. For homologically trivial wormhole contacts the contact is unstable against splitting.

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