Does Riemann Zeta Code for Generic Coupling Constant Evolution?

M. Pitkänen,

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Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Contents

Abstract

A general model for the coupling constant evolution is proposed. The analogy of Riemann zeta and fermionic zeta $\zeta_F(s)/\zeta_F(2s)$ with complex square root of a partition function natural in Zero Energy Ontology suggests that the the poles of $\zeta_F(ks)$, $k = 1/2$, correspond to complexified critical temperatures identifiable as inverse of Kähler coupling strength itself having interpretation as inverse of critical temperature. One can actually replace the argument s of ζ_F with Möbius transformed argument $w = (as + b)/(cs + d)$ with a, b, c, d real numbers, rationals, or even integers. For α_K $w = (s + b)/2$ is proper choices and gives zeros of $\zeta(s)$ and $s = 2 - b$ as poles. The identification $\alpha_K = \alpha_{U(1)}$ leads to a prediction for α_{em}, which deviates by .7 per cent from the experimental value at low energies (atomic scale) if the experimental value of the Weinberg angle is used. The conjecture generalizes also to weak, color and gravitational interactions when general Möbius transformation leaving upper half-plane invariant is allowed. One ends up with a general model predicting successfully the entire electroweak coupling constant evolution successfully from the values of fine structure constant at atomic or electron scale and in weak scale.

1   Introduction

During years I have made several attempts to understand coupling evolution in TGD framework.

1. The first idea dates back to the discovery of WCW Kähler geometry defined by Kähler function defined by Kähler action (this happened around 1990) [K3]. The only free parameter of the theory is Kähler coupling strength α_K analogous to temperature parameter α_K postulated to be is analogous to critical temperature. Whether only single value or entire spectrum of of values α_K is possible, remained an open question.

   About decade ago I realized that Kähler action is *complex* receiving a real contribution from space-time regions of Euclidian signature of metric and imaginary contribution from the Minkoswkian regions. Euclidian region would give Kähler function and Minkowskian regions analog of QFT action of path integral approach defining also Morse function. Zero energy ontology (ZEO) [K7] led to the interpretation of quantum TGD as complex square root of thermodynamics so that the vacuum functional as exponent of Kähler action could be identified as a complex square root of the ordinary partition function. Kähler function would correspond to the real contribution Kähler action from Euclidian space-time regions. This led to ask whether also Kähler coupling strength might be complex: in analogy with the complexification of gauge coupling strength in theories allowing magnetic monopoles. Complex α_K could allow to explain CP breaking. I proposed that instanton term also reducing to Chern-Simons term could be behind CP breaking

2. p-Adic mass calculations for 2 decades ago [K4] inspired the idea that length scale evolution is discretized so that the real version of p-adic coupling constant would have discrete set of values labelled by p-adic primes. The simple working hypothesis was that Kähler coupling strength is renormalization group (RG) invariant and only the weak and color coupling strengths depend on the p-adic length scale. The alternative ad hoc hypothesis considered was that gravitational constant is RG invariant. I made several number theoretically motivated ad hoc guesses about coupling constant evolution, in particular a guess for the formula for gravitational coupling in terms of Kähler coupling strength, action for CP_2 type vacuum extremal, p-adic length scale as dimensional quantity [K11]. Needless to say these attempts were premature and a hoc.

3. The vision about hierarchy of Planck constants $h_{eff} = n \times h$ and the connection $h_{eff} = h_{gr} = GMm/v_0$, where $v_0 < c = 1$ has dimensions of velocity [K8] forced to consider very seriously the hypothesis that Kähler coupling strength has a spectrum of values in one-one correspondence with p-adic length scales. A separate coupling constant evolution associated with h_{eff} induced by $\alpha_K \propto 1/\hbar_{eff} \propto 1/n$ looks natural and was motivated by the idea that Nature is theoretician friendly: when the situation becomes non-perturbative, Mother Nature comes in rescue and an h_{eff} increasing phase transition makes the situation perturbative again.

Quite recently the number theoretic interpretation of coupling constant evolution [K9] [L2] in terms of a hierarchy of algebraic extensions of rational numbers inducing those of p-adic number fields encouraged to think that $1/\alpha_K$ has spectrum labelled by primes and values of h_{eff}. Two coupling constant evolutions suggest themselves: they could be assigned to length scales and angles which are in p-adic sectors necessarily discretized and describable using only algebraic extensions involve roots of unity replacing angles with discrete phases.

4. Few years ago the relationship of TGD and GRT was finally understood [K6]. GRT space-time is obtained as an approximation as the sheets of the many-sheeted space-time of TGD are replaced with single region of space-time. The gravitational and gauge potential of sheets add together so that linear superposition corresponds to set theoretic union geometrically. This forced to consider the possibility that gauge coupling evolution takes place only at the level of the QFT approximation and α_K has only single value. This is nice but if true, one does not have much to say about the evolution of gauge coupling strengths.

5. The analogy of Riemann zeta function with the partition function of complex square root of thermodynamics suggests that the zeros of zeta have interpretation as inverses of complex temperatures $s = 1/\beta$. Also $1/\alpha_K$ is analogous to temperature. This led to a radical idea to be discussed in detail in the sequel.

   Could the spectrum of $1/\alpha_K$ reduce to that for the zeros of Riemann zeta or - more plausibly - to the spectrum of poles of fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ giving for $k = 1/2$ poles as zeros of zeta and as point $s = 2$? ζ_F is motivated by the fact that fermions are the only fundamental particles in TGD and by the fact that poles of the partition function are naturally associated with quantum criticality whereas the vanishing of ζ and varying sign allow no natural physical interpretation.

   The poles of $\zeta_F(s/2)$ define the spectrum of $1/\alpha_K$ and correspond to zeros of $\zeta(s)$ and to the pole of $\zeta(s/2)$ at $s = 2$. The trivial poles for $s = 2n$, $n = 1, 2, ..$ correspond naturally to the values of $1/\alpha_K$ for different values of $h_{eff} = n \times h$ with n even integer. Complex poles would correspond to ordinary QFT coupling constant evolution. The zeros of zeta in increasing order would correspond to p-adic primes in increasing order and UV limit to smallest value of poles at critical line. One can distinguish the pole $s = 2$ as extreme UV limit at which QFT approximation fails totally. CP_2 length scale indeed corresponds to GUT scale.

6. One can test this hypothesis. $1/\alpha_K$ corresponds to the electroweak U(1) coupling strength so that the identification $1/\alpha_K = 1/\alpha_{U(1)}$ makes sense. One also knows a lot about the evolutions of $1/\alpha_{U(1)}$ and of electromagnetic coupling strength $1/\alpha_{em} = 1/[cos^2(\theta_W)\alpha_{U(1)}]$. What does this predict?

   It turns out that at p-adic length scale $k = 131$ ($p \simeq 2^k$ by p-adic length scale hypothesis, which now can be understood number theoretically [K9]) fine structure constant is predicted with .7 per cent accuracy if Weinberg angle is assumed to have its value at atomic scale! It is difficult to believe that this could be a mere accident because also the prediction evolution of $\alpha_{U(1)}$ is correct qualitatively. Note however that for $k = 127$ labelling electron one can reproduce fine structure constant with Weinberg angle deviating about 10 per cent from the measured value of Weinberg angle. Both models will be considered.

7. What about the evolution of weak, color and gravitational coupling strengths? Quantum criticality suggests that the evolution of these couplings strengths is universal and independent of the details of the dynamics. Since one must be able to compare various evolutions and combine them together, the only possibility seems to be that the spectra of gauge coupling strengths are given by the poles of $\zeta_F(w)$ but with argument $w = w(s)$ obtained by a global conformal transformation of upper half plane - that is Möbius transformation (see `https://en.wikipedia.org/wiki/Mbius_transformation`) with real coefficients (element of $GL(2, R)$) so that one as $\zeta_F((as+b)/(cs+d))$. Rather general arguments force it to be and element of $GL(2, Q)$, $GL(2, Z)$ or maybe even $SL(2, Z)$ ($ad - bc = 1$) satisfying additional constraints. Since TGD predicts several scaled variants of weak and color interactions, these copies could be perhaps parameterized by some elements of $SL(2, Z)$ and by a scaling factor K.

Could one understand the general qualitative features of color and weak coupling contant evolutions from the properties of corresponding Möbius transformation? At the critical line there can be no poles or zeros but could asymptotic freedom be assigned with a pole of $cs + d$ and color confinement with the zero of $as + b$ at real axes? Pole makes sense only if Kähler action for the preferred extremal vanishes. Vanishing can occur and does so for massless extremals characterizing conformally invariant phase. For zero of $as + b$ vacuum function would be equal to one unless Kähler action is allowed to be infinite: does this make sense?. One can however hope that the values of parameters allow to distinguish between weak and color interactions. It is certainly possible to get an idea about the values of the parameters of the transformation and one ends up with a general model predicting the entire electroweak coupling constant evolution successfully.

To sum up, the big idea is the identification of the spectra of coupling constant strengths as poles of $\zeta_F((as + b/)(cs + d))$ identified as a complex square root of partition function with motivation coming from ZEO, quantum criticality, and super-conformal symmetry; the discretization of the RG flow made possible by the p-adic length scale hypothesis $p \simeq k^k$, k prime; and the assignment of complex zeros of ζ with p-adic primes in increasing order. These assumptions reduce the coupling constant evolution to four real rational or integer valued parameters (a, b, c, d). In the sequel this vision is discussed in more detail.

2 Fermionic Zeta As Partition Function And Quantum Criticality

Riemann zeta has formal interpretation as a partition function $\zeta = Z_B = \prod 1/(1 - p^s)$ for a gas of bosons with energies coming as integer multiples of $log(p)$, for given mode labelled by prime p. I have proposed different interpretation based on the fermionic zeta ζ_F based on its representation as a product

$$\zeta_F = \prod_p (1 + p^s)$$

of single fermion partition functions associated with fermions with energy $log(p)$ (by Fermi statistics the fermion number is 0 or 1). In this framework the *poles* (not zeros!) of the fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ (the value of k turns out to be $k = 1/2$) (this identity is trivial to deduce) correspond to $s/2$, where s is either trivial or non-trivial zero of zeta (denominator), or the pole of zeta at $s = 1$ (numerator). Trivial poles are negative integers $s = -1 - 2, -3...$ suggesting an interpretation as conformal weights. This interpretation is proposed also for the nontrivial poles.

ζ_F emerges naturally in TGD, where the only fundamental (to be distinguished from elementary) particles are fermions. The assignment of physics to *poles* rather than zeros of ζ_F is also natural. The interpretation inspired by the structure of super-symplectic algebra is as conformal weights associated with the representations of extended super-conformal symmetry associated with super-symplectic algebra defining symmetries of TGD at the level of "World of Classical Worlds" (WCW).

"Conformal confinement" states that the sum of conformal weights of particles in given state is real. I discovered the idea for decade ago but gave it up to end up with it again. The fractal structure of superconformal algebra conforms with quantum criticality: infinite hierarchy of symmetry breakings to sub-symmetry isomorphic to original one! The conformal structure is infinitely richer than the ordinary one since the algebra in question has infinite number of generating elements labelled by all zeros of zeta rather than a handful of conformal weights ($n = -2, ... + 2$ for Virasoro algebra). Kind of Mandelbrot fractal is in question. There is however deviation from the ordinary conformal symmetry since real conformal weights can have only one sign (for generating elements all negative conformal weights $n = -1, -2, -.$ are realized as poles of $1/\zeta(2s)$ but $n = 1$ realized as pole of $\zeta(s)$ is the only positive conformal weight). Situation is therefore not quite identical with that in conformal field theories although also conformal field theories realizes only positive conformal weights (positivity is a convention) and have also some tachyonic conformal weights which are negative.

The problem of all attempts to interpret zeros of zeta relates to the fact that zeros are *not* purely imaginary but possess the troublesome real part $Re(s) = 1/2$. This led me to consider coherent states instead of eigenstates of Hamiltonian in my proposal, which I christened a strategy for proving Riemann hypothesis [K5], [L1]. Zeta has phase at the critical line so the interpretation as a partition function can be only formal. So called Z function defined at critical line and obtained by extracting the phase of zeta out, is real at critical line.

In TGD framework the solution of these problems is provided by zero energy ontology (ZEO). Quantum theory is "complex square root" of thermodynamics and means that partition function becomes a complex entity having also a phase. The well-known function

$$\xi(s) = \frac{1}{2}\pi^{-s/2}s(s-1)\Gamma(s/2)(\zeta(s)$$

assignable to Riemann zeta having same zeros and basic symmetries has at critical line phase equal ± 1 except at zeros where the phase can be defined only as a limit depending the direction from which the zero is approached. Fermionic partion function $\zeta_F(s)$ has a complex phase and it is not clear whether it makes sense to assign with it the analog of $\xi(s)$. Ordinary partition function is modulus squared for the generalized partition function.

Why does the partition function interpretation does demand poles?

1. In ordinary thermodynamics the vanishing of partition function makes sense only at the limit of zero temperature when all Boltzmann weights approach to zero. By subtracting the energy of the lowest energy state from the energies the partition function becomes non-vanishing also in this case. Hence the idea that partition function vanishes does not look very attractive. The varying sign is even worse problem.

2. Since the temperature interpreted as $1/s$ in the partition function is not infinite could mean that one has analog of Hagedorn temperature (see `http://tinyurl.com/pvkbrum`): the degeneracy of states increases exponentially with temperature and at Hagedorn temperature compensates the s exponential decreases of Boltzmann weights so that partition function is sum of infinite number of terms approaching to unity. Hagedorn temperature relates by strong form of holography to magnetic flux tubes behaving as strings with infinite number of degrees of freedom. One would have quantum critical system possessing supersymplectic symmetry and other superconformal symmetries predicted by TGD [K2, K1, K10].

3. The temperature is complex for non-trivial zeros. This requires a generalization of thermodynamics by making partition function complex. Modulus squared of this function takes the role of an ordinary partition function. One can allow in the case of Kähler action the replacement of argument s with $ks+b$ without giving up the basic features of U(1) coupling constant evolution. Here one can allow rational numbers k and b. The inverse temperature for $\zeta_F(ks+b)$ is identified as $\beta = 1/T = k(s+b)$. It turns out that in the model for coupling constant evolution the scaling factor $k = 1/2$ is required. b is not completely fixed.

   Complex temperature is indeed the natural quantity to consider in ZEO. The real part of temperature at critical line equals to $Re(\beta) = (s+b)/4k$, with b rational or integer for $\zeta_F(w = k(s+b))$ at poles assignable with the zeros of $\zeta(2k(s+b))$ in denominator. Imaginary part

$$Im\,[\beta] = \frac{1}{T} = \frac{1}{2k}(b + \ frac12 + iy) \tag{2.1}$$

   of the inverse temperature does not depend on b. Infinite number of critical temperatures is predicted and a discrete coupling constant evolution takes place already at the level of basic quantum TGD rather than emerging only at the QFT limit - I have also considered the possibility that coupling constant evolution emerges at the QFT limit only [K11]. One could even allow Möbius transformation with real coefficients in the argument of ζ_F and that this could allow the understanding of the evolutions of weak and colour coupling constants.

   $\zeta_F(w)$ at $s = -(n - b)/k$ are also present. For $s = 1/T$ they would correspond to negative temperatures $\beta = (-n + b)/k$? In the real context and for Hamiltonian with a fixed sign this

looks weird. Preferred extremals can be however dominated by either electric or magnetic fields and the sign of the action density depends on this.

4. Interestingly, in p-adic thermodynamics p-adic temperatures has just the values $T = -1/n$ if one defines p-adic Boltzmann weight as $exp(-E/T) \to p^{-E/T}$, with $E = n \geq 0$ conformal weight. The condition that weight approaches zero requires that T identified in this is as real integer negative for p-adic thermodynamics! Trivial poles would correspond to p-adic thermodynamics and non-trivial poles to ordinary real thermodynamics! Note that the earlier convention is that $T = 1/n$ is positive: the change of the sign is just a convention. Could the hierarchy of p-adic thermodynamics labelled by p-adic primes corresponds to the sequence of critical zeros of zeta? Number theoretic vision indeed leads to this proposal [L2], [K9].

The factor $1/(1 - p^n)$ at the real poles $s = -2n$ would exist p-adically in p-adic number field Q_p so that the factors of zeta would correspond to adelic decomposition of the partition function. At critical line in turn $1/1 + p^{1/2+iy}$ would exist for zeros y for which p^{iy} is root of unity (note that $p^{1/2}$ is somewhat problematic for Q_p: does it make sense to speak about an extension of Q_p containing $sqrtp$ or is the extension just the same p-adic number field but with different definition of norm?). That p^{iy} is root of unity for some set $C(p)$ of zeros y associated with p was proposed in [L2], [K9]. Now $C(p)$ would consist of single zero $y = y(p)$.

2.1 Could The Spectrum Of Kähler Couplings Strength Correspond To Poles Of $\zeta_F(s/2)$?

The idea that the spectrum of conformal weights for supersymplectic algebra is given by the poles of ζ_F is not new [L2].

Poles of $\zeta_F(ks)$ ($k = /2$ turns out to be the correct choice) have also interpretation as complexified temperatures. Kähler action can be interpreted as a complexified partition function and the inverse $1/\alpha_K$ of Kähler coupling appears in the role of critical inverse temperature β. The original hypothesis was that Kähler coupling strength has only single value. The hierarchy of quantum criticalities and its assignment with number theoretical hierarchy of algebraic extensions of rationals led to consider the possibility that Kähler coupling strength has a spectrum corresponding to a hierarchy of critical temperatures. Quantum criticality and Hagedorn temperature for magnetic flux tubes as string like objects are indeed key elements of TGD.

The hypothesis to be studied is that the values $1/\alpha_K$ correspond to poles of

$$\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$$

with the identification $1/\alpha_K = ks$. The model for coupling constant evolution however favors $k = 1/2$ predicting that poles correspond to zeros of zeta in the denominator of ζ_F and $s = 2$ in its numerator. For $k = 1/2$ only even negative integers would appear in the spectrum and there would be pole at $s = 2$. Here one onr also allow the sift $ks \to ks + b$, b integer without shifting the imaginary parts of poles crucial for the coupling constant evolution. This induces a shift $Re[s] \to kRe[s] + b$ for the real parts of poles.

For nontrivial poles this requires the replacement of temperature with a complex temperature. Therefore also $1/\alpha_K$ becomes complex. This is just what the ZEO inspired idea about quantum theory as complex square root of thermodynamics suggests. Kähler action is also complex already for real values of $1/\alpha_K$ since Euclidian *resp.* Minkowskian regions give real/imaginary contribution to the Kähler action.

The poles of ζ_F would appear both as spectrum of complex critical temperatures $\beta = 1/T = 1/\alpha_K$ and as spectrum of supersymplectic conformal weights. ζ_F is complex along the critical line containing the complex poles. This makes sense only in ZEO. ξ function associated with ζ is real at critical line but the problems are vanishing at finite temperature, indefinite sign, and also the fact that partition function interpretation fails at positive real axis. This does not conform with the intuitive picture about partition function defined in terms of Boltzmann weights.

2.2 The Identification Of $1/\alpha_K$ As Inverse Temperature Identified As Pole Of ζ_F

Let us list the general assumptions of the model based on the identification of $1/\alpha_K$ as a complexified inverse temperature in turn identified as zero of ζ_F.

1. I have earlier considered the number theoretical vision based on the assumption that vacuum functional identified as exponent of Kähler action receiving real/imaginary contributions from Euclidian/Minkowskian space-time regions exists simultaneously in all number fields. This is in spirit with the idea of integrability meaning that functional integral reduces to a sum over exponents of Kähler action associated with stationary points. What is nice that by the Kähler property of WCW metric Gaussian and metric determinants cancel [K3, K9] and one indeed obtains a discrete sum over exponentials making sense also in p-adic sectors, where ordinary integration does not make sense. Number theoretic universality is realized if one allows the extension of rationals containing also some roots of e if the exponent reduces to a product of root of unity and product of rational powers of e (e^p is ordinary p-adic number) and integer powers of primes p. It is perhaps needless to emphasize the importance of this result.

   The criticism is obvious: how does one know, which preferred extremals have a number theoretically universal action exponent? For calculational purposes it might not be necessary to know this. The easy option would be that all preferred extremals are number theoretically universal: this cannot be however the case if the values of $1/\alpha_K$ correspond to zeros of ζ. Second option is that in the sum over preferred extremals those which do not have a number theoretically universal exponent give a vanishing net contribution and are effectively absent. The situation brings in mind the reduction of momentum spectrum of a particle in a box to momenta equal to $k = n2\pi/L$, L the length of the box. The contributions of other plane waves in integrals vanish since they are dropped away by boundary conditions.

   Strong form of number theoretic universality requires that the exponent of Kähler action reduces to a product of rational power of some prime p or $e^{m/n}$ and a root of unity [K9], [L2]. This might be too strong a condition and weaker condition allows also powers of p mapped to real sector and vice versa by canonical identification. One could pose root of unity condition for the phase of $exp(S_K)$ as a boundary condition at the ends of causal diamond (CD) stating that some integer power of the exponent of Kähler action for the given value of α_K is real. If $exp(K)$ contains $e^{m/n}$ factor but no p^n factors, the reality of the nth power of $exp(i\pi K)$ would reveal this. Single p^n factor in absence of $e^{m/n}$ factor could be detected by requiring that the exponent $exp(iyK)$ is real for some y (imaginary part of zero of zeta with p^{iy} a root of unity).

2. The assumption that $1/\alpha_K$ corresponds to a nontrivial zero of zeta has strong constraints on the values of the reduced Kähler action $S_{K,red} = \alpha_K S_K$ for which the classical field equations do not depend on α_K at all. The reason is that the S_K must be proposal to $1/\alpha_K$ to achieve number theoretical universality. Number theoretical universality thus implies that preferred extremals depend on $1/\alpha_K$ - this is something very quantal. The proportionality $1/\alpha_K$ to $h_{eff} = n \times h$ is highly suggestive. It does not destroy number theoretical universality for given preferred extremal.

3. $1/\alpha_K$ has form $1/\alpha_K = s = a+ib = (1/2k)(1/2+iy/2)$ for nontrivial poles, $1/\alpha_K = s = -n/k$ for trivial poles of $1/\zeta(2s)$, and $1/\alpha_K = s = 1/k$ for the pole of ζ. $k = 1/2$ is the physically preferred choice.

   Kähler action can be written as a sum of Euclidian and Minkowskian contributions: $K = K_E + iK_M$. For non-trivial poles in the case of $1/\alpha_K = ks$ one has

   $$K = s \times (K_E + iK_M) = \frac{1}{k} \times \left[\frac{K_E}{2} - yK_M + i(\frac{K_M}{2} + yK_E) \right] \quad . \qquad (2.2)$$

   Here $K_{red} = K_E + iK_M$ is *reduced* Kähler action. This option generalizes directly the original proposal.

4. For trivial poles $s = -n/k$ and $s = 1/k$ one has

$$K = \frac{s}{k} \times K_{red} = \frac{s}{k} \times (K_E + iK_M) \ . \tag{2.3}$$

5. For real poles universality holds true without additional conditions since the multiplication of $1/\alpha_K$ by the scaling factor $-n_2/n_1$ does not spoil number theoretical universality. One can of course consider this condition. It predicts that the K_{red} is scaled by n_1/n_2 in the transition $n_2 \to n_1$. For nontrivial poles K_{red} is scaled by the complex ratio s_2/s_1.

   An attractive possibility is that the hierarchy of Planck constants corresponds to this RG evolution. n would correspond to the number of sheets in the n-sheeted covering for which sheets co-incide at the ends of space-time at the boundaries of CD. Therefore p-adic and $h_{eff} = n \times h$ hierarchies would find a natural interpretation in terms of zeros of ζ_F. To avoid confusion let us make clear that the values of $n = h_{eff}/h$ would not correspond to trivial poles.

Number theoretical universality could be realized in terms of RG invariance leaving the vacuum functional invariant but deforming the vacuum extremal. The hierarchy of Planck constants and p-adic length scale hierarchy could be interpreted as RG flows along real axis and critical line.

1. The grouping of poles to 4 RG orbits corresponding to non-trivial poles $y > 0$ and $y < 0$, to poles $s = -n/k < 0$, and $s = 1/k$ looks natural. The differential equations for RG evolution of Kähler action would be replaced with a difference equation relating the values of Kähler action for two subsequent critical poles of ζ_F.

2. Number theoretical universality allows to relate Minkowskian and Euclidian contributions K_M and K_E to each other. Earlier I have not even tried to deduce any correlation between them although the boundary conditions at light-like wormhole throats at which the signature of the induced metric changes, probably give strong constraints.

   The strongest form of the number theoretical universality condition assumes

$$K_{red} = K_{red,E} + iK_{red,M} = \alpha_K K_1 = \frac{K_1}{s} = K(\alpha_K = 1) \ , \quad s = \frac{1}{\alpha_K} \ . \tag{2.4}$$

   K_1 satisfies the number theoretic universality meaning that $exp(K_1) = expK(\alpha_K = 1)$ reduces to a product of powers primes, root of e and root of unity.

   This ansatz has the very remarkable property that α_K disappears from the vacuum functional completely so that the RG action can be regarded as a symmetry leaving vacuum function invariant. This operation however changes the preferred extremal and reduced Kähler action so that the situation is non-classical. RG orbit would start from the pole $s = 1$ and contain complex poles.

3. The large CP breaking suggested by complexity of α_K would disappear at the level of vacuum functional and appears only at the level of preferred extremals. If this is to conform with the quantum classical correspondence, correlation functions, which must break CP symmetry receive this breaking from preferred extremals. $s = 1/2k$ and complex poles belong to the same orbit. This ansatz is not necessary for poles $s = 1/k$ and $s = -n/k$ for which number theoretic universality conditions are satisfied irrespective of the value of s.

4. A more realistic looking solution is obtained by assuming that complex poles correspond to separate orbit or even that positive and negative values of y correspond to separate orbits. RG flow would begin from the lowest zero of zeta at either side of real axis. This gives

$$K_{red} = \frac{\alpha_K}{\alpha_{K,0}} \times K_{red}(\alpha_{K,0}) \ . \tag{2.5}$$

Also now the vacuum functional is invariant and preferred extremal changes in RG evolution. In accordance with quantum classical correspondence one has however a breaking of CP symmetry also at the level of vacuum functional due to the complexity of $\alpha_{K,0}$ unless $K_{red}(\alpha_{K,0})$ is proportional to $\alpha_{K,0}$.

Remark: The above arguments must be modified if one includes to the action cosmological volume term strongly suggested by twistor lift of TGD.

3 About Coupling Constant Evolution

p-Adic mass calculations inspired the hypothesis that the continuous coupling constant evolution in QFTs reduces in TGD framework to a discrete p-adic coupling constant evolution but assuming that α_K is absolute RG invariant. Therefore the hypothesis that the evolution of $1/\alpha_K$ defined by the non-trivial poles of ζ_F corresponds to the p-adic coupling constant evolution deserves a serious consideration.

1. p-Adic length scale hypothesis in the strong form states that primes $p \simeq 2^k$, k prime, correspond to physically preferred p-adic length scales. This would suggest that non-trivial zeros $s_1, s_2, s_3, ..$ taken in increasing order for magnitude correspond to primes $k = 2, 3, 5, 7...$ as suggested also in [L2], [K9]. This allows to assign to each zero s_n a unique prime: $p \leftrightarrow y(p)$ and this suggests more precise of the earlier hypothesis to state that $p^{iy(p)}$ is root of unity. The class of zeros associated with p would contain single zero.

   Discrete p-adic length scale evolution would thus correspond to the evolution of non-trivial zeros. The evolution associated with the hierarchy of Planck constants could only multiple Kähler action with integer. To make this more concrete one must consider detailed physical interpretation.

2. $1/\alpha_K$ corresponds to $U(1)$ coupling of standard model: $\alpha_K = \alpha(U(1)) \equiv 1/\alpha_1$. Kähler action could be seen as analogous to a Hamiltonian associated with electroweak U(1) symmetry. $U(1)$ gauge theory is not asymptotically free and this correspond to the fact that $Im(1/\alpha_K) = y$ approaches in UV to the lowest zero $y = 14.12...$ In IR y diverges, which conforms with $U(1)$ gauge theory symmetry.

   Electromagnetic coupling corresponds to

   $$\frac{1}{\alpha_{em}} = \frac{1}{\alpha_K cos^2(\theta_W)} \quad . \tag{3.1}$$

   The challenge is to understand also the evolution of $cos^2(\theta_W)$ allowing in turn to understand the entire electroweak evolution.

3. The values of electroweak couplings at the length scale of electron ($k = 127$ or at 4 times longer length scale $k = 131$ ($L(131) = .1$ Angstrom) are well-known and this provides a killer test for the model. Depending on whether one assumes fine structure constant to correspond to $L(127)$ associated with electron or to 4 times long length scale $L(131)$ one has too options. $L(131)$ allows to reproduce fine structure constant with a value of $p = sin^2(\theta_W)$ deviating only .7 per cent from its measured value in this length scale! If this is not a mere nasty accident, Riemann zeta might code the entire electroweak physics and perhaps even strong interactions!

   The first guess is that UV asymptotia for the Weinberg angle is same as for GUTS: $p \to 3/8$ for $p = 2$ giving $1/\alpha_{em} \to 22.61556016$. IR asymptotia corresponds to $p \to 0$ implying $1/\alpha_{em} = 1/\alpha_K$. Notice that the evolution is rather fast in extreme UV. In extreme IR it becomes slow. It turns out that the UV behavior of Weinberg angle does not conform with this naive expectation.

4. Since p-adic length scale is proportional to $1/p^{1/2}$ it is enough to obtain RG evolution for coupling constnt as function of p. One obtains reasonably accurate understanding about the evolution by deducing an estimate for pdy/dp. This is obtained as $pdy/dp = (dy/dN)(dN/dk)p(dk/dp)$.

- $p \simeq 2^k$ implies $k \simeq log(p)/log(2)$ and $pdk/dp \simeq 1/log(2)$.
- The approximate formula for the number $N(y)$ of zeros smaller than y is given by

$$N(y) \sim u \times log(u) \ , \ \ u = \frac{y}{2\pi}$$

giving

$$\frac{dN}{dy} \sim \frac{1}{2\pi} \times (log(u) - 1), \ \ u = \frac{y}{2\pi} \ .$$

- The number $\pi(k)$ of primes smaller than k is given by

$$N(k) \sim \frac{k}{log(k)}$$

giving

$$\frac{dN(y)}{dk} \sim \frac{1}{log(k)} - \frac{1}{log(k)^2} \ .$$

By combining the formulas, one obtains

$$p\frac{dy}{dp} \ = \ \beta = \frac{2\pi}{log(2)} \times (\frac{1}{log(y/2\pi)} - 1) \times (\frac{1}{log(k)} - \frac{1}{log(k)^2}) \ , \ \ k = \frac{log(p)}{log(2)} \ .$$

(3.2)

The beta function for the evolution as function of p-adic length scale differs by factor 2 from this one. Note that also double logarithms appear in the formula. Note that beta function depends on y logarithmically making the equation rather nonlinear. This dependence can be shifted to the left hand side and by replacing y which appropriation chosen function of it one obtains

$$p\frac{dN(y)}{dp} \ = \ \beta_1 = \frac{1}{log(k)} - \frac{1}{log(k)^2} \ , \ \ k = \frac{log(p)}{log(2)} \ .$$

(3.3)

5. Coupling constant evolution would take place at the level of single space-time sheet. Observations involve averaging over space-time sheet sizes characterized by p-adic length scales so that a direct comparison with experimental facts is not quite easy and requires a concrete statistical model.

The entire electroweak $U(1)$ coupling constant evolution would be predicted exactly from number theory. Physics would represent mathematics rather than vice versa. Concerning experimental testing a couple of remarks are in order.

1. An open question is how much many-sheetedness of space-time affects situation: one expects kind of statistical average of say Weinberg angles over p-adic length scales coming from a superposition over space-time sheets of many-sheeted space-time. Space-time with single sheet is not easy to construct experimentally although mathematically it is extremely simple system as compared to the space-time of GRT.

2. The discreteness of the coupling constant evolution at fundamental level is one testable prediction. There is no continuous flow but sequence of phases with fixed point behavior with discrete phase transitions between them. At QFT limit one expects that continuous coupling constant evolution emerges is statistical average.

3. Later it will be found that the entire electroweak evolution can be predicted and this prediction is certainly testable.

3.1 General Description Of Coupling Strengths In Terms Of Complex Square Root Of Thermodynamics

The above picture is unsatisfactory in the sense that it says nothing about the evolution of other electroweak couplings and of color coupling strength. Does number theory fix also them rather than only $U(1)$ coupling? And what about color coupling strength α_s?

Here quantum TGD as a complex square root of thermodynamics vision helps.

1. Kähler action reduces for preferred extremals to Abelian Chern-Simons action localized at the ends of space-time surfaces at boundaries of causal diamond (CD) and possibly contains terms also at light-like orbits of partonic 2-surfaces. This corresponds to almost topological QFT property of TGD.

2. Kähler action contains additional boundary terms which serve as analogs for Lagrangian multipler terms fixing the numbers of various particles in thermodynamics. Now they fix the values of isometry charges for instance, or force the symplectic charges for a sub-algebra to vanish.

   Lagrangian multiplies can be written in the form μ_i/T in ordinary thermodynamics: μ_i denotes the chemical potentials assignable to particle of type i. Number theoretical universality strongly favors similar representation now. For instance, this would give

$$\frac{1}{\alpha_{em}} = \frac{\mu_{em}}{\alpha_K} \quad , \quad \mu_{em} = \frac{1}{cos^2(\theta_W)} \quad . \tag{3.4}$$

   In the same manner SU(2) coupling strength given by

$$\frac{1}{\alpha_W} = \frac{\mu_W}{\alpha_K} = \frac{cot^2(\theta_W)}{\alpha_K} \tag{3.5}$$

   would define $cot^2(\theta_W)$ as analog of chemical potential.

3. In the case of weak interactions Chern-Simons term for induced SU(2) gauge potentials as a boundary term would be the analog of Kähler action having interpretation as Lagrangian multiplier term. In color degrees of freedom also an analog of Chern-Simons term would be in question and would be associated with the classical color gauge field defined by $H_A J$, where H_A is Hamiltonian of color isometry in CP_2 and J is induced Kähler form.

4. The conditions for number theoretical universality would become more complex as also RG invariance interpreted in terms of number theoretical universality.

This picture assuming a linear relationship between generic coupling strength α and α_K in terms of chemical potential is not yet general enough.

3.2 Does ζ_F With $GL(2,Q)$ Transformed Argument Dictate The Evolution Of Other Couplings?

It seems that one cannot avoid dynamics totally. The dynamics at (quantum) criticality is however universal. This raises the hope that the evolution of coupling constant is universal and does not depend on the details of the dynamics at all. This could also explain the marvellous successes of QED and standard model

At criticality the dynamics reduces to conformal invariance by quantum criticality, and this inspires the idea about the values of coupling constant strength as poles of a meromorphic function obtained from ζ_F by a conformal transformation of the argument. After all, what one must understand is the relationship between $1/\alpha_W$ and $1/\alpha_K$, and the linear relationship between them can be seen as a simplifying assumption and an approximation.

The values of generic coupling strength - call it just α (to be not confused with α_{em}) without specifying the interaction - would still correspond to poles of $\zeta_F(s)$ but with a transformed argument

s. Conformal transformation would relate various coupling constant evolutions to each other and allow to combine them together in a unique manner. Discreteness is of course absolutely essential. The analysis of the situation leads to a surprisingly simple picture about the coupling constant evolutions for weak and color coupling strengths.

1. By the symmetry of ζ_F under the reflection with respect to x-axis one can restrict the consideration to globally defined conformal transformations of the upper half plane identifiable as Möbius tranformations $w = (as+b)/(cs+d)$ with the real matrix coefficients (a,b,c,d). One can express the transformation as a product of an overall scaling by factor k and $GL(2,R)$ transformation with $ad - bc = 1$. Number theoretical universality demands that k and the coefficients a, b, c, d of $GL(2,R)$ matrix are real rationals. Number theoretically $GL(2,Q)$ is attractive and one can consider also the possibility that the transformation matrix $GL(2,Z)$ matrix with a, b, c, d integers. $SL(2,Z)$ is probably too restrictive option.

2. The Möbius transformation $w = (as + b)/(cs + d)$ acts on zeros of ζ mapping the discrete coupling constant evolution for $1/\alpha_K$ to that for $1/\alpha_W$ or $1/\alpha_s$. The transformed coupling constant depends logarithmically on p-adic length scale via $1/\alpha_K$ supporting the interpretation in terms of RG flow induced by that for $1/\alpha_K$ - something very natural since Kähler action is in special role in TGD framework since it determines the dynamics of preferred extremals.

3. Asymptotically (long length scales) one has $w \to a/c$ for $a \neq 0$ so that both at critical line and real axis one has accumulation of critical points to $w = a/c$! Thus for the option allowing only very large value of coupling strength in IR one has

$$w = K \times \frac{as + b}{cs + d} \quad , \quad ad - bc = 1 \quad \text{(Option 1)} \ . \tag{3.6}$$

$a/c = 0$ $(a = 0)$ corresponds to a diverging coupling strength (for color interactions and for weak interactions for vanishing Weinberg angle) and corresponds to $w = K \times b/cs + d$. $ad - bc = 1$ gives $b = -c = 1$ and if one accepts the IR divergence of coupling constant, one has

$$w = \frac{K}{-s + d} \quad \text{(Option 2)} \ . \tag{3.7}$$

The only free parameters are the rational $K > 0$ and integer d. w has pole at $s = d$ mapped to 1 by ζ_F.

To gain physical insight consider the situation at real axes.

1. The real poles $s = -n/k$ and $s = 1/k$ are mapped to poles on real axes and the reflection symmetry with respect to x-axis is respected. Negative poles would be thus mapped to negative poles for $d \in 0, 1$ and $k < 0$. One could also require that the pole $s = 1$ is mapped to positive pole. For option 2 it is mapped to $w = +\infty$.

2. For option 1 this is true if one has $cs + d < 0$ and $as + b > 0$. The other manner to satisfy the conditions is $cs + d > 0$ and $as + b < 0$ for $s = -1, -2, ...$ By replacing the (a, b, c, d) with $(-a, -b, -c, -d)$ these conditions can be transformed to each other so that it is enough to consider the first conditions. The first form of the condition requires $c > 0$ and $a < 0$.

The condition that $s = 1/k$ goes to a positive pole gives $c/k + d > 0$ and $a/k + b > 0$. Altogether this gives for the two Options the conditions

$$w = K \times \frac{as + b}{cs + d} < 0 \ ,$$
$$k > 0 \ , a < 0 \ , \ c > 0 \ , \ \frac{c}{k} + d > 0 \ , \ \frac{a}{k} + b > 0 \ . \ \text{(Option 1)} \ ,$$

$$\tag{3.8}$$

and

$$w \;=\; \frac{K}{-s + \frac{1}{k}} < 0 \;\;,\;\; k > 0 \;\;.\;\; \text{(Option 2)}$$

(3.9)

3. For option 2 $s = 1/k$ phase is mapped to $w = +\infty$. Coupling strength vanishes in this phase: this brings in mind the asymptotic freedom for QCD realized at extreme UV? In long scales α would behave like $1/\alpha_K$ and diverge suggesting that Option 2 provides at least an idealized description of QCD. The scaling parameter K would remain the only free parameter.

   For option 1 α can become arbitrary large in long scales but remains finite. The analog of asymptotically free phase is replaced with that having non-vanishing inverse coupling strength $w = (a + b)/(c + d)$. The interpretation could be in terms of weak coupling constant evolution. The non-vanishing of the parameter a would distinguish between weak and strong coupling constant evolution.

   By feeding in information about the evolution of weak and color coupling strengths, one can deduce information about the values of K and a.

Whether the analogs of weak and Chern-Simons actions can satisfy the number theoretical universality, when the transformation is non-linear is far from obvious since the induced gauge fields are not independent.

3.3 Questions About Coupling Constant Evolution

The simplest hypothesis conforming with the general form of Yang-Mills action is $1/\alpha_K = s$, where s is zero of zeta. With the identification $1/\alpha_K = 1/\alpha_{U(1)}$ this predicts the evolution of U(1) coupling and one obtains excellent prediction in p-adic length scale $k = 131$ ($L(131 \simeq 10^{-11}$ meters).

3.3.1 How general is the formula for $1/\alpha_K$?

Is the simplest linear form for $1/\alpha_K$ general enough? Consider first the most general form of $2\pi/\alpha_K$ taking as input the fact that its imaginary is equal to $1/\alpha_{U(1)}$ and corresponds to imaginary part y of zero of zeta at critical line.

Linear Möbius transformations $w = (as + b)/d$ with real coefficients do not affect $Im[s]$ and therfore the inverse of the imaginary part of the Kähler coupling strength which corresponds to th inverse of the measured U(1) coupling strength. The general formula for complex Kähler coupling strength would be

$$w = s + \frac{b}{d}$$

(3.10)

in the case of $SL(2, Q)$ giving $Re[1/\alpha_K] = 1/2 + b/d$. This would correspond to the analog of the inverse temperature appearing in the real exponent of Kähler function. For $SL(2, Z)$ on obtains

$$w = s + b \;\;,\;\; b \in Z \;\;.$$

(3.11)

This gives $Re[1/\alpha_K] = 1/2 + b$.

3.3.2 Does the reduction to Chern-Simons term give constraints

The coefficient of non-Abelian Chern-Simons action is quantized to integer and one can wonder whether this has any implications in TGD framework.

1. The Minkowskian term in Kähler action reduces to Abelian Chern-Simons term for Kähler action. In non-Abelian case the coefficient of Chern-Simons action (see `http://tinyurl.com/y7nfaj67`) is $k_1/4\pi$, where k_1 is integer.

   In Abelian case the triviality of gauge transformations does not give any condition on the phase factor so that in principle no conditions are obtained. One can however look what this condition gives. The coefficient of Chern-Simons term coming from in Kähler action is $1/(8\pi\alpha_K)$. For non-Abelian Chern-Simons theory with n fermions one obtains action $k \to k - n/2$. Depending on gauge group k_1 can vanish modulo 2 or 4. For the zeros at the real axes this would give the condition

$$\frac{s}{2} = s + \frac{b}{d} = Re[\frac{1}{\alpha_K}] = 2k_1 \ , \ \ s = -2n < 0 \ \text{ or } \ s = 2 \ , \tag{3.12}$$

   which is identically satisfied for integer valued b/d. Thus it seems that $SL(2, Z)$ is forced by the Chern-Simons argument in the case of Kähler action, which is however not too convincing for U(1).

   For non-trivial zeros it is not at all clear whether one certainly cannot apply the condition since there is also a contribution yS_E to the imaginary part. In any case, the condition would be

$$\frac{Re[s]}{2} = 1/2 + \frac{b}{d} = Re[\frac{1}{\alpha_K}] = 2k_1 \ . \tag{3.13}$$

   b/d must be half odd integer to satisfy the condition so that one would have $SL(2, Z)$ instead of $SL(2, Q)$. This is however in conflict with the Chern-Simons condition at real axis.

2. $w = s + b/d$ implies that the trivial poles $s = -2n$, $n > 0$, at the real axes are shifted to $s = -2n + b/d$ and become fractional. The poles at $s = 2$ is shifted to $2 + b/d$.

In the non-Abelian case one expects also Chern-Simons term but now emerging as an analog of Lagrange multiplier term rather than fundamental action reducing to Chern-Simons term. For $w = (as+b)(cs+d)$ the poles at real axis are mapped to rational numbers $w = (am+b)/(cm+d)$, $m = -2n$ or $m = 2$. Chern-Simons action would suggest integers. Gauge transformations would transform the action by a phase which is a root of unity. Vacuum functional is ZEO an analog of wave function as a square root of action exponential. Can one allow the wave function to be a finitely-many valued section in bundle?

3.3.3 Does the evolution along real axis corresponds to a confining or topological phase?

At real axis the imaginary part of s vanishes. Since it corresponds to the inverse of the gauge coupling strength, one can ask whether the proper interpretation is in terms of confining phase in which gauge coupling is literally infinite and it does not make sense to speak of perturbation theory. Instead one would have a phase in which Minkowski part of the Kähler action contributes only to the imaginary Chern-Simons term but not to the real part of the action. Topological QFT also based on Chern-Simons action also suggests itself.

The vanishing of gauge coupling strength is not a catastrophe now since the real part is non-vanishing. What looks strange that this phase is obtained also for Kähler coupling strength. Could this interpreted in terms of the fact that induced gauge potentials are not independent dynamical degrees of freedom but expressible in terms of CP_2 coordinates.

The spectrum of $1/\alpha_K$ at real axis has the $-2n + \frac{b}{d}$ and $2 + \frac{b}{d}$ and is integer or half-odd integer valued by the conditions on Chern-Simons action. One could make the entire spectrum integer value by a proper choice of b/d.

Integer valuedness forced by Chern-Simons condition leads to ask whether the situation could relate to hierarchy of Planck constants. This cannot be the case. One can assign to each value of y

p-adic coupling constant labelled by prime k $(p \simeq 2^k)$ a hierarchy of Planck constants $h_{eff} = n \times h$. If number theoretical universality is realized for $n = 1$, it is realized for all values of n and one can say that one has $1/\alpha = n/\alpha$ fora generic coupling strength α.

p-Adic temperature $T = 1/n$ using $log(p)$ as a unit correspond to the temperature parameter defined by α_K: the values of both are inverse integers. p-Adic thermodynamics might therefore provide a proper description for the confining phase as also the success of p-adic mass calculations encourages to think.

The sign of $1/\alpha_K$ is not fixed for the simplest option. The shift by $\frac{b}{d}$ could fix the sign to be negative. There is however no absolute need for a fixed sign since in Minkowskian regions the sign of Kähler action density depends on whether magnetic or electric fields dominate. In Euclidian regions the sign is always positive. Since the real part of Kähler action receives contributions from both Euclidian and Minkowskian regions it can can well have both signs so that for preferred extremals the signs of the real part of Kähler coupling strength and proper Kähler action compensate each other.

4 A Model For Electroweak Coupling Constant Evolution

In the following a model for electroweak coupling constant evolution using as inputs Weinberg angle at p-adic length scale $k = 127$ of electron or at four times longer scale $k = 131$ and in weak length scale $k = 89$ is developed.

4.1 Evolution Of Weinberg Angle

Concerning the electroweak theory, a key question is whether the notion of Weinberg angle still makes sense or whether one must somehow generalize the notion. Experimental data plus the prediction for $1/\alpha_{U(1)}$ as zero of zeta suggest that Weinberg angle varies. For instance, the value of $1/\alpha_{U(1)}$ for $k = 89$ corresponds to weak length scale and is 87.4 whereas fine structure constant is around 127. This gives $sin^2(\theta_W) \sim .312$, which is larger than standard model value.

1. Assume that the coupling constant evolutions for $1/\alpha_{em}$ and $1/\alpha_W$ correspond to different Möbius transformations acting in a nonlinear manner to s. Tangent of Weinberg angle is defined as the ratio of weak and U(1) coupling constants: $tan(\theta_W) = g_W/g_{U(1)}$ and it expresses the vectorial character of electromagnetic coupling. One can write

$$sin^2(\theta_W) = \frac{1}{1 + X} \;\; , \;\; X = \frac{\alpha_{U(1)}}{\alpha_W} \;\; . \tag{4.1}$$

One can write the ansätze for for the coupling strengths as imaginary parts of complexified ones:

$$\begin{aligned} \frac{1}{\alpha_{U(1)}} &= Im[s + b] = y \;\; , \;\; s = \frac{1}{2} + iy \\ \frac{1}{\alpha_W} &= Im[\frac{a_W s + b_W}{c_W s + d_W}] = \frac{Dy}{c^2(\frac{1}{4} + y^2) + cd + d^2)} \;\; , \\ D &= ad - bc \;\; . \end{aligned}$$

$$\tag{4.2}$$

Here $GL(2, Q)$ matrices are assumed and determinant $D = ad - bc$ is allowed to differ from unity. From this one obtains for the Weinberg angle the expression

$$sin^2(\theta_W(y)) = \;\; = \;\; \frac{1}{1 + [\frac{c^2}{D}(y^2 + \frac{1}{4}) + \frac{d}{c} + (\frac{d}{c})^2]} \;\; , \;\; D = ad - bc \;\; .$$

As the physical intuition suggests, Weinberg angle approaches zero at long length scales ($y \to \infty$). The value at short distance limit (the lowest zero $y_0 = 14.13$ at critical line) assignable to $p = 2$ is given by

$$sin^2(\theta_W(y_1)) \quad = \quad \frac{1}{1 + \frac{c^2}{D}[(y_1^2 + \frac{1}{4} + \frac{d}{c} + \frac{d}{c})^2]} \quad .$$

Note that Weinberg angle decreases monotonically with y. The choices for which c^2/D are equivalent but the parameters (a, b, c, d) can be chosen nearer to integers for large enough D.

2. How to fix the parameters D, c, d?

(a) The first guess $D = ad - bc = 1$ would reduces the unknown parameters to c, d. This does not however allow even approximately integer valued parameters a, b, cd.

(b) The GUT value of Weinberg angle at this limit is $sin^2(\theta_W) = 3/8$. TGD suggests that the values of Weinberg angle correspond to Pythagorean triangles (see http://tinyurl.com/o7c4pkt). The lowest primitive Pythagorean triangle (side lengths are coprimes, (see http://tinyurl.com/j6ojlko) corresponds to the triplet (3,4,9) forming the trunk of the 3-tree formed by the primitive Pythagorean triangles with 3 triangles emanating at each node) and to $sin^2(\theta_W) = 9/25$ slightly smaller than the GUT value. The problem is that y_0 is not a rational number and for rational values of c, d the equation for Weinberg angule cannot be satisfied.

(c) An alternative more reliable option is to use as input Weinberg angle at intermediate boson length scale $k = 89$ which corresponds to $y(24) = 87.4252746$. The value of fine structure constant at Z^0 boson length scale is about $1/\alpha_{em}(89) \simeq 127$. From this one would obtain

$$sin^2(\theta_W(k = 89)) = 1 - \frac{y_{24}}{\alpha_{em}(89)} = 1 - \frac{\alpha_{U(1)}(24)}{\alpha_{em}(89)} \simeq 0.3116, \quad . \tag{4.3}$$

One can obviously criticize the rather large value of the Weinberg angle forced by the value of $y(24)$ as being smaller than the experimental value. Experiments however suggests that Weinberg angle starts to increase after Z^0 pole. Gauge theory limit corresponds to a limit at which the sheets of many-sheeted are lumped together and one obtains a statistical average and the contributions of longer scale might increase the value of $1/\alpha_{U(1)}(24)$ and therefore reduce the value of the effective Weinberg angle.

(d) Another input is the value of fine structure constant either at $k = 127$ corresponding to electron's p-adic length scale or at $k = 131$ ($L(131) = 10^{-11}$ meters and four times the p-adic length scale of electron) fixed by the condition that fine structure constant $\alpha_{em} = \alpha_{(U(1))}cos^2(\theta_W)$ corresponds its low energy value $1/\alpha_{em} = 137.035999139$ assigned often to electron length scale. From $y(32(= 1/\alpha_{U(1)} = 105.446623$ or $y(31) = 103.725538$ and $1/\alpha_{em}(131) = 137.035999139$ one can estimate the value of Weinberg angle as

$$sin^2(\theta_W(k = 131)) \quad = \quad 1 - \frac{y_{32}}{\alpha_{em}(131)} \simeq 0.23052 \text{ or}$$

$$sin^2(\theta_W(k = 130)) \quad = \quad 1 - \frac{y_{32}}{\alpha_{em}(127)} \quad .$$

$$\tag{4.4}$$

It turns out that the first option does not work unless one assumes $1/alpha_{em}(k = 89) \leq 125.5263$ rather than $1/alpha_{em}(k = 89) \simeq 127$. The deviation is about 1-2 per cent. Second option works with a minimal modification for $1/alpha_{em}(k = 89) \simeq 127$.

(e) The value of $y(1)$ is $y_1 = 14.13472$. The two latter conditions give rise to the following series of equations

$$
\begin{aligned}
X(k) &= cot^2(\theta_W)(k) = \frac{c^2}{D}(y^2(k) + A) \ , \quad A = \frac{1}{4} + \frac{d}{c} + (\frac{d}{c})^2 \ , \\
\frac{X(24)}{X(K)} &\equiv Y = \frac{cot^2(\theta_W)(24)}{cot^2(\theta_W)(K)} = \frac{y^2(24) + A}{y^2(K) + A} \ , \\
A &= \frac{Y(y^2(K) - y^2(24))}{1 - Y} \ .
\end{aligned}
$$

$$(4.5)$$

Here K is either $K = 31$ or $K = 32$ corresponding to the p-adic length scale $k = 127$ or 131. It turns out that only $K = 31$ works fo $1/\alpha_{em}(89) = 127$.

Also following parameters can be expressed in terms of the data.

$$
\begin{aligned}
\frac{c^2}{D} &= \frac{cot^2(\theta_W)(K)}{y^2(K) + A} \ , \\
\frac{d}{c} &= \frac{1}{2}\left(-1 + \sqrt{A}\right) \ , \\
sin^2(\theta_W)(1) &= \frac{1}{1 + X(1)} \ , \quad X(1) = \frac{c^2}{D}\left(y^2(1) + A\right) \ .
\end{aligned}
$$

$$(4.6)$$

If the parameters a, b, c, d are integers, the equations cannot be satisfied exactly. For $K = 32$ it turns out that parameter A is negative for $1/alpha_{em}(k = 89) \leq 125.5263$. For $K = 31$ still negative and small so that $A = 0$ is the natural choice breaking slightly the conditions. **Table 1** represent both options.

(f) For $D = 1$ one has $c^2 \simeq 0.0002894$, which is very near to zero and not an integer. c must be non-vanishing to obtain a running Weinberg angle. For the general value of D the role c is taken by $c^2 D$ as an invariant fixed by the input data. $c \to c = 2$ requires $D = 1 \to int(4/c^2) = 138$. $D = 139$ almost equally good. One has $d/c = -0.5$ for $A = 0$ so that one would have $d = -1, c = 2$ for mimimum option. The condition $ad - bc = -a - 2b = D$ allows to estimate the values of the integer valued parameters a and b and get additional constraint on integer D. The values are not completely unique without additional conditions, say $b = 1$. This would give $a = -D + 2 = -137$ for $D = 139$ (one cannot avoid association with the famous "137"!).

3. Consider now the physical predictions. The evolution of Weinberg angle is depicted in the tables **1** and **2** for $k = 127$ model whereas tables **3** and **4** give the predictions of $k = 131$ model. The value of Weinberg angle at electron scale $k = 127$ is predicted to be $sin^2(\theta_w) \simeq 0.2430$ deviating from its measured value by 5 per cent. For $k = 131$ the Weinberg angle deviates .7 per cent from the measured value but the value of $1/\alpha_{em}(k = 89)$ is about 1 per cent too small.

The expression for the predicted value of Weinberg angle at p-adic length scale $p = 2$ is $sin^2(\theta_W)_{p=2} \simeq 0.9453368487$, which is near to its maximal value and much larger than the $sin^2(\theta_W)_{p=2} \simeq 0.375$ of GUTs. This prediction was a total surprise but could be consistent with the new physics predicted by TGD predicting several scaled up copies of hadron physics above weak scale.

A related surprise at the high energy end was that $1/\alpha_{em}$ begins to increase again at $k = 13$ and is near to fine structure constant at $k = 11$! As if asymptotic freedom would apply to all couplings except $U(1)$ coupling. This behavior is due to the approach of $cos^2(\theta_W)$ to zero. One can of course ask whether $sin^2(\theta_W) = 1$ could be obtained for a suitable choice of the parameters. This can be achieved only for $y(1) = 0$ which is not possible since A the parameter A cannot be negative.

To sum up, experimental input allows to fix electroweak coupling constant evolution completely. The problematic feature of $k = 127$ model is the possibly too large value of Weinberg theta at low energies. The predicted scaled up copies of hadron physics could explain why Weinberg angle must increase at high energies. At electron length scale the 5 per cent too high value is somewhat disturbing. The many-sheeted space-time requiring lumping together of sheets to get space-time of General Relativity might help to understand why measured Weinberg angle is smaller than predicted. Average over sheets of different sizes could be in question.

4.2   Test For The Model Of Electroweak Coupling Constant Evolution

One can check whether the values of 100 lowest non-trivial zeros are consistent with their assignment with primes k in $p \simeq 2^k$ and whether the model is consistent with the value of fine structure constant $1/\alpha_{em} = 137.035999139$ and experimental value $P = .2312$ of Weinberg angle assigned either with electron's p-adic length scale $k = 127$ or $k = 131$ (0.1 Angstroms).

The tables below summarize the values of $1/\alpha_K$ identified as imaginary part of Riemann zero and $\alpha_{em} = \alpha_K(1 - P)$ for the model already discussed. P is .7 per cent smaller than the experimental value $P = .2312$ for $k = 131$. This agreement is excellent but it turns out that the model works only if fine structure constant corresponds to $\alpha_{em}(k)$ in electron length scale $k = 127$.

For $k = 127$ one obtains fine structure constant correctly for $P = 0.243078179077$ about 10 per cent larger than the experimental value. The predicted value of α_K at scale $k = 127$ changes from $\alpha_K = \alpha_{em}$ to $\alpha(U(1))$ due the presence of $cos^2(\theta_W) = .77$. One can wonder whether this is consistent with the p-adic mass calculations and the condition on CP_2 coming from the string tension of cosmic strings.

The predicted value of α_K changes at electron length scale by the introduction of $cos(\theta_W)$ factor. The formula for the p-adic mass squared involves second order contribution which cannot be predicted with certainty. This contribution is 20 per cent at maximum so that the change of α_K by 10 per cent can be tolerated.

Galactic rotation velocity spectrum gives also constraint on the string tension of cosmic strings and in this manner also to the value of the inverse $1/R$ of CP_2 radius to which p-adic mass scales are proportional. The size scale or large voids corresponds roughly to $k = 293$. From **Table 2** one has $1/\alpha_K = 167.2$. If the condition $\alpha_K \simeq \alpha_{em}$ holds true in long length scales, the scaling of $1/\alpha_K = 1/\alpha_{em}$ used earlier would be given by $r \simeq 167/137$ and would increase the string tension of cosmic strings by factor 1.2. This could be compensated by scaling $R^2_{CP_2}$ by the same factor. CP_2 mass scale would be scaled by factor $1/\sqrt{1.2} \simeq .9$. Also this can be tolerated. Note that maximal value cosmic string tension is assumed making sense only for the ideal cosmic strings with 2-D M^4 projection. Thickening of cosmic strings reduces their tension since magnetic energy per length is reduced.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 1	14.1347251	2	0.945336	258.5784
2	21.0220396	3	0.886600	185.3802
3	25.0108575	5	0.846706	163.1566
4	30.4248761	7	0.788698	143.9880
5	32.9350615	11	0.761068	137.8428
6	37.5861781	13	0.709786	129.5121
7	40.9187190	17	0.673584	125.3579
8	43.3270732	19	0.647955	123.0727
9	48.0051508	23	0.599889	119.9796
10	49.7738324	29	0.582401	119.1907
11	52.9703214	31	0.551851	118.1982
12	56.4462476	37	0.520249	117.6574
13	59.3470440	41	0.495203	117.5663
14	60.8317785	43	0.482855	117.6301
15	65.1125440	47	0.449024	118.1767
16	67.0798105	53	0.434344	118.5877
17	69.5464017	59	0.416691	119.2275
18	72.0671576	61	0.399493	120.0105
19	75.7046906	67	0.376117	121.3444
20	77.1448400	71	0.367315	121.9326
21	79.3373750	73	0.354389	122.8874
22	82.9103808	79	0.334500	124.5836
23	84.7354929	83	0.324876	125.5111
24	87.4252746	89	0.311321	126.9464
25	88.8091112	97	0.304627	127.7144
26	92.4918992	101	0.287691	129.8480
27	94.6513440	103	0.278326	131.1552
28	95.8706342	107	0.273213	131.9102
29	98.8311942	109	0.261303	133.7912
30	101.317851	113	0.251824	135.4198
31	103.725538	127	0.243078	137.0359
32	105.446623	131	0.237073	138.2133
33	107.168611	137	0.231264	139.4088
34	111.029535	139	0.218919	142.1486
35	111.874659	149	0.216337	142.7587

Table 1: Table represents the first 35 zeros of zeta identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the proposed model for $sin^2(\theta_W)$.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 36	114.320220	151	0.209095	144.5436
37	116.226680	157	0.203677	145.9543
38	118.790782	163	0.196690	147.8767
39	121.370125	167	0.189990	149.8379
40	122.946829	173	0.186049	151.0495
41	124.256818	179	0.182861	152.0633
42	127.516683	181	0.175248	154.6123
43	129.578704	191	0.170659	156.2431
44	131.087688	193	0.167407	157.4452
45	133.497737	197	0.162390	159.3794
46	134.756509	199	0.159853	160.3964
47	138.116042	211	0.153349	163.1322
48	139.736208	223	0.150345	164.4624
49	141.123707	227	0.147838	165.6068
50	143.111845	229	0.144348	167.2548
51	146.000982	233	0.139481	169.6662
52	147.422765	239	0.137170	170.8597
53	150.053520	241	0.133037	173.0796
54	150.925257	251	0.131706	173.8183
55	153.024693	257	0.128579	175.6036
56	156.112909	263	0.124167	178.2452
57	157.597591	269	0.122123	179.5214
58	158.849988	271	0.120436	180.6009
59	161.188964	277	0.117374	182.6243
60	163.030709	281	0.115040	184.2239
61	165.537069	283	0.111970	186.4094
62	167.184439	293	0.110016	187.8511
63	169.094515	307	0.107811	189.5277
64	169.911976	311	0.106886	190.2468
65	173.411536	313	0.103056	193.3360
66	174.754191	317	0.101639	194.5256
67	176.441434	331	0.099898	196.0238
68	178.377407	337	0.097952	197.7472
69	179.916484	347	0.096444	199.1206
70	182.207078	349	0.094262	201.1698

Table 2: Table represents the zeros y_n of zeta in the range $n \in [35, 70]$ identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the proposed model for $sin^2(\theta_W)$.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 1	14.1347251	2	0.943414	249.7949
2	21.0220396	3	0.882868	179.4744
3	25.0108575	5	0.841896	158.1927
4	30.4248761	7	0.782535	139.9074
5	32.9350615	11	0.754350	134.0732
6	37.5861781	13	0.702190	126.2089
7	40.9187190	17	0.665488	122.3238
8	43.3270732	19	0.639563	120.2072
9	48.0051508	23	0.591074	117.3933
10	49.7738324	29	0.573475	116.6964
11	52.9703214	31	0.542785	115.8544
12	56.4462476	37	0.511110	115.4580
13	59.3470440	41	0.486058	115.4744
14	60.8317785	43	0.473724	115.5892
15	65.1125440	47	0.439988	116.2700
16	67.0798105	53	0.425376	116.7369
17	69.5464017	59	0.407825	117.4423
18	72.0671576	61	0.390747	118.2878
19	75.7046906	67	0.367570	119.7045
20	77.1448400	71	0.358853	120.3232
21	79.3373750	73	0.346062	121.3225
22	82.9103808	79	0.326403	123.0862
23	84.7354929	83	0.316902	124.0459
24	87.4252746	89	0.303530	125.5263
25	88.8091112	97	0.296931	126.3164
26	92.4918992	101	0.280251	128.5057
27	94.6513440	103	0.271035	129.8435
28	95.8706342	107	0.266007	130.6152
29	98.8311942	109	0.254301	132.5350
30	101.317851	113	0.244992	134.1945
31	103.725538	127	0.236408	135.8390
32	105.446623	131	0.230518	137.0359
33	107.168611	137	0.224822	138.2504
34	111.029535	139	0.212726	141.0304
35	111.874659	149	0.210197	141.6489

Table 3: Table represents the first 35 zeros of zeta identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the $k = 131$ model for $sin^2(\theta_W)$.

REFERENCES

Books related to TGD

[K1] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#quthe`, 2006.

[K2] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#compl1`, 2006.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 36	114.320220	151	0.203108	143.4576
37	116.226680	157	0.197806	144.8861
38	118.790782	163	0.190972	146.8316
39	121.370125	167	0.184423	148.8150
40	122.946829	173	0.180571	150.0397
41	124.256818	179	0.177456	151.0641
42	127.516683	181	0.170022	153.6387
43	129.578704	191	0.165542	155.2850
44	131.087688	193	0.162368	156.4981
45	133.497737	197	0.157474	158.4494
46	134.756509	199	0.154999	159.4751
47	138.116042	211	0.148658	162.2333
48	139.736208	223	0.145730	163.5739
49	141.123707	227	0.143287	164.7270
50	143.111845	229	0.139887	166.3872
51	146.000982	233	0.135146	168.8158
52	147.422765	239	0.132897	170.0175
53	150.053520	241	0.128873	172.2522
54	150.925257	251	0.127578	172.9957
55	153.024693	257	0.124534	174.7923
56	156.112909	263	0.120242	177.4499
57	157.597591	269	0.118254	178.7336
58	158.849988	271	0.116613	179.8194
59	161.188964	277	0.113635	181.8541
60	163.030709	281	0.111367	183.4623
61	165.537069	283	0.108383	185.6594
62	167.184439	293	0.106483	187.1085
63	169.094515	307	0.104341	188.7935
64	169.911976	311	0.103443	189.5162
65	173.411536	313	0.099722	192.6201
66	174.754191	317	0.098346	193.8152
67	176.441434	331	0.096655	195.3201
68	178.377407	337	0.094766	197.0512
69	179.916484	347	0.093302	198.4305
70	182.207078	349	0.091184	200.4884

Table 4: Table represents the zeros y_n of zeta in the range $n \in [35, 70]$ identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the $k = 131$ model for $sin^2(\theta_W)$.

[K3] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#kahler`, 2006.

[K4] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/padphys.html#mless`, 2006.

[K5] Pitkänen M. Riemann Hypothesis and Physics. In *TGD as a Generalized Number Theory*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#riema`, 2006.

[K6] Pitkänen M. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdclass.html#tgdgrt`, 2006.

[K7] Pitkänen M. Why TGD and What TGD is? In *Topological Geometrodynamics: an Overview*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdlian.html#WhyTGD`, 2013.

[K8] Pitkänen M. Criticality and dark matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/neuplanck.html#qcritdark`, 2014.

[K9] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#numbervision`, 2014.

[K10] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistorstory`, 2016.

[K11] Pitkänen M. TGD View about Coupling Constant Evolution. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#ccevolution`, 2018.

Articles about TGD

[L1] Pitkänen M. A Strategy for Proving Riemann Hypothesis. Available at: `http://www.emis.math.ca/EMIS/journals/AMUC/`, 2003.

[L2] Pitkänen M. Could one realize number theoretical universality for functional integral? Available at: `http://tgdtheory.fi/public_html/articles/ntu.pdf`, 2015.