

Introduction to "Physics in Many-Sheeted Space-Time: Part I"

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1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).
2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces as analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2 , which are the only 4-manifolds allowing twistor space with Kähler structure [A7]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes

for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of electromagnetic fields are nonvanishing. The correlations functions for weak fields are nonvanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.
6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement

theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $\hbar_{eff}/\hbar = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A1] [B4, B2, B3]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral

is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.
5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B1]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.
6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

1.2.1 TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_{\times} CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A3, A6, A2, A5].

The identification of the space-time as a sub-manifold [A4, A9] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First,

the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H -spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H -metric to four-surface define classical electro-weak, color gauge fields and metric in X^4 .

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

1.2.2 TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3-surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very “stringy”. By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

1.2.3 Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a “topological condensate” containing matter as particle like 3-surfaces “glued” to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the “topological condensate” there could be “vapor phase” that is a “gas” of particle like 3-surfaces and string like objects (counterpart of the “baby universes” of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** <http://tgdtheory.fi/appfigures/manysheeted.jpg> or **Fig.** ?? in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle

(EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

1.3.1 Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other things this leads to models for cell membrane, nerve pulse, and EEG.

1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

1.4.1 World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H . "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.
2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.
3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory ¹

¹There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions

1.4.2 Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factor coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

1.4.3 WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H .

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.
2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H

or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H . The modes of D_H define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

1.4.4 The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H . The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H . This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.5 Construction of scattering amplitudes

1.5.1 Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A8, A10, A11]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B+C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle

sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word “world of classical worlds” (WCW) instead of rather formal “configuration space”. I hope that “WCW” does not induce despair in the reader having tendency to think about the technicalities involved!

1.5.2 Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). “Small” SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.
2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.
3. Also “big” SFRs (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.
4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.
5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

1.5.3 The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A given M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.
3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.
4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K11]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. \circ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space (“world of classical worlds”, WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name “TGD as a generalized number theory”. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

1.6.1 The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name “TGD as a generalized number theory”. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the “physics as generalized number theory” thread.
3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called “physics as a generalized number theory”. In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

1.6.2 Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, $M^8 - H$ duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and $M^8 - H$ duality is analogous to momentum-position duality of ordinary wave mechanics.
2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

The complexification of M^8 , identified as complexified octonions, would provide a realization of this picture and $M^8 - H$ duality would map the algebraic physics in M^8 to the ordinary physics in $M^4 \times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantum state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their $M^8 - H$ duals in M_c^8 are defined for both reals and various p-adic number fields. This is true if they are

defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.
6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p = 3$).

The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

1.6.3 p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired “Universe as Computer” vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduce the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M_c^8$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P . These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P , the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** <http://tgdtheory.fi/appfigures/cat.jpg> or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginings) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K10].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.6.4 Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M_c^8$, where M_c^8 is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M_c^8 , equivalently E_c^8 , can be regarded as complexified octonions. M_c^8 has a subspace M_c^4 containing M^4 .

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k . i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M_c^8 .

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

1.7.1 Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v) , which are analogous to z and \bar{z} . Any analytic map $u \rightarrow f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2 . E^2 has some complex coordinates with imaginary unit defined by i .

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

1.7.2 Number theoretic holography in M_c^8

$Y^4 \subset M_c^8$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4 .

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space

distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2 . M_c^4 point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P . The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M_c^4 \subset M_c^8$, which are analogs of hyperbolic spaces H^3 . The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3 .

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3 ?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 , when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}}) \quad (1.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \rightarrow Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \rightarrow 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

1.7.3 Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M_c^8$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4 .

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M_c^4 \rightarrow SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \in SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0 , which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y . This simplifies the construction dramatically.
2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique

and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.
4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g . If M^4 coordinates are possible for Y^4 , the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4 . This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.
5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4 ? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

1.7.4 What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M_c^4 \subset M_c^8$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3 . This is true if the $g(y)$ defines the same CP_2 point for a given component X_i^3 of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X_i^3 and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2 : at least the isometries are the same. The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

1.7.5 Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2 , one obtains an explicit formula from the generalization of $M^8 - H$ duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M_c^8 and $M^4 \times CP_2$?

1. The selection of $SU(3) \subset G_2$ for ordinary $M^8 - H$ duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3 \subset M_c^8$ and its image $H^3 \subset H$ vanish. One would have color confinement at the mass shells H_i^3 , where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3) \subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of $M^8 - H$ duality as something more fundamental than the ordinary $M^8 - H$ duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3 \times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1) \times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields. There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1 \oplus 3 \oplus \bar{3}$. The automorphism property requires that 1 can be transformed to 3 or $\bar{3}$ to themselves: this requires that the decomposition contains $3 \oplus \bar{3}$. Furthermore, it must be possible to transform 3 and $\bar{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8 \oplus 3 \oplus \bar{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4 . The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3) \times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x) \times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X_i^3 .

1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

1.8.1 Dark Matter as Large \hbar Phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant \hbar replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr} . Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K19].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n \times h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n . Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that fermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

1.8.2 Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range

associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K16, K17, K15]) support the view that dark matter might be a key player in living matter.

1.8.3 Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff} .

1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

1.9.1 Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K24]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A7]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2 . The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2 .

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2 . The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2 .

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

$M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [?].

1.9.2 Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4 . Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.
2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [?]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?
3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see <http://tinyurl.com/yyhwvqbq>) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holger-Bek Nielsen, and Leonard Susskind to N-particle amplitude (see <http://tinyurl.com/yyvkv7as>) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?
4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts

in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance width.

In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebra (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width. QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise

to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

The dominating term in the sum over sums of resonances in t -channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$ -behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

2 Bird's Eye of View about the Topics of "Physics in Many-sheeted Space-time: Part I

This book is mostly devoted to what might be called classical TGD.

1. In a well-defined sense classical TGD defined as the dynamics of space-time surfaces determining them as kind of generalized Bohr orbits can be regarded as an exact part of quantum theory and assuming quantum classical correspondence has served as an extremely valuable guideline in the attempts to interpret TGD, to form a view about what TGD really predicts, and to guess what the underlying quantum theory could be and how it deviates from standard quantum theory.
2. The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales. Also long ranged classical color and electro-weak fields are an unavoidable prediction.
3. It took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p -adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the embedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.
4. The new view about energy and time justified by the notion of zero energy ontology (ZEO) means that the sign of inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric past. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

2.1 The Implications Deriving From The Topology Of Space-Time Surface And From The Properties Of Induced Gauge Fields

1. The general properties of Kähler action, in particular its vacuum degeneracy and failure of the classical determinism in the conventional sense, have rather far reaching implications. Space-time surfaces as a generalization of Bohr orbit provide not only a representation of quantum states but also sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.
2. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights [K2]. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new

kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

This general picture serves as a cornerstone of also TGD inspired view about cosmology and astrophysics. For obvious reasons the newest ideas developed during last year and still developing (in particular, the vision about dark matter) are not discussed in full depth yet.

2.1.1 Many-Sheeted Cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

2.1.2 Basic deviations from standard cosmology

The most important differences between TGD based and standard cosmology are following.

1. Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.
2. TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.
3. The new view about energy means that inertial energy is negative for space-time sheets with negative time orientation and that the density of inertial energy vanishes in cosmological length scales. Therefore any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.
4. Dark matter hierarchy with dynamical quantized Planck constant $h_{eff} = nh_0$, where h_0 is the minimum value of Planck constant ($h = 6h_0$ is the proposal) implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4 . Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.
5. Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations of the microwave background reflect the quantum criticality of the critical period rather than amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

This picture can be criticized since it is based on Kähler action. Twistor lift however predicts that action contains a small volume term proportional to dynamical cosmological constant. This means that Robertson-Walker cosmologies are not possible as preferred extremals except at the limit of vanishing cosmological constant corresponds to infinitely large space-time sheets and is also possible in principle.

2.1.3 Cosmic strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically. Concerning the understanding of cosmic strings a decisive breakthrough came through the identification of gravitational four-momentum as the

difference of inertial momenta associated with matter and antimatter and the realization that the net inertial energy of the Universe vanishes. This forced to conclude cosmological constant in TGD Universe is non-vanishing. p-Adic length fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet. The recent value of the cosmological constant comes out correctly. The gravitational energy density described by the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and of magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

2.1.4 Dark Matter and Quantization of Gravitational Planck Constant

The notion of gravitational Planck constant having gigantic value is perhaps the most radical idea related to the astrophysical applications of TGD. D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant \hbar replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2 -fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$ -sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

2.2 The organization Of “Physics in Many-sheeted Space-time: Part I”

The book ”Physics in Many-sheeted Space-time: Part I consists of 2 parts.

1. In the first part the notion of many-sheeted space-time is discussed. In the first two chapters the extremals of Kähler action are discussed. This discussion is somewhat out-of-date since twistor lift of TGD adds to the Kähler action a volume term. All known extremals of Kähler action with non-vanishing induced Kähler form are however also minimal surfaces. Quantum criticality suggests that extremal has no dependence on Kähler coupling strength. This is achieved for minimal surface extremals of Kähler action. As a matter fact, minimal surface property would be lost at 2-D singular surfaces identifiable as string world sheets and 3-D surface identifiable as orbits of partonic 2-surfaces.

Recently it has become clear that minimal surface extremals are obtained as analogs of Bohr orbits and thus satisfying holography required by general coordinate invariance for any general coordinate invariant action constructible in terms of the induced geometry. These surfaces would obey a generalization of 2-D holomorphy so that one can write a general form of these

extremals just as in the case of ordinary complex surfaces. Hence classical TGD would be an exactly solvable theory.

2. In the second part the relationship between TGD and GRT is discussed. The discussion proceeds in the same order as time-line of TGD itself rather than starting from the final outcome.

- (a) The first chapter is devoted TGD-GRT relationship. The understanding of QFT-GRT limit of TGD required several decades. The key idea is that QFT limit means that the sheets of many-sheeted space-time are replaced with single region of M^4 made slightly curved so that the topology of the many-sheeted space-time is lost. The gauge potentials in this space are identified as sums of the induced gauge potentials associated with space-time sheets having same M^4 projection. Similar identification of gravitational field as deviation from flat Minkowski metric is performed. One can say that in many-sheeted space-time the fields of separate systems reside at different space-time sheets: systems have field identity - field body. A test particle touching the sheets carrying the fields however experiences the sum of their effects. At QFT limit sum of the effects is replaced with the sum of the fields. This distinction plays a key role in TGD inspired quantum biology.

- (b) Second chapter is devoted to various gravitational anomalies.

- (c) The third chapter is devoted to the Nottale's formula for gravitational Planck constant assignable to gravitational flux tubes. Gravitational Planck constant plays a key role in TGD based view about quantum gravity and dark matter and its proper interpretation has been a long standing problem. Second problem concerns the origin of Newton's constant G .

The original identification of Planck length was as CP_2 size R . It however turned out that R is about $10^{3.5}$ longer than Planck length. How does Planck length emerge from quantum criticality? Twistor lift and hierarchy of Planck constants led to a possible solution of the problem. One can indeed identify R as fundamental length and analog of Planck length. Newton's constant G would not be given as $G = R^2/\hbar$ but as $G = R^2/\hbar_{CP_2}$, where $\hbar_{M^4} = n_2\hbar_0$ is the number of sheets of space-time surface as covering of M^4 . The value of n_{gr} in $\hbar_{eff} = n_{gr}\hbar_0$ appearing in Nottale's formula is identified as $n = n_1n_2$, where n_1 is the number of sheets of space-time surface as covering of CP_2 .

- (d) In chapter "Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life" the new view about space-time is applied in attempt to understand the mysteries associated with Cambrian Explosion and to the finding that the continents seem to fit nicely together if the radius of Earth is taken one half of its recent value. The explanation is in terms of rapid expansion of Earth radius induced by reduction of length scale dependent cosmological constant in turn inducing an expansion of the space-time sheet by factor of order 2.

- (e) The Shnoll effect is an "impossible" effect appearing for a very large variety of systems. In nuclear physics the fluctuations for the number of decays per time interval fluctuates but the spectrum should consist of a single peak. It is however multi-peaked, as if the nucleus had several variants with different decay rates. The weird finding is that the peak spectrum correlates with the distances of the Earth from Sun and Moon as its periodic variations demonstrate. Here the hierarchy of Planck constants, in particular the notion of gravitational Planck constant, could provide an explanation.

3 Sources

The eight online books about TGD [K26, K25, K18, K13, K5, K12, K7, K21] and nine online books about TGD inspired theory of consciousness and quantum biology [K23, K4, K14, K3, K6, K8, K9, K20, K22] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (<http://tinyurl.com/ybv8dt4n>) contains a lot of material about TGD. In particular, a TGD glossary at <http://tinyurl.com/yd6jf3o7>).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (<http://tinyurl.com/ycyrxj4o>) founded by Lian Sidorov and in *Prespacetime Journal* (<http://tinyurl.com/ycvktjhn>), *Journal of Consciousness Research and Exploration* (<http://tinyurl.com/yba4f672>), and *DNA Decipher Journal* (<http://tinyurl.com/y9z52khg>), all of them founded by Huping Hu. One can find the list about the articles published at <http://tinyurl.com/ybv8dt4n>. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

3.1 PART I: THE NOTION OF MANY-SHEETED SPACE-TIME

3.1.1 Extremals of the Kähler Action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the geometry of the “world of classical worlds” (WCW) and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to associative and co-associative regions. The reduction of the classical theory to the level of the Kähler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [?]. It is not clear whether criticality and associativity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. In ZEO one can also consider the possibility that there is no selection of preferred extremal at all! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting them, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. Actually all the discussions of this chapter are about extremals in general so that the attribute “preferred” is not relevant for them.
3. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = \exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.
4. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $\exp(K)$ is analogous to the exponent $\exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated

by the spin glass analogy. Coupling constant evolution would be replaced by effective discrete evolution with respect to p-adic length scale and angle variable defined by the phases appearing in the algebraic extension of p-adic numbers in question.

5. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with Kähler-Dirac action defines classical theory: this is in complete accordance with the proposed definition of the WCW spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2 , gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.
2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.
3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariance from $SO(3, 1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual

breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

3.1.2 Identification of the Preferred extremals of Kähler Action

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute “preferred” really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [?]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n -multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

3.1.3 Holography and Hamilton-Jacobi Structure as a 4-D generalization of 2-D complex structure

Hamilton-Jacobi structure is a key notion of Topological Geometrodynamics proposed to generalize 2-D complex structure to 4-D case. Space-time surfaces $X^4 \subset H = M^4 \times CP_2$, as analogs of Bohr orbits realizing almost deterministic holography, are simultaneous extremals of both volume action and Kähler action if X^4 is determined by the simultaneous vanishing of 2 complex valued functions of H coordinates. In the simplest situation, the functions depend on a light-like coordinate of $M^2 \subset M^4$, complex coordinate of E^2 orthogonal to M^2 , and 2 complex coordinates of CP_2 .

The complex structures of 2-D Riemann surface are parameterized by the moduli space (Teichmüller parameters). The same should be true in 4-D case. The proposal is that Hamilton-Jacobi structure, defining an integrable distribution of partonic 2-surfaces and string world sheets, slices X^4 by partonic 2-surfaces X^2 and string world sheets Y^2 . The complex (hypercomplex) structure for X^2 would depend on point of Y^2 (X^2). Space-time would define a 4-D surface in the product of Teichmüller space for the partonic 2-surfaces and its analog for string world sheets.

3.1.4 About Hydrodynamical and Thermodynamical Interpretations of TGD

This chapter is collected from the material related to the relationship between TGD and hydrodynamics on one hand and TGD and thermodynamics on the other hand. What I have called hydrodynamics ansatz is a proposal for what the preferred extremals of Kähler action might be. The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. The basic condition is the vanishing of the contraction of the conserved Kähler current j with the induced Kähler gauge potential A implying the reduction of the Kähler action to 3-D contributions coming from the boundaries between space-time regions of Minkowskian and Euclidian signature.

Hydrodynamical interpretation demands that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. Otherwise the flow line would resemble those for a gas of particles moving randomly. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when the weak electro-weak duality is applied as boundary conditions. This allows also a definition of non-constant quantal order parameters depending on the spatial coordinates transversal to the flow lines.

Kiehn and others have studied Beltrami flows as integrable flows for which the flow lines define coordinate lines. In $D=3$ this requires that the rotor of the flow vector is parallel to the flow vector stating that Lorentz force vanishes. In $D=4$ the condition states that Lorentz 4-force vanishes so that also dissipation is absent. This kind of extremals are of special interest as asymptotic self-organization patterns: in fact all preferred extremals might satisfy these conditions. 3-D Beltrami flows are highly interesting topologically since the flow lines can get knotted. Their 4-D counterparts would have flow lines replaced with world sheets which can develop 2-knots. String world sheets carrying induced spinor fields are fundamental objects in TGD framework and they could indeed get knotted.

Kiehn has worked with both Beltrami flows developed what he calls topological thermodynamics (TTD). This work is rather interesting from TGD point of view and the relationship between TTD and TGD is discussed in this chapter.

3.1.5 General View About Physics in Many-Sheeted Space-Time

This chapter, which is second part of a summary about the recent view about many-sheeted space-time, provides a summary of the developments in TGD that have occurred during last few years (the year I am writing this is 2007). The view is out-of-date in some respects. The most important steps of progress are following ones.

1. Parton level formulation of quantum TGD

The formulation of quantum TGD at partonic level identifying fundamental objects as light-like 3-surfaces having also interpretation as random light-like orbits of 2-D partons having arbitrarily large size. This picture reduces quantum TGD to an almost-topological quantum field theory and leads to a dramatic understanding of S-matrix. A generalization of Feynman diagrams emerges obtained by replacing lines of Feynman diagram with light-like 3-surfaces meeting along their ends at vertices. This picture is different from that of string models and means also a generalization of the view about space-time and 3-surface since these surfaces cannot be assumed to be a smooth manifold anymore.

The condition that the formulation in terms of light-like 3-surfaces is equivalent with that using pairs of space-like 3-surfaces at the ends of causal diamonds leads to strong form of holography stating that partonic 2-surfaces and their tangent space-data code for physics. It has turned out that fermionic string model in 4-D space-time emerges naturally from TGD. This is not yet taken into account in these considerations of the chapter.

2. Zero energy ontology

In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T , and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T .

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued “modulus” and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach. M-matrices in turn form orthogonal rows of U-matrix which is defined between zero energy states whereas S and M-matrices are defined by entanglement coefficients between positive and negative energy parts of zero energy states.

3. Fusion of real and p-adic physics to single one

The fusion of p-adic physics and real physics to single coherent whole requires generalization of the number concept obtained by gluing reals and various p-adic number fields along common algebraic numbers. This leads to a new vision about how cognition and intentionality make themselves visible in real physics via long range correlations realized via the effective p-adicity of real physics. The success of the p-adic length scale hypothesis and p-adic mass calculations suggest that cognition and intentionality are present already at elementary particle level. This picture leads naturally to an effective discretization of the real physics at the level of S-matrix and relying on the notion of number theoretic braid.

It has turned out that the notion of braid emerges naturally from the localization of spinor modes to 2-D surfaces in the generic case. Braids correspond to the orbits of the strings ends at given space-time sheet.

4. Dark matter hierarchy and hierarchy of Planck constants

Dark matter revolution with levels of the hierarchy labeled by values of Planck constant suggests a further generalization of the notion of embedding space and thus of space-time - at least as an effective mathematical tool. One can say that embedding space is a book like structure obtained by gluing together infinite number of copies of the embedding space like pages of a book: two copies characterized by singular discrete bundle structure are glued together along 4-dimensional set of common points. These points have physical interpretation in terms of quantum criticality. Particle states belonging to different sectors (pages of the book) can interact via field bodies representing space-time sheets which have parts belonging to two pages of this book.

It has turned out that the hierarchy of effective Planck constants $h_{eff} = n \times h$ follows from the quantum criticality implied by the non-determinism of Kähler action and that one can relate it to an infinite hierarchy of breakings of conformal symmetries acting on the orbits of light-like 3-surfaces leaving the space-like ends of space-time surface at boundaries of CD invariant. Hierarchy of conformal algebras corresponds to sub-algebras of conformal algebras with conformal weights coming as multiples of n .

5. Equivalence Principle and evolution of gravitational constant

The views about Equivalence Principle (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Gravitational constant, cosmological constant, and various gauge couplings emerge as predictions. Planck length should be related to CP_2 size by a dimensionless numerical factor predicted by the theory. These constants need not be universal constants: cosmological constant is certainly

very large for the Euclidian variant of GRT space-time. These constants could also depend on p-adic length scale. p-Adic coupling constant evolution suggests itself as a discretized variant of coupling constant evolution and p-adic scales would relate naturally to the size scales of causal diamonds: perhaps the integer n characterizing the multiple of CP_2 scale giving the distance between the tips of CD has p-adic prime p or its power as a divisor.

At the level of single space-time sheet and CD it is not possible to talk about coupling constant evolution since Kähler action and Kähler-Dirac action contain no coupling constants.

This description however gives rise to p-adic coupling constant evolution since the process of lumping together the sheets of the many-sheeted space-time gives a result which depends on the size scale of CD . If the non-deterministic dynamics of Kähler action for the maxima of Kähler function mimics p-adic non-determinism then one has hopes about p-adic coupling constant evolution. The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CD s whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This “prediction” is of course subject to criticism.

6. Renormalization group equations for gauge couplings at space-time level

In classical TGD only Kähler coupling constant appears explicitly but does not affect the classical dynamics. Other gauge couplings do not appear at all in classical dynamics since the definition of classical fields absorbs them as normalization constants. This suggests that the notion of continuous coupling constant evolution at space-time level is not needed in quantum TGD proper and emerges only at the QFT limit when space-time is replaced with general relativistic effective space-time.

For the known extremals of Kähler action gauge couplings are RG invariants inside single space-time sheet, which supports the view that discrete p-adic coupling constant evolution replacing the ordinary continuous coupling constant evolution emerges only when space-time sheets are lumped together to define GRT space-time. This evolution would have as parameters the p-adic length scale characterizing the causal diamond (CD) associated with particle and the phase factors characterizing the algebraic extension of p-adic numbers involved.

The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CD s whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This “prediction” is of course subject to criticism.

7. Quantitative g for the values of coupling constants

All quantitative statements about coupling constants are bound to be guesswork as long as explicit formulas for M-matrix elements are lacking. p-Adic length scale hypothesis provides one guideline for the guesses. Second guideline is provided by number theoretical universality. Third guideline is general physical intuition. What is done can be however seen as exercises perhaps giving some familiarity with the basic notions.

The latest progress in the understanding of p-adic coupling constant evolution comes from a formula for Kähler coupling strength α_K in terms of Dirac determinant of the Kähler-Dirac operator associated with Kähler action.

The formula for α_K fixes its number theoretic anatomy and also that of other coupling strengths. The assumption that simple rationals (p-adicization) are involved can be combined with the input from p-adic mass calculations and with an old conjecture for the formula of gravitational constant allowing to express it in terms of CP_2 length scale and Kähler action of topologically condensed CP_2 type vacuum extremal. The prediction is that α_K is renormalization group invariant and equals to the value of fine structure constant at electron length scale characterized by M_{127} . Although Newton's constant is proportional to p-adic length scale squared it can be RG invariant thanks to exponential reduction due to the presence of the exponent of Kähler action associated with the two CP_2 type vacuum extremals representing the wormhole contacts associated with graviton. The number theoretic anatomy of R^2/G allows to consider two options. For the first one only M_{127} gravitons are possible number theoretically. For the second option gravitons corresponding to $p \simeq 2^k$ are possible.

A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

In this chapter the above topics are discussed in detail. Also the possible role of so called supersymplectic gauge bosons in the understanding of non-perturbative phase of QCD and black-hole physics is discussed.

3.1.6 Hydrodynamics and CP_2 Geometry

The chapter is one of the earliest attempts to apply TGD to macroscopic physics and must be taken as such. The chapter begins with a brief summary of the basic notions related to many-sheeted space-time. A generalization of hydrodynamics to a p-adic hierarchy of hydrodynamics is considered and a mechanism of energy transfer between condensate levels is identified. It is suggested that TGD based generalization of Hawking-Bekenstein law holds even in macroscopic length scales and that hydrodynamical vortices behave in some aspects like elementary particles. TGD leads to a formulation of a general theory of phase transitions: the new element is the presence of several condensate levels.

It has much later become clear that the vision about elementary particles Euclidian space-time regions defining lines of generalized Feynman diagrams generalizes to macroscopic scales and that every macroscopic body should accompany such space-time sheet and thus in some aspects behave like elementary particle.

A topological model for the generation of the hydrodynamical turbulence is proposed. The basic idea is that hydrodynamical turbulence can be regarded as a spontaneous Kähler magnetization leading to the increase the value of Kähler function and therefore of the probability of the configuration. Kähler magnetization is achieved through the formation of a vortex cascade via the decay of the mother vortex by the emission of smaller daughter vortices. Vortices with various values of the fractal quantum number and with sizes related by a discrete scaling transformation appear in the cascade. The decay of the vortices takes place via the so called phase slippage process.

An encouraging result is the prediction for the size distribution of the vortices: the prediction is practically identical with that obtained from the model of Heisenberg but on rather different physical grounds. The model is rather insensitive to the p-adic scaling of vortices in the transition as long as it is smaller than $\lambda = 2^{-5}$. The model is also consistent with the assumption that the decay of a vortex to smaller vortices corresponds to a phase transition from a given level of dark matter hierarchy to a lower level so that the value of \hbar is reduced by a factor $\lambda = v_0/n \simeq 2^{-11}/n$, $n = 1, 2, \dots$ so that Compton length scales as well as sizes of vortices are reduced by this factor.

3.1.7 Macroscopic Quantum Phenomena and CP_2 Geometry

Topological field quantization is applied to a unified description of three macroscopic quantum phases: super conductors, super fluids and quantum Hall phase. The basic observation is that the formation of connections identified as join along boundaries bonds makes possible the formation of macroscopic quantum system from topological field quanta having size of the order of the coherence length ξ for ordinary phase. The presence of the connections makes possible supra flow and the presence of two levels of the topological condensate explains the two-fluid picture of super fluids. In standard physics, the order parameter is constant in the ground state. In TGD context, the non-simply connected topology of the 3-surface makes possible ground states with a covariantly constant order parameter characterized by the integers telling the change of the order parameter along closed homotopically nontrivial loops. Later an alternative identification of connections as Kähler magnetic flux tubes carrying magnetic monopole flux has emerged but does not change the general vision.

The role of the ordinary magnetic field in super conductivity is proposed to be taken by the Z^0 magnetic field in super fluidity and the mathematical descriptions of super conductors and super fluids become practically identical. The generalization of the quantization condition for the magnetic flux to a condition involving also a velocity circulation, plays a central role in the description of both phases and suggests a new description of the rotating super fluid and some new effects. A classical explanation for the fractional Quantum Hall effect in terms of the topological

field quanta is proposed. Quantum Hall phase is very similar to the supra phases: an essential role is played by the generalized quantization condition and the hydrodynamic description of the Hall electrons. The role of Z^0 magnetic field is suggested by large parity breaking effects in biology.

The results obtained support the view that in condensed matter systems topological field quanta with size of the order of $\xi \simeq 10^{-8} - 10^{-7}$ meters are of special importance. This new length scale is expected to have also applications to less exotic phenomena of the condensed matter physics (the description of the conductors and di-electrics and ferromagnetism) and in hydrodynamics (the failure of the hydrodynamic approximation takes place at this length scale). These field quanta of course, correspond to only one condensate level and many length scales are expected to be present.

3.2 PART II: TGD AND GRT

3.2.1 The Relationship Between TGD and GRT

In this chapter the recent view about TGD as Poincare invariant theory of gravitation is discussed. Radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

1. Equivalence Principle and GRT limit of TGD

The views about Equivalence Principle (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

2. The problem of cosmological constant

A further implication of dark matter hierarchy is that astrophysical systems correspond to stationary states analogous to atoms and do not participate to cosmic expansion in a continuous manner but via discrete quantum phase transitions in which gravitational Planck constant increases. By quantum criticality of these phase transitions critical cosmologies are excellent candidates for the modeling of these transitions. Imbeddable critical (and also over-critical) cosmologies are unique apart from a parameter determining their duration and represent accelerating cosmic expansion so that there is no need to introduce cosmological constant.

It indeed turns out possible to understand these critical phases in terms of quantum phase transition increasing the size of large modeled in terms of cosmic strings. A possible mechanism driving the strings to the boundaries of large voids could be repulsive interaction due to net charges of strings. Also repulsive gravitational acceleration could do this. In this framework cosmological constant like parameter does not characterize the density of dark energy but that of dark matter identifiable as quantum phases with large Planck constant.

A concrete interpretation for the dark matter is as Kähler magnetic energy of Kähler magnetic flux tubes, which are outcome of the expansion of primordial cosmic strings. Dark matter in turn corresponds to particles with non-standard value of Planck constant given by $h_{eff} = n \times$ residing at the Kähler magnetic flux tubes. The GRT limit of TGD allows a description of dark energy in terms of cosmological constant in Einstein's equations.

A further problem is that the naive estimate for the cosmological constant is predicted to be

by a factor 10^{120} larger than its value deduced from the accelerated expansion of the Universe. In TGD framework the resolution of the problem comes naturally from the fact that large voids are quantum systems which follow the cosmic expansion only during the quantum critical phases.

p-Adic fractality predicting that cosmological constant is reduced by a power of 2 in phase transitions occurring at times $T(k) \propto 2^{k/2}$, which correspond to p-adic time scales. These phase transitions would naturally correspond to quantum phase transitions increasing the size of the large voids during which critical cosmology predicting accelerated expansion naturally applies. On the average $\Lambda(k)$ behaves as $1/a^2$, where a is the light-cone proper time. This predicts correctly the order of magnitude for observed value of Λ .

3. Topics of the chapter

The topics discussed in the chapter are following.

1. The basic principles of GRT (General Coordinate Invariance, Equivalence Principle, and Machian Principle) are discussed from TGD point of view.
2. The theory assuming that the most important solution is applied to the vacuum extremal embeddings of Reissner-Nordström and Schwarzschild metric.
3. A model for the final state of star indicates that Z^0 force, presumably created by dark matter, might have an important role in the dynamics of the compact objects. During year 2003, more than decade after the formulation of the model, the discovery of the connection between supernovas and gamma ray bursts provided strong support for the predicted axial magnetic and Z^0 magnetic flux tube structures predicted by the model for the final state of a rotating star. Two years later the interpretation of the predicted long range weak forces as being caused by dark matter emerged.

The progress in understanding of hadronic mass calculations has led to the identification of what I call super-symplectic bosons and their super-counterparts as basic building blocks of hadrons. This notion leads also to a microscopic description of neutron stars and black-holes in terms of highly entangled string like objects in Hagedorn temperature and in very precise sense analogous to gigantic hadrons.

4. There is a brief summary about cosmic strings, which form a corner stone of TGD inspired cosmology.
5. The idea of entropic gravity is not consistent with what is already known about the quantal behavior of neutrons in the Earth's gravitational field. The discussion of entropic gravity in TGD framework however leads to fresh ideas about GRT limit of TGD and is therefore included.

3.2.2 TGD and Possible Gravitational Anomalies

In this chapter the applications of TGD to various real or potential anomalies of GRT approach are discussed.

1. In the first section Allais effect as a possible evidence for large \hbar dark gravitons is discussed.
2. TGD inspired model of gravimagnetism is studied. There are claims about strong gravimagnetism and these claims are considered in terms large \hbar hypothesis.
3. The dependence of operationally defined light velocity on space-time sheet distinguishes between the sub-manifold gravity of TGD and the abstract manifold gravity GRT. Possible evidence for the effect is discussed. These effects are discussed in several sections. Also the time dilation effect caused by the warping of space-time sheet in absence of matter is considered.
4. There are also some considerations not strictly related to anomalies such as possible interpretations of Machian Principle in TGD framework.

3.2.3 About the Nottale's formula for \hbar_{gr} and the relation between Planck length and CP_2 length

Nottale's formula for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula but

the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In this chapter a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed. In ZEO the non-changing parts of zero energy states are assigned to the passive boundary of CD and β_0 should be assigned to it.

There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_H$. The identification $\beta_0 = L_{M^4}/L_H$ does not allow the identification of $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for β_0 by approximating the space-time surface as Robertson-Walker cosmology expected to be a good approximation near the passive light-like boundary of CD. The resulting formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in this region! Could G have several values? This suggests the generalization $G = l_P^2/\hbar \rightarrow G = R^2/\hbar_{eff}$ so that G would indeed have a spectrum and that Planck length l_P would be equal to CP_2 radius R so that only one fundamental length would be associated with twistorialization. Ordinary Newton's constant would be given by $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$.

The second topic of the chapter relates to the the fact that the measurements of G give differing results with differences between measurements larger than the measurement accuracy. This suggests that there might be some new physics involved. In TGD framework the hierarchy of Planck constants $h_{eff} = nh_0$, $h = 6h_0$ together with the condition that theory contains CP_2 size scale R as only fundamental length scale, suggest the possibility that Newtons constant is given by $G = R^2/\hbar_{eff}$, where R replaces Planck length ($l_P = \sqrt{\hbar G} \rightarrow l_P = R$) and \hbar_{eff}/h is in the range $10^6 - 10^7$. The spectrum of Newton' constant is consistent with Newton's equations if the scaling of \hbar_{eff} inducing scaling G is accompanied by opposite scaling of M^4 coordinates in $M^4 \times CP_2$: dark matter hierarchy would correspond to discrete hierarchy of scales given by breaking of scale invariance. In the special case $h_{eff} = h_{gr} = GMm/v_0$ quantum critical dynamics as gravitational fine structure constant $(v_0/c)/4\pi$ as coupling constant and it has no dependence of the value of G or masses M and m .

In this chapter I consider a possible interpretation for the finding of a Chinese research group measuring two different values of G differing by 47 ppm in terms of varying h_{eff} . Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration g at surface of Earth induced by a change of h_{eff} due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of G_{eff} at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

3.2.4 Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life

TGD inspired quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the gigantic value of the gravitational Planck constant h_{gr} characterizing space-time mediating gravitational interactions between two masses or gravitational self interactions. This assumption provides explanation for the apparent cosmological constant. As a matter fact, gigantic value of h_{gr} . By Equivalence principle and independence of gravitational acceleration on mass it is enough to assume that only microscopic systems have the

gravitational flux tube contacts with central mass. In this case the value range of h_{gr} is consistent with the identification as $h_{eff} = n \times h$ introduced with motivations coming from biology and in TGD framework following from the non-determinism of Kähler action.

Also planets are predicted to expand in a stepwise manner allowing to imagine a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering almost the entire surface of Earth but with radius which was one half of the recent one.

This leads also to a rather fascinating vision about biology. The mysterious Cambrian Explosion in which a large number of new species emerged suddenly (realized already Darwin as the strongest objection against his theory) could be understood if the life would have gone to underground lakes and seas formed during the expansion period as fractures were formed and the underground cavities expanded and were filled with water. This would have allowed the life to escape cosmic radiation, meteoric bombardment, and the extremely cold climate during Proterozoic period preceding the Cambrian Explosion and migrate back as highly developed life forms as the period of glaciations ended.

Before the Proterozoic era the radius of Earth would have been one half of its recent value and started to grow with gradually accelerating rate. This forces to rewrite the entire geological and climate history of Earth during the Proterozoic period.

1. The postulated physically implausible cyclic appearance of single connected super-continent containing all land mass can be given up and replaced with a single continent containing large inland seas. There is no need to postulate the existence of series of super-oceans whose ocean floor would have subducted totally so that no direct information about them would exist nowadays.
2. The dominating model for pre-Cambrian climate is so called Snowball Earth model inspired by the finding that signatures of glaciations have been found at regions of Earth, which should have been near Equator during the Proterozoic. Snowball model has several difficulties: in particular, there is a lot of evidence that a series of ordinary glaciations was in question. For $R/2$ option the regions located to Equator would have actually been near North Pole so that the glaciations would have indeed been ordinary glaciations proceeding from the poles. A killer prediction is the existence of non-glaciated regions at apparent southern latitudes around about 45 degrees and there is evidence for these indeed exists! The model makes also testable paleomagnetic killer predictions. In particular, during periods when the magnetic dipole in the direction of rotation axis the directions of the magnetic fields for $R/2$ model are predicted to be same at South Pole and apparent Equator and opposite for the standard option.

3.2.5 A Possible Explanation of Shnoll Effect

Shnoll and collaborators have discovered strange repeating patterns of random fluctuations of physical observables such as the number n of nuclear decays in a given time interval. Periodically occurring peaks for the distribution of the number $N(n)$ of measurements producing n events in a series of measurements as a function of n is observed instead of a single peak. The positions of the peaks are not random and the patterns depend on position and time varying periodically in time scales possibly assignable to Earth-Sun and Earth-Moon gravitational interaction.

These observations suggest a modification of the expected probability distributions but it is very difficult to imagine any physical mechanism in the standard physics framework. Rather, a universal deformation of predicted probability distributions could be in question requiring something analogous to the transition from classical physics to quantum physics.

TGD gives hints about the nature of the modification.

1. TGD inspired quantum measurement theory proposes a description of the notion of finite measurement resolution in terms of inclusions of so called hyper-finite factors of type II_1 (HFFs) and closely related quantum groups parameterized by quantum phase $q_m = \exp(i\pi/m)$. Canonical identification mapping p-adic integers to their real counterparts is central element of TGD. For $m = p$ one can consider also the quantum variant of p-adic integer n mapped to n_R by canonical identification. There are 2 candidates for quantum-p-adics de-

pending on whether the binary digits are interpreted as quantum integers as such or mapped to a product of quantum counterparts of their prime factors.

2. Adelic physics provides a possible unification of real number based physics as physics of sensory experience and various p-adics physics as physics of cognition and predicts a hierarchy of Planck constants $h_{eff} = nh_0$ and suggests the identification of preferred p-adic prime p as a ramified prime of extension of rationals associated with the adele.

p-Adicization or perhaps even quantum-padicization could explain the findings of Shnoll.

1. The universality of the modified distribution $P(n)$ would reduce to the interpretation of the integer n in the distribution $P(n|\lambda)$ of counts as a p-adic integer or its counterpart mapped by canonical identification to a real number n_R appearing as argument of $P(n|\lambda)$. Same can be applied to $n!$. The fractality implied by the quantum criticality of TGD Universe suggests that $P(n)$ should be approximately scaling invariant under $n \rightarrow p^k n$.
2. TGD can be regarded formally as complex square root of thermodynamics, which suggest the representation $P(n) = |\Psi(n)|^2$, where $\Psi(n)$ would be wave function in the space of counts expressible as product of classical part and “quantum factor”. One could have wave functions in the space of counts n expressible as superpositions of “plane waves” q_m^{kn} , with k playing the role of momentum.

A more concrete model relies on wave function proportional to $(kn)_{q_p} \propto q_m^{kn} + q_m^{-kn}$ - analog to a superposition of plane waves with momenta k propagating to opposite directions in the space of counts reduced effectively to a box $0 \leq n < p = m$ representing modulo p counter. One would have effectively wave functions in finite field G_p . The symmetries of quantum factor would correspond to a multiplication or shift of k by element r of F_p .

Various additional rational-valued parameters characterizing the probability distribution can be mapped to (possibly quantum-) p-adics mapped to reals by canonical identification. The parameters taking care of the converge such as the parameter λ in Poisson distribution must be mapped to a power of p in p-adic context.

The model can be applied to explain the findings of Shnoll.

1. The model makes rather detailed predictions about the periodically occurring positions of the peaks of $P(n)$ as function of p based on number theoretical considerations and in principle allows to determine these parameters for given distribution. There is p -periodicity due to the fact that the lowest binary digit of n_R gives first approximation to n_R .
2. The slow variation of the p-adic prime p and integer $m = p$ characterizing quantum integers could explain the slow variation of the distributions with position and time. The periodic variations occurring with both solar and sidereal periods could be understood in two manners. The value of p could be characterized by the sum a_{net} of gravitational accelerations assignable to Earth-Sun and Earth-Moon systems and could vary. If the value of p is outcome of state function process, it is not determined by deterministic dynamics but should have a distribution. If this distribution is peaked around one particular value, one can understand the findings of Shnoll.
3. An alternative explanation would be based on slow dependence of quantum factor of $\Psi(n)$ on gravitational parameters and on time. For instance, the momentum k defining the standing wave in the space of counts modulo p could change so that the peaks of the diffraction pattern would be permuted.

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