

Introduction to "TGD as a Generalized Number Theory: Part II"

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Contents

1	Basic Ideas Of Topological Geometrodynamics (TGD)	3
1.1	Basic Vision Very Briefly	3
1.2	Two Visions About TGD And Their Fusion	6
1.2.1	TGD as a Poincare invariant theory of gravitation	6
1.2.2	TGD as a generalization of the hadronic string model	6
1.2.3	Fusion of the two approaches via a generalization of the space-time concept	7
1.3	Basic Objections	8
1.3.1	Topological field quantization	8
1.4	P-Adic Variants Of Space-Time Surfaces	8
1.5	The Threads In The Development Of Quantum TGD	9
1.5.1	Quantum TGD as spinor geometry of World of Classical Worlds	9
1.5.2	TGD as a generalized number theory	12
1.6	Hierarchy Of Planck Constants And Dark Matter Hierarchy	16
1.6.1	Dark matter as large \hbar phases	16
1.6.2	Hierarchy of Planck constants from the anomalies of neuroscience and biology	16
1.6.3	Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?	17
1.6.4	Dark matter as a source of long ranged weak and color fields	18
1.7	Twistors in TGD and connection with Veneziano duality	18
1.7.1	Twistor lift at space-time level	18
1.7.2	Twistor lift at the level of scattering amplitudes and connection with Veneziano duality	19
2	Bird's Eye of View about the Topics of the Book	22
2.1	Organization of "Physics as Generalized Number Theory: Part II"	23

3	Sources	24
4	The contents of the book	24
4.1	PART I: $M^8 - H$ DUALITY	24
4.1.1	Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part I	24
4.1.2	Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part II	26
4.1.3	Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III	28
4.1.4	Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III	28
4.1.5	Breakthrough in understanding of $M^8 - H$ duality	29
4.1.6	Could quantum randomness have something to do with classical chaos?	30
4.2	PART II: RIEMANN ZETA AND PHYSICS	31
4.2.1	Riemann hypothesis and physics	31
4.2.2	Does Riemann Zeta Code for Generic Coupling Constant Evolution?	32
4.2.3	TGD View about Coupling Constant Evolution	33
4.2.4	About the role of Galois groups in TGD framework	33
4.2.5	Some Questions about Coupling Constant Evolution	34
4.3	PART III: MISCELLANEOUS TOPICS	34
4.3.1	What p-adic icosahedron could mean? And what about p-adic manifold?	34
4.3.2	TGD and Non-Standard Numbers	35
4.3.3	Infinite Primes and Motives	36
4.3.4	Langlands Program and TGD	39
4.3.5	Langlands Program and TGD: Years Later	42
4.3.6	Quantum Arithmetics and the Relationship between Real and p-Adic Physics	42
4.3.7	Quantum Adeles	44
4.3.8	About Absolute Galois Group	45

1 Basic Ideas Of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

1.1 Basic Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).
2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in imbedding space metric and parallel translation using spinor connection of imbedding space.

The induction procedure applies to octonionic structure and the conjecture is that for preferred extremals the induced octonionic structure is quaternionic: again one just projects the octonion units. I have proposed that one can lift space-time surfaces in H to the Cartesian product of the twistor spaces of M^4 and CP_2 , which are the only 4-manifolds allowing twistor space with Kähler structure [A7]. Now the twistor structure would be induced in some sense, and should co-incide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of space-time surface. This poses constraint on the imbedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows

a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogeneity in background space-time surface which looks like the space-time of general relativity in long length scales.

One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. A possible resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics. A possible objection is that the Kähler-Dirac gamma matrices do not define an integrable distribution of 2-planes defining string world sheet.

An even stronger condition would be that the induced classical gauge fields at string world sheet vanish: this condition is allowed by the topological description of particles. The CP_2 projection of string world sheet would be 1-dimensional. Also the number theoretical condition that octonionic and ordinary spinor structures are equivalent guaranteeing that fermionic dynamics is associative leads to the vanishing of induced gauge fields.

The natural action would be given by string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant would be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality condition. The hierarchy of Planck constants $\hbar_{eff}/\hbar = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of super string theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations

for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

Whether also induced spinor fields associated with Kähler-Dirac action and de-localized inside entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using imbeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces serve are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A1] [?, ?, ?]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.
4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the imbedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.
5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are

introduced: Kähler geometry and associated spinor structure are enough. “Topological” in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [?]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of everyday world represent non-trivial topology of space-time in TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchies of Planck constants $h_{eff} = n \times h$ reducing to the quantum criticality of TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 41 years for the realization of this dream and this has resulted 24 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.2 Two Visions About TGD And Their Fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

1.2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A3, A6, A2, A5].

The identification of the space-time as a sub-manifold [A4, A9] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H -spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H -metric to four-surface define classical electro-weak, color gauge fields and metric in X^4 .

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

1.2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very “stringy”. By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models. Scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

1.2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a “topological condensate” containing matter as particle like 3-surfaces “glued” to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the “topological condensate” there could be “vapor phase” that is a “gas” of particle like 3-surfaces and string like objects (counterpart of the “baby universes” of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see **Fig.** <http://tgdtheory.fi/appfigures/manysheeted.jpg> or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell’s theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

1.3.1 Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other things this leads to models for cell membrane, nerve pulse, and EEG.

1.4 P-Adic Variants Of Space-Time Surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. One might say that TGD space-time is adelic. Also the hierarchy of Planck constants forces

a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.5 The Threads In The Development Of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.
2. The discussions with Tony Smith initiated a fourth thread which deserves the name “TGD as a generalized number theory”. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the “physics as generalized number theory” thread.
3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called “physics as a generalized number theory”. In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K17, K12, K8, K23, K15, K22, K21, K14] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K16, K3, K9, K2, K4, K5, K6, K13, K20] are warmly recommended to the interested reader.

1.5.1 Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was “Do not quantize”. The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH (“world of classical worlds”, WCW) consisting of all possible 3-surfaces in H . “All possible” means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A8, A10, A11]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.
2. During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word “world of classical worlds” (WCW) instead of rather formal “configuration space”. I hope that “WCW” does not induce despair in the reader having tendency to think about the technicalities involved!
3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory ¹
4. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in propagators. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense.

In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

¹There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as Kähler action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also Kähler Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K18]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. \circ tells that S acts at the active boundary of CD only. It turns out possible to construct a general representation for the U-matrix reducing its construction to that of S-matrix. S-matrix has interpretation as exponential of the Virasoro generator L_{-1} of the Virasoro algebra associated with super-symplectic algebra.

5. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The Kähler-Dirac gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.
6. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factor coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only “almost” since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

1.5.2 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space (“world of classical worlds”, WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name “TGD as a generalized number theory”. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. *p-Adic TGD and fusion of real and p-adic physics to single coherent whole*

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired “Universe as Computer” vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduce the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, imbedding space, and WCW.

The notion of p-adic manifold [K29] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheet as chart map provided a possible solution of the basic challenge of relating real and p-adic classical physics. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, “thought bubbles”). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach and this leads to problems with symmetries: canonical identification does not commute with symmetries.

It is now clear that much more elegant approach based on abstraction exists [K28]. The map of real preferred extremals to p-adic ones is not induced from a local correspondence between points but is global. Discretization occurs only for the parameters characterizing string world sheets and partonic 2-surfaces so that they belong to some algebraic extension of rationals. Restriction to these 2-surfaces is possible by strong form of holography. Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** <http://tgdtheory.fi/appfigures/cat.jpg> or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing

string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginings) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K7].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative (“quaternionic” is equivalent with “associative”) surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K26]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.
2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K26]? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4 . This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.
3. $M^8 - H$ duality can be generalized to a duality $H \rightarrow H$ if the images of the associative surface in M^8 is associative surface in H . One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.
4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.
5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as a sub-space of

complexified octonions (addition of commuting imaginary unit i) their tangent space or normal space is quaternionic- and thus maximally associative or co-associative. These surfaces can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K26] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either quaternionic or co-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2 . The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the “free” gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.
2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.
3. For gamma matrix option induced rather than Kähler-Dirac gamma matrices must be in question since Kähler-Dirac gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.6 Hierarchy Of Planck Constants And Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

1.6.1 Dark matter as large \hbar phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant \hbar replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr} . Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K25].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n \times h_{gr}$. The large value of h_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n . Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that fermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

1.6.2 Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K10, K11, K19]) support the view that dark matter might be a key player in living matter.

1.6.3 Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant \hbar_0 is assigned with a multiple singular covering of the imbedding space [K24]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invari-

ance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see **Fig.** <http://tgdtheory.fi/appfigures/planckhierarchy.jpg> or **Fig. ??** the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n .

1.6.4 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff} .

1.7 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

1.7.1 Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K27]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A7]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2 . The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2 .

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of imbeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on

length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2 . The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2 .

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the imbedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

$M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L3].

1.7.2 Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their

boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

One can assign 4-momentum both to the spinor harmonics of the imbedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4 . Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?
4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.
2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adèle [L1]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?
3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see <http://tinyurl.com/yyhwvqb>) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see <http://tinyurl.com/yyv7as>) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it

seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebra (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

The dominating term in the sum over sums of resonances in t -channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$ -behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

2 Bird's Eye of View about the Topics of the Book

The focus of this book is the number theoretical vision about physics. This vision involves three loosely related parts.

1. The fusion of real physics and various p-adic physics to a single larger whole by generalizing the number concept by fusing real numbers and various p-adic number fields along common rationals. Extensions of p-adic number fields can be introduced by gluing them along common algebraic numbers to reals. Algebraic continuation of the physics from rationals and their extensions to various number fields (completion of rational physics to physics in various number fields) is the key idea and the challenge is to understand whether how one could achieve this dream. A very profound implication is that purely local p-adic physics codes for the p-adic fractality of long length scale real physics and vice versa. As a consequence one can understand the origins of p-adic length scale hypothesis and ends up with a very concrete view about space-time correlates of cognition.

The fusion of various p-adic physics to single coherent whole leads to what I call adelic physics [L1, L2].

2. Second part of the vision involves what the classical number fields defined as subspaces of their complexifications with Minkowskian signature of the metric. The hypothesis is that allowed space-time surfaces correspond to quaternionic sub-manifolds of complexified octonionic space. The proposed interpretation of quaternionicity would in terms of being zero for the real or imaginary part of octonionic polynomial with rational or perhaps even algebraic coefficients.

Real/imaginary part refers to a composition of octonion to quaternion and imaginary unit multiplying second quaternion analogous to the decomposition of ordinary complex number to real and imaginary parts. Space-time surface would correspond to imaginary roots (in the sense that they are proportional to the imaginary unit i commuting with the octonionic units). It is argued that this notion of quaternionicity is equivalent with the assumption that the tangent space or normal of space-time surface in M^8 at each point is quaternionic.

Besides this one assumes that one can assign to each point of space-time surface a complex plane M_c^2 as subspace of the quaternionic plane M_c^4 . These planes could even depend on point of space-time surface and define an integrable distribution - kind of string world sheet.

Quaternionicity of the tangent plane in this sense allows to map the space-time surface in M^8 to a space-time surface in $H = M^4 \times CP_2$. This involves a projection to M^4 in the decomposition $M^8 = M^4 \times C_2$ and the assignment to the point of space-time surface point of CP_2 labelling its tangent space.

It is not clear whether one can assign also to each point of space-time surface in H a quaternionic tangent or normal in the tangent space M^8 of H . In the case in H this plane could be the tangent/normal plane defined by the modified gamma matrices or induced gamma matrices. These two planes co-incide with each other only for action defined by the metric determinant. Hence the basic variational principle of TGD would have deep number theoretic content. Reduction to a closed form would also mean that classical TGD would define a generalized topological field theory with Noether charges defining topological invariants.

3. The third part of the vision involves infinite primes, which can be identified in terms of an infinite hierarchy of second quantized arithmetic quantum fields theories on one hand, and as having representations as space-time surfaces analogous to zero surfaces of polynomials on the other hand. In this framework space-time surface would represent an infinite number. This vision leads also the conclusion that single point of space-time has an infinitely complex structure since real unity can be represented as a ratio of infinite numbers in infinitely many manners each having its own number theoretic anatomy. Thus single space-time point is in principle able to represent in its structure the quantum state of the entire universe. This number theoretic variant of Brahman=Atman identity also means that Universe is an algebraic hologram.

Besides this holy trinity I will discuss also loosely related topics. Included are the possible applications of the category theory in TGD and in TGD inspired theory of consciousness; various TGD inspired considerations related to Riemann hypothesis - in particular, a strategy for proving Riemann hypothesis using a modification of Hilbert-Polya conjecture replacing quantum states with coherent states of a unique conformally invariant physical system; topological quantum computation in TGD Universe; and TGD inspired approach to Langlands program.

2.1 Organization of “Physics as Generalized Number Theory: Part II”

The book consists of 3 parts.

1. In the 1st part $M^8 - H$ duality is discussed. It states that the purely algebraic physics (no variational principle nor partial differential equations) based on algebraic surfaces in complexified M^8 regarded as complexified octonions is dual to the physics defined by preferred extremals (presumably minimal surfaces) in $H = M^4 \times CP_2$. Quantum criticality would bring in infinite number of constraints analogous to gauge conditions implying that space-time surfaces in H are analogs of Bohr orbits. The dynamics based on variational principle in H would be equivalent with purely algebraic physics in M^8 .
2. The 2nd part includes various TGD inspired considerations related to Riemann hypothesis - in particular, a strategy for proving Riemann hypothesis using a modification of Hilbert-Polya conjecture replacing quantum states with coherent states of a unique conformally invariant physical system. The proposal that zeros of Riemann zeta could correspond to complex values of coupling constant is also discussed. Although the values of the coupling parameter fit rather nicely with those of U(1) coupling strength for electro-weak interactions, I have more or less given up this conjecture in favor of much more convincing conjecture justifiable from a model of coupling constant evolution reducing to that for the length scale dependent cosmological constant taking the role of cutoff parameter and emerging from the twistor lift of TGD. For this option the values of coupling constant are labelled by the zeros of zeta but are not so directly related to them.

3. In lack of better title I have have referred the contents of the 3rd part as “miscellaneous topics”. These topics touch the boundaries of my mathematical understanding and skills, and I do not regard these chapters as core TGD. The first chapter represents the first serious attempt to define the notion of p-adic manifold. It started from the question what p-adic variant of icosahedron could mean. Later I realized that it is better to approach the problems from the perspective of TGD inspired physics rather than trying to mimick what mathematicians have done. Much simpler and physically more attractive approach emerges from the notion of cognitive representation based on extensions of rationals defining a hierarchy if adeles. There are also chapters about TGD and non-standard numbers and infinite primes and motives. The last chapter is about Langlands program and TGD.

3 Sources

The eight online books about TGD [K17, K12, K23, K15, K8, K22, K21, K14] and nine online books about TGD inspired theory of consciousness and quantum biology [K16, K3, K9, K2, K4, K5, K6, K13, K20] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (<http://tinyurl.com/ybv8dt4n>) contains a lot of material about TGD. In particular, a TGD glossary at <http://tinyurl.com/yd6jf3o7>).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (<http://tinyurl.com/ycyrxj4o> founded by Lian Sidorov and in *Prespacetime Journal* (<http://tinyurl.com/ycvktjhn>), *Journal of Consciousness Research and Exploration* (<http://tinyurl.com/yba4f672>), and *DNA Decipher Journal* (<http://tinyurl.com/y9z52khg>), all of them founded by Huping Hu. One can find the list about the articles published at <http://tinyurl.com/ybv8dt4n>. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

4 The contents of the book

4.1 PART I: $M^8 - H$ DUALITY

4.1.1 Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part I

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces.

In the sequel I shall consider the following topics.

1. I will discuss basic notions of algebraic geometry such as algebraic variety, surface, and curve, all rational point of variety central for TGD view about cognitive representation, elliptic curves and surfaces, and rational and potentially rational varieties. Also the notion of Zariski topology and Kodaira dimension are discussed briefly. I am not a mathematician and what hopefully saves me from horrible blunders is physical intuition developed during 4 decades of TGD.
2. It will be shown how $M^8 - H$ duality could reduce TGD at fundamental level to octonionic algebraic geometry. Space-time surfaces in M^8 would be algebraic surfaces identified as zero loci for imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial of complexified octonionic variable o_c decomposing as $o_c = q_c^1 + q_c^2 I^4$ and projected to a Minkowskian subspace M^8 of complexified O . Single real valued polynomial of real variable with algebraic coefficients would determine space-time surface! As proposed already earlier, spacetime surfaces would form commutative and associative algebra with addition, product and functional composition.

One can interpret the products of polynomials as correlates for free many-particle states with interactions described by added interaction polynomial, which can vanish at boundaries of CDs thanks to the vanishing in Minkowski signature of the complexified norm $q_c \bar{q}_c$ appearing in $RE(P)$ or $IM(P)$ caused by the quaternionic non-commutativity. This leads to the same picture as the view about preferred extremals reducing to minimal surfaces near boundaries of CD. Also zero energy ontology (ZEO) could emerge naturally from the failure of number field property for quaternions at light-cone boundaries.

The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients can give rise to associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

The hierarchy of notions involved is well-ordering for 1-D structures, commutativity for complex numbers, and associativity for quaternions. This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy-Riemann conditions are linear and constant value manifolds are 1-D and thus well-ordered. Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

In fact, all algebras obtained by Cayley-Dickson construction adding imaginary units to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and $M^8 - H$ correspondence could generalize.

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k , $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i . In the generic case this gives 3-D surface.

In this generic case $M^8 - H$ duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H , and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

One can also have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i . Space-time surface would be analogous to a polynomial with a multiple root. The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces

except at 3-D boundary surfaces. These regions correspond to interaction regions in which Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes to the level of complex/co-complex surfaces assigned with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces?

I have proposed commutativity or co-commutativity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commutative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

4.1.2 Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part II

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to octonionic algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces.

The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients can give rise to associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

The hierarchy of notions involved is well-ordering for 1-D structures, commutativity for complex numbers, and associativity for quaternions. This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy Riemann conditions are linear and constant value manifolds are 1-D and thus well-ordered. Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

In fact, all algebras obtained by Cayley-Dickson construction adding imaginary units to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and $M^8 - H$ correspondence could generalize.

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k , $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i . In the generic case this gives 3-D surface.

In this generic case $M^8 - H$ duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H , and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

One can also have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i . Space-time surface would be analogous to a polynomial with a multiple root. The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces except at 3-D boundary surfaces. These regions correspond to interaction regions in which Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes to the level of complex/co-complex surfaces assigned with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces?

I have proposed commutativity or co-commutativity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commutative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

4. The super variant of the octonionic geometry relying on octonionic triality makes sense and the geometry of the space-time variety correlates with fermion and antifermion numbers assigned with it. This new view about super-geometry involving also automatic SUSY breaking at the level of space-time geometry.

Also a sketchy proposal for the description of interactions is discussed.

1. The surprise that $RE(P) = 0$ and $IM(P) = 0$ conditions have as singular solutions light-cone interior and its complement and 6-spheres $S^6(t_n)$ with radii t_n given by the roots of the real $P(t)$, whose octonionic extension defines the space-time variety X^4 . The intersections $X^2 = X^4 \cap S^6(t_n)$ are tentatively identified as partonic 2-varieties defining topological interaction vertices.

The idea about the reduction of zero energy states to discrete cognitive representations suggests that interaction vertices at partonic varieties X^2 are associated with the discrete set of intersection points of the sparticle lines at light-like orbits of partonic 2-surfaces belonging to extension of rationals.

2. CDs and therefore also ZEO emerge naturally. For CDs with different origins the products of polynomials fail to commute and associate unless the CDs have tips along real (time) axis. The first option is that all CDs under observation satisfy this condition. Second option allows general CDs.

The proposal is that the product $\prod P_i$ of polynomials associated with CDs with tips along real axis the condition $IM(\prod P_i) = 0$ reduces to $IM(P_i) = 0$ and criticality conditions guaranteeing associativity and provides a description of the external particles. Inside these CDs $RE(\prod P_i) = 0$ does not reduce to $RE(P_i) = 0$, which automatically gives rise to geometric interactions. For general CDs the situation is more complex.

3. The possibility of super octonionic geometry raises the hope that the twistorial construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY generalizes to TGD in rather straightforward manner to a purely geometric construction. Functional integral over WCW would reduce to summations over polynomials with coefficients in extension of rationals and criticality conditions on the coefficients could make the summation well-defined by bringing in finite measurement resolution.

Scattering diagrams would be determined by points of space-time variety, which are in extension of rationals. In adelic physics the interpretation is as cognitive representations.

1. Cognitive representations are identified as sets of rational points for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8 - and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces. For the earlier work related to adelic TGD and cognitive representations see [?]
2. Some aspects related to homology charge (Kähler magnetic charge) and genus-generation correspondence are discussed. Both topological quantum numbers are central in the proposed model of elementary particles and it is interesting to see whether the picture is internally consistent and how algebraic variety property affects the situation. Also possible problems related to $h_{eff}/h = n$ hierarchy []adelicphysics realized in terms of n -fold coverings of space-time surfaces are discussed from this perspective.

4.1.3 Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III

Cognitive representations are the basic topic of the third chapter related to $M^8 - H$ duality. Cognitive representations are identified as sets of points in extension of rationals for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8 - and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces.

The notion is applied in various cases and the connection with $M^8 - H$ duality is rather loose.

1. Extensions of rationals are essentially coders of information. There the possible analogy of extensions of rationals with genes deserves discussion. Extensions, which are not extensions of extensions would be analogous to genes. The notion of conserved gene as number theoretical analogy for Galois extensions as the Galois group of extension which is normal subgroup of Galois extension.
2. The possible physical meaning of the notion of perfectoid introduced by Peter Scholze is discussed in the framework of p-adic physics. Extensions of p-adic numbers involving roots of the prime defining the extension are involved and are not considered previously in TGD framework. There there possible physical meaning deserves discussion.
3. The construction of cognitive representation reduces to a well-known mathematical problem of finding the points of space-time surface with imbedding space coordinates in given extension of rationals. The work of Kim and Coates represents new ideas in this respect and there is a natural connection with TGD.
4. One expects that large cognitive representations are winners in the number theoretical fight for survival. Strong form of holography suggests that it is enough to consider cognitive representations restricted to string world sheets and partonic 2-surfaces. If the 2-surface possesses large group of symmetries acting in extension of rationals, one can have large cognitive representations as orbit of point in extension. Examples of highly symmetric 2-D surfaces are geodesic spheres assignable to partonic 2-surfaces and cosmic strings and elliptic curves assignable with string world sheets and cosmic strings.

5. Rationals and their extensions give rise to a unique discretizations of space-time surface (for instance) - cognitive representation - having interpretation in terms of finite measurement resolution. There are however many open questions. Should one allow only octonionic polynomials defined as algebraic continuations of real polynomials or should one allow also analytic functions and regard polynomials as approximations. Zeta functions are especially interesting analytic functions and Defekind zetas characterize extensions of rationals and one can pose physically motivated questions about them.

4.1.4 Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III

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4.1.5 Breakthrough in understanding of $M^8 - H$ duality

This article represents a critical re-examination of $M^8 - H$ duality, which is one of the cornerstones of Topological Geometroynamics (TGD). The original version of $M^8 - H$ duality assumed that space-time surfaces in M^8 can be identified as associative or co-associative surfaces. If the surface has associative tangent or normal space and contains a complex or co-complex surface, it can be mapped to a 4-surface in $H = M^4 \times CP_2$.

Later emerged the idea that octonionic analyticity realized in terms of real polynomials P algebraically continued to polynomials of complexified octonion could fulfill the dream. The vanishing of the real part $Re_Q(P)$ (imaginary part $Im_Q(P)$) in the quaternionic sense would give rise to an associative (co-associative) space-time surface.

The realization of the general coordinate invariance motivated the notion of strong form of holography (SH) in H allowing realization of a weaker form of $M^8 - H$ duality by assuming that associativity/co-associativity conditions are needed only at 2-D string world sheet and partonic 2-surfaces and possibly also at their light-like 3-orbits.

The outcome of the re-examination was a positive surprise. Although no interesting associative space-time surfaces are possible, every distribution of normal associative planes (co-associativity) is integrable. Another positive surprise was that Minkowski signature is the only possible option. Equivalently, the image of M^4 as real co-associative sub-space of O_c (complex valued octonion norm squared is real valued for them) by an element of local $G_{2,c}$ or its subgroup $SU(3,c)$ gives a real co-associative space-time surface. The conjecture is that the polynomials P determine these surfaces as roots of $Re_Q(P)$. These surfaces also possess co-complex 2-D sub-manifolds allowing the mapping to H to H by $M^8 - H$ duality as a whole. SH would not be needed and would be replaced with number theoretic holography determining space-time surface from its roots and selection of real subspace of O_c characterizing the state of motion of a particle.

Octonionic Dirac equation as analog of momentum space variant of ordinary Dirac equation forces the interpretation of M^8 as an analog of momentum space and Uncertainty Principle forces to modify the map $M^4 \subset M^8 \rightarrow M^4 \subset H$ from identification to inversion. The equations for $Re_Q(P) = 0$ reduce to simultaneous roots of the real polynomials defined by the odd and even parts of P having interpretation as complex values of mass squared mapped to light-cone proper time constant surfaces in H . This leads to the idea that the formulation of scattering amplitudes at M^8 levels provides the counterpart of momentum space description of scattering whereas the formulation at the level of H provides the counterpart of space-time description.

This picture combined with zero energy ontology leads also to a view about quantum TGD at the level of M^8 . Local $G_{2,c}$ element has properties suggesting a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by P . The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

The new view about $M^8 - H$ duality differs from the earlier one rather dramatically so that a summary of the differences is added to the end of paper.

4.1.6 Could quantum randomness have something to do with classical chaos?

Tim Palmer has proposed that classical chaos and quantum randomness might be related. It came as a surprise to me that these two notions could have deep relationship in TGD framework.

1. Strong form of Palmer's idea stating that quantum randomness reduces to classical chaos certainly fails but one can consider weaker forms of the idea. Even these variants fail in Copenhagen interpretation since strictly speaking there is no classical reality, only wave function coding for the knowledge about the system. Bohr orbits should be more than approximation and in TGD framework space-time surface as preferred extremal of action is analogous to Bohr orbit and classical physics defined by Bohr orbits is an exact part of quantum theory.
2. In the zero energy ontology (ZEO) of TGD the idea works in weaker form and has very strong implications for the more detailed understanding of ZEO and $M^8 - M^4 \times CP_2$ duality. Ordinary ("big") state functions (BSFRs) meaning the death of the system in a universal sense and re-incarnation with opposite arrow of time would involve quantum criticality accompanied by classical chaos assignable to the correspondence between geometric time and subjective time identified as sequence of "small" state function reductions (SSFRs) as analogs of weak measurements. The findings of Mineev et al give strong support for this view and

Libet's findings about active aspects of consciousness can be understood if the act of free will corresponds to BSFR.

M^8 picture identifies 4-D space-time surfaces X^4 as roots for "imaginary" or "real" part of octonionic polynomial P_2P_1 obtained as a continuation of real polynomial $P_2(L-r)P_1(r)$, whose arguments have origin at the tips of B and A and roots at the light-cone boundaries associated with tips. Causal diamond (CD) is identified intersection of future and past directed light-cones A and B . In the sequences of SSFRs $P_2(L-r)$ assigned to B varies and $P_1(r)$ assigned to A is unaffected. L defines the size of CD as distance $\tau = 2L$ between its tips.

Besides 4-D space-time surfaces there are also brane-like 6-surfaces corresponding to roots $r_{i,k}$ of $P_i(r)$ and defining "special moments in the life of self" having $t_i = r_{i,k}$ ball as M^4_+ projection. The number of roots and their density increases rapidly in the sequence of SSFRs. The condition that the largest root belongs to CD gives a lower bound to its size L as largest root. Note that L increases.

Concerning the approach to chaos, one can consider three options.

Option I: The sequence of steps consisting of unitary evolutions followed by SSFR corresponds to a functional factorization at the level of polynomials as sequence $P_2 = Q_1 \circ Q_2 \circ \dots \circ Q_n$. If the size of CD is assumed to increase, also the tip of active boundary of CD must shift so that the argument of P_2 $r - L$ is replaced in each iteration step to with updated argument with larger value of L .

Option II: A completely unexpected connection with the iteration of analytic functions and Julia sets, which are fractals assigned also with chaos interpreted as complexity emerges. In a reasonable approximation quantum time evolution by SSFRs could be induced by an iteration of a polynomial or even an analytic function: $P_2 = P_2 \rightarrow P_2^{\circ 2} \rightarrow \dots$. For $P_2(0) = 0$ the roots of the iterate consists of inverse images of roots of P_2 by $P_2^{\circ -k}$ for $k = 0, \dots, N - 1$.

Suppose that M^8 and X^4 are complexified and thus also $t = r$ and "real" X^4 is the projection of X^4_c to real M^8 . Complexify also the coefficients of polynomials P . If so, the Mandelbrot and Julia sets (<http://tinyurl.com/cplj9pe> and <http://tinyurl.com/cvnr83g>) characterizing fractals would have a physical interpretation in ZEO.

One approaches chaos in the sense that the $N - 1$:th inverse images of the roots of P_2 belonging to filled Julia set approach to points of Julia set of P_2 as the number N of iterations increases. Minimal L would increase with N if CD is assumed to contain all roots. The density of the roots in Julia set increases near L since the size of CD is bounded by the size Julia set. One could perhaps say that near the $t = L$ in the middle of CD the life of self when the size of CD has become almost stationary, is the most intensive.

Option III: A conservative option is to consider also real polynomials $P_2(r)$ with real argument r . Only non-negative real roots r_n are of interest whereas in the general case one considers all values of r . For a large N the new roots with possibly one exception would approach to the real Julia set obtained as a real projection of Julia set for complex iteration.

How the size L of CD is determined and when can BSFR occur?

Option I: If L is minimal and thus given by the largest (non-exceptional) root of iterate of P_2 in Julia set, it is bound to increase in the iteration (this option is perhaps too deterministic). L should smaller than the sizes of Julia sets of both A and B since the iteration gives no roots outside Julia sets.

Could BSFR become probable when L as the largest allowed root for iterate P_2 is larger than the size of Julia set of A ? There would be no more new "special moments in the life of self" and this would make death (in universal sense) and re-incarnation with opposite arrow of time probable. The size of CD could decrease dramatically in the first iteration for P_1 if it is determined as the largest allowed root of P_1 : the re-incarnated self would have childhood.

Option II: The size of CD could be determined in SSFR statistically as an allowed root of P_2 . Since the density of roots increases, one would have a lot of choices and quantum criticality and fluctuations of the order of clock time $\tau = 2L$: the order of subjective time would not anymore correspond to that for clock time. BSFR would occur for the same reason as for the first option.

4.2 PART II: RIEMANN ZETA AND PHYSICS

4.2.1 Riemann hypothesis and physics

Riemann hypothesis states that the nontrivial zeros of Riemann Zeta function lie on the critical line $Re(s) = 1/2$. Since Riemann zeta function allows a formal interpretation as thermodynamical partition function for a quantum field theoretical system consisting of bosons labeled by primes, it is interesting to look Riemann hypothesis from the perspective of physics. The complex value of temperature is not however consistent with thermodynamics. In zero energy ontology one obtains quantum theory as a square root of thermodynamics and this objection can be circumvented and a nice argument allowing to interpret RH physically emerges.

Conformal invariance leads to a beautiful generalization of Hilbert-Polya conjecture allowing to interpret RH in terms of coherent states rather than energy eigenstates of a Hamiltonian. In zero energy ontology the interpretation is that the coherent states in question represent Bose-Einstein condensation at criticality. Zeros of zeta correspond to coherent states orthogonal to the coherent state characterized by $s = 0$, which has finite norm, and therefore does not represent Bose-Einstein condensation.

Quantum TGD and also TGD inspired theory of consciousness provide additional view points to the hypothesis and suggests sharpening of Riemann hypothesis, detailed strategies of proof of the sharpened hypothesis, and heuristic arguments for why the hypothesis is true. These considerations are however highly speculative and are represented at the end of the chapter.

1. Super-conformal invariance and generalization of Hilbert-Polya hypothesis

Super-conformal invariance inspires a strategy for proving the Riemann hypothesis. The vanishing of the Riemann Zeta reduces to an orthogonality condition for the eigenfunctions of a non-Hermitian operator D^+ having the zeros of Riemann Zeta as its eigenvalues. The construction of D^+ is inspired by the conviction that Riemann Zeta is associated with a physical system allowing super-conformal transformations as its symmetries and second quantization in terms of the representations of the super-conformal algebra. The eigenfunctions of D^+ are analogous to coherent states of a harmonic oscillator and in general they are not orthogonal to each other. The states orthogonal to a vacuum state (having a negative norm squared) correspond to the zeros of Riemann Zeta. The physical states having a positive norm squared correspond to the zeros of Riemann Zeta at the critical line. Riemann hypothesis follows both from the hermiticity and positive definiteness of the metric in the space of states corresponding to the zeros of ζ . Also conformal symmetry in appropriate sense implies Riemann hypothesis and after one year from the discovery of the basic idea it became clear that one can actually construct a rigorous twenty line long analytic proof for the Riemann hypothesis using a standard argument from Lie group theory.

2. Zero energy ontology and RH

A further approach to RH is based on zero energy ontology and is consistent with the approach based on the notion of coherent state. The postulate that all zero energy states for Riemann system are zeros of zeta and critical in the sense being non-normalizable (Bose-Einstein condensation) combined with the fact that $s = 1$ is the only pole of ζ implies that the all zeros of ζ correspond to $Re(s) = 1/2$ so that RH follows from purely physical assumptions. The behavior at $s = 1$ would be an essential element of the argument. The interpretation as a zero energy counterpart of a coherent state seems to makes sense also now. Note that in ZEO coherent state property is in accordance with energy conservation. In the case of coherent states of Cooper pairs same applies to fermion number conservation. With this interpretation the condition would state orthogonality with respect to the coherent zero energy state characterized by $s = 0$, which has finite norm and does not represent Bose-Einstein condensation. This would give a connection for the proposal for the strategy for proving Riemann Hypothesis by replacing eigenstates of energy with coherent states.

3. Miscellaneous ideas

During years I have also considered several ideas about Riemann hypothesis which I would not call miscellaneous. I have moved them to the end of the chapter because of the highly speculative nature.

4.2.2 Does Riemann Zeta Code for Generic Coupling Constant Evolution?

A general model for the coupling constant evolution is proposed. The analogy of Riemann zeta and fermionic zeta $\zeta_F(s)/\zeta_F(2s)$ with complex square root of a partition function natural in Zero Energy Ontology suggests that the poles of $\zeta_F(ks)$, $k = 1/2$, correspond to complexified critical temperatures identifiable as inverse of Kähler coupling strength itself having interpretation as inverse of critical temperature. One can actually replace the argument s of ζ_F with Möbius transformed argument $w = (as + b)/(cs + d)$ with a, b, c, d real numbers, rationals, or even integers. For α_K $w = (s + b)/2$ is proper choices and gives zeros of $\zeta(s)$ and $s = 2 - b$ as poles. The identification $\alpha_K = \alpha_{U(1)}$ leads to a prediction for α_{em} , which deviates by .7 per cent from the experimental value at low energies (atomic scale) if the experimental value of the Weinberg angle is used. The conjecture generalizes also to weak, color and gravitational interactions when general Möbius transformation leaving upper half-plane invariant is allowed. One ends up with a general model predicting successfully the entire electroweak coupling constant evolution successfully from the values of fine structure constant at atomic or electron scale and in weak scale.

4.2.3 TGD View about Coupling Constant Evolution

New results related to the TGD view about coupling constant evolution are discussed. The results emerge from the discussion of the recent claim of Atyiah that fine structure constant could be understood purely mathematically. The new view allows to understand the recently introduced TGD based construction of scattering amplitudes based on the analog of micro-canonical ensemble as a cognitive representation for the much more complex construction of full scattering amplitudes using real numbers rather than p-adic number fields. This construction utilizes number theoretic discretization of space-time surface inducing that of “world of classical worlds” (WCW) and makes possible adelization of quantum TGD.

The understanding of coupling constant evolution has been one of most longstanding problems of TGD and I have made several proposals during years.

Could number theoretical constraints fix the evolution? Adelization suffers from serious number theoretical problem due to the fact that the action exponentials do not in general exist p-adically for given adele. The solution of the problem turned out to be trivial. The exponentials disappear from the scattering amplitudes! Contrary to the first beliefs, adelization does not therefore seem to determine coupling constant evolution.

TGD view about cosmological constant turned out to be the solution of the problem. The formulation of the twistor lift of Kähler action led to a rather detailed view about the interpretation of cosmological constant as an approximate parameterization of the dimensionally reduced 6-D Kähler action (or energy) allowing also to understand how it can decrease so fast as a function of p-adic length scale. In particular, a dynamical mechanism for the dimensional reduction of 6-D Kähler action giving rise to the induction of the twistor structure and predicting this evolution emerges.

In standard QFT view about coupling constant evolution ultraviolet cutoff length serves as the evolution parameter. TGD is however free of infinities and there is no cutoff parameter. It turned out cosmological constant replaces this parameter and coupling constant evolution is induced by that for cosmological constant from the condition that the twistor lift of the action is not affected by small enough modifications of the moduli of the induced twistor structure. The moduli space for them corresponds to rotation group $SO(3)$. This leads to explicit evolution equations for α_K , which can be studied numerically.

The approach is also related to the view about coupling constant evolution based on the inclusions of hyper-finite factors of type II_1 , and it is proposed that Galois group replaces discrete subgroup of $SU(2)$ leaving invariant the algebras of observables of the factors appearing in the inclusion.

4.2.4 About the role of Galois groups in TGD framework

This article was inspired by the inverse problem of Galois theory. Galois groups are realized as number theoretic symmetry groups realized physically in TGD a symmetries of space-time surfaces. Galois confinement as an analog of color confinement is proposed in TGD inspired quantum biology.

Two instances of the inverse Galois problem, which are especially interesting in TGD, are following:

Q1: Can a given finite group appear as Galois group over Q ? The answer is not known.

Q2: Can a given finite group G appear as a Galois group over some EQ? Answer to Q2 is positive as will be found and the extensions for a given G can be explicitly constructed.

The TGD based formulation based on $M^8 - H$ duality in which space-time surface in complexified M^8 are coded by polynomials with rational coefficients involves the following open question.

Q: Can one allow only polynomials with coefficients in Q or should one allow also coefficients in EQs?

The idea allowing to answer this question is the requirement that TGD adelic physics is able to represent all finite groups as Galois groups of Q or some EQ acting physical symmetry group.

If the answer to **Q1** is positive, it is enough to have polynomials with coefficients in Q . It not, then also EQs are needed as coefficient fields for polynomials to get all Galois groups. The first option would be the more elegant one.

In the sequel the inverse problem is considered from the perspective of TGD. Galois groups, in particular simple Galois groups, play a fundamental role in the TGD view of cognition. The TGD based model of the genetic code involves in an essential manner the groups A_5 (icosahedron), which is the smallest simple and non-commutative group, and A_4 (tetrahedron). The identification of these groups as Galois groups leads to a more precise view about genetic code.

4.2.5 Some Questions about Coupling Constant Evolution

In this chapter questions related to the hierarchy of Planck constants and p-adic coupling constant evolution (CCE) in the TGD framework are considered.

1. Is p-adic length scale hypothesis (PLS) correct in this recent form and can one deduce this hypothesis or its generalization from the basic physics of TGD defined by Kähler function of the "world of classical worlds" (WCW)? The fact, that the scaling of the roots of polynomial does not affect the algebraic properties of the extension strongly suggests that p-adic prime does not depend on purely algebraic properties of EQ. In particular, the proposed identification of p as a ramified prime of EQ could be wrong.

Number theoretical universality suggests the formula $\exp(\Delta K) = p^n$, where ΔK is the contribution to Kähler function of WCW for a given space-time surface inside causal diamond (CD).

2. The understanding of p-adic length scale evolution is also a problem. The "dark" CCE would be $\alpha_K = g_K^2/2h_{eff} = g_K^2/2nh_0$, and the PLS evolution $g_K^2(k) = g_K^2(max)/k$ should define independent evolutions since scalings commute with number theory. The total evolution $\alpha_K = \alpha_K(max)/nk$ would induce also the evolution of other coupling strengths if the coupling strengths are related to α_K by Möbius transformation as suggested.
3. The formula $h_{eff} = nh_0$ involves the minimal value h_0 . How could one determine it? p-Adic mass calculations for $h_{eff} = h$ lead to the conclusion that the CP_2 scale R is roughly $10^{7.5}$ times longer than Planck length l_P . Classical argument however suggests $R \simeq l_P$. If one assumes $h_{eff} = h_0$ in the p-adic mass calculations, this is indeed the case for $h/h_0 = (R/CP_2)/l_P)^2$. This ratio follows from number theoretic arguments as $h/h_0 = n_0 = (7!)^2$. This gives $\alpha_K = n_0/kn$, and perturbation theory can converge even for $n = 1$ for sufficiently long p-adic length scales. Gauge coupling strengths are predicted to be practically zero at gravitational flux tubes so that only gravitational interaction is effectively present. This conforms with the view about dark matter.
4. Nottale hypothesis predicts gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ ($\beta_0 = v_0/c$ is velocity parameter), which has gigantic values. Gravitational fine structure constant is given by $\alpha_{gr} = \beta_0/4\pi$. Kepler's law $\beta^2 = GM/r = r_S/2r$ suggests length scale evolution $\beta^2 = xr_S/2L_N = \beta_{0,max}^2/N^2$, where x is proportionality constant, which can be fixed.

Phase transitions changing β_0 are possible at $L_N/a_{gr} = N^2$ and these scales correspond to radii for the gravitational analogs of the Bohr orbits of hydrogen. p-Adic length scale

hierarchy is replaced by that for the radii of Bohr orbits. The simplest option is that β_0 obeys a CCE induced by α_K .

This picture conforms with the existing applications and makes it possible to understand the value of β_0 for the solar system, and is consistent with the application to the superfluid fountain effect.

4.3 PART III: MISCELLANEOUS TOPICS

4.3.1 What p-adic icosahedron could mean? And what about p-adic manifold?

The original focus of this chapter was p-adic icosahedron. The discussion of attempt to define this notion however leads to the challenge of defining the concept of p-adic sphere, and more generally, that of p-adic manifold, and this problem soon became the main target of attention since it is one of the key challenges of also TGD.

There exists two basic philosophies concerning the construction of both real and p-adic manifolds: algebraic and topological approach. Also in TGD these approaches have been competing: algebraic approach relates real and p-adic space-time points by identifying the common rationals. Finite pinary cutoff is however required to avoid totally wild fluctuations and has interpretation in terms of finite measurement resolution. Canonical identification maps p-adics to reals and vice versa in a continuous manner but is not consistent with p-adic analyticity nor field equations unless one poses a pinary cutoff. It seems that pinary cutoff reflecting the notion of finite measurement resolution is necessary in both approaches. This represents a new notion from the point of view of mathematics.

1. One can try to generalize the theory of real manifolds to p-adic context. The basic problem is that p-adic balls are either disjoint or nested so that the usual construction by gluing partially overlapping spheres fails. One attempt to solve the problem relies on the notion of Berkovich disk obtained as a completion of p-adic disk having path connected topology (non-ultrametric) and containing p-adic disk as a dense subset. This plus the complexity of the construction is heavy price to be paid for path-connectedness. A related notion is Bruhat-Tits tree defining kind of skeleton making p-adic manifold path connected. The notion makes sense for the p-adic counterparts of projective spaces, which suggests that p-adic projective spaces (S^2 and CP_2 in TGD framework) are physically very special.
2. Second approach is algebraic and restricts the consideration to algebraic varieties for which also topological invariants have algebraic counterparts. This approach looks very natural in TGD framework - at least for imbedding space. Preferred extremals of Kähler action can be characterized purely algebraically - even in a manner independent of the action principle - so that they might make sense also p-adically.

Number theoretical universality is central element of TGD. Physical considerations force to generalize the number concept by gluing reals and various p-adic number fields along rationals and possible common algebraic numbers. This idea makes sense also at the level of space-time and of “world of classical worlds” (WCW).

Algebraic continuation between different number fields is the key notion. Algebraic continuation between real and p-adic sectors takes place along their intersection, which at the level of WCW (“world of classical worlds”) correspond to surfaces allowing interpretation both as real and p-adic surfaces for some value(s) of prime p . The algebraic continuation from the intersection of real and p-adic WCWs is not possible for all p-adic number fields. For instance, real integrals as functions of parameters need not make sense for all p-adic number fields. This apparent mathematical weakness can be however turned to physical strength: real space-time surfaces assignable to elementary particles can correspond only some particular p-adic primes. This would explain why elementary particles are characterized by preferred p-adic primes. The p-adic prime determining the mass scale of the elementary particle could be fixed number theoretically rather than by some dynamical principle formulated in real context (number theoretic anatomy of rational number does not depend smoothly on its real magnitude!).

Although Berkovich construction of p-adic disk does not look promising in TGD framework, it suggests that the difficulty posed by the total disconnectedness of p-adic topology is real. TGD in

turn suggests that the difficulty could be overcome without the completion to a non-ultrametric topology. Two approaches emerge, which ought to be equivalent.

The TGD inspired solution to the construction of path connected effective p-adic topology is based on the notion of canonical identification mapping reals to p-adics and vice versa in a continuous manner. The trivial but striking observation was that canonical identification satisfies triangle inequality and thus defines an Archimedean norm allowing to induce real topology to p-adic context. Canonical identification with finite measurement resolution defines chart maps from p-adics to reals and vice versa and preferred extremal property allows to complete the discrete image to hopefully space-time surface unique within finite measurement resolution so that topological and algebraic approach are combined. Finite resolution would become part of the manifold theory. p-Adic manifold theory would also have interpretation in terms of cognitive representations as maps between realities and p-adicities.

4.3.2 TGD and Non-Standard Numbers

The chapter represents a comparison of ultrapower fields (loosely surreals, hyper-reals, long line) and number fields generated by infinite primes having a physical interpretation in Topological Geometro-dynamics.

Ultrapower fields are discussed in very physicist friendly manner in the articles of Elemer Rosinger and these articles are taken as a convenient starting point. The physical interpretations and principles proposed by Rosinger are considered against the background provided by TGD. The construction of ultrapower fields is associated with physics using the close analogies with gauge theories, gauge invariance, and with the singularities of classical fields.

Non-standard numbers are compared with the numbers generated by infinite primes and it is found that the construction of infinite primes, integers, and rationals has a close similarity with construction of the generalized scalars. The construction replaces at the lowest level the index set $\Lambda = \mathbb{N}$ of natural numbers with algebraic numbers \mathbb{A} , Frechet filter of \mathbb{N} with that of \mathbb{A} , and \mathbb{R} with unit circle S^1 represented as complex numbers of unit magnitude. At higher levels of the hierarchy generalized -possibly infinite and infinitesimal- algebraic numbers emerge. This correspondence maps a given set in the dual of Frechet filter of \mathbb{A} to a phase factor characterizing infinite rational algebraically so that correspondence is like representation of algebra.

The basic difference between two approaches to infinite numbers is that the counterpart of infinitesimals is infinitude of real units with complex number theoretic anatomy: one might loosely say that these real units are exponentials of infinitesimals.

4.3.3 Infinite Primes and Motives

In this chapter the goal is to find whether the general mathematical structures associated with twistor approach, superstring models and M-theory could have a generalization or a modification in TGD framework. The contents of the chapter is an outcome of a rather spontaneous process, and represents rather unexpected new insights about TGD resulting as outcome of the comparisons.

1. *Infinite primes, Galois groups, algebraic geometry, and TGD*

In algebraic geometry the notion of variety defined by algebraic equation is very general: all number fields are allowed. One of the challenges is to define the counterparts of homology and cohomology groups for them. The notion of cohomology giving rise also to homology if Poincare duality holds true is central. The number of various cohomology theories has inflated and one of the basic challenges to find a sufficiently general approach allowing to interpret various cohomology theories as variations of the same motive as Grothendieck, who is the pioneer of the field responsible for many of the basic notions and visions, expressed it.

Cohomology requires a definition of integral for forms for all number fields. In p-adic context the lack of well-ordering of p-adic numbers implies difficulties both in homology and cohomology since the notion of boundary does not exist in topological sense. The notion of definite integral is problematic for the same reason. This has led to a proposal of reducing integration to Fourier analysis working for symmetric spaces but requiring algebraic extensions of p-adic numbers and an appropriate definition of the p-adic symmetric space. The definition is not unique and the interpretation is in terms of the varying measurement resolution.

The notion of infinite has gradually turned out to be more and more important for quantum TGD. Infinite primes, integers, and rationals form a hierarchy completely analogous to a hierarchy of second quantization for a super-symmetric arithmetic quantum field theory. The simplest infinite primes representing elementary particles at given level are in one-one correspondence with many-particle states of the previous level. More complex infinite primes have interpretation in terms of bound states.

1. What makes infinite primes interesting from the point of view of algebraic geometry is that infinite primes, integers and rationals at the n :th level of the hierarchy are in 1-1 correspondence with rational functions of n arguments. One can solve the roots of associated polynomials and perform a root decomposition of infinite primes at various levels of the hierarchy and assign to them Galois groups acting as automorphisms of the field extensions of polynomials defined by the roots coming as restrictions of the basic polynomial to planes $x_n = 0$, $x_n = x_{n-1} = 0$, etc...
2. These Galois groups are suggested to define non-commutative generalization of homotopy and homology theories and non-linear boundary operation for which a geometric interpretation in terms of the restriction to lower-dimensional plane is proposed. The Galois group G_k would be analogous to the relative homology group relative to the plane $x_{k-1} = 0$ representing boundary and makes sense for all number fields also geometrically. One can ask whether the invariance of the complex of groups under the permutations of the orders of variables in the reduction process is necessary. Physical interpretation suggests that this is not the case and that all the groups obtained by the permutations are needed for a full description.
3. The algebraic counterpart of boundary map would map the elements of G_k identified as analog of homotopy group to the commutator group $[G_{k-2}, G_{k-2}]$ and therefore to the unit element of the abelianized group defining cohomology group. In order to obtain something analogous to the ordinary homology and cohomology groups one must however replace Galois groups by their group algebras with values in some field or ring. This allows to define the analogs of homotopy and homology groups as their abelianizations. Cohomotopy, and cohomology would emerge as duals of homotopy and homology in the dual of the group algebra.
4. That the algebraic representation of the boundary operation is not expected to be unique turns into blessing when one keeps the TGD as almost topological QFT vision as the guide line. One can include all boundary homomorphisms subject to the condition that the anticommutator $\delta_k^i \delta_{k-1}^j + \delta_k^j \delta_{k-1}^i$ maps to the group algebra of the commutator group $[G_{k-2}, G_{k-2}]$. By adding dual generators one obtains what looks like a generalization of anticommutative fermionic algebra and what comes in mind is the spectrum of quantum states of a SUSY algebra spanned by bosonic states realized as group algebra elements and fermionic states realized in terms of homotopy and cohomotopy and in abelianized version in terms of homology and cohomology. Galois group action allows to organize quantum states into multiplets of Galois groups acting as symmetry groups of physics. Poincare duality would map the analogs of fermionic creation operators to annihilation operators and vice versa and the counterpart of pairing of k :th and $n - k$:th homology groups would be inner product analogous to that given by Grassmann integration. The interpretation in terms of fermions turns however to be wrong and the more appropriate interpretation is in terms of Dolbeault cohomology applying to forms with homomorphic and antihomomorphic indices.
5. The intuitive idea that the Galois group is analogous to 1-D homotopy group which is the only non-commutative homotopy group, the structure of infinite primes analogous to the braids of braids of braids of ... structure, the fact that Galois group is a subgroup of permutation group, and the possibility to lift permutation group to a braid group suggests a representation as flows of 2-D plane with punctures giving a direct connection with topological quantum field theories for braids, knots and links. The natural assumption is that the flows are induced from transformations of the symplectic group acting on $\delta M_{\pm}^2 \times CP_2$ representing quantum fluctuating degrees of freedom associated with WCW ("world of classical worlds"). Discretization of WCW and cutoff in the number of modes would be due to the

finite measurement resolution. The outcome would be rather far reaching: finite measurement resolution would allow to construct WCW spinor fields explicitly using the machinery of number theory and algebraic geometry.

6. A connection with operads is highly suggestive. What is nice from TGD perspective is that the non-commutative generalization homology and homotopy has direct connection to the basic structure of quantum TGD almost topological quantum theory where braids are basic objects and also to hyper-finite factors of type II_1 . This notion of Galois group makes sense only for the algebraic varieties for which coefficient field is algebraic extension of some number field. Braid group approach however allows to generalize the approach to completely general polynomials since the braid group make sense also when the ends points for the braid are not algebraic points (roots of the polynomial).

This construction would realize the number theoretical, algebraic geometrical, and topological content in the construction of quantum states in TGD framework in accordance with TGD as almost TQFT philosophy, TGD as infinite-D geometry, and TGD as generalized number theory visions.

2. *p-Adic integration and cohomology*

This picture leads also to a proposal how p-adic integrals could be defined in TGD framework.

1. The calculation of twistorial amplitudes reduces to multi-dimensional residue calculus. Motivic integration gives excellent hopes for the p-adic existence of this calculus and braid representation would give space-time representation for the residue integrals in terms of the braid points representing poles of the integrand: this would conform with quantum classical correspondence. The power of 2π appearing in multiple residue integral is problematic unless it disappears from scattering amplitudes. Otherwise one must allow an extension of p-adic numbers to a ring containing powers of 2π .
2. Weak form of electric-magnetic duality and the general solution ansatz for preferred extremals reduce the Kähler action defining the Kähler function for WCW to the integral of Chern-Simons 3-form. Hence the reduction to cohomology takes places at space-time level and since p-adic cohomology exists there are excellent hopes about the existence of p-adic variant of Kähler action. The existence of the exponent of Kähler gives additional powerful constraints on the value of the Kähler fuction in the intersection of real and p-adic worlds consisting of algebraic partonic 2-surfaces and allows to guess the general form of the Kähler action in p-adic context.
3. One also should define p-adic integration for vacuum functional at the level of WCW. p-Adic thermodynamics serves as a guideline leading to the condition that in p-adic sector exponent of Kähler action is of form $(m/n)^r$, where m/n is divisible by a positive power of p-adic prime p . This implies that one has sum over contributions coming as powers of p and the challenge is to calculate the integral for $K = \text{constant}$ surfaces using the integration measure defined by an infinite power of Kähler form of WCW reducing the integral to cohomology which should make sense also p-adically. The p-adicization of the WCW integrals has been discussed already earlier using an approach based on harmonic analysis in symmetric spaces and these two approaches should be equivalent. One could also consider a more general quantization of Kähler action as sum $K = K_1 + K_2$ where $K_1 = r \log(m/n)$ and $K_2 = n$, with n divisible by p since $\exp(n)$ exists in this case and one has $\exp(K) = (m/n)^r \times \exp(n)$. Also transcendental extensions of p-adic numbers involving $n + p - 2$ powers of $e^{1/n}$ can be considered.
4. If the Galois group algebras indeed define a representation for WCW spinor fields in finite measurement resolution, also WCW integration would reduce to summations over the Galois groups involved so that integrals would be well-defined in all number fields.

3. *Floer homology, Gromov-Witten invariants, and TGD*

Floer homology defines a generalization of Morse theory allowing to deduce symplectic homology groups by studying Morse theory in loop space of the symplectic manifold. Since the

symplectic transformations of the boundary of $\delta M_{\pm}^4 \times CP_2$ define isometry group of WCW, it is very natural to expect that Kähler action defines a generalization of the Floer homology allowing to understand the symplectic aspects of quantum TGD. The hierarchy of Planck constants implied by the one-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates leads naturally to singular coverings of the imbedding space and the resulting symplectic Morse theory could characterize the homology of these coverings.

One ends up to a more precise definition of vacuum functional: Kähler action reduces Chern-Simons terms (imaginary in Minkowskian regions and real in Euclidian regions) so that it has both phase and real exponent which makes the functional integral well-defined. Both the phase factor and its conjugate must be allowed and the resulting degeneracy of ground state could allow to understand qualitatively the delicacies of CP breaking and its sensitivity to the parameters of the system. The critical points with respect to zero modes correspond to those for Kähler function. The critical points with respect to complex coordinates associated with quantum fluctuating degrees of freedom are not allowed by the positive definiteness of Kähler metric of WCW. One can say that Kähler and Morse functions define the real and imaginary parts of the exponent of vacuum functional.

The generalization of Floer homology inspires several new insights. In particular, space-time surface as hyper-quaternionic surface could define the 4-D counterpart for pseudo-holomorphic 2-surfaces in Floer homology. Holomorphic partonic 2-surfaces could in turn correspond to the extrema of Kähler function with respect to zero modes and holomorphy would be accompanied by super-symmetry.

Gromov-Witten invariants appear in Floer homology and topological string theories and this inspires the attempt to build an overall view about their role in TGD. Generalization of topological string theories of type A and B to TGD framework is proposed. The TGD counterpart of the mirror symmetry would be the equivalence of formulations of TGD in $H = M^4 \times CP_2$ and in $CP_3 \times CP_3$ with space-time surfaces replaced with 6-D sphere bundles.

4. *K-theory, branes, and TGD*

K-theory and its generalizations play a fundamental role in super-string models and M-theory since they allow a topological classification of branes. After representing some physical objections against the notion of brane more technical problems of this approach are discussed briefly and it is proposed how TGD allows to overcome these problems. A more precise formulation of the weak form of electric-magnetic duality emerges: the original formulation was not quite correct for space-time regions with Euclidian signature of the induced metric. The question about possible TGD counterparts of R-R and NS-NS fields and S, T, and U dualities is discussed.

5. *p-Adic space-time sheets as correlates for Boolean cognition*

p-Adic physics is interpreted as physical correlate for cognition. The so called Stone spaces are in one-one correspondence with Boolean algebras and have typically 2-adic topologies. A generalization to p-adic case with the interpretation of p pinary digits as physically representable Boolean statements of a Boolean algebra with $2^n > p > p^{n-1}$ statements is encouraged by p-adic length scale hypothesis. Stone spaces are synonymous with profinite spaces about which both finite and infinite Galois groups represent basic examples. This provides a strong support for the connection between Boolean cognition and p-adic space-time physics. The Stone space character of Galois groups suggests also a deep connection between number theory and cognition and some arguments providing support for this vision are discussed.

4.3.4 Langlands Program and TGD

Number theoretic Langlands program can be seen as an attempt to unify number theory on one hand and theory of representations of reductive Lie groups on one hand. So called automorphic functions to which various zeta functions are closely related define the common denominator. Geometric Langlands program tries to achieve a similar conceptual unification in the case of function fields. This program has caught the interest of physicists during last years.

TGD can be seen as an attempt to reduce physics to infinite-dimensional Kähler geometry and spinor structure of the “world of classical worlds” (WCW). If TGD can be regarded also as a generalized number theory, it is difficult to escape the idea that the interaction of Langlands

program with TGD could be fruitful. I of course hasten to confess that I am not number theorists nor group theorists and that the following considerations are just speculations inspired by TGD.

More concretely, TGD leads to a generalization of number concept based on the fusion of reals and various p-adic number fields and their extensions implying also a generalization of manifold concept, which inspires the notion of number theoretic braid crucial for the formulation of quantum TGD. TGD leads also naturally to the notion of infinite primes and rationals. The identification of Clifford algebra of WCW in terms of hyper-finite factors of type II₁ in turn inspires further generalization of the notion of imbedding space and the idea that quantum TGD as a whole emerges from number theory. The ensuing generalization of the notion of imbedding space predicts a hierarchy of macroscopic quantum phases characterized by finite subgroups of $SU(2)$ and by quantized Planck constant. All these new elements serve as potential sources of fresh insights.

1. *The Galois group for the algebraic closure of rationals as infinite symmetric group?*

The naive identification of the Galois groups for the algebraic closure of rationals would be as infinite symmetric group S_∞ consisting of finite permutations of the roots of a polynomial of infinite degree having infinite number of roots. What puts bells ringing is that the corresponding group algebra is nothing but the hyper-finite factor of type II₁ (HFF). One of the many avatars of this algebra is infinite-dimensional Clifford algebra playing key role in Quantum TGD. The projective representations of this algebra can be interpreted as representations of braid algebra B_∞ meaning a connection with the notion of number theoretical braid.

2. *Representations of finite subgroups of S_∞ as outer automorphisms of HFFs*

Finite-dimensional representations of $Gal(\overline{Q}/Q)$ are crucial for Langlands program. Apart from one-dimensional representations complex finite-dimensional representations are not possible if S_∞ identification is accepted (there might exist finite-dimensional l-adic representations). This suggests that the finite-dimensional representations correspond to those for finite Galois groups and result through some kind of spontaneous breaking of S_∞ symmetry.

1. Sub-factors determined by finite groups G can be interpreted as representations of Galois groups or, rather infinite diagonal imbeddings of Galois groups to an infinite Cartesian power of S_n acting as outer automorphisms in HFF. These transformations are counterparts of global gauge transformations and determine the measured quantum numbers of gauge multiplets and thus measurement resolution. All the finite approximations of the representations are inner automorphisms but the limit does not belong to S_∞ and is therefore outer. An analogous picture applies in the case of infinite-dimensional Clifford algebra.
2. The physical interpretation is as a spontaneous breaking of S_∞ to a finite Galois group. One decomposes infinite braid to a series of n-braids such that finite Galois group acts in each n-braid in identical manner. Finite value of n corresponds to IR cutoff in physics in the sense that longer wave length quantum fluctuations are cut off. Finite measurement resolution is crucial. Now it applies to braid and corresponds in the language of new quantum measurement theory to a sub-factor $\mathcal{N} \subset \mathcal{M}$ determined by the finite Galois group G implying non-commutative physics with complex rays replaced by \mathcal{N} rays. Braids give a connection to topological quantum field theories, conformal field theories (TGD is almost topological quantum field theory at parton level), knots, etc..
3. TGD based space-time correlate for the action of finite Galois groups on braids and for the cutoff is in terms of the number theoretic braids obtained as the intersection of real partonic 2-surface and its p-adic counterpart. The value of the p-adic prime p associated with the parton is fixed by the scaling of the eigenvalue spectrum of the modified Dirac operator (note that renormalization group evolution of coupling constants is characterized at the level free theory since p-adic prime characterizes the p-adic length scale). The roots of the polynomial would determine the positions of braid strands so that Galois group emerges naturally. As a matter fact, partonic 2-surface decomposes into regions, one for each braid transforming independently under its own Galois group. Entire quantum state is modular invariant, which brings in additional constraints.
4. Braiding brings in homotopy group aspect crucial for geometric Langlands program. Another global aspect is related to the modular degrees of freedom of the partonic 2-surface, or more

precisely to the regions of partonic 2-surface associated with braids. $Sp(2g, R)$ (g is handle number) can act as transformations in modular degrees of freedom whereas its Langlands dual would act in spinorial degrees of freedom. The outcome would be a coupling between purely local and global aspects which is necessary since otherwise all information about partonic 2-surfaces as basic objects would be lost. Interesting ramifications of the basic picture about why only three lowest genera correspond to the observed fermion families emerge.

3. Correspondence between finite groups and Lie groups

The correspondence between finite and Lie group is a basic aspect of Langlands.

1. Any amenable group gives rise to a unique sub-factor (in particular, compact Lie groups are amenable). These groups act as genuine outer automorphisms of the group algebra of S_∞ rather than being induced from S_∞ outer automorphism. If one gives up uniqueness, it seems that practically any group G can define a sub-factor: G would define measurement resolution by fixing the quantum numbers which are measured. Finite Galois group G and Lie group containing it and related to it by Langlands correspondence would act in the same representation space: the group algebra of S_∞ , or equivalently configuration space spinors. The concrete realization for the correspondence might transform a large number of speculations to theorems.
2. There is a natural connection with McKay correspondence which also relates finite and Lie groups. The simplest variant of McKay correspondence relates discrete groups $G \subset SU(2)$ to ADE type groups. Similar correspondence is found for Jones inclusions with index $\mathcal{M} : \mathcal{N} \leq 4$. The challenge is to understand this correspondence.
 - (a) The basic observation is that ADE type compact Lie algebras with n -dimensional Cartan algebra can be seen as deformations for a direct sum of n $SU(2)$ Lie algebras since $SU(2)$ Lie algebras appear as a minimal set of generators for general ADE type Lie algebra. The algebra results by a modification of Cartan matrix. It is also natural to extend the representations of finite groups $G \subset SU(2)$ to those of $SU(2)$.
 - (b) The idea would be that is that n -fold Connes tensor power transforms the direct sum of n $SU(2)$ Lie algebras by a kind of deformation to a ADE type Lie algebra with n -dimensional Cartan Lie algebra. The deformation would be induced by non-commutativity. Same would occur also for the Kac-Moody variants of these algebras for which the set of generators contains only scaling operator L_0 as an additional generator. Quantum deformation would result from the replacement of complex rays with \mathcal{N} rays, where \mathcal{N} is the sub-factor.
 - (c) The concrete interpretation for the Connes tensor power would be in terms of the fiber bundle structure $H = M_\pm^4 \times CP_2 \rightarrow H/G_a \times G_b$, $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$, which provides the proper formulation for the hierarchy of macroscopic quantum phases with a quantized value of Planck constant. Each sheet of the singular covering would represent single factor in Connes tensor power and single direct $SU(2)$ summand. This picture has an analogy with brane constructions of M-theory.

4. Could there exist a universal rational function giving rise to the algebraic closure of rationals?

One could wonder whether there exists a universal generalized rational function having all units of the algebraic closure of rationals as roots so that S_∞ would permute these roots. Most naturally it would be a ratio of infinite-degree polynomials.

With motivations coming from physics I have proposed that zeros of zeta and also the factors of zeta in product expansion of zeta are algebraic numbers. Complete story might be that non-trivial zeros of Zeta define the closure of rationals. A good candidate for this function is given by $(\xi(s)/\xi(1-s)) \times (s-1)/s$, where $\xi(s) = \xi(1-s)$ is the symmetrized variant of ζ function having same zeros. It has zeros of zeta as its zeros and poles and product expansion in terms of ratios $(s-s_n)/(1-s+s_n)$ converges everywhere. Of course, this might be too simplistic and might give only the algebraic extension involving the roots of unity given by $exp(i\pi/n)$. Also products

of these functions with shifts in real argument might be considered and one could consider some limiting procedure containing very many factors in the product of shifted ζ functions yielding the universal rational function giving the closure.

5. *What does one mean with S_∞ ?*

There is also the question about the meaning of S_∞ . The hierarchy of infinite primes suggests that there is entire infinity of infinities in number theoretical sense. Any group can be formally regarded as a permutation group. A possible interpretation would be in terms of algebraic closure of rationals and algebraic closures for an infinite hierarchy of polynomials to which infinite primes can be mapped. The question concerns the interpretation of these higher Galois groups and HFFs. Could one regard these as local variants of S_∞ and does this hierarchy give all algebraic groups, in particular algebraic subgroups of Lie groups, as Galois groups so that almost all of group theory would reduce to number theory even at this level?

Be it as it may, the expressive power of HFF:s seem to be absolutely marvellous. Together with the notion of infinite rational and generalization of number concept they might unify both mathematics and physics!

4.3.5 Langlands Program and TGD: Years Later

Langlands correspondence is for mathematics what unified theories are for physics. The number theoretic vision about TGD has intriguing resemblances with number theoretic Langlands program. There is also geometric variant of Langlands program. I am of course amateur and do not have grasp about the mathematical technicalities and can only try to understand the general ideas and related them to those behind TGD. Physics as geometry of WCW ("world of classical worlds") and physics as generalized number theory are the two visions about quantum TGD: this division brings in mind geometric and number theoretic Langlands programs. This motivates re-consideration of Langlands program from TGD point of view. I have written years ago a chapter about this earlier but TGD has evolved considerably since then so that it is time for a second attempt to understand what Langlands is about.

By Langlands correspondence the representations of $G \times Gal$ and G should correspond to each other. The analogy with the representations of Lorentz group suggests that the representations of G should have "spin" for some compact subgroup acting from left or right such that the dimension of this representation is same as the representation of non-commutative Galois group.

Automorphic functions are indeed typically functions in G , which reduce to a function invariant under left and/or right action of a compact or even discrete subgroups H_1 and H_2 or more generally, belong to a finite-dimensional unitary representation of $H_1 \times H_2$ in $H_1 \backslash G / H_2$. Therefore they can be said to have $H_1 \times H_2$ quantum numbers analogous to spin if interpreted as "field modes" in the space of double cosets $H_1 g H_2$. This would conform with the vision about physics as generalized number theory. If I have understood correctly, the question is whether a finite-dimensional representation of H_1 or H_2 could correspond to a finite-dimensional representation of Galois group at the number theory side.

4.3.6 Quantum Arithmetics and the Relationship between Real and p-Adic Physics

This chapter considers possible answers to the basic questions of the p-adicization program, which are following.

Some of the basic questions of the p-adicization program are following.

1. Is there some kind of duality between real and p-adic physics? What is its precise mathematic formulation? In particular, what is the concrete map of p-adic physics in long scales (in real sense) to real physics in short scales? Can one find a rigorous mathematical formulation of the canonical identification induced by the map $p \rightarrow 1/p$ in binary expansion of p-adic number such that it is both continuous and respects symmetries or one must accept the finite measurement resolution.

Few years after writing this the answer to this question is in terms of the notion of p-adic manifold. Canonical identification serving as its building brick however allows many variants and it seems that quantum arithmetics provides one further variant

2. What is the origin of the p-adic length scale hypothesis suggesting that primes near power of two are physically preferred? Why Mersenne primes seem to be especially important (p-adic mass calculations suggest this)?

This chapter studies some ideas but does not provide a clearcut answer to these questions. The notion of quantum arithmetics obtained is central in this approach.

The starting point of quantum arithmetics is the map $n \rightarrow n_q$ taking integers to quantum integers: $n_q = (q^n - q^{-n})/(q - q^{-1})$. Here $q = \exp(i\pi/n)$ is quantum phase defined as a root of unity. From TGD point of view prime roots $q = \exp(i\pi/p)$ are of special interest. Also prime power roots $q = \exp(i\pi/p^n)$ of unity are of interest. Quantum phase can be also generalized to complex number with modulus different from unity.

One can consider several variants of quantum arithmetics. One can regard finite integers as either real or p-adic. In the intersection of “real and p-adic worlds” finite integers can be regarded both p-adic and real.

1. If one regards the integer n real one can keep some information about the prime decomposition of n by dividing n to its prime factors and performing the mapping $p \rightarrow p_q$. The map takes prime first to finite field $G(p, 1)$ and then maps it to quantum integer. Powers of p are mapped to zero unless one modifies the quantum map so that p is mapped to p or $1/p$ depending on whether one interprets the outcome as analog of p-adic number or real number. This map can be seen as a modification of p-adic norm to a map, which keeps some information about the prime factorization of the integer. Information about both real and p-adic structure of integer is kept.
2. For p-adic integers the decomposition into prime factors does not make sense. In this case it is natural to use pinary expansion of integer in powers of p and perform the quantum map for the coefficients without decomposition to products of primes $p_1 < p$. This map can be seen as a modification of canonical identification.
3. If one wants to interpret finite integers as both real and p-adic then one can imagine the definition of quantum integer obtained by de-compositing n to a product of primes, using pinary expansion and mapping coefficients to quantum integers looks natural. This map would keep information about both prime factorization and also about pinary series of factors. One can also decompose the coefficients to prime factors but it is not clear whether this really makes sense since in finite field $G(p, 1)$ there are no primes.

One can distinguish between two basic options concerning the definition of quantum integers.

1. For option I the prime number decomposition of integer is mapped to its quantum counterpart by mapping the primes l to quantum primes $l_q = (q^l - q^{-l})/(q - q^{-1})$, $q = \exp(i\pi/p)$ so that image of product is product of images. Sums are *not* mapped to sums as is easy to verify. p is mapped to zero for the standard definition of quantum integer. Now p is however mapped to itself or $1/p$ depending on whether one wants to interpret quantum integer as p-adic or real number. Quantum integers generate an algebra with respect to sum and product.
2. Option II one uses pinary expansion and maps the prime factors of coefficients to quantum primes. There seems to be no point in decomposing the pinary coefficients to their prime factors so that they are mapped to standard quantum integers smaller than p .

The quantum primes l_q act as generators of Kac-Moody type algebra defined by powers p^n such that sum is completely analogous to that for Kac-Moody algebra: $a + b = \sum_n a_n p^n + \sum_n b_n p^n = \sum_n (a_n + b_n) p^n$. For p-adic numbers this is not the case.

3. For both options it is natural to consider the variant for which one has expansion $n = \sum_k n_k p^{kr}$, $n_k < p^r$, $r = 1, 2, \dots$. p^k would serve as cutoff.

The notion of quantum matrix group differing from ordinary quantum groups in that matrix elements are commuting numbers makes sense. This group forms a discrete counterpart of ordinary quantum group and its existence suggested by quantum classical correspondence. The existence of this group for matrices with unit determinant is guaranteed by mere ring property since the inverse matrix involves only arithmetic product and sum.

1. The quantum counterparts of special linear groups $SL(n, F)$ exists always. For the covering group $SL(2, C)$ of $SO(3, 1)$ this is the case so that 4-dimensional Minkowski space is in a very special position. For orthogonal, unitary, and orthogonal groups the quantum counterpart exists only if the number of powers of p for the generating elements of the quantum matrix group satisfies an upper bound characterizing the matrix group.
2. For the quantum counterparts of $SO(3)$ ($SU(2)/SU(3)$) the orthogonality conditions state that at least some multiples of the prime characterizing quantum arithmetics is sum of three (four/six) squares. For $SO(3)$ this condition is strongest and satisfied for all integers, which are not of form $n = 2^{2r}(8k + 7)$. The number $r_3(n)$ of representations as sum of squares is known and $r_3(n)$ is invariant under the scalings $n \rightarrow 2^{2r}n$. This means scaling by 2 for the integers appearing in the square sum representation.

The findings about quantum $SO(3)$ suggest a possible explanation for p-adic length scale hypothesis and preferred p-adic primes.

1. The idea to be studied is that the quantum matrix group which is discrete is in some sense very large for preferred p-adic primes. If cognitive representations correspond to the representations of quantum matrix group, the representational capacity of cognitive representations is high and this kind of primes are survivors in the algebraic evolution leading to algebraic extensions with increasing dimension. The simple estimates of this chapter restricting the consideration to finite fields ($O(p) = 0$ approximation) do not support this idea in the case of Mersenne primes.
2. An alternative idea is that number theoretic evolution leading to algebraic extensions of rationals with increasing dimension favors p-adic primes which do not split in the extensions to primes of the extension. There is also a nice argument that infinite primes which are in one-one correspondence with prime polynomials code for algebraic extensions. These primes code also for bound states of elementary particles. Therefore the stable bound states would define preferred p-adic primes as primes which do not split in the algebraic extension defined by infinite prime. This should select Mersenne primes as preferred ones.

4.3.7 Quantum Adeles

Quantum arithmetics provides a possible resolution of a long-lasting challenge of finding a mathematical justification for the canonical identification mapping p-adics to reals playing a key role in TGD - in particular in p-adic mass calculations. p-Adic numbers have p-adic binary expansions $\sum a_n p^n$ satisfying $a_n < p$. of powers p^n to be products of primes $p_1 < p$ satisfying $a_n < p$ for ordinary p-adic numbers. One could map this expansion to its quantum counterpart by replacing a_n with their counterpart and by canonical identification map $p \rightarrow 1/p$ the expansion to real number. This definition might be criticized as being essentially equivalent with ordinary p-adic numbers since one can argue that the map of coefficients a_n to their quantum counterparts takes place only in the canonical identification map to reals.

One could however modify this recipe. Represent integer n as a product of primes l and allow for l all expansions for which the coefficients a_n consist of primes $p_1 < p$ but give up the condition $a_n < p$. This would give 1-to-many correspondence between ordinary p-adic numbers and their quantum counterparts.

It took time to realize that $l < p$ condition might be necessary in which case the quantization in this sense - if present at all - could be associated with the canonical identification map to reals. It would correspond only to the process taking into account finite measurement resolution rather than replacement of p-adic number field with something new, hopefully a field. At this step one might perhaps allow $l > p$ so that one would obtain several real images under canonical identification.

One can however imagine a third generalization of number concept. One can replace integer n with n -dimensional Hilbert space and sum $+$ and product \times with direct sum \oplus and tensor product \otimes and introduce their co-operations, the definition of which is highly non-trivial. This procedure yields also Hilbert space variants of rationals, algebraic numbers, p-adic number fields, and even complex, quaternionic and octonionic algebraics. Also adeles can be replaced with their Hilbert space counterparts. Even more, one can replace the points of Hilbert spaces with Hilbert

spaces and repeat this process, which is very similar to the construction of infinite primes having interpretation in terms of repeated second quantization. This process could be the counterpart for construction of n^{th} order logics and one might speak of Hilbert or quantum mathematics. The construction would also generalize the notion of algebraic holography and provide self-referential cognitive representation of mathematics.

This vision emerged from the connections with generalized Feynman diagrams, braids, and with the hierarchy of Planck constants realized in terms of coverings of the imbedding space. Hilbert space generalization of number concept seems to be extremely well suited for the purposes of TGD. For instance, generalized Feynman diagrams could be identifiable as arithmetic Feynman diagrams describing sequences of arithmetic operations and their co-operations. One could interpret \times_q and $+_q$ and their co-algebra operations as 3-vertices for number theoretical Feynman diagrams describing algebraic identities $X = Y$ having natural interpretation in zero energy ontology. The two vertices have direct counterparts as two kinds of basic topological vertices in quantum TGD (stringy vertices and vertices of Feynman diagrams). The definition of co-operations would characterize quantum dynamics. Physical states would correspond to the Hilbert space states assignable to numbers. One prediction is that all loops can be eliminated from generalized Feynman diagrams and diagrams are in projective sense invariant under permutations of incoming (outgoing legs).

4.3.8 About Absolute Galois Group

Absolute Galois Group defined as Galois group of algebraic numbers regarded as extension of rationals is very difficult concept to define. The goal of classical Langlands program is to understand the Galois group of algebraic numbers as algebraic extension of rationals - Absolute Galois Group (AGG) - through its representations. Invertible adèles -ideles - define Gl_1 which can be shown to be isomorphic with the Galois group of maximal Abelian extension of rationals (MAGG) and the Langlands conjecture is that the representations for algebraic groups with matrix elements replaced with adèles provide information about AGG and algebraic geometry.

I have asked already earlier whether AGG could act as symmetries of quantum TGD. The basis idea was that AGG could be identified as a permutation group for a braid having infinite number of strands. The notion of quantum adèle leads to the interpretation of the analog of Galois group for quantum adèles in terms of permutation groups assignable to finite 1 braids. One can also assign to infinite primes braid structures and Galois groups have lift to braid groups.

Objects known as dessins d'enfant provide a geometric representation for AGG in terms of action on algebraic Riemann surfaces allowing interpretation also as algebraic surfaces in finite fields. This representation would make sense for algebraic partonic 2-surfaces, and could be important in the intersection of real and p-adic worlds assigned with living matter in TGD inspired quantum biology, and would allow to regard the quantum states of living matter as representations of AGG. Adèles would make these representations very concrete by bringing in cognition represented in terms of p-adics and there is also a generalization to Hilbert adèles.

REFERENCES

Mathematics

- [A1] Yangian symmetry. Available at: <http://en.wikipedia.org/wiki/Yangian>.
- [A2] Pope CN. Eigenfunctions and $Spin^c$ Structures on CP_2 , 1980.
- [A3] Hanson J Eguchi T, Gilkey B. *Phys Rep*, 66, 1980.
- [A4] Eisenhart. *Riemannian Geometry*. Princeton University Press, 1964.
- [A5] Pope CN Gibbons GW. CP_2 as gravitational instanton. *Comm Math Phys*, 55, 1977.
- [A6] Pope CN Hawking SW. Generalized Spin Structures in Quantum Gravity. *Phys Lett*, (1), 1978.

- [A7] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc*, 8(43):133–151, 1981.. Available at: <http://tinyurl.com/pb8zpqo>.
- [A8] Milnor J. *Topology form Differential Point of View*. The University Press of Virginia, Virginia, 1965.
- [A9] Spivak M. *Differential Geometry I,II,III,IV*. Publish or Perish, Boston, 1970.
- [A10] Thom R. *Comm Math Helvet*, 28, 1954.
- [A11] Wallace. *Differential Topology*. W. A. Benjamin, New York, 1968.

Cosmology and Astro-Physics

- [E1] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity, 2003. Available at: <http://arxiv.org/abs/astro-ph/0310036>.

Books related to TGD

- [K1] Pitkänen M. *Topological Geometroynamics*. 1983.
- [K2] Pitkänen M. *Bio-Systems as Conscious Holograms*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/holography.html>, 2006.
- [K3] Pitkänen M. *Bio-Systems as Self-Organizing Quantum Systems*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/bioselforg.html>, 2006.
- [K4] Pitkänen M. *Genes and Memes*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/genememe.html>, 2006.
- [K5] Pitkänen M. *Magnetospheric Consciousness*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/magnconsc.html>, 2006.
- [K6] Pitkänen M. *Mathematical Aspects of Consciousness Theory*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/mathconsc.html>, 2006.
- [K7] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness*. Available at: <http://tgdtheory.fi/pdfpool/nmpc.pdf>, 2006.
- [K8] Pitkänen M. *Physics in Many-Sheeted Space-Time*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdclass.html>, 2006.
- [K9] Pitkänen M. *Quantum Hardware of Living Matter*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/bioware.html>, 2006.
- [K10] Pitkänen M. Quantum Model for Bio-Superconductivity: I. In *TGD and EEG*. Available at: <http://tgdtheory.fi/pdfpool/biosupercondI.pdf>, 2006.
- [K11] Pitkänen M. Quantum Model for Bio-Superconductivity: II. In *TGD and EEG*. Available at: <http://tgdtheory.fi/pdfpool/biosupercondII.pdf>, 2006.
- [K12] Pitkänen M. *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdgeom.html>, 2006.
- [K13] Pitkänen M. *TGD and EEG*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdeeg.html>, 2006.
- [K14] Pitkänen M. *TGD and Fringe Physics*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/freenergy.html>, 2006.
- [K15] Pitkänen M. *TGD as a Generalized Number Theory*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdnumber.html>, 2006.

- [K16] Pitkänen M. *TGD Inspired Theory of Consciousness*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdconsc.html>, 2006.
- [K17] Pitkänen M. *Topological Geometro-dynamics: an Overview*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdview.html>, 2006.
- [K18] Pitkänen M. Number theoretic vision, Hyper-finite Factors and S-matrix. In *Towards M-Matrix: part I*. Available at: <http://tgdtheory.fi/pdfpool/UandM.pdf>, 2012.
- [K19] Pitkänen M. Quantum Mind and Neuroscience. In *TGD based view about living matter and remote mental interactions*. Available at: <http://tgdtheory.fi/pdfpool/lianPN.pdf>, 2012.
- [K20] Pitkänen M. *TGD Based View About Living Matter and Remote Mental Interactions*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdlian.html>, 2012.
- [K21] Pitkänen M. *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/neuplanck.html>, 2013.
- [K22] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/padphys.html>, 2013.
- [K23] Pitkänen M. *Quantum TGD*. Online book. Available at: <http://www.tgdtheory.fi/tgdhtml/tgdquantum.html>, 2013.
- [K24] Pitkänen M. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy: Part I*. Available at: <http://tgdtheory.fi/pdfpool/Planck.pdf>, 2019.
- [K25] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time: Part II*. Available at: <http://tgdtheory.fi/pdfpool/astro.pdf>, 2019.
- [K26] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory: Part I*. Available at: <http://tgdtheory.fi/pdfpool/visionb.pdf>, 2019.
- [K27] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix: Part II*. Available at: <http://tgdtheory.fi/pdfpool/twistorstory.pdf>, 2019.
- [K28] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory: Part I*. Available at: <http://tgdtheory.fi/pdfpool/numbervision.pdf>, 2019.
- [K29] Pitkänen M. What p-Adic Icosahedron Could Mean? And What about p-Adic Manifold? In *TGD as a Generalized Number Theory: Part I*. Available at: <http://tgdtheory.fi/pdfpool/picosahedron.pdf>, 2019.

Articles about TGD

- [L1] Pitkänen M. Philosophy of Adelic Physics. Available at: http://tgdtheory.fi/public_html/articles/adelephysics.pdf, 2017.
- [L2] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer. Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.
- [L3] Pitkänen M. TGD view about coupling constant evolution. Available at: http://tgdtheory.fi/public_html/articles/ccevolution.pdf, 2018.