Topological Geometrodynamics: Basic Visions

M. Pitkänen,

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Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

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Abstract

In this article I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics.
2. Second vision is number theoretical and involves three threads. The first thread relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept. Second thread is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically. The third threadrelies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units.
3. The third vision is based on TGD inspired theory of consciousness, which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics.

The basic aspects of quantum classical correspondence are discussed. Strong form of General Coordinate Invariance implies strong form of holography and effective 2-dimensionality. Weak form of electric magnetic duality and simple general condition on preferred extremals of Kähler action imply that TGD indeed reduces to almost topological QFT defined by Chern-Simons terms located at space-like at ends of CDs and light-like 3-surfaces defined by the orbits of partonic 2-surfaces defining wormhole throats at which the signature of induced metric changes. A further reduction of action to sum of areas of minimal surfaces is conjectured on basis of effective 2-dimensionality. Feynman diagrams have direct interpretation in terms of space-time topology and ZEO leads to a dramatic simplification of the Feynman diagrammatics and suggest a close connection with twistorial diagrams. Induced gauge field concept makes impossible the superposition of classical fields in TGD Universe. This is a grave objection circumvented by simple observation: only the superposition of their effects is observed and many-sheeted space-time implies it.

1 Introduction

Originally Topological Geometrodynamics (TGD) was proposed as a solution of the problems related to the definition of conserved four-momentum in General Relativity. It was assumed that physical space-times are representable as 4-D surfaces in certain higher-dimensional space-time having symmetries of the empty Minkowski space of Special Relativity. This is guaranteed by the decomposition $H = M^4 \times S$, where S is some compact internal space. It turned out that the choice $S = CP_2$ is unique in the sense that it predicts the symmetries of the standard model and provides a realization for Einstein's dream of geometrizing of fundamental interactions at classical level. TGD can be also regarded as a generalization of super string models obtained by replacing strings with light-like 3-surfaces or equivalently with space-like 3-surfaces: the equivalence of these identification implies quantum holography.

The construction of quantum TGD turned out to be much more than mere technical problem of deriving S-matrix from path integral formalism. A new ontology of physics (many-sheeted space-time, zero energy ontology, generalization of the notion of number, and generalization of quantum theory based on spectrum of Planck constants giving hopes to understand what dark matter and dark energy are) and also a generalization of quantum measurement theory leading to a theory of consciousness and model for quantum biology providing new insights to the mysterious ability of living matter to circumvent the constraints posed by the second law of thermodynamics were needed. The construction of quantum TGD involves a handful of different approaches consistent with a similar overall view, and one can say that the construction of M-matrix, which generalizes the S-matrix of quantum field theories, is understood to a satisfactory degree although it is not possible to write even in principle explicit Feynman rules except at quantum field theory limit [K6].

In this article I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics [K11].

2. Second vision is number theoretical [K15] and involves three threads.

   (a) The first thread [K17] relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept.

   (b) Second thread [K18] is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically.

   (c) The third thread [K16] relies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units. This implies number theoretical Brahman=Atman identity or number theoretical holography when one consider hyper-octonionic infinite primes.

   (d) The third vision is based on TGD inspired theory of consciousness [K19], which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics. The basic notions at quantum jump identified as as a moment of consciousness and self. Negentropy Maximization Principle (NMP) defines the fundamental variational principle and reproduces standard quantum measurement theory and predicts second law but also some totally new physics in the intersection of real and p-adic worlds where it is possible to define a hierarchy of number theoretical variants of Shannon entropy which can be also negative. In this case NMP favors the generation of entanglement and state function reduction does not mean generation of randomness anymore. This vision has obvious almost applications to biological self-organization.

My aim is to provide a bird's eye of view and my hope is that reader would take the attitude that details which cannot be explained in this kind of representation are not essential for the purpose of getting a feeling about the great dream behind TGD. I have also commented various ideas from the point of view of Quantum Mind program.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [?].

2 Quantum Physics As Infinite-Dimensional Geometry

The first vision in its original form is a the generalization of Einstein's program for the geometrization of physics by replacing space-time with the WCW identified roughly as the space of 4-surfaces in $H = M^4 \times CP_2$. Later generalization due to replacement of H with book like structures from by real and p-adic variants of H emerged. A further book like structure of imbedding space emerged via the introduction of the hierarchy of Planck constants. These generalizations do not however add anything new to the basic geometric vision.

2.1 Geometrization Of Fermionic Statistics In Terms Of WCW Spinor Structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields defined at space-time surface.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. Ramond model [B3] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   WCW spinor fields can have arbitrary fermion number and there are good hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field. One non-trivial implication is bosonic emergence: elementary bosons correspond to fermion anti-fermion bound states associated with the wormhole contacts (pieces of CP_2 type vacuum extremals) with throats carrying fermion and anti-fermion numbers. Fermions correspond to single throats associated with topologically condensed CP_2 type vacuum extremals.

2. The classical theory for the bosonic fields is an essential part of WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \quad ,$$

   where J_{AB} denotes the matrix elements of the Kähler form of WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2.2 Construction Of WCW Clifford Algebra In Terms Of Second Quantized Induced Spinor Fields

The construction of WCW spinor structure must have a direct relationship to quantum physics as it is usually understood. The second quantization of the space-time spinor fields is needed to define the anti-commutative gamma matrices of WCW: this means a geometrization of Fermi statistics [K24] in the sense that free fermionic quantum fields at space-time surface correspond to purely classical Clifford algebra of WCW. This is in accordance with the idea that physics at WCW level is purely classical apart from the notion of quantum jump.

The identification of the correct variational principle for the dynamics of space-time spinor fields identified as induced spinor fields has involved many trials and errors. Ironically, the final outcome was almost the most obvious guess: the so called Kähler-Dirac action. What was difficult to discover was that the well-definedness of em charge requires that the modes of K-D equation are localized at 2-D string world sheets. The same condition results also from the condition that octonionic and ordinary spinor structures are equivalent for the modes of the induced spinor field and also from the condition that quantum deformations of fermionic oscillator operator algebra requiring 2-dimensionality can be realized as realization of finite measurement resolution. Fermionic string model therefore emerges from TGD.

The notion of measurement resolution realized in terms of the inclusions of hyper-finite factors of type II_1 and having discretization using rationals or algebraic extensions of rationals have been one of the key challenges of quantum TGD. Quantum classical correspondence suggests with measurement interaction term defined as Lagrange multiplier terms stating that classical charges belonging to Cartan algebra are equal to their quantal counterparts after state function reduction for space-time surfaces appearing in quantum superposition [K24]. This makes sense if classical charges parametrize zero modes. State function reduction would mean state function collapse in zero modes.

Kähler function equals to the real part of Kähler action coming from Euclidian space-time regions for a preferred extremal whereas Minkowski regions give an exponent of phase factor responsible for quantum interferences effects. The conjecture is that preferred extremals by internal consistency conditions are critical in the sense that they allows infinite number of vanishing second variations having interpretation as conformal deformations respecting light-likeness of the partonic orbits. Criticality is realize classically as vanishing of the super-symplectic charges for sub-algebra of the entire super-symplectic algebra. This realizes the notion of quantum criticality-one of guiding principles of quantum TGD-at space-time level.

Recently this idea has become very concrete.

1. There is an infinite hierarchy of quantum criticalities identified as a hierarchy of breakings of conformal symmetry in the sense that the gauge symmetry for the super-symplectic algebra having natural conformal structure is broken to a dynamical symmetry: gauge degrees of freedom are transformed to physical ones.

2. The sub-algebras of the supersymplectic algebra isomorphic with the algebra itself are parametrized by integer n: the conformal weights for the sub-algebra are n-multiples for those of the entire algebra. This predicts an infinite number of infinite hierarchies characterized by sequences of integers $n_{i+1} = \prod_{k\leq i} m_k$. The integer n_i characterizes the effective value of Planck constant $h_{eff} = n_i$ for a given level of hierarchy and the interpretation is in terms of dark matter. The increase of n_i takes place spontaneously since it means reduction of criticality. Both the value of n_i and the numbers of string world sheets associated with 3-surfaces at the ends of CD and connecting partonic 2-surfaces characterize measurement resolution.

3. The symplectic hierarchies correspond to hierarchies of inclusions for HFFs [K23] and finite measurement resolution is a property of both zero energy state and space-time surface. The original idea about addition of measurement interaction terms to the Kähler action does not seem to be needed.

Number theoretical approach in turn leads to the conclusion that space-time surfaces are either associative or co-associative in the sense that the induced gamma matrices at each point of space-time surface in their octonionic representation define a quaternionic or co-quaternionic algebra and therefore have matrix representation. The conjecture is that these identifications of space-time dynamics are consistent or even equivalent. The string sheets at which spinor modes are localized can be regarded as commutative surfaces.

The recent understanding of the Kähler-Dirac action has emerged through a painful process and has strong physical implications.

1. Kähler-Dirac equation at string world sheets can be solved exactly just as in string models. At the light-like boundaries the limit of K-D equation holds true and gives rise to the analog of massless Dirac equation but for K-D gamma matrices. One could have a 1-D boundary term defined by the induced Dirac equation at the light-like boundaries of string world sheet. If it is there, the modes are solutions with light-like 8-momentum which has light-like projection to space-time surface. This would give rise to a fermionic propagator in the construction of scattering amplitudes mimicking Feynman diagrammatics: note that the M^4 projection of the momentum need not be light-like.

2. The space-time super-symmetry generalizes to what might be called $\mathcal{N} = \infty$ supersymmetry whose least broken sub-symmetry reduces to $\mathcal{N} = 2$ broken super-symmetry generated by right-handed neutrino and ant-ineutrino [K6] . The generators of the super-symmetry correspond to the oscillator operators of the induced spinor field at space-time sheet and to the super-symplectic charges. Bosonic emergence means dramatic simplifications in the formulation of quantum TGD.

3. It is also possible to generalize the twistor program to TGD framework if one accepts the use of octonionic representation of the gamma matrices of imbedding space and hyper-quaternionicity of space-time surfaces [K33]: what one obtains is 8-D generalization of the twistor Grassmann approach allowing non-light-like M^4 momenta. Essential condition is that octonionic and ordinary spinor structures are equivalent at string world sheets.

2.3 ZEO And WCW Geometry

In the ZEO quantum states have vanishing net values of conserved quantum numbers and decompose to superposition of pairs of positive and negative energy states defining counterparts of initial and final states of a physical event in standard ontology.

2.3.1 *ZEO*

ZEO was forced by the interpretational problems created by the vacuum extremal property of Robertson-Walker cosmologies imbedded as 4-surfaces in $M^4 \times CP_2$ meaning that the density of inertial mass (but not gravitational mass) for these cosmologies was vanishing meaning a conflict with Equivalence Principle. The most feasible resolution of the conflict comes from the realization that GRT space-time is obtained by lumping the sheets of many-sheeted space-time to M^4 endowed with effective metric. Vacuum extremals could however serve as models for GRT space-times such that the effective metric is identified with the induced metric [K20]. This is true if space-time is genuinely single-sheeted. In the models of astrophysical objects and cosmology vacuum extremals have been used [K14].

In zero energy ontology physical states are replaced by pairs of positive and negative energy states assigned to the past *resp.* future boundaries of causal diamonds defined as pairs of future and past directed light-cones ($\delta M^4_\pm \times CP_2$). The net values of all conserved quantum numbers of zero energy states vanish. Zero energy states are interpreted as pairs of initial and final states of a physical event such as particle scattering so that only events appear in the new ontology. It is possible to speak about the energy of the system if one identifies it as the average positive energy for the positive energy part of the system. Same applies to other quantum numbers.

The matrix ("M-matrix") representing time-like entanglement coefficients between positive and negative energy states unifies the notions of S-matrix and density matrix since it can be regarded as a complex square root of density matrix expressible as a product of real squared of density matrix and unitary S-matrix. The system can be also in thermal equilibrium so that thermodynamics becomes a genuine part of quantum theory and thermodynamical ensembles cease to be practical fictions of the theorist. In this case M-matrix represents a superposition of zero energy states for which positive energy state has thermal density matrix.

ZEO combined with the notion of quantum jump resolves several problems. For instance, the troublesome questions about the initial state of universe and about the values of conserved quantum numbers of the Universe can be avoided since everything is in principle creatable from vacuum. Communication with the geometric past using negative energy signals and time-like entanglement are crucial for the TGD inspired quantum model of memory and both make sense in zero energy ontology. ZEO leads to a precise mathematical characterization of the finite resolution of both quantum measurement and sensory and cognitive representations in terms of inclusions of von Neumann algebras known as hyperfinite factors of type II_1. The space-time correlate for the finite resolution is discretization which appears also in the formulation of quantum TGD.

2.3.2 *Causal diamonds*

The imbedding space correlates for ZEO are causal diamonds (CDs) CD serves as the correlate zero energy state at imbedding space-level whereas space-time sheets having their ends at the light-like boundaries of CD are the correlates of the system at the level of 4-D space-time. Zero energy state can be regarded as a quantum superposition of space-time sheets with fermionic and other quantum numbers assignable to the partonic 2-surfaces at the ends of the space-time sheets.

1. The basic construct in the ZEO is the space $CD \times CP_2$, where the causal diamond CD is defined as an intersection of future and past directed light-cones with time-like separation between their tips regarded as points of the underlying universal Minkowski space M^4. In ZEO physical states correspond to pairs of positive and negative energy states located at the boundaries of the future and past directed light-cones of a particular CD.

2. CDs form a fractal hierarchy and one can glue smaller CDs within larger CDs. Also unions of CDs are possible.

3. Without any restrictions CDs would be parametrized by the position of say lower tip of CD and by the relative M^4 coordinates of the upper tip with respect to the lower one so that the moduli space would be $M^4 \times M^4_+$. p-Adic length scale hypothesis follows if the values of temporal distance T between tips of CD come in powers of 2^n: $T = 2^n T_0$. This would reduce the future light-cone M^4_+ reduces to a union of hyperboloids with quantized value of light-cone proper time. A possible interpretation of this distance is as a quantized cosmic time. Also the quantization of the hyperboloids to a lattices of discrete points classified by discrete sub-groups of Lorentz group is an attractive proposal and the quantization of cosmic redshifts provides some support for it.

ZEO forces to replaced the original WCW by a union of WCWs associated with CDs and their unions. This does not however mean any problems of principle since Clifford algebras are simply tensor products of the Clifford algebras of CDs for the unions of CDs.

2.3.3 *Generalization of S-matrix in ZEO*

ZEO forces the generalization of S-matrix with a triplet formed by U-matrix, M-matrix, and S-matrix. The basic vision is that quantum theory is at mathematical level a complex square root of thermodynamics. What happens in quantum jump was already discussed.

1. M-matrices are matrices between positive and negative energy parts of the zero energy state and correspond to the ordinary S-matrix. M-matrix is a product of a hermitian square root - call it H - of density matrix ρ and universal S-matrix S. There is infinite number of different Hermitian square roots H_i of density matrices assumed to define orthogonal matrices with respect to the inner product defined by the trace: $Tr(H_i H_j) = 0$. One can interpret square roots of the density matrices as a Lie algebra acting as symmetries of the S-matrix. The most natural identification is in terms of super-symplectic algebra or as its sub-algebra. Since these operators should not change the vanishing quantum number of zero energy states, a natural identification would be as bilinears of the generators of super-symplectic generators associated with the opposite boundaries of CD and having vanishing net quantum numbers.

2. One can consider a generalization of M-matrices so that they would be analogous to the elements of Kac-Moody algebra. These M-matrices would involve all powers of S.

   (a) The orthogonality with respect to the inner product defined by $\langle A|B\rangle = Tr(AB)$ requires the conditions $Tr(H_1H_2S^n) = 0$ for $n \neq 0$ and H_i are Hermitian matrices appearing as square root of density matrix. H_1H_2 is hermitian if the commutator $[H_1, H_2]$ vanishes. It would be natural to assign n:th power of S to the CD for which the scale is n times the CP_2 scale.

   (b) Trace - possibly quantum trace for hyper-finite factors of type II_1) is the analog of integration and the formula would be a non-commutative analog of the identity $\int_{S^1} exp(in\phi)d\phi = 0$ and pose an additional condition to the algebra of M-matrices.

   (c) It might be that one must restrict M matrices to a Cartan algebra and also this choice would be a process analogous to state function reduction. Since density matrix becomes an observable in TGD Universe, this choice could be seen as a direct counterpart for the choice of a maximal number of commuting observables which would be now hermitian square roots of density matrices. Therefore ZEO gives good hopes of reducing basic quantum measurement theory to infinite-dimensional Lie-algebra.

The collections of M-matrices defined as time reversals of each other define the sought for two natural state basis.

1. As for ordinary S-matrix, one can construct the states in such a manner that either positive or negative energy part of the state has well defined particle numbers, spin, etc... resulting in state function preparation. Therefore one has two kinds of M-matrices: $M_K^{\pm}$ and for both of these the above orthogonality relations hold true. This implies also two kinds of U-matrices call them $U^{\pm}$. The natural assumption is that the two M-matrices differ only by Hermitian conjugation so that one would have $M_K^- = (M_K^+)^\dagger$.

   One can assign opposite arrows of geometric time to these states and the proposal is that the arrow of time is a result of a process analogous to spontaneous magnetization. The possibility that the arrow of geometric time could change in quantum jump has been already discussed.

2. Unitary U-matrix $U^{\pm}$ is induced from a projector to the zero energy state basis $|K^{\pm}\rangle$ acting on the state basis $|K^{\mp}\rangle$ and the matrix elements of U-matrix are obtained by acting with the representation of identity matrix in the space of zero energy states as $I = \sum_K |K^+\rangle\langle K^+|$ on the zero energy state $|K^-\rangle$ (the action on K^+ is trivial!) and gives

$$U_{KL}^{+} = Tr(M_K^+ M_L^+) \ .$$

   Note that finite measurement resolution requires that the trace operation is q-trace rather than ordinary trace.

3. As the detailed discussion of the anatomy of quantum jump demonstrated, the first step in state function reduction is the choice of $M_K^{\pm}$ meaning the choice of the hermitian square root of a density matrix. A quantal selection of the measured observable takes place. This step is followed by a choice of "initial" state analogous to state function preparation and a choice of the "final state" analogous to state function reduction. The net outcome is the transition $|K^{\pm}\rangle \rightarrow |L^{\pm}\rangle$. It could also happen that instead of state function reduction as third step unitary process $U^{\mp}$ (note the change of the sign factor!) takes place and induces the change of the arrow of geometric time.

4. As noticed, one can imagine even higher level choices and this would correspond to the choice of the commuting set of hermitian matrices H defining the allowed square roots of density matrices as a set of mutually commuting observables.

5. The original naive belief that the unitary U-matrix has as its rows orthonormal M-matrices turned out to be wrong. One can deduce the general structure of U-matrix from first principles by identifying it as a time evolution operator in the space of moduli of causal diamonds relating to each other M-matrices. Inner product for M-matrices gives the matrix elements of

U-matrix. S-matrix can be identified as a representation for the exponential of the Virasoro generator L_{-1} for the super-symplectic algebra. The detailed construction of U-matrix in terms of M-matrices and S-matrices depending on CD moduli is discussed in [K27].

2.4 Quantum Criticality, Strong Form Form of Holography, and WCW Geometry

Quantum TGD and WCW geometry in particular can be understood in terms of two principles: Quantum Criticality (QC) and Strong form of Holography (SH).

2.4.1 *Quantum Criticality*

In its original form QC stated that the Kähler couplings strength appearing in the exponent of vacuum functional identifiable uniquely as the exponent of Kähler function defining the Kähler metric of WCW defines the analog of partition function of a thermodynamical system. Later it became clear that Kähler action in Minkowskian space-time regions is imaginary (by $\sqrt{g}$ factor) so that the exponent become that of complex number. The interpretation in ZEO is in terms of quantum TGD as "square root of thermodynamics" vision. Minkowskian Kähler action is the analog of action of quantum field theories.

TGD should be unique. The analogy with thermodynamics implies that Kähler coupling strength α_K is analogous to temperature. The natural guess is that it corresponds to a critical temperature at which a phase transition between two phases occurs. It is of course possible that there are several critical values of α_K.

QC is physically very attractive since it would give maximally complex Universe. At quantum criticality long range fluctuations would be present and make possible macroscopic quantum coherence especially relevant for life.

In 2-D critical systems conformal symmetry provides the mathematical description of criticality and in TGD something similar but based on a huge generalization of the conformal symmetries is expected. Ordinary conformal symmetries are indeed replaced by super-symplectic isometries, by the generalized conformal symmetries acting on light-cone boundary and on light-like orbits of partonic 2-surfaces, and by the ordinary conformal symmetries at partonic 2-surfaces and string world sheets carrying spinors. Even a quaternionic generalization of conformal symmetries must be considered.

2.4.2 *Strong Form of Holography*

Strong form of holography (SH) is the second big principle. It is strongly suggested by the strong form of general coordinate invariance (SGCI) stating that the fundamental objects can be taken to be either the light-like orbits of partonic 2-surfaces or space-like 3-surfaces at the ends of causal diamonds (CDs). This would imply that partonic 2-surfaces at their intersection at the boundaries of CDs carry the data about quantum states.

As a matter fact, one must include also string world sheets at which fermions are localized - this for instance by the condition that em charge is well-defined. String world sheets carry vanishing induced W boson fields (they would mix different charge states) and the Kähler-Dirac gamma matrices are parallel to them. These conditions give powerful integrability conditions and it remains to be seen whether solutions to them indeed exist.

The best manner to proceed is to construct preferred extremals using SH - that is by assuming just string world sheets and partonic 2-surfaces intersecting by discrete point set as given, and finding the preferred extremals of Kähler action containing them and satisfying the boundary conditions at string world sheets and partonic 2-surfaces.

If this construction works, it must involve boundary conditions fixing the space-time surfaces to very high degree. Due to the non-determinism of Kähler action implied by its huge vacuum degeneracies, one however expects a gauge degeneracy. QC indeed suggests non-determinism. By 2-D analogy one expects the analogs of conformal symmetries acting as gauge symmetries. The proposal is that the fractal hierarchy of mutually isomorphic sub-algebras of super-symplectic algebra (and possibly of all conformal algebras involved) having conformal weights, which are n-ples of those for the entire algebra act as gauge symmetries so that the Noether charges for this sub-algebra would vanish. This would be the case at the ends of preferred extremals at both boundaries of CDs. This almost eliminates the classical degrees of freedom outside string world sheets and partonic 2-surfaces, and thus realizes the strong form of holography. In the fermionic sector the fermionic super-symplectic charges in the sub-algebra annihilate the physical states: this is a generalization of Super-Virasoro and Super Kac-Moody conditions.

In the phase transitions increasing the value of n the sub-algebra of gauge symmetries is reduced and gauge degrees of freedom become physical ones. By QC this transition occurs spontaneously. TGD Universe is like ball at the top of hill at the top of ....: ad infinitum and its evolution is endless dropping down. In TGD inspired theory of consciousness, one can understand living systems as systems fighting to stay at given level of criticality.

One could say that the conformal subalgebra is analogous to that defined by functions of $w = z^n$ act as conformal symmetries. One can also see the space-time surfaces at the level n as analogous to Riemann surface for function $f(z) = z^{1/n}$ conformal gauge symmetries as those defined by functions of z. This brings in n sheets not connected by conformal gauge symmetries. Hence the conformal equivalence classes of sheets give rise n-fold physical degeneracy. An effective description for this would be in terms of n-fold singular covering of the imbedding space introduced originally but this is only an auxiliary concept.

A natural interpretation of the hierarchy of conformal criticalities is as a hierarchy of Planck constants $h_{eff} = n \times h$. The identification is suggested by the interpretation of n as the number of sheets in the singular covering of the space-time surface for which the sheets at the ends of space-time surface (the 3-surfaces at boundaries of CD) co-incide. The n sheets increase the action by a factor n and this is equivalent with the replacement $h \rightarrow h_{eff} = n \times h$.

The hierarchy of Planck constants allows to consider several interpretations.

1. If one regards the sheets of the covering as distinct, one has single critical value of g_K^2 and of h. This is the fundamental interpretation and justifies the subscript "$_{eff}$" in $h_{eff} = n \times h$.

2. If the sheets of the covering are are lumped to a single sheet (this is done for all sheets of the many-sheeted space-time in General Relativity approximation), there are two possible interpretations. There is single critical value of g_K^2 and a hierarchy of Planck constants $h_{eff} = n \times h$ giving rise to $\alpha_K(n) = g_K^2/2h_{eff}$. Alternatively, there is single value of Planck constant and a hierarchy of critical values $\alpha_K(n) = (g_K^2/2h)/n$ having an accumulation point at origin (zero temperature).

2.4.3 *Non-commutative imbedding space and strong form of holography*

The precise formulation of strong form of holography (SH) is one of the technical problems in TGD. A comment in FB page of Gareth Lee Meredith led to the observation that besides the purely number theoretical formulation based on commutativity also a symplectic formulation in the spirit of non-commutativity of imbedding space coordinates can be considered. One can however use only the notion of Lagrangian manifold and avoids making coordinates operators leading to a loss of General Coordinate Invariance (GCI).

Quantum group theorists have studied the idea that space-time coordinates are non-commutative and tried to construct quantum field theories with non-commutative space-time coordinates (see `http://tinyurl.com/z3m8sny`). My impression is that this approach has not been very successful. In Minkowski space one introduces antisymmetry tensor J_{kl} and uncertainty relation in linear M^4 coordinates m^k would look something like $[m^k, m^l] = l_P^2 J^{kl}$, where l_P is Planck length. This would be a direct generalization of non-commutativity for momenta and coordinates expressed in terms of symplectic form J^{kl}.

1+1-D case serves as a simple example. The non-commutativity of p and q forces to use either p or q. Non-commutativity condition reads as $[p, q] = \hbar J^{pq}$ and is quantum counterpart for classical Poisson bracket. Non-commutativity forces the restriction of the wave function to be a function of p or of q but not both. More geometrically: one selects Lagrangian sub-manifold to which the projection of J_{pq} vanishes: coordinates become commutative in this sub-manifold. This condition can be formulated purely classically: wave function is defined in Lagrangian sub-manifolds to which the projection of J vanishes. Lagrangian manifolds are however not unique and this leads to problems in this kind of quantization. In TGD framework the notion of "World of Classical Worlds" (WCW) allows to circumvent this kind of problems and one can say that quantum theory is purely classical field theory for WCW spinor fields. "Quantization without quantization" would have Wheeler stated it.

GCI poses however a problem if one wants to generalize quantum group approach from M^4 to general space-time: linear M^4 coordinates assignable to Lie-algebra of translations as isometries do not generalize. In TGD space-time is surface in imbedding space $H = M^4 \times CP_2$: this changes the situation since one can use 4 imbedding space coordinates (preferred by isometries of H) also as space-time coordinates. The analog of symplectic structure J for M^4 makes sense and number theoretic vision involving octonions and quaternions leads to its introduction. Note that CP_2 has naturally symplectic form.

Could it be that the coordinates for space-time surface are in some sense analogous to symplectic coordinates (p_1, p_2, q_1, q_2) so that one must use either (p_1, p_2) or (q_1, q_2) providing coordinates for a Lagrangian sub-manifold. This would mean selecting a Lagrangian sub-manifold of space-time surface? Could one require that the sum $J_{\mu\nu}(M^4) + J_{\mu\nu}(CP_2)$ for the projections of symplectic forms vanishes and forces in the generic case localization to string world sheets and partonic 2-surfaces. In special case also higher-D surfaces - even 4-D surfaces as products of Lagrangian 2-manifolds for M^4 and CP_2 are possible: they would correspond to homologically trivial cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, which are not anymore vacuum extremals but minimal surfaces if the action contains besides Käction also volume term.

But why this kind of restriction? In TGD one has strong form of holography (SH): 2-D string world sheets and partonic 2-surfaces code for data determining classical and quantum evolution. Could this projection of $M^4 \times CP_2$ symplectic structure to space-time surface allow an elegant mathematical realization of SH and bring in the Planck length l_P defining the radius of twistor sphere associated with the twistor space of M^4 in twistor lift of TGD? Note that this can be done without introducing imbedding space coordinates as operators so that one avoids the problems with general coordinate invariance. Note also that the non-uniqueness would not be a problem as in quantization since it would correspond to the dynamics of 2-D surfaces.

The analog of brane hierarchy for the localization of spinors - space-time surfaces; string world sheets and partonic 2-surfaces; boundaries of string world sheets - is suggestive. Could this hierarchy correspond to a hierarchy of Lagrangian sub-manifolds of space-time in the sense that $J(M^4) + J(CP_2) = 0$ is true at them? Boundaries of string world sheets would be trivially Lagrangian manifolds. String world sheets allowing spinor modes should have $J(M^4) + J(CP_2) = 0$ at them. The vanishing of induced W boson fields is needed to guarantee well-defined em charge at string world sheets and that also this condition allow also 4-D solutions besides 2-D generic solutions.

This condition is physically obvious but mathematically not well-understood: could the condition $J(M^4) + J(CP_2) = 0$ force the vanishing of induced W boson fields? Lagrangian cosmic string type minimal surfaces $X^2 \times Y^2$ would allow 4-D spinor modes. If the light-like 3-surface defining boundary between Minkowskian and Euclidian space-time regions is Lagrangian surface, the total induced Kähler form Chern-Simons term would vanish. The 4-D canonical momentum currents would however have non-vanishing normal component at these surfaces. I have considered the possibility that TGD counterparts of space-time super-symmetries could be interpreted as addition of higher-D right-handed neutrino modes to the 1-fermion states assigned with the boundaries of string world sheets [K28].

Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would couple only to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [K1] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of weak group to electromagnetic gauge group.

An alternative - but of course not necessarily equivalent - attempt to formulate SH would be in terms of number theoretic vision. Space-time surfaces would be associative or co-associative depending on whether tangent space or normal space in imbedding space is associative - that is quaternionic. These two conditions would reduce space-time dynamics to associativity and commutativity conditions. String world sheets and partonic 2-surfaces would correspond to maximal commutative or co-commutative sub-manifolds of imbedding space. Commutativity (co-commutativity) would mean that tangent space (normal space as a sub-manifold of space-time surface) has complex tangent space at each point and that these tangent spaces integrate to 2-surface. SH would mean that data at these 2-surfaces would be enough to construct quantum states. String world sheet boundaries would in turn correspond to real curves of the complex 2-surfaces intersecting partonic 2-surfaces at points so that the hierarchy of classical number fields would have nice realization at the level of the classical dynamics of quantum TGD. The analogy with branes and super-symmetry force to consider two options.

2.4.4 *Two options for fundamental variational principle*

One ends up to two options for the fundamental variational principle.

Option I: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K28].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option II: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If the induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes!

If string world sheets are generalized Lagrangian sub-manifolds, only the induced em field would be non-vanishing and electroweak symmetry breaking would be a fundamental prediction. This however requires that M^4 has the analog of symplectic structure suggested also by twistorialization. This in turn provides a possible explanation of CP breaking and matter-antimatter asymmetry. In this case 4-D spinor modes do not define space-time super-symmetries.

The latter option conforms with SH and would mean that the theory is amazingly simple. String world sheets together with number theoretical space-time discretization meaning small breaking of SH would provide the basic data determining classical and quantum dynamics. The Galois group of the extension of rationals defining the number-theoretic space-time discretization would act as a covering group of the covering defined by the discretization of the space-time surface, and the value of $h_{eff}/h = n$ would correspond to the dimension of the extension dividing the order of its Galois group. The phase transitions reducing n would correspond to spontaneous symmetry breaking leading from Galois group to a subgroup and the transition would replace n with its factor.

The ramified primes of the extension would be preferred primes of given extension. The extensions for which the number of p-adic space-time surfaces representable also as a real algebraic continuation of string world sheets to preferred extrenal is especially large would be physically favored as also corresponding ramified primes. In other words, maximal number of p-adic imaginations would be realizable so that these extensions and corresponding ramified primes would be winners in the number-theoretic fight for survival. Whether this conforms with p-adic length scale hypothesis, remains an open question.

2.4.5 *Consequences*

The outcome is a precise identification of preferred extremals and therefore also a precise definition of Kähler function as Kähler action in Euclidian space-time regions: the Kähler action in Minkowskian regions takes the role of action in quantum field theories and emerges because one has complex square root of thermodynamics. The outcome is a vision combining several big ideas thought earlier to be independent.

1. Effective 2-dimensionality, which was already 30 years ago realized to be unavoidable but meant a catastrophe with the physical understanding that I had at that time. Now it is the outcome of SH implied by SGCI.

2. QC is very naturally realized in terms of generalized conformal symmetries and implies a fractal hierarchy of quantum criticalities, and gives as a side product the hierarchy of Planck constants, which emerged originally from purely physical considerations rather than from TGD. Also the hierarchy of inclusions of hyper-finite factors is a natural outcome as well as the interpretation in terms of measurement resolutions (increasing when n increases by integer factor).

3. The reduction of quantum TGD proper by SH so that only data at partonic 2-surfaces and string world sheets are used to construct the scattering amplitudes. This allows to realized number theoretical universality both at the level of space-time and WCW using algebraic continuation of the physics from an algebraic extension of rationals to real and p-adic number fields. This adelic picture together with Negentropy Maximization Principle (NMP) allows to understand the preferred p-adic primes and deduce a generalization of p-adic length scale hypothesis.

2.5 Hyper-Finite Factors And The Notion Of Measurement Resolution

The work with TGD inspired model [K22, K5] for topological quantum computation [B2] led to the realization that von Neumann algebras [A9], in particular so called hyper-finite factors of type II_1 [A5], seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. Later came the realization that the Clifford algebra of WCW defines a canonical representation of hyper-finite factors of type II_1 and that WCW spinor fields give rise to HFFs of type III_1 encountered also in relativistically invariant quantum field theories [K23].

2.5.1 *Philosophical ideas behind von Neumann algebras*

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A5].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD.

2.5.2 *Von Neumann, Dirac, and Feynman*

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [K24] based on the notion of delta function, plus the emergence of generalized Feynman graphs [K32], the possibility to formulate the notion of delta function rigorously in terms of distributions [A11, A6], and the emergence of path integral approach [A13] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A4, A14] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A2, A15] relate closely to type II_1 factors. In topological quantum computation [B2] based on braid groups [A17] modular S-matrices they play an especially important role.

In algebraic quantum field theory [A7] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [A1, A8].

2.5.3 *Hyper-finite factors in quantum TGD*

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type II_1 and III_1- the latter appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of WCW is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K14]. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

2.5.4 *Hyper-finite factors and M-matrix*

HFFs of type III_1 provide a general vision about M-matrix [K23].

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in ZEO.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). ZEO requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem [A12], which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in ZEO: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

The concrete construction of M-matrix utilizing the idea of bosonic emergence (bosons as fermion anti-fermion pairs at opposite throats of wormhole contact) meaning that bosonic propagators reduce to fermionic loops identifiable as wormhole contacts leads to generalized Feynman rules for M-matrix in which Kähler-Dirac action containing measurement interaction term defines stringy propagators [K3]. This M-matrix should be consistent with the above proposal.

2.5.5 *Connes tensor product as a realization of finite measurement resolution*

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product [A3] but do not fix M-matrix as was the original optimistic belief.

1. In ZEO $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ averaged counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$ averaged probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

2.5.6 *Number theoretical braids as space-time correlates for finite measurement resolution*

Finite measurement resolution has discretization as a space-time counterpart. In the intersection of real and p-adic worlds defines as partonic 2-surfaces with a mathematical representation allowing interpretation in terms of real or p-adic number fields one can identify points common to real and p-adic worlds as rational points and common algebraic points (in preferred coordinates dictated by symmetries of imbedding space). Quite generally, one can identify rational points and algebraic points in some extension of rationals as points defining the initial points of what might be called number theoretical braid beginning from the partonic 2-surface at the past boundary of CD and connecting it with the future boundary of CD. The detailed definition of the braid inside light-like 3-surface is not relevant if only the information at partonic 2-surface is relevant for quantum physics.

Number theoretical braids are especially relevant for topological QFT aspect of quantum TGD. The topological QFT associated with braids accompanying light-like 3-surfaces having interpretation as lines of generalied Feynman diagrams should be important part of the definition of amplitudes assigned to generalized Feynman diagrams. The number theoretic braids relate also closely to a symplectic variant of conformal field theory emerges very naturally in TGD framework (symplectic symmetries acting on $\delta M^4_\pm \times CP_2$ are in question) and this leads to a concrete proposal for how to to construct n-point functions needed to calculate M-matrix [K3]. The mechanism guaranteeing the predicted absence of divergences in M-matrix elements can be understood in terms of vanishing of symplectic invariants as two arguments of n-point function coincide.

2.5.7 *Quantum spinors and fuzzy quantum mechanics*

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities [K23]. For quantum spinors state function reduction to spin eigenstates cannot be performed unless quantum deformation parameter $q = exp(i2\pi/n)$ equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". Therefore the probability for either spin state becomes a quantized observable. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

2.5.8 *Concrete realization of finite measurement resolution*

The recent view about the realization of finite measurement resolution is surprisingly concrete.

1. The hierarchy of Planck constants $h_{eff} = n \times h$ relates to a hierarchy of criticalities and hierarchy of measurement resolutions since each breaking of symplectic conformal symmetries transforms some gauge degrees of freedom to physical ones making possible improved resolution. For the conformal symmetries associated with the spinor modes the identification as unbroken gauge symmetries is the natural one and conforms with the interpretation as counterparts of gauge symmetries. The hierarchies of conformal symmetry breakings can be identified as hierarchies of inclusions of HFFs. Criticality would generate dark matter phase characterized by n.

   The conformal sub-algebra realized as gauge transformations corresponds to the included algebra gets smaller as n increases so that the measurement resolution improves. The integer n would naturally characterize the inclusions of hyperfinite factors of type II_1 characterized by quantum phase $exp(2\pi/n)$. Finite measurement resolution is expected to give rise to the quantum group representations of symmetries, q-special functions, and q-derivative replacing ordinary derivative and reflecting the presence of discretization.

In p-adic context representation of angle by phases coming as roots of unity corresponds to this as also the hierarchy of effective p-adic topologies reflecting the fact that finite measurement resolution makes well-orderedness of real numbers as un-necessary luxury and one can use much simpler p-adic mathematics. An excellent example is provided by p-adic mass calculations where number theoretical existence arguments fix the predictions of the model based on p-adic thermodynamics to a high degree.

2. Also the numbers of partonic 2-surfaces and string world sheets connecting them give rise to a physical realization of the finite measurement resolution since fermions at string world sheets represent the space-time geometry physically in finite measurement resolution realized also as a hierarchy of geometries for WCW (via the representation of WCW Kähler metric in terms of anti-commutators of super charges). Finite measurement resolution is a property of physical system formed by the observer and system studied: the system studied changes when the resolution changes.

3. This representation is automatically discrete at the level of partonic 2-surfaces, 1-D at their light-like orbits and 4-D in space-time interior. The discretization can be induced from discretization at the level of imbedding space as is done in the definition of p-adic variants of space-time surfaces [K30].

   For $D > 0$ the discretization could also take place more abstractly for the parameters characterizing the functions (say coefficients of polynomials) characterizing string boundaries, string world sheets and partonic 2-surfaces, 3-surfaces, and 4-D space-time surfaces. Clearly, an abstraction hierarchy is involved. Similar discretization applied to the parameters characterizing the functions defining the 3-surfaces makes sense at the level of WCW. The discretization is obviously analogous to a choice of gauge and p-adicization suggests that rational numbers and their algebraic extensions give rise to a natural discretization allowing easy algebraic continuation of scattering amplitudes between different number fields.

3 Physics As A Generalized Number Theory

Physics as a generalized number theory vision involves actually three threads: p-adic ideas [K17], the ideas related to classical number fields [K18], and the ideas related to the notion of infinite prime [K16].

3.1 Fusion Of Real And P-Adic Physics To A Coherent Whole

p-Adic number fields were not present in the original approach to TGD. The success of the p-adic mass calculations (summarized in the first part of [K29]) made however clear that one must generalize the notion of topology also at the infinitesimal level from that defined by real numbers so that the attribute "topological" in TGD gains much more profound meaning than intended originally. It took a decade to get convinced that the identification of p-adic physics as a correlate of cognition is the most plausible interpretation [K10].

Another idea has been that that p-adic topology of p-adic space-time sheets somehow induces the effective p-adic topology of real space-time sheets. This idea could make physical sense but is not necessary in the recent adelic vision.

The discovery of the properties of number theoretic variants of Shannon entropy led to the idea that living matter could be seen as as something in the intersection of real and p-adic worlds and gave additional support for this interpretation. If even elementary particles reside in this intersection and effective p-adic topology applies for real partonic 2-surfaces, the success of p-adic mass calculations can be understood. The precise identification of this intersection has been a long-standing problem and only quite recently a definite progress has taken place [K31].

The original view about physics as the geometry of WCW is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [K15].

1. The first element is a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common rationals and algebraics to form a larger adelic structure (see **Fig. ??** in the appendix of this book).

2. At the level of imbedding space this gluing could be seen as a gluing of real and p-adic variants of the imbedding space together along common points in an algebraic extension of rationals inducing those for p-adic fields to what could be seen as a book like structure. General Coordinate Invariance (GCI) restricted to rationals or their extension requires preferred coordinates for $CD \times CP_2$ and this kind coordinates can be fixed by isometries of H. The coordinates are however not completely unique since non-rational isometries produce new equally good choices.

3. The manner to get rid of these problems is a more abstract formulation at the level of WCW: a discrete collection of space-time surface instead of a discrete collection of points of space-time surface. In the recent formulation based on strong form of holography identifying the back of the book as string world sheets and partonic 2-surfaces with parameters in some algebraic extension of rationals, the problems with GCI seem to disappear since the equations for the 2-surfaces in the intersection can be interpreted in any number field. One also gets rid of the ugly discretization at space-time level needed in the notion of p-adic manifold [K30] since it is performed at the level of parameters characterizing 2-D surfaces. By conformal invariance these parameters could be conformal moduli so that infinite-D WCW would effectively reduce to finite-D spaces.

4. The possibility to assign a p-adic prime to the real space-time sheets is required by the success of the elementary particle mass calculations and various applications of the p-adic length scale hypothesis. The original idea was that the non-determinism of Kähler action corresponds to p-adic non-determinism for some primes. It has been however difficult to make this more concrete.

   Rational numbers are common to reals and all p-adic number fields. One can actually assign to any algebraic extension of rationals extensions of p-adic numbers and construct corresponding adeles. These extensions can be arranged according to the complexity and I have already earlier proposed that this hierarchy gives rie to an evolutionary hierarchy.

   How the existence of preferred p-adic primes characterizing space-time surfaces emerge was solved only quite recently [K31]. The solution relies on p-adicization based on strong holography motivating the idea the idea that string world sheets and partonic surfaces with parameters in algebraic extensions of rationals define the intersection of reality and various p-adicities. The algebraic extension possesses preferred primes as primes, which are ramified meaning that their decomposition to a product of primes of the extension contains higher than first powers of its primes (prime ideals is the more precise notion).

   These primes are obviously natural candidates for the primes characterizing string world sheets number theoretically and it might even happen that strong form of holography is possible only for these primes. The weak form of NMP [K8] allows also to justify a generalization of p-adic length scale hypothesis. Primes near but below powers of primes are favoured since they allow exceptionally large negentropy gain so that state function reductions to tend to select them. Therefore the adelic approach combined with strong form of holography seems to be a rather promising approach.

   p-Adic continuations of 2-surfaces to 4-surfaces identifiable as imaginations would be due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K10]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

The interpretation for discretization the level of partonic 2-surfaces could be in terms of cognitive, sensory, and measurement resolutions rather than fundamental discreteness of the space-time. At the level of partonic 2-surface the discretization reduces to the naively expected one: the corners of string world sheets at partonic 2-surface defined the end points of string and orbits of string ends carrying fermion number. This discretization has concrete physical interpretation. Clearly a co-dimension rule holds. Discretization of n-D object consist of n-2-D objects.

What looks rather counter intuitive first is that transcendental points of p-adic space-time sheets are at spatiotemporal infinity in real sense so that the correlates of cognition cannot be localized to any finite spatiotemporal volume unlike those of sensory experience. The description of cognition in this manner predicts p-adic fractality of real physics meaning chaos in short scales combined with long range correlations: p-adic mass calculations represent one example of p-adic fractality.

The realization of this program at the level of WCW is far from trivial. Kähler-Dirac equation and classical field equations make sense but quantities expressible as space-time integrals - in particular Kähler action- do not make sense p-adically. Therefore one can ask whether only the partonic surfaces in the intersection of real and p-adic worlds should be allowed. Also this restricted theory would be highly non-trivial physically.

3.2 Classical Number Fields And Associativity And Commutativity As Fundamental Law Of Physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [A10]) are involved [K18] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the imbedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K18]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of Kähler-Dirac action the identification of space-time surface as a hyper-quaternionic sub-manifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a hyper-quaternionic sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K24, K33].

3.2.1 *How to achieve associativity in the fermionic sector?*

In the fermionic sector an additional complication emerges. The associativity of the tangent- or normal space of the space-time surface need not be enough to guarantee the associativity at the level of Kähler-Dirac or Dirac equation. The reason is the presence of spinor connection. A possible cure could be the vanishing of the components of spinor connection for two conjugates of quaternionic coordinates combined with holomorphy of the modes.

1. The induced spinor connection involves sigma matrices in CP_2 degrees of freedom, which for the octonionic representation of gamma matrices are proportional to octonion units in Minkowski degrees of freedom. This corresponds to a reduction of tangent space group $SO(1,7)$ to G_2. Therefore octonionic Dirac equation identifying Dirac spinors as complexified octonions can lead to non-associativity even when space-time surface is associative or co-associative.

2. The simplest manner to overcome these problems is to assume that spinors are localized at 2-D string world sheets with 1-D CP_2 projection and thus possible only in Minkowskian regions. Induced gauge fields would vanish. String world sheets would be minimal surfaces in $M^4 \times D^1 \subset M^4 \times CP_2$ and the theory would simplify enormously. String area would give rise to an additional term in the action assigned to the Minkowskian space-time regions and for vacuum extremals one would have only strings in the first approximation, which conforms with the success of string models and with the intuitive view that vacuum extremals of Kähler action are basic building bricks of many-sheeted space-time. Note that string world sheets would be also symplectic covariants.

   Without further conditions gauge potentials would be non-vanishing but one can hope that one can gauge transform them away in associative manner. If not, one can also consider the possibility that CP_2 projection is geodesic circle S^1: symplectic invariance is considerably reduces for this option since symplectic transformations must reduce to rotations in S^1.

3. The fist heavy objection is that action would contain Newton's constant G as a fundamental dynamical parameter: this is a standard recipe for building a non-renormalizable theory. The very idea of TGD indeed is that there is only single dimensionless parameter analogous to critical temperature. One can of coure argue that the dimensionless parameter is $\hbar G/R^2$, R CP_2 "radius".

   Second heavy objection is that the Euclidian variant of string action exponentially damps out all string world sheets with area larger than $\hbar G$. Note also that the classical energy of Minkowskian string would be gigantic unless the length of string is of order Planck length. For Minkowskian signature the exponent is oscillatory and one can argue that wild oscillations have the same effect.

   The hierarchy of Planck constants would allow the replacement $\hbar \rightarrow \hbar_{eff}$ but this is not enough. The area of typical string world sheet would scale as h_{eff} and the size of CD and gravitational Compton lengths of gravitationally bound objects would scale as $\sqrt{h_{eff}}$ rather than $\hbar_{eff} = GMm/v_0$, which one wants. The only way out of problem is to assume $T \propto (\hbar/h_{eff})^2 \times (1/h_{bar}G)$. This is however un-natural for genuine area action. Hence it seems that the visit of the basic assumption of superstring theory to TGD remains very short.

3.2.2 *Is super-symmetrized Kähler-Dirac action enough?*

Could one do without string area in the action and use only K-D action, which is in any case forced by the super-conformal symmetry? This option I have indeed considered hitherto. K-D Dirac equation indeed tends to reduce to a lower-dimensional one: for massless extremals the K-D operator is effectively 1-dimensional. For cosmic strings this reduction does not however take place. In any case, this leads to ask whether in some cases the solutions of Kähler-Dirac equation are localized at lower-dimensional surfaces of space-time surface.

1. The proposal has indeed been that string world sheets carry vanishing W and possibly even Z fields: in this manner the electromagnetic charge of spinor mode could be well-defined. The vanishing conditions force in the generic case 2-dimensionality.

   Besides this the canonical momentum currents for Kähler action defining 4 imbedding space vector fields must define an integrable distribution of two planes to give string world sheet. The four canonical momentum currents $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ identified as imbedding 1-forms can have only two linearly independent components parallel to the string world sheet. Also the Frobenius conditions stating that the two 1-forms are proportional to gradients of two imbedding space coordinates Φ_i defining also coordinates at string world sheet, must be satisfied. These conditions are rather strong and are expected to select some discrete set of string world sheets.

2. To construct preferred extremal one should fix the partonic 2-surfaces, their light-like orbits defining boundaries of Euclidian and Minkowskian space-time regions, and string world sheets. At string world sheets the boundary condition would be that the normal components of canonical momentum currents for Kähler action vanish. This picture brings in mind strong form of holography and this suggests that might make sense and also solution of Einstein equations with point like sources.

3. The localization of spinor modes at 2-D surfaces would would follow from the well-definedness of em charge and one could have situation is which the localization does not occur. For instance, covariantly constant right-handed neutrinos spinor modes at cosmic strings are completely de-localized and one can wonder whether one could give up the localization inside wormhole contacts.

4. String tension is dynamical and physical intuition suggests that induced metric at string world sheet is replaced by the anti-commutator of the K-D gamma matrices and by conformal invariance only the conformal equivalence class of this metric would matter and it could be even equivalent with the induced metric. A possible interpretation is that the energy density of Kähler action has a singularity localized at the string world sheet.

   Another interpretation that I proposed for years ago but gave up is that in spirit with the TGD analog of AdS/CFT duality the Noether charges for Kähler action can be reduced to integrals over string world sheet having interpretation as area in effective metric. In the case of magnetic flux tubes carrying monopole fluxes and containing a string connecting partonic 2-surfaces at its ends this interpretation would be very natural, and string tension would characterize the density of Kähler magnetic energy. String model with dynamical string tension would certainly be a good approximation and string tension would depend on scale of CD.

5. There is also an objection. For M^4 type vacuum extremals one would not obtain any non-vacuum string world sheets carrying fermions but the successes of string model strongly suggest that string world sheets are there. String world sheets would represent a deformation of the vacuum extremal and far from string world sheets one would have vacuum extremal in an excellent approximation. Situation would be analogous to that in general relativity with point particles.

6. The hierarchy of conformal symmetry breakings for K-D action should make string tension proportional to $1/h_{eff}^2$ with $h_{eff} = h_{gr}$ giving correct gravitational Compton length $\Lambda_{gr} = GM/v_0$ defining the minimal size of CD associated with the system. Why the effective string tension of string world sheet should behave like $(\hbar/\hbar_{eff})^2$?

   The first point to notice is that the effective metric $G^{\alpha\beta}$ defined as $h^{kl}\Pi_k^\alpha\Pi_l^\beta$, where the canonical momentum current $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ has dimension $1/L^2$ as required. Kähler action density must be dimensionless and since the induced Kähler form is dimensionless the canonical momentum currents are proportional to $1/\alpha_K$.

   Should one assume that α_K is fundamental coupling strength fixed by quantum criticality to $\alpha_K = 1/137$? Or should one regard g_K^2 as fundamental parameter so that one would have $1/\alpha_K = \hbar_{eff}/4\pi g_K^2$ having spectrum coming as integer multiples (recall the analogy with inverse of critical temperature)?

   The latter option is the in spirit with the original idea stating that the increase of h_{eff} reduces the values of the gauge coupling strengths proportional to α_K so that perturbation series converges (Universe is theoretician friendly). The non-perturbative states would be critical states. The non-determinism of Kähler action implying that the 3-surfaces at the boundaries of CD can be connected by large number of space-time sheets forming n conformal equivalence classes. The latter option would give $G^{\alpha\beta} \propto h_{eff}^2$ and $det(G) \propto 1/h_{eff}^2$ as required.

7. It must be emphasized that the string tension has interpretation in terms of gravitational coupling on only at the GRT limit of TGD involving the replacement of many-sheeted space-time with single sheeted one. It can have also interpretation as hadronic string tension or effective string tension associated with magnetic flux tubes and telling the density of Kähler magnetic energy per unit length.

   Superstring models would describe only the perturbative Planck scale dynamics for emission and absorption of $h_{eff}/h = 1$ on mass shell gravitons whereas the quantum description of bound states would require $h_{eff}/n > 1$ when the masses. Also the effective gravitational constant associated with the strings would differ from G.

   The natural condition is that the size scale of string world sheet associated with the flux tube mediating gravitational binding is $G(M+m)/v_0$, By expressing string tension in the form $1/T = n^2\hbar G_1$, $n = h_{eff}/h$, this condition gives $\hbar G_1 = \hbar^2/M_{red}^2$, $M_{red} = Mm/(M+m)$. The effective Planck length defined by the effective Newton's constant G_1 analogous to that appearing in string tension is just the Compton length associated with the reduced mass of the system and string tension equals to $T = [v_0/G(M+m)]^2$ apart from a numerical constant ($2G(M+m)$ is Schwartschild radius for the entire system). Hence the macroscopic stringy description of gravitation in terms of string differs dramatically from the perturbative one. Note that one can also understand why in the Bohr orbit model of Nottale [E1] for the planetary system and in its TGD version [K13] v_0 must be by a factor 1/5 smaller for outer planets rather than inner planets.

3.2.3 *Are 4-D spinor modes consistent with associativity?*

The condition that octonionic spinors are equivalent with ordinary spinors looks rather natural but in the case of Kähler-Dirac action the non-associativity could leak in. One could of course give up the condition that octonionic and ordinary K-D equation are equivalent in 4-D case. If so, one could see K-D action as related to non-commutative and maybe even non-associative fermion dynamics. Suppose that one does not.

1. K-D action vanishes by K-D equation. Could this save from non-associativity? If the spinors are localized to string world sheets, one obtains just the standard stringy construction of conformal modes of spinor field. The induce spinor connection would have only the holomorphic component A_z. Spinor mode would depend only on z but K-D gamma matrix Γ^z would annihilate the spinor mode so that K-D equation would be satisfied. There are good hopes that the octonionic variant of K-D equation is equivalent with that based on ordinary gamma matrices since quaternionic coordinated reduces to complex coordinate, octonionic quaternionic gamma matrices reduce to complex gamma matrices, sigma matrices are effectively absent by holomorphy.

2. One can consider also 4-D situation (maybe inside wormhole contacts). Could some form of quaternion holomorphy [A16] [K33] allow to realize the K-D equation just as in the case of super string models by replacing complex coordinate and its conjugate with quaternion and its 3 conjugates. Only two quaternion conjugates would appear in the spinor mode and the corresponding quaternionic gamma matrices would annihilate the spinor mode. It is essential that in a suitable gauge the spinor connection has non-vanishing components only for two quaternion conjugate coordinates. As a special case one would have a situation in which only one quaternion coordinate appears in the solution. Depending on the character of quaternionion holomorphy the modes would be labelled by one or two integers identifiable as conformal weights.

   Even if these octonionic 4-D modes exists (as one expects in the case of cosmic strings), it is far from clear whether the description in terms of them is equivalent with the description using K-D equation based ordinary gamma matrices. The algebraic structure however raises hopes about this. The quaternion coordinate can be represented as sum of two complex coordinates as $q = z_1 + Jz_2$ and the dependence on two quaternion conjugates corresponds to the dependence on two complex coordinates z_1, z_2. The condition that two quaternion complexified gammas annihilate the spinors is equivalent with the corresponding condition for Dirac equation formulated using 2 complex coordinates. This for wormhole contacts. The possible generalization of this condition to Minkowskian regions would be in terms Hamilton-Jacobi structure.

Note that for cosmic strings of form $X^2 \times Y^2 \subset M^4 \times CP_2$ the associativity condition for S^2 sigma matrix and without assuming localization demands that the commutator of Y^2 imaginary units is proportional to the imaginary unit assignable to X^2 which however depends on point of X^2. This condition seems to imply correlation between Y^2 and S^2 which does not look physical.

To summarize, the minimal and mathematically most optimistic conclusion is that Kähler-Dirac action is indeed enough to understand gravitational binding without giving up the associativity of the fermionic dynamics. Conformal spinor dynamics would be associative if the spinor modes are localized at string world sheets with vanishing W (and maybe also Z) fields guaranteeing well-definedness of em charge and carrying canonical momentum currents parallel to them. It is not quite clear whether string world sheets are present also inside wormhole contacts: for CP_2 type vacuum extremals the Dirac equation would give only right-handed neutrino as a solution (could they give rise to $N = 2$ SUSY?).

The construction of preferred extremals would realize strong form of holography. By conformal symmetry the effective metric at string world sheet could be conformally equivalent with the induced metric at string world sheets.

Dynamical string tension would be proportional to $\hbar/h_{eff}^2$ due to the proportionality $\alpha_K \propto 1/h_{eff}$ and predict correctly the size scales of gravitationally bound states for $\hbar_{gr} = \hbar_{eff} = GMm/v_0$. Gravitational constant would be a prediction of the theory and be expressible in terms of α_K and R^2 and $\hbar_{eff}$ ($G \propto R^2/g_K^2$).

In fact, all bound states - elementary particles as pairs of wormhole contacts, hadronic strings, nuclei [K9], molecules, etc. - are described in the same manner quantum mechanically. This is of course nothing new since magnetic flux tubes associated with the strings provide a universal model for interactions in TGD Universe. This also conforms with the TGD counterpart of AdS/CFT duality.

3.3 Infinite Primes And Quantum Physics

The hierarchy of infinite primes (and of integers and rationals) [K16] was the first mathematical notion stimulated by TGD inspired theory of consciousness. The construction recipe is equivalent with a repeated second quantization of a super-symmetric arithmetic quantum field theory with bosons and fermions labeled by primes such that the many-particle states of previous level become the elementary particles of new level. At a given level there are free many particles states plus counterparts of many particle states. There is a strong structural analogy with polynomial primes. For polynomials with rational coefficients free many-particle states would correspond to products of first order polynomials and bound states to irreducible polynomials with non-rational roots.

The hierarchy of space-time sheets with many particle states of space-time sheet becoming elementary particles at the next level of hierarchy. For instance, the description of proton as an elementary fermion would be in a well defined sense exact in TGD Universe. Also the hierarchy of n:th order logics are possible correlates for this hierarchy.

This construction leads also to a number theoretic generalization of space-time point since a given real number has infinitely rich number theoretical structure not visible at the level of the real norm of the number a due to the existence of real units expressible in terms of ratios of infinite integers. This number theoretical anatomy suggest a kind of number theoretical Brahman=Atman identity stating that the set consisting of number theoretic variants of single point of the imbedding space (equivalent in real sense) is able to represent the points of WCW or maybe even quantum states assignable to causal diamond. One could also speak about algebraic holography.

The hierarchy of algebraic extensions of rationals is becoming a fundamental element of quantum TGD. This hierarchy would correspond to the hierarchy of quantum criticalities labelled by integer $n = h_{eff}/h$, and n could be interpreted as the product of ramified primes of the algebraic extension or its power so that number theoretic criticality would correspond to quantum criticality. The idea is that ramified primes are analogous to multiple roots of polynomial and criticality indeed corresponds to this kind of situation.

Infinite primes at the n:th level of hierarchy representing analogs of bound states correspond to irreducible polynomials of n-variables identifiable as polynomials of z_n with coefficients, which are polynomials of $z_1, .., z_{n-1}$. At the first level of hierarchy one has irreducible polynomials of single variable and their roots define irreducible algebraic extensions of rationals. Infinite integers in turn correspond to products of reducible polynomials defining reducible extensions. The infinite integers at the first level of hierarchy would define the hierarchy of algebraic extensions of rationals in turn defining a hierarchy of quantum criticalities. This observation could generalize to the higher levels of hierarchy of infinite primes so that infinite primes would be part of quantum TGD although in much more abstract sense as thought originally.

4 Physics As Extension Of Quantum Measurement Theory To A Theory Of Consciousness

TGD inspired theory of consciousness could be seen as a generalization of quantum measurement theory to make observer, which in standard quantum measurement theory remains an outsider, a genuine part of physical system subject to laws of quantum physics. The basic notions are quantum jump identified as moment of consciousness and the notion of self [K7]: in zero energy ontology these notions might however reduce to each other. Negentropy Maximization Principle [K8] defines the dynamics of consciousness and as a special case reproduces standard quantum measurement theory.

4.1 Quantum Jump As Moment Of Consciousness

TGD suggests that the quantum jump between quantum histories could identified as moment of consciousness and could therefore be for consciousness theory what elementary particle is for physics [K7].

This means that subjective time evolution corresponds to the sequence of quantum jumps $\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f$ consisting of unitary process followed by state function process. Originally U was thought to be the TGD counterpart of the unitary time evolution operator $U(-t,t)$, $t \rightarrow \infty$, associated with the scattering solutions of Schrödinger equation. It seems however impossible to assign any real Schrödinger time evolution with U. In zero energy ontology U defines a unitary matrix between zero energy states and is naturally assignable to intentional actions whereas the ordinary S-matrix telling what happens in particle physics experiment (for instance) generalizes to M-matrix defining time-like entanglement between positive and negative energy parts of zero energy states. One might say that U process corresponds to a fundamental act of creation creating a quantum superposition of possibilities and the remaining steps generalizing state function reduction process select between them.

4.2 Negentropy Maximization Principle And The Notion Of Self

Negentropy Maximization Principle (NMP [K8]) defines the variational principle of TGD inspired theory of consciousness. It has developed considerably during years. The notion of negentropic entanglement (NE) and Zero Energy Ontology (ZEO) have been main stimuli in this process.

1. U-process is followed by a sequence of state function reductions. Negentropy Maximization Principle (NMP [K8]) in its original form stated that in a given quantum state the most quantum entangled subsystem-complement pair can perform the quantum jump to a state with vanishing entanglement. More precisely: the reduction of the entanglement entropy in the quantum jump is as large as possible. This selects the pair in question and in case of ordinary entanglement entropy leads the selected pair to a product state. The interpretation of the reduction of the entanglement entropy as a conscious information gain makes sense. The sequence of state function reductions decomposes at first step the entire system to two parts in such a manner that the reduction entanglement entropy is maximal. This process repeats itself for subsystems. If the subsystem in question cannot be divided into a pair of entangled free system the process stops since energy conservation does not allow it to occur (binding energy).

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. Everything is consciousness but consciousness can be lost if self develops bound state entanglement during U process so that state function reduction to smaller un-entangled pieces is impossible.

2. The existence of number theoretical entanglement entropies in the intersection of real and various p-adic worlds forced to modify this picture. These entropies can be negative and therefore are actually positive negentropies representing conscious or potentially conscious information.

   The reduction process can stop also if the self in question allows only decompositions to pairs of systems with negentropic entanglement (NE). This does not require that that the system forms a bound state for any pair of subsystems so that the systems decomposing it can be free (no binding energy). This defines a new kind of bound state not describable as a jail defined by the bottom of a potential well. Subsystems are free but remain correlated by NE (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book).

   The consistency with quantum measurement theory demands that quantum measurement leads to an eigen-space of the density matrix so that the outcome of the state function reduction would be characterized by a possibly higher-dimensional projection operator. This would define strong form of NMP. The condition that negentropy gain (rather than final state negentropy) is maximal fixed the sub-system complement pair for which the reduction occurs.

3. Strong form of NMP would mean very restricted form of free will: we would live in the best possible world. The weak form of NMP allows the outcome of state function reduction to be a lower-dimensional subspace of the space defined by the projector. This form of NMP allows free will, event also ethics and moral can be understood if one assumes that NE means experience with positive emotional coloring and has interpretation as information (Akashic records) [K21]. Weak form of NMP allows also to predict generalization of p-adic length scale hypothesis [K31]. Hence weak NMP is much more feasible than strong form of NMP.

It is not at all obvious that NMP is consistent with the second law and it is quite possible that second law holds true only if one restricts the consideration to the visible matter sector with ordinary value of Planck constant.

1. The ordinary state function reductions - as opposed to those generating negentropic entanglement - imply dissipation crucial for self organization and quantum jump could be regarded as the basic step of an iteration like process leading to the asympotic self-organization patterns. One could regard dissipation as a Darwinian selector as in standard theories of self-organization. NMP thus predicts that self organization and hence presumably also fractalization can occur inside selves. NMP would favor the generation of negentropic entanglement. This notion is highly attractive since it could allow to understand how quantum self-organization generates larger coherent structures.

2. State function reduction for NE is not deterministic for the weak form of NMP but on the average sense negentropy assignable to dark matter sectors increases. This could allow to understand how living matter is able to develop almost deterministic cellular automaton like behaviors.

3. A further implication of NMP is that Universe generates information about itself represented in terms of NE: if one is not afraid of esoteric associations one could call this information Akashic records. This ia not in obvious conflict with second law since the entropy in the case of second law is ensemble entropy assignable to single particle in thermodynamical description.

   The simplest assumption is that the information measured by number theoretic negentropy is experienced during the state function reduction sequence at fixed boundary of CD defining self.

Weak NMP provides an understanding of life, which is the mirror image of that believed to be provided by the second law. Life in the standard Universe would be a thermodynamical fluctuation - the needed size of this fluctuation has been steadily increasing and it seems that it will eventually fill the entire Universe! Life in TGD Universe is a necessity implied by NMP and the attribute "weak" makes possible the analogs of thermodynamical fluctuations in opposite effects meaning that the world is not the best possible one. On the other hand, weak form of NMP implies evolution as selection of preferred p-adic primes since the free will allows also larger negentropy gains than strong form of NMP.

4.3 Life As Islands Of Rational/Algebraic Numbers In The Seas Of Real And P-Adic Continua?

NMP and negentropic entanglement demanding entanglement probabilities which are equal to inverse of integer, is the starting point. Rational and even algebraic entanglement coefficients make sense in the intersection of real and p-adic words, which suggests that in some sense life and conscious intelligence reside in the intersection of the real and p-adic worlds.

What could be this intersection of realities and p-adicities?

1. The facts that fermionic oscillator operators are correlates for Boolean cognition and that induced spinor fields are restricted to string world sheets and partonic 2-surfaces suggests that the intersection consists of these 2-surfaces.

2. Strong form of holography allows a rather elegant adelization of TGD by a construction of space-time surfaces by algebraic continuations of these 2-surfaces defined by parameters in algebraic extension of rationals inducing that for various p-adic number fields to real or p-adic number fields. Scattering amplitudes could be defined also by a similar algebraic contination. By conformal invariance the conformal moduli characterizing the 2-surfaces would defined the parameters.

This suggests a rather concrete view about the fundamental quantum correlates of life and intelligence.

1. For the minimal option life would be effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests. There are good reasons to expect that only the data from the intersection of real and p-adic string world sheets partonic two-surfaces appears in U-matrix so that the data localizable to strings connecting partonic 2-surfaces would dictate the scattering amplitudes.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving [K5]. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua. Life as a critical phenomenon in the number theoretical sense would be one aspect of quantum criticality of TGD Universe besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K12].

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

Later progress in understanding of quantum TGD allows to refine and simplify this view dramatically. The idea about p-adic-to-real transition for space-time sheets as a correlate for the transformation of intention to action has turned out to be un-necessary and also hard to realize mathematically. In adelic vision real and p-adic numbers are aspects of existence in all length scales and mean that cognition is present at all levels rather than emerging. Intentions have interpretation in terms of state function reductions in ZEO and there is no need to identify p-adic space-time sheets as their correlates.

4.4 Two Times

The basic implication of the proposed view is that subjective time and geometric time of physicist are not the same [K7]. This is not a news actually. Geometric time is reversible, subjective time irreversible. Geometric future and past are in completely democratic position, subjective future does not exist at all yet. One can say that the non-determinism of quantum jump is completely outside space-time and Hilbert space since quantum jumps replaces entire 4-D time evolution (or rather, their quantum superposition) with a new one, re-creates it. Also conscious existence defies any geometric description. This new view resolves the basic problem of quantum measurement theory due to the conflict between determinism of Schrödinger equation and randomness of quantum jump. The challenge is to understand how these two times correlate so closely as to lead to their erratic identification.

With respect to geometric time the contents of conscious experience is naturally determined by the space-time region inside CD in zero energy ontology. This geometro-temporal integration should have subjecto-temporal counterpart. The experiences of self are determined partially by the mental images assignable to sub-selves (having sub-CDs as imbedding space correlates) and the quantum jump sequences associated with sub-selves define a sequence of mental images.

The view about the experience of time has changed.

1. The original hypothesis was that self experiences these sequences of mental images as a continuous time flow. If the mental images define the contents fo consciousness completely, self would experience in absence of mental images experience of "timelessness". This could be seen to be in accordance with the reports of practitioners of various spiritual practices. One must be however extremely cautious and try to avoid naive interpretations.

2. ZEO forces to modify this view: the experience about the flow of time and its arrow corresponds to a sequence of repeated state function reductions leaving the state at fixed boundary of CD invariant: in standard quantum theory the entire state would remain invariant but now the position of the upper boundary of CD and state at it changes. Perhaps the experiences of meditators are such that the upper boundary of CD is more or less stationary during them.

What happens when consciousness is lost?

1. The original vision was that self loses consciousness in quantum jump generating entropic entanglement and experience an expansion of consciousness if the resulting entanglement is negentropic.

2. The recent vision is that the first state function reduction to the opposite boundary of CD means for self death followed by re-incarnation at the opposite boundary.

The assumption that the integration of experiences of self involves a kind of averaging over sub-selves of sub-selves guarantees that the sensory experiences are reliable despite the fact that quantum nondeterminism is involved with each quantum jump.

The measurement of density matrix defined by the $MM^\dagger$, where M is the M-matrix between positive and negative energy parts of the zero energy state would correspond to the passive aspects of consciousness such as sensory experiencing. U would represent at the fundamental level volition as a creation of a quantum superposition of possibilities. What follows it would be a selection between them. The volitional choice between macroscopically differing space-time sheets representing different maxima of Kähler function could be basically responsible for the active aspect of consciousness. The fundamental perception-reaction feedback loop of biosystems would result from the combination of the active and passive aspects of consciousness represented by U and M.

4.5 How Experienced Time And The Geometric Time Of Physicist Relate To Each Other?

The relationship between experienced time and time of physicist is one of the basic puzzles of modern physics. In the proposed framework they are certainly two different things and the challenge is to understand why the correlation between them is so strong that it has led to their identification. One can imagine several alternative views explaining this correlation [K21, K2] and it is better to keep mind open.

4.5.1 *Basic questions*

The flow of subjective time corresponds to quantum jump sequences for sub-selves of self having interpretation as mental images. If mind is completely empty of mental images subjectively experienced time ceases to exists. This leaves however several questions to be answered.

1. Why the contents of conscious of self comes from a finite space-time region looks like an easy question. If the contents of consciousness for sub-selves representing mental images is localized to the sub-CDs with indeed have defined temporal position inside CD assigned with the self the contents of consciousness is indeed from a finite space-time volume. This implies a new view about memory. There is no need to store again and again memories to the "brain now" since the communications with the geometric past by negative energy signals and also time-like negentropic quantum entanglement allow the sharing of the mental images of the geometric past.

2. There are also more difficult questions. Subjective time has arrow and has only the recent and possibly also past. The subjective past could in principle reduce to subjective now if conscious experience is about 4-D space-time region so that memories would be always geometric memories. How these properties of subjective time are transferred to apparent properties of geometric time? How the arrow of geometric time is induced? How it is possible that the locus for the contents of conscious experience shifts or at least seems to be shifted quantum jump by quantum jump to the direction of geometric future? Why the sensory mental images are located in a narrow time interval of about .1 seconds in the usual states of consciousness (not that sensory memories are possible: scent memories and phantom pain in leg could be seen as examples of vivid sensory memory)?

4.5.2 *The recent view about arrow of time*

The basic intuitive idea about the explanation for the arrow of psychological time has been the same from the beginning - diffusion inside light-cone - but its detailed realization has required understanding of what quantum TGD really is. The replacement of ordinary positive energy ontology with zero energy ontology (ZEO) has played a crucial role in this development. The TGD based vision about how the arrow of geometric time is by no means fully developed and final. It however seems that the most essential aspects have been understood now.

1. What seems clear now is the decisive role of ZEO and hierarchy of CDs, and the fact that the quantum arrow of geometric time is coded into the structure of zero energy states to a high extent. The still questionable but attractively simple hypothesis is that U matrix two basis with opposite quantum arrows of geometric time: is this assumption really consistent with what we know about the arrow of time? If this is the case, the question is how the relatively well-defined quantum arrow of geometric time implies the experienced arrow of geometric time. Should one assume the arrow of geometric time separately as a basic property of the state function reduction cascade or more economically- does it follow from the arrow of time for zero energy states or only correlate with it?

2. The state function reductions can occur at both boundaries of CD. If the reduction occurs at given boundary is immediately followed by a reduction at the opposite boundary, the arrow of time alternates: this does not conform with intuitive expectations: for instance, this would imply that there are two selves assignable to the opposite boundaries!

   Zero energy states are however de-localized in the moduli space CDs (size of CD plus discrete subgroup of Lorentz group defining boosts of CD leaving second tip invariant). One has quantum superposition of CDs with difference scales but with fixed upper or lower boundary belonging to the same light-cone boundary after state function reduction. In standard quantum measurement theory the repetition of state function reduction does not change the state but now it would give rise to the experienced flow of time. Zeno effect indeed requires that state function reductions can occur repeatedly at the same boundary. In these reductions the wave function in moduli degrees of freedom of CD changes. This implies "dispersion" in the moduli space of CDs experienced as flow of time with definite arrow. This view lead to a precise definition of self as sequence of quantum jumps to the reducing to the same boundary of CD.

3. This approach codes also the arrow of time at the space-time level: the average space-time sheet in quantum superposition increases in size as the average position of the "upper boundaries" of CDs drift towards future state function reduction by state function reduction.

4. In principle the arrow of time can temporarily change but it would seem that this can occur in very special circumstances and probably takes place in living matter routinely. Phase conjugate laser beam is a non-biological example about reversal of the arrow of time. The act of volition would correspond to the first state function reduction to the opposite boundary so that the reversal of time arrow at some level of the hierarchy of selves would take place in the act of volition.

Usually it is thought that the increase of ensemble entropy implied by second law gives rise to the arrow of observed time. In TGD framework NMP replaces second law as a fundamental principle and at the level of ensembles implies it. The negentropy assignable to entanglement increases by NMP if one accept the number of number theoretic Shannon entropy.

Could the increase of entanglement negentropy define the arrow of time? Negentropy is assignable to the fixed boundary of CD and characterizes self. The sequence of repeated state function reductions cannot therefore increase negentropy. Negentropy would increase only in the state function reduction a the opposite boundary of CD and the increased negentropy would be associated the re-incarnated self. The increase of negentropy would be forced by NMP and also the size scale of CD would increase.

This would be certainly consistent with evolution. The prediction is that a given CD corresponds to an entire family CDs coming integer multiples $n = h_{eff}/h$ of a minimal size. During state function reduction sequence to fixed boundary of CD the average size defined by average value of n and p-adic length scale involved would increase in statistical sense. One can consider also the possibility that there is sharp localization to given value of n.

The periods of repeated state function reductions would be periods of coherence (sustained mental image, subself) and decoherence would be implied by the first state function to the opposite boundary of CD forced by NMP to eventually to occur. At the level of action principle the increase of h_{eff} means gradual reduction of string tension $T \propto 1/\hbar_{eff}G$ and generation of gravitationally bound states of increasing size with binding realized in terms of strings connecting the partonic 2-surfaces. Gravitation, biology, and evolution would be very intimately related.

5 Implications Of Quantum Classical Correspondence

Quantum Classical Correspondence has been of the guiding principles in the construction of Quantum TGD. Recall that at the level of WCW Quantum TGD is a theory of purely classical spinor fields. In ZEO the modes of sub- WCW spinor fields associated with a given CD have by effective 2-dimensionality as their arguments collections of partonic 2-surfaces and their 4-D tangent space data. U-matrix, M-matrices, and S-matrix are in principle reducible to the properties of the basis of WCW spinor fields [K27].

Quantum classical correspondence assumes that classical dynamics defined by the preferred extremals of Kähler action define an exact part of Quantum TGD. More generally, all quantum notions - even the quantum jump sequence characterizing the contents of consciousness - must have space-time counterpart: this representation is analogous to written language.

The notion of WCW Kähler geometry combined with GCI allows to identify classical space-time surfaces as analogs of Bohr orbits as preferred extremals of Kähler action. What "preferred" means is of course a highly non-trivial question. Assuming that light-like 3-surfaces and space-like 3-surfaces at the ends of CDs give rise to same theory implies effective 2-dimensionality and strong form of holography having dramatic implications for the theory.

5.1 Strong Form Of Holography And Effective 2-Dimensionality

Strong form of holography reduces to strong form of General Coordinate Invariance.

1. The starting point is the vision about geometrization of quantum physics in terms of the geometry of WCW , the space of 3-surfaces of H. Quantum states correspond to classical WCW spinor fields and WCW spinors (spinor field at given point of WVW, which is 3-surface!). No quantization occurs at WCW level: spinor fields are classical. WCW spinor corresponds to fermionic Fock states with fermionic oscillator operators associated with free second quantized induced spinor fields (spinor fields of H) at 3-surfaces and extended to 4-surfaces. This "second quantization" has purely geometric meaning and makes possible WCW spinor geometry.

2. General Coordinate Invariance is one of the fundamental symmetries and states that 4-D general coordinate transformations act as gauge symmetries. This requires that the definition of WCW metric assigns to 3-D surface a 4-D space-time surface. This space-time surface is analogous to Bohr orbit and defines the "classical physics" associated with the 3-surface but satisfying the analogs of Bohr quantization rules. This space-time surface is a preferred extremal of Kähler action and the value of Kähler action for the Euclidian regions of space-time surface defines Kähler function defining the Kähler metric of WCW .

   The value of Kähler action for Minkowskian regions of space-time surface defines a complex phase in vacuum functional and plays a role of Morse function and is also analogous to the action in ordinary quantum field theory: in particular it makes possible interference effects at the level of vacuum functional central in quantum field theories.

   Obviously the effective reduction of 4-D theory to 3-D theory corresponds to holography. In ordinary QFT approach to TGD this would not take place since one performs path integral over all space-time surfaces. In fact, the total failure of this approach led to the generalization of Einstein's geometrization program of classical physics to a geometrization of quantum physics in terms of WCW geometry.

   Characterizing the mathematical conditions satisfied by the preferred extremals of Kähler action precisely is still one of the basic mathematical challenges and several conjectures have been made during years.

3. GCI makes possible to fix the gauge by choosing the 3-surfaces in some especially convenient manner. One choice is as unions of space-like 3-surfaces at the light-like boundaries of CDs. Second choice is as wormhole throats which are light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian and behaving in many respects like causal horizons and black hole horizons. Which of the choices is correct or are both correct? If both choices are correct one ends up with the strong form of GCI: the intersections of 3-D light-like wormhole throats with the 3-D space-like ends of space-time surface defining partonic 2-surfaces and their 4-D tangent space data carry information about quantum states.

   Strong form of GCI implies strong form of holography. Already partonic 2-surfaces and their 4-D tangent space data are enough. This does not mean genuine 2-dimensionality and reduction to a string theory since tangent space data are needed. Also the breaking of strict determinism for Kähler action implies that the effective 2-dimensionality is true only in some length scales.

5.2 Weak Form Of Electric Magnetic Duality

The notion of electric-magnetic duality [B1] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of WCW . Quite recently a considerable step of progress took place in the understanding of this notion [K4]. What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces [K3].

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in the length scale defined by their Compton length. Since the other end of flux tube carries neutrino pair it is however essentially invisible at low energies so that there is no obvious conflict with experimental facts.

The hierarchy of Planck constants means that the Compton lengths of dark elementary particles can be macroscopic so that their character as magnetic flux tubes with monopoles at ends could make itself manifest in condensed and living matter.

5.3 TGD As Almost Topological QFT

TGD as almost topological QFT is one of those idea that one cannot be sure of. I think it emerged around 2005. I have been even ready to give it up but it experienced re-incarnation as I discovered the weak form of electric-magnetic duality.

1. Holography in the sense that data at 3-D surfaces code for the quantum state is an idea which emerged already at 1990 or so since 3-surfaces are indeed basic objects in quantum TGD. General Coordinate Invariance indeed implies this and the highly non-trivial implication is that space-time surface associated with a given 3-surface is analogous to Bohr orbit. Therefore semiclassical quantization is an exact part of quantum TGD.

2. The question is whether it is light-like 3-surfaces or space-like 3-surfaces at the ends of space-time sheet defined by CD can be identified as the 3-surfaces that carry the data. Strong form of General Coordinate Invariance states that both choices are equally good. Only the intersections of these surfaces at the boundaries of CDs and their 4-D tangent spaces carry

the data. This implies effective 2-dimensionality and strongly suggests conformal invariance and coset representation meaning that the actions of conformal generators of light-like 3-surface and those associated with the boundary of CD cancel each other. This implies EP in generalized form.

3. Already effective 3-dimensionality suggests but does not imply that the Kähler action reduces to 3-D Chern-Simons term. If this occurs, the theory simplifies enormously calculationally and there are good hopes of calculating even without knowing details about preferred extremals. Chern-Simons action defines a topological QFT for braids and braids indeed replaced the 3-D light-like orbits of partonic 2-surfaces in TGD Universe in finite measurement resolution.

5.3.1 The reduction of Kähler action to 3-D integrals

To achieve reduction to Chern-Simons term the Kähler action for preferred extremals must reduce to a total divergence. This is achieved if in the decomposition of action to a total divergence and term $j \cdot A$, where j is Kähler current the latter term vanishes: $j \cdot A = 0$. This takes place in the following situations.

1. Empty space Maxwell equations $j = 0$ stating the vanishing of Kähler current hold true.

2. j and A are light-like and in the same direction so that their product vanishes. This is true for so called "massless extremals" (topological light rays).

3. j is proportional to the instanton current $j = \Phi j_I$, $j_I = \epsilon^{\alpha\beta\gamma\delta} A_\beta J_{\gamma\delta}$ so that $j \cdot A$ vanishes identically. Conservation of the Kähler current requires that the proportional factor Φ must satisfy $d\Phi \cdot j_I + \Phi I = 0$ where I is instanton density. $d\Phi$ is either orthogonal to j_I or both $d\Phi$ and j_I are light-like and have same direction.

   This kind of proportionality might hold true also for other isometry currents and would mean "topologicalization" of conserved currents in accordance with the idea about almost topological QFT.

One also ends up with the proposal that preferred extremals are such that the flow lines of isometry currents integrate to coordinate lines globally. This kind of flow is known as Beltrami flow. This would mean that they define the analog of hydrodynamic flow in which the orbits of particles do not cross each other and there are no collisions. The analog of quantum flow (no collisions - no dissipation) would be in question and one could assign to the flow an order parameter of a supra phase varying only along the flow lines. The basic condition for a flow J to define Beltrami flow read as $J \wedge dJ = 0$, where J is the 1-form defined by the current (covariant form of current depending on induced metric).

5.3.2 Reduction to Chern-Simons term by the weak form of electric-magnetic duality

The proportionality of Kähler current to instanton current implies the reduction of action to 3-D terms but not yet a reduction to Chern-Simons terms implying almost topological QFT property.

1. This is guaranteed if one assumed what I have called weak form of electric-magnetic duality. This duality generalizes the Montonen-Olive electric-magnetic duality and would hold at wormhole throats and space-like 3-surfaces at the ends of space-time sheets but not necessarily elsewhere. It would imply that Kähler flux equals to magnetic flux so that Kähler electric charge is quantized. There are good reasons to assume that this charge corresponds to fermion number so that all wormhole throats carrying fermion number would be magnetic monopoles carrying Kähler magnetic charge equal to fermion number. Physical particles would correspond to multi-monopole states with vanishing total Kähler magnetic charge.

2. It is important to notice that the weak form of electric-magnetic duality at the space-like 3-surfaces and wormhole throats involves the induced metric of the space-time sheet so that metric does not disappear from the theory although Kähler action reduces to Chern-Simons term. This gives a precise content to the attribute "almost". The reduction to Chern-Simons

terms would mean enormous calculational simplification of the theory and raises the hope that the theory could be calculable.

3. This also fixes to a high degree the view about leptons and hadrons. For instance, leptons should be string like objects formed by Kähler magnetically charged wormhole throats connected by magnetic flux tubes. Analogous picture applies to gauge bosons consisting of wormhole contacts with throats carrying fermion and anti-fermion numbers respectively. Hadrons could correspond multimonopole states.

5.3.3 Morse, Kähler, and me

First year physics student would immediately say that $\sqrt{g_4}$ is imaginary in the space-time regions with Minkowskian signature of the induced metric and real otherwise. For me it took 33 years to finally accept this trivial fact as a fact but finally I had to give up! This simple fact implies that Minkowskian regions give imaginary exponent of Chern-Simons term and Euclidian regions real exponent of Chern-Simons term [K26]. Under rather natural assumptions the two Chern-Simons terms are identical and would be obtained as an exponent of Chern-Simons term multiplied by complex number.

The imaginary exponent gives rise to interference effects typical for gauge theories and defining the core mechanism of quantum field theories and implies that stationary phase approximation makes sense. Stationary phase approximation is important also in topological QFTs and Chern-Simons term plays the role of Morse function in topological QFTs classifying the topological of 4-manifolds. The real exponent defines Kähler function and guarantees the convergence of the functional integral and guarantees that it exists as a genuine mathematical object.

5.3.4 Could Kähler action reduce to a 2-D integral?

Effective 2-dimensionality suggests a further dimensional reduction in the sense that Chern-Simons terms might allow expression as 2-dimensional integrals. If this idea is accepted, the only natural option is a reduction to a sum of areas of string world sheets with dynamical string tension. I have indeed developed a detailed proposal concerning the identification of this string world sheet [K33]. String world sheets indeed emerge naturally in quantum TGD and have as their boundary the space-like braid strands at the ends of space-time surfaces and light-like braid strands at the light-like 3-surfaces. Knotting of string world sheets is possible in 4-D space-time whereas braid strands link and knot at 3-surfaces so that quantum TGD would provide a theory of ordinary knots and 2-knots. This adds additional aspect to the statement that TGD is almost topological QFT.

5.4 Generalized Feynman Diagrams And Braids

The notion of generalized Feynman diagram [K25, K33, K32] has been developing rapidly during last five years. This progress has been boosted by several developments. The basic observation is that the regions of space-time surface with Euclidian signature of induced metric can be identified as generalized Feynman diagrams. Same interpretation applies by holography also to the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian. Additional boosts are due to ZEO allowing to interpreted the Feynman diagrams as a characterization of zero energy states. Also strong form of holography, bosonic energence, finite measurement resolution realized as discretization allows to replaced space-time sheets with string world sheets with the ends of string world sheets realized as braid strands, the realization that knotting of these strings is possible and could play a key role, and the connection with twistor approach have been important stimuli. The special role of 10 Hz frequency assignable to electron suggests that generalized Feynman diagrams could be relevant also for TGD inspired biology.

ZEO together with the notion of bosonic emergence leads to a new view about Feynman diagrams. The new element is that all physical states consist basically of wormhole throats which carry light-like four-momentum. Even virtual momenta are light-like and space-like four-momenta are obtained for wormhole contacts for which the energies of light-like states are of opposite sign. This leads to very powerful constraints on loop diagrams and there are good reasons to believe that both UV and IR divergences are absent.

Finite measurement resolution allows to assign braid strands to the light-like 3-surfaces and string world sheets to the 4-surfaces and one can also identify the braid strands as lines of generalized Feynman diagrams. It is possible to distinguish between light-like braids assignable to the light-like 3-surfaces and space-like braids connecting different partonic 2-surfaces at the ends of the space-time surface at the boundary of CD. Braids have also direct biological significance. DNA as topological quantum computer [K5] utilizes both kinds of braidings.

5.5 The Superposition Of Classical Fields In TGD Universe

Living system as conscious hologram is one of the basic visions. What one means with classical fields, their interference, and their interaction with elementary particles is an very essential aspect of what it is to be a hologram and a clarification to this issued emerged only during last year. As a matter fact, basic objection against TGD is that the interference of classical fields in the usual sense is not possible in TGD Universe!

In TGD Universe gauge fields are replaced with topological field quanta. Examples are topological light rays, magnetic/electric flux tubes and sheets, and flux quanta carrying both magnetic and electric fields. Flux quanta form a fractal hierarchy in the sense that there are flux quanta inside flux quanta. It is natural to assume quantization of Kähler magnetic flux. Braiding and reconnection are the basic topological operations for flux quanta.

The basic question is how the basic notions assigned with the classical gauge and gravitational fields understood in standard sense generalize in TGD framework.

1. Superposition and interference of the classical fields is very natural in Maxwell electrodynamics and certainly experimentally verified phenomena. Also the notion of hologram relies crucially on the notion of interference. How can one describe the effects explained in terms of superposition of fields in a situation in which the theory is extremely non-linear and all classical gauge fields are expressible in terms of CP_2 coordinates and their gradients? It is also rather clear that the preferred extremals for Kähler action decompose to space-time regions representing space-time correlates for quanta. The superposition of classical fields in Maxwellian sense is impossible.

2. How can one cope with this situation? The answer is based on simple observation: only the *effects* of the classical fields superpose. There is no need for the fields to superpose. Together with the notion of many-sheeted space-time this leads to elegant description of interference effects without any need to assume that linearization is a good approximation.

3. Topological quantization brings in also braiding and reconnection of magnetic flux tubes as basic operations for classical fields. These operations for flux tubes have also Maxwellian counterparts at the level of field lines. Braiding and reconnection are in a central role in TGD Universe and especially so in in TGD inspired theory of consciousness and quantum biology. The challenge is to build a coherent overall phenomenological view about the role of topologically quantized classical fields in biology and neuroscience. For instance, one can ask what is the precise formulation for the notion of conscious hologram and whether magnetic flux tubes could serve as correlates of entanglement (or at least negentropic entanglement suggested by the number theoretic vision and identified as a basic signature of living matter).

4. Topological quantization and the notion of magnetic body are especially important in TGD inspired model of EEG. The attempt to understand the findings of Persinger from the study of what is known as God helmet leads to a considerable progress in the understanding the possible role of topologically quantized classical fields in biology and neuro-science.

The replacement of superposition of fields with superposition of their effects allows to understand also how the many-sheeted space-time of TGD relates to the space-time of general relativity. GRT space-time seen as an effective space-time obtained by replacing Minkowskian regions of many-sheeted space-time with a region of Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical form of Equivalence Principle realized in terms of Einstein's equation for the GRT limit in long length scales at least.

One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also the gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets and standard model follows naturally at the QFT limit of TGD. TGD can be seen as a microscopic theory with space-time sheets carrying extremely simple "archetypal" field patterns and arranged in complex manner to many-sheeted space-time topologies whereas GRT and QFT represents long length scale limit with topological complexity replaced with the complexity of fields patterns.

For the induced gauge fields coupling constants are absorbed into the definition of gauge potentials so that coupling constant evolution does not make sense in TGD space-time. Coupling constant evolution characterizing quantum field theories makes in TGD framework sense as the dependence of various vertices on the size scale of causal diamond (CD). This discrete coupling constant evolution becomes at QFT and GRT limits continuous coupling constant evolution.

REFERENCES

Mathematics

[A1] Schroer B. Lectures on Algebraic Quantum Field Theory and Operator Algebras. Available at: http://xxx.lanl.gov/abs/math-ph/0102018, 2001.

[A2] Kassel C. *Quantum Groups.* Springer Verlag, 1995.

[A3] Kreimer D Connes A. *Hofp algebras, renormalization, and non-commutative geometry*, volume 1999. Kluwer, 1998.

[A4] Witten E. Quantum field theory and the Jones polynomial. *Comm Math Phys*, 121:351–399, 1989.

[A5] Jones FR. *Braid groups, Hecke algebras and type II_1 factors.* 1983.

[A6] Shapiro ZYa Gelfand IM, Minklos RA. *Representations of the rotation and Lorentz groups and their applications.* Pergamon Press, 1963.

[A7] Kastler D Haag R. An Algebraic Approach to Quantum Field Theory. *J Math Phys*, 5:848–861, 1964.

[A8] Borchers HJ. On Revolutionizing QFT with Tomita's Modular Theory. *J Math Phys.* Available at: http://www.lqp.uni-goettingen.de/papers/99/04/99042900.html, pages 3604–3673, 2000.

[A9] Dixmier J. *Von Neumann Algebras.* North-Holland, Amsterdam, 1981.

[A10] Baez JC. The Octonions. *Bull Amer Math Soc.* Available at: http://math.ucr.edu/home/baez/Octonions/octonions.html, 39(2002), 2001.

[A11] Schwartz L. Generalisation de la Notion de Fonction, de Derivation, de Transformation de Fourier et Applications Mathematiques et Physiques. *Publications de l'Institut de Mathematique de l'Universite de Strasbourg, Vols 9-10, Paris: Hermann*, 1945.

[A12] Takesaki M. *Tomita's Theory of Modular Hilbert Algebras and Its Applications*, volume 128. Springer, Berlin, 1970.

[A13] Rivers RJ. *Path Integral Methods in Quantum Field Theory.* Cambridge University Press, Cambridge, 1987.

[A14] Sawin S. Links, Quantum Groups, and TQFT's. Available at: http://arxiv.org/abs/q-alg/9506002, 1995.

[A15] Jones V. In and around the origin of quantum groups. Available at: http://arxiv.org/abs/math/0309199, 2003.

[A16] Vandoren S Wit de B, Rocek M. Hypermultiplets, Hyperkähler Cones and Quaternion-Kähler Geometry. Available at: http://arxiv.org/pdf/hep-th/0101161.pdf, 2001.

[A17] Ge ML Yang CN. *Braid Group, Knot Theory, and Statistical Mechanics.* World Scientific, 1989.

Theoretical Physics

[B1] Montonen Olive Duality. Available at: http://en.wikipedia.org/wiki/Montonen-Olive_duality.

[B2] Wang Z Freedman M, Larsen H. A modular functor which is universal for quantum computation. *Comm Math Phys* . Available at: http://arxiv.org/abs/quant-ph/0001108, 1(2):605–622, 2002.

[B3] Schwartz JH. *Super strings. The first 15 years of Superstring Theory.* World Scientific, 1985.

Cosmology and Astro-Physics

[E1] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity. Available at: http://arxiv.org/abs/astro-ph/0310036, 2003.

Books related to TGD

[K1] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. In *Towards M-matrix.* Online book. Available at: http://www.tgdtheory.fi/public_html/pdfpool/append.pdf, 2006.

[K2] Pitkänen M. About Nature of Time. In *TGD Inspired Theory of Consciousness.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html#timenature, 2006.

[K3] Pitkänen M. Construction of Quantum Theory: M-matrix. In *Towards M-Matrix.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#towards, 2006.

[K4] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#compl1, 2006.

[K5] Pitkänen M. DNA as Topological Quantum Computer. In *Genes and Memes.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/genememe.html#dnatqc, 2006.

[K6] Pitkänen M. Does the QFT Limit of TGD Have Space-Time Super-Symmetry? In *Towards M-Matrix.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#susy, 2006.

[K7] Pitkänen M. Matter, Mind, Quantum. In *TGD Inspired Theory of Consciousness.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html#conscic, 2006.

[K8] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html#nmpc, 2006.

[K9] Pitkänen M. Nuclear String Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy.* Online book. Available at: http://www.tgdtheory.fi/tgdhtml/neuplanck.html#nuclstring, 2006.

[K10] Pitkänen M. p-Adic Physics as Physics of Cognition and Intention. In *TGD Inspired Theory of Consciousness*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html#cognic, 2006.

[K11] Pitkänen M. *Quantum Physics as Infinite-Dimensional Geometry*. Online book.Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html, 2006.

[K12] Pitkänen M. Quantum Theory of Self-Organization. In *Bio-Systems as Self-Organizing Quantum Systems*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/bioselforg.html#selforgac, 2006.

[K13] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdclass.html#astro, 2006.

[K14] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdclass.html#cosmo, 2006.

[K15] Pitkänen M. *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html, 2006.

[K16] Pitkänen M. TGD as a Generalized Number Theory: Infinite Primes. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#visionc, 2006.

[K17] Pitkänen M. TGD as a Generalized Number Theory: p-Adicization Program. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#visiona, 2006.

[K18] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#visionb, 2006.

[K19] Pitkänen M. *TGD Inspired Theory of Consciousness*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html, 2006.

[K20] Pitkänen M. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdclass.html#tgdgrt, 2006.

[K21] Pitkänen M. Time and Consciousness. In *TGD Inspired Theory of Consciousness*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html#timesc, 2006.

[K22] Pitkänen M. Topological Quantum Computation in TGD Universe. In *Genes and Memes*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/genememe.html#tqc, 2006.

[K23] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html#vNeumann, 2006.

[K24] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#cspin, 2006.

[K25] Pitkänen M. Knots and TGD. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#knotstgd, 2011.

[K26] Pitkänen M. Motives and Infinite Primes. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#infmotives, 2011.

[K27] Pitkänen M. Construction of Quantum Theory: More about Matrices. In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#UandM, 2012.

[K28] Pitkänen M. SUSY in TGD Universe. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html#susychap, 2012.

[K29] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html, 2013.

[K30] Pitkänen M. What p-Adic Icosahedron Could Mean? And What about p-Adic Manifold? In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#picosahedron, 2013.

[K31] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#numbervision, 2014.

[K32] Pitkänen M. From Principles to Diagrams. In *Towards M-Matrix*. Online book.Available at: http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#diagrams, 2016.

[K33] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistorstory, 2016.