p-Adic Particle Massivation: Hadron Masses

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Contents

Abstract

In this chapter the results of the calculation of elementary particle masses will be used to construct a model predicting hadron masses.

1. *Topological mixing of quarks*

In TGD framework CKM mixing is induced by topological mixing of quarks (that is 2-dimensional topologies characterized by genus). Number theoretical constraints on topological mixing can be realized by assuming that topological mixing leads to a thermodynamical equilibrium. This gives an upper bound of 1200 for the number of different U and D matrices and the input from top quark mass and $\pi^+ - \pi^0$ mass difference implies that physical U and D matrices can be constructed as small perturbations of matrices expressible as direct sum of essentially unique 2×2 and 1×1 matrices. The maximally entropic solutions can be found numerically by using the fact that only the probabilities p_{11} and p_{21} can be varied freely. The solutions are unique in the accuracy used, which suggests that the system allows only single thermodynamical phase.

The matrices U and D associated with the probability matrices can be deduced straightforwardly in the standard gauge. The U and D matrices derived from the probabilities determined by the entropy maximization turn out to be unitary for most values of n_1 and n_2. This is a highly non-trivial result and means that mass and probability constraints together with entropy maximization define a sub-manifold of $SU(3)$ regarded as a sub-manifold in 9-D complex space. The choice $(n(u), n(c)) = (4, n)$, $n < 9$, does not allow unitary U whereas $(n(u), n(c)) = (5, 6)$ does. This choice is still consistent with top quark mass and together with $n(d) = n(s) = 5$ it leads to a rather reasonable CKM matrix with a value of CP breaking invariant within experimental limits. The elements V_{i3} and V_{3i}, $i = 1, 2$ are however roughly twice larger than their experimental values deduced assuming standard model. V_{31} is too large by a factor 1.6. The possibility of scaled up variants of light quarks could lead to too small experimental estimates for these matrix elements. The whole parameter space has not been scanned so that better candidates for CKM matrices might well exist.

2. *Higgs contribution to fermion masses is negligible*

There are good reasons to believe that Higgs expectation for the fermionic space-time sheets is vanishing although fermions couple to Higgs. Much later good reasons for believing that Higgs expectaton does not play any role in massivation in TGD famework have emerged. Thus p-adic thermodynamics would explain fermion masses completely. This together with the fact that the prediction of the model for the top quark mass is consistent with the most recent limits on it, fixes the CP_2 mass scale with a high accuracy to the maximal one obtained if second order contribution to electron's p-adic mass squared vanishes. This is very strong constraint on the model.

3. *The p-adic length scale of quark is dynamical*

The assumption about the presence of scaled up variants of light quarks in light hadrons leads to a surprisingly successful model for pseudo scalar meson masses using only quark masses and the assumption mass squared is additive for quarks with same p-adic length scale and mass for quarks labelled by different primes p. This conforms with the idea that pseudo scalar mesons are Goldstone bosons in the sense that color Coulombic and magnetic contributions to the mass cancel each other. Also the mass differences between hadrons containing different numbers of strange and heavy quarks can be understood if s, b and c quarks appear as several scaled up versions.

This hypothesis yields surprisingly good fit for meson masses but for some mesons the predicted mass is slightly too high. The reduction of CP_2 mass scale to cure the situation is not possible since top quark mass would become too low. In case of diagonal mesons for which quarks correspond to same p-adic prime, quark contribution to mass squared can be reduced by ordinary color interactions and in the case of non-diagonal mesons one can require that quark contribution is not larger than meson mass.

4. *Super-symplectic bosons at hadronic space-time sheet can explain the constant contribution to baryonic masses*

Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD.

Classically this contribution would naturally be assigned with the Kähler magnetic energy of color magnetic flux tubes connecting valence quarks. A possible quantal identification of

this contribution is in terms of super-symplectic gluons predicted by TGD. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent. Super-symplectic gluons also provide a possible solution to the spin puzzle of proton.

Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight.

In the case of mesons pion could contain super-symplectic boson of first generation preventing the large negative contribution of the color magnetic spin-spin interaction to make pion a tachyon. For heavier bosons super-symplectic boson need not to be assumed. The preferred role of pion would relate to the fact that its mass scale is below QCD Λ.

5. *Description of color magnetic spin-spin splitting in terms of conformal weight*

What remains to be understood are the contributions of color Coulombic and magnetic interactions to the mass squared. There are contributions coming from both ordinary gluons and super-symplectic gluons and the latter is expected to dominate by the large value of color coupling strength.

Conformal weight replaces energy as the basic variable but group theoretical structure of color magnetic contribution to the conformal weight associated with hadronic space-time sheet ($k = 107$) is same as in case of energy. The predictions for the masses of mesons are not so good than for baryons, and one might criticize the application of the format of perturbative QCD in an essentially non-perturbative situation.

The comparison of the super-symplectic conformal weights associated with spin 0 and spin 1 states and spin 1/2 and spin 3/2 states shows that the different masses of these states could be understood in terms of the super-symplectic particle contents of the state correlating with the total quark spin. The resulting model allows excellent predictions also for the meson masses and implies that only pion and kaon can be regarded as Goldstone boson like states. The model based on spin-spin splittings is consistent with the model.

To sum up, the model provides a satisfcatory understanding of baryon and meson masses. This success is highly non-trivial since the fit involves only the integers characterizing the p-adic length scales of quarks and the integers characterizing color magnetic spin-spin splitting plus p-adic thermodynamics and topological mixing for super-symplectic gluons. The next challenge would be to predict the correlation of hadron spin with super-symplectic particle content in case of long-lived hadrons.

1 Introduction

In this chapter the results of the calculation of elementary particle masses will be used to construct a model predicting hadron masses. The new elements are a revised identification for the p-adic length scales of quarks and the realization that number theoretical constraints on topological mixing can be realized by assuming that topological mixing leads to a thermodynamical equilibrium. This gives an upper bound of 1200 for the number of different U and D matrices and the input from top quark mass and $\pi^+ - \pi^0$ mass difference implies that physical U and D matrices can be constructed as small perturbations of matrices expressible as a direct sum of essentially unique 2×2 and 1×1 matrices.

The assumption about the presence of scaled up variants of light quarks in light hadrons leads to a surprisingly successful model for pseudo scalar meson masses in terms of only quark masses. This conforms with the idea that at least light pseudo scalar mesons are Goldstone bosons in the sense that color Coulombic and magnetic contributions to the mass cancel each other. Also the mass differences between baryons containing different numbers of strange quarks can be understood if s quark appears as three scaled up versions. The earlier model for the purely hadronic contributions to hadron masses simplifies dramatically and only the color Coulombic and magnetic contributions to color conformal weight are needed.

1.1 Construction Of U And D Matrices

The basic constraint on the topological mixing that the modular contributions to the conformal weight defining the mass squared remain integer valued in the proper units: if this condition does not hold true, the order of magnitude for the real counterpart of the p-adic mass squared corresponds to 10^{-4} Planck masses.

Number theory gives strong constraints on CKM matrix. p-Adicization requires that U and D matrix elements are algebraic numbers. A strong constraint would be that the mixing probabilities are rational numbers implying that matrices defined by the moduli of U and D involve only square roots of rationals. The phases of matrix elements should belong to a finite extension of complex rationals.

Little can be said about the details of the dynamics of topological mixing. Nothing however prevents for constructing a thermodynamical model for the mixing. A thermodynamical model for U and D matrices maximizing the entropy defined by the mixing probabilities subject to the constraints fixing the values of n_{q_i} and the sums of row/column probabilities to one gives a thermodynamical ensemble with two quantized temperatures and two quantized chemical potentials. The resulting polynomial equations allow at most 1200 different solutions so that the number of U and D matrices is relatively small. The fact that matrix elements are algebraic numbers guarantees that the matrices are continuable to p-adic number fields as required.

The detailed study of quark mass spectrum leads to a tentative identification $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$ of the modular contributions of conformal weights of quarks: note that in absence of mixing the contributions would be $(0, 9, 60)$ for both U and D type quarks. That b and t quark masses are nearly maximal and thus mix very little with lighter quarks is forced by the masses of t quark and $t\bar{t}$ meson. The values of n_{q_i} for light quarks follow by considering $\pi^+ - \pi^0$ mass difference.

One might consider the possibility that n_{q_i} for slightly dynamical and can vary in light mesons in order to guarantee that $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ give identical modular contributions to the conformal weight in states which are linear combinations of quark pairs. It turns out that unitarity does not allow the choices $(n_1 = 4, n_2 < 9)$, and that the choice $(n_d, n_s) = (5, 5)$, $(n_u, n_c) = (5, 6)$ is the unique choice producing a realistic CKM matrix. The requirement that quark contribution to pseudo scalar meson mass is smaller than meson mass is possible to satisfy and gives a constraint on CP_2 mass scale consistent with the prediction of leptonic masses when second order p-adic contribution to lepton mass is allowed to be non-vanishing.

The small mixing with b and t quarks is natural since the modular conformal weight of unmixed state having spectrum $\{0, 9, 60\}$ is analogous to energy so that Boltzmann weight for $n(g = 3)$ thermal excitation is small for $g = 1, 2$ ground states.

The maximally entropic solutions can be found numerically by using the fact that only the probabilities p_{11} and p_{21} can be varied freely. The solutions are unique in the accuracy used, which suggests that the system allows only single thermodynamical phase.

The matrices U and D associated with the probability matrices can be deduced straightforwardly in the standard gauge. The U and D matrices derived from the probabilities determined by the entropy maximization turn out to be unitary for most values of n_1 and n_2. This is a highly non-trivial result and means that mass and probability constraints together with entropy maximization define a sub-manifold of $SU(3)$ regarded as a sub-manifold in 9-D complex space. The choice $(n_u, n_c) = (4, n)$, $n < 9$, does not allow unitary U whereas $(n_u, n_c) = (5, 6)$ does. This choice is still consistent with top quark mass and together with $n_d = n_s = 5$ it leads to a rather reasonable CKM matrix with a value of CP breaking invariant within experimental limits. The elements V_{i3} and V_{3i}, $i = 1, 2$ are however roughly twice larger than their experimental values deduced assuming standard model. V_{31} is too large by a factor 1.6. The possibility of scaled up variants of light quarks could lead to too small experimental estimates for these matrix elements. The whole parameter space has not been scanned so that better candidates for CKM matrices might well exist.

1.2 Observations Crucial For The Model Of Hadron Masses

The evolution of the model for hadron masses involves several key observations made during the more decade that I have been working with p-adic mass calculations.

1.2.1 The p-adic mass scales of quarks are dynamical

The existence of scaled up variants of quarks is suggested by various anomalies such as Aleph anomaly [C14] and the strange bumpy structure of the distribution of the mass of the top quark candidate. This leads to the idea that the the integer $k(q)$ characterizing the p-adic mass scale of quark is different for free quarks and bound quarks and that $k(q)$ can depend on hadron. Hence one can understand not only the notions of current quark mass and constituent quark mass but reproduce also the p-adic counterpart of Gell-Mann-Okubo mass formula. Indeed, the assumption about scaled up variants of u, d, s, and even c quarks in light hadrons leads to an excellent fit of meson masses with quark contribution explaining almost all of meson mass.

1.2.2 Quarks give dominating contribution to the masses of pseudo-scalar mesons

The interpretation is that color Coulombic and color magnetic interaction conformal weights (rather than interaction energies) cancel each other in a approximation for pseudo-scalar mesons in accordance with the idea that pseudo scalar mesons are massless as far as color interactions are considered. In the case of baryons the assumption that s quark appears in three different scaled up versions (which are Λ, $\{\Sigma, \Xi\}$, and Ω) allows to understand the mass differences between baryons with different s quark content. The dominating contribution to baryon mass has however remained hitherto unidentified.

1.2.3 What it means that Higgs like contribution to fermion masses is negligible?

The failure of the simplest form of p-adic thermodynamics for intermediate gauge bosons led to the unsatisfactory conclusion that p-adic thermodynamics is not enough and the coupling to Higgs bosons contributes to the gauge boson masses. This option had its own problems.

1. There are good, purely topological - reasons to believe that Higgs expectation for the fermionic space-time sheets is vanishing although fermions couple to Higgs. p-Adic thermodynamics would explain fermion masses completely: this indeed turns out to be the case within experimental uncertainties. The absence of Higgs contribution to fermion masses would however mean asymmetry between fermions and bosons unless also boson masses have some other origin.

2. The recent view about elementary particles is as pairs of wormhole contacts connected by magnetic flux tubes carrying monopole flux. The modes of Kähler-Dirac equation are localized at 2-D surfaces: string world sheets and possibly also partonic 2-surfaces and flux tubes are accompanied by oneor more string world sheets. Therefore there is a strong temptation to assign an additional contribution to mass squared. The flux tube would give the dominating contribution to gauge boson masses and only a small contribution to fermion masses. One can even consider the possibility of Regge trajectories for gauge bosons.

The fact that the prediction of the model for the top quark mass is consistent with the most recent limits on it [C20], fixes the CP_2 mass scale with a high accuracy to the maximal one obtained if second order contribution to electron's p-adic mass squared vanishes. This is very valuable constraint on the model.

1.2.4 Mass squared is additive for quarks with same p-adic prime

An essential element of the new understanding is that mass squared (conformal weight) is additive for quarks with the same p-adic length scale whereas mass is additive for quarks with different values of p. For instance, the masses of heavy $q\overline{q}$ mesons are equal to $\sqrt{2}\times m(q)$ rather than $2m(q)$. Since $k = 107$ for hadronic space-time sheet, for quarks with $k(q) \neq 107$, additivity holds true for the quark and color contributions for mass rather than mass squared.

This hypothesis yields surprisingly good fit for meson masses but for some mesons the predicted mass is slightly too high. The reduction of CP_2 mass scale to cure the situation is not possible since top quark mass would become too low. In case of diagonal mesons for which quarks correspond to same p-adic prime, quark contribution to mass squared can be reduced by ordinary color interactions and in case of non-diagonal mesons one can require that quark contribution is not larger than meson mass.

1.2.5 A remark about terminology

Before continuing a remark about terminology is in order.

1. In the generalized coset construction the symplectic algebra of $\delta M^4_\pm \times CP_2$ and Super-Kac Moody algebras at light-like partonic surfaces X^3 are lifted to hyper-complex algebras inside the causal diamond of $M^4 \times CP_2$ carrying the zero energy states. SKM is identified as a sub-algebra of SC and the differences of SC and SKM Super-Virasoro generators annihilate the physical states. All purely geometric contributions and their super-counterparts can be regarded as SC contributions. The fermionic contributions in electro-weak and spin degrees of freedom responsible also for color partial waves are trivially one and same. One could say that there is no other contribution than SC which can be however divided into a contribution from imbedded SKM subalgebra and a genuine SC contribution.

2. In the coset construction a tachyonic ground state of negative SC conformal weight from which SKM generators create massless states must have a negative conformal weight also in SKM sense. Therefore the earlier idea that genuine SC generators create the ground states with a negative conformal weight assignable to elementary particles does not work anymore: the negative conformal weight must be due to SKM generators with conformal weight which is most naturally of form $h = -1/2 + iy$.

3. Super-symplectic contribution with a positive conformal weight can be regarded also as a product of genuine SC contribution with a vanishing conformal weight and a contribution having also interpretation as SKM contribution. What motivates the term "super-symplectic bosons" used in the sequel is that in a non-perturbative situation this contribution is most naturally calculated by regarding it as a super-symplectic contribution. This contribution is highly constrained since it comes solely from generators which are color octets and singlets have spin one or spin zero. Genuine SC contribution with a zero conformal weight comes from the products of super-Hamiltonians in higher representations of $SU(3) \times SO(3)$ containing both positive and negative conformal weights compensating each other. This contribution must have vanishing color quantum numbers and spin since otherwise Dirac operators of H in SKM and SC degrees of freedom could not act on it in the same manner. Note that gluons do not correspond to SKM generators but to pairs of quark and antiquark at throats of a wormhole contact.

1.2.6 Super-symplectic bosons at hadronic space-time sheet can explain the constant contribution to baryonic masses

Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD.

A possible identification of this contribution is in terms of super-symplectic gluons predicted by TGD. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent. super-symplectic gluons also provide a possible solution to the spin puzzle of proton.

Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight.

In the case of mesons pion could contain super-symplectic boson of first generation preventing the large negative contribution of the color magnetic spin-spin interaction to make pion a tachyon. For heavier bosons super-symplectic boson is not absolutely necessary but a very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.

1.2.7 Color magnetic spin-spin splitting formulated in terms of conformal weight

What remains to be understood are the contributions of color Coulombic and magnetic interactions to the mass squared. There are several delicate points to be taken into account.

1. The QCD based formula for the color magnetic interaction energy fails completely since the dependence of color magnetic spin-spin splittings on quark mass scale is nearer to logarithmic dependence on p-adic length scale than being of form $1/m(q_i)m(q_j) \propto L(k_i)L(k_j)$. This finding supports the decade old idea that the proper notion is not color interaction energy but color conformal weight. A model based on this assumption is constructed assuming that all pseudo-scalars are Goldstone boson like states. The predictions for the masses of mesons are not so good than for baryons, and one might criticize the application of the format of perturbative QCD in an essentially non-perturbative situation.

2. The comparison of the super-symplectic conformal weights associated with spin 0 and spin 1 states and spin 1/2 and spin 3/2 states shows that the different masses of these states could be understood in terms of the super-symplectic particle contents of the state correlating with the total quark spin. The resulting model allows excellent predictions also for the meson masses and implies that only pion and kaon can be regarded as Goldstone boson like states. The model based on spin-spin splittings is consistent with the model.

To sum up, the model provides an excellent understanding of baryon and meson masses. This success is highly non-trivial since the fit involves only the integers characterizing the p-adic length scales of quarks and the integers characterizing color magnetic spin-spin splitting plus p-adic thermodynamics and topological mixing for super-symplectic gluons. The next challenge would be to predict the correlation of hadron spin with super-symplectic particle content in the case of long-lived hadrons.

1.3 A Possible Model For Hadron

These findings suggest that the following model for hadrons deserves a testing. Hadron can be characterized in terms of $k \geq 113$ partonic 2-surfaces $X^2(q_i)$ connected by join along boundaries bonds (JABs, flux tubes) to $k = 107$ 2-surface $X^2(H)$ corresponding to hadron. These flux tubes which for $k = 113$ have size much larger than hadron can be regarded as "field bodies" of quarks which themselves have sub-hadronic size. Color flux tubes between quarks are replaced with pairs of flux tubes from $X^2(q_1)) \rightarrow X^2(H) \rightarrow X^2(q_2)$ mediating color Coulombic and magnetic interactions between quarks. In contrast to the standard model, mesons are characterized by two flux tubes rather than only one flux tube. Certainly this model gives nice predictions for hadron masses and even the large color Coulomb contribution to baryon masses can be deduced from $\rho-\pi$ mass splitting in a good approximation.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L2].

2 Quark Masses

The prediction or quark masses is more difficult due the facts that the deduction of even the p-adic length scale determining the masses of these quarks is a non-trivial task, and the original identification was indeed wrong. Second difficulty is related to the topological mixing of quarks. The new scenario leads to a unique identification of masses with top quark mass as an empirical input and the thermodynamical model of topological mixing as a new theoretical input. Also CKM matrix is predicted highly uniquely.

2.1 Basic Mass Formulas

By the earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as a unit, color conformal weight

$h_c \equiv m^2_{CP_2}$ is integer for $p\ mod\ = \pm 1$ for U type quark belonging to $(p+1,p)$ type representation and obeying $h_c(U) = (p^2+3p+2)/3$ and for $p\ mod\ 3 = 1$ for D type quark belonging $(p, p+2)$ type representation and obeying $h_c(D) = (p^2+4p+4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

In the recent case the minimal $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This requires super-symplectic operators O with a net conformal weight $h_{sc} = -3$ to compensate the anomalous color just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc.

Consider now the mass squared values for quarks. For $h(D) = 0$ and $h(U) = -1$ and using $m_0^2/3$ as a unit the expression for the thermal contribution to the mass squared of quark is given by the formula

$$\begin{aligned} M^2 &= (s+X)\frac{m_0^2}{p} \ , \\ s(U) &= 5 \ , \ s(D) = 8 \ , \\ X &\equiv \frac{(3Yp)_R}{3} \ , \end{aligned} \tag{2.1}$$

where the second order contribution Y corresponds to renormalization effects coming and depending on the isospin of the quark.

With the above described assumptions one has the following mass formula for quarks

$$\begin{array}{lll} M^2(q) = A(q)\frac{m_0^2}{p(q)} \ , & & \\ A(u) = 5 + X_U(p(u) \ , & A(c) = 14 + X_U(p(c)) \ , & A(t) = 65 + X_U(p(t)) \ , \\ A(d) = 8 + X_D(p(d)) \ , & A(s) = 17 + X_D(p(s)) \ , & A(b) = 68 + X_D(p(b)) \ . \end{array} \tag{2.2}$$

p-Adic length scale hypothesis allows to identify the p-adic primes labeling quarks whereas topological mixing of U and D quarks allows to deduce topological mixing matrices U and D and CKM matrix V and precise values of the masses apart from effects like color magnetic spin orbit splitting, color Coulomb energy, etc..

Integers n_{q_i} satisfying $\sum_i n(U_i) = \sum_i n(D_i) = 69$ characterize the masses of the quarks and also the topological mixing to high degree. The reason that modular contributions remain integers is that in the p-adic context non-trivial rationals would give CP_2 mass scale for the real counterpart of the mass squared. In the absence of mixing the values of integers are $n_d = n_u = 0$, $n_s = n_c = 9$, $n_b = n_t = 60$.

The fact that CKM matrix V expressible as a product $V = U^\dagger D$ of topological mixing matrices is near to a direct sum of 2×2 unit matrix and 1×1 unit matrix motivates the approximation $n_b \simeq n_t$.

The model for topological mixing matrices and CKM matrix predicts U and D matrices highly uniquely and allows to understand quark and hadron masses in surprisingly detailed level.

The large masses of top quark and of $t\bar{t}$ meson encourage to consider a scenario in which $n_t = n_b = n \le 60$ holds true.

1. $n_d = n_u = 60$ is not allowed by number theoretical conditions for U and D matrices and by the basic facts about CKM matrix but $n_t = n_b = 59$ allows almost maximal masses for b and t. This is not yet a complete hit. The unitarity of the mixing matrices and the construction of CKM matrix to be discussed in the next section forces the assignments

$$(n_d, n_s, n_b) = (5, 5, 59) \ , \quad (n_u, n_c, n_t) = (5, 6, 58) \ . \tag{2.3}$$

fixing completely the quark masses apart from a possible few per cent renormalization effects of hadronic mass scale in topological condensation which seem to be present and will be discussed later [1]. Note that top quark mass is still rather near to its maximal value.

2. The constraint that quark contribution to pion mass does not exceed pion mass implies the constraint $n(d) \leq 6$ and $n(u) \leq 6$ in accordance with the predictions of the model of topological mixing. It is important to notices that $u-d$ mass difference does not affect $\pi^+ - \pi^0$ mass difference and the quark contribution to $m(\pi)$ is predicted to be $\sqrt{(n_d + n_u + 13)/24} \times$ 136.9 MeV for the maximal value of CP_2 mass (second order p-adic contribution to electron mass squared vanishes).

2.2 The P-Adic Length Scales Associated With Quarks And Quark Masses

The identification of p-adic length scales associated with the quarks has turned to be a highly non-trivial problem. The reasons are that for light quarks it is difficult to deduce information about quark masses for hadron masses and that the unknown details of the topological mixing (unknown until the advent of the thermodynamical model) made possible several p-adic length scales for quarks. It has also become clear that the p-adic length scale can be different form free quark and bound quark and that bound quark p-adic scale can depend on hadron.

Two natural constraints have however emerged from the recent work.

1. Quark contribution to the hadron mass cannot be larger than color contribution and for quarks having $k_q \neq 107$ quark contribution to mass is added to color contribution to the mass. For quarks with same value of k conformal weight rather than mass is additive whereas for quarks with different value of k masses are additive. An important implication is that for diagonal mesons $M = q\overline{q}$ having $k(q) \neq 107$ the condition $m(M) \geq \sqrt{2}m_q$ must hold true. This gives strong constraints on quark masses.

2. The realization that scaled up variants of quarks explain elegantly the masses of light hadrons allows to understand large mass splittings of light hadrons without the introduction of strong isospin-isospin interaction.

The new model for quark masses is based on the following identifications of the p-adic length scales.

1. The nuclear p-adic length scale $L_e(k)$, $k = 113$, corresponds to the p-adic length scale determining the masses of u, d, and s quarks. Note that $k = 113$ corresponds to a so called Gaussian Mersenne. The interpretation is that quark massivation occurs at nuclear space-time sheet at which quarks feed their em fluxes. At $k = 107$ space-time sheet, where quarks feed their color gauge fluxes, the quark masses are vanishing in the first p-adic order. This could be due to the fact that the p-adic temperature is $T_p = 1/2$ at this space-time sheet so that the thermal contribution to the mass squared is negligible. This would reflect the fact that color interactions do not involve any counterpart of Higgs mechanism.

   p-Adic mass calculations turn out to work remarkably well for massive quarks. The reason could be that M_{107} hadron physics means that *all* quarks feed their color gauge fluxes to $k = 107$ space-time sheets so that color contribution to the masses becomes negligible for heavy quarks as compared to Super-Kac Moody and modular contributions corresponding to em gauge flux fed to $k > 107$ space-time sheets in case of heavy quarks. Note that Z^0 gauge flux is fed to space-time sheets at which neutrinos reside and screen the flux and their size corresponds to the neutrino mass scale. This picture might throw some light to the question of whether and how it might be possible to demonstrate the existence of M_{89} hadron physics.

   One might argue that $k = 107$ is not allowed as a condensation level in accordance with the idea that color and electro-weak gauge fluxes cannot be fed at the space-time space time sheet since the classical color and electro-weak fields are functionally independent. The

[1]As this was written I had not realized that there is also a Higgs contribution which tends to increase top quark mass

identification of η' meson as a bound state of scaled up $k = 107$ quarks is not however consistent with this idea unless one assumes that $k = 107$ space-time sheets in question are separate.

2. The requirement that the masses of diagonal pseudo-scalar mesons of type $M = q\overline{q}$ are larger but as near as possible to the quark contribution $\sqrt{2}m_q$ to the valence quark mass, fixes the p-adic primes $p \simeq 2^k$ associated with c, b quarks but not t since toponium does not exist. These values of k are "nominal" since k seems to be dynamical. c quark corresponds to the p-adic length scale $k(c) = 104 = 2^3 \times 13$. b quark corresponds to $k(b) = 103$ for $n(b) = 5$. Direct determination of p-adic scale from top quark mass gives $k(t) = 94 = 2 \times 47$ so that secondary p-adic length scale is in question.

3. Top quark is experimentally in a unique position since toponium does not exist and top quark mass is that of free top. The prediction for top quark mass (see **Table 1**) is 167.8 GeV for $Y_t = Y_e = 0$ (second order contributions to mass vanish) and 169.1 GeV for $Y_t = 1$ and $Y_e = 0$ (maximal possible mass for top). The experimental estimate for m_t remained for a long time somewhat higher than the prediction but the estimates have gradually reduced. The previous experimental average value was $m(t) = 169.1$ GeV with the allowed range being [164.7, 175.5] GeV [C4, C20]. The fine tuning $Y_e = 0, Y_t = 1$ giving 169.1 GeV is somewhat un-natural. The most recent value obtained by CDF and discussed in detail by Tommaso Dorigo [C21] is $m_t = 165.1 \pm 3.3 \pm 3.1$ GeV. This is value is consistent with the lower bound predicted by TGD for $Y_e = Y_t = 0$ and increase of Y_t increases the value of the predicted mass. Clearly, TGD passes the stringent test posed by top quark.

4. There are good reasons to believe that the p-adic mass scale of quark is different for free quark and bound state quark and that in case of bound quark it can also depend on hadron. This would explain the notions of valence (constituent) quark and current quark mass as masses of bound state quark and free quark and leads also to a TGD counterpart of Gell-Mann-Okubo mass formula.

1. Constituent quark masses

Constituent quark masses correspond to masses derived assuming that they are bound to hadrons. If the value of k is assumed to depend on hadron one obtains nice mass formula for light hadrons as will be found later. **Table 1** summarizes constituent quark masses labelled by k_q deduced from the masses of diagonal mesons.

q	d	u	s	c	b	t
n_q	4	5	6	6	59	58
s_q	12	10	14	11	67	63
$k(q)$	113	113	113	104	103	94
$m(q)/GeV$	.105	.092	.105	2.191	7.647	167.8

Table 1: Constituent quark masses predicted for diagonal mesons assuming $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$, maximal CP_2 mass scale($Y_e = 0$), and vanishing of second order contributions.

2. Current quark masses

Current quark masses would correspond to masses of free quarks which tend to be lower than valence quark masses. Hence k could be larger in the case of light quarks. The table of quark masses in Wikipedia [C4] gives the value ranges for current quark masses depicted in the **Table 2** together with TGD predictions for the spectrum of current quark masses.

Some comments are in order.

1. The long p-adic length associated with light quarks seem to be in conflict with the idea that quarks have sizes smaller than hadron size. The paradox disappears when one realized that

q	d	u	s
$m(q)_{exp}/MeV$	4-8	1.5-4	80-130
$k(q)$	(122, 121, 120)	(125, 124, 123, 122)	(114, 113, 112)
$m(q)/MeV$	(4.5, 6.6, 9.3)	(1.4, 2.0, 2.9, 4.1)	(74, 105, 149)
q	c	b	t
$m(q)_{exp}/MeV$	1150-1350	4100-4400	1691
$k(q)$	(106, 105)	(105, 104)	92
$m(q)/MeV$	(1045, 1477)	(3823, 5407)	167.8

Table 2: The experimental value ranges for current quark masses [C4] and TGD predictions for their values assuming $(n_d, n_s, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (5, 6, 58)$, $Y_e = 0$, and vanishing of second order contributions.

$k(q)$ characterizes the electromagnetic "field body" of quark having much larger size than hadron.

2. u and d current quarks correspond to a mass scale not much higher than that of electron and the ranges for mass estimates suggest that u could correspond to scales $k(u) \in (125, 124, 123, 122) = (5^3, 4 \times 31, 3 \times 41, 2 \times 61)$, whereas d would correspond to $k(d) \in (122, 121, 120) = (2 \times 61, 11^2, 3 \times 5 \times 8)$.

3. The TGD based model for nuclei based on the notion of nuclear string leads to the conclusion that exotic copies of $k = 113$ quarks having $k = 127$ are present in nuclei and are responsible for the color binding of nuclei [K6, L1] , [L1].

4. The predicted values for c and b masses are slightly too low for $(k(c), k(b)) = (106, 105) = (2 \times 53, 3 \times 5 \times 7)$. Second order Higgs contribution could increase the c mass into the range given in [C4] but not that of b.

One can talk about constituent and current quark masses simultaneously only if they correspond to dual descriptions. $M^8 - H$ duality [K3] has been indeed suggested to relate the old fashioned low energy description of hadrons in terms of $SO(4)$ symmetry (Skyrme model) and higher energy description of hadrons based on QCD. In QCD description the mass of say baryon would be dominated by the mass associated with super-symplectic quanta carrying color. In $SO(4)$ description constituent quarks would carry most of the hadron mass.

2.3 Are Scaled Up Variants Of Quarks Also There?

The following arguments suggest that p-adically scaled up variants of quarks might appear not only at very high energies but even in low energy hadron physics.

2.3.1 Aleph anomaly and scaled up copy of b quark

The prediction for the b quark mass is consistent with the explanation of the Aleph anomaly [C14] inspired by the finding that neutrinos seem to condense at several p-adic length scales [C15]. A reasonable estimate for $k(b)$ is $k(b) = 103$ from the estimates for its mass in range 4.3-4.7 GeV. It must be emphasized that this estimate is uncertain and even $k_b = 105$ giving two times smaller mass can be considered. If b quark condenses at $k(b) = 96$ level, the predicted mass is $m(b, 96) = 52.3$ GeV for $n_b = 59$ for the maximal CP_2 mass consistent with η' mass. If the mass of the particle candidate is defined experimentally as one half of the mass of resonance, b quark mass is actually by a factor $\sqrt{2}$ higher and scaled up b corresponds to $k(b) = 96 = 2^5 \times 3$. The prediction is consistent with the estimate 55 GeV for the mass of the Aleph particle and gives additional support for the model of topological mixing. Also the decay characteristics of Aleph particle are consistent with the interpretation as a scaled up b quark.

2.3.2 Scaled variants of top quark

To begin with notice that the recent estimate for top quark mass is around 172 GeV and in p-adic mass calculatiosn top quark mass corresponds to $k(t) = 94$. Tony Smith has emphasized the fact that the distribution for the mass of the top quark candidate has a clear structure suggesting the existence of several states, which he interprets as excited states of top quark [C23]. According to the figures **??** and **??** representing published FermiLab data, this structure is indeed clearly visible.

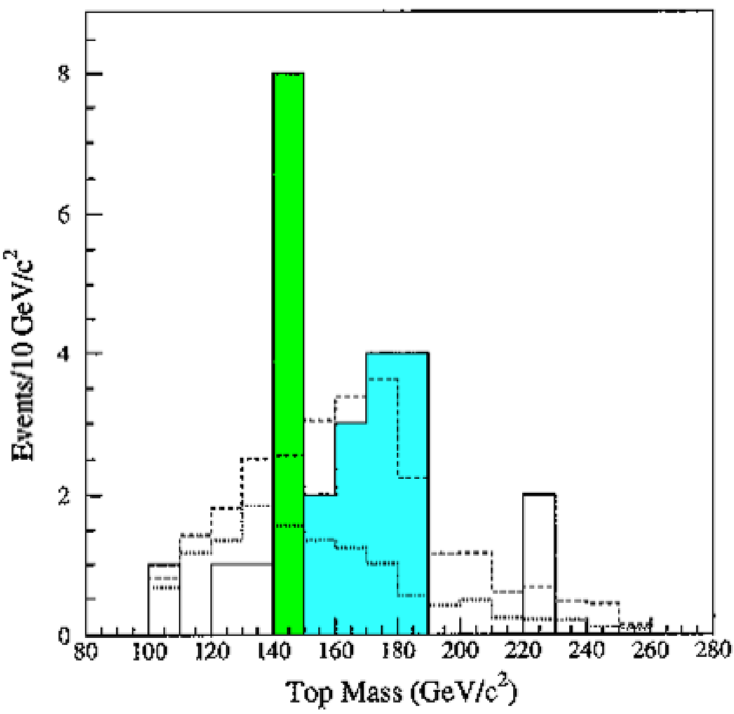

Figure 1: Fermilab semileptonic histogram for the distribution of the mass of top quark candidate (FERMILAB-PUB-94/097-E).

There is evidence for a sharp peak in the mass distribution of the top quark in 140-150 GeV range (**Fig. 1**). There is also a peak slightly below 120 GeV, which could correspond to a p-adically scaled down variant t quark with $k = 93$ having mass 121.6 GeV for $(Y_e = 0, Y_t = 1)$. There is also a small peak also around 265 GeV which could relate to $m(t(95)) = 243.2$ GeV. There top could appear at least for the p-adic scales $k = 93, 94, 95$ as also u and d quarks seem to appear as current quarks.

2.3.3 Scaled up variants of d, s, u, c in top quark mass scale

The fact that all neutrinos seem to appear as scaled up versions in several scales, encourages to look whether also u, d, s, and c could appear as scaled up variants transforming to the more stable variants by a stepwise increase of the size scale involving the emission of electro-weak gauge bosons. In the following the scenario in which t and b quarks mix minimally is considered.

1. For $k = 92$, the masses would be $m(q, 92)$ =134, 140, 152, 152 GeV in the order q= u, c, d, s so that all these quarks might appear in the critical region where the top quark mass has been wandering.

2. For $k = 91$ copies would have masses $m(q, 91)$ =189, 198, 256, 256 GeV in the order q= u, c, d, s. The masses of u and c are somewhat above the value of latest estimate 170 GeV for top quark mass [C20].

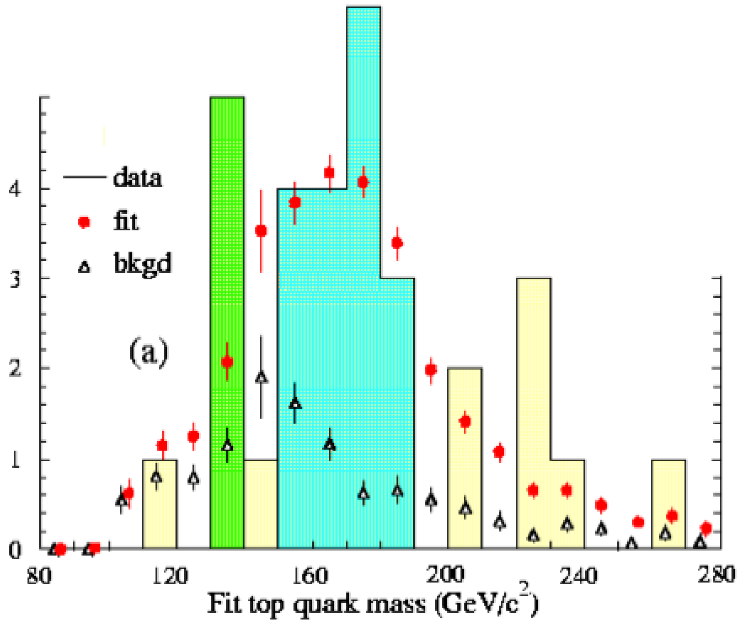

Figure 2: Fermilab D0 semileptonic histogram for the distribution of the mass of top quark candidate (hep-ex/9703008, April 26, 1994)

q	$m(92)/GeV$	$m(91)/GeV$	$m(90)/GeV$
u	134	189	267
d	152	216	304
c	140	198	280
s	152	216	304

Table 3: The masses of $k = 92, 91$ and $k = 90$ scaled up variants of u, d, c, s quarks assuming same integers n_{q_i} as for ordinary quarks in the scenario $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$ and maximal CP_2 mass consistent with the η' mass.

Note that it is possible to distinguish between scaled up quarks of M_{107} hadron physics and the quarks of M_{89} hadron physics since the unique signature of M_{89} hadron physics would be the increase of the scale of color Coulombic and magnetic energies by a factor of 512. As will be found, this allows to estimate the masses of corresponding mesons and baryons by a direct scaling. For instance, M_{89} pion and nucleon would have masses 71.7 GeV and 481 GeV.

It must be added that the detailed identifications are sensitive to the exact value of the CP_2 mass scale. The possibility of at most 2.5 per cent downward scaling of masses occurs is allowed by the recent value range for top quark mass.

2.3.4 Fractally scaled up copies of light quarks and low mass hadrons?

One can of course ask, whether the fractally scaled up quarks could appear also in low lying hadrons. The arguments to be developed in detail later suggest that u, d, and s quark masses could be dynamical in the sense that several fractally scaled up copies can appear in low mass hadrons and explain the mass differences between hadrons.

In this picture the mass splittings of low lying hadrons with different flavors would result from fractally scaled up excitations of s and also u and d quarks in case of mesons. This notion would also throw light into the paradoxical presence of two kinds of quark masses: constituent quark masses and current quark masses having much smaller values than constituent quarks masses. That color spin-spin splittings are of same order of magnitude for all mesons supports the view that color gauge fluxes are fed to $k = 107$ space-time sheet.

The alert reader has probably already asked whether also proton mass could be understood in terms of scaled up copies of u and d quarks. This does not seem to be the case, and an argument predicting with 23 per cent error proton mass scale from $\rho - \pi$ and $\Delta - N$ color magnetic splittings emerges.

To sum up, it seems quite possible that the scaled up quarks predicted by TGD have been observed for decade ago in FermiLab about that the prevailing dogmas has led to their neglect as statistical fluctuations. Even more, scaled up variants of s quarks might have been in front of our eyes for half century! Phenomenon is an existing phenomenon only if it is an understood phenomenon.

2.3.5 The mystery of two Ω_b baryons

Tommaso Dorigo has three interesting postings [C22] about the discovery of Ω_b baryon containing two strange quarks and one bottom quark. Ω_b has been discovered -even twice. This is not a problem. The problem is that the masses of these Ω_bs differ quite too much. D0 collaboration discovered Ω_b with a significance of 5.4 sigma and a mass of 6165 $\pm$ 16.4 MeV [C9]. Later CDF collaboration announced the discovery of the same particle with a significance of 5.5 sigma and a mass of 6054.4 $\pm$ 6.9 MeV. Both D0 and CDF agree that the particle is there at better than 5 sigma significance and also that the other collaboration is wrong. They cant both be right Or could they? In some other Universe that that of standard model and all its standard generalizations, maybe in some less theoretically respected Universe, say TGD Universe?

The mass difference between the two Ω_b candidates is 111 MeV, which represents the mass scale of strange quark. TDG inspired model for quark masses relies on p-adic thermodynamics and predicts that quarks can appear in several p-adic mass scales forming a hierarchy of half octaves - in other words mass scales comes as powers of square root of two. This property is absolutely essential for the TGD based model for masses of even low lying baryons and mesons where strange quarks indeed appear with several different p-adic mass scales. It also explains the large difference of the mass scales assigned to current quarks and constituent quarks. Light variants of quarks appear also in nuclear string model where nucleons are connected by color bonds containing light quark and antiquark at their ends.

Ω_b contains two strange quarks and the mass difference between the two candidates is of order of mass of strange quark. Could it be that both Ω_b s are real and the discrepancy provides additional support for p-adic length scale hypothesis? The prediction of p-adic mass calculations for the mass of s quark is 105 MeV (see **Table 1**) so that the mass difference can be understood if the second s-quark in Ω_b has mass which is twice the "standard" value. Therefore the strange finding about Ω_b could give additional support for quantum TGD. Before buying a bottle of champaigne, one should however understand why D0 and CDF collaborations only one Ω_b instead of both of them.

3 Topological Mixing Of Quarks

The requirement that hadronic mass spectrum is physical requires mixing of U and D type boundary topologies. In this section quark masses and the mixing of the boundary topologies are considered on the general level and CKM matrix is derived using the existing empirical information plus the constraints on the quark masses to be derived from the hadronic mass spectrum in the later sections.

3.1 Mixing Of The Boundary Topologies

In TGD the different mixings of the boundary topologies for U and D type quarks provide the fundamental mechanism for CKM mixing and also CP breaking. In the determination of CKM matrix one can use following conditions.

1. Mass squared expectation values in order $O(p)$ for the topologically mixed states must be integers and the study of the hadron mass spectrum leads to very stringent conditions on

the values of these integers. Physical values for these integers imply essentially correct value for Cabibbo angle provided U and D matrices differ only slightly from the mixing matrices mixing only the two lowest generations.

2. The matrices U and D describing the mixing of U and D type boundary topologies are unitary in the p-adic sense. The requirement that the moduli squared of the matrix elements are rational numbers, is very attractive since it suggests equivalence of p-adic and real probability concepts and therefore could solve some conceptual problems related to the transition from the p-adic to real regime. It must be however immediately added that rationality assumption for the probabilities defined by S-matrix turns out to be non-physical. It turns out that the mixing scenario reproducing a physical CKM matrix is consistent with the rationality of the moduli squared of the matrix elements of U and D matrices but not with the rationality of the matrix elements themselves. The phase angles appearing in U and D matrix can be rational and in this case they correspond to Pythagorean triangles. In principle the rationality of the CKM matrix is possible.

3. The requirements that Cabibbo angle has correct value and that the elements $V(t,d)$ and $V(u,b)$ of the CKM matrix have small values not larger than 10^{-2} fixes the integers n_i characterizing quark masses to a very high degree and in a good approximation one can estimate the angle parameters analytically. remains open at this stage. The requirement of a realistic CKM matrix leads to a scenario for the values of n_i, which seems to be essentially unique.

The mass squared constraints give for the D matrix the following conditions

$$\begin{aligned} 9|D_{12}|^2+60|D_{13}|^2 &= n_1(D)\equiv n_d \ , \\ 9|D_{22}|^2+60|D_{23}|^2 &= n_2(D)\equiv n_s \ , \\ 9|D_{32}|^2+60|D_{33}|^2 &= n_3(D)\equiv n_b = 69-n_2(D)-n_1(D) \ . \end{aligned} \tag{3.1}$$

The third condition is not independent since the sum of the conditions is identically true by unitarity.

For U matrix one has similar conditions:

$$\begin{aligned} 9|U_{12}|^2+60|U_{13}|^2 &= n_1(U)\equiv n_u \ , \\ 9|U_{22}|^2+60|U_{23}|^2 &= n_2(U)\equiv n_c \ , \\ 9|U_{32}|^2+60|U_{33}|^2 &= n_3(U)\equiv n_t = 69-n_2(U)-n_1(U) \ . \end{aligned} \tag{3.2}$$

The integers n_d, n_s and n_u, n_c characterize the masses of the physical quarks and the task is to derive the values of these integers by studying the spectrum of the hadronic masses. The second task is to find unitary mixing matrices satisfying these conditions.

The general form of U and D matrices can be deduced from the standard parameterization of the CKM matrix given by

$$V = \left[\begin{array}{ccc} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3-s_2s_3exp(i\delta_{CP}) & c_1c_2s_3+s_2c_3exp(i\delta_{CP}) \\ -s_1s_2 & c_1s_2c_3+c_2s_3exp(i\delta_{CP}) & c_1s_2s_3-c_2c_3exp(i\delta_{CP}) \end{array}\right] \tag{3.3}$$

This form of the CKM matrix is always possible to achieve by multiplying each U and D type quark fields with a suitable phase factor: this induces a multiplication U and D from left by a diagonal phase factor matrix inducing the multiplication of the columns of U and D by phase factors:

$$\begin{aligned} &U\rightarrow U\times d(\phi_1,\phi_2,\phi_3) \ , \\ &D\rightarrow D\times d(\chi_1,\chi_2,\chi_3) \ , \\ &d(\phi_1,\phi_2,\phi_3)\equiv diag(exp(i\phi_1),exp(i\phi_2),exp(i\phi_3)) \ . \end{aligned}$$

The multiplication of the columns by the phase factors affects CKM matrix defined as

$$V = U^\dagger D \rightarrow d(-\phi_1, -\phi_2, -\phi_3) V d(\chi_1, \chi_2), \chi_3) \ . \tag{3.4}$$

By a suitable choice of the phases, the first row and column of V can be made real. The multiplication of the rows of U and D from the left by the same phase factors does not affect the elements of V. One can always choose D to be of the same general form as the CKM matrix but must allow U to have nontrivial phase overall factors on the second and third row so that the most general U matrix is parameterized by six parameters.

Mass squared conditions give two independent conditions on the values of the moduli of the matrix elements of U and D. This eliminates two coordinates so that the most general D matrix can be chosen to depend on 2 parameters, which can be taken to be $r_{11} \equiv |D_{11}|$ and $r_{21} \equiv |D_{21}|$. U matrix contains also the overall phase angles associated with the second and third row and hence depends on four parameters altogether.

3.2 The Constraints On U And D Matrices From Quark Masses

The new view about quark masses allows a surprisingly simple model for U and D matrices predicting in the lowest order approximation that the probabilities defined by these matrices are identical and that the integers characterizing the masses of U and D type quarks are identical.

3.2.1 The constraints on $|U|$ and $|D|$ matrices from quark masses

The understanding of quark masses pose strong constraints on U and D matrices. The constraints are identical in the approximation that V-matrix is identity matrix and read in the case of D-matrix as

$$\begin{aligned} n_d &= 13 = P^D_{12} \times 9 + P^D_{13} \times 60 \ , \\ n_s &= 31 = P^D_{22} \times 9 + P^D_{23} \times 60 \ . \end{aligned} \tag{3.5}$$

The conditions for b quark give nothing new. The extreme cases when only $g = 1$ or $g = 2$ contributes to n_q gives the bounds

$$\begin{aligned} \frac{15}{36} &\leq P^D_{13} \leq \frac{15}{60} , \\ \frac{22}{60} &\leq P^D_{23} \leq \frac{31}{60} \ . \end{aligned} \tag{3.6}$$

3.2.2 Unitarity conditions

The condition $D = VU$ and the fact that V is in not too far from unit matrix being in a good approximation a direct sum of 2×2 matrix and 1×1 identity matrix, imply together that U an D cannot differ much from each other. At least the probabilities defined by the moduli squared of matrix elements are near to each other.

1. Instead of trying numerically to solve U and D matrices by a direct numerical search, it is more appropriate to try to deduce estimates for the probabilities $P^U_{ij} = |U_{ij}|^2$ and $P^D_{ij} = |D_{ij}|^2$ determined by the moduli squared of the matrix elements and satisfying the unitarity conditions $\sum_j P^X_{ij} = 1$ and $\sum_i P^X_{ij} = 1$.

2. The formula $D = UV$ using the fact that V_{i3} is small for $i = 1, 2$ implies $|D_{i3}| \simeq |U_{i3}|$. By probability conservation also the condition $|D_{33}| \simeq |U_{33}|$ must hold true so that the third columns of U and D are same in a reasonable approximation.

1. Parametrization of $|U|$ and $|D|$ matrices

The following parameterization is natural for the matrices P_{ij}^X.

$$\begin{aligned}
P_{12}^D &= \tfrac{k_D}{9} \ , \qquad & P_{13}^D &= \tfrac{n_d - k_D}{9} \ , \\
P_{22}^D &= \tfrac{l_D}{9} \ , \qquad & P_{23}^D &= \tfrac{n_s - l_D}{60} \ , \\
P_{32}^D &= \tfrac{9-k_D-l_D}{9} \ , \qquad & P_{33}^D &= \tfrac{60-n_s-n_d-k_D-l_D}{60} \ .
\end{aligned} \tag{3.7}$$

A similar parameterization holds true for P_{ij}^U but with $n_d = n_u$ and $n_s = n_c$ but possibly different values of k_U and l_U. Since $l_D \ll n_s$ is expected to hold true, P_{23}^D is in a good approximation equal to $P_{23}^D = n_s/60 = 31/60$. Same applies to P_{23}^U.

$k_X = 2$ (k_X need not be an integer) gives a good first estimate for mixing probabilities of u and d quark. Thus only the parameter l_X remains free if $k_D = 2$ is accepted.

The approximation $P_{i3}^U = P_{i3}^D$ motivated by the near unit matrix property of V, gives the parameterization

$$P_{12}^D = P_{12}^U = \frac{k}{9} \ , \ P_{13}^D = P_{13}^U \frac{n_d - k}{60} \ . \tag{3.8}$$

2. Constraints from CKM matrix in $|U| = |D|$ approximation

The condition $D_{12} = (UV)_{12}$ when fed to the condition

$$P_{12}^U = P_{12}^D \tag{3.9}$$

using the approximation $k_D = k_U = k$ $l_D = l_U = l$ gives

$$|U_{i2}|^2 - |U_{i1}V_{12} + U_{i2}V_{22} + U_{i3}V_{32}|^2 = 0 \ . \tag{3.10}$$

$i = 1, 2, 3$ In the approximation that the small V_{32} term does not contribute, this gives

$$|U_{i1}V_{12} + U_{i2}V_{22})|^2 = |U_{i2}|^2 \ . \tag{3.11}$$

By dividing with $|U_{i1}|^2|V_{22}|^2$ and using the approximation $|V_{22}|^2 = 1$ this gives

$$\begin{aligned}
v_i^2 &+ 2u_iv_i \times cos(\Psi_i) = 0 \ , \\
\Psi_i &= arg(V_{i2}) - arg(V_{32}) + arg(U_{i1}) - arg(U_{i2}) \ . \\
u_i &= |\frac{U_{i2}}{U_{i1}}| \ , \ v_i = |\frac{V_{i2}}{V_{22}}| \ .
\end{aligned} \tag{3.12}$$

This gives

$$\begin{aligned}
cos(\Psi_i) &= -\frac{v_i}{2u_i} = -\frac{v_i}{2}\sqrt{\frac{9x_i}{k_i}} \ , \\
x_i &= P_{ii}^D = 1 - \frac{k_i}{9} - \frac{n(i) - k(i)}{60} \ , \\
k(1) &= k \ , \ k(2) = l \ , \ n(1) = n_d, \ n(2) \equiv n_s \ .
\end{aligned} \tag{3.13}$$

The condition $|cos(\Psi)| \leq 1$ is trivially satisfied. For $n_d = 13$ and $k = 2$ the condition gives $x = .59$ and $cos(\Psi_1) = .185$. $k = 1.45$ gives $x = .65$ and $cos(\Psi) = .226$, which is rather near to V_{12}.

$\lvert V_{13}\rvert \equiv \lvert V_{ub}\rvert = (0.087 \pm 0.075)V_{cb}$: $0.42 \cdot 10^{-3} < \lvert V_{ub}\rvert < 6.98 \cdot 10^{-3}$
$\lvert V_{23}\rvert \equiv \lvert V_{cb}\rvert = (41.2 \pm 4.5) \cdot 10^{-3}$
$\lvert V_{31}\rvert \equiv \lvert V_{td}\rvert = (9.6 \pm 0.9) \cdot 10^{-3}$
$\lvert V_{32}\rvert \equiv \lvert V_{ts}\rvert = (40.2 \pm 4.4) \cdot 10^{-3}$
$s_{Cab} = 0.226 \pm 0.002$

Table 4: The experimental constraints on the absolute values of the CKM matrix elements.

3.3 Constraints From CKM Matrix

Besides the constraints from hadron masses, there are constraints from CKM matrix $V = U^\dagger D$ on U and D matrices.

1. The fact that CKM matrix is near unit matrix implies that U and D matrix are near to each other and the assumption $n(U_i) = n(D_i)$ predicting quark masses correctly is consistent with this.

2. Cabibbo angle allows to derive the estimate for the difference $|U_{11}| - |D_{11}|$. Together with other conditions this difference fixes the scenario essentially uniquely.

3. The requirement that CP breaking invariant J has a correct order of magnitude gives a very strong constraint on D matrix. The smallness of J implies that V is nearly orthogonal matrix and same assumption can be made about U and D matrices.

4. The requirement that the moduli the first row (column) of CKM matrix are predicted correctly makes it possible to deduce for given D (U) U (D) matrix essentially uniquely. Unitarity requirement poses very strong additional constraints. It must be emphasized that the constraints from the moduli of the CKM alone are sufficient to determine U and D matrices and hence also quark masses and hadron masses to very high degree.

1. Bounds on CKM matrix elements

The most recent experimental information [C5] concerning CKM matrix elements is summarized in **Table 4**

$$\begin{aligned} s_1 &= .226 \pm .002 \ , \\ s_1 s_2 &= V_{31} = (9.6 \pm .9) \cdot 10^{-3} \ , \\ s_1 s_3 &= V_{13} = (.087 \pm .075) \cdot V_{23} \ , \\ V_{23} &= (40.2 \pm 4.4) \cdot 10^{-3} \ . \end{aligned} \tag{3.14}$$

The remaining parameter is $sin(\delta)$ or equivalently the CP breaking parameter J:

$$J = Im(V_{11}V_{22}\overline{V}_{12}\overline{V}_{21}) = c_1c_2c_3s_2s_3s_1^2 sin(\delta) \ , \tag{3.15}$$

where the upper bound is for $sin(\delta) = 1$ and the previous average values of the parameters s_i, c_i (note that the poor knowledge of s_3 affects on the upper bound for J considerably). Unitary triangle [C8] gives for the CP breaking parameter the limits

$$1.0 \times 10^{-4} \leq J \leq 1.7 \times 10^{-4} \ . \tag{3.16}$$

2. CP breaking in $M - \overline{M}$ systems as a source of information about CP breaking phase

Information about the value of $sin(\delta)$ as well as on the range of possible top quark masses comes from CP breaking in $K - \bar{K}$ and $B - \bar{B}$ systems.

The observables in $K_L \rightarrow 2\pi$ system [C17]

$$
\begin{aligned}
\eta_{+-} &= \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} = \epsilon + \frac{\epsilon'}{1+\omega/\sqrt{2}} \ , \\
\eta_{00} &= \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} = \epsilon - 2\frac{\epsilon'}{1-\sqrt{2}\omega} \ , \\
\omega &\sim \frac{1}{20} \ , \\
\epsilon &= (2.27 \pm .02)\cdot 10^{-3} \cdot exp(i43.7^o) \ , \\
|\frac{\epsilon'}{\epsilon}| &= (3.3 \pm 1.1)\cdot 10^{-3} \ .
\end{aligned} \tag{3.17}
$$

The phases of ϵ and ϵ' are in good approximation identical. CP breaking in $K-\bar{K}$ mass matrix comes from the CP breaking imaginary part of $\bar{s}d \rightarrow s\bar{d}$ amplitude M_{12} (via the decay to intermediate W^+W^- pair) whereas $K^0\bar{K}^0$ mass difference Δm_K comes from the real part of this amplitude: the calculation of the real part cannot be done reliably for kaon since perturbative QCD does not work in the energy region in question. On can however relate the real part to the known mass difference between K_L and K_S: $2Re(M_{12}) = \Delta m_K$.

Using the results of [C17]) one can express ϵ and ϵ'/ϵ in the following numerical form

$$
\begin{aligned}
|\epsilon| &= \frac{1}{\sqrt{2}}\frac{Im(M_{12}^{sd})}{\Delta m_K} - .05\cdot|\frac{\epsilon'}{\epsilon}| = 2J(22.2B_K\cdot X(m_t) - .28B_K') \ , \\
|\frac{\epsilon'}{\epsilon}| &= C\cdot J\cdot B_K' \ , \\
X(m_t) &= \frac{H(m_t)}{H(m_t = 60\ GeV)} \ , \\
H(m_t) &= -\eta_1 F(x_c) + \eta_2 F(x_t)K + \eta_3 G(x_c, x_t) \ , \\
x_q &= \frac{m(q)^2}{m_W^2} \ , \\
K &= s_2^2 + s_2 s_3 cos(\delta) \ .
\end{aligned} \tag{3.13}
$$

Here the values of QCD parameters η_i depend on top mass slightly. B_K' and B_K are strong interaction matrix elements and vary between 1/3 and 1. The functions F and G [C17] are given by

$$
\begin{aligned}
F(x) &= x\left[\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2}\right] + \frac{3}{2}(\frac{x}{x-1})^3 log(x) \ , \\
G(x,y) &= xy\left[\frac{1}{x-y}\left[\frac{1}{4} + \frac{3}{2}\frac{1}{1-x} - \frac{3}{4}\frac{1}{(1-x)^2}\right] log(x) + (y \rightarrow x) - \frac{3}{4}\frac{1}{(1-x)(1-y)}\right] \ .
\end{aligned} \tag{3.12}
$$

One can solve parameter B_K' by requiring that the value of ϵ'/ϵ corresponds to the experimental mean value:

$$
B_K' = \frac{1}{C\times J}\frac{\epsilon'}{\epsilon} \ . \tag{3.13}
$$

The most recent measurements by KTeV collaboration in Fermi Lab [C1] give for the ratio $|\epsilon'/\epsilon|$ the value $|\epsilon'/\epsilon| = (28 \pm 1)\times 10^{-4}$. The proposed standard model explanation for the large value of B_K' is that s-quark has running mass about $m_s(m_c) \simeq .1$ GeV at m_c [C16]. The explanation is marginally consistent with the TGD prediction $m(s) = 127$ MeV for the mass of s quark. Also the effects caused by the predicted higher gluon generations having masses around 33 GeV can increase the value of ϵ'/ϵ by a factor 3 in the lowest approximation since the corrections involve sum over three different one-gluon loop diagrams with gluon mass small respect to intermediate boson mass scale [K4].

A second source of information comes from $B-\bar{B}$ mass difference. At the energies in question perturbative QCD is expected to be applicable for the calculation of the mass difference and mass difference is predicted correctly if the mass of the top quark is essentially the mass of the observed top candidate [C6].

3. U and D matrices could be nearly orthogonal matrices

The smallness of the CP breaking phase angle δ_{CP} means that V is very near to an orthogonal matrix. This raises the hope that in a suitable gauge also U and D are nearly orthogonal matrices and would be thus almost determined by single angle parameter θ_X, $X=U,D$. Cabibbo angle $s_c = sin(\theta_c) = .226$ which is not too far from $sin^2(\theta_W)\simeq .23$ and appears in V matrix rotating the rows of U to those of D. In very vague sense this angle would characterize between the difference of angle parameters characterizing U and D matrices. If U is orthogonal matrix then the decomposition

$$V = V_1V_2 = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1c_2 & c_1c_2 & s_2exp(i\delta_{CP}) \\ -s_1s_2 & c_1s_2 & -c_2exp(i\delta_{CP}) \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{bmatrix} \tag{3.14}$$

suggests that CP breaking can be visualized as a process in which first s and b quarks are slightly mixed to s' and b' by V_2 ($s_3 \simeq 1.4\times 10^{-2}$) after which V_1 induces a slightly CP-breaking mixing of d and s' with b' ($s_2\simeq .04$).

4. How the large mixing between u and c results

The prediction that u quark spends roughly 1/3 of time in $g=0$ state looks bizarre and it is desirable to understand this from basic principles. The basic observations are following.

1. V matrix is in good approximation direct sum of 2×2 matrix inducing relatively large rotation with $sin(\theta_c)\simeq .23$ and unit matrix. In particular, V_{i3} are very small for $i=1,2$. Using the formula $D=UV$ one finds that $|U_{i3}| = |D_{i3}|$ in a good approximation for $i=1,2$ and by unitarity also for $I=3$. Thus the third columns of U and D are identical in a good approximation.

2. Assume that also U_{i3} and D_{i3} are small for $i=1,2$. A stronger assumption is that even the contribution of D_{13} and U_{13} are so small that they do not affect u and d masses. This implies

$$\begin{aligned} n_d &= 9|D_{12}|^2 + 60|D_{13}|^2 \simeq 9|D_{12}|^2 \ , \\ n_u &\simeq 9|U_{12}|^2 \ . \end{aligned} \tag{3.14}$$

   Unitarity implies in this approximation

$$\begin{aligned} |U_{11}|^2 \leq 1-\frac{n_u}{9} = \frac{1}{3} \ , \\ |D_{11}|^2 \leq 1-\frac{n_d}{9} = \frac{5}{9} \ . \end{aligned} \tag{3.14}$$

3. It might be that there are also solutions for which mixing of u resp. d quark is mostly with t resp. b quarks but numerical experimentation does not favor this idea since CP breaking becomes extremely small. Since mixing presumably involves topology change, it seems obvious that topological mixing involving a creation or annihilation of two handles is improbable.

4 Construction Of U, D, And CKM Matrices

In this section it will be found that various mathematical and experimental constraints on U and D matrices determine them essentially uniquely.

4.1 The Constraints From CKM Matrix And Number Theoretical Conditions

The requirement that U, D and V allow an algebraic continuation to finite-dimensional extensions of various p-adic number fields provides a very strong additional constraints. The mathematical problem is to understand how many unitary V matrices acting on U as $U \rightarrow D = UV$ respect the number theoretic constraints plus the constraints $n_u = n_d + 2$ and $n_c = n_d - 2$.

It is instructive to what happens in much simpler 2-dimensional case. In this case the conditions boil down to the conditions on $n(i)$ imply $|U| = |D|$ and this condition is equivalent with (say) the condition $|U_{11}| = D_{11}$. U and D can be parameterized as

$$U = \left(\begin{array}{cc} cos(\theta)exp(i(\psi) & sin(\theta)exp(i\phi) \\ -sin(\theta)exp(-i\phi) & cos(\theta)exp(-i\psi) \end{array} \right) \ .$$

If $cos(\theta)^2$ and $sin(\theta)^2$ are rational numbers, $exp(i\theta)$ is associated with a Gaussian integer. A more general requirement is that $exp(i\theta)$ belongs to a finite-dimensional extension of rational numbers and thus corresponds to a products of a phase associated with Gaussian integer and a phase in a finite-dimensional algebraic extension of rational numbers.

Eliminating the trivial multiplicative phases gives a set of matrices U identifiable as a double coset space $X^2 = SU(2)/U(1)_R \times U(1)_L$. The value of $cos(\theta) = |U_{11}|$ serving as a coordinate for X^2 is respected by the right multiplication with V. Eliminating trivial $U(1)_R$ phase multiplication, the space of V: s reduces to $S^2 = SU(2)/U(1)_R$. The condition that $cos(\theta)$ is not changed leaves one parameter set of allowed matrices V.

The translation of these results to 3-dimensional case is rather straightforward. In the 3-dimensional case the probabilities P_{i2}, P_{i3}, $i = 1,2$ characterize a general matrix $|U|$, and V can affect these probabilities subject to constraints on $n(I)$. When trivial phases affecting the probabilities are eliminated, the matrices U correspond naturally to points of the 4-dimensional double coset space $X^4 = SU(3)/(U(1) \times U(1))_R \times U(1) \times U(1))_L$ having dimension $D = 4$.

The two constraints on the probabilities mean that allowed solutions for given values of $n(I)$ define a 2-dimensional surface X^2 in X^4. The allowed unitary transformations V must be such that they move U along this surface. Certainly they exist since X^2 can be regarded as a local section in $SU(3) \rightarrow X^2$ bundle obtained as a restriction of $SU(3) \rightarrow X^4$ bundle. The action of V on rows of U is ordinary unitary transformation plus a 2-dimensional unitary transformation preserving the Hermitian degenerate lengths $L_i = 9|U_{i2}|^2 + 60|U_{i3}|^2 = n_i$ defining the sub-bundle $SU(3) \rightarrow X^2$. Note for $L_1 = 0$ ($L_2 = 0$) the situation becomes 2-dimensional and solutions correspond to points in S^2. Thus these points seem to represent a conical singularity of X^2.

The 2-dimensionality of the solution space means that two moduli (probabilities) of any row or column of U or D matrix characterize the matrix apart from the non-uniqueness due to the gauge choice allowing $U(1)_L \times U(1)_R$ transformation of U. Of course, discrete sign degeneracy might be present.

A highly non-trivial problem is whether the set X^2 contains rational points and what is the number of these points. For instance, Fermat's theorem says that no rational solutions to the equation $x^n + y^n - z^n = 0$ exist for $n > 2$. The fact that the degenerate situation allows infinite number of rational solutions suggest that they exist also in the general case. Note also that the additional conditions are second order polynomial equations with rational coefficients so that $SU(3, Q)$ should contain non-trivial solutions to the equations.

It is possible to write $|U|$ in a form containing minimal number of square roots:

$$\begin{array}{lll} |U_{11}| = \sqrt{n_u}\frac{p_1}{N_1} \ , & |U_{12}| = \sqrt{\frac{n_u}{9}}\frac{r_1}{N_1} \ , & |U_{13}| = \sqrt{\frac{n_u}{60}}\frac{s_1}{N_1} \ , \\ |U_{21}| = \sqrt{n_c}\frac{p_2}{N_2} \ , & |U_{22}| = \sqrt{\frac{n_c}{9}}\frac{r_2}{N_2} \ , & |U_{23}| = \sqrt{\frac{n_c}{60}}\frac{s_2}{N_2} \ , \\ |U_{31}| = \sqrt{n_t}\frac{p_3}{N_3} \ , & |U_{32}| = \sqrt{\frac{n_t}{9}}\frac{r_3}{N_3} \ , & |U_{23}| = \sqrt{\frac{n_t}{60}}\frac{s_3}{N_3} \ . \end{array} \qquad (4.1)$$

Completely analogous expression holds true for D. r_i, s_i and N_i are integers, and the defining equations reduce in both cases to equations generalizing those satisfied by Pythagorean triangles

$$\begin{aligned} r_1^2+s_1^2 &= N_1^2 \ , \\ r_2^2+s_2^2 &= N_2^2 \ , \\ r_3^2+s_3^2 &= N_3^2 \ . \end{aligned} \tag{4.0}$$

The square roots of n_i are also eliminated from the unitarity conditions which become equations with rational coefficients for the phases appearing in U and D. Hence there are good hopes that even rational solutions to the conditions might exist.

4.2 How Strong Number Theoretic Conditions One Can Pose On U And D Matrices?

It is not quite clear how strong the number theoretic conditions on U and D matrices are. An attractive working hypothesis is that mixing probabilities are rational. This leaves a lot of freedom concerning the mixing matrices themselves since square roots of rationals, Pythagorean phases, and finite roots of unity can appear in the mixing matrices.

1. The most stringent requirement would be that U and D matrices are rational unitary matrices. p-Adicization without algebraic extension allows only matrices for which various phases and trigonometric functions are products of Pythagorean phases. This option will be found to be too restrictive. The minimal extension allows square roots requiring a finite-dimensional extension of p-adic numbers: geometrically this means a generalization Pythagorean triangles to triangles for which short sides are integer valued and long side is square root of integer. Pythagorean phases and their generalizations span infinite discrete subgroups of SU(3).

2. Both the phases and also cosines and sines appearing in the mixing matrices could be restricted to algebraic roots of unit that is of form $exp(i2\pi/N)$ requiring finite algebraic extension of rationals and p-adic numbers. Roots of unity could define finite discrete subgroup of SU(3) implying rather stringent conditions on the model. Root of unity option is highly suggestive in light of the most recent developments (more than decade after development of the model) related to the p-adicization in terms of harmonic analysis in symmetric spaces relying on the counterparts of plane waves defined in terms of roots of unity and leading to a p-adic version of real symmetric space [K7]. Finite roots of unity define as a special case discrete subgroups of SU(3) implying rather stringent conditions on the model. For instance, in case of SU(2) these finite groups are well-known.

4.3 Could Rational Unitarity Make Sense?

In this section the considerations are restricted mostly to rational unitarity which at the time of writing of this chapter looked more attractive than the allowance of algebraic roots of unity. The number theoretic conditions following from the rational unitarity on the moduli of the U and D matrices are not completely independent of the parameterization used. The reason is that the products of the parameters in some algebraic extension of the rationals can combine to give a rational number. The safest parameterization to use is the one based on the moduli of the U and D matrix.

4.3.1 Parameterization of moduli in the case of rational unitarity

If one assumes rationality for the mixing matrix then all moduli can be written in the form

$$|D_{ij}| = \frac{n_{ij}}{N} \ . \tag{4.1}$$

If only moduli squared are required to be rational, the condition is replaced with a milder one:

$$|D_{ij}| = \frac{n_{ij}}{\sqrt{N}} \ . \tag{4.2}$$

Here $\sqrt{N}$ belongs to square root allowing algebraic extension of the p-adic numbers but is not an integer itself. An even milder condition is

$$|D_{ij}| = \sqrt{\frac{n_{ij}}{N}} . \tag{4.3}$$

The following arguments show that only this option or more general option allowing roots of unity with rational mixing probabilities is allowed. These options is also natural in light or preceding general considerations.

4.3.2 Unitary and mass conditions modulo 8 for rational unitarity

For $p_{ij} = (\sqrt{\frac{n_{ij}}{N}})^k$, $k = 1$ or 2, the requirement that the rows are unit vectors implies

$$\begin{aligned} \sum_j n_{i,j}^k &= N^k \ , \\ k &= 1 \text{ or } 2 \ . \end{aligned} \tag{4.3}$$

The problem of finding vectors with integer valued components and with a given integer valued length squared m ($k = 2$ case) is a well known and well understood problem of the number theory [A2]. The basic idea is to write the conditions modulo 8 and use the fact that the square of odd (even) integer is 1 (0 or 4) modulo 8. The result is that one must have

$$m \in \{1, 2, 3, 5, 6\} \ , \tag{4.4}$$

for the conditions to possess nontrivial solutions. For $m = N$ case this is the only condition needed. In $m = N^2$ case the condition implies that N must be odd.

Using this result one can write the mass squared conditions modulo 8 for $k = 2$ as

$$\begin{aligned} 3n_{i,2}^2 + 4n_{i,3}^2 &= n_i X \ , \\ X &= 1 \text{ for } m = N^2 \ , \\ X &\in \{1, 2, 3, 5, 6\} \text{ for } m = N \ . \end{aligned} \tag{4.3}$$

Here modulo 8 arithmetics is understood. In $m = N^2$ case one must have $n_i \in \{0, 3, 4\}$ modulo 8. These conditions are not satisfied in general. For $m = N$ conditions allow considerably more general set of solutions. By summing the equations and using probability conservation one however obtains $7N = 5N$ implying $2N = 0$ so that the non-allowed value $N = 4$ or 0 results.

For $k = 1$ no obvious conditions result on the values of n_i and only this option is allowed by mass conditions for the physical masses.

4.3.3 Rational unitarity cannot hold true for U and D matrices separately

The mixing scenario is not consistent with the assumption that the matrix elements of U and D matrix are complex rational numbers. If this were the case then matrix elements had to be proportional to a common denominator $1/N$ such that N is odd integer (otherwise the conditions stating that the unit vector property of the rows is not satisfied). The conditions

$$\begin{aligned} \sum_j r_{ij} &= 1 \ , \\ 9r_{12} + 60r_{13} &= n_d \ , \\ 9r_{22} + 60r_{23} &= n_s \ , \\ 9r_{32} + 60r_{33} &= n_b \ , \\ r_{ij} &= \frac{n_{ij}}{N_i} \ , \end{aligned} \tag{4.-1}$$

can be written modulo 8 as

$$\begin{aligned} \sum_j n_{ij}^k &= N^k \ , \\ n_{12}^k + 4n_{13}^k &= n_d N^k \ , \\ n_{22}^k + 4n_{23}^k &= n_s N^k \ , \\ n_{32}^k + 4n_{33}^k &= n_b N^k \ , \\ r_{ij} &= (\frac{n_{ij}}{N})^{k/2} \ , \ k = 1 \text{ or } 2 \ . \end{aligned} \qquad (4.-5)$$

1. Consider first the case $k = 2$. For odd n $n^2 = 1$ holds true and for even n $n^2 = 4$ or 0 holds true. It is easy to see that the conditions can be satisfied only of all integers are proportional to 4 but this cannot be possible since it would be possible since n_{ij} an N cannot contain common factors. Thus at least an extension allowing square roots is needed. Quite generally from $N^2 = 1$ mod 8 the above equations give

   $$n_{q_i} \ mod \ 8 \in \{0, 3, 4, 7\} \ .$$

   This condition fails to be satisfied by in the general case.

2. For the option $k = 1$ for which only the probabilities are rational the sum of all three equations gives $5N = 5N$ so that equations are consistent.

The result favors the possibility that roots of unity are the basic building bricks of the mixing matrices. This does not exclude the possibility that mixing probabilities are rational numbers.

4.3.4 Rational unitarity for phase factors

The phase factors associated with the rows of the mixing matrix are rational provided the corresponding angles correspond to Pythagorean triangles. It must be however emphasized that roots of unit are highly suggestive in the recent vision about p-adicization. Combining this property with the orthogonality conditions for the rows of the U matrix, one obtains highly nontrivial conditions relating the integers characterizing the sides of the Pythagorean triangle to the integers n_{ij}. The requirement that the imaginary parts of the inner product vanish, gives the conditions

$$\frac{s_{i,2}}{s_{i3}} = \frac{n_{13}n_{i3}}{n_{12}n_{22}} \ , \ i = 2, 3 \ . \qquad (4.-4)$$

Combining this conditions with the general representation for the sines of the Pythagorean triangle

$$sin(\phi) = \frac{2rs}{r^2 + s^2} \text{ or } \frac{r^2 - s^2}{r^2 + s^2} \ , \qquad (4.-3)$$

one obtains conditions relating the integers appearing characterizing the triangle to the integers on the right hand side.

An interesting possibility is that the lengths of the hypothenusae of the triangles associated with $s(i, 2)$ $((r(i), s(i)))$ and s_{i3} $((r_1(i), s_1(i)))$ are the same and sines correspond to the products $2rs$:

$$\begin{aligned} r^2(i) + s^2(i) &= r_1^2(i) + s_1^2(i) \ , \\ s_{i,2} &= 2r(i)s(i)/(r^2(i) + s^2(i)) \ , \\ s_{i,3} &= 2r_1(i)s_1(i)/(r_1^2(i) + s_1^2(i)) \ . \end{aligned} \qquad (4.-4)$$

In this case the conditions give

$$\frac{r(i)s(i)}{r_1(i)s_1(i)} = \frac{n_{13}n_{i3}}{n_{12}n_{22}} . \tag{4.-3}$$

The conditions are satisfied if one has

$$\begin{aligned} r(i)s(i) &= n_{13}n_{i3} \ , \\ r_1(i)s_1(i) &= n_{12}n_{22} \ . \end{aligned} \tag{4.-3}$$

This implies that $r(i)$ and $s(i)$ are products of the factors contained in the product $n_{13}n_{i3}$. Analogous conclusion applies to $r_1(i)$ and $s_1(i)$.

Additional number theoretic conditions are obtained from the requirement that the real parts of the inner products between first row and second and third rows vanish:

$$n_{11}n_{i1} + c_{i,2}n_{12}n_{i2} + c_{i,3}n_{13}n_{i3} = 0 \ , \ i = 2,3 \ . \tag{4.-2}$$

4.4 The Parameterization Suggested By The Mass Squared Conditions

To understand the consequences of the mass squared conditions, it is useful to use a parameterization, which is more natural for the treatment of the mass squared conditions than the standard parameterization:

$$U = \left[\begin{array}{ccc} r_{11} & r_{12} & r_{13} \\ r_{21}x_2 & r_{22}x_2exp(i\phi_{22}) & r_{23}x_2exp(i\phi_{23}) \\ r_{31}x_3 & r_{32}x_3exp(i\phi_{32}) & r_{33}x_3exp(i\phi_{33}) \end{array} \right]$$

$$\begin{aligned} x_2 &= exp(i\phi_2) \ , \\ x_3 &= exp(i\phi_3) \ . \end{aligned} \tag{4.-1}$$

In case of D matrix, the phase factors x_2 and x_3 can be chosen to be trivial. As far as the treatment of the mass conditions and unitarity conditions for the rows is considered, one can restrict the consideration to the case, when the overall phase factors are trivial. The remaining parameters are not independent and one on can deduce the formulas relating the moduli r_{ij} as well as the phase angles ϕ_{ij} to the parameters r_{11} and r_{12}. In general, the resulting parameters are not real and unitarity is broken.

Mass squared conditions and the requirement that the rows are unit vectors:

$$\begin{aligned} 9r_{i2}^2 + 60r_{i3}^2 &= n_i \ , \ i = 1,2 \ , \\ \sum_k r_{ik}^2 &= 1 \ , \end{aligned} \tag{4.-1}$$

allows one to express r_{i2} and r_{i3} in terms of r_{i1}

$$\begin{aligned} r_{i2} &= \sqrt{[-\frac{n_i}{51} + \frac{20}{17}(1 - r_{i1}^2)]} \ , \\ r_{i3} &= \sqrt{[\frac{n_i}{51} - \frac{3}{17}(1 - r_{i1}^2)]} \ . \end{aligned} \tag{4.-1}$$

The requirement that the rows are orthogonal to each other, relates the phase angles ϕ_{ij} in terms to r_{11} and r_{21}. Using the notations $sin(\phi_{ij}) = s_{ij}$ and $cos(\phi_{ij}) = c_{ij}$, one has

$$\begin{array}{ll} c_{i2} = \frac{a_i}{b_i} \ , & c_{i3} = -\frac{(A_{1i}+c_{i2}A_{2i})}{A_{3i}} \ , \\ s_{i2} = \epsilon(i)\sqrt{1-c_{i2}^2} \ , & s_{i3} = -\frac{A_{2i}}{A_{3i}}s_{i2} \ , \\ A_{1i} = r_{11}r_{i1} \ , & A_{2i} = r_{12}r_{i2} \\ A_{3i} = r_{13}r_{i3} \ , & \epsilon(i) = \pm 1 \ . \\ a_i = A_{3i}^2 - A_{1i}^2 - A_{2i}^2 \ , & b_i = 2A_{1i}A_{2i} \ , \end{array} \tag{4.0}$$

The sign factors $\epsilon(i)$ are not completely free and must be chosen so that the second and third row are orthogonal.

The mass conditions imply the following bounds for the parameters r_{i1}

$$\begin{aligned} m_i &\leq r_{i1} \leq M_i \ , \\ m_i &= \sqrt{1-\frac{n_i}{9}} \text{ for } n_i \leq 9 \ , \\ m_i &= 0 \text{ for } n_i \geq 9 \ , \\ M_i &= \sqrt{1-\frac{n_i}{60}} \ . \end{aligned} \quad (4.\text{-}2)$$

The boundaries for the regions of the solution manifold in (r_{11}, r_{21}) plane can be understood as follows. For given values of r_{11} and r_{21} there are in general two solutions corresponding to the sign factor $\epsilon(i)$ appearing in the equations defining the solutions of the mass squared conditions. This means just that complex conjugation gives a new solution from a given one. These two branches become degenerate, when the phase factors become ± 1 so that (s_{i2}, s_{i3}) vanishes for $i = 2$ or $i = 3$. Thus the curves at which one has $(s_{i2} = 0, s_{i3} = 0)$ define the boundaries of the projection of the solution manifold to (r_{11}, r_{21}) plane. At the boundaries the orthogonality conditions reduce to the form

$$\begin{aligned} &r_{11}r_{i1} + \epsilon(i,2)r_{12}r_{i2} + \epsilon_{i3}r_{13}r_{i3} = 0 \ , \quad i = 2 \text{ or } 3 \ , \\ &\epsilon_{22} = \epsilon_{32} \ , \\ &\epsilon_{23} = -\epsilon_{33} \end{aligned} \quad (4.\text{-}1)$$

where ϵ_{ij} corresponds to the value of the cosine of the phase angle in question. Consistency requires that either second or third row becomes real on the boundary of the unitarity region and that the matrices reduce to orthogonal matrices at the tip of the region allowed by unitarity.

4.5 Thermodynamical Model For The Topological Mixing

What would be needed is a physical model for the topological mixing allowing to deduce U and D matrices from first principles. The physical mechanism behind the mixing is change of the topology of X^2 in the dynamical evolution defined by the light like 2-surface X_l^3 defining parton orbit. This suggests that the topology changes $g \rightarrow g \pm 1$ dominate the dynamics so that matrix elements U_{13} and D_{13} should be indeed small so that the weird looking result $P_{11}^U \simeq 1/3$ follows from the requirement $n_u = 6$. This model however suggests that the matrix elements U_{23} and D_{23} could be large unlike in the original model for U and D matrices.

4.5.1 Solution of thermodynamical model

A possible approach to the construction of mixing matrices is based on the idea that the interactions causing the mixing lead to a thermal equilibrium so that the entropies for the ensemble defined by the probabilities p_{ij}^U and p_{ij}^D matrix is maximized (the subscripts U and D are dropped in the sequel).

1. The elements in the three rows of the mixing matrix represent probabilities for three states of the system with energies $(E_{i1}, E_{i2}, E_{i3}) = (0, 9, 60)$ and average energy is fixed to $\langle E \rangle = 69$.

2. There are usual constraints from probability conservation for each row plus two independent constraints from columns. The latter constraints can be regarded as a constraint on a second quantity equal to 1 for each column and brings in variable analogous to chemical potential besides temperature.

The constraint from mass squared for the third row follows from these constraints. The independent constraints can be chosen to be the following ones

$$\sum_j p_{ij} - 1 = 0 \ , \ i = 1,2,3 \qquad \sum_i p_{ij} - 1 = 0, \ j = 1,2 \ ,$$
$$9p_{i2} + 60p_{i3} - n_{q_i} = 0 \ , \ i = 1,2 \ . \tag{4.0}$$

The obvious notations $(q_1, q_2) = (d, s)$ and $(q_1, 2_2) = (u, c)$ are introduced. The conditions on mass squared are completely analogous to the conditions fixing the energy of the ensemble and thus its temperature, and thermodynamical intuition suggests that the probabilities p_{ij} decrease exponentially as function of E_j in the absence of additional constraints coming from the probability conservation for the columns and meaning presence of chemical potential.

The variational principle maximizing entropy in presence of these constraints can be expressed as

$$\begin{aligned} L &= S + S_c \\ S &= \sum_{i,j} p_{ij} \times log(p_{ij}) \\ S_c &= \sum_i \lambda_i (\sum_j p_{ij} - 1) + \sum_{j=1,2} \mu_j (\sum_i p_{ij} - 1) + \sum_{i=1,2} \sigma_i (9p_{i2} + 60p_{i3} - n_{q_i}) \ . \end{aligned} \tag{4.-2}$$

The variational equation is

$$\partial_{p_{ij}} L = 0 \ , \tag{4.-1}$$

and gives the probabilities as

$$\begin{array}{lll} p_{11} = \frac{1}{Z_1} \ , & p_{12} = \frac{xx_1^3}{Z_1} & p_{13} = \frac{yx_1^{20}}{Z_1} \ , \\ p_{21} = \frac{1}{Z_2} \ , & p_{22} = \frac{xx_2^3}{Z_2} & p_{13} = \frac{yx_2^{20}}{Z_2} \ , \\ p_{31} = \frac{1}{Z_3} \ , & p_{32} = \frac{x}{Z_3} & p_{33} = \frac{y}{Z_3} \ , \end{array} \tag{4.-1}$$

Here the parameters x, y, x_1, x_2 are defined as

$$\begin{array}{ll} x = exp(-\mu_2) \ , & y = exp(-\mu_3) \ , \\ x_1 = exp(-3\sigma_1) \ , & x_2 = exp(-3\sigma_2) \ . \end{array} \tag{4.-1}$$

whereas the row partition functions Z_i are defined as

$$Z_1 = 1 + xx_1^3 + yx_1^{20} \ , \quad Z_2 = 1 + xx_2^3 + yx_2^{20} \ , \quad Z_3 = 1 + x + y \ . \tag{4.0}$$

Note that the parameters λ_i have been eliminated. There are four parameters $\mu_2, \mu_3, \sigma_2, \sigma_3$ and 2 conditions from columns and 2 mass conditions so that the number of solutions is discrete and only finite number of U and D matrices are possible in the thermodynamical approximation.

4.5.2 Mass squared conditions

The mass squared conditions read as

$$9xx_1^3 + 60yx_1^{20} = n(q_1)Z_1 \ , \quad 9xx_2^3 + 60yx_2^{20} = n(q_2)Z_2 \ . \tag{4.1}$$

These equations allow to solve y as a simple linear function of x

$$y = \frac{n(q_1)-xx_1^3(9-n(q_1))}{(60-n(q_1))x_1^{20}} \equiv kx + l \ , \quad y = \frac{n(q_2)-xx_2^3(9-n(q_2))}{(60-n(q_2)x_2^{20}} \quad . \tag{4.2}$$

The identification of the two expressions for y allows to solve x_1 in terms of x_2 using equation of form $x_1^{20} - bx_1^3 + c = 0$:

$$\begin{aligned} & \left[60 - n(q_2)x_2^{20}\right]\left[n(q_1) - xx_1^3(9 - n(q_1))\right] \\ = & \left[60 - n(q_1))x_1^{20}\right]\left[n(q_2) - xx_2^3(9 - n(q_2)\right] \quad . \end{aligned} \tag{4.2}$$

In the most general case the equation allows 20 roots $x_1 = x_2(x_1)$.

4.5.3 Probability conservation

Probability conditions give additional information. By solving $1/Z_3$ from the first column gives

$$Z_1Z_2Z_3 - Z_1Z_2 - Z_2Z_3 - Z_1Z_3 \quad = \quad 0 \ , \tag{4.3}$$

(4.4)

This equation is a polynomial equation for in x_1 and x_2 with degree 20 and together with Eq. 4.2 having same degree determines and (x_1, x_2) the possible values of x_1 and x_2 as function of x. The number of real positive roots is at most $20^2 = 400$.

Probability conservation for the second column gives

$$x\left[(1 - x_1^3)Z_2 + (1 - x_2^3)Z_1\right] + (1 - x)Z_1Z_2 = 0 \ . \tag{4.5}$$

The row partition functions Z_i are linear functions of x and y and mass squared conditions give $y = kx + l$ (see Eq. 4.2) so that a third order polynomial equation for x results and gives the roots as functions of control parameters x_1 and x_2. Either 1 or 3 real roots are obtained for x. The values of x_1 and x_2 are determined by the probability constraint Eq. 4.4 for the first column and Eq. 4.2 relating x_1 and x_2.

4.5.4 The analogy with spontaneous magnetization

Physically the situation is analogous to a spontaneous symmetry breaking with y representing the external magnetizing field and x linear magnetization or vice versa. x_1 and x_2 are control parameters characterizing the interaction between spins. For single real root for x no spontaneous magnetization occurs but for 3 real roots there are two directions of spontaneous magnetization plus unstable state. In the recent case the roots must be positive. Since the maximal number of roots for (x_1, x_2) is 400, the maximal number of real roots is 1200. The trivial solution to the conditions is $p_{11} = 1, p_{22} = 1$, $p_{33} = 1$ with $x = y = 0$ represents corresponds to the absence of external magnetizing field and of magnetization.

4.5.5 Catastrophe theoretic description of the system

In the catastrophe theoretic approach one can see that situation as a cusp catastrophe with x as a behavior variable and x_1, x_2 in the role of control variables. In the standard parameterization of the cusp catastrophe [A1] the conditions correspond to the equation

$$x^3 - a - bx \quad = \quad 0 \ , \tag{4.5}$$

In the recent case a more general polynomial $P_3(x)$ easily transformable to the standard form is in question. The coefficients of the polynomial $P_3(x) = Dx^3 + Cx^2 + Bx + A$ are

$$\begin{aligned}
A &= Q(x_1)Q(x_2) \ , \\
B &= P(x_1)Q(x_2) + P(x_2)Q(x_1) + R(x_2) + R(x_1) \ , \\
C &= P(x_1)R(x_2) + P(x_2)R(x_1) - R(x_1)Q(x_2) - R(x_2)Q(x_1) \ , \\
D &= R(x_1)R(x_2) \ , \\
P(u) &= 1-u^3 \ , \ Q(u) = 1 + lu^{20} \ , \ R(u) = u^3 + ku^{20} \ . \qquad (4.2)
\end{aligned}$$

The trivial scaling transformation $A \to A/D = \hat{A}$, $B \to B/D = \hat{B}$, $C \to C/D = \hat{C}$ and the shift $x \to x + \hat{C}/3$ casts the equation in the standard form and gives

$$\begin{aligned}
a &= -\hat{A} + \frac{\hat{C}^3}{9} \ , \\
b &= -\hat{B} + \frac{\hat{C}^2}{3} \ . \qquad (4.1)
\end{aligned}$$

The curve

$$a = \pm 2(\frac{b}{3})^{3/2} \ , b \geq 0 \qquad (4.2)$$

represents the bifurcation set for the solutions. For $b \geq 0$, $|a| \leq (\frac{b}{3})^{3/2}$ three roots are obtained for x. $a = b = 0$ corresponds to the tip of the cusp. Three solutions result under the conditions

$$\begin{aligned}
\frac{\hat{C}^2}{3} &\geq 3\hat{B} \ , \\
(-\hat{B} + \frac{\hat{C}^2}{3})^3 &\leq \frac{(-\hat{A} + \frac{\hat{C}^3}{9})^2}{4} \ , \\
\hat{A} &= \frac{Q(x_1)Q(x_2)}{R(x_1)R(x_2)} \ , \\
\hat{B} &= \frac{P(x_1)Q(x_2) + P(x_2)Q(x_1) + R(x_2) + R(x_1)}{R(x_1)R(x_2)} \ , \\
\hat{C} &= \frac{P(x_1)R(x_2) + P(x_2)R(x_1) - R(x_1)Q(x_2) - R(x_2)Q(x_1)}{R(x_1)R(x_2)} \ , \\
P(u) &= 1-u^3 \ , \ Q(u) = 1 + lu^{20} \ , \ R(u) = u^3 + ku^{20} \ . \qquad (4.\text{-}2)
\end{aligned}$$

The boundaries of the regions are defined by polynomial equations for x_1 and x_2.. The two mass squared conditions and the probability conservation for the first row select a discrete set of parameter combinations.

One might ask whether U and D matrices could correspond to different solutions of these equations for same values of n_{q_i}. This cannot be the case since this would predict too large $u-d$ mass difference. Orthogonalization conditions for the rows should determine the phases more or less uniquely and could force CP breaking. The requirement that probabilities are rational valued implies that x_1, x_2, x and y are rational and poses very strong additional conditions to the solutions. The roots should correspond to very special solutions possessing symmetries so that the solutions of polynomial equations give probabilities as rational numbers. Note however that the solutions of polynomial equations with integer coefficients are in question and the solutions are algebraic numbers: this is enough as far as the p-adicization of the theory is considered.

4.5.6 Maximization of entropy solving constraint equations explicitly

The mass squared conditions allow to express the probabilities p_{ij} in terms of p_{11} and p_{21} (for instance) and this allows a rather concise representation for the solution to the maximization the entropy of topological mixing. The key formulas are following.

$$\begin{aligned} p_{31} &= 1 - p_{11} - p_{12} \ , \\ p_{i2} &= -\frac{n_i}{51} + \frac{20}{17}(1-p_{i1}) \ , \ i = 1,2 \ , \\ p_{i3} &= \frac{n_i}{51} - \frac{3}{17}(1-p_{i1}) \ , \ i = 1,2 \ . \end{aligned} \tag{4.-3}$$

Expressing entropy directly in terms of p_{11} and p_{21}, the conditions for the maximization of entropy imply the equations

$$log(p_{ij})X^{ij} = 0 \ , \ log(p_{ij})Y^{ij} = 0 \ , \tag{4.-2}$$

where a summation over repeated indices is carried out. The matrices $X^{ij} = \partial_{p_{11}} p_{ij}$ and $Y^{ij} = \partial_{p_{21}} p_{ij}$ are given by

$$\begin{aligned} X &= \begin{pmatrix} 1 & -\frac{20}{17} & \frac{3}{17} \\ 0 & 0 & 0 \\ -1 & \frac{20}{17} & -\frac{3}{17} \end{pmatrix} \\ Y &= \begin{pmatrix} 0 & 0 & 0 \\ 1 & -\frac{20}{17} & \frac{3}{17} \\ -1 & \frac{20}{17} & -\frac{3}{17} \end{pmatrix} \end{aligned} \tag{4.-3}$$

The equations can be transformed into the form

$$\prod_{ij} p_{ij}^{X_{ij}} = 1 \ , \ \prod_{ij} p_{ij}^{Y_{ij}} = 1 \ . \tag{4.-2}$$

When written explicitly, these equations read as

$$\begin{aligned} &\tfrac{p_{11}}{1-p_{11}-p_{21}} \times (\tfrac{-n_1+60(1-p_{11})}{-n_3+60(p_{11}+p_{21})})^{-20/17} \times (\tfrac{n_1-9(1-p_{11})}{n_3-9(p_{11}+p_{21})})^{3/17} = 1 \ , \\ &\tfrac{p_{21}}{1-p_{11}-p_{21}} \times (\tfrac{-n_2+60(1-p_{21})}{-n_3+60(p_{11}+p_{21})})^{-20/17} \times (\tfrac{n_2-9(1-p_{21})}{n_3-9(p_{11}+p_{21})})^{3/17} = 1 \ . \end{aligned} \tag{4.-2}$$

The equations can be cast into polynomial equations in p_{11} and p_{21} by taking 17: th power of both equations. This gives polynomial equations of degree $d = 17 + 20 + 3 = 40$. The total number of solutions to the equations is at most $40 \times 40 = 1600$. The previous estimate gave upper bound $3 \times 20 \times 20 = 1200$ for the number of solution. It might be that some symmetry is involved and reduces the upper bound by a factor $3/4$.

The solutions can be sought using gradient dynamics in which system in (p_{11}, p_{21}) plane drifts in the force field defined by the gradient ∇S of the entropy $S = -\sum_{ij} p_{ij} log(p_i j)$ and ends up to the maximum of S, $S = -\sum_{ij} p_{ij} log(p_i j)$.

$$\begin{aligned} \tfrac{dp_{11}}{dt} &= \partial_{p_{11}} S = -X^{ij} log(p_{ij}) \ , \\ \tfrac{dp_{21}}{dt} &= \partial_{p_{21}} S = -Y^{ij} log(p_{ij}) \ , \end{aligned} \tag{4.-2}$$

The conditions that the probabilities are positive give the constraints

$$\begin{aligned} 1 - \frac{n_1}{9} &\leq p_{11} \leq 1 - \frac{n_1}{60} \ , \\ 1 - \frac{n_2}{9} &\leq p_{21} \leq 1 - \frac{n_2}{60} \ , \\ 0 &\leq p_{21} \leq 1 - p_{11} \ , \\ \frac{69 - n_1 - n_2}{60} - p_{11} &\leq p_{21} \leq \frac{69 - n_1 - n_2}{9} - p_{11} \end{aligned} \tag{4.-5}$$

on the region containing the solutions.

4.6 U And D Matrices From The Knowledge Of Top Quark Mass Alone?

As already found, a possible resolution to the problems created by top quark is based on the additivity of mass squared so that top quark mass would be about 230 GeV, which indeed corresponds to a peak in mass distribution of top candidate, whereas $t\bar{t}$ meson mass would be 163 GeV. This requires that top quark mass changes very little in topological mixing. It is easy to see that the mass constraints imply that for $n_t = n_b = 60$ the smallness of V_{i3} and $V(3i)$ matrix elements implies that both U and D must be direct sums of 2×2 matrix and 1×1 unit matrix and that V matrix would have also similar decomposition. Therefore $n_b = n_t = 59$ seems to be the only number theoretically acceptable option. The comparison with the predictions with pion mass led to a unique identification $(n_d, n_b, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (4, 6, 59)$.

4.6.1 U and D matrices as perturbations of matrices mixing only the first two genera

This picture suggests that U and D matrices could be seen as small perturbations of very simple U and D matrices satisfying $|U| = |D|$ corresponding to $n = 60$ and having $(n_d, n_b, n_b) = (4, 5, 60)$, $(n_u, n_c, n_t) = (4, 5, 60)$ predicting V matrix characterized by Cabibbo angle alone. For instance, CP breaking parameter would characterize this perturbation. The perturbed matrices should obey thermodynamical constraints and it could be possible to linearize the thermodynamical conditions and in this manner to predict realistic mixing matrices from first principles. The existence of small perturbations yielding acceptable matrices implies also that these matrices be near a point at which two different matrices resulting as a solution to the thermodynamical conditions coincide.

D matrix can be deduced from U matrix since $9|D_{12}|^2 \simeq n_d$ fixes the value of the relative phase of the two terms in the expression of D_{12}.

$$\begin{aligned}
|D_{12}|^2 &= |U_{11}V_{12} + U_{12}V_{22}|^2 \\
&= |U_{11}|^2|V_{12}|^2 + |U_{12}|^2|V_{22}|^2 \\
&+ 2|U_{11}||V_{12}||U_{12}||V_{22}|cos(\Psi) = \frac{n_d}{9} \ , \\
\Psi &= arg(U_{11}) + arg(V_{12}) - arg(U_{12}) - arg(V_{22}) \ .
\end{aligned} \tag{4.-8}$$

Using the values of the moduli of U_{ij} and the approximation $|V_{22}| = 1$ this gives for $cos(\Psi)$

$$\begin{aligned}
\cos(\Psi) &= \frac{A}{B} \ , \\
A &= \frac{n_d - n_u}{9} - \frac{9 - n_u}{9}|V_{12}|^2 \ , \\
B &= \frac{2}{9|V_{12}|}\sqrt{n_u(9 - n_u)} \ .
\end{aligned} \tag{4.-9}$$

The experimentation with different values of n_d and n_u shows that $n_u = 6, n_d = 4$ gives $cos(\Psi) = -1.123$. Of course, $n_u = 6, n_d = 4$ option is not even allowed by $n_t = 60$. For $n_d = 4, n_u = 5$ one has $cos(\Psi) = -0.5958$. $n_d = 5, n_u = 6$ corresponding to the perturbed solution gives $cos(\Psi) = -0.6014$.

Hence the initial situation could be $(n_u = 5, n_s = 4, n_b = 60)$, $(n_d = 4, n_s = 5, n_t = 60)$ and the physical U and D matrices result from U and D matrices by a small perturbation as one unit of t (b) mass squared is transferred to u (s) quark and produces symmetry breaking as $(n_d = 5, n_s = 5, n_b = 59)$, $(n_u = 6, n_c = 4, n_t = 59)$.

The unperturbed matrices $|U|$ and $|D|$ would be identical with $|U|$ given by

$$|U_{11}| = |U_{22}| = \tfrac{2}{3} \ , \quad |U_{12}| = |U_{21}| = \tfrac{\sqrt{5}}{3} \ , \tag{4.-8}$$

The thermodynamical model allows solutions reducing to a direct sum of 2×2 and 1×1 matrices, and since $|U|$ matrix is fixed completely by the mass constraints, it is trivially consistent with the thermodynamical model.

4.6.2 Direct search of U and D matrices

The general formulas for p^U and p^D in terms of the probabilities p_{11} and p_{21} allow straightforward search for the probability matrices having maximum entropy just by scanning the (p_{11}, p_{21}) plane constrained by the conditions that all probabilities are positive and smaller than 1. In the physically interesting case the solution is sought near a solution for which the non-vanishing probabilities are $p_{11} = p_{22} = (9 - n_1)/9$, $p_{12} = p_{21} = n_1/9$, $p_{33} = 1$, $n_1 = 4$ or 5. The inequalities allow to consider only the values $p_{11} \geq (9 - n_1)/9$.

1. Probability matrices p^U and p^D

The direct search leads to maximally entropic p^D matrix with $(n_d, n_s) = (5, 5)$:

$$p^D = \begin{pmatrix} 0.4982 & 0.4923 & 0.0095 \\ 0.4981 & 0.4924 & 0.0095 \\ 0.0037 & 0.0153 & 0.9810 \end{pmatrix} \; , \; p_0^D = \begin{pmatrix} 0.5556 & 0.4444 & 0 \\ 0.4444 & 0.5556 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; . \tag{4.-8}$$

p_0^D represents the unperturbed matrix p_0^D with $n(d = 4), n_s = 5$ and is included for the purpose of comparison. The entropy $S(p^D) = 1.5603$ is larger than the entropy $S(p_0^D) = 1.3739$. A possible interpretation is in terms of the spontaneous symmetry breaking induced by entropy maximization in presence of constraints.

A maximally entropic p^U matrix with $(n_u, n_c) = (5, 6)$ is given by

$$p^U = \begin{pmatrix} 0.5137 & 0.4741 & 0.0122 \\ 0.4775 & 0.4970 & 0.0254 \\ 0.0088 & 0.0289 & 0.9623 \end{pmatrix} \tag{4.-8}$$

The value of entropy is $S(p^U) = 1.7246$. There could be also other maxima of entropy but in the range covering almost completely the allowed range of the parameters and in the accuracy used only single maximum appears.

The probabilities p_{ii}^D *resp.* p_{ii}^U satisfy the constraint $p(i,i) \geq .492$ *resp.* $p_{ii} \geq .497$ so that the earlier proposal for the solution of proton spin crisis must be given up and the solution discussed in [K2] remains the proposal in TGD framework.

2. Near orthogonality of U and D matrices

An interesting question whether U and D matrices can be transformed to approximately orthogonal matrices by a suitable $(U(1) \times U(1))_L \times (U(1) \times U(1))_R$ transformation and whether CP breaking phase appearing in CKM matrix could reflect the small breaking of orthogonality. If this expectation is correct, it should be possible to construct from $|U|$ ($|D|$) an approximately orthogonal matrix by multiplying the matrix elements $|U_{ij}|$, $i, j \in \{2, 3\}$ by appropriate sign factors. A convenient manner to achieve this is to multiply $|U|$ ($|D|$) in an element wise manner ($(A \circ B)_{ij} = A_{ij} B_{ij}$) by a sign factor matrix S.

1. In the case of $|U|$ the matrix $U = S \circ |U|$, $S(2,2) = S(2,3) = S(3,2) = -1$, $S_{ij} = 1$ otherwise, is approximately orthogonal as the fact that the matrix $U^T U$ given by

$$U^T U = \begin{pmatrix} 1.0000 & 0.0006 & -0.0075 \\ 0.0006 & 1.0000 & -0.0038 \\ -0.0075 & -0.0038 & 1.0000 \end{pmatrix}$$

is near unit matrix, demonstrates.

2. For D matrix there are two nearly orthogonal variants. For $D = S \circ |D|$, $S(2,2) = S(2,3) = S(3,2) = -1$, $S_{ij} = 1$ otherwise, one has

$$D^T D = \begin{pmatrix} 1.0000 & -0.0075 & 0.0604 \\ -0.0075 & 1.0000 & 0.0143 \\ 0.0604 & 0.0143 & 1.0000 \end{pmatrix} .$$

The choice $D = S \circ D$, $S(2,2) = S(2,3) = S(3,3) = -1$, $S_{ij} = 1$ otherwise, is slightly better

$$D^T D = \begin{pmatrix} 1.0000 & -0.0075 & 0.0604 \\ -0.0075 & 1.0000 & 0.0143 \\ 0.0601 & 0.0143 & 1.0000 \end{pmatrix} .$$

3. The matrices U and D in the standard gauge

Entropy maximization indeed yields probability matrices associated with unitary matrices. 8 phase factors are possible for the matrix elements but only 4 are relevant as far as the unitarity conditions are considered. The vanishing of the inner products between row vectors, gives 6 conditions altogether so that the system seems to be over-determined. The values of the parameters s_1, s_2, s_3 and phase angle δ in the "standard gauge" can be solved in terms of r_{11} and r_{21}.

The requirement that the norms of the parameters c_i are not larger than unity poses non-trivial constraints on the probability matrices. This should should be the case since the number of unitarity conditions is 9 whereas probability conservation for columns and rows gives only 5 conditions so that not every probability matrix can define unitary matrix. It would seem that that the constraints are satisfied only if the the 2 mass squared conditions and 2 conditions from the entropy maximization are equivalent with 4 unitarity conditions so that the number of conditions becomes 5+4=9. Therefore entropy maximization and mass squared conditions would force the points of complex 9-dimensional space defined by 3×3 matrices to a 9-dimensional surface representing group $U(3)$ so that these conditions would have a group theoretic meaning.

The formulas

$$\begin{aligned} r_{i2} &= \sqrt{[-\frac{n_i}{51} + \frac{20}{17}(1 - r_{i1}^2)]} \ , \\ r_{i3} &= \sqrt{[\frac{n_i}{51} - \frac{3}{17}(1 - r_{i1}^2)]} \ . \end{aligned} \tag{4.-8}$$

and

$$U = \begin{bmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 exp(i\delta) & c_1 c_2 s_3 + s_2 c_3 exp(i\delta) \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 exp(i\delta) & c_1 s_2 s_3 - c_2 c_3 exp(i\delta) \end{bmatrix} \tag{4.-7}$$

give

$$\begin{aligned} c_1 = r_{11} \ , \quad & c_2 = \frac{r_{21}}{\sqrt{1-r_{11}^2}} \ , \\ s_3 = \frac{r_{13}}{\sqrt{1-r_{11}^2}} \ , \quad & cos(\delta) = \frac{c_1^2 c_2^2 c_3^2 + s_2^2 s_3^2 - r_{22}^2}{2 c_1 c_2 c_3 s_2 s_3} \ . \end{aligned} \tag{4.-6}$$

Preliminary calculations show that for $n_1 = n_2 = 5$ case the matrix of moduli allows a continuation to a unitary matrix but that for $n_1 = 4, n_2 = 6$ the value of $cos(\delta)$ is larger than one. This would suggest that unitarity indeed gives additional constraints on the integers n_i. The unitary (in the numerical accuracy used) $(n_d, n_s) = (5,5)$ D matrix is given by

$$D = \begin{pmatrix} 0.7059 & 0.7016 & 0.0975 \\ -0.7057 & 0.7017 - 0.0106i & 0.0599 + 0.0766i \\ -0.0608 & 0.0005 + 0.1235i & 0.4366 - 0.8890i \end{pmatrix} .$$

The unitarity of this matrix supports the view that for certain integers n_i the mass squared conditions and entropy maximization reduce to group theoretic conditions. The numerical experimentation shows that the necessary condition for the unitarity is $n_1 > 4$ for $n_2 < 9$ whereas for $n_2 \geq 9$ the unitarity is achieved also for $n_1 = 4$.

4.6.3 Direct search for CKM matrices

The standard gauge in which the first row and first column of unitary matrix are real provides a convenient representation for the topological mixing matrices: it is convenient to refer to these representations as U_0 and D_0. The possibility to multiply the rows of U_0 and D_0 by phase factors $(U(1) \times U(1))_R$ transformations) provides 2 independent phases affecting the values of $|V|$. The phases $exp(i\phi_j)$, $j = 2, 3$ multiplying the second and third row of D_0 can be estimated from the matrix elements of $|V|$, say from the elements $|V_{11}| = cos(\theta_c) \equiv v_{11}$, $sin\theta_c = .226 \pm .002$ and $|V_{31}| = (9.6 \pm .9) \cdot 10^{-3} \equiv v_{31}$. Hence the model would predict two parameters of the CKM matrix, say s_3 and δ_{CP}, in its standard representation.

The fact that the existing empirical bounds on the matrix elements of V are based on the standard model physics raises the question about how seriously they should be taken. The possible existence of fractally scaled up versions of light quarks could effectively reduce the matrix elements for the electro-weak decays $b \rightarrow c + W$, $b \rightarrow u + W$ *resp.* $t \rightarrow s + W$, $t \rightarrow d + W$ since the decays involving scaled up versions of light quarks can be counted as decays $W \rightarrow bc$ *resp.* $W \rightarrow tb$. This would favor too small experimental estimates for the matrix elements V_{i3} and V_{3i}, $i = 1, 2$. In particular, the matrix element $V_{31} = V_{td}$ could be larger than the accepted value.

Various constraints do not leave much freedom to choose the parameters n_{q_i}. The preliminary numerical experimentation shows that the choice $(n_d, n_s) = (5, 5)$ and $(n_u, n_c) = (5, 6)$ yields realistic U and D matrices. In particular, the conditions $|U(1,1)| > .7$ and $|D(1,1)| > .7$ hold true and mean that the original proposal for the solution of spin puzzle of proton must be given up. In [K2] an alternative proposal based on more recent findings is discussed. Only for this choice reasonably realistic CKM matrices have been found.

1. The requirement that the parameters $|V_{11}|$ (or equivalently, Cabibbo angle and $|V_{31}|$ are produced correctly, yields CKM matrices for which CP breaking parameter J is roughly one half of its accepted value. The matrix elements $V_{23} \equiv V_{cb}$, $V_{32} \equiv V_{tc}$, and $V_{13} \equiv V_{ub}$ are roughly twice their accepted value. This suggests that the condition on V_{31} should be loosened.

2. The following equations summarize the results of the search requiring that

   (a) the value of the Cabibbo angle s_{Cab} is within the experimental limits $s_{Cab} = .223 \pm .002$,

   (b) $V_{31} = (9.6 \pm .9) \cdot 10^{-3}$, is allowed to have value at most twice its upper bound,

   (c) V_{13} whose upper bound is determined by probability conservation, is within the experimental limits $.42 \cdot 10^{-3} < |V_{ub}| < 6.98 \cdot 10^{-3}$ whereas $V_{23} \simeq 4 \times 10^{-3}$ should come out as a prediction,

   (d) the CP breaking parameter satisfies the condition $|(J - J_0)/J_0| < .6$, where $J_0 = 10^{-4}$ represents the lower bound for J (the experimental bounds for J are $J \times 10^4 \in (1 - 1.7)$).

The pairs of the phase angles (ϕ_1, ϕ_2) defining the phases $(exp(i\phi_1), exp(i\phi_2))$ are listed below

$$
\begin{array}{ll}
class\ \ 1: & \begin{array}{ccccc} \phi_1 & 0.1005 & 0.1005 & 4.8129 & 4.8129 \\ \phi_2 & 0.0754 & 1.4828 & 4.7878 & 6.1952 \end{array} \\
class\ \ 2: & \begin{array}{ccccc} \phi_1 & 0.1005 & 0.1005 & 4.8129 & 4.8129 \\ \phi_2 & 2.3122 & 5.5292 & 0.7414 & 3.9584 \end{array}
\end{array}
\qquad (4.\text{-}6)
$$

The phase angle pairs correspond to two different classes of U, D, and V matrices. The U, D and V matrices inside each class are identical at least up to 11 digits(!). Very probably the phase angle pairs are related by some kind of symmetry.

The values of the fitted parameters for the two classes are given by

	$\vert V_{11}\vert$	$\vert V_{31}\vert$	$\vert V_{13}\vert$	$J/10^{-4}$
class 1	0.9740	0.0157	0.0069	.93953
class 2	0.9740	0.0164	0.0067	1.0267

V_{31} is predicted to be about 1.6 times larger than the experimental upper bound and for both classes V_{23} and V_{32} are roughly too times too large. Otherwise the fit is consistent with the experimental limits for class 2. For class 1 the CP breaking parameter is 7 per cent below the experimental lower bound. In fact, the value of J is fixed already by the constraints on V_{31} and V_{11} and reduces by a factor of one half if V_{31} is required to be within its experimental limits.

U, D and $|V|$ matrices for class 1 are given by

$$U = \begin{bmatrix} 0.7167 & 0.6885 & 0.1105 \\ -0.6910 & 0.7047 - 0.0210i & 0.0909 + 0.1310i \\ -0.0938 & 0.0696 + 0.1550i & 0.1747 - 0.9653i \end{bmatrix}$$

$$D = \begin{bmatrix} 0.7059 & 0.7016 & 0.0975 \\ -0.6347 - 0.3085i & 0.6358 + 0.2972i & 0.0203 + 0.0951i \\ -0.0587 - 0.0159i & -0.0317 + 0.1194i & 0.6534 - 0.7444i \end{bmatrix}$$

$$|V| = \begin{bmatrix} 0.9740 & 0.2265 & 0.0069 \\ 0.2261 & 0.9703 & 0.0862 \\ 0.0157 & 0.0850 & 0.9963 \end{bmatrix} \tag{4.-8}$$

U, D and $|V|$ matrices for class 2 are given by

$$U = \begin{bmatrix} 0.7167 & 0.6885 & 0.1105 \\ -0.6910 & 0.7047 - 0.0210i & 0.0909 + 0.1310i \\ -0.0938 & 0.0696 + 0.1550i & 0.1747 - 0.9653i \end{bmatrix}$$

$$D = \begin{bmatrix} 0.7059 & 0.7016 & 0.0975 \\ -0.6347 - 0.3085i & 0.6358 + 0.2972i & 0.0203 + 0.0951i \\ -0.0589 - 0.0151i & -0.0302 + 0.1198i & 0.6440 - 0.7525i \end{bmatrix}$$

$$|V| = \begin{bmatrix} 0.9740 & 0.2265 & 0.0067 \\ 0.2260 & 0.9704 & 0.0851 \\ 0.0164 & 0.0838 & 0.9963 \end{bmatrix} \tag{4.-10}$$

What raises worries is that the values of $|V_{23}| = |V_{cb}|$ and $|V_{32}| = |V_{ts}|$ are roughly twice their experimental estimates. This, as well as the discrepancy related to V_{31}, might be understood in terms of the electro-weak decays of b and t to scaled up quarks causing a reduction of the branching ratios $b \to c + W$, $t \to s + W$ and $t \to t + d$. The attempts to find more successful integer combinations n_i has failed hitherto. The model for pseudo-scalar meson masses, the predicted relatively small masses of light quarks, and the explanation for $t\bar{t}$ meson mass supports this mixing scenario.

5 Hadron Masses

Besides the quark contributions already discussed, hadron mass squared can contain several other contributions and the task is to find a model allowing to identify and estimate these contributions. There are several guidelines for the numerical experimentation.

1. Conformal weight, that is mass squared, is assumed to be additive for quarks corresponding to the same p-adic prime. For instance, in case of $q\bar{q}$ mesons the mass would be $\sqrt{2}m(q)$ and the contribution of $k = 113$ u, d, s quarks to nucleon mass would be $\sqrt{3} \times 100$ MeV and thus surprisingly small. For CD meson quark masses would be additive.

2. Old fashioned quark model explains reasonably well hadron masses in terms of constituent quark masses. Effective 2-dimensionality of partons suggests an interpretation for the constituent quark as a composite structure formed by the current quark identified as a partonic 2-surface X^2 characterized by $k(q)$ and by join along boundaries bond/flux tube, kind of a gluonic "rubber band" characterized by $k = 107$ and connecting X^2 to the $k = 107$ hadronic 2-surface $X^2(H)$ representing hadron. $X^2(q_i)$ could be perhaps regarded as a hole in $k = k(q)$ 3-surface. The 2-dimensional visualization for a 3-dimensional topological condensation would become much more than a mere visualization. This view about hadrons brings in mind unavoidably the surreal 2-dimensional structures formed by organs like retina. Of course, effective 2-dimensionality allows to characterize the entire Universe as an extremely complex fractal 2-surface.

   The large mass of the constituent quark would be due to the color Coulombic and spin-spin interaction conformal weights of join along boundaries bond. Quark mass and the mass due to the color interaction conformal weight would be additive unless $k = 107$ for the quark (it seems that for η' this is indeed the case!). Classical color gauge fluxes would flow between $k = 107$ and $k \neq 107$ space-time sheets along the bonds. Color dynamics would take place at $k = 107$ space-time sheet in the sense that color gauge flux between quarks q_1 and q_2 flows first from $X^2((k(q_1)$ to the hadronic 2-surface $X^2(k = 107)$ and then back to $X^2(k(q_2))$. The induced Kähler field is always accompanied by a classical color gauge field and the classical color gauge flux would represent non-perturbative aspects of color interactions at space-time level.

3. A crucial observation is that the mass of η meson is rather precisely 4 times the pion mass whereas the mass of its spin excited companion ω is very nearly the same as the mass of ρ meson. This suggests that u, d quarks correspond to $k = 109$ inside η but to $k = 113$ inside ω. This inspires the idea that the p-adic mass scale of quarks is dynamical and sensitive to small perturbations as the fact that for CP_2 type extremals the operators corresponding to different p-adic primes reduce to one and same operator forces to suspect. If k characterizes the length scale associated with the elementary particle horizon as $\sqrt{k}$ multiple of CP_2 length scale, quark mass would be characterized by the size of elementary particle horizon sensitive to the dynamics in hadronic mass scale.

   The physical states would result as small perturbations of this degenerate ground state and the value of $k(q)$ would be sensitive to the perturbation. A rather nice fit for meson and baryon masses results by assuming that the p-adic length scale of the quark is dynamical.

4. In the case of pseudo-scalar mesons the scaled up versions of light quarks identifiable as constituent quarks, turn out to explain almost all of the pseudo scalar meson mass, and this inspires a new formulation for the old vision about pseudo-scalar mesons as Goldstone bosons. At least light pseudo-scalar mesons are Goldstone bosons in the sense that the color Coulombic and spin-spin interaction energies cancel in a good approximation so that quarks at $k \neq 107$ space-time sheets are responsible for most of the meson mass. The assumption that only $k(s)$ is dynamical for light baryons is enough to understand the mass differences between baryons having different numbers of strange quarks.

5. Color magnetic spin-spin interaction energies are indeed surprisingly constant among baryons. Also for mesons spin-spin interaction energies vary much less than the scaling of quark masses would predict on basis of QCD formula. This motivates the replacement of the interaction energy with interaction conformal weight in the case of color interactions. The interaction conformal weight is assignable to $k = 107$ space-time sheet, and the fact that spin-spin splittings of also heavy hadrons can be measured in few hundred MeVs, supports this identification. The mild dependence of color Coulombic conformal weight and spin-spin interaction conformal weight on hadron would be due to their dependence on the primes $k(q_i)$ and $k = 107$ characterizing space-time sheets connected by the color bonds $q_i \rightarrow 107$ and $107 \rightarrow q_j$.

6. The values for the parameters s^c_{ij} and S_{ij} characterizing color Coulombic and color magnetic interaction conformal weights can be deduced from the mass squared differences for hadrons

and assuming definite values for the parameters $k(q_i)$ characterizing quark masses. It seems that no other sources to meson mass (or at least pion mass) are needed.

7. In the case of nucleons the understanding of nucleon mass requires a large additional contribution about 780 MeV since quarks contribute only about 160 MeV to the mass of nucleon. This contribution can be assumed to be same for all baryons as the possibility to understand baryon mass differences in terms of quark masses demonstrates. The most plausible identification of this contribution is in terms of 2- or 3-particle state formed by super-symplectic gluons assignable to $k = 107$ hadronic space-time sheet and having conformal weight $s = 16$ corresponding to mass 934.2 MeV (rather near to nucleon mass and η' mass). This leads to a vision about non-perturbative aspects of color interactions and allows to understand baryon masses with accuracy better than one per cent. Also a connection with hadronic string model emerges and hadronic string tension is predicted correctly.

5.1 The Definition Of The Model For Hadron Masses

The defining assumptions of the model of hadron masses are following. CP_2 mass defines the overall elementary particle mass scale. Electron mass determines this mass only in certain limits.

5.1.1 Model for hadronic quarks

The numerical construction of U and D matrices using the thermodynamical model for the topological mixing justifies the assumptions $n_d = n_s = 5, n_b = 59$ and $n_u = 5, n_c = 6, n_t = 58$.

Quarks can appear both as free quarks and bound state quarks and the value of $k(q)$ is in general different for free and bound state quarks and can depend on hadron in case of bound state quarks. This allows to understand satisfactorily the masses of low lying hadrons.

5.1.2 Quark mass contribution to the mass of the hadron

Quark mass squared is p-adically additive for quarks with same value of p-adic prime. In the case of meson one has

$$m_M^2(p_1 = p_2) = m_{q_1}^2 + m_{q_2}^2 \ . \tag{5.1}$$

m_q denotes constituent quark mass which is larger than current quark mass due to the smaller value of k.

Masses are additive for different values of p.

$$m_M(p_1 \neq p_2) = m_{q_1} + m_{q_2} \ . \tag{5.2}$$

The generalization of these formulas to the cse of baryons is trivial.

5.1.3 Super-symplectic gluons and non-perturbative aspects of hadron physics

At least in the case of light pseudo-scalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudo-scalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of WCW degrees of freedom ("world of classical worlds") in which super-symplectic algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a possible solution of proton spin puzzle emerges [K4].

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-symplectic gluons. For $g = 0$ these gluons are massless and in absence of topological mixing could form a contribution analogous to sea or Bose-Einstein condensate. For $g = 1$ their mass can be calculated. It turns out the model assuming same topological mixing as in case of U quarks leads to excellent understanding of baryon masses assuming that hadron spin correlates with the super-symplectic particle content of the hadronic space-time sheet.

5.1.4 Top quark mass as a fundamental constraint

CP_2 mass is an important parameter of the model. The vanishing second order contribution to electron mass gives an upper bound for CP_2 mass. The bound $Y_e \leq .7357$ can be derived from the requirement that it is possible to reproduce τ mass in p-adic thermodynamics. Maximal second order contribution corresponds to a minimal CP_2 mass reduced by a factor $\sqrt{5/6} = .9129$ from its maximal value. There is a natural mechanism making second order contribution negligible. Leptonic masses tend to be predicted to be few per cent too high [K3] if the second order contribution from p-adic thermodynamics to the electron mass vanishes, which suggests that second order contribution might be there.

For $Y_e = 0$ and $Y_t = 1$ the most recent experimental best estimate 169.1 GeV [C20] for top quark mass is reproduced exactly. Even $Y_t = 0$ allows a prediction in the allowed range. For too large Y_e top quark mass is predicted to be too small unless one allows first order Higgs contribution to the top quark mass. This means that CP_2 mass can be scaled down from its maximal value at most 2.5 per cent. This translates to the condition $Y_e < .26$. It is possible to understand quark masses satisfactorily by assuming that Higgs contribution is second order p-adically and even negligible. In fact, there are good arguments suggesting that Higgs does not develop vacuum expectation at fermionic space-time sheets [K3]. If this is the case, top quark mass gives a very strong constraint to the model.

The super-symplectic color interactions associated with $k = 107$ space-time sheet give rise to the dominant reduction of the color conformal weight having interpretation in terms of color magnetic and electric conformal weights. Canonical correspondence implies that this contribution is always non-negative. Therefore the simple additive formula can lead to a situation in which the contribution of quarks to the meson mass can be slightly larger than meson mass and it is not obvious whether it is possible to reduce this contribution by any means since the reduction of CP_2 mass scale makes top quark mass too small.

For diagonal mesons for which quarks have the same value of p-adic prime, ordinary color interaction between quarks can contribute negative conformal weight reducing the contribution to the mass squared. In the case of non-diagonal mesons it is not clear whether this kind of color interaction exists. This kind of gluons would correspond to pairs of light-like partonic 3-surfaces for which throats correspond to different values of p-adic prime p. These are in principle possible but could couple weakly to matter. It seems that the parameters of the model, essentially CP_2 mass scale strongly constrained by the top quark mass, allow the quark contributions of non-diagonal mesons to be below the mass of the meson.

The fact that standard QCD model for color binding energies works rather well for heavy mesons suggests that the notion of negative color binding energy might make sense and could explain the discrepancy. The mixing of real and p-adic physics descriptions is however aesthetically very unappealing but might be the only way out of the problem. The p-adic counterpart of this description in case of heavy diagonal mesons would be based on the introduction of a negative color Coulombic contribution to the conformal weight of quark pair.

5.1.5 Smallness of isospin splittings

The smallness of isospin splittings inside $I_s = 1/2$ doublets poses an further constraint. $d_{113} - u_{113}$ mass difference is about $\Delta m_{d-u} = 13$ MeV and larger than typical isospin splitting. The repulsive

Coulomb interaction between quarks typically tends to reduce the mass differences due to Δm_{d-u} and the sign of Δm_{d-u} explains the "wrong" sign of n-p mass difference equal to $\Delta m_{n-p} = 1.3$ MeV. Non-diagonal hadrons containing scaled up u and d quarks would have anomalously large isospin splittings. On the other hand, for a diagonal meson containing b quark and scaled up u and d quark isospin splitting is proportional to $(m_d^2 - m_u^2)/m_b$ and small. B meson corresponds to this kind of situation.

5.2 The Anatomy Of Hadronic Space-Time Sheet

Although the presence of the hadronic space-time sheet having $k = 107$ has been obvious from the beginning, the questions about its anatomy emerged only quite recently after the vision about the spectrum of Kähler coupling strength had emerged [K10, K4].

In the case of pseudo-scalar mesons quarks give the dominating contribution to the meson mass. This is not true for spin 1/2 baryons and the dominating contribution must have some other origin. TGD allows to identify this contribution in terms of states created by purely bosonic generators of super-symplectic algebra and having as a space-time correlate CP_2 type vacuum extremals topologically condensed at $k = 107$ hadronic space-time sheet (or having this space-time sheet as field body). Proton and neutron masses are predicted with.5 per cent accuracy and $\Delta - N$ mass splitting with.6 per cent accuracy. A further outcome is a possible solution to the spin puzzle of proton proposed already earlier [K4].

5.2.1 Quark contribution cannot dominate light baryon mass

The first guess would be that the masses give dominating contribution to the mass of baryon. Since mass squared is additive, this would require rather large quark masses for proton and neutron. $k(d) = k(u) = k(s) = 108$ would give $(m(d), m(u), m(s)) = (571.3, 520.4, 616.6)$ MeV and $(m(n), m(p)) = (961.1, 931.7)$ MeV to be compared with the actual masses $(m(n), m(p) = (939.6, 938.3)$ MeV. The difference looks too large to be explainable in terms of Coulombic self-interaction energy. $\lambda - n$ mass splitting would be 27.6 MeV for $k(s) = 108$ which is much smaller than the real mass splitting 176.0 MeV. For $k(s) = 110$ one would have 120.0 MeV.

5.2.2 Does $k = 107$ hadronic space-time sheet give the large contribution to baryon mass?

In the sigma model for baryons the dominating contribution to the mass of baryon results as a vacuum expectation value of scalar field and light pseudo-scalar mesons are analogous to Goldstone bosons whose masses are basically due to the masses of light quarks.

This would suggest that $k = 107$ gluonic/hadronic space-time sheet gives a large contribution to the mass squared of baryon. p-Adic thermodynamics allows to expect that the contribution to the mass squared is in a good approximation of form

$$\Delta m^2 = nm^2(107) \ ,$$

where $m^2(107)$ is the minimum possible p-adic mass mass squared and n a positive integer. One has $m(107) = 2^{10}m(127) = 2^{10}m_e/\sqrt{(5)} = 233.55$ MeV for $Y_e = 0$ favored by the top quark mass.

1. $n = 11$ predicts $(m(n), m(p)) = (944.5, 939.3)$ MeV for $k = 113$ quarks: the actual masses are $(m(n), m(p) = (939.6, 938.3)$ MeV. Coulomb repulsion between u quarks could reduce the p-n difference to a realistic value.

2. $\lambda - n$ mass splitting would be 184.7 MeV for $k(s) = 111$ to be compared with the real difference which is 176.0 MeV. Note however that color magnetic spin-spin splitting requires that the ground state mass squared is larger than $11m_0^2(107)$.

5.2.3 What is responsible for the large ground state mass of the baryon?

The observations made above do not leave much room for alternative models. The basic problem is the identification of the large contribution to the mass squared coming from the hadronic space-time sheet with $k = 107$. This contribution could have the energy of classical color field as a space-time correlate.

1. The assignment of the energy to the vacuum expectation value of sigma boson does not look very promising since the very existence sigma boson is questionable and it does not relate naturally to classical color gauge fields. More generally, since no gauge symmetry breaking is involved, the counterpart of Higgs mechanism as a development of a coherent state of scalar bosons does not look a plausible idea.

2. One can however consider the possibility of a Bose-Einstein condensate or of a more general many-particle state of massive bosons possibly carrying color quantum numbers. A many-boson state of exotic bosons at $k = 107$ space-time sheet having net mass squared

$$m^2 = nm_0^2(107) \ , \ n = \sum_i n_i$$

could explain the baryonic ground state mass. Note that the possible values of n_i are predicted by p-adic thermodynamics with $T_p = 1$.

5.2.4 Glueballs cannot be in question

Glueballs [C2, C3] define the first candidate for the exotic boson in question. There are however several objections against this idea.

1. QCD predicts that lightest glue-balls consisting of two gluons have $J^{PC} = 0^{++}$ and 2^{++} and have mass 1650 MeV [C3]. If one takes QCD seriously, one must exclude this option. One can also argue that light glue balls should have been observed long ago and wonder why their Bose-Einstein condensate is not associated with mesons.

2. There are also theoretical objections in TGD framework.

   i) Can one really apply p-adic thermodynamics to the bound states of gluons? Even if this is possible, can one assume the p-adic temperature $T_p = 1$ for them if $T_p < 1$ holds true for gauge bosons consisting of fermion-anti-fermion pairs [K10, K4].

   ii) Baryons are fermions and one can argue that they must correspond to single space-time sheet rather than a pair of positive and negative energy space-time sheets required by the glueball Bose-Einstein condensate realized as wormhole contacts connecting these space-time sheets. This argument should be taken with a big grain of salt.

5.2.5 Do exotic colored bosons give rise to the ground state mass of baryon?

The objections listed above lead to an identification of bosons responsible for the ground state mass, which looks much more promising.

1. Super-symplectic gluons

TGD predicts exotic bosons and their super-conformal partners. The bosons created by the purely bosonic part of super-symplectic algebra [K1, K9], whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. The super-partners of the super-symplectic bosons have quantum numbers of a right handed neutrino and have no electro-weak couplings. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry.

Exotic bosons are single-sheeted structures meaning that they correspond to a single wormhole throat associated with a CP_2 type vacuum extremal. The assignment of these bosons to hadronic space-time having $k = 107$ is an attractive idea. The only contribution to the mass would come from the genus and $g = 0$ state would be massless in absence of topological mixing. In this case $g = 0$ bosons could condense on the ground state and define the analog of gluonic contribution to the parton sea. If they mix situation changes.

In the following calculations it is assumed that the contributions to mass from different p-adic primes sum up linearly whereas for contributions with same value of p-adic prime mass squared is additive. This rule is required if one wants to understand the mass differences of mesons and baryons in terms of mass differences due to quark flavor and the dependence of the p-adic length scale of quark on hadron. If one assumes that all contributions to masses sum up quadratically, unreasonably large quark mass differences are required. The objection from QCD based approach is that quarks contribute less than 2 per cent to the mass of the hadron. In TGD sea quarks would correspond to large value of p-adic prime and only their contribution would be so small whereas the contribution of the valence quarks would be of the order of largest quark mass present.

$g = 1$ unmixed super-symplectic boson would have mass squared $9m_0^2(k)$ (mass would be 700.7 MeV). For a ground state containing two $g = 1$ exotic bosons, one would have ground state mass squared $M_0^2 = 18m_0^2$ corresponding to $(m(n), m(p)) = (1160.8, 1155.6)$ MeV. Negative color Coulombic conformal weight and color magnetic spin-spin splitting can reduce the mass of the system. Electromagnetic Coulomb interaction energy can reduce the p-n mass splitting to a realistic value.

1. Color magnetic spin-spin splitting for baryons gives a test for this hypothesis. The splitting of the conformal weight is by group theoretic arguments of the same general form as that of color magnetic energy and given by $(m^2(N), m^2(\Delta)) = (18m_0^2 - X, 18m_0^2 + X)$ in absence of topological mixing. $n = 11$ for nucleon mass implies $X = 7$ and $m(\Delta) = 5m_0(107) = 1338$ MeV to be compared with the actual mass $m(\Delta) = 1232$ MeV. The prediction is too large by about 8.6 per cent.

2. If one allows negative color Coulombic conformal weight $\Delta s = -2$ the mass squared reduces by 2 units. The alternative is topological mixing one can have $m^2 = 8m_0^2$ instead of $9m_0^2$. This gives $m(\Delta) = 1240$ MeV so that the error is only.6 per cent. The mass of topologically mixed exotic boson would be 660.6 MeV and equals to m_{104}.

One must consider also the possibility that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks in which the conformal weights would be (5, 6, 58) for the three lowest generations.

1. For this option the ground state of baryon could consist of 2 gluons of lowest generation and one gluon of second generation $(5 + 5 + 6 = 16)$.

2. If the mixing is same as for D type quarks with weights (5, 5, 59), one can have only $s = 15$ state. It turns out that this option allows to predict hadron masses with amazing precision if one assumes correlation between hadron spin and its super-symplectic particle content.

3. For this option one can even consider the possibility that super-symplectic gluons are are able to represent also color Coulombic conformal weight so that model would simply considerably.

The conclusion is that a many-particle state of super-symplectic bosons could be responsible for the ground state mass of baryon. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.

2. A connection with hadronic string model

Hadronic string model provides a phenomenological description of the non-perturbative aspects of hadron physics, and TGD was born also as a generalization of the hadronic string model. Hence one can ask whether something resembling hadronic string model might emerge from the super-symplectic sector. TGD allows string like objects but the fundamental string tension is gigantic, roughly a factor 10^{-8} of that defined by Planck constant. The hypothesis motivated by the p-adic length scale hypothesis is that vacuum extremals deformed to non-vacuum extremals give to a hierarchy of string like structures with string tension $T \propto 1/L_p^2$, L_p the p-adic length scale. The challenge has been the identification of quantum counterpart of this picture.

The fundamental mass formula of the string model relates mass squared and angular momentum of the stringy state. It has the form

$$M^2 = kJ \ , \ k \simeq .9 \ GeV^2 \ . \tag{5.3}$$

A more general formula is $M^2 = kn$.

n_0	5	9	16	18
M_0^2/GeV^2	.273	.490	0.872	0.982

Table 5: The prediction for the hadronic string tension for some values of the mass squared of super-symplectic particle used to construct hadronic excitations.

This kind of formula results from the additivity of the conformal weight (and thus mass squared) if one constructs a many particle state from $g = 1$ super-symplectic bosons with a thermal mass squared $M^2 = M_0^2 n$, $M_0^2 = n_0 m_{107}^2$. The angular momentum of the building blocks has some spectrum fixed by Virasoro conditions. If the basic building block has angular momentum J_0 and mass squared M_0^2, one obtains $M^2 = M_0^2 J$, $k = M_0^2$, $J = nJ_0$. The values of n are even in old fashioned string model for a Regge trajectory with a fixed parity. $J_0 = 2$ implies the same result so that basic unit might be called "strong graviton".

One can consider several candidates for the values of n_0. In the absence of topological mixing one has $n_0 = 9$ for super-symplectic gluons. The bound state of two super-symplectic $g = 1$ bosons with mass squared $M_0^2 = 16m_{107}^2$ (two units of color binding conformal weight) could be responsible for the ground state mass of baryons. If topological mixing occurs and is same as for U type quarks then also a bound state of 2 gluons of first generation and 1 gluon of second generation gives $M_0^2 = 16m_{107}^2$.

Table 5 summarizes the prediction for the string tension in various cases. The identification of the basic excitations as many-particle states from from bound states of super-symplectic gluons with $M_0^2 = 16m_{107}^2$ predicts the nominal value of the.9 GeV with 3 per cent accuracy.

Pomeron [C18] represented an anomaly of the hadronic string model as a hadron like particle which was not accompanied by a Regge trajectory. A natural interpretation would be as a space-time sheet containing valence quarks as a structure connected by color flux tubes to single structure. There is recent quite direct experimental evidence for the existence of Pomeron [C12, C11, C13] in proton photon collisions: Pomeron seems to leave the hadronic space-time sheet for a moment and collide with photon after which it topologically condenses back to the hadronic space-time sheet. For a more detailed discussion see [K4].

This picture allows also to consider a possible mechanism explaining spin puzzle of proton and I have already earlier considered an explanation in terms of super-symplectic spin [K4] assuming that state is superposition of ordinary $(J = 0, J_q = 1/2)$ state and $(J = 2, J_q = 3/2)$ state in which super-symplectic bound state has spin 2.

3. Some implications

If one accepts this picture, it becomes possible to derive general mass formulas also for the baryons of scaled up copies of QCD possibly associated with various Mersenne primes and Gaussian Mersennes. In particular, the mass formulas for leptobaryons, in particular "electro-baryons", can be deduced [K8]. Good estimates for the masses of the mesons and baryons of M_{89} hadron physics are also obtained by simple scaling of the ordinary hadron masses by factor 512. Scaled up isospin splittings would be one signature of M_{89} hadron physics: for instance, n-p splitting of 1.3 MeV would scale up to 665.6 MeV.

5.2.6 What about mesons?

The original short-lived belief was that only baryons are accompanied by a pair of super-symplectic bosons condensed at hadronic $k = 107$ space-time sheet. By noticing that color magnetic spin-spin splitting requires an additional contribution to the conformal weight of meson cancelled by spin-spin splitting conformal weight in the case of pseudo-scalar mesons to first order in p, one ends up with the conclusion that also mesons could possess the hadronic space-time sheet.

It is however unclear whether one must included besides the quantized contribution of super-symplectic gluons also color Coulombic contribution having interpretation as perturbative contribution. These contributions are of same form and one could argue that only super-symplectic contribution should be allowed. This would mean very strong quantization rules.

It however turns out that the contribution of super-symplectic massive boson is necessarily only in the case of $\pi - \rho$ system and produces mere nuisance in the case of heavier mesons. The special role of $\pi - \rho$ system could be understood in terms of color confinement which would make pion $k = 107$ tachyon without the presence of additional mass squared.

Assuming topological mixing of super-symplectic bosons to be same as for U type quarks, the super-symplectic contribution must correspond to a conformal weight of 5 units in the case of pion and thus to *single* super-symplectic boson with $m^2 = 5m_{107}^2$ instead of $9m_{107}^2$ as for $g = 1$ super-symplectic bosons. A possible interpretation is in terms of $g = 0$ boson which has suffered a topological mixing. That 5 units of conformal weight result also in the topological mixing of u and d quarks supports this option and forces to ask whether also super-symplectic topological mixing is same inside baryons and mesons. If it is same for U type quarks and super-symplectic bosons one has $(s_1, s_2, s_3) = (5, 6, 58)$ for the super-symplectic gluons. As noticed, $S_{SC} = 16$ for baryons is obtained if one has a bound state of 2 bosons of first generation and one boson of second generation giving $s_{SC} = 5 + 5 + 6 = 16$. One can wonder how tightly the super-symplectic gluons are associated with baryonic valence quarks.

5.3 Pseudoscalar Meson Masses

The requirement that all contributions to the meson masses have p-adic origin allows to fix the model uniquely and also constraints on the value of the parameter Y_e emerge. In the following only pseudo-scalar mesons will be considered.

5.3.1 Light pseudo-scalar mesons as analogs of Goldstone bosons

Fractally scaled up versions of light quarks allow a rather simple model for hadron masses. In the old fashioned $SU(3)$ based quark model η meson is regarded as a combination $u\overline{u} + d\overline{d} - 2s\overline{s}$. The basic observation is that η mass is rather precisely 4 times the mass of π whereas the mass of ω is very near to ρ mass. This suggests that η results by a fractal scaling of quark masses obtained by the replacement $k(q) = 113 \rightarrow 109$ for the quarks appearing in η. This inspires the idea that mesonic quarks are scaled up variants of light quarks and at least light pseudo-scalar mesons are almost Goldstone bosons in the sense that quark contribution to the mass is as large as possible but smaller than meson mass. This idea must of course be taken as an interesting ansatz and in the end of the chapter it will be found that this idea might work only in the case of pion and kaon systems.

5.3.2 Quark contributions to meson masses

Table 6 summarizes the predictions for quark contributions to the masses of mesons assuming $k(q)$ depending on meson and assuming $Y_e = 0$ guaranteeing maximum value of top quark mass.

1. The quark contribution to pion mass is predicted to be 140 MeV, which is by few percent above the pion mass. Ordinary color interactions between pionic quarks can however reduce the conformal weight of pion by one unit. The reduction of CP_2 mass scaled cannot be considered since it would reduce top quark mass to 163.3 GeV which is slightly below the favored range of values [C20].

2. The success of the fit requires that spin-spin splitting cancels the mass of super-symplectic boson in a good approximation for pseudo-scalar mesons. This would be in accordance with the Goldstone boson interpretation of pseudo-scalar mesons in the sense that color contribution to the mass from $k = 107$ space-time sheet vanishes in the lowest p-adic order.

3. In the case of η *resp.* η' meson it has been assumed that the states have form $(u\overline{u}+\overline{d}-2s\overline{s})/\sqrt{6}$ *resp.* $(u\overline{u} + \overline{d} + s\overline{s})/\sqrt{3}$.

4. B mesons have anomalously large coupling to $\eta' K$ and $\eta' X$ [C10], which indicates an anomalously large coupling of η' to gluons [C7]. The interpretation has been in terms of a considerable mixing η' with gluon-gluon bound state.

   η' mass is only 2.5 per cent higher than the mass $4m_{107}$ of super-symplectic boson B_{SC} associated with the hadronic space-time sheet of hadron. Large mixing scenario is however

$Meson$	$scaled\ quarks$	$m_q(M)/MeV$	m_{exp}/MeV
π^0	d_{113}, u_{113}	140.0	135.0
π^+	d_{113}, u_{113}	140.0	139.6
K^0	d_{114}, s_{109}	495.5	497.7
K^+	u_{114}, s_{109}	486.3	493.7
η	$u_{109}, d_{109}, s_{109}$	522.2	548.9
η'	$u_{107}, d_{107}, s_{107}, c_{107}$	1144.2	957.6
$\eta' = B_{SC} + \sum_i q_i\overline{q}_i$	q_{118}	959.2	957.6
η_c	c_{104}	3098	2980
D^0	c_{105}, u_{113}	1642	1865
D^+	c_{105}, d_{113}	1654	1870
Υ	b_{103}	10814	9460
B	$b_{104}, d_{104}, u_{104}$	5909	5270

Table 6: Summary of the model for contribution of quarks to the masses of mesons containing scaled up u, d, and s quarks. The model assumes the maximal value of CP_2 mass allowed by η' mass and the condition $Y_e = 0$ favored by top quark mass.

not consistent with the existence of Φ with nearly the same mass. This encourages to consider the possibility that η' corresponds to a super-symplectic boson B_{SC} plus quark pair with $k(d) = k(u) = k(s) = k(c) = 118$ with maximal mixing. In this case the contribution of quarks to the mass would be 25.1 MeV and one would have $m(\eta') = 959.2$ MeV which coincides with the actual mass with 1 per mille accuracy. Note that this model predicts identical couplings to various quark pairs as does also the model assuming that $\eta' - \Phi$ system is singlet with respect to flavor $SU(3)$ (having no fundamental status in TGD).

It is clear from the above table that the quark contributions to the masses of π, η' and B are slightly above the meson masses. In the case of B the discrepancy is largest and about 12 per cent. If one assumes that all contributions to the mass have p-adic origin, they are necessarily non-negative.

1. In the case of diagonal mesons the ordinary color interactions can reduce the contribution of quark masses to the mass of the meson. In the case of η' baruyonic super-symplectic gluon B_{SC} could resolve the problem.

2. In the case of non-diagonal mesons the only possible solution of the problem is that $Y_e > 0$ holds true so that mass scale is reduced by a factor $1 - P = \sqrt{5/(5 + Y_e)}$ giving $Y_e \simeq .056$. The prediction for top quark mass is reduced by 1.1 per cent to 167.2 GeV which belongs to the allowed range [C20].

3. In the case of B meson one is forced to assume $k_b = k_d = k_u = 104$ although it would be possible to achieve smaller quark contribution by an alternative choice. This choice explains the observed very small isospin splitting and diagonality allows the ordinary color interaction to reduce the quark contribution to the B meson mass.

4. At the end of the chapter an alternative scenario in which quark masses give in good approximation only the masses of pion and kaon will be considered.

5.3.3 An example about how the mesonic mass formula works

The mass of the B_c meson (bound state of b and c quark) has been measured with a precision by CDF (see the blog posting by Tommaso Dorigo [C19]) and is found to be $M(B_c) = 6276.5 \pm 4.8$ MeV. Dorigo notices that there is a strange "crackpottian" co-incidence involved. Take the masses of the fundamental mesons made of $c\overline{c}$ (Ψ) and $b\overline{b}$ (Υ), add them, and divide by two. The value of mass turns out to be 6278.6 MeV, less than one part per mille away from the B_c mass!

The general p-adic mass formulas and the dependence of k_q on hadron explain the co-incidence. The mass of B_c is given as $m(B_c) = m(c, k_c(B_c)) + m(b, k_b(B_c))$, whereas the masses of Ψ and

Υ are given by $m(\Psi) = \sqrt{2}m(c, k_\Psi)$ and $m(\Upsilon) = \sqrt{2}m(b, k_\Upsilon)$. Assuming $k_c(B_c) = k_c(\Psi)$ and $k_b(B_c) = k_b(\Upsilon)$ would give $m(B_c) = [m(\Psi) + m(\Upsilon)]/\sqrt{2}$ which is by a factor $\sqrt{2}$ higher than the prediction of the "crackpot" formula. $k_c(B_c) = k_c(\Psi) + 1$ and $k_b(B_c) = k_b(\Upsilon) + 1$ however gives the correct result.

As such the formula makes sense but the one part per mille accuracy must be an accident in TGD framework.

1. The predictions for Ψ and Υ masses are too small by 2 *resp.* 5 per cent in the model assuming no effective scaling down of CP_2 mass.
2. The formula makes sense if the quarks are effectively free inside hadrons and the only effect of the binding is the change of the mass scale of the quark. This makes sense if the contribution of the color interactions, in particular color magnetic spin-spin splitting, to the heavy meson masses are small enough. Ψ and η_c correspond to spin 1 and spin 0 states and their masses by 3.7 per cent ($m(\eta_c) = 2980$ MeV and $m(\Psi) = 3096.9$) so that color magnetic spin-spin splitting is measured using per cent as natural unit.

5.4 Baryonic Mass Differences As A Source Of Information

The first step in the development of the model for the baryon masses was the observations that $B-n$ mass differences can be understood if baryons are assumed to contain scaled versions of strange and heavy quarks. The deduction of precise values of $k(q)$ is however not quite straightforward since the color magnetic contribution to the mass affects the situation. Thus a working hypothesis worth of studying is that ground state contribution is same for all baryons and that for spin 1/2 baryons quark contribution to the mass added to this contribution is near as possible to the real mass but smaller than it.

The purpose of the following explicit is to to convince the reader that baryon mass difference can be indeed understood in terms of quark mass differences. This of course requires that quark space-time sheet is not the hadronic $k = 107$ space-time sheet. Otherwise quadratic mass formula applies.

1. $\Lambda - n$ mass difference is 176 MeV and $(k(s) = 111, k(d) = 114)$ for λ would predict the mass difference $m(\lambda) - m(n) = m_q(\lambda) - m_q(n)$, where one has $m_q(\lambda) = m(s_{111}) + \sqrt{2}m(d_{114}) - m(n)$, $m_q(n) = \sqrt{m(u_{113})^2 + 2m(d_{113})^2)}$. The prediction equals to 141 MeV. It is possible to achieve smaller discrepancy but more precise considerations support this identification. Note that the spin-spin interaction energy is same if u and d quark form the paired quark system which is in $J = 0$ or $J = 1$ state so that the mass difference indeed can be regarded as quark mass difference.
2. $\Sigma - n$ mass difference is 257 MeV. If sigma contains s_{111}, u_{114} and d_{114}, the mass difference is predicted to be $m_q(\Sigma) - m_q(n)$, $m_q(\Sigma) = m(s_{111}) + \sqrt{2}m(d_{114})$ and comes out as 228 MeV.
3. If Ξ contains two s_{110} quarks and u_{113} (d_{113}), he mass difference comes out as 351 MeV to be compared with the experimental value 381 MeV.
4. Even single hadron, such as Ω, could contain several scaled up variants of s quark. $s_{108} + 2s_{111}$ decomposition would give mass difference 718 MeV to be compared with the real mass difference 734 MeV.
5. For Λ_c the mass is 2282 MeV. For $k(c) = 105$ instead of $k(c) = 104$ the predicted $\Lambda_c - n$ mass difference is 1341 MeV whereas the experimental difference is 1344 MeV.
6. For Λ_b the mass is 5425 MeV. For $k(b) = 104$ instead of $k(b) = 103$ the predicted $\Lambda_b - n$ mass difference is 4403 MeV. The experimental difference is 4485 MeV.

Baryon	*s content*	$\Delta m/MeV$	$\Delta m_{exp}/MeV$
Λ	s_{111}	141	176
Σ	s_{110}	228	257
Ξ	$s_{110}+s_{111}$	351	381
Ω	$s_{108}+2s_{110}$	718	734
Λ_c	$c_{105}, d_{112}, u_{112}$	1341	1344
Λ_b	$b_{105}, u_{106}, d_{106}$	4403	4485

Table 7: Summary of the model for the quark contribution to the masses of baryons containing strange quarks deduced from mass differences and neglecting second order contributions to the mass. Δm denotes the predicted $B-n$ mass difference $m(B)-m(n)$. The subscript "exp" refer to experimental value of the quantity in question.

5.5 Color Magnetic Spin-Spin Splitting

Color magnetic hyperfine splitting makes it possible to understand the $\rho-\pi$, $K^\star-K$, $\Delta-N$, etc. mass differences [B1]. That the order of magnitude for the splittings remains same over the entire spectrum of hadrons serves as a support for the idea that color fluxes are fed to $k=107$ space-time sheet. This would suggest that color coupling strength does not run for the physical states and runs only for the intermediate states appearing in parton description of the hadron reactions. A possible manner to see the situation in terms of intermediate states feeding color gauge flux to space-time sheets with $k>107$ so that the additive color Coulombic interaction conformal weights $s(q_i,q_j)$ would depend only on the integers $k(q_i), k(q_j)$. It will be found that the dependence is roughly of form $1/(k(q_i)+k(q_j))$, which brings in mind a logarithmic dependence of α_s on p-adic length scales involved.

There are two approaches to the problem of estimating spin-spin splitting: the first one is based on spin-spin interaction energy and the second one on spin-spin interaction conformal weight. The latter one turns out to be the only working one.

5.5.1 The model based on spin-spin interaction energy fails

Classical model would apply real number based physics to estimate the splittings and calculate color magnetic interaction energies. Standard QCD approach predicts that the color magnetic interaction energy is of form

$$\Delta E = S\sum_{pairs}\frac{\bar{s}_i\cdot\bar{s}_j}{m_i m_j r_{ij}^3} . \tag{5.4}$$

The mass differences for hadrons allow to deduce information about the nature of color magnetic interaction and make some conclusions about the applicability this model.

1. For mesons the spin-spin splitting various from 630 MeV for $\rho-\pi$ system to 120 MeV $\Psi-\eta_c$ excludes the classical model predicting that the splitting should be proportional to $1/m(q_1)m(q_2)$ (variation by a factor $2^{113-106}=128$ instead of 5 would be predicted if the size of the hadron remains same). Also the predicted ratio of K^*-K splitting to $\rho-\pi$ splitting would be $1/4$ rather than.63. The ratio of $\eta-\omega$ splitting to $\rho-\pi$ splitting would be $1/16$ rather than.34. The ratio of $\Phi-\eta\prime$ splitting to $\rho-\pi$ splitting would be $1/32\simeq .03$ instead of .11.

   The inspection of the spin-spin interaction energies would suggest that the interaction energy scales $E(i,j)$ obey roughly the formula

   $$E(i,j)\sim \tfrac{5}{2}\times\tfrac{1}{(\Delta k(q_1)+\Delta k(q_2))} = 5\times\tfrac{1}{X}$$

   $$X = log_2([L(113)/L(k(q_1))]\times[L(113)/L(k(q_2))])$$

   $$\Delta k(q) = 113-k(q)$$

rather than being proportional to $2^{-k(q_1)-k(q_1)}$. The hypothesis that p-adic length scale $L(k)$ of order CP_2 length scale range corresponds to the size of elementary particle horizon associated with wormhole contacts feeding gauge fluxes of the CP_2 type extremal representing particle to the larger space-time sheet with $p \simeq 2^k$ might allow to understand this dependence.

2. $\Delta - N$, $\Sigma^* - \Sigma$, and $\Xi^* - \Xi$ mass differences are 291 MeV, 194 MeV, 220 MeV. If strange quark inside Σ corresponds to $k = 110$, the ratio of $\Sigma^* - \Sigma$ splitting to $\Delta - N$ splitting is predicted to be by a factor 1.17 larger than experimental ratio. $\Xi^* - \Xi$ splitting assuming $k(s) = 109$ the ratio would be.19 and quite too small. Assuming that s, u, d quarks have more or less same mass, the model would predict reasonably well the ratios of the splittings. Either the idea about scaled up variants of s is wrong or the notion of interaction energy must be replaced with interaction conformal weight in order to calculate the effects of color interactions to hadron masses.

5.5.2 The modelling of color magnetic spin-spin interaction in terms of conformal weight

The model based on the notion of interaction conformal weight generalizes the formula for color magnetic interaction energy to the p-adic context so that color magnetic interaction contributes directly to the conformal weight rather than rest mass. The effect is so large that it must be p-adically first order (the maximal contribution in second order to hadron mass would be however only 224 MeV) and the generalization of the mass splitting formula is rather obvious:

$$\Delta s = \sum_{pairs} S_{ij}\bar{s}_i \cdot \bar{s}_j \ . \tag{5.5}$$

The coefficients S_{ij} depend must be such that integer valued Δs results and CP_2 masses are avoided: this makes the model highly predictive. Coefficients can depend both on quark pair and on hadron since the size of hadron need not be constant. In any case, very limited range of possibilities remains for the coefficients.

This might be understood if the color flux tube carrying color magnetic flux and connecting quark to $k = 107$ hadronic space-time sheet is also characterized by a value of $k \geq 113$. This fixes practically completely the model in the case of mesons. If the interaction strengths $s_c(i, j)$ characterizing color Coulombic interaction conformal weight between two quarks depends only on the flux tube pair connecting the quarks via $k = 107$ space-time sheet via the integers $k(q_i)$, the model contains only very few parameters.

5.5.3 The modelling of color magnetic- spin-spin splitting in terms of super-symplectic boson content

The recent variant for the model of the color magnetic spin-spin splitting replacing energy with conformal weight is considerably simpler than the earlier one. Still one can argue that a model using perturbative QCD as a format is not the optimal one in a genuinely non-perturbative situation.

The explicit comparison of the super-symplectic conformal weights associated with spin 0 and spin 1 states on one hand and spin 1/2 and spin 3/2 states on the other hand is carried out at the end of the chapter. The comparison demonstrates that the difference between these states could be understood in terms of super-symplectic particle contents of the states by introducing only single additional negative conformal weight s_c describing color Coulomb binding. s_c is constant for baryons($s_c = -4$) and in the case of mesons non-vanishing only for pions ($s_c = -5$) and kaons ($s_c = -12$). This leads to an excellent prediction for the masses also in the meson sector since pseudo-scalar mesons heavier than kaon are not Golstone boson like states in this model.

The correlation of the spin of quark-system with the particle content of the super-symplectic sector increases dramatically the predictive power of the model since the allowed conformal weights of super-symplectic bosons are (5, 6, 58). One can even consider the possibility that also exotic hadrons with different super-symplectic particle content exist: this means a natural generalization of the notion of Regge trajectories. This description will be summarized at the end of the chapter.

5.6 Color Magnetic Spin-Spin Interaction And Super-Symplectic Contribution To The Mass Of Hadron

Since $k = 107$ contribution to hadron mass is always non-negative, spin-spin interaction conformal weight and also color Coulombic conformal weight must be subtracted from some additional contribution both in the case of pseudo-scalars and spin 1/2 baryons.

5.6.1 Baryonic case

In the case of baryons the additional contribution could be identified as a 2-particle state of super-symplectic bosons with mass squared $9m_{107}^2$ in case of baryons. The net mass is $s_{CS} = 18m_{107}^2$. The study of $N-\Delta$ system shows that color Coulomb interaction energy for single super-symplectic structural unit corresponds to $\Delta s_{SC} = -2$ in the case of nucleon system so that one has $s_{SC} = 18 \rightarrow 16$. If topological mixing for super-symplectic bosons is same as for U type quarks with conformal weights (5, 6, 58), the already discussed three-particle state of would give $s_{SC} = 5 + 5 + 6 = 16$.

The basic requirement is that the sum of spin-spin interaction conformal weight and s_{CS} reduces to the conformal weight corresponding to the difference of nucleon mass and quark contribution to 774.6 MeV and corresponds to $s = 11$.

One might hope that the situation could be the same for all baryons but it is safer to introduce an additional color Coulombic conformal weight $s_c(B)$ which vanishes for $N - \Delta$ system and hope that it is small as suggested by the fact that quark contributions explain quite satisfactorily the mass differences of baryons. This conformal weight could be assigned to the interaction of quarks via super-symplectic gluons and would represent a correction to the simplest model. Strictly speaking, the term "*color Coulombic* " should be taken as a mere convenient letter sequence.

5.6.2 Pseudo scalars

In the case of pseudo-scalars the situation is not so simple. What is clear that quark masses determine the meson mass in good accuracy.

In this case s_{CS} can be determined uniquely from the requirement that in case of pion it is cancelled the conformal weight characterizing $\rho - \pi$ color magnetic spin-spin splitting:

$$s_{SC} = |\Delta s(\pi, spin - spin)| \ . \tag{5.6}$$

This gives $s_{SC} = 21/4 \simeq 5$.

The conformal weights assignable to gluons and super-symplectic gluon must compensate the negative color magnetic spin-spin splitting making pion a tachyon. The following options represent the extremes

1. A positive color Coulombic conformal weight assignable to ordinary gluons compensates the negative conformal weight .

2. Conformal weight of the lightest topologically mixed super-symplectic gluon takes care of the compensation.

The interpretation as a bound state of unmixed super-symplectic $g = 1$ with $n = 9$ and massless $g = 0$ gluon would require binding conformal weight by 4 units i which looks somewhat strange. The masslessness of $g = 0$ gluon does not support the formation of this kind of bound state. An alternative option is in terms of topological mixing in which case $g = 0$ boson should receive 5 units of conformal weight which is near to the.

Explicit calculations demonstrate that for mesons heavier than pion the role of s_c is to compensate s_{SC}. This suggests that the boson of lowest generation is present only inside $\pi - \rho$ system and prevents the large and negative color magnetic spin-spin interaction conformal weight to make pion a tachyon. The special role of pion could be understood in terms of a phase transition to color confining phase. Note however that the mass of η' could be understood by assuming baryonic super-symplectic boson of conformal weight $s_{SC} = 16$ and fully mixed $k = 118$ quarks.

5.6.3 Formulas for $s_c(H)$ for mesons

There are two options to consider. For option I one has $s_{SC} = 5$ for all mesons. For option II s_{CS} vanishes for all mesons except π and ρ. For option I one obtains the formula

$$s_c(M) = -s_{SC} - \Delta s(M_0, spin - spin) = -5 + |\Delta s(M_0, spin - spin)| \ . \tag{5.6}$$

For option II one has

$$\begin{aligned} s_c(M) &= -5 + |\Delta s(M_0, spin - spin)| \ , \quad M = \pi, \rho \ , & (5.6) \\ s_c(M) &= |\Delta s(M_0, spin - spin)| \ , \quad M \neq \pi, \rho \ . & (5.7) \end{aligned}$$

M_0 refers to the pseudo-scalar.

5.6.4 Formulas for $s_c(H)$ for baryons

In the case of spin 1/2 baryons the requirement that the sum of color Coulombic and color magnetic conformal weights is same as for nucleons fixes the values of $s_c(B)$:

$$\begin{aligned} s_c(B) &= s_0 - s_{SC} - \Delta s(B_{1/2}, spin - spin) = -5 + |\Delta s(B_{1/2}, spin - spin)| \ , \\ s_{SC} &= 16 \ , \\ s_0 &= S(m(n) - m_q(n), 107) \ , \\ m_q(n) &= \sqrt{2m_d^2 + m_u^2} \ , \\ S(x, 107) &\equiv [(\frac{x}{m_{107}})^2] \ . \end{aligned} \tag{5.4}$$

$s_0 = 11$ corresponds to the contribution difference of (say) neutron mass and quark contribution to the nucleon mass. s_{CS} corresponds to the conformal weight due to super-symplectic bosons. In the defining formula for $S(x, 107)$ $[x]$ denotes the integer closest to x.

5.6.5 The conformal weights associated with spin-spin splitting

The general formula for the spin-spin splitting allowing to determine the parameters S_{ij} from the masses of a pair $H^\star - H$ of hadrons (say $\rho - \pi$ or $\Delta - N$). The parameters can be deduced from the observation that the mass difference $m(M^*) - m(M)$ for mesons corresponds to the difference of spin-splitting contributions to the mass:

$$\Delta s(M^*) - \Delta s(M) = S(m(M^*) - m(M), 107) \ . \tag{5.5}$$

For baryons one has

$$\begin{aligned} \Delta s(B^*) - \Delta s(B) &= X_1 - X_0 \ , \\ X_1 &= S(m(B^*) - m_q(B), 107) = \ , \\ X_0 &= S(m(B) - m_q(B), 107) \ . \end{aligned} \tag{5.4}$$

Here $m_q(B) = m_q(B^*)$ denotes the quark contribution to the nucleon mass. The possibility to understand the mass differences of spin 1/2 baryons in terms of differences for$m_q(B)$ inspires the hypothesis that X_0 is constant also for baryons (it vanishes for mesons). If so X_0 can be determined from neutron mass as

$$\begin{aligned} X_0 &= S(m(n) - m_q(n), 107) \ , \\ m_q(n) &= \sqrt{2m_d^2 + m_u^2)} \ . \end{aligned} \tag{5.4}$$

Here $m_q(n)$ is the contribution of quarks to neutron mass.

These formulas are *not* identical with those used in the earlier calculations and the difference is due to the fact that $k = 107$ contributions and quark contributions are calculated separately unless quarks correspond $k = 107$. The formula allows to calculate second order contribution to the mass splitting.

p-Adicization brings in additional constraints. The requirement that the predicted mass of spin 1 boson and spin 3/2 fermion is not larger than than the experimental mass can pose strong constraints the scaling factor $\sqrt{5/(5+Y_e)}$ in the case of non-diagonal hadrons unless one is willing to modify the model for spin-spin splittings. It was already found that in case of $\rho - \pi$ system this implies that top quark mass is at the lower limit of the allowed mass interval. One cannot take these constraints so seriously as the constraints that quark mass contribution is lower than meson mass in the case the quarks do not correspond to $k = 107$.

5.6.6 General mass formula

The general formula for the mass of hadron can be written as a sum of perturbative and non-perturbative contributions as

$$m(H) = m_P + m_{NP} \ . \tag{5.5}$$

Preceding considerations lead to a simple formula for the non-perturbative contribution m_{NP} to the masses of spin 0 and spin 1 doublet of mesons:

$$\begin{aligned} m_{NP}(M) &= \sqrt{s_{NP}(M)} \times m_{107} \ , \\ s_{NP}(M_0) &= 0 \ , \\ s_{NP}(M_1) &= S(m(M^*) - m(M), 107) \ . \end{aligned} \tag{5.4}$$

For spin 1/2 and 3/2 doublet of baryons one has

$$\begin{aligned} m_{NP}(B) &= \sqrt{s_{NP}(B)} \times m_{107} \ , \\ s_{NP}(B_{1/2}) &= S(m_n - \sqrt{2m_d^2 + m_u^2}, 107) \ , \\ s_{NP}(B_{3/2}) &= S(m(B^*) - m_q(B), 107) \ . \end{aligned} \tag{5.3}$$

Perturbative contribution m_P contains in the lowest order approximation only the contribution of quark masses. In the case of diagonal mesons also a contribution from the ordinary color-Coulombic force and color magnetic spin-spin splitting can be present. For heavy mesons this contribution seems necessary since pure quark contribution is exceeds by few per cent the mass of meson.

5.6.7 Spin-spin interaction conformal weights for baryons

Consider now the determination of S_{ij} in the case of baryons. The general splitting pattern for baryons resulting from color Coulombic, and spin-spin interactions is given by the following equations summarize spin-spin splittings for baryons in a form of a table.

baryon	J	J_{12}	Δs^{spin}
N	$\frac{1}{2}$	0	$-\frac{3}{4}S_{d,d}$
Δ	$\frac{3}{2}$	1	$\frac{3}{4}S_{d,d}$
Λ	$\frac{1}{2}$	0	$-\frac{3}{4}S_{d,d}$
Σ	$\frac{1}{2}$	0	$-\frac{3}{4}S_{d,d}$
Σ^*	$\frac{1}{2}$	0	$\frac{1}{4}S_{d,d}$ $+\frac{1}{2}S_{d,s}$
Ξ	$\frac{1}{2}$	0	$-\frac{3}{4}S_{s,s}$
Ξ^*	$\frac{1}{2}$	0	$\frac{1}{4}S_{s,s}$ $+\frac{1}{2}S_{d,s}$
Ω	$\frac{3}{2}$	1	$\frac{3}{4}S_{s,s}$

(5.3)

Spin-spin splittings are deduced from the formulas

$$\begin{aligned} \Delta s^{spin} &= S_{q_1,q_)}(\frac{J_{12}(J_{12}+1)}{2} - \frac{3}{4}) \ , \\ &+ \frac{1}{4}(S_{q_1,q_3} + S_{q_2,q_3})(J(J+1) - J_{12}(J_{12}+1) - \frac{3}{4}) \ , \end{aligned} \tag{5.2}$$

where J_{12} is the angular momentum eigenvalue of the "first two quarks", whose value is fixed by the requirement that magnetic moments are of correct sign.

The masses determine the values of the parameters uniquely if one assumes that color binding energy is constant as indeed suggested by the very notion of M_{107} hadron physics. The requirement is that the mass difference squared for $\Delta - N$, $\Sigma^* - \Sigma$, and $\Xi^* - \Xi$ come out correctly.

Consider now the values of S_{ij} for the models assuming $k = 113$ light quarks and dynamical $k(s)$.

1. For $N - \Delta$ system the equation is

$$S_{d_{113},d_{113}} = \frac{1}{3}S(m(\Delta) - m_q(N), 107) - S(m(N) - m_q(N), 107) \ .$$

   Here $m_q(N)$ refers to the quark contribution to the baryon mass.

2. For $\Sigma^* - \Sigma$ system the basic equation can be written as

$$S_{d_{114},s_{110}} = 2[S(m(\Sigma^*) - m_q(\Sigma), 107) - S(m(\Sigma) - m_q(\Sigma), 107) - S(d_{114}, d_{114})] \ .$$

   One must make some assumption in order to find a unique solution. The simplest assumption is that $S_{d_{114},d_{114}} = S_{d_{114},s_{110}}$. This implies

$$S_{d_{114},d_{114}} = \frac{1}{3}[S(m(\Sigma^*) - m_q(\Sigma), 107) - S(m(\Sigma) - m_q(\Sigma), 107)] \ .$$

3. In the case of $\Xi^* - \Xi$ system the equation is

$$S_{s_{110},s_{110}} = -\frac{1}{2}S_{d_{113},s_{110}} + [S(m(\Xi^*) - m_q(\Xi), 107) - S(m(\Xi) - m_q(\Xi), 107)] \ .$$

   If one assumes $S_{s_{110},s_{110}} = S_{d_{113},s_{110}}$ one obtains

$$S_{s_{110},s_{110}} = \frac{1}{3}[S(m(\Xi^*) - m_q(\Xi), 107) - S(m(\Xi) - m_q(\Xi), 107)] \ .$$

The resulting values of the parameters characterizing baryonic spin-spin splittings are represented by the following equations.

$S_{d_{113},d_{113}}$	$S_{d_{114},d_{114}}$	$S_{d_{114},s_{110}}$	$S_{s_{110},s_{110}}$	$S_{d_{113},s_{110}}$
7	6	6	$\frac{8}{3}$	$\frac{8}{3}$

(5.3)

The mass squared unit used is m_0^2 and $k = 107$ defines the p-adic length scale used. The elements of $S_{i,j}$ between different p-adic primes are assumed to be vanishing. The matrix elements are quite near to each other which raises the hope that the model indeed makes sense.

Color Coulombic binding conformal weights are given by the expression $s_c = -5 + |\Delta s(B_{1/2}, spin-spin)|$. The weights are represented by the following equations.

$baryon$	N	Σ	Ξ
s_c	$\frac{1}{4}$	$-\frac{1}{2}$	-3

(5.3)

Some remarks are in order.

1. A good sign is that the values of s_c are small as compared to the value of $s_{CS} = 18$ in all baryons so that only a small correction is in question.

2. The magnitude of s_c increases with the mass of baryon which does not conform with the expectations raised by ordinary QCD evolution. Could this mean that asymptotic freedom means that the color interaction between quarks occurs increasingly via super-symplectic gluons? For $N - \Delta$ system the actual value of s_c should vanish.

3. One might worry about the fact the color binding conformal weights are not integral valued. The total conformal weights determining the mass squared are however integers.

5.6.8 Spin-spin interaction conformal weights for mesons

The values of mesonic interaction strengths $S_{i,j}$ can in principle deduced from the observed mass splittings. The following equations summarize the spin-spin splitting pattern for mesons in a form of table.

$meson$	Δs^{spin}
π	$-\frac{3}{4}S_{d,d}$
ρ	$\frac{1}{4}S_{d,d}$
η	$-\frac{3}{4}S_{d,d}$
ω	$\frac{1}{4}S_{d,d}$
$K^{\pm}, K^0(CP=1)$	$-\frac{3}{4}S_{d,s}$
$K^0(CP=-1)$	$-\frac{3}{4}S_{d,s}$
$K^{*,\pm}, K^{*,0}(CP=1)$	$\frac{1}{4}S_{d,s}$
$K^{*,0}(CP=-1)$	$\frac{1}{4}S_{d,s}$
η'	$-\frac{3}{4}S_{s,s}$
Φ	$\frac{1}{4}S_{s,s}$
η_c	$-\frac{3}{4}S_{c,c}$
Ψ	$\frac{1}{4}S_{c,c}$
$D^{\pm}, D^0(CP=1)$	$-\frac{3}{4}S_{d,c}$
$D^0(CP=-1)$	$-\frac{3}{4}S_{d,c}$
$D^{*,\pm}, D^{*0}(CP=1)$	$\frac{1}{4}S_{d,c}$
$D^{*0}(CP=-1)$	$\frac{1}{4}S_{d,c}$

(5.3)

Consider the spin-spin interaction for mesons.

1. For $\rho - \pi$ system one has

$$S_{d_{113},d_{113}} = s(m(\rho) - m_q(\pi)) \ .$$

   Using $s(\rho) = 14$ and $s(\pi) = 0$ gives $S(d_{113}, d_{113}) = 13$.

2. $\omega - \eta$ system one obtains

$$S_{q_{109},q_{109}} = S(m(\omega) - m_q(\eta), 107)$$

3. $K^\star - K$-splitting gives $S_{d_{114},s_{109}} = S(m(K^*) - m_q(K), 107)$.

4. $\Phi - \eta'$ splitting gives $S_{q_{107},q_{107}} = S(m(\Phi) - m_q(\eta'), 107)$.

5. $D^* - D$ mass splitting gives $S_{d_{113},c_{105}} = S(m(D^*) - m_q(D), 107)$.

6. $\Psi - \eta_c$ mass difference gives $S_{c_{104},c_{104}} = s(m(\Psi) - m_q(\eta_c), 107)$.

The results for the spin-spin interaction strengths S_{ij} are summarized in the table below. q_{109} refers to u, d, and s quarks.

$S_{d_{113},d_{113}}$	$S_{q_{109},q_{109}}$	$S_{q_{107},q_{107}}$	$S_{d_{114},s_{109}}$	$S_{d_{113},c_{105}}$	$S_{c_{104},c_{104}}$
7	1	0	3	2	0

(5.3)

Note that interaction strengths tend to be weaker for mesons than for baryons. For scaled up quarks the value of interaction strength tends to decrease and is smaller for non-diagonal than diagonal interactions. Since the values of $k(q_i)$ maximize the quark contribution to hadron masses, the interaction strength produce a satisfactory mass fit for hadrons with errors of not larger than about five cent.

The color Coulombic binding conformal weights for meson states are given by the following equations.

meson	π	K	η	η'	D	ψ
$s_c(I)$	$+1/4$	$-4-1/4$	-6	$-3-1/4$	$-4-1/2$	-5
$s_c(II)$	$1/4$	$3/4$	1	$1+3/4$	$1/2$	0

(5.3)

For option I $g = 1$ boson is present in all mesons. The magnitude of s_c increases with the mass of the meson and more or less compensates $s_{CS} = 5$. This forces to consider the possibility that only pion contains the super-symplectic boson compensating the large and negative spin-spin interaction conformal weight making the state tachyon otherwise. The alternative possibility is that positive color Coulomb energy due to sea gluons does this. For option II s_c is relatively small and positive for this option.

5.7 Summary About The Predictions For Hadron Masses

The following tables summarize the predictions for baryon masses following from the proposed model with optimal choices of the integers $k(q)$ characterizing the mass scales of quarks and requiring that the predicted isospin splittings are of same order than the observed splittings. This condition is non-trivial: for instance, in case of B meson the smallness of splitting forces the condition $k(b) = k(d) = k(u) = 104$ so that mass squared is additive and the large contribution of b quark minimizes the isospin splitting.

$Meson$	$quarks$	$m_{pr}(M)/MeV$	m_{exp}/MeV
π^0	d_{113}, u_{113}	140.0	135.0
π^+	d_{113}, u_{113}	140.0	139.6
ρ^0	d_{113}, u_{113}	756	772
ρ^+	d_{113}, u_{113}	756	770
K^0	d_{114}, s_{109}	496	498
K^+	u_{114}, s_{109}	486	494
K^{0*}	d_{114}, s_{109}	896	900
K^{+*}	u_{114}, s_{109}	892	891
η	$u_{109}, d_{109}, s_{109}$	522	549
ω^0	$u_{109}, d_{109}, s_{109}$	817	783
η'	$u_{107}, d_{107}, s_{107}, c_{107}$	1144	958
Φ	$u_{107}, d_{107}, s_{107}, c_{107}$	1144	1019
η_c	c_{104}	3098	2980
D^0	c_{105}, u_{113}	1642	1865
D^+	c_{105}, d_{113}	1655	1870
D^{*0}	c_{105}, u_{114}	1971	2007
D^{*+}	c_{105}, d_{114}	1985	2010
F	$c_{105}, s(106)$	1954	2021
Υ	b_{103}	10814	9460
B	$b_{104}, d_{104}, u_{104}$	5909	5270

Table 8: The prediction of meson masses. The model assumes the maximal value of CP_2 mass allowed by η' mass and the condition $Y_e = 0$ favored by top quark mass.

5.7.1 Meson masses assuming that all pseudo-scalars are Goldstone bosons

In the case of meson masses the predictions for masses are not so good as for baryons. Errors are at worst about 5 per cent. For non-diagonal mesons the predicted masses are smaller than actual masses and in the case of kaons excellent. Also the prediction of pion mass is good but about 5 per cent too large. In the case of diagonal mesons ordinary color interactions could reduce the predicted masses in case that they are larger than actual ones.

5.7.2 Meson masses assuming that only pion and kaon are Goldstone bosons

The Golstone boson interpretation does not seem completely satisfactory. In order to make progress one can check whether the masses associated with super-symplectic bosons could serve as basic units for pseudo-scalar and vector boson masses. A more general fit would be based on the use of fictive boson B_{107} with mass m_{107} as a basic unit in $k = 107$ contribution to the mass. **Table ??** gives very accurate formulas for the meson masses in terms of the scale m_{107} and quark contribution to the masses.

Table **Table ??** demonstrates following.

1. For mesons heavier than kaons, the masses can be expressed effective sums of masses for quarks and many-particle state formed by super-symplectic bosons allowed by the topological mixing of U type quarks. For lighter mesons it is not possible to express the masses in terms of the conformal weights of quarks and super-symplectic gluons. This suggests that one must introduce positive color Coulombic conformal weight as the analog of positive Coulomb energy. The simplest assumption is that this conformal weight compensates for the color-magnetic spin-spin splitting in the case pf pseudo-scalars and to the Goldstone boson option. This option indeed looks more reasonable.

2. For $\pi - \rho$ *resp.* $K - K^*$ systems the masses can be expressed using effective $7B_{107}$ state state *resp.* $3B_{107}$ state. Second order contribution to the conformal weight from the super-

$Meson$	$quarks$	$m_{pr}(M)/MeV$	m_{exp}/MeV
π^0	d_{113}, u_{113}	140.0	135.0
π^+	d_{113}, u_{113}	140.0	139.6
ρ^0	$7B_{107} + d_{113}, u_{113}$	758	772
ρ^+	$7B_{107} + d_{113}, u_{113}$	758	770
K^0	d_{114}, s_{109}	496	498
K^+	u_{114}, s_{109}	486	494
K^{0*}	$3B_{107} + d_{114}, s_{109}$	901	900
K^{+*}	$3B_{107} + u_{114}, s_{109}$	891	891
η	$B_{SC,1} + u_{118}, d_{118}, s_{118}$	548	549
ω^0	$2B_{SC,1} + u_{118}, d_{118}, s_{118}$	803	783
η'	$2B_{SC,1} + B_{SC,2} + q_{118}$	959	958
Φ	$2B_{SC,1} + B_{SC,2} + q_{118}$	959	1019
η_c	$2B_{SC,1} + c_{105}$	2929	2980
Ψ	$3B_{SC,1} + c_{105}$	3098	3100
D^0	$2B_{SC,1} + c_{106}, u_{118}$	1853	1865
D^+	$2B_{SC,1} + c_{106}, d_{118}$	1850	1870
D^{*0}	$3B_{SC,1} + c_{106}, u_{118}$	2019	2007
D^{*+}	$3B_{SC,1} + c_{106}, d_{118}$	2016	2010
F	$3B_{SC,2} + c_{105}, s(113)$	2010	2021
Υ	$B_{SC,3} + b_{104}$	9441	9460
$B^{\pm}$	$3B_{SC,2} + b_{105}, d_{105}, u_{105}$	5169	5270

Table 9: Table demonstrates that scalar and vector meson masses can be effectively regarded as expressible in terms of quark contribution and contribution coming from many particle states of super-symplectic bosons $B_{SC,k}$, $k = 1, 2, 3$, with conformal weights (5, 6, 58) associated also with U type quarks. B_{107} denotes effective super-symplectic boson with conformal weight 1 and mass $m_{107} = 233.6$ MeV. $Y_e = 0$ favored by top quark mass is assumed.

symplectic color interaction can explain the too small mass of ρ and too large mass of π if it interferes with the corresponding quark mass contribution.

3. For pseudo-scalars heavier than kaon the mass of the super-symplectic meson is not completely compensated by spin-spin splitting for the pseudo-scalar state so that Goldstone boson interpretation does not make sense anymore. In the case of heavy mesons the predicted masses of pseudo-scalars are slightly below the actual mass.

4. The predicted masses are not larger than actual masses (ω_0 is the troublemaker) if one assumes 2.5 per cent reduction of CP_2 mass scale for which top quark mass is at the lower bound of the allowed mass range.

5. Color magnetic spin-spin splitting parameters can be deduced from the differences of super-symplectic conformal weights for pseudo-scalar and spin one boson. There is however no absolute need for this perturbative construct.

6. One can consider the possibility that the super-symplectic boson content is actual and correlates with the spin of quark-antiquark system for mesons heavier than kaons. The point would be that the representability in terms of super-symplectic bosons would make the model for the color magnetic spin-spin splittings highly predictive. This interpretation makes sense in the case of $\pi - \rho$ and $K - K^*$ systems only if one introduces negative color Coulombic conformal weight s_c. For heavier mesons only this contribution would be second order in p which is more or less consistent with the view about color coupling evolution. $\pi - \rho$ would correspond to B_1 ($s = 5$) and $2B_2$ ($s = 12$) ground states with color Coulombic conformal weight $s_c = -12$. $K - K^*$ would correspond to $2B_2$ ($s = 12$) and $3B_1$ with $s_c = -12$. The presence of ground state bosons saves π and K from becoming tachyons.

Whatever the correct physical interpretation of the mass formulas represented by **Table ??** is, it is clear that m_{107} defines a fundamental mass scale also for meson systems.

5.7.3 Baryon masses

One can ask whether the representability of spin-spin splitting in terms of super-symplectic conformal boson content is possible also in the case of baryons so that perturbative formulas altogether would not be necessary. The physical interpretation would be that the total spin of baryonic quarks correlates with the content of super-symplectic bosons. The existence of this kind of representation would be one step towards understanding of also spin-spin splitting from first principles.

This is indeed the case if one accepts negative color Coulombic conformal weight $s_c = -4$. What is disturbing is that the sign of the corresponding parameter is positive for mesons and compensates color magnetic spin-spin splitting for pseudo-scalars (for Goldstone option). Spin 1/2 ground states would correspond to $3B_1$ with conformal weight $s = 15$, one B_1 for each valence quark. Spin 3/2 states would correspond to $5B_1$ with $s = 25$ in the case of Δ, to $2B_1 + B_2$ in the case of Σ^* with $s = 23$, and to $B_1 + 3B_2$ with $s = 24$ in case of Ξ^*.

From **Table 10** for the predicted baryon masses one finds that the predicted masses are slightly below the experimental masses for all baryons except for some baryons in $N - \Delta$ multiplet and for Ω. The reduction of the CP_2 mass scale by a factor of order per cent consistent with what is known about top quark mass cures this problem (also ordinary color interactions could take of the problem).

In principle the quark contribution to the hadron mass is measurable. Suppose that color binding conformal weight can be assigned to the color interaction in super-symplectic degrees of freedom alone. Above the "ionization" energy, which corresponds to the contribution of quarks to the mass of hadron, valence quark space-time sheet can separate from the hadronic space-time sheet in the collisions of hadrons. This threshold might be visible in the collision cross sections for say nucleon-nucleon collisions. For nucleons this energy corresponds to 170 MeV.

5.8 Some Critical Comments

The number theoretical model for quark masses and topological mixing matrices and CKM matrix as well as the simple model for hadron masses give strong support for the belief that the general

$Baryon$	$quarks$	$m_{pr}(B)/MeV$	m_{exp}/MeV
p	$3B_1 + u_{113}, d_{113}$	942.3	938.3
n	$3B_1 + u_{113}, d_{113}$	949.8	939.6
Δ^{++}	$5B_1 + u_{113}$	1230	1231
Δ^{+}	$5B_1 + u_{113}, d_{113}$	1238	1235
Δ^{0}	$5B_1 + u_{113}, d_{113}$	1245	1237
Δ^{-}	$5B_1 + d_{113}$	1253	≤ 1238
Λ	$3B_1 + u_{114}, d_{114}, s_{111}$	1090	1116
Σ^{+}	$3B_1 + u_{114}, s_{110}$	1165	1189
Σ^{0}	$3B_1 + u_{114}, d_{113}, s_{110}$	1171	1192
Σ^{-}	$3B_1 + d_{114}, s_{110}$	1178	1197
Σ^{*+}	$2B_1 + 2B_2 + u_{114}, s_{110}$	1381	1385
Ξ^{0}	$2B_1 + 2B_2 + u_{113}, s_{110}, s_{111}$	1301	1315
Ξ^{-}	$3B_1 + d_{113}, s_{110}$	1288	1321
Ξ^{*0}	$B_1 + 3B_2 + u_{113}, s_{110}$	1531	1532
Ξ^{*-}	$B_1 + 3B_2 + d_{113}, s_{110}$	1505	1535
Ω^{-}	$3B_1 + s_{108}, s_{111}$	1667	1672
Λ_c	$3B_1 + d_{110}, u_{110}, c_{106}$	2261	2282
Λ_b	$3B_1 + d_{108}, u_{108}, b_{105}$	5390	5425

Table 10: The predictions for baryon masses assuming $Y_e = 0$. One can represent color magnetic spin-spin splittings by assuming different super-symplectic boson content for spin 1/2 and spin 3/2 states. The number of super-symplectic bosons need not be however different for them.

vision is correct. One must bear in mind that the scenario need not be final so that the basic objections deserve an explicit articulation.

5.8.1 Is the canonical identification the only manner to map mass squared values to their real counterparts

In p-adic thermodynamics p-adic particle mass squared is mapped to its real counterpart by the canonical identification. If the $O(p)$ contribution corresponds to non-trivial rational number, the real mass is of order CP_2 mass. This allows to eliminate a large number of exotics. In particular, it implies that the modular contribution to the mass squared must be of form np rather than $(r/s)p$. This assumption is absolutely crucial in the model of topological mixing matrices and CKM matrix.

One can however question the use of the standard form of the canonical identification to map p-adic mass squared to its real counterpart. The requirement that p-adic and real S-matrix elements (in particular coupling constants) are related in a realistic manner, forces a modification of the canonical identification. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R \rightarrow R_p}$ in which the expansion of rational number $q = r/s = \sum r_n p^n / \sum s_n p^n$ is replaced with the rational number $q_1 = r_1/s_1 = \sum r_n p^{-n} / \sum s_n p^{-n}$ interpreted as a p-adic number:

$$q \;=\; \frac{r}{s} = \frac{\sum_n r_n p^n}{\sum_m s_n p^n} \rightarrow q_1 = \frac{\sum_n r_n p^{-n}}{\sum_m s_n p^{-n}} = \frac{I(r)}{I(s)} \quad . \tag{5.4}$$

The nice feature of this variant of the canonical identification is that it respects quantitative behavior of amplitudes, respects symmetries, and maps unitary matrices to unitary matrices if the matrix elements correspond to rationals (or generalized rationals in algebraic extension of rationals) if the p-adic integers involved are smaller than p. At the limit of infinitely large p this is always satisfied.

Quite generally, the thermodynamical contribution to the particle mass squared is in the lowest p-adic order of form rp/s, where r is the number of excitations with conformal weight 1 and s

the number of massless excitations with vanishing conformal weight. The real counterpart of mass squared for the ordinary canonical identification is of order CP_2 mass by $r/s = R + r_1 p + ...$ with $R < p$ near to p. Hence the states for which massless state is degenerate become ultra heavy if r is not divisible by s. For the new variant of canonical identification these states would be light.

Even worse, the new form does not require the modular contribution to the p-adic mass squared to be of form np. Some other justification for this assumption would be needed. The first guess is that the conditions on mass squared plus probability conservation might not be consistent with unitarity unless the modular contribution to the mass squared remains integer valued in the mixing (note that all integer values are not possible). Direct numerical experimentation however shows that that this is not the case.

The predicted integer valued contributions to the mass squared are minimal in the case of u and d quarks and very nearly maximal in the case t and b quarks. This suggests a possible way out of the difficulty. Perhaps the rational valued p-adic mass squared of u and d quarks are minimal and those of b and t quarks maximal or nearly maximal. This might also allow to improve the prediction for the CKM matrix.

The objection against the use of the new variant of canonical identification is that the predictions of p-adic thermodynamics for mass squared are not rational numbers but infinite power series. p-Adic thermodynamics itself however defines a unique representation of probabilities as ratios of generalized Boltzmann weights and partition function and thus the variant of canonical identification might indeed generalize. If this representation generalizes to the sum of modular and Virasoro contributions, then the new form of canonical identification becomes very attractive. Also an elegant model for the masses of intermediate gauge bosons results if $O(p)$ contribution to mass squared is allowed to be a rational number.

5.8.2 Uncertainties related to the CP_2 length scale

The uncertainties related to the CP_2 length scale mean that one cannot take the detailed model for hadron masses too literally unless one takes the recent value of top quark mass at face value and requiring $(Y_e = 0, Y_t = 1)$ in rather high accuracy. This constraint allows at most 2.5 per cent reduction of the fundamental mass scale and baryonic masses suggest a 1 per cent reduction. The accurate knowledge of top quark mass is therefore of fundamental importance from the point of view of TGD.

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