Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

M. Pitkänen,

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Email: matpitka6@gmail.com.
`http://tgdtheory.com/public_html/`.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

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Abstract

Contrary to the original expectations, TGD seems to allow a generalization of the space-time SUSY to its 8-D variant with masslessness in 4-D sense replaced with masslessness in 8-D sense. The algebra in question is the Clifford algebra of fermionic oscillator operators associated with given partonic 2-surface. In terms of these algebras one can in turn construct generators super-symplectic algebra as stringy Noether charges and also other super-conformal algebras and even their Yangians used to create quantum states. This also forces to generalize twistor approach to give 8-D counterparts of ordinary 4-D twistors.

The 8-D analog of super Poincare algebra emerges at the fundamental level through the anti-commutation relations of the fermionic oscillator operators. For this algebra $\mathcal{N} = \infty$ holds true. Most of the states in the representations of this algebra are massive in $4-D$ sense. The restriction to the massless sector gives the analog of ordinary SUSY with a finite value of $\mathcal{N}$ - essentially as the number of massless states of fundamental fermions to be distinguished from elementary fermions. The addition of a fermion in particular mode defines particular super-symmetry. This super-symmetry is broken due to the dynamics of the Kähler-Dirac operator, which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a restriction to a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of a fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

   The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the super-charges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in D=10 and D=11 as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

   In superstring theory the hermiticity of super generator G_0 giving as its square scaling generator L_0 is strong argument in favor if Majorana spinors since G_0 appears as a propagator. In TGD framework the counterparts of G_0 in quark and lepton sector carry fermion number so that identification as a propagator does not make sense. The recent formulation of scattering amplitudes in terms of Yangian algebra allows to circumvent the problem. Fundamental propagators are fermion propagators for fermions massless in 8-D sense.

2. The spinor components of imbedding space spinors identifiable with physical helicities and with fixed fermion number correspond to the generators of the SUSY algebra at QFT limit. This SUSY is broken due to electroweak and color interactions. Right-handed neutrinos do not have these interactions but there is a mixing with left-handed neutrinos due to the mixing of M^4 and CP_2 gamma matrices in the Kähler-Dirac gamma matrices appearing in the K-D action. Therefore also the $\mathcal{N} = 2$ sub-SUSY generated by right-handed neutrinos is broken.

In this chapter the details of the above general picture are discussed. A brief summary of the basic aspects of SUSY is included and the constraints on the formulation of the SUSY limit are discussed in detail. The formulation itself is left to the reader possessing the needed technical skills. In principle there seems to be no reason preventing the formulation in terms of super fields: the only new elements relate to the fact that baryon and lepton number are conserved now so that Majorana spinors are replaced with Weyl spinors combining to from Dirac spinors.

1 Introduction

Contrary to the original expectations, TGD seems to allow the analog of the space-time supersymmetry. This became clear with the increased understanding of both Kähler action and Kähler-Dirac action [K16, K3]. It is however far from clear whether SUSY type QFT can define the QFT limit of TGD and whether this kind of formulation is the optimal one.

1.1 Is The Analog Of Space-Time SUSY Possible In TGD?

The basic question is whether the huge algebras with super-conformal structure acting as symmetries of quantum TGD give rise to a SUSY algebra at space-time level (meaning super-Poincare symmetry). A more technical question is whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT or whether one must generalize this approach just as it seems necessary to generalize the notion of twistor by replacing masslessness in 4-D sense with masslessness in 8-D sense.

1. From the beginning it was clear that super-conformal symmetry is realized in TGD but differs in many respects from the more standard realizations such as $\mathcal{N}=1$ SUSY realized in MSSM [B4] involving Majorana spinors in an essential manner.

   Note that the belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry can be used as an objection against TGD. Besides Majorana spinors Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D=8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in D=10 and D=11 as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

2. In TGD framework the covariantly constant right-handed neutrino generates the supersymmetry at the level of CP_2 geometry. The original idea was that the construction of super-partners would be more or less equivalent with the addition of *covariantly constant* right-handed neutrino and antineutrinos to the state. It was however not clear whether space-time supersymmetry is realized at all since one could argue that that by covariant constancy these states are just gauge degrees of freedom or that SUSY is only realized for the spinor harmonics of imbedding space with 8-D notion of masslessness. Much later it became clear that covariantly constant right handed neutrino indeed represents gauge degree of freedom at *space-time level.*

3. A more general general SUSY algebra is generated by the modes of the Kähler-Dirac operator at partonic 2-surface being also Clifford algebra. This algebra can be associated with the ends of the boundaries of string world sheets and each string defines its own sub-algebra of oscillator operators.

   (a) At first it would seem that the value of $\mathcal{N}$ can be very large - even infinite as the fact that fermionic oscillator operators are labelled by conformal weight. It is however the number of *massless states* in M^4 sense, which determines the value of $\mathcal{N}$ for SUSY in M^4: for the full theory the analog of SUSY in H $\mathcal{N}=\infty$ could make sense. Indeed, super-symplectic generators bring in the analog of wave function of fermion at partonic 2-surfaces and constant wave functions and therefore massless states are expected to be favored by Uncertainty Principle. The dimension of SUSY algebra is expected to just the number of spinor components of the imbedding space spinor possessing physical imbedding space helicity.

   A more general situation is that the conformal gauge algebra is its sub-algebra isomorphic to the entire algebra having conformal weights coming as n-ples of those for the full algebra. The conformal gauge symmetry would be broken so that only the super-symplectic generators for which the conformal weight is proportional to fixed integer $n \in \{1, 2, ...\}$ annihilate the physical states. This increases the value of $\mathcal{N}$ and a possible interpretation is in terms of improved measurement resolution. N would also correspond to the value of Planck constant $h_{eff}/n = N$ and N would label phases of dark matter and also a hierarchy of criticalities. As N increases, super-conformal gauge degrees of freedom are transformed to physical ones. This kind of situation might be possible for quantum deformations of the oscillator operator algebra characterized by quantum phase as $q = exp(i2\pi/N)$ and possible by the 2-dimensionality of string world sheets.

An alternative manner to see the situation is as a fractionization of conformal weights due to the emergence of N-fold coverings of space-time surfaces analogous to coverings of complex plane defined by analytic function $z^{1/N}$. Only the states with integer conformal weights would be annihilated by the original conformal algebra and quantum group would describe the situation.

The SUSY in standard sense is expected to be broken. First, the notion of masslessness is generalized: fermions associated with the boundaries of string world sheets have light-like 8-momentum and therefore can be massive in 4-D sense: this allows to generalize twistor description to massive case [K22]. The ordinary 4-D SUSY is expected to emerge only as an approximate description in massless sector (as it also appears in dimensional reduction). Secondly, standard SUSY characterizes the QFT description obtained by replacing many-sheeted space-time time with a slightly curved region of Minkowski space.

(b) SUSY algebra is replaced with Clifford algebra at the level of partonic 2-surfaces and the generators can be identified as fermionic oscillator operations at the end points of fermionic lines, which are light-like geodesics. Light-like four-momenta in anti-commutation relations are replaced with 8-D light-like momenta demanding a generalization of twistor approach. The octonionic realization of twistors is a very attractive possibility in this framework and quaternionicity condition guaranteeing associativity leads to twistors which are almost equivalent with ordinary 4-D twistors.

The space-time super-symmetry means addition of fermion to the state assign to a partonic surface and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible. The non-hermitian character of super conformal generator $G \neq G^\dagger$ made impossible the naive generalization of stringy rules to TGD framework since they involve G as the analog of fermionic propagator. This problem disappears in the twistor Yangian approach [K22].

(c) The notion of super-field does not seem natural in the full TGD framework but would be replaced with a Yangian of the super-symplectic algebra and related conformal algebras with generators identified as Noether charges assignable to strings connecting partonic 2-surfaces. Multi-locality coded by Yangian in the scale of partonic surfaces is a new element. There is also the hierarchy of Planck constants interpreted in terms of dark matter and Zero Energy Ontology.

1.2 What Happens When Many-Sheeted Space-Time Is Approximated With Minkowski Space?

The question is what happens when one replaces many-sheeted space-time with a region of Minkowski space and identifies gauge potentials as sum of the induced gauge potentials?

1. It is plausible that gauge theory like description is a good approximation. But what happens to the SUSY? Can one replace 8-D light-likeness with 4-D light-likeness and describe massivation in terms of Higgs mechanism and analogous - not very successful - mechanisms for 4-D SUSY? It is quite possible that this is not possible: 4-D QFT approximation taken partonic 2-surfaces to points might miss too much of physics and too much elegance.

2. Should one try to find a generalization of ordinary 4-D SUSY allowing the description of massive particles in terms of 8-D light-likeness? This would allow also to understand baryons

and lepton number conservation as 8-D chiral symmetry, to avoid Majorana spinors, and would force a new view about QCD color. Maybe the attempt to describe things by QFT or even ordinary string model is like an attempt to describe quantum physics using classical mechanics. To my opinion generalization of twistor approach from 4-D to 8-D context based on the notion of super-symplectic Yangian is a more promising approach than sticking to effective field theory thinking [K22].

The first guess - much before the understanding of the Kähler-Dirac equation and the role of right-handed neutrino - was that it might be possible to formulate even quantum TGD proper in terms of super-field defined in the world of classical worlds (WCW). Super-fields could provide in this framework an elegant book-keeping apparatus for the elements of local Clifford algebra of WCW extended to fields in the $M^4 \times CP_2$, whose points label the positions of the tips of the causal diamonds CDs). At this moment I feel skeptic about this approach.

1.3 What SUSY QFT Limit Could Mean?

What the actual construction of SUSY QFT limit means depends on how strong approximations one wants to make.

1. The minimal approach to SUSY QFT limit is based on an approximation assuming only the super-multiplets generated from fundamental fermions by right-handed neutrino or both right-handed neutrino and its antineutrino.
2. Elementary are particles are composed of fundamental fermions so that the super-multiplets are more complex for them. One of the key predictions of TGD is that elementary particles can be regarded as bound states of fermions and anti-fermions located at the throats of two wormhole contacts. As a special case this implies bosonic emergence meaning that it QFT limit can be defined in terms of Dirac action.

1.4 Scattering Amplitudes As Sequences Of Algebraic Operations

The attempts to generalize twistor Grassmannian approach in TGD framework led to a revival an old idea about scattering amplitudes as representations of sequences of algebraic operations connecting two sets of algebraic objects. Any two sequences connecting same sets would give rise to same scattering amplitudes. One might say that instead of mathematics representing physics physics represents mathematics.

1. In Yangian approach fundamental vertices correspond to product and co-product for the generators of Yangian of super-symplectic algebra with charges identified in terms of Noether charges assignable to strings connecting partonic 2-surfaces [K22]. Scattering amplitudes are obtained by the analog of Wick contraction procedure in which fermion lines connecting different vertices would be obtained. This also allows creation of fermion pairs from vacuum with members at opposite throats of wormhole contact defining the fundamental boson propagators. This picture about bosonic emergence is similar to the earlier one.
2. Yangian approach has huge symmetries since the duality symmetry of string models generalizes in the sense that one can freely move the ends of the lines and snip off loops in this manner. The fact that all diagram representing computation connecting same initial and final states are equivalent implies huge number of constraints and it is clear that ordinary Feynman diagrammatics cannot satisfy these constraints. Twistor diagrammatics could however do so since it has turned out that twistor diagrams indeed have symmetries analogous to this kind of symmetry. It seems however that one must generalized 4-D twistors to 8-D ones so that the twistor Yangian approach looks like the most promising approach at this moment: if of course applies to full theory rather than only in massless sector of the theory.

The plan of the chapter reflects partially my own needs. I had to learn space-time supersymmetry at the level of the basic formalism and the best manner to do it was to write it out. As the vision about fermions in TGD crystallized it became also clear that SUSY QFT in Feynman graph formulation does not catch the simplicity of what I identify as fundamental formulation of TGD. Therefore I dropped a lot of material in the original chapter.

1. The chapter begins with a brief summary of the basic concepts of SUSYs without doubt revealing my rather fragmentary knowledge about these theories. The original belief was that super-field formalism could be generalized to TGD framework. At this moment I however believe that Yangian approach is more realistic one for reasons already mentioned. Therefore I have dropped the section about the formalism proposed earlier. I have also dropped material about various attempts to understand the role right-handed neutrinos. The chapter in its recent form is about whether SUSY limit could emerge from TGD. Just general conditions are formulated since I do not have the expertise to formulate the theory in detail.

2. The Clifford algebra of fermionic oscillator operators assignable to the ends of strings connecting partonic 2-surfaces replaces SUSY algebra, and anti-commutation relations realize the analog of super Poincare symmetry. Since the number of conformal weights is infinite, one would naively expect $\mathcal{N} = \infty$ SUSY. States are however created by super-symplectic generators bringing in the analog of wave function of fermion at partonic 2-surface rather fermionic oscillator operators. Also conformal gauge invariance conditions are satisfied, and this is expected to change the situation. For ideal measurement resolution only the fermionic oscillator operators with vanishing conformal weight are expected to remain effective. The description of finite measurement resolution in terms of quantum variant of fermionic anti-commutation relations is expected to increase the number of conformal weights so that $\mathcal{N}$ increases for dark matter. Right-handed neutrino and its antineutrino would define the least broken sub-algebra of SUSY.

3. Twistors have become a part of the calculational arsenal of SUSY gauge theories, and TGD leads to a proposal how to avoid the problems caused by massive particles by using the notion of masslessness in 8-D sense and the notion of induced octo-twistor [K22]. The equivalence of octonionic spinor structure with the ordinary one leads also to the localization of spinors to string world sheets and fermions at light-like geodesics at their boundaries at partonic 2-surfaces. Already the fundamental formulation keeps just the knowledge that particle moves along light-like geodesic of $M^4 \times CP_2$ and strings connect partonic 2-surfaces. Could QFT limit could be formulated as SUSY in $M^4 \times S^1$ allowing to describe massive particles as massless particles in $M^4 \times S^1$? Or could simplified string model type description in $M^4 \times S^1$ make sense?

4. With the improved understanding of Kähler-Dirac equation one can develop arguments that $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY generated by right-handed neutrino emerges naturally in TGD framework and corresponds to the addition of a collinear right-handed neutrino and and antineutrino to the state representing massless particle.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L2]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L3].

2 SUSY Briefly

The Tasi 2008 lectures by Yuri Shirman [B6] provide a modern introduction to 4-dimensional $\mathcal{N} = 1$ super-symmetry and super-symmetry breaking. In TGD framework the super-symmetry is 8-dimensional super-symmetry induced to 4-D space-time surface and one $\mathcal{N} = 2N$ can be large so that this introduction is quite not enough for the recent purposes. This section provides only a brief summary of the basic concepts related to SUSY algebras and SUSY QFTs and the breaking of super-symmetry is mentioned only by passign. I have also listed the crucial basic facts about $\mathcal{N} > 1$ super-symmetry [B1, B3] with emphasis in demonstrating that for 8-D super-gravity with one time-dimension super-charges are non-Hermitian and that Majorana spinors are absent as required by quantum TGD.

2.1 Weyl Fermions

Gamma matrices in chiral basis.

$$\begin{array}{llll}
\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \ , & \gamma_5 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix} \ , & & \\
\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ , & \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , & \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , & \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ , \\
\overline{\sigma}^0 = \sigma^0 \ , & \overline{\sigma}^i = -\sigma^i \ . & &
\end{array} \tag{2.1}$$

Note that Pauli sigma matrices can be interpreted as matrix representation for hyper-quaternion units.

Dirac spinors can be expressed in terms of Weyl spinors as

$$\Psi = \begin{pmatrix} \eta^\alpha \\ \overline{\chi}^*_{\dot\alpha} \end{pmatrix} \ . \tag{2.2}$$

Note that does not denote complex conjugation and that complex conjugation transforms non-dotted and dotted indices to each other. η and $\overline{\chi}$ are both left handed Weyl spinors and transform according to complex conjugate representations of Lorentz group and one can interpret $\overline{\chi}$ as representing that charge conjugate of right handed Dirac fermion.

Spinor indices can be lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot\alpha\dot\beta}$ and one has

$$\begin{array}{ll}
\eta^\alpha\eta_\alpha = 0 \ , & \overline{\chi}^*_{\dot\alpha}\overline{\chi}^{\dot\alpha *} = 0 \ , \\
\eta\overline{\chi} = \overline{\chi}\eta = \epsilon^{\alpha\beta}\eta_\alpha\overline{\chi}_\beta \ , & \eta^*\overline{\chi}^* = \overline{\chi}^*\eta^* = \epsilon^{\alpha\beta}\eta^*_\alpha\overline{\chi}^*_\beta \ .
\end{array} \tag{2.3}$$

Left-handed and right handed spinors can be combined to Lorentz vectors as

$$\eta^*_{\dot\alpha}\sigma^{\mu\dot\alpha\alpha}\eta_\alpha = -\eta^{*\alpha}\sigma^\mu_{\alpha\dot\alpha}\eta^{*\dot\alpha} \ . \tag{2.4}$$

The SUSY algebra at QFT limit differs from the SUSY algebra defining the fundamental anti-commutators of the fermionic oscillator operators for the induced spinor fields since the Kähler-Dirac gamma matrices defined by the Kähler action are replaced with ordinary gamma matrices. This is quite a dramatic difference and raises two questions.

The Dirac action

$$L = i\overline{\Psi}\partial_\mu\gamma^\mu\Psi - m\overline{\Psi}\Psi \tag{2.5}$$

for a massive particle reads in Weyl representation as

$$L = i\eta^*\partial_\mu\sigma^\mu\eta + i\overline{\chi}^*\partial_\mu\overline{\sigma}^\mu\overline{\chi} - m\overline{\chi}\eta - m\overline{\chi}^*\eta^* \ . \tag{2.6}$$

2.2 SUSY Algebras

In the following 4-D SUSY algebras are discussed first following the representation of [B6]. After that basic results about higher-dimensional SUSY algebras are listed with emphasis on 8-D case.

2.2.1 $D = 4$ SUSY algebras

Poincare SUSY algebra contains as super-generators transforming as Weyl spinors transforming in complex conjugate representations of Lorentz group. The basic anti-commutation relations of Poincare SUSY algebra in Weyl fermion basis can be expressed as

$$\begin{aligned}
\{Q_\alpha, Q_{\dot\beta}\} &= 2\sigma^\mu_{\alpha\dot\beta}P_\mu \ , \\
\{Q_\alpha, Q_\beta\} &= \{Q_{\dot\alpha}, Q_{\dot\beta}\} = 0 \ , \\
[Q_\alpha, P_\mu] &= [Q_{\dot\alpha}, P_\mu] = 0 \ .
\end{aligned} \tag{2.7}$$

By taking a trace over spinor indices one obtains expression for energy as $P^0 = \sum_i Q_i\overline{Q}_i + \overline{Q}_iQ_i$. Since super-generators must annihilated super-symmetric ground states, the energy must vanish for them.

This algebra corresponds to simplest $\mathcal{N} = 1$ SUSY in which only left-handed fermion appears. For $\mathcal{N} = 1$ SUSY the super-charges are are hermitian whereas in TGD framework supercharges carry fermion number. This implies that super-charges come in pairs of super charge so that $\mathcal{N} = 2N$ must hold true and its hermitian conjugate and only the second half of super-charges can annihilate vacuum state. Weyl spinors must also come as pairs of right- and left-handed spinors.

The construction generalizes in a straightforward manner to allow arbitrary number of fermionic generators. The most general anti-commutation relations in this case are

$$\begin{aligned}
\{Q_{i\alpha}, Q^j_{\dot\beta}\} &= 2\delta^j_i\sigma^\mu_{\alpha\dot\beta}P_\mu \ , \\
\{Q_{i\alpha}, Q_{j\beta}\} &= \epsilon_{\alpha\beta}Z_{ij}, \\
\{Q_{\dot\alpha}, Q_{\dot\beta}\} = \epsilon^{\dot\alpha\dot\beta}Z^*_{ij} \ . &
\end{aligned} \tag{2.8}$$

The complex constants are called central charges because they commute with all generators of the super-Poincare group.

2.2.2 Higher-dimensional SUSY algebras

The character of supersymmetry is sensitive to the dimension D of space-time and to the signature of the space-time metric higher dimensions [B1]. The available spinor representations depend on k; the maximal compact subgroup of the little group of the Lorentz that preserves the momentum of a massless particle is $Spin(d-1)\times Spin(D-d-1)$, where d is the number of spatial dimensions $D-d$ is the number time dimensions and k is defined as $k = 2d - D$. Due to the mod 8 Bott periodicity of the homotopy groups of the Lorentz group, really we only need to consider $k = 2d - D$ modulo 8. In TGD framework one has $D = 8$, $d = 7$ and $k = 6$.

For any value of k there is a Dirac representation, which is always of real dimension $N = [2^{1+[(2d-k)/2]}$ where $[x]$ is the greatest integer less than or equal to x. For TGD this of course gives $2^5 = 32$ corresponding to complex 8-component quark and lepton like spinors. For $-2 \leq k \leq 2$ not realized in TGD there is a real Majorana spinor representation, whose dimension is $N/2$. When k is even (TGD) there is a Weyl spinor representation, whose real dimension is $N/2$. For $k\ mod\ 8 = 0$ (say in super-string models) there is a Majorana-Weyl spinor, whose real dimension is $N/4$. For $3 \leq k \leq 5$ so called symplectic Majorana spinor with dimension $D/2$ and for $k = 4$ symplectic Weyl-Majorana spinors with dimension $D/4$ is possible. The matrix Γ_{D+1} defined as the product of all gamma matrices has eigenvalues $\pm(-1)^{-k/2}$. The eigenvalue of Γ_{D+1} is the chirality of the spinor. CPT theorem implies that the for $D\ mod\ 4 = 0$ the numbers of left and right handed super-charges are same. For $D\ mod\ 4 = 2$ the numbers of left and right handed chiralities can be different and corresponding SUSYs are classified by $\mathcal{N} = (\mathcal{N}_L, \mathcal{N}_R)$, where $\mathcal{N}_L$ and $\mathcal{N}_R$ are the numbers of left and right handed super charges. Note that in TGD the chiralities are ± 1 and correspond to quark and leptons like spinors.

TGD does not allow super-symmetry with Majorana particles. It is indeed possible to have non-hermitian super-charges [B3] in dimension $D = 8$. In $D = 8$ SUGRA with one time dimension super-charges ar non-hermitian and Majorana particles are absent. Also in $D = 4$ SUGRA predicts super-charges are non-hermitian super-charges but Majorana particles are present.

1. $D = 8$ super-gravity corresponds to $\mathcal{N} = 2$ and allows complex super-charges $Q^i_\alpha \in 8$ and their hermitian conjugates $\overline{Q}^i_\alpha \in \overline{8}$. The group of R symmetries is $U(2)$. Bosonic fields consists the metric g_{mn}, seven real scalars, six vectors, three 2-form fields and one 3-form field. Fermionic fields consist of two Weyl (left) gravitini $\psi^{\alpha i}$, six Weyl (right) spinors plus their hermitian conjugates of opposite chirality. There are no Majorana fermions.

2. $D = 4, \mathcal{N} = 8$ SUGRA is second example allowing complex non-hermitian super-charges. The supercharges $Q^i_\alpha \in 2$ and their hermitian conjugates $\overline{Q}^i_{\dot\alpha} \in \overline{2}$. R-symmetry group is $U(8)$. Bosonic fields are metric g_{mn}, 70 real scalars and 28 vectors. Fermionic fields are 8 Majorana gravitini $\Psi^{a,i}_m$ and 56 Majorana spinors.

For $\mathcal{N} = 2N$ and at least $D = 8$ with one time dimension the super charges can be assumed to come in hermitian conjugate pairs and the non-vanishing anti-commutators can be expressed as

$$\begin{aligned} \{Q^\dagger_{i\alpha}, Q^j_{\dot\beta}\} &= 2\delta^j_i \sigma^\mu_{\alpha\dot\beta} P_\mu \ , \\ \{Q^\dagger_{i\alpha}, Q_{j\beta}\} &= \epsilon_{\alpha\beta} Z_{ij} , \\ \{Q^\dagger_{\dot\alpha}, Q_{\dot\beta}\} = \epsilon^{\dot\alpha\ dot\beta} Z^*_{ij} \ . \end{aligned} \qquad (2.9)$$

In this case Z_{ij} is anti-hermitian matrix. 8-D chiral invariance (separate conservation of lepton and quark numbers) suggests strongly that that the condition $Z_{ij} = 0$ must hold holds true. A given pair of super-charges is analogous to creation and annihilation operators for a given fermionic chirality. In TGD framework opposite chiralities correspond to quark and lepton like spinors.

2.2.3 Representations of SUSY algebras in dimension $D = 4$

The physical components of super-fields correspond to states in the irreducible representations of SUSY algebras. The representations can be constructed by using the basic anti-commutation relations for $Q_{i\alpha}$ and $Q_{j\dot\alpha}$, $i, j \in \{1, ..., \mathcal{N}\}$, $\alpha, \dot\alpha \in \{1, 2\}$. The representations can be classified to massive and massless ones. Also the presence of central charges affects the situation. A given irreducible representation is characterized by its ground state and R-parity assignments distinguish between representations with the same spin content, say fermion and its scalar super-partner and Higss with its fermionic super-partner.

1. In the massive case one obtains in the rest system just fermionic creation operators and $2^{\mathcal{N}}$ annihilation operators. The number of states created from a vacuum state with spin s_0 is $2\mathcal{N}$ and maximum spin is $s_0 + \mathcal{N}/2$. For instance, for $\mathcal{N} = 1$ and $s_0 = 0$ one obtains for 4 states with spins $J \leq 1/2$. Renormalizability requires massive matter to have $s \leq 1/2$ so that only $\mathcal{N} = 1$ is possible in this case. For particles massless at fundamental level and getting their masses by symmetry breaking this kind of restriction does not apply.

2. In the massless case only one half of fermionic oscillator operators have vanishing anti-commutators corresponding to the fact that for massless state only the second helicity is physical. This implies that the number of states is only $2^{\mathcal{N}}$ and the helicities vary from λ_0 to $\lambda_0 + \mathcal{N}/2$. For $\mathcal{N} = 1$ the representation is 2-dimensional.

3. In the presence of central charges $Z_{ij} = -Z_{ji}$ the representations are in general massive (Z_{ij} has dimensions of mass), $U(N)$ acts as symmetries of Z, and since Z^2 is symmetric its diagonalizability implies that Z matrix can be cast by a unitary transformation into a direct sum of 2-D antisymmetric real matrices multiplied by constants Z_i. Therefore the super-algebra can be cast in diagonal form with anti-commutators proportional to $M \pm Z_m$ with $M - Z_m \geq 0$ by unitarity. This implies the celebrated Bogomol'nyi bound $M \geq max\{Z_n\}$. For this value of varying mass parameter it is possible to have reduction of the dimension of the representation by one half. If the eigenvalues Z_n are identical the number of states is reduced to that for a massless representation. This multiplet is known as short BPS multiplet. Although BPS multiplets are massive (mass is expressible in terms of Higgs expectation value) they form multiplets shorter than the usual massive SUSY multiplets.

2.3 Super-Space

The heuristic view about super-space [B2] is as a manifold with D local bosonic coordinates x^μ and $\mathcal{N}D/2$ complex anti-commuting spinor coordinates θ^α_i and their complex conjugates $\overline{\theta}^i_{\dot\alpha} = (\theta^\alpha_i)^*$. For $\mathcal{N} = 1$, which is relevant to minimally super-symmetric standard model (MSSM), the spinors

θ can also chosen to be real that is Majorana spinors, so that one has 4 bosonic and four real coordinates. In TGD framework one must however use Weyl spinors.

The anti-commutation relations for the super-coordinates are

$$\{\theta_\alpha,\theta_\beta\} = \{\theta_{\dot\alpha},\theta_{\dot\beta}\} = \{\theta_\alpha,\theta_{\dot\beta}\} = 0 \ . \tag{2.10}$$

The integrals over super-space in 4-D $\mathcal{N}=1$ case are defined by the following formal rules which actually state that super-integration is formally analogous to derivation.

$$\begin{aligned} \int d\theta = \int d\overline{\theta} = \int d\theta\overline{\theta} = \int d\overline{\theta}\theta = 0 \ , \\ \int d\theta^\alpha d\theta_\beta = \delta^\alpha_\beta \ , \ \int d\overline{\theta}_{\dot\alpha} d\overline{\theta}_{\dot\beta} = \delta^{\dot\beta}_{\dot\alpha} \ , \\ \int d^2\theta\theta^2 = \int d^2\overline{\theta\theta}^2 \ , \ \int d^4\theta\theta^2\overline{\theta}^2 = 1 \ . \end{aligned} \tag{2.11}$$

Here the shorthand notations

$$\begin{aligned} d^2\theta &\equiv -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^\alpha d\theta^\beta \ , \\ d^2\overline{\theta} &\equiv -\frac{1}{4}\epsilon^{\dot\alpha\dot\beta}d\theta_{\dot\alpha}d\ theta_{\dot\beta} \ , \\ d^4\theta &\equiv d^2\theta d^2\overline{\theta} \ . \end{aligned} \tag{2.12}$$

are used.

The generalization of the formulas to $D>4$ and $\mathcal{N}>1$ cases is trivial. In infinite-dimensional case relevant for the super-symmetrization of the WCW geometry in terms of local Clifford algebra of WCW to be proposed later the infinite number of complex theta parameters poses technical problems unless one defines super-space functions properly.

2.3.1 Chiral super-fields

Super-multiplets can be expressed as single super-field define in super-space. Super-field can be expanded as a Taylor series with respect to the theta parameters. In 4-dimensional $\mathcal{N}=1$ case one has

$$\Phi(x^\mu,\theta,\overline{\theta}) = \phi(x^\mu) + \theta\eta(x^\mu) + \overline{\theta}\eta^\dagger(x^\mu) + \overline{\theta\ sigma}^\alpha\theta V_\alpha(x^\mu) + \theta^2F(x^\mu) + \overline{\theta^2}\overline{F}(x^\mu) ... + \theta^2\overline{\theta}^2 D(x^\mu) \tag{2.13}$$

The action of super-symmetries on super-fields can be expressed in terms of super-covariant derivatives defined as

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\ dot\alpha}\frac{\partial}{\partial_\mu} \ , \ \overline{D}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\ dot\alpha}\frac{\partial}{\partial_\mu} \ . \tag{2.14}$$

This allows very concise realization of super-symmetries.

General super-field defines a reducible representation of super-symmetry. One can construct irreducible representations of super-fields a pair of chiral and antichiral super-fields by posing the condition

$$\overline{D}_{\dot\alpha}\Phi = 0 \ , \quad D_\alpha\Phi^\dagger = 0 \ . \tag{2.15}$$

The hermitian conjugate of chiral super-field is anti-chiral.

Chiral super-fields can be expressed in the form

$$\Phi = \Phi(\theta, y^{\mu}) \ \ , \ \ y^{\mu} = x^{\mu} + i\overline{\theta}\sigma^{\mu}\theta \ \ , y^{\mu\dagger} = x^{\mu} - i\overline{\theta}\sigma^{\mu}\theta \ \ . \tag{2.16}$$

These formulas generalize in a rather straightforward manner to $D > 4$ and $\mathcal{N} > 1$ case.

It is easy to check that any analytic function of a chiral super-field, call it $W(\Phi)$, is a chiral super-field. In super-symmetries its θ^2 component transforms by a total derivative so that the action defined by the super-space integral of $W(\phi)$ is invariant under super-symmetries. This allows to construct super-symmetric actions using $W(\Phi)$ and $W(\Phi^{\dagger})$. The so called super-potential is defined using the sum of $W(\Phi) + W(\Phi^{\dagger})$.

Analytic functions of does not give rise to kinetic terms in the action. The observation $\theta^2\overline{\theta}^2$ component of a real function of chiral super-fields transforms also as total derivative under super-symmetries allows to circumvent this problem by introducing the notion of Kähler potential $K(\Phi, \Phi^{\dagger})$ as a real function of chiral super-field and its conjugate. In he simplest case one has

$$K = \sum_i \Phi_i^{\dagger}\Phi_i \ \ . \tag{2.17}$$

$L_K = \int K d^4\theta$ gives rise to simples super-symmetric action for left-handed fermion and its scalar super-partner.

Kähler potential allows an interpretation as a Kähler function defining the Kähler metric for the manifold defined by the scalars ϕ_i. This Kähler metric depends in the general case on ϕ_i and appears in the kinetic term of the super-symmetric action. Super-potential in turn can be interpreted as a counterpart of real part of a complex function which can be added to the Kähler function without affect the Kähler metric. This geometric interpretation suggests that in TGD framework every complex coordinate ϕ_i of WCW defines a chiral super-field whose bosonic part.

2.3.2 Wess-Zumino model as simple example

Wess-Zumino model without interaction term serves as a simple illustration of above formal considerations. The action density of Wess-Zumino Witten model can be deduced by integration Kähler potential $K = \Phi^{\dagger}\Phi$ for chiral super fields over theta parameters. The result is

$$L = \partial_u\phi^*\partial^{\mu}\phi + i\eta^*\partial^{\mu}\eta + F^*F \ \ . \tag{2.18}$$

The action of super-symmetry

$$\delta\Phi = \epsilon^{\alpha}D_{\alpha}\Phi \ \ , \ \ \delta\Phi^{\dagger} = \overline{\epsilon}^{\dot{\alpha}}\overline{D}_{\dot{\alpha}}\Phi \ \ , \ \ \epsilon_{\dot{\alpha}} = \epsilon^{*\alpha} \tag{2.19}$$

gives the transformation formulas

$$\delta\phi = \epsilon^{\alpha}\eta_{\alpha} \ \ , \ \ \delta\eta = -i\eta^{*\dot{\alpha}}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\phi + \epsilon_{\alpha}F \ \ , \ \ \delta F = -i\epsilon_{\dot{\alpha}}\overline{\sigma}^{\mu\dot{\alpha}\alpha} \ partial_{\mu}\eta_{\alpha} \tag{2.20}$$

plus their hermitian conjugates. The corresponding Noether current is indeed hermitian since the transformation parameters ϵ^{α} and $\overline{\epsilon}_{\dot{\alpha}} = \epsilon^{*\alpha}$ appear in it and cannot be divided away. This conserved current has as such no meaning and the statement that ground state is annihilated by the corresponding super-charge means that vacuum field configuration rather than Fock vacuum remains invariant under supersymmetries. Rather, the breaking of super-symmetry by adding a super-potential implies that F develops vacuum expectation and the vacuum solution ($\phi = 0, \eta = 0, F = constant$) of field equations is not anymore invariant super super-symmetries.

The non-hermitian parts of the super current corresponding to different fermion numbers are separately conserved and corresponding super-charges are non-Hermitian and together with other charges define a super-algebra which to my best understanding is not equivalent with the super-algebra defined by allowing the presence of anti-commuting parameters ϵ. The situation is similar in TGD where one class of non-hermitian super-currents correspond to the modes of the induced spinor fields contracted with $\overline{\Psi}$ and their conjugates. The octonionic solution ansatz for the induced spinor field allows to express the solutions in terms of two complex scalar functions so that the super-currents in question would be analogous to those of $\mathcal{N} = 2$ SUSY and one might see the super-symmetry of quantum TGD extended super-symmetry obtained from the fundamental $\mathcal{N} = 2$ super-symmetry.

2.3.3 Vector super-fields and supersymmetric variant of YM action

Chiral super-fields allow only the super-symmetrization of Dirac action. The super-symmetrization of YM action requires the notion of a hermitian vector super field $V = V^{\dagger}$, whose components correspond to vector bosons, their super-counterparts and additional degrees of freedom which cannot be dynamical. These degrees of freedom correspond gauge degrees of freedom.

In the Abelian case the gauge symmetries are realized as $V \rightarrow V + \Lambda + \Lambda^{\dagger}$, where Λ is a chiral super-field. These symmetries induce gauge transformations of the vector potential. Their action on chiral super-fields is $\Phi \rightarrow exp(-q\Lambda)\Phi$, $\Phi^{\dagger} \rightarrow \Phi^{\dagger}exp(-\Lambda^{\dagger})$. In non-Abelian case the realization is as $exp(V) \rightarrow exp(-\Lambda^{\dagger})exp(V)exp(\Lambda)$ so that the modified Kähler potential $K(\Phi^{\dagger}, exp(qV)\Phi)$ remains invariant.

One can assign to V a gauge invariant chiral spinor super-field as

$$\begin{aligned} W_{\alpha} &= -\frac{1}{4}\overline{D}^2(e^V D_{\alpha}e^{-V}) \ , \\ \overline{D}^2 &= \epsilon^{\dot{\alpha}\dot{\beta}}\overline{D}_{\dot{alpha}}\overline{D}_{\dot{\beta}} \end{aligned} \tag{2.21}$$

defining the analog of gauge field. $\overline{D}^2$ eliminates all terms the exponent of $\overline{\theta}$ is higher than that of θ since these would spoil the chiral super-field property (the anti-commutativity of super-covariant derivatives $\overline{D}_{\dot{\alpha}}$ makes this obvious). D_{α} in turn eliminates from the resulting scalar part so that one indeed has chiral spinor super-field. In higher dimensions and for larger value of $\mathcal{N}$ the definition of W_{α} must be modified in order to achieve this: what is needed is the product of all derivatives $\overline{D}_{i\dot{\alpha}}$.

The analytic functions of chiral spinor super-fields are chiral super-fields and θ^2 component of $W^{\alpha}W_{\alpha}$ transforms as a total derivatives. The super-symmetric Lagrangian of U(1) theory can be written as

$$L = \frac{1}{4g^2}\left(\int d^2\theta W^{\alpha}W_{\alpha} + \int d^2\overline{\theta}W^{\dagger}_{\dot{\alpha}}W^{\dagger}_{\dot{\alpha}}\right) \ . \tag{2.22}$$

Note that in standard form of YM action $1/2g^2$ appears.

2.3.4 R-symmetry

R-symmetry is an important concomitant of super-symmetry. In $\mathcal{N} = 1$ case R-symmetry performs a phase rotation $\theta \rightarrow e^{i\alpha}\theta$ for the super-space coordinate θ and an opposite phase rotation for the differential $d\theta$. For $\mathcal{N} > 1$ R-symmetries are $U(N)$ rotations. R-symmetry is an additional symmetry of the Lagrangian terms due to Kähler potential since both $d^4\theta$ (and its generalization) as well as Kähler potential are real. Also super-symmetric YM action is R-invariant. R-symmetry is a symmetry of if super-potential W only if it has super-charge $Q_R = 2$ ($Q_R = 2\mathcal{N}$) in order to compensate the super-charge of $d^{2\mathcal{N}}\theta$.

2.4 Non-Renormalization Theorems

Super-symmetry gives powerful constraints on the super-symmetric Lagrangians and leads to non-renormalization theorems.

The following general results about renormalization of supersymmetric gauge theories hold true (see [B6], where a heuristic justification of the non-renormalization theorems and explicit formulas are discussed).

1. Super-potential is not affected by the renormalization.

2. Kähler potential is subject to wave function renormalization in all orders. The renormalization depends on the parameters with dimensions of mass. In particular, quadratic divergences to masses cancel.
3. Gauge coupling suffers renormalization only by a constant which corresponds to one-loop renormalization. Any renormalization beyond one loop is due to wave function renormalization of the Kähler potential and it is possible to calculate the beta function exactly.

It is interesting to try to see these result from TGD perspective.

1. In TGD framework super-potential interpreted as defining the modification of WCW Kähler function, which does not affect Kähler metric and would reflect measurement interaction. The non-renormalization of W would mean that the measurement interaction is not subject to renormalization. The interpretation is in terms of quantum criticality which does not allow renormalization of the coefficients appearing in the measurement interaction term since otherwise Kähler metric of WCW would be affected.
2. The wave function renormalization of Kähler potential would correspond in TGD framework scaling of the WCW Kähler metric. Quantum criticality requires that Kähler function remains invariant. Also since no parameters with dimensions of mass are available, there is temptation to conclude that wave function renormalization is trivial.
3. Only the gauge coupling would be suffer renormalization. If one however believes in the generalization of bosonic emergence it is Kähler function which defines the SUSY QFT limit of TGD so that gauge couplings follow as predictions and their renormalization is a secondary -albeit real- effect having interpretation in terms of the dependence of the gauge coupling on the p-adic length scale. The conclusion would be that at the fundamental level the quantum TGD is RG invariant.

3 Does TGD Allow The Counterpart Of Space-Time Super-symmetry?

The question whether TGD allows space-time super-symmetry or something akin to it has been a longstanding problem. A considerable progress in the respect became possible with the better understanding of the Kähler-Dirac equation.

3.1 Kähler-Dirac Equation

Before continuing one must briefly summarize the recent view about Kähler-Dirac equation.

1. The localization of the induced spinor fields to 2-D string world sheets is crucial. It is demanded both by the well-definedness of em charge and by number theoretical constraints. Induced W boson fields must vanish, and the Frobenius integrability conditions guaranteeing that the K-D operator involves no covariant derivatives in directions normal to the string world sheet must be satisfied.
2. The Kähler-Dirac equation (or Kähler Dirac equation) reads as

   $$D_K\Psi = 0 \ . \tag{3.1}$$

   in the interior of space-time surface. The boundary variation of K-D equation gives the term

   $$\Gamma^n\Psi = 0 \tag{3.2}$$

   at the light-like orbits of partonic 2-surfaces. Clearly, Kähler-Dirac gamma matrix Γ^n in normal direction must be light-like or vanish.

3. To the boundaries of string world sheets at the orbits of partonic 2-surfaces one assigns 1-D Dirac action in induced metric line with length as bosonic counterpart. By field equations both actions vanish, and one obtains light-like geodesic carrying light-like 8-momentum. Algebraic variant of massless 8-D Dirac equation is satisfied for the 8-momentum parallel to 8-velocity.

   The boundaries of the string world sheets are thus pieces of light-like M^8 geodesics and different fermion lines should have more or less parallel M^4 momenta for the partonic 2-surface to preserve its size. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach encourages also by the very special twistorial properties of M^4 and CP_2.

   One can wonder how this relates to braiding which is one of the key ingredients of TGD. Is the braiding possible unless it is induced by particle exchanges so that the 8-momentum changes its direction and partonic 2-surface replicates. In principle it should be possible to construct the orbits of partonic 2-surfaces in such a manner that braiding occurs. Situation is the reverse of the usual in which one has fixed 3-manifold in which one constructs braid.

4. One can construct preferred extremals by starting from string world sheets satisfying the vanishing of normal components of canonical momentum currents as analogs of boundary conditions. One can also fix 3-D space-like surfaces and partonic orbits and pose the vanishing of super-symplectic charges for a sub-algebra with conformal weights coming as multiples of fixed integer n as conditions selecting preferred extremals.

5. The quantum numbers characterizing zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse. Thermodynamics would naturally couple to the space-time geometry via the thermodynamical or quantum averages of the quantum numbers.

3.2 Development Of Ideas About Space-Time SUSY

Let us first summarize the recent overall view about space-time super-symmetry for TGD discussed in detail in chapter " WCW spinor structure" and also in [K16].

1. Right-handed covariantly constant neutrino spinor ν_R defines a super-symmetry in CP_2 degrees of freedom in the sense that CP_2 Dirac equation is satisfied by covariant constancy and there is no need for the usual ansatz $\Psi = D\Psi_0$ giving $D^2\Psi = 0$. This super-symmetry allows to construct solutions of Dirac equation in CP_2 [A2, A4, A1, A3].

2. In $M^4 \times CP_2$ this means the existence of massless modes $\Psi = \not{p}\Psi_0$, where Ψ_0 is the tensor product of M^4 and CP_2 spinors. For these solutions M^4 chiralities are not mixed unlike for all other modes which are massive and carry color quantum numbers depending on the CP_2 chirality and charge. As matter fact, massless right-handed neutrino covariantly constant in CP_2 spinor mode is the only color singlet. The mechanism leading to non-colored states for fermions is based on super-conformal representations for which the color is neutralized [K9, K9]. The negative conformal weight of the vacuum also cancels the enormous contribution to mass squared coming from mass in CP_2 degrees of freedom.

3. All spinor modes define conserved fermion super-currents and also the super-symplectic algebra has a fermion representation as Noether currents at string world sheets. WCW metric can be constructed as anti-commutators of super-symplectic Noether currents and one obtains a generalization of AdS/CFT duality to TGD framework from the possibility to express Kähler also in terms of Kähler function (and thus Kähler action). The fact that that super-Poincare anti-commutator vanishes for oscillator operators associated with covariantly constant right-handed neutrino and anti-neutrino implies that it corresponds to a pure gauge degree of freedom.

4. The natural conjecture is that the TGD analog space-time SUSY is generated by the Clifford algebra of the second quantized fermionic oscillator operators at string world sheets. This algebra in turn generalizes to Yangian. The oscillator operators indeed allow the 8-D analog of super-Poincare anti-commutation relations at the ends of 1-D light-like geodesics defined by the boundaries of string world sheets belonging to the orbits of partonic 2-surfaces and carrying 8-D light-like momentum.

   For incoming on mass shell particles one can identify the M^4 part of 8-momentum as gravitational for momentum equal to the inertial four-momentum assignable to imbedding space spinor harmonic for incoming on mass shell state. The square of E^4 momentum giving mass squared corresponds to the eigenvalue of CP_2 d'Alembertian.

   8-D light-like momentum forces an 8-D generalization of the twistor approach and M^4 and CP_2 are indeed unique in that they allow twistor space with Kähler structure [A5]. The conjecture is that integration over virtual momenta restricts virtual momenta to 8-D light-like momenta but the polarizations of virtual fermions are non-physical.

5. The 8-D generalization of SUSY describes also massive states and one has $\mathcal{N} = \infty$. Ordinary 4-D SUSY is obtained by restricting the states to the massless sector of the theory. The value of $\mathcal{N}$ is finite in this case and corresponds to the value of massless modes for fundamental fermions. Quark and lepton type spinor components with physical helicity for fermions and anti-fermions define the basis of the SUSY algebra as Clifford algebra of oscillator operators with anti-commutators analogous to those associated with super Poincare algebra. Therefore the generators of SUSY correspond to the $4+4$ components of imbedding space spinor modes (quarks and leptons) with vanishing conformal weight so that analogs of $\mathcal{N} = 4$ SUSY are obtained in quark and lepton sectors.

   The SUSY is broken due to the electro-weak and color interactions between the fundamental fermions. For right-handed neutrinos these interactions are not present but the mixing with left handed neutrino due to the mixing of M^4 and CP_2 gamma matrices in Kähler-Dirac gamma matrices at string world sheets implies SUSY breaking also now: also R-parity is broken.

   Basically a small mixing with the states with CP_2 mass is responsible for the generation of mass and breaking of SUSY. p-Adic thermodynamics describes this mixing. SUSY is broken at QFT limit also due the replacement of the many-sheeted space-time with single slightly curved region of M^4.

6. The SUSY in question is not the conventional $\mathcal{N} = 1$ SUSY. Space-time (in the sense of Minkowski space M^4) $\mathcal{N} = 1$ SUSY in the conventional sense of the word is impossible in TGD framework since it would require require Majorana spinors. In 8-D space-time with Minkowski signature of metric Majorana spinors are definitely ruled out by the standard argument leading to super string model. Majorana spinors would also break the separate conservation of lepton and baryon numbers in TGD framework. What is remarkable is that in 8-D space-time one obtains naturally SUSY with Dirac spinors.

3.3 Summary About TGD Counterpart Of Space-Time SUSY

This picture allows to define more precisely what one means with the approximate super-symmetries in TGD framework.

1. One can in principle construct many-fermion states containing both fermions and anti-fermions at fermion lines located at given light-like parton orbit. The four-momenta of states related by super-symmetry need not be same. Super-symmetry breaking is present and has as the space-time correlate the deviation of the Kähler-Dirac gamma matrices from the ordinary M^4 gamma matrices. In particular, the fact that $\hat{\Gamma}^\alpha$ possesses CP_2 part in general means that different M^4 chiralities are mixed: a space-time correlate for the massivation of the elementary particles.

2. For right-handed neutrino super-symmetry breaking is expected to be smallest but also in the case of the right-handed neutrino mode mixing of M^4 chiralities takes place and breaks the

TGD counterpart of super-symmetry. Maybe the correct manner to interpret the situation is to speak about 8-D massless states for which the counterpart of SUSY would not be broken but mass splittings are possible.

3. The fact that all helicities in the state are physical for a given light-like 3-surface has important implications. For instance, the addition of a right-handed antineutrino to right-handed (left-handed) electron state gives scalar (spin 1) state. Also states with fermion number two are obtained from fermions. For instance, for e_R one obtains the states $\{e_R, e_R\nu_R\overline{\nu}_R, e_R\overline{\nu}_R, e_R\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,0,1)$. For e_L one obtains the states $\{e_L, e_L\nu_R\overline{\nu}_R, e_L\overline{\nu}_R, e_L\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,1,0)$. In the case of gauge boson and Higgs type particles -allowed by TGD but not required by p-adic mass calculations- gauge boson has 15 super partners with fermion numbers $[2,1,0,-1,-2]$.

The cautious conclusion is that the recent view about quantum TGD allows the analog of super-symmetry, which is necessary broken and for which the multiplets are much more general than for the ordinary super-symmetry. Right-handed neutrinos might however define something resembling ordinary super-symmetry to a high extent. The question is how strong prediction one can deduce using quantum TGD and proposed super-symmetry.

1. For a minimal breaking of super-symmetry only the p-adic length scale characterizing the super-partner differs from that for partner but the mass of the state is same. This would allow only a discrete set of masses for various super-partners coming as half octaves of the mass of the particle in question. A highly predictive model results.

2. The quantum field theoretic description could be based on QFT limit of TGD, which I have formulated in terms of bosonic emergence. The idea was that his formulation allows to calculate the propagators of the super-partners in terms of fermionic loops. Similar description of exchanged boson as fermionic loop emerges also in the proposed identification of scattering amplitudes as representations of algebraic computations in Yangian using product and co-product as fundamental vertices assignable to partonic 2-surfaces at which 3-surfaces replicate.

3. This TGD variant of space-time super-symmetry resembles ordinary super-symmetry in the sense that selection rules due to the right-handed neutrino number conservation and analogous to the conservation of R-parity hold true (the mixing of right-handed neutrino with the left-handed one breaks R-parity). The states inside super-multiplets have identical electroweak and color quantum numbers but their p-adic mass scales can be different. It should be possible to estimate reaction reaction rates using rules very similar to those of super-symmetric gauge theories.

4. It might be even possible to find some simple generalization of standard super-symmetric gauge theory to get rough estimates for the reaction rates. There are however problems. The fact that spins $J = 0,1,2,3/2,2$ are possible for super-partners of gauge bosons forces to ask whether these additional states define an analog of non-stringy strong gravitation. Note that graviton in TGD framework corresponds to a pair of wormhole throats connected by flux tube (counterpart of string) and for gravitons one obtains 2^8-fold degeneracy.

3.4 SUSY Algebra Of Fermionic Oscillator Operators And WCW Local Clifford Algebra Elements As Super-fields

Whether TGD allows space-time supersymmetry has been a long-standing question. Majorana spinors appear in $N = 1$ super-symmetric QFTs- in particular minimally super-symmetric standard model (MSSM). Majorana-Weyl spinors appear in M-theory and super string models. An undesirable consequence is chiral anomaly in the case that the numbers of left and right handed spinors are not same. For $D = 11$ and $D = 10$ these anomalies cancel, which led to the breakthrough of string models and later to M-theory. The probable reason for considering these dimensions is that standard model does not predict right-handed neutrino (although neutrino mass suggests that right handed neutrino exists) so that the numbers of left and right handed Weyl-spinors are not the same.

In TGD framework the situation is different. Covariantly constant right-handed neutrino spinor acts as a super-symmetry in CP_2. One might think that right-handed neutrino in a well-defined sense disappears from the spectrum as a zero mode so that the number of right and left handed chiralities in $M^4 \times CP_2$ would not be same. For light-like 3-surfaces covariantly constant right-handed neutrino does not however solve the counterpart of Dirac equation for a non-vanishing four-momentum and color quantum numbers of the physical state. Therefore it does not disappear from the spectrum anymore and one expects the same number of right and left handed chiralities.

In TGD framework the separate conservation of baryon and lepton numbers excludes Majorana spinors and also the the Minkowski signature of $M^4 \times CP_2$ makes them impossible. The conclusion that TGD does not allow super-symmetry is however wrong. For $\mathcal{N} = 2N$ Weyl spinors are indeed possible and if the number of right and left handed Weyl spinors is same super-symmetry is possible. In 8-D context right and left-handed fermions correspond to quarks and leptons and since color in TGD framework corresponds to CP_2 partial waves rather than spin like quantum number, also the numbers of quark and lepton-like spinors are same.

The physical picture suggest a new kind of approach to super-symmetry in the sense that the anti-commutations of fermionic oscillator operators associated with the modes of the induced spinor fields define a structure analogous to SUSY algebra in 8-D sense. Massless modes of spinors in 1-1 corresponds with imbedding space spinors with physical helicity are in 1-1 correspondence with the generators of SUSY at space-time level giving $\mathcal{N} = 4 + 4$. Right handed neutrino modes define a sub-algebra for which the SUSY is only slightly broken by the absence of weak interactions and one could also consider a theory containing a large number of $\mathcal{N} = 2$ super-multiplets corresponding to the addition of right-handed neutrinos and antineutrinos at the wormhole throat.

Masslessness condition is essential if super-symmetric quantum field theories and at the fundamental level it can be generalized to masslessness in 8-D sense in terms of Kähler-Dirac gamma matrices using octonionic representation and assuming that they span local quaternionic sub-algebra at each point of the space-time sheet. SUSY algebra has standard interpretation with respect to spin and isospin indices only at the partonic 2-surfaces so that the basic algebra should be formulated at these surfaces: in fact, out that the formulation is needed only at the ends of fermion lines. Effective 2-dimensionality would require that partonic 2-surfaces can be taken to be ends of any light-like 3-surface Y_l^3 in the slicing of the region surrounding a given wormhole throat.

3.4.1 Super-algebra associated with the Kähler-Dirac action

Anti-commutation relations for fermionic oscillator operators associated with the induced spinor fields are naturally formulated in terms of the Kähler-Dirac gamma matrices. The canonical anti-commutation relations for the fermionic oscillator operators at light-like 3-surfaces or at their ends can be formulated as anti-commutation relations for SUSY algebra. The algebra creating physical states is super-symplectic algebra whose generators are expressed as Noether charges assignable to strings connecting partonic 2-surfaces.

Lepton and quark like spinors are now the counterparts of right and left handed Weyl spinors. Spinors with dotted and un-dotted indices correspond to conjugate representations of $SO(3,1) \times SU(4)_L \times SU(2)_R$. The anti-commutation relations make sense for sigma matrices identified as 6-dimensional matrices $1_6, \gamma_7, \gamma_1, ...\gamma_6$.

Consider first induced spinor fields at the boundaries of string world sheets at the orbits of wormhole throats. Dirac action for induced spinor fields and its bosonic counterpart defined by line-length are required by the condition that one obtains fermionic propagators massless in 8-D sense.

1. The localization of induced spinor fields to string world sheets and the addition of 1-D Dirac action at the boundaries of string world sheets at the orbits of partonic 2-surfaces reduces the quantization to that at the end of the fermion line at partonic 2-surface located at the boundary of CD. Therefore the situation reduces to that for point particle.

2. The boundary is by the extremization of line length a geodesic line of imbedding space, which can be characterized by conserved four-momentum and conserved angular momentum like charge - call it hypercharge Y. The square of 8-velocity vanishes: $v_4^2 - (v^\phi)^2 = 0$ and one can choose $v_4^2 = 1$. 8-momentum is proportional to 8-velocity expressible as (v^k, v^ϕ).

3. Dirac equation gives $\Gamma^t \partial_t \Psi = (\gamma_k v^k + \gamma_\phi) v^\phi) \partial_t \Psi = 0$. The non-trivial solution corresponds to $\partial_t \Psi = i\omega\Psi$ and the light-likeness condition. The value of parameter ω defines the mass scale and quantum classical correspondences suggests that ω^2 gives the mass squared identifiable as the eigenvalue of CP_2 Laplacian for spinor modes.

4. Anti-commutation relations must be fixed at either end of fermion line for the oscillator operators associated with the modes of induced spinor field at string world sheet labelled by integer value conformal weight and spin and weak isospin for the H-spinor involved. These anti-commutation relations must be consistent with standard canonical quantization allowing in turn to assign Noether charges to super-symplectic algebra defined as integrals over string world sheet. The identification of WCW gamma matrices as these charges allows to calculate WCW metric as their anti-commutators.

5. The oscillator operators for the modes with different values of conformal weight vanish. Standard anti-commutation relations in massive case are completely fixed and correspond to just Kronecker delta for conformal weights, spin, and isospin.

Space-time supersymmetry and the need to generalize 4-D twistors to 8-D ones suggest the anti-commutation relations obeyed by 8-D analogs of massless Weyl spinors and thus proportional to $p_8^k \sigma_k$, where p_8^k is the 8-momentum associated with the end of the fermion line and σ_k are the 8-D analogs of 2×2 sigma matrices.

1. This requires the introduction of octonionic spinor structure with gamma matrices represented in terms of octonionic units and introducing octonionic gamma matrices. The natural condition is that the octonionic gamma matrices are equivalent with the ordinary one. This is true if fermions are localied at time-like or light-like geodesic lines of imbedding space since they represent- not only quaternionic, but even hypercomplex sub-manifolds of imbedding space. This allows ordinary matrix representations for the gamma matrices at fermion lines.

2. One can avoid the problems with the non-associativity also at string world sheets possible caused by the Kähler Dirac gamma matrices if the two Kähler Dirac gamma matrices span commutative subspace of complexified octonions. The sigma matrices appearing in induced gauge potentials could be second source of non-associativity. By assuming that the solutions are holomorphic spinors (just as in string models) and that in the gauge chosen only holomorphic or anti-holomorphic components of gauge boson fields are non-vanishing, one avoids these problems.

3. It must be admitted that the constraints on string world sheets are strong: vanishing W induced gauge fields, Frobenius integrability conditions, and the condition that K-D gamma matrices span a commutative sub-space of complexified octonions, and I have not really proven that they can be satisfied.

The super-generators of space-time SUSY are proportional to fermionic oscillator operators obeying the canonical anti-commutation relations. It is not quite clear to me whether the proportionality constant can be taken to be equal to one although intuition suggests this strongly. The anti-commutations can contain only the light-like 8-velocity at the right hand side carrying information about the direction of the fermion line.

One can wonder in how strong sense the strong form of holography is realized.

1. Is the only information about the presence of strings at the level of scattering amplitudes the information coded by the anti-commutation relations at their end points? This would be the case if the fermion super-conformal charges vanish or create zero norm states for non-vanishing conformal weights. It could however happen that also the super-conformal generators associated with a sub-algebra of conformal algebra with weights coming as integer multiples of the entire algebra do this. At least this should be the case for the super-symplectic algebra.

2. Certainly one must assume that the 8-velocities associated with the ends of the fermionic string are independent so that strings would imply bi-locality of the dynamics.

3.4.2 Summing up the anti-commutation relations

In leptonic sector one would have the anti-commutation relations

$$\begin{aligned} \{a^{\dagger}_{m\dot{\alpha}}, a^{n}_{\beta}\} &= 2\delta^{n}_{m} D_{\dot{\alpha}\beta} \ , \\ D &= (p_{\mu} + \sum_{a} Q^{a}_{\mu})\sigma^{\mu} \ . \end{aligned} \tag{3.3}$$

In quark sector σ^{μ} is replaced with $\overline{\sigma}^{\mu}$ obtained by changing the signs of space-like sigma matrices in leptonic sector. p_{μ} and Q^{a}_{μ} are the projections of momentum and color charges in Cartan algebra to the space-time surface and their values correspond to those assignable to the fermion line and related by quantum classical correspondence to those associated with incoming spinor harmonic.

The anti-commutation relations define a generalization of the ordinary equal-time anti-commutation relations for fermionic oscillator operators to a manifestly covariant form. Extended SUSY algebra suggest that the anti-commutators could contain additional central charge term proportional to $\delta_{\alpha\beta}$ but the 8-D chiral invariance excludes this term.

In the octonionic representation of the sigma matrices matrix indices cannot be present at the right handed side without additional conditions. Octonionic units however allow a representation as matrices defined by the structure constants failing only when products of more than two octonions are considered. For the quaternionic sub-algebra this does not occur. Both spinor modes and and gamma matrices must belong to the local hyper-quaternionic sub-algebra and do trivially so for fermion lines and string. Octonionic representation reduces $SO(7,1)$ so G_2 as a tangent space group. Similar reduction for 7-dimensional compact space takes place also M-theory.

In standard SUSY local super-fields having values in the Grassmann algebra generated by theta parameters appear. In TGD framework this would mean allowance of many-fermion states at single space-time point and this is perhaps too heavy an idealization since partonic 2-surfaces are the fundamental objects. Multi-stringy generators in the extension of super-symplectic algebra to Yangian is a more natural concept in TGD framework since one expects that partonic 2-surfaces involve several strings connecting them to other partonic 2-surfaces. Super-symplectic charges would be Noether charges assignable to these strings and quantum states would be created by these charges from vacuum. Scattering amplitudes would be defined in terms of Yangian algebra [K22]. Only at QFT limit one can hope that super-field formalism works.

4 Understanding Of The Role Of Right-Handed Neutrino In Supersymmetry

The development of the TGD view about space-time SUSY has been like a sequence of questions loves -doesn't love- loves.... From the beginning it was clear that right-handed neutrino could generate super-conformal symmetry of some kind, and the natural question was whether it generates also space-time SUSY. Later it became clear that all fermion oscillator operators can be interpreted as super generators for the analog of space-time SUSY. After that the challenge was to understand whether all spin-isospin states of fermions correspond super generators.

$\mathcal{N}=1$ SUSY was excluded by separate conservation of B and L but $\mathcal{N}=2$ variant of this symmetry could be considered and could be generated by massless right-handed neutrino and antineutrino mode.

The new element in the picture was the physical realization of the SUSY by adding fermions - in special case right-handed neutrino - to the state associated with the orbit of partonic 2-surface. An important realization was the necessity to localized spinors to string world sheet and the assignment of fernionic oscillator operator with boundaries of string world sheets at them. Variational principles implies that the fermions have light-like 8-momenta and that the fermion lines are light-like geodesics in 8-D sense. This leads to a precise view about the quantization of induced spinor fields. Fermionic oscillator operator algebra would generate Clifford algebra replacing the SUSY algebra and one would obtain the analog of super Poincare algebra from anti-commutation relations.

4.1 Basic Vision

As already explained, the precise meaning of SUSY in TGD framework has been a long-standing head ache. In TGD framework SUSY is inherited from super-conformal symmetry at the level of WCW [K4, K3]. The SUSY differs from $\mathcal{N} = 1$ SUSY of the MSSM and from the SUSY predicted by its generalization and by string models. Allowing only right-handed neutrinos as SUSY generators, one obtains the analog of the $\mathcal{N} = 4$ SUSY in bosonic sector but there are profound differences in the physical interpretation. The most general view is that all fermion modes with vanishing conformal weights define super charges.

1. One could understand SUSY in very general sense as an algebra of fermionic oscillator operators acting on vacuum states at partonic 2-surfaces. Oscillator operators are assignable to braids ends and generate fermionic many particle states. SUSY in this sense is badly broken and the algebra corresponds to rather large $\mathcal{N}$. The restriction to covariantly constant right-handed neutrinos (in CP_2 degrees of freedom) gives rise to the counterpart of ordinary SUSY, which is more physically interesting at this moment.

2. Right handed neutrino and antineutrino are not Majorana fermions. This is necessary for separate conservation of lepton and baryon numbers. For fermions one obtains the analog $\mathcal{N} = 2$ SUSY.

3. Bosonic emergence means the construction of bosons as bound states of fermions and antifermions at opposite throats of wormhole contact. Later it became clear that all elementary particles emerge as bound states of fundamental fermions located at the wormhole throats of a pair of wormhole contacts. Two wormhole contacts are required by the assumption wormhole contacts carry monopole magnetic flux stabilizing them.

   This reduces TGD SUSY to that for fundamental fermions. This difference is fundamental and means deviation from the $\mathcal{N} = 4$ SUSY, where SUSY acts on gauge boson states. Bosonic representations are obtained as tensor products of representations assigned to the opposite throats of wormhole contacts. One can also have several fermion lines at given throat but these states are expected to be exotic.

   Further tensor products with representations associated with the wormhole ends of magnetic flux tubes are needed to construct physical particles. This represents a crucial difference with respect to standard approach, where one introduces at the fundamental level both fermions and bosons or gauge bosons as in $\mathcal{N} = 4$ SUSY. Fermionic $\mathcal{N} = 2$ representations are analogous to “short” $\mathcal{N} = 4$ representations for which one half of super-generators annihilates the states.

4. If stringy super-conformal symmetries act as gauge transformations, the analog of $\mathcal{N} = 4$ SUSY is obtained in both quark and lepton sector. This extends to $\mathcal{N} = 8$ SUSY if parton orbits can carry both quarks and leptons. Lepto-quark is the simplest state of this kind.

5. The introduction of both fermions and gauge bosons as fundamental particles leads in quantum gravity theories and string models to $d = 10$ condition for the target space, spontaneous compactification, and eventually to the landscape catastrophe.

   For a supersymmetric gauge theory (SYM) in d-dimensional Minkowski space the condition that the number of transversal polarization for gauge bosons given by $d - 2$ equals to the number of fermionic states made of Majorana fermions gives $d - 2 = 2^k$, since the number of fermionic spinor components is always power of 2.

   This allows only $d = 3, 4, 6, 10, 16, ...$ Also the dimensions $d + 1$ are actually possible since the number of spinor components for d and $d + 1$ is same for d even. This is the standard argument leading to super-string models and M-theory. It it lost - or better to say, one gets rid of it - if the basic fields include only fermion fields and bosonic states are constructed as the tensor products of fermionic states. This is indeed the case in TGD, where spontaneous compactification plays no role and bosons are emergent.

6. Spontaneous compactification leads in string model picture from $\mathcal{N} = 1$ SUSY in say $d = 10$ to $\mathcal{N} > 1$ SUSY in $d = 4$ since the fermionic multiplet reduces to a direct sum of

fermionic multiplets in $d = 4$. In TGD imbedding space is not dynamical but fixed by internal consistency requirements, and also by the condition that the theory is consistent with the standard model symmetries. The identification of space-time as 4-surface makes the induced spinor field dynamical and the notion of many-sheeted space-time allows to circumvent the objections related to the fact that only 4 field like degrees of freedom are present.

4.2 What Is The Role Of The Right-Handed Neutrino?

Whether right-handed neutrinos generate a supersymmetry in TGD has been a long standing open question. $\mathcal{N} = 1$ SUSY is certainly excluded by fermion number conservation but already $\mathcal{N} = 2$ defining a "complexification" of $\mathcal{N} = 1$ SUSY is possible and could generate right-handed neutrino and its antiparticle. Right-handed neutrinos should however possess a non-vanishing light-like momentum since the fully covariantly constant right-handed neutrino generates zero norm states.

The general view about the preferred extremals of Kähler action and application of the conservation of em charge to the Kähler-Dirac equation have led to a rather detailed view about classical and TGD and allowed to build a bridge between general vision about super-conformal symmetries in TGD Universe and field equations. This vision is discussed in detail in [K16].

1. Many-sheeted space-time means that single space-time sheet need not be a good approximation for astrophysical systems. The GRT limit of TGD can be interpreted as obtained by lumping many-sheeted space-time time to Minkowski space with effective metric defined as sum M^4 metric and sum of deviations from M^4 metric for various space-time sheets involved [K15]. This effective metric should correspond to that of General Relativity and Einstein's equations would reflect the underlying Poincare invariance. Gravitational and cosmological constants follow as predictions and EP is satisfied.

2. The general structure of super-conformal representations can be understood: super-symplectic algebra is responsible for the non-perturbative aspects of QCD and determines also the ground states of elementary particles determining their quantum numbers. The hierarchy of breakings of conformal symmetry as gauge gauge symmetry would explain dark matter. The sub-algebra for which super-conformal symmetry remains gauge symmetry would be isomorphic to the original algebra and generated by generators for which conformal weight is multiple of integer $n = h_{eff}/h$. This would would be true for super-symplectic algebra at least and possible for all other conformal algebras involved.

3. Super-Kac-Moody algebras associated with isometries and holonomies dictate standard model quantum numbers and lead to a massivation by p-adic thermodynamics: the crucial condition that the number of tensor factors in Super-Virasoro represention is 5 is satisfied.

4. One can understand how the Super-Kac-Moody currents assignable to stringy world sheets emerging naturally from the conservation of em charge defined as their string world sheet Hodge duals gauge potentials for standard model gauge group and also their analogs for gravitons. Also the conjecture Yangian algebra generated by Super-Kac-Moody charges emerges naturally.

5. One also finds that right handed neutrino is in a very special role because of its lacking couplings in electroweak sector and its role as a generator of the least broken SUSY. The most feasible option is that all modes of the induced spinor field are restricted to 2-D string world sheets. If covariantly constant right-handed neutrino could be de-localized completely it cannot generate ordinary kind of gauge super-symmetry. It is not yet completely clear whether the modes of the induced spinor field are localized at string world sheets also inside the Euclidian wormhole contacts defining the lines of the generalized Feynman diagrams.

   Intermediate gauge boson decay widths require that sparticles are either heavy enough or dark in the sense of having non-standard value of Planck constant. Darkness would provide an elegant explanation for their non-observability. It should be emphasized that TGD predicts that all fermions act as generators of badly broken super-symmetries at partonic 2-surfaces but these super-symmetries could correspond to much higher mass scale as that associated with the de-localized right-handed neutrino. The following piece of text summarizes the argument.

6. Ordinary SUSY means that apart from kinematical spin factors sparticles and particles behave identically with respect to standard model interactions. These spin factors would allow to distinguish between particles and sparticles. This requires strong correlations between fermion and right-handed neutrino: in fact, they should be at rest with respect to each other. Right-handed neutrinos have vanishing color and electro-weak quantum numbers. How it is possible to have sparticles as bound states with ordinary particle and right-handed neutrino?

   The localization of induced spinor fields to string world sheets suggests a solution to the problem.

   (a) The localization forces the fermions to move in parallel although they have no interactions. The 8-momenta and 8-velocities of fermion are light-like and they move along light-like 8-geodesics. Since the size of the partonic 2-surface should not change much. If all fundamental fermions involved are massive one can assume that they are at rest and in this manner geometrically stable state.

   (b) If one has massive fermion and massless right-handed neutrino, they should be at rest with respect to each other. What looks paradoxical that one cannot reduce the velocity to exactly zero in any coordinate system since covariantly constant right-handed neutrino represents a pure gauge degree of freedom. It is of course possible to assume that the relative velocity is some sufficiently low velocity. One can also argue that sparticles are unstable and that this is basically due to a geometric instability implied by the non-parallel 3-momenta of fundamental fermions.

   (c) If one assumes that the 4-momentum squared corresponds to that associated with the imbedding space spinor harmonics, one can to estimate the mass of the sparticle once the energy of the right-handed neutrino is fixed. This argument applies also to n-fermion states at associated with the wormhole contact pairs.

   (d) p-Adic mass calculations however give to mass squared also other contributions that that coming from the spinor harmonic, in particular negative ground state contribution and that the mass squared of the fundamental fermion vanishes for lowest states which would therefore have vanishing CP_2 velocity. Why the light-like four-momentum of the resulting state should not characterize the fermion line? In this picture p-adic thermal excitations would make the state unstable. One could in fact turn this argument to an explanation for why the stable physical particles must parallel 4-momenta.

   (e) What is still not well-understood is the tachyonic contribution to four-momentum. One possibility is that wormhole contact gives imaginary contribution to four-momentum. Second possibility is that the generating super-symplectic conformal weights are the negatives for the zeros of zeta. For non-trivial zeros the real part of the conformal would be -1/2.

So called massless extremals (MEs) define massless represent classical field pattern moving with light velocity and preserving its shape. This suggests that particle represented as a magnetic flux tube structure carrying monopole flux with two wormhole contacts and sliced between two MEs could serve as a starting point in attempts to understand the role of right handed neutrinos and how $\mathcal{N}=2$ or $\mathcal{N}=4$ type SYM emerges at the level of space-time geometry.

4.3 The Impact From LHC And Evolution Of TGD Itself

The missing energy predicted standard SUSY seems to be absent at LHC. The easy explanation would be that the mass scale of SUSY is unexpectedly high, of order 1-10 TeV. This would however destroy the original motivations for SUSY. The arguments developed in the following manner.

1. One must distinguish between imbedding space spinor harmonics and the modes of the induced spinor field. Right-handed neutrino with vanishing color quantum numbers and thus

covariantly constant in CP_2 is massless. All other modes of the induced spinor field are massive and in according to the p-adic mass calculations negative conformal weight of the ground state and the presence of Kac-Moody and super-symplectic generators make possible massless states having thermal excitations giving to the state a thermal mass. Right-handed neutrino can mix with left-handed neutrino ad can get mass. One can assign to any fermion a super-multiplet with 4 members.

One cannot assign full super-4-plet also to non-colored right handed neutrino itself: the multiplet would contain only 3 states. The most natural possibility is that the ground state is now a color excitation of right-handed neutrino and massless non-colored right-handed neutrinos give rise to the 4-plet. The colored spinor mode at imbedding space level is however a mixture or left- and right handed neutrinos.

2. In TGD framework the natural first guess is that right-handed neutrinos carrying four-momentum can give rise to missing energy. The assumption that fermions correspond to color partial waves in H implies that color excitations of the right handed neutrino that would appear in asymptotic states are necessarily colored. It could happen that these excitations are color neutralized by super-conformal generators. If this is not the case, these neutrinos would be like quarks and color confinement would explain why they cannot be observed as asymptotic states in macroscopic scales.

   Second possibility is that SUSY itself is generated by color partial waves of right-handed neutrino, octet most naturally. This option is however not consistent with the above model for one-fermion states and their super-partners.

4.4 Supersymmetry In Crisis

Supersymmetry is very beautiful generalization of the ordinary symmetry concept by generalizing Lie-algebra by allowing grading such that ordinary Lie algebra generators are accompanied by super-generators transforming in some representation of the Lie algebra for which Lie-algebra commutators are replaced with anti-commutators. In the case of Poincare group the super-generators would transform like spinors. Clifford algebras are actually super-algebras. Gamma matrices anti-commute to metric tensor and transform like vectors under the vielbein group (SO(n) in Euclidian signature). In supersymmetric gauge theories one introduced super translations anti-commuting to ordinary translations.

Supersymmetry algebras defined in this manner are characterized by the number of super-generators and in the simplest situation their number is one: one speaks about $\mathcal{N} = 1$ SUSY and minimal super-symmetric extension of standard model (MSSM) in this case. These models are most studied because they are the simplest ones. They have however the strange property that the spinors generating SUSY are Majorana spinors- real in well-defined sense unlike Dirac spinors. This implies that fermion number is conserved only modulo two: this has not been observed experimentally. A second problem is that the proposed mechanisms for the breaking of SUSY do not look feasible.

LHC results suggest MSSM does not become visible at LHC energies. This does not exclude more complex scenarios hiding simplest $\mathcal{N} = 1$ to higher energies but the number of real believers is decreasing. Something is definitely wrong and one must be ready to consider more complex options or totally new view abot SUSY.

What is the analog of SUSY in TGD framework? I must admit that I am still fighting to gain understanding of SUSY in TGD framework [K17]. That I can still imagine several scenarios shows that I have not yet completely understood the problem but I am working hardly to avoid falling to the sin of sloppying myself.

At the basic level one has super-conformal invariance generated in the fermion sector by the super-conformal charges assignable to the strings emanating from partonic 2-surfaces and connecting them to each other. For elementary particles one has 2 wormhole contacts and 4 wormhole throats. If the number of strings is just one, one has symplectic super-conformal symmetry, which is already huge. Several strings must be allowed and this leads to the Yangian variant of super-conformal symmetry, which is multi-local (multi-stringy).

One can also say that fermionic oscillator operators generate infinite-D super-algebra. One can restrict the consideration to lowest conformal weights if spinorial super-conformal invariance acts as gauge symmetry so that one obtains a finite-D algebra with generators labelled by electro-weak quantum numbers of quarks and leptons. This super-symmetry is badly broken but contains the algebra generated by right-handed neutrino and its conjugate as sub-algebra.

The basic question is whether covariantly constant right handed neutrino generators $\mathcal{N} = \in$ SUSY or whether the SUSY is generated as approximate symmetry by adding massless right-handed neutrino to the state thus changing its four-momentum. The problem with the first option is that it the standard norm of the state is naturally proportional to four-momentum and vanishes at the limit of vanishing four-momentum: is it possible to circumvent this problem somehow? In the following I summarize the situation as it seems just now.

1. In TGD framework $\mathcal{N} = 1$ SUSY is excluded since B and L and conserved separately and imbedding space spinors are not Majorana spinors. The possible analog of space-time SUSY should be a remnant of a much larger super-conformal symmetry in which the Clifford algebra generated by fermionic oscillator operators giving also rise to the Clifford algebra generated by the gamma matrices of the "world of classical worlds" (WCW) and assignable with string world sheets. This algebra is indeed part of infinite-D super-conformal algebra behind quantum TGD. One can construct explicitly the conserved super conformal charges accompanying ordinary charges and one obtains something analogous to $\mathcal{N} = \infty$ super algebra. This SUSY is however badly broken by electroweak interactions.

2. The localization of induced spinors to string world sheets emerges from the condition that electromagnetic charge is well-defined for the modes of induced spinor fields. There is however an exception: covariantly constant right handed neutrino spinor ν_R: it can be de-localized along entire space-time surface. Right-handed neutrino has no couplings to electroweak fields. It couples however to left handed neutrino by induced gamma matrices except when it is covariantly constant. Note that standard model does not predict ν_R but its existence is necessary if neutrinos develop Dirac mass. ν_R is indeed something which must be considered carefully in any generalization of standard model.

4.4.1 *Could covariantly constant right handed neutrinos generate SUSY?*

Could covariantly constant right-handed spinors generate exact $\mathcal{N} = 2$ SUSY? There are two spin directions for them meaning the analog $\mathcal{N} = 2$ Poincare SUSY. Could these spin directions correspond to right-handed neutrino and antineutrino. This SUSY would not look like Poincare SUSY for which anti-commutator of super generators would be proportional to four-momentum. The problem is that four-momentum vanishes for covariantly constant spinors! Does this mean that the sparticles generated by covariantly constant ν_R are zero norm states and represent super gauge degrees of freedom? This might well be the case although I have considered also alternative scenarios.

4.4.2 *What about non-covariantly constant right-handed neutrinos?*

Both imbedding space spinor harmonics and the Kähler-Dirac equation have also right-handed neutrino spinor modes not constant in M^4 and localized to the partonic orbits. If these are responsible for SUSY then SUSY is broken.

1. Consider first the situation at space-time level. Both induced gamma matrices and their generalizations to Kähler-Dirac gamma matrices defined as contractions of imbedding space gamma matrices with the canonical momentum currents for Kähler action are superpositions of M^4 and CP_2 parts. This gives rise to the mixing of right-handed and left-handed neutrinos. Note that non-covariantly constant right-handed neutrinos must be localized at string world sheets.

   This in turn leads neutrino massivation and SUSY breaking. Given particle would be accompanied by sparticles containing varying number of right-handed neutrinos and antineutrinos localized at partonic 2-surfaces.

2. One an consider also the SUSY breaking at imbedding space level. The ground states of the representations of extended conformal algebras are constructed in terms of spinor harmonics

of the imbedding space and form the addition of right-handed neutrino with non-vanishing four-momentum would make sense. But the non-vanishing four-momentum means that the members of the super-multiplet cannot have same masses. This is one manner to state what SUSY breaking is.

4.4.3 *What one can say about the masses of sparticles?*

The simplest form of massivation would be that all members of the super-multiplet obey the same mass formula but that the p-adic length scales associated with them are different. This could allow very heavy sparticles. What fixes the p-adic mass scales of sparticles? If this scale is CP_2 mass scale SUSY would be experimentally unreachable. The estimate below does not support this option.

One can consider the possibility that SUSY breaking makes sparticles unstable against phase transition to their dark variants with $h_{eff} = n \times h$. Sparticles could have same mass but be non-observable as dark matter not appearing in same vertices as ordinary matter! Geometrically the addition of right-handed neutrino to the state would induce many-sheeted covering in this case with right handed neutrino perhaps associated with different space-time sheet of the covering.

This idea need not be so outlandish at it looks first.

1. The generation of many-sheeted covering has interpretation in terms of breaking of conformal invariance. The sub-algebra for which conformal weights are n-tuples of integers becomes the algebra of conformal transformations and the remaining conformal generators do note represent gauge degrees of freedom anymore. They could however represent conserved conformal charges still.
2. This generalization of conformal symmetry breaking gives rise to infinite number of fractal hierarchies formed by sub-algebras of conformal algebra and is also something new and a fruit of an attempt to avoid sloppy thinking. The breaking of conformal symmetry is indeed expected in massivation related to the SUSY breaking.

The following poor man's estimate supports the idea about dark sfermions and the view that sfermions cannot be very heavy.

1. Neutrino mixing rate should correspond to the mass scale of neutrinos known to be in eV range for ordinary value of Planck constant. For $h_{eff}/h = n$ it is reduced by factor $1/n$, when mass kept constant. Hence sfermions could be stabilized by making them dark.
2. A very rough order of magnitude estimate for sfermion mass scale is obtained from Uncertainty Principle: particle mass should be higher than its decay rate. Therefore an estimate for the decay rate of sfermion could give a lower bound for its mass scale.
3. Assume the transformation $\nu_R \rightarrow \nu_L$ makes sfermion unstable against the decay to fermion and ordinary neutrino. If so, the decay rate would be dictated by the mixing rate and therefore to neutrino mass scale for the ordinary value of Planck constant. Particles and sparticles would have the same p-adic mass scale. Large h_{eff} could however make sfermion dark, stable, and non-observable.

4.4.4 *A rough model for the neutrino mixing in TGD framework*

The mixing of neutrinos would be the basic mechanism in the decays of sfermions. The following argument tries to capture what is essential in this process.

1. Conformal invariance requires that the string ends at which fermions are localized at wormhole throats are light-like curves. In fact, light-likeness gives rise to Virasosoro conditions.
2. Mixing is described by a vertex residing at partonic surface at which two partonic orbits join. Localization of fermions to string boundaries reduces the problem to a problem completely analogous to the coupling of point particle coupled to external gauge field. What is new that orbit of the particle has edge at partonic 2-surface. Edge breaks conformal invariance since one cannot say that curve is light-like at the edge. At edge neutrino transforms from right-handed to left handed one.

3. In complete analogy with $\overline{\Psi}\gamma^t A_t\Psi$ vertex for the point-like particle with spin in external field, the amplitude describing $nu_R - \nu_L$ transition involves matrix elements of form $\overline{\nu}_R\Gamma^t(CP_2)Z_t\nu_L$ at the vertex of the CP_2 part of the Kähler-Dirac gamma matrix and classical Z^0 field.

   How Γ^t is identified? The Kähler-Dirac gamma matrices associated with the interior need not be well-defined at the light-like surface and light-like curve. One basis of weak form of electric magnetic duality the Kähler-Dirac gamma matrix corresponds to the canonical momentum density associated with the Chern-Simons term for Kähler action. This gamma matrix contains only the CP_2 part.

The following provides as more detailed view.

1. Let us denote by $\Gamma^t_{CP_2}(in/out)$ the CP_2 part of the Kähler-Dirac gamma matrix at string at at partonic 2-surface and by Z^0_t the value of Z^0 gauge potential along boundary of string world sheet. The direction of string line in imbedding space changes at the partonic 2-surface. The question is what happens to the Kähler-Dirac action at the vertex.

2. For incoming and outgoing lines the equation

$$D(in/out)\Psi(in/out) = p^k(in,out)\gamma_k\Psi(in/out) \ ,$$

   where the Kähler-Dirac operator is $D(in/out) = \Gamma^t(in/out)D_t$, is assumed. ν_R corresponds to "in" and ν_R to "out". It implies that lines corresponds to massless M^4 Dirac propagator and one obtains something resembling ordinary perturbation theory.

   It also implies that the residue integration over fermionic internal momenta gives as a residue massless fermion lines with non-physical helicities as one can expect in twistor approach. For physical particles the four-momenta are massless but in complex sense and the imaginary part comes classical from four-momenta assignable to the lines of generalized Feynman diagram possessing Euclidian signature of induced metric so that the square root of the metric determinant differs by imaginary unit from that in Minkowskian regions.

3. In the vertex $D(in/out)$ could act in $\Psi(out/in)$ and the natural idea is that $\nu_R - \nu_L$ mixing is due to this so that it would be described the classical weak current couplings $\overline{\nu}_R\Gamma^t_{CP_2}(out)Z^0_t(in)\nu_L$ and $\overline{\nu}_R\Gamma^t_{CP_2}(out)Z^0_t(in)\nu_L$.

To get some idea about orders of magnitude assume that the CP_2 projection of string boundary is geodesic circle thus describable as $\Phi = \omega t$, where Φ is angle coordinate for the circle and t is Minkowski time coordinate. The contribution of CP_2 to the induced metric g_{tt} is $\Delta g_{tt} = -R^2\omega^2$.

1. In the first approximation string end is a light-like curve in Minkowski space meaning that CP_2 contribution to the induced metric vanishes. Neutrino mixing vanishes at this limit.

2. For a non-vanishing value of ωR the mixing and the order of magnitude for mixing rate and neutrino mass is expected to be $R \sim \omega$ and $m \sim \omega/h$. p-Adic length scale hypothesis and the experimental value of neutrino mass allows to estimate m to correspond to p-adic mass to be of order eV so that the corresponding p-adic prime p could be $p \simeq 2^{167}$. Note that $k = 127$ defines largest of the four Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ appearing in the length scale range 10 nm -2.5 μm. Hence the decay rate for ordinary Planck constant would be of order $R \sim 10^{14}$/s but large value of Planck constant could reduced it dramatically. In living matter reductions by a factor 10^{-12} can be considered.

To sum up, the space-time SUSY in TGD sense would differ crucially from SUSY in the standard sense. There would no Majorana spinors and sparticles could correspond to dark phase of matter with non-standard value of Planck constant. The signatures of the standard SUSY do not apply to TGD. Of course, a lot of professional work would be needed to derive the signatures of TGD SUSY.

4.5 Right-Handed Neutrino As Inert Neutrino?

There is a very interesting posting by Jester in Resonaances with title "How many neutrinos in the sky?" (see `http://tinyurl.com/y8scxzqr`) [C1]. Jester tells about the recent 9 years WMAP data [C3] and compares it with earlier 7 years data. In the earlier data the effective number of neutrino types was $N_{eff} = 4.34 \pm 0.87$ and in the recent data it is $N_{eff} = 3.26 \pm 0.35$. WMAP alone would give $N_{eff} = 3.89 \pm 0.67$ also in the recent data but also other data are used to pose constraints on N_{eff}.

To be precise, N_{eff} could include instead of fourth neutrino species also some other weakly interacting particle. The only criterion for contributing to N_{eff} is that the particle is in thermal equilibrium with other massless particles and thus contributes to the density of matter considerably during the radiation dominated epoch.

Jester also refers to the constraints on N_{eff} from nucleosynthesis (see `http://tinyurl.com/y8fkfn5y`) , which show that $N_{eff} \sim 4$ us slightly favored although the entire range $[3, 5]$ is consistent with data.

It seems that the effective number of neutrinos could be 4 instead of 3 although latest WMAP data combined with some other measurements favor 3. Later a corrected version e `http://tinyurl.com/y9er8szf`) of the eprint appeared [C3] telling that the original estimate of N_{eff} contained a mistake and the correct estimate is $N_{eff} = 3.84 \pm 0.40$.

An interesting question is what $N_{eff} = 4$ could mean in TGD framework?

1. One poses to the modes of the Kähler-Dirac equation the following condition: electric charge is conserved in the sense that the time evolution by Kähler-Dirac equation does not mix a mode with a well-defined em charge with those with different em charge. The implication is that all modes except pure right handed neutrino are restricted at string world sheets. The first guess is that string world sheets are minimal surfaces of space-time surface (rather than those of imbedding space). One can also consider minimal surfaces of imbedding space but with effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices. This would give a direct physical meaning for this somewhat mysterious effective metric.

   For the neutrino modes localized at string world sheets mixing of left and right handed modes takes place and they become massive. If only 3 lowest genera for partonic 2-surfaces are light, one has 3 neutrinos of this kind. The same applies to all other fermion species. The argument for why this could be the case relies on simple observation [K2]: the genera g=0, 1, 2 have the property that they allow for all values of conformal moduli Z_2 as a conformal symmetry (hyper-ellipticity). For $g > 2$ this is not the case. The guess is that this additional conformal symmetry is the reason for lightness of the three lowest genera.

2. Only purely right-handed neutrino is completely de-localized in 4-volume so that one cannot assign to it genus of the partonic 2-surfaces as a topological quantum number and it effectively gives rise to a fourth neutrino very much analogous to what is called sterile neutrino. De-localized right-handed neutrinos couple only to gravitation and in case of massless extremals this forces them to have four-momentum parallel to that of ME: only massless modes are possible. Very probably this holds true for all preferred extremals to which one can assign massless longitudinal momentum direction which can vary with spatial position.

3. The coupling of ν_R is to gravitation alone and all electroweak and color couplings are absent. According to standard wisdom de-localized right-handed neutrinos cannot be in thermal equilibrium with other particles. This according to standard wisdom. But what about TGD?

   One should be very careful here: de-localized right-handed neutrinos is proposed to give rise to SUSY (not $\mathcal{N} = 1$ requiring Majorana fermions) and their dynamics is that of passive spectator who follows the leader. The simplest guess is that the dynamics of right handed neutrinos at the level of amplitudes is completely trivial and thus trivially supersymmetric. There are however correlations between four-momenta.

   (a) The four-momentum of ν_R is parallel to the light-like momentum direction assignable to the massless extremal (or more general preferred extremal). This direct coupling to the geometry is a special feature of the Kähler-Dirac operator and thus of sub-manifold gravity.

(b) On the other hand, the sum of massless four-momenta of two parallel pieces of preferred extremals is the - in general massive - four-momentum of the elementary particle defined by the wormhole contact structure connecting the space-time sheets (which are glued along their boundaries together since this is seems to be the only manner to get rid of boundary conditions requiring vacuum extremal property near the boundary). Could this direct coupling of the four-momentum direction of right-handed neutrino to geometry and four-momentum directions of other fermions be enough for the right handed neutrinos to be counted as a fourth neutrino species in thermal equilibrium? This might be the case!

One cannot of course exclude the coupling of 2-D neutrino at string world sheets to 4-D purely right handed neutrinos analogous to the coupling inducing a mixing of sterile neutrino with ordinary neutrinos. Also this could help to achieve the thermal equilibrium with 2-D neutrino species.

4.6 Experimental Evidence For Sterile Neutrino?

Many physicists are somewhat disappointed to the results from LHC: the expected discovery of Higgs has been seen as the main achievement of LHC hitherto. Much more was expected. To my opinion there is no reason for disappointment. The exclusion of the standard SUSY at expected energy scale is very far reaching negative result. Also the fact that Higgs mass is too small to be stable without fine tuning is of great theoretical importance. The negative results concerning heavy dark matter candidates are precious guidelines for theoreticians. The non-QCD like behavior in heavy ion collisions and proton-ion collisions is bypassed my mentioning something about AdS/CFT correspondence and non-perturbative QCD effects. I tend to see these effects as direct evidence for M_{89} hadron physics [K10].

In any case, something interesting has emerged quite recently. Resonaances tells that the recent analysis (see `http://tinyurl.com/ycf4vbkq`) [C2] of X-ray spectrum of galactic clusters claims the presence of monochromatic 3.5 keV photon line. The proposed interpretation is as a decay product of sterile 7 keV neutrino transforming first to a left-handed neutrino and then decaying to photon and neutrino via a loop involving W boson and electron. This is of course only one of the many interpretations. Even the existence of line is highly questionable.

One of the poorly understood aspects of TGD is right-handed neutrino, which is obviously the TGD counterpart of the inert neutrino.

1. The old idea is that covariantly constant right handed neutrino could generate $\mathcal{N}=2$ supersymmetry in TGD Universe. In fact, all modes of induced spinor field would generate superconformal symmetries but electroweak interactions would break these symmetries for the modes carrying non-vanishing electroweak quantum numbers: they vanish for ν_R. This picture is now well-established at the level of WCW geometry [K19]: super-conformal generators are labelled angular momentum and color representations plus two conformal weights: the conformal weight assignable to the light-like radial coordinate of light-cone boundary and the conformal weight assignable to string coordinate. It seems that these conformal weights are independent. The third integer labelling the states would label genuinely Yangian generators: it would tell the poly-locality of the generator with locus defined by partonic 2-surface: generators acting on single partonic 2-surface, 2 partonic 2-surfaces, ...

2. It would seem that even the SUSY generated by ν_R must be badly broken unless one is able to invent dramatically different interpretation of SUSY. The scale of SUSY breaking and thus the value of the mass of right-handed neutrino remains open also in TGD. In lack of better one could of course argue that the mass scale must be CP_2 mass scale because right-handed neutrino mixes considerably with the left-handed neutrino (and thus becomes massive) only in this scale. But why this argument does not apply also to left handed neutrino which must also mix with the right-handed one!

3. One can of course criticize the proposed notion of SUSY: wonder whether fermion + extremely weakly interacting ν_R at same wormhole throat (or interior of 3-surface) can behave as single coherent entity as far spin is considered [K17] ?

4. The condition that the modes of induced spinor field have a well-defined electromagnetic charge eigenvalue [K16] requires that they are localized at 2-D string world sheets or partonic 2-surfaces: without this condition classical W boson fields would mix the em charged and neutral modes with each other. Right-handed neutrino is an exception since it has no electroweak couplings. Unless right-handed neutrino is covariantly constant, the Kähler-Dirac gamma matrices can however mix the right-handed neutrino with the left handed one and this can induce transformation to charged mode. This does not happen if each Kähler-Dirac gamma matrix can be written as a linear combination of either M^4 or CP_2 gamma matrices and Kähler-Dirac equation is satisfied separately by M^4 and CP_2 parts of the Kähler-Dirac equation.

5. Is the localization of the modes other than covariantly constant neutrino to string world sheets a consequence of dynamics or should one assume this as a separate condition? If one wants similar localization in space-time regions of Euclidian signature - for which CP_2 type vacuum extremal is a good representative - one must assume it as a separate condition. In number theoretic formulation string world sheets/partonic 2-surfaces would be commutative/co-commutative sub-manifolds of space-time surfaces which in turn would be associative or co-associative sub-manifolds of imbedding space possessing (hyper-)octonionic tangent space structure. For this option also right-handed neutrino would be localized to string world sheets. Right-handed neutrino would be covariantly constant only in 2-D sense.

   One can consider the possibility that ν_R is de-localized to the entire 4-D space-time sheet. This would certainly modify the interpretation of SUSY since the number of degrees of freedom would be reduced for ν_R.

6. Non-covariantly constant right-handed neutrinos could mix with left-handed neutrinos but not with charged leptons if the localization to string world sheets is assumed for modes carrying non-vanishing electroweak quantum numbers. This would make possible the decay of right-handed to neutrino plus photon, and one cannot exclude the possibility that ν_R has mass 7 keV.

   Could this imply that particles and their spartners differ by this mass only? Could it be possible that practically unbroken SUSY could be there and we would not have observed it? Could one imagine that sfermions have annihilated leaving only states consisting of fundamental fermions? But shouldn't the total rate for the annihilation of photons to hadrons be two times the observed one? This option does not sound plausible.

   What if one assumes that given sparticle is charactrized by the same p-adic prime as corresponding particle but is dark in the sense that it corresponds to non-standard value of Planck constant. In this case sfermions would not appear in the same vertex with fermions and one could escape the most obvious contradictions with experimental facts. This leads to the notion of shadron: shadrons would be [K17] obtained by replacing quarks with dark squarks with nearly identical masses. I have asked whether so called X and Y bosons having no natural place in standard model of hadron could be this kind of creatures.

The interpretation of 3.5 keV photons as decay products of right-handed neutrinos is of course totally ad hoc. Another TGD inspired interpretation would be as photons resulting from the decays of excited nuclei to their ground state.

1. Nuclear string model [K11] predicts that nuclei are string like objects formed from nucleons connected by color magnetic flux tubes having quark and antiquark at their ends. These flux tubes are long and define the "magnetic body" of nucleus. Quark and antiquark have opposite em charges for ordinary nuclei. When they have different charges one obtains exotic state: this predicts entire spectrum of exotic nuclei for which statistic is different from what proton and neutron numbers deduced from em charge and atomic weight would suggest. Exotic nuclei and large values of Planck constant could make also possible cold fusion [K6].

2. What the mass difference between these states is, is not of course obvious. There is however an experimental finding [C4] (see *Analysis of Gamma Radiation from a Radon Source: Indications of a Solar Influence* at `http://tinyurl.com/d9ymwm3`) that nuclear decay rates oscillate with a period of year and the rates correlate with the distance from Sun. A possible explanation is that the gamma rays from Sun in few keV range excite the exotic nuclear states with different decay rate so that the average decay rate oscillates [K11]. Note that nuclear excitation energies in keV range would also make possible interaction of nuclei with atoms and molecules.

3. This allows to consider the possibility that the decays of exotic nuclei in galactic clusters generates 3.5 keV photons. The obvious question is why the spectrum would be concentrated at 3.5 keV in this case (second question is whether the energy is really concentrated at 3.5 keV: a lot of theory is involved with the analysis of the experiments). Do the energies of excited states depend on the color bond only so that they would be essentially same for all nuclei? Or does single excitation dominate in the spectrum? Or is this due to the fact that the thermal radiation leaking from the core of stars excites predominantly single state? Could $E = 3.5$ keV correspond to the maximum intensity for thermal radiation in stellar core? If so, the temperature of the exciting radiation would be about $T \simeq E/3 \simeq 1.2 \times 10^7$ K. This in the temperature around which formation of Helium by nuclear fusion has begun: the temperature at solar core is around 1.57×10^7 K.

4.7 Delicacies of the induced spinor structure and SUSY mystery

The discussion of induced spinor structure leads to a modification of an earlier idea (one of the many) about how SUSY could be realized in TGD in such a manner that experiments at LHC energies could not discover it and one should perform experiments at the other end of energy spectrum at energies which correspond to the thermal energy about .025 eV at room temperature. I have the feeling that this observation could be of crucial importance for understanding of SUSY.

4.7.1 Induced spinor structure

The notion of induced spinor field deserves a more detailed discussion. Consider first induced spinor structures.

1. Induced spinor field are spinors of $M^4 \times CP_2$ for which modes are characterized by chirality (quark or lepton like) and em charge and weak isospin.

2. Induced spinor spinor structure involves the projection of gamma matrices defining induced gamma matrices. This gives rise to superconformal symmetry if the action contains only volume term.

   When Kähler action is present, superconformal symmetry requires that the modified gamma matrices are contractions of canonical momentum currents with imbedding space gamma matrices. Modified gammas appear in the modified Dirac equation and action, whose solution at string world sheets trivializes by super-conformal invariance to same procedure as in the case of string models.

3. Induced spinor fields correspond to two chiralities carrying quark number and lepton number. Quark chirality does not carry color as spin-like quantum number but it corresponds to a color partial wave in CP_2 degrees of freedom: color is analogous to angular momentum. This reduces to spinor harmonics of CP_2 describing the ground states of the representations of super-symplectic algebra.

   The harmonics do not satisfy correct correlation between color and electroweak quantum numbers although the triality t=0 for leptonic waves and t=1 for quark waves. There are two manners to solve the problem.

   (a) Super-symplectic generators applied to the ground state to get vanishing ground states weight instead of the tachyonic one carry color and would give for the physical states correct correlation: leptons/quarks correspond to the same triality zero(one partial wave irrespective of charge state. This option is assumed in p-adic mass calculations [K9].

(b) Since in TGD elementary particles correspond to pairs of wormhole contacts with weak isospin vanishing for the entire pair, one must have pair of left and right-handed neutrinos at the second wormhole throat. It is possible that the anomalous color quantum numbers for the entire state vanish and one obtains the experimental correlation between color and weak quantum numbers. This option is less plausible since the cancellation of anomalous color is not local as assume in p-adic mass calculations.

The understanding of the details of the fermionic and actually also geometric dynamics has taken a long time. Super-conformal symmetry assigning to the geometric action of an object with given dimension an analog of Dirac action allows however to fix the dynamics uniquely and there is indeed dimensional hierarchy resembling brane hierarchy.

1. The basic observation was following. The condition that the spinor modes have well-defined em charge implies that they are localized to 2-D string world sheets with vanishing W boson gauge fields which would mix different charge states. At string boundaries classical induced W boson gauge potentials guarantee this. Super-conformal symmetry requires that this 2-surface gives rise to 2-D action which is area term plus topological term defined by the flux of Kähler form.

2. The most plausible assumption is that induced spinor fields have also interior component but that the contribution from these 2-surfaces gives additional delta function like contribution: this would be analogous to the situation for branes. Fermionic action would be accompanied by an area term by supersymmetry fixing modified Dirac action completely once the bosonic actions for geometric object is known. This is nothing but super-conformal symmetry.

   One would actually have the analog of brane-hierarchy consisting of surfaces with dimension D= 4,3,2,1 carrying induced spinor fields which can be regarded as independent dynamical variables and characterized by geometric action which is D-dimensional analog of the action for Kähler charged point particle. This fermionic hierarchy would accompany the hierarchy of geometric objects with these dimensions and the modified Dirac action would be uniquely determined by the corresponding geometric action principle (Kähler charged point like particle, string world sheet with area term plus Kähler flux, light-like 3-surface with Chern-Simons term, 4-D space-time surface with Kähler action).

3. This hierarchy of dynamics is consistent with SH only if the dynamics for higher dimensional objects is induced from that for lower dimensional objects - string world sheets or maybe even their boundaries orbits of point like fermions. Number theoretic vision [K20] suggests that this induction relies algebraic continuation for preferred extremals. Note that quaternion analyticity [K21] means that quaternion analytic function is determined by its values at 1-D curves.

4. Quantum-classical correspondences (QCI) requires that the classical Noether charges are equal to the eigenvalues of the fermionic charges for surfaces of dimension $D = 0, 1, 2, 3$ at the ends of the CDs. These charges would not be separately conserved. Charges could flow between objects of dimension $D + 1$ and D - from interior to boundary and vice versa. Four-momenta and also other charges would be complex as in twistor approach: could complex values relate somehow to the finite life-time of the state?

   If quantum theory is square root of thermodynamics as zero energy ontology suggests, the idea that particle state would carry information also about its life-time or the time scale of CD to which is associated could make sense. For complex values of α_K there would be also flow of canonical and super-canonical momentum currents between Euclidian and Minkowskian regions crucial for understand gravitational interaction as momentum exchange at imbedding space level.

5. What could be the physical interpretation of the bosonic and fermionic charges associated with objects of given dimension? Condensed matter physicists assign routinely physical states to objects of various dimensions: is this assignment much more than a practical approximation or could condensed matter physics already be probing many-sheeted physics?

4.7.2 SUSY and TGD

From this one ends up to the possibility of identifying the counterpart of SUSY in TGD framework [K17, K8].

1. In TGD the generalization of much larger super-conformal symmetry emerges from the super-symplectic symmetries of WCW. The mathematically questionable notion of super-space is not needed: only the realization of super-algebra in terms of WCW gamma matrices defining super-symplectic generators is necessary to construct quantum states. As a matter of fact, also in QFT approach one could use only the Clifford algebra structure for super-multiplets. No Majorana condition on fermions is needed as for $\mathcal{N} = 1$ space-time SUSY and one avoids problems with fermion number non-conservation.
2. In TGD the construction of sparticles means quite concretely adding fermions to the state. In QFT it corresponds to transformation of states of integer and half-odd integer spin to each other. This difference comes from the fact that in TGD particles are replaced with point like particles.
3. The analog of $\mathcal{N} = 2$ space-time SUSY could be generated by covariantly constant right handed neutrino and antineutrino. Quite generally the mixing of fermionic chiralities implied by the mixing of M^4 and CP_2 gamma matrices implies SUSY breaking at the level of particle masses (particles are massless in 8-D sense). This breaking is purely geometrical unlike the analog of Higgs mechanism proposed in standard SUSY.

There are several options to consider.

1. The analog of brane hierarchy is realized also in TGD. Geometric action has parts assignable to 4-surface, 3-D light like regions between Minkowskian and Euclidian regions, 2-D string world sheets, and their 1-D boundaries. They are fixed uniquely. Also their fermionic counterparts - analogs of Dirac action - are fixed by super-conformal symmetry. Elementary particles reduce so composites consisting of point-like fermions at boundaries of wormhole throats of a pair of wormhole contacts.

   This forces to consider 3 kinds of SUSYs! The SUSYs associated with string world sheets and space-time interiors would certainly be broken since there is a mixing between M^4 chiralities in the modified Dirac action. The mass scale of the broken SUSY would correspond to the length scale of these geometric objects and one might argue that the decoupling between the degrees of freedom considered occurs at high energies and explains why no evidence for SUSY has been observed at LHC. Also the fact that the addition of massive fermions at these dimensions can be interpreted differently. 3-D light-like 3-surfaces could be however an exception.

2. For 3-D light-like surfaces the modified Dirac action associated with the Chern-Simons term does not mix M^4 chiralities (signature of massivation) at all since modified gamma matrices have only CP_2 part in this case. All fermions can have well-defined chirality. Even more: the modified gamma matrices have no M^4 part in this case so that these modes carry no four-momentum - only electroweak quantum numbers and spin. Obviously, the excitation of these fermionic modes would be an ideal manner to create spartners of ordinary particles consting of fermion at the fermion lines. SUSY would be present if the spin of these excitations couples - to various interactions and would be exact.

   What would be these excitations? Chern-Simons action and its fermionic counterpart are non-vanishing only if the CP_2 projection is 3-D so that one can use CP_2 coordinates. This strongly suggests that the modified Dirac equation demands that the spinor modes are covariantly constant and correspond to covariantly constant right-handed neutrino providing only spin.

   If the spin of the right-handed neutrino adds to the spin of the particle and the net spin couples to dynamics, $\mathcal{N} = 2$ SUSY is in question. One would have just action with unbroken SUSY at QFT limit? But why also right-handed neutrino spin would couple to dynamics if only CP_2 gamma matrices appear in Chern-Simons-Dirac action? It would seem that it is independent degree of freedom having no electroweak and color nor even gravitational couplings by its covariant constancy. I have ended up with just the same SUSY-or-no-SUSY that I have had earlier.

3. Can the geometric action for light-like 3-surfaces contain Chern-Simons term?

   (a) Since the volume term vanishes identically in this case, one could indeed argue that also the counterpart of Kähler action is excluded. Moreover, for so called massless extremals of Kähler action reduces to Chern-Simons terms in Minkowskian regions and this could happen quite generally: TGD with only Kähler action would be almost topological QFT as I have proposed. Volume term however changes the situation via the cosmological constant. Kähler-Dirac action in the interior does not reduce to its Chern-Simons analog at light-like 3-surface.

   (b) The problem is that the Chern-Simons term at the two sides of the light-like 3-surface differs by factor $\sqrt{-1}$ coming from the ratio of $\sqrt{g_4}$ factors which themselves approach to zero: oOne would have the analog of dipole layer. This strongly suggests that one should not include Chern-Simons term at all.

   Suppose however that Chern-Simons terms are present at the two sides and α_K is real so that nothing goes through the horizon forming the analog of dipole layer. Both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions would decouple completely but currents would flow to the analog of dipole layer. This is not physically attractive.

   The canonical momentum current and its super counterpart would give fermionic source term $\Gamma^n \Psi_{int,\pm}$ in the modified Dirac equation defined by Chern-Simons term at given side $\pm$: $\pm$ refers to Minkowskian/Euclidian part of the interior. The source term is proportional to $\Gamma^n \Psi_{int,\pm}$ and Γ^n is in principle mixture of M^4 and CP_2 gamma matrices and therefore induces mixing of M^4 chiralities and therefore also 3-D SUSY breaking. It must be however emphasized that Γ^n is singular and one must be consider the limit carefully also in the case that one has only continuity conditions. The limit is not completely understood.

   (c) If α_K is complex there is coupling between the two regions and the simplest assumption has been that there is no Chern-Simons term as action and one has just continuity conditions for canonical momentum current and hits super counterpart.

   The cautious conclusion is that 3-D Chern-Simons term and its fermionic counterpart are absent.

4. What about the addition of fermions at string world sheets and interior of space-time surface ($D = 2$ and $D = 4$). For instance, in the case of hadrons $D = 2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. Let us consider the interior ($D = 4$). For instance, inn the case of hadrons $D = 2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. The smallness of cosmological constant implies that the contribution to the four-momentum from interior should be rather small so that an interpretation in terms of broken SUSY might make sense. There would be mass $m \sim .03$ eV per volume with size defined by the Compton scale $\hbar/m$. Note however that cosmological constant has spectrum coming as inverse powers of prime so that also higher mass scales are possible.

   This interpretation might allow to understand the failure to find SUSY at LHC. Sparticles could be obtained by adding interior right-handed neutrinos and antineutrinos to the particle state. They could be also associated with the magnetic body of the particle. Since they do not have color and weak interactions, SUSY is not badly broken. If the mass difference between particle and sparticle is of order $m = .03$ eV characterizing dark energy density ρ_{vac}, particle and sparticle could not be distinguished in higher energy physics at LHC since it probes much shorter scales and sees only the particle. I have already earlier proposed a variant of this mechanism but without SUSY breaking.

To discover SUSY one should do very low energy physics in the energy range $m \sim .03$ eV having same order of magnitude as thermal energy $kT = 2.6 \times 10^{-2}$ eV at room temperature 25 °C. One should be able to demonstrate experimentally the existence of sparticle with mass differing by about $m \sim .03$ eV from the mass of the particle (one cannot exclude higher mass scales since Λ is expected to have spectrum). An interesting question is whether the sfermions associated with standard fermions could give rise to Bose-Einstein condensates whose existence in the length scale of large neutron is strongly suggested by TGD view about living matter.

4.8 Conclusions

The conclusion that the standard SUSY ($\mathcal{N} = 1$ SUSY with Majorana spinors) is absent in TGD Universe and also in the real one looks rather feasible in light of various arguments discussed in this chapter and also conforms with the LHC data. A more general SUSY with baryon and lepton conservation and Dirac spinors is however possible in TGD framework.

During the attempts to understand SUSY several ideas have emerged and the original discussions are retained as such in this chapter. It is interesting to see that their fate is if standard SUSY has no TGD counterpart.

1. One of the craziest ideas was that spartners indeed exists and even with the same p-adic mass scale but might be realized as dark matter. Same mass scale is indeed a natural prediction if right-handed neutrino and particle have same mass scale. Therefore even the mesons of ordinary hadron physics would be accompanied by smesons - pairs of squark and anti-squark. In fact, this is what the most recent form of the theory predicts: unfortunately there is no manner to experimentally distinguish between fermion and pseudo-sfermion if ν_R is zero momentum state lacking even gravitational interactions.

2. There are indications that charmonium as exotic states christened as X and Y mesons and the question was that they could correspond to mesons built either from colored excitations of charged quark and antiquark or from squark and anti-squark. The recent view leaves only the option based on colored excitations alive. The states in question would be analogous to pairs of color excitations of leptons introduced to explain various anomalies in leptonic sector [K14]. The question was whether lepto-hadrons could correspond to bound states of colored sleptons and have same p-adic mass scale as leptons have [K14]. The original form of lepto-hadron hypothesis remains intact.

3. Evidence that pion and also other hadrons have what could be called infrared Regge trajectories has been reported, and one could ask whether these trajectories could include spion identified as a bound state of squarks. Also this identification is excluded and the proposed identification in terms of stringy states assignable to long color magnetic flux tubes accompanying hadron remains under consideration. IR Regge trajectories would serve as a signature for the non-perturbative aspects of hadron physics.

4. The latest idea along these lines is that spartners are obtained by adding right-handed neutrinos to the interior of space-time surface assignable to the particle. SUSY would not be detectable at high energies, which would explain the negative findings at LHC. Spartners could be discovered at low energy physics perhaps assignable to the magnetic bodies of particles: the mass scale could be as low .03 eV determined by cosmological constant in the scale of cosmology. Note however that cosmological constant has spectrum coming as inverse powers of prime.

5 SUSY Algebra At QFT Limit

The first expectation is that QFT limit TGD corresponds to a situation in which given space-time surface is representable as a graph for some map $M^4 \rightarrow CP_2$. This assumption is essential for the understanding of how the QFT limit of TGD emerges when many-sheeted space-time is replaced with a piece of Minkowski space in macroscopic scales and how gauge potentials of standard model relate to the induced gauge potentials. Already at elementary particle scales this assumption fails if they are regarded as pairs of wormhole contacts at distance characterized by Compton length: two sheetedness is involved in an essential manner.

This assumption is not actually needed in zero energy ontology if M^4 is assumed to label the positions of either tip of CD rather than points of the space-time sheet. The position of the other tip of CD relative to the first one could be interpreted in terms of Robertson-Walker coordinates for quantum cosmology [K13].

An intuitively plausible idea is that particle space-time sheets with Euclidian signature of the induced metric are replaced with world-lines. Fermions can be said to propagate along the boundaries of string world sheets so that this approximation would force all fermion lines of the parton orbit to form single line. Intuitively this might correspond to the replacement of multi-stringy Yangian [K22] with a super-field.

Strings bring in bi-locality at fundamental level and the hierarchy of Planck constants implies this non-locality in arbitrarily long length scales. The formation of gravitational bound states would involve gigantic values of Planck constant $h_{eff} = n \times h$ and macroscopic quantum coherence in astrophysical scales [K7, K18, K12]. This requires a generalization of quantum theory itself and of course challenges the idea that SUSY limit of TGD could make sense except in special situations.

What is essential for QFT limit is that only perturbations around single maximum of Kähler function are considered. If several maxima are important, one must include a weighting defined by the values of the exponent of Kähler function. The huge symmetries of WCW geometry are expected to make the functional integral over perturbations calculable.

5.1 Minimum Information About Space-Time Sheet And Particle Quantum Numbers Needed To Formulate SUSY Algebra

The basic problem is how to feed just the essential information about quantum states and space-time surfaces to the definition of the QFT limit.

1. The information about quantum numbers of particles must be fed also to the QFT approximation. It is natural to start from the classical description of point like fermions in H in terms of light-like geodesics of H at the light-like parton orbits carrying light-like 8-momentum: action principle indeed leads to this picture. Momentum and color charges serve as natural quantum numbers besides electroweak quantum numbers. The conserved color charges associated with CP_2 geodesics need not correspond to the usual color charges since they correspond to center of mass rotational motion in CP_2 degrees of freedom. Ordinary color charges correspond to the spinorial partial wavs assignable to CP_2 type extremals.

   The propagators of fundamental fermions massless in 8-D sense are the basic building bricks of the scattering amplitudes in the fundamental formulation of TGD. Elementary particles emerge as bound states of fundamental fermions, and one might of hope that the scattering amplitudes might allow also at the QFT limit a formulation involving only fundamental fermions. The basic vertices would correspond to product and co-product for super-symplectic Yangian and these 3-vertices should correspond to gauge theory vertices. The basic building brick of gauge boson would be wormhole contact with throats carrying fermion and antifermion. It might be that the QFT limit requires the introduction of boson fields. Both fermions and bosons consist of at least two wormhole contacts.

2. Should one interpret QFT limit as a QFT in X^4 representable as a graph for a map $M^4 \rightarrow CP_2$, or in M^4, or perhaps in $M^4 \times CP_2$? In zero energy ontology the proper interpretation is in terms of QFT in M^4 defining the coordinates of the M^4 projection of space-time point. Minimal Kaluza-Klein type extension to $M^4 \times S^1$ might be required in order to take into account the geodesic motion of fundamental fermions in CP_2 degrees of freedom.

3. What information about space-time surface is needed?

   (a) One can in principle feed all information about space-time sheet without losing Poincare invariance since momentum operators do not act on space-time coordinates. The description becomes however in-practical even if one restricts the consideration to the maxima of Kähler function.

(b) Partonic two-surfaces X^2 are identified as intersections of 3-D light-like wormhole throats with the boundary of CD characterizes basic building bricks of elementary particles and elementary particle itself corresponds to space-like 3-surface at the boundary of CD. The minimal approach would use only cm degrees of freedom for the 3-surface characterizing the particle. A better accuracy would be obtained by using cm coordinates for the partonic 2-surfaces. Even better approximation would be obtained by using the positions fermions associated with given partonic 2-surface.

(c) The ends of fermion lines defined by the boundaries of string world sheets represent necessary information but correspond to single point of M^4 in QFT approximation. The conformal moduli of the partonic 2-surface are very relevant and the elementary particle vacuum functional in the moduli space [K2] depending on the genus of the partonic 2-surface codes for a relevant information. This information could be compressed to genus its genus characterizing fermion generations plus a rule stating that the particles in the same 3-vertex have same genus and that bosons are superpositions over different genera. Only the three lowest genera have been observed and this can be understood in terms of hyper-ellipticity [K2].

(d) Some information about zero modes characterized by the induced Kähler form invariant under quantum fluctuations assignable to Hamiltonians of $\delta M^4_\pm \times CP_2$ at boundaries of CD is certainly needed: here the identification of Kähler potential as the Kähler function of WCW is highly attractive hypothesis.

5.2 The Physical Picture Behind The Realization Of SUSY Algebra At Point Like Limit

The challenge is to deduce SUSY algebra in the approximation that particle like 3-surfaces are replaced by points. The basic physical constraint on the realization of the SUSY algebra come from the condition that one must be able to describe also massive particles as members of SUSY multiplets. This should make possible also 8-D counterpart of twistorialization in terms of octonionic gamma matrices reducing to quaternionic ones using representation of octonion units in terms of the structure constants of the octonionic algebra. The general structure of Kähler-Dirac action suggests how to proceed. $p^k\gamma_k$ should be replaced with a simplified version of its 8-D variant in $M^4 \times CP_2$ and the CP_2 part of this operator should describe the massivation.

1. Fermion lines correspond to light-like geodesics of imbedding space. For particles which are massless in M^4, the geodesic circle defining CP_2 projection must contract to a point.

2. The generalization of the Dirac operator appearing in commutation relations reads as

$$\begin{array}{c} p^k\gamma_k \quad \rightarrow \quad D = p^k\gamma_k + Q\gamma_k\frac{ds^k}{ds} \quad , \\ s_{kl}\frac{ds^k}{dt}\frac{ds^l}{dt} = 1 \quad . \end{array} \tag{5.1}$$

   Mass shell condition fixes the value of Q

$$Q \quad = \quad \pm m \quad . \tag{5.2}$$

   For geodesic circle the angle coordinate to be angle parameterizing the geodesic circle is the natural variable and the gamma matrices can be taken to be just single constant gamma matrix along the geodesic circle.

3. Imbedding space spinors have anomalous color charge equal to -1 unit for lepton and 1/3 units for quarks. Mass shell condition is satisfied if Q is proportional to anomalous hypercharge and mass of the particle in turn determined by p-adic thermodynamics. Quantum classical correspondence suggests that the square of CP_2 part of 8-momentum equals to the eigenvalue of CP_2 spinor Laplacian given the mass square of the spinor mode for an incoming particle.

4. Particle mass m should relate closely to the frequencies characterizing general extremals. Quite generally, one can write in cylindrical coordinates the general expressions of CP_2 angle variables Ψ and Φ as $(\Psi, \Phi) = (\omega_1 t + k_1 z + n_1\phi ..., \omega_2 t + k_2 z + n_2\phi ...)$. Here... denotes Fourier expansion [L1], [L1]: this corresponds to Cartan algebra of Poincare group with energy, one momentum component and angular momentum defining the quantum numbers. One can say that the frequencies define a warping of M^4 for $(\Psi, \Phi) = (\omega_1 t, \omega_2 t)$. The frequencies characterizing the warping of the canonically imbedded M^4 should closely relate to the mass of the particle. This raises the question whether the replacement of S^1 with $S^1 \times S^1$ is appropriate.

5. Twistor description is also required. Generalization of ordinary twistors to octotwistor with quaternionicity condition as constraint allows to describe massive particles using almost-twistors. For massive particle the unit octonion corresponding to momentum in rest frame, the octonion defined by the polarization vector $\epsilon_k\gamma_k$, and the tangent vector $\gamma_k ds^k/ds$ (analog of polarization vector in CP_2) generate quaternionic sub-algebra. For massless particle momentum and polarization generate quaternionic sub-algebra as M^4 tangent space.

The SUSY algebra at QFT limit differs from the SUSY algebra defining the fundamental anti-commutators of the fermionic oscillator operators for the induced spinor fields since the Kähler-Dirac gamma matrices defined by the Kähler action are replaced with ordinary gamma matrices. The canonical commutation relations are however those between Ψ and its canonical momentum density $\overline{\Psi}\Gamma^t_{K-D}$ with the same right-hand side as usually (for quantum variant quantum phase appears in the anti-commutation relations). Hence the general form of anti-commutation relations are not changed in the transition and SUSY character is preserved if present in the fundamental formulation.

5.3 Explicit Form Of The SUSY Algebra At QFT Limit

The explicit form of the SUSY algebra follows from the proposed picture.

1. Spinor modes at X^2 correspond to the generators of the algebra. Effective 2-D property implies that spinor modes at partonic 2-surface can be assumed to have well-defined weak isospin and spin and be proportional to constant spinors.

2. The anti-commutators of oscillator operators define SUSY algebra. In leptonic sector one has

$$\begin{aligned} \{a^\dagger_{m\dot{\alpha}}, a^n_\beta\} &= \delta^n_m D_{\dot{\alpha}\beta} \ , \\ D &= (p^k\sigma_k + Q^a\sigma_a) \ . \end{aligned} \tag{5.3}$$

   Q^a denote color charges. The notions are same as in the case of WCW Clifford algebra. In quark sector one has opposite chirality and σ is replaced with $\hat{\sigma}$. Both the ordinary and octonionic representations of sigma matrices are possible.

5.4 How The Representations Of SUSY In TGD Differ From The Standard Representations?

The minimal super-sub-algebra generated by right-handed neutrino and antineutrino are the most interesting at low energies, and it is interesting to compare the naturally emerging representations of SUSY to the standard representations appearing in super-symmetric YM theories.

The basic new element is that it is possible to have short representations of SUSY algebra for massive states since particles are massless in 8-D sense. The mechanism causing the massivation remains open and p-adic thermodynamics can be responsible for it. Higgs mechanism could however induce small corrections to the masses.

The SUSY representations of SYM theories are constructed from $J = 0$ ground state (chiral multiplet for $\mathcal{N} = 1$ hyper-multiplet for $\mathcal{N} = 2$: more logical naming convention would be just scalar multiplet) and $J = 1/2$ ground state for vector multiplet in both cases. $\mathcal{N} = 2$ multiplet decomposes to vector and chiral multiplets of $\mathcal{N} = 1$ SUSY. Hyper-multiplet decomposes into two chiral multiplets which are hermitian conjugates of each other. The group of R-symmetries is $SU(2)_R \times U(1)_R$. In TGD framework the situation is different for two reasons.

1. The counterparts of ordinary fermions are constructed from $J = 1/2$ ground state with standard electro-weak quantum numbers associated with wormhole throat rather than $J = 0$ ground state.
2. The counterparts of ordinary bosons are constructed from $J = 0$ and $J = 1$ ground states assigned to wormhole contacts with the electroweak quantum numbers of Higgs and electroweak gauge bosons. If one poses no restrictions on bound states, the value of $\mathcal{N}$ is effectively doubled from that for representation associated with single wormhole throat.

These differences are allowed by general SUSY symmetry which allow the ground state to have arbitrary quantum numbers. Standard SYM theories however correspond to different representations so that the formalism used does not apply as such.

Consider first the states associated with single wormhole throat. The addition of right-handed neutrinos and their antineutrinos to a state with the constraint that $p^k\gamma^k$ annihilates the state at partonic 2-surface X^2 would mean that the helicities of the two super-symmetry generators are opposite. In this respect the situation is same as in the case of ordinary SUSY.

1. If one starts from $J = 0$ ground state, which could correspond to a bosonic state generated by WCW Hamiltonian and carrying $SO(2) \times SU(3)_c$ quantum numbers one obtains the counterparts of chiral/hyper- multiplets. These states have however vanishing electro-weak quantum numbers and do not couple to ordinary quarks neither.
2. If one starts $J = 1/2$ ground state one obtains the analog of the vector multiplet as in SYM but but belonging to a fundamental representation of rotation group and weak isospin group rather than to adjoint representation. For $\mathcal{N} = 1$ one obtains the analog of vector chiral multiplet but containing spins $J = 1/2$ and $J = 1$. For $\mathcal{N} = 2$ on obtains two chiral multiplets with $(J, F, R) = (1, 2, 1)$ and $(J, F, R) = (1/2, 1, 0)$ and $(J, F, R) = (0, 0, -1)$ and $(-1/2, 1, 0) = (0, 0, 0)$.
3. It is possible to have standard SUSY multiplet if one assumes that the added neutrino has always fermionic number opposite that the fermion in question. In this case on obtains $\mathcal{N} = 1$ scalar multiplet. This option could be defended by stability arguments and by the fact that it does not put right-handed neutrino itself to a special role.

For the states associated with wormhole contact zero energy ontology allows to consider two non-equivalent options. The following argument supports the view that gauge bosons are obtained as wormhole throats only if the throats correspond to different signs of energy.

1. For the first option the both throats correspond to positive energies so that spin 1 bosons are obtained only if the fermion and anti-fermion associated with throats have opposite M^4 chirality in the case that they are massless (this is important!). This looks somewhat strange but reflects the fact that $J = 1$ states constructed from fermion and anti-fermion with same chirality and parallel 4-momenta have longitudinal polarization. If the ground state has longitudinal polarization the spin of the state is due to right-handed neutrinos alone: in this case however spin 1 states would have fermion number 2 and -2.
2. If the throats correspond to positive and negative energies the momenta are related by time reflection and physical polarizations for the negative energy anti-fermion corespond to non-physical polarizations of positive energy anti-fermion. In this case physical polarizations are obtained.

If one assumes that the signs of the energy are opposite for the wormhole throats, the following picture emerges.

1. If fermion and anti-fermion correspond to $\mathcal{N} = 2$-dimensional representation of super-symmetry, one expects $2\mathcal{N} = 4$ gauge boson states obtained as a tensor product of two hyper-multiplets if bound states with all possible quantum number combinations are possible. Taking seriously the idea that only the bound states of fermion and anti-fermion are possible, one is led to consider the idea that the wormhole throats carry representations of $\mathcal{N} = 1$ super-symmetry generated by M^4 Weyl spinors with opposite chiralities at the two wormhole throats (right-handed neutrino and its antineutrino). This would give rise to a vector representation and eliminate a large number of exotic quantum number combinations such as the states with fermion number equal to two and also spin two states. This idea makes sense a also for a general value of $\mathcal{N}$. Bosonic representation could be also seen as the analog of short representation for $\mathcal{N} = 2N$ super-algebra reducing to a long representation $\mathcal{N} = N$. Short representations occur quite generally for the massive representations of SUSY and super-conformal algebras when 2^r generators annihilate the states [B5].

   Note that in TGD framework the fermionic states of vector and hyper multiplets related by $U(2)_R$ R-symmetry differ by a $\nu_R\overline{\nu}_R$ pair whose members are located at the opposite throats of the wormhole contact.

2. If no restrictions on the quantum numbers of the boson like representation are posed, zero energy ontology allows to consider also an alternative interpretation. $\mathcal{N} = 4$ (or more generally, $\mathcal{N} = 2N$-) super-algebra could be interpreted as a direct sum of positive and negative energy super-algebras assigned to the opposite wormhole throats. Boson like multiplets could be interpreted as a long representation of the full algebra and fermionic representations as short representations with states annihilated either by the positive or negative energy part of the super-algebra. The central charges Z_{ij} must vanish in order to have a trivial representations with $p^k = 0$. This is expected since the representations are massless in the generalized sense.

3. Standard $\mathcal{N} = 2$ multiplets are obtained if one assume that right-handed neutrino has always opposite fermion number than the fermion at the throat. The arguments in favor of this option have been already given.

5.5 SUSY after LHC

As we now know, SUSY was not found at LHC and the basic motivation for SUSY at LHC energies has disappeared. The popular article "Where Are All the 'Sparticles' That Could Explain What's Wrong with the Universe?" (see `http://tinyurl.com/y6n5cjhv`) tells about the situation. The title is however strange. There is nothing wrong with the Universe. Theoreticians stubbornly sticking to a wrong theory are the problem.

Could it be that the interpretation of SUSY has been wrong? For instance, the minimal $\mathcal{N} = 1$ SUSY predicts typically Majorana neutrinos and non-conservation of fermion number. This does not conform with my own physical intuition. Perhaps we should seriously reconsider the notion of supersymmetry itself and ask what goes wrong with it.

Can TGD framework provide any new insights?

1. TGD can be seen as a generalization of superstring models, which emerged years before superstring models came in fashion. In superstring models supersymmetry is extended to super-conformal invariance and could give badly broken SUSY as space-time symmetry. SUSY in standard QFT framework requires massless particles and this requires generalization of the Higgs mechanism. The proposals are not beautiful - this is most diplomatic manner to state it.

   In TGD framework super-conformal symmetries generalize dramatically since light-like 3-D surfaces - in particular light-cone boundary and boundaries of causal diamond (CD) have one light-like direction and are metrically 2-D albeit topologically 3-D. One outcome is modification of AdS/CFT duality - which turned out to be a disappointment - to a more realistic duality in which 2-D surfaces of space-time regarded itself as surface in $H = M^4 \times CP_2$ are basic objects. The holography in question is very much like strong form of ordinary holography and is akin to the holography assigned with blackhole horizons.

2. The generators of supersymmetries are fermionic oscillator operators and the Fock states can be regarded as members of SUSY multiplets but having totally different physical interpretation. At elementary particle level these many fermion states are realized at partonic 2-surfaces carrying point-like fermions assignable to lepton and quark like spinors associated with single fermion generations. There is infinite number of modes and most of them are massive.

   This gives rise to infinite super-conformal multiplets in TGD sense. Ordinary light elementary particles could correspond to partonic 2-surfaces carrying only fermion number at most ± 1.

3. By looking the situation from the perspective of 8-D imbedding space $M^4 \times CP_2$ situation gets really elegant and simple.

   8-D twistorialization [K23] requires massless states in 8-D sense and these can be massive in 4-D sense. Super-conformal invariance for 8-D masslessness is infinite-D variant of SUSY: all modes of fundamental fermions generate supersymmetries. The counterpart SUSY algebra is generated by the fermionic oscillator operators for induced spinor fields. All modes independently of their 4-D mass are generators of supersymmetries. M^4 chirality conservation of 4-D SUSY requiring 4-D masslessness is replaced by 8-D chirality conservation implying a separate conservation of baryon and lepton numbers. Quark-lepton symmetry is possible since color quantum numbers are not spin-like but realized as color partial waves in cm degrees of freedom of particle like geometric object.

   No breaking of superconformal symmetry in the sense of ordinary SUSYs is needed. p-Adic thermodynamics causes massivation of massless (in 4-D sense) states of spectrum via mixing with very heavy excitations having mass scale determined by CP_2 mass.

   One could say that the basic mistake of colleagues - who have been receiving prizes for impressively many breakthroughs during last years - is the failure to realize that 4-D spinors must be replaced with 8-D ones. This however requires 8-D imbedding space and space-time surfaces and one ends up to TGD by requiring standard models symmetries or just the existence of twistor lift of TGD. All attempts to overcome the problems lead to TGD. Colleagues do not seem like this at all so that they prefer to continue as hitherto. And certainly this strategy has been an amazing professional success.

What about the counterpart of space-time supersymmetry - SUSY - in TGD framework? The question whether TGD allows space-time SUSY or not has bothered me for a long time, and I have considered SUSY from TGD point of view in [K8, K17, K5]. In the following I summarize my recent views, which reflect the increased understanding of twistor lift and cosmological constant and of preferred extremals as minimal surfaces having 2-D string world sheets as singularities analogous to edges [L4, L5, L6] [K23].

1. The analog of SUSY would be generated by massless or light modes of induced spinor fields. Space-time SUSY would correspond to the lightest slowly varying modes for the induced spinor fields being in 1-1-correspondence with the components of H-spinors. The number $\mathcal{N}$ associated with SUSY is quite large as the number of components of H-spinors. The corresponding fermionic oscillator operators generate repsesentations of Clifford algebra and SUSY multiplets are indeed such.

   If space-time surface is canonically imbedded Minkowski space M^4, no SUSY breaking occurs. This is however an unrealistic situation. For general preferred extremal right- and left handed components of spinors mix, which causes in turn massivation and breaking of SUSY in 4-D sense.

   Could right-handed neutrino be an exception. It does not couple to electroweak and color gauge potentials. Does this mean that ν_R and its antiparticle generate exact $\mathcal{N}=2$ SUSY? No: ν_R has small coupling to CP_2 parts of induced gamma matrices mixing neutrino chiralities and this coupling causes also SUSY breaking. This coupling is completely new and not present in standard QFTs since they do not introduce induced spinor structure forced by the notion of sub-manifold gemetry.

   Even worse, one can argue that right-handed neutrino is "eaten" as right- and left-handed massless neutrinos combine to massive neutrino unless one has canonically imbedded M^4.

There fate resembles that of charge Higgs components. One could still however say that one has an analog of broken SUSY generated by massive lepton and quark modes. But it would be better to talk about 8-D supersymmetry.

2. The situation is now however so simple as this. TGD space-time is many-sheeted and one has a hierarchy of space-time sheets in various scales labelled by p-adic primes labelling also particles and by the value of Planck constant $h_{eff} = n \times h_0$.

   Furthermore, spinors can be assigned to 4-D space-time interiors, to 2-D string world sheets, to their light-like 1-D boundaries at 3-D light-like orbits of partonic 2-surfaces, or even with the partonic orbits. 2-D string world sheets are analogous to edges of 3-D object and action receives "stringy" singular contribution from them because of edge property. Same applies to the boundaries of string world sheets location at the light-like orbits of partonic 2-surfaces. Think of a cloth, which has folds which move along it as an analog. Space-time interior is a minimal surface in 4-D sense except at 2-D folds and string world sheets and their boundaries are also minimal surfaces.

   Therefore one has many kinds of fermions: 4-D space-time fermions, 2-D string world sheet fermions possibly associated with hadrons (there presence might provide new insights to the spin puzzle of proton), and 1-D boundary fermions for these as point-like particles and naturally identifiable as basic building bricks of ordinary elementary particles. Perhaps even 3-D fermions associated with light-like partonic orbits can be considered. All these belong to the spectrum and the situation is very much like in condensed matter physics, where people talk fluently about edge states.

3. In TGD framework ordinary elementary particles are assigned with the light-like boundaries of string world sheets. Right-handed neutrino and antineutrino generate $\mathcal{N} = 2$ SUSY for massless states assignable as light-like curves at light-like orbits of partonic 2-surfaces. This implies badly broken SUSY and it seems that one cannot talk about SUSY at all in the conventional sense. These states are however massless in 8-D sense, not in 4-D sense!

   In TGD framework one can however consider an analogy of SUSY for which massless ν_R modes in 4-D space-time interior - rather than at orbits of partonic 2-surfaces - generate supersymmetry. One could say that the many particle state, rather than particle has a spartner. Think of any system - it can contain larger number of ordinary particles forming a single quantum coherent entity to which one an assign space-time sheet. One can assign to this system space-time shet a right-handed neutrino, antineutrino, or both. This gives the superpartner of the system. The presence of ν_R is not seen in the same manner in interactions as in SUSY theories.

This picture [L4, L5, L6] is an outcome of a work lasted for decades, not any ad hoc model. One can say that classical aspects of TGD (exact part of quantum theory in TGD framework) are now well understood. To sum up, the simplest realizations of SUSY in TGD sense are following and the best manner to look at them is from the perspective 8-D masslessness.

1. Massless 4-D supersymmetry generated by ν_R. Other fermions which are massive because of their electroweak and color interactions not possessed by ν_R. Also ν_R generates small mass. These spartners are not however visible in elementary particle physics but belong to condensed matter physics.

2. Massive neutrino and other fermions but no supersymmetry generatig ν_R anymore since it is "eaten". This would be realized as very badly broken SUSY in 4-D sense and the spartners would be very massive. At the partonic 2-surfaces, this option forced by Uncertainty Principle.

REFERENCES

Mathematics

[A1] Pope CN. Eigenfunctions and $Spin^c$ Structures on CP_2, 1980.

[A2] Hanson J Eguchi T, Gilkey B. *Phys Rep*, 66:1980, 1980.

[A3] Pope CN Gibbons GW. CP_2 as gravitational instanton. *Comm Math Phys*, 55, 1977.

[A4] Pope CN Hawking SW. Generalized Spin Structures in Quantum Gravity. *Phys Lett*, (1), 1978.

[A5] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc.* Available at: `http://tinyurl.com/pb8zpqo`, 8(43):133–151, 1981.

Theoretical Physics

[B1] Higher-dimensional super-gravity. Available at: `http://wapedia.mobi/en/Higher-dimensional_supergravity?t=3`.

[B2] Super-space. Available at: `http://en.wikipedia.org/wiki/Superspace`.

[B3] Susy Summary. Available at: `http://bolvan.ph.utexas.edu/~vadim/Classes/01f/396T/table.pdf`.

[B4] The Minimally Super-Symmetric Standard Model. Available at: `http://en.wikipedia.org/wiki/Minimal_Supersymmetric_Standard_Model`.

[B5] Binetruy P. *Supersymmetry: Theory, Experiment and Cosmology.* Oxford University Press, Oxford, 2006.

[B6] Shriman Y. TASI 2008 Lectures: Introduction to Supersymmetry and Supersymmetry Breaking. Available at: `http://arxiv.org/abs/0907.0039`, 2009.

Particle and Nuclear Physics

[C1] How many neutrinos in the sky? Available at: `http://resonaances.blogspot.fi/2013/01/how-many-neutrinos-in-sky.html`.

[C2] Bulbul E et al. Detection of an unidentified emission line in the stacked x-ray spectrum of galaxy clusters. Available at: `http://arxiv.org/abs/1402.2301`, 2014.

[C3] Hinshaw G et al. Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results. Available at: `http://arxiv.org/abs/1212.5226`.

[C4] Sturrock PA et al. Analysis of Gamma Radiation from a Radon Source: Indications of a Solar Influence. Available at: `http://arxiv.org/abs/1205.0205`, 2012.

Books related to TGD

[K1] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. In *Towards M-matrix.* Online book. Available at: `http://www.tgdtheory.fi/public_html/pdfpool/append.pdf`, 2006.

[K2] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics.* Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/padphys.html#elvafu`, 2006.

[K3] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Towards M-Matrix.* Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#quthe`, 2006.

[K4] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry.* Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#compl1`, 2006.

[K5] Pitkänen M. Could $\mathcal{N} = 2$ Super-conformal Theories Be Relevant For TGD? In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#N2sconf`, 2006.

[K6] Pitkänen M. Dark Nuclear Physics and Condensed Matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/neuplanck.html#exonuclear`, 2006.

[K7] Pitkänen M. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/neuplanck.html#Planck`, 2006.

[K8] Pitkänen M. Does the QFT Limit of TGD Have Space-Time Super-Symmetry? In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#susy`, 2006.

[K9] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/padphys.html#mless`, 2006.

[K10] Pitkänen M. New Particle Physics Predicted by TGD: Part I. In *p-Adic Physics*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/padphys.html#mass4`, 2006.

[K11] Pitkänen M. Nuclear String Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/neuplanck.html#nuclstring`, 2006.

[K12] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdclass.html#astro`, 2006.

[K13] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdclass.html#cosmo`, 2006.

[K14] Pitkänen M. The Recent Status of Lepto-hadron Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/neuplanck.html#leptc`, 2006.

[K15] Pitkänen M. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdclass.html#tgdgrt`, 2006.

[K16] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#cspin`, 2006.

[K17] Pitkänen M. SUSY in TGD Universe. In *p-Adic Physics*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/padphys.html#susychap`, 2012.

[K18] Pitkänen M. Criticality and dark matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/neuplanck.html#qcritdark`, 2014.

[K19] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#wcwnew`, 2014.

[K20] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#numbervision`, 2014.

[K21] Pitkänen M. From Principles to Diagrams. In *Towards M-Matrix*. Online book.Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#diagrams`, 2016.

[K22] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistorstory`, 2016.

[K23] Pitkänen M. The Recent View about Twistorialization in TGD Framework. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#smatrix`, 2018.

Articles about TGD

[L1] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. Available at: `http://tgdtheory.fi/pdfpool/append.pdf`, 2006.

[L2] Pitkänen M. CMAP representations about TGD. Available at: `http://www.tgdtheory.fi/cmaphtml.html`, 2014.

[L3] Pitkänen M. CMAP representations about TGD, and TGD inspired theory of consciousness and quantum biology. Available at: `http://www.tgdtheory.fi/tgdglossary.pdf`, 2014.

[L4] Pitkänen M. TGD view about coupling constant evolution. Available at: `http://tgdtheory.fi/public_html/articles/ccevolution.pdf`, 2018.

[L5] Pitkänen M. Does coupling constant evolution reduce to that of cosmological constant? Available at: `http://tgdtheory.fi/public_html/articles/ccevoTGD.pdf`, 2019.

[L6] Pitkänen M. More about the construction of scattering amplitudes in TGD framework. Available at: `http://tgdtheory.fi/public_html/articles/scattampl.pdf`, 2019.