Cognitive representations for partonic 2-surfaces, string world sheets, and string like objects

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Abstract

Cognitive representations are identified as points of space-time surface $X^4 \subset M^4 \times CP_2$ having imbedding space coordinates in the extension of of rationals defined by the polynomial defined by the M^8 pre-image of X^4 under $M^8 - H$ correspondence. Cognitive representations have become key piece in the formulation of scattering amplitudes and in TGD view about consciousness and cognition. One might argue that number theoretic evolution as increase of the dimension of the extension of rationals favors space-time surfaces with especially large cognitive representations since the larger the number of points in the representation is, the more faithful the representation is. Strong form of holography (SH) suggests that it is enough to consider cognitive representations restricted to partonic 2-surfaces and string world sheets. What kind of 2-surfaces are the cognitively fittest one? It would not be surprising if surfaces with large symmetries acting in extension were favored and elliptic curves with discrete 2-D translation group indeed turn out to be assigable string world sheets as singularities and string like objects. In the case of partonic 2-surfaces geodesic sphere of CP_2 is similar object.

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1 Introduction

Cognitive representations are identified as points of space-time surface $X^4 \subset M^4 \times CP_2$ having imbedding space coordinates in the extension of of rationals defined by the polynomial defined by the M^8 pre-image of X^4 under $M^8 - H$ correspondence [L2, L3, L11, L8, L7, L5]. Cognitive representations have become key piece in the formulation of scattering amplitudes [L9] . One might argue that number theoretic evolution as increase of the dimension of the extension of rationals favors space-time surfaces with especially large cognitive representations since the larger the number of points in the representation is, the more faithful the representation is.

One can pose several questions if one accepts the idea that space-time surfaces with large cognitive representations are survivors.

1. Preferred p-adic primes are proposed to correspond to the ramified primes of the extension [L13]. The proposal is that the p-adic counterparts of space-time surfaces are identifiable as imaginations whereas real space-time surfaces correspond to realities. p-Adic space-time surfaces would have the imbedding space points in extension of rationals as common with

real surfaces and large number of these points would make the representation realistic. Note that the number of points in extension does not depend on p-adic prime.

Could some extensions have an especially high number of points in the cognitive representation so that the corresponding ramified primes could be seen as survivors in number theoretical fight for survival, so to say? Galois group of the extension acts on cognitive representation. Galois extension of an extension has the Galois group of the original extension as normal subgroup so that ormal Galois group is analogous to a conserved gene.

2. Also the type of extremal matters. For instance, for instance canonically imbedded M^4 and CP_2 contain all points of extension. These surfaces correspond to the vanishing of real or imaginary part (in quaternionic sense) for a linear octonionic polynomial $P(o) = o$! As a matter of fact, this is true for all known preferred extremals under rather mild additional conditions. Boundary conditions posed at both ends of CD in ZEO exclude these surfaces and the actual space-time surfaces are expected to be their deformations.

3. Could the surfaces for which the number of points in cognitive representation is high, be the ones most easily discovered by mathematical mind? The experience with TGD supports positive answer: in TGD the known extremals [K3] are examples of such mathematical objects! If so, one should try to identify mathematical objects with high symmetries and look whether they allow TGD realization.

4. One must also specify more precisely what cognitive representation means. Strong form of holography (SH) states that the information gives at 2-D surfaces - string world sheets and partonic 2-surfaces - is enough to determine the space-time surfaces. This suggests that it is enough to consider cognitive representation restricted to these 2-surfaces. What kind of 2-surfaces are the cognitively fittest one? It would not be surprising if surfaces with large symmetries acting in extension were favored and elliptic curves with discrete 2-D translation group indeed turn out to be assigable string world sheets as singularities and string like objects. In the case of partonic 2-surfaces geodesic sphere of CP_2 is similar object.

2 Assigning large cognitive representations to partonic 2-surfaces, string world sheets, and cosmic strings

By SH partonic 2-surfaces and string world sheets are expected to give rise to cognitive representations. Under what conditions these representations are large?

2.1 Partonic 2-surfaces as seats of cognitive representations

One can start from SH and look the situation more concretely. The situation for partonic 2-surfaces has been considered already earlier [L12, L6] but deserves a separate discussion.

1. Octonionic polynomials allow special solutions for which the entire polynomial vanishes. This happens at 6-sphere S^6 at the boundary of 8-D light-cone. S^6 is analogous to brane and has radius $R = r_n$, which is a root of the real polynomial with rational coefficients algebraically continued to the octonionic polynomial.

   S^6 has the ball B^3 of radius r_n of the light-cone M^4_+ with time coordinate $t = r_n$ as analog of base space and sphere S^3 of E^4 with radius $R = \sqrt{r_n^2 - r^2}$, r the radial coordinate of B^3 as an analog of fiber. The analog of the fiber contracts to a point at the boundary of the light-cone. The points with B^3 projection and E^4 coordinates in extension of rationals belong to the cognitive representation. The condition that $R^2 = x_i x^i = r_n^2 - r^2$ is square of a number of extension is rather mild and allows infinite number of solutions.

2. The 4-D space-time surfaces X^4 are obtained as generic solutions of $Im(P(o)) = 0$ or $Re(P(o)) = 0$. Their intersection with S^6 - partonic 2-surface X^2 - is 2-D. The assumption is that the incoming and outgoing 4-D space-time surfaces representing orbits of particles in topological sense are glued together at X^2 and possibly also in their interiors. X^2 serves

as an analog of vertex for 3-D particles. This gives rise to topological analogs of Feynman diagrams.

In the generic case the number of points in cognitive representation restricted to X^2 is finite unless the partonic 2-surface X^2 is special - say correspond to a geodesic spere of S^6.

3. The discrete isometries and conformal symmetries of the cognitive representation restricted to X^2 possibly represented as elements of Galois group might play a role. For $X^2 = S^2$ the finite discrete subgroups of $SO(3)$ giving rise to finite tesselations and appearing in ADE correspondence might be relevant. For genera $g = 01, 2$ conformal symmetry Z_2 is always possible but for higher genera only in the case of hyper-elliptic surfaces- this used to explain why only $g = 0, 1, 2$ correspond to observed particles [K1] whereas higher genera could be regarded as many-particle states of handles having continuous mass spectrum. Torus is an exceptional case and one can ask whether discrete subgroup of its isometries could be realized.

4. In TGD inspired theory of consciousness [L4, L6] the moments $t = r_n$ corresponds to "very special moments in the life of self". They would be also cognitively very special - kind of eureka moments with a very large number of points in cognitive representation. The question is whether these surfaces might be relevant for understanding the nature of mathematical consciousness and how the mathematical notions emerge at space-time level.

2.2 Ellipticity

Surfaces with discrete translational symmetries is a natural candidate for a surface with very large cognitive representation. Are their analogs possible? The notions of elliptic function, curve, and surface suggest themselves as a starting point.

1. Elliptic functions (`http://tinyurl.com/gpugcnh`) have 2-D discrete group of translations as symmetries and are therefore doubly periodic and thus identifiable as functions on torus.

   Weierstrass elliptic functions $\mathcal{P}(z; \omega_1, \omega_2)$ (`http://tinyurl.com/ycu8oa4r`) are defined on torus and labelled by the conformal equivalence class $\lambda = \omega_1/\omega_2$ of torus identified as the ratio $\lambda = \omega_1/\omega_2$ of the complex numbers ω_i defining the periodicities of the lattice involved. Functions $\mathcal{P}(z; \omega_1, \omega_2)$ are of special interest as far as elliptic curves are considered and defines an imbedding of elliptic curve to CP_2 as will be found.

   If the periods are in extension of rationals then values in the extension appear infinitely many times. Elliptic functions are not polynomials. Although the polynomials giving rise to octonionic polynomials could be replaced by analytic functions it seems that elliptic functions are not the case of primary interest. Note however that the roots r_n could be also complex and could correspond to values of elliptic function forming a lattice.

2. Elliptic curves (`http://tinyurl.com/lovksny`) are defined by the polynomial equation

$$y^2 = P(x) = x^3 + ax + b \ . \qquad (2.1)$$

   An algebraic curve of genus 1 allowing 2-D discrete translations as symmetries is in question. If a point of elliptic curve has coordinates in extension of rationals then 2-D discrete translation acting in extension give rise to infinite number of points in the cognitive representation. Clearly, the 2-D vectors spanning the lattice defining the group must be in extension of rationals.

One can indeed define commutative sum $P + Q$ for the points of the elliptic curve. The detailed definition of the group law and its geometric illustration can be found in Wikipedia article (`http://tinyurl.com/lovksny`).

1. Consider real case for simplicity so that elliptic curve is planar curve. $y^2 = P(x) = x^3 + ax + b$ must be non-negative to guarantee that y is real. $P(x) \geq 0$ defines a curve in upper (x, y) plane extending from some negative value x_{min} corresponding to $y^2 = P(x_{min}) = 0$ to the

right. Given value of y can correspond to 3 real roots or 1 real root of $P_y(x) = y^2 - P(x)$. At the two extrema of $P_y(x)$ 2 real roots co-incide. The graph of $y = \pm\sqrt{P(x)}$ is reflection symmetric having two branches beginning from $(x_{min}, y = 0)$.

2. The negative $-P$ is obtained by reflection with respect to x-axis taking y_P to $-y_P$. Neutral element O is identified as point a infinity (assuming compactification of the plane to a sphere) which goes to itself under reflection $y \rightarrow -y$.

3. One assigns to the points P and Q of the elliptic curve a line $y = sx + d$ containing them so that one has $s = (y_p - y_Q)/(x_P - x_Q)$. In the generic case the line intersects the elliptic curve also at third point R since $P_{y=sx+d}(x)$ is third order polynomial having three roots (x_P, x_Q, x_R). It can happen that 2 roots are complex and one has 1 real root. At criticalityfor the transiton from 3 to 1 real roots one has $x_Q = x_R$.

   Geometrically one can distinguish between 4 cases.

   - The roots P, Q, R of $P_{y=sx+d}(x)$ are different and finite: one defines the sum as $P+Q = -R$.
   - $P \neq Q$ and $Q = R$ (roots Q and R are degenerate): $P + Q + Q = O$ giving $R = -P/2$.
   - P and Q are at a line parallel to y-axis and one has $R = O$: $P + Q + O = O$ and $P = -Q$.
   - P is double root of $P_{y=sx+d}(x)$ with tangent parallel to y-axis at the point $(x_{min}, y = 0)$ at which the elliptic curve begins so that one has $R = O$: $P+P+O = O$ gives $P = -P$. This corresponds to torsion.

4. Elliptic surfaces (see `http://tinyurl.com/yc33a6dg`) define a generalization of elliptic curves and are defined for 4-D complex manifolds. Fiber is required to be smooth and has genus 1.

2.3 String world sheets and elliptic curves

In twistor lift of TGD space-time surfaces identifiable as minimal surfaces with singularities, which are string world sheets and partonic 2-surfaces. Preferred extremal property means that space-time surfaces are extremals of both Kähler action and volume action except at singularities.

Are string world sheets with very large number of points in cognitive representation possible? One has right to expect that string world sheets allow special kind of symmetries allowing large, even infinite number of points at the limit of large sheet and related by symmetries acting in the extension of rationals. If one of the points is in the extension, also other symmetry related points are in the extension. For a non-compact group, say translation one would have infinite number of points in the representation but the finite size of CD would pose a limitation to the number of points.

String world sheets are good candidates for the realization of elliptic curves.

1. The general conjecture is that preferred extremals allow what I call Hamilton-Jacobi structure for M^4 [K2]. The distribution of tangent spaces having decomposition $M^4(x) = M^2(x) \times E^2(x)$ would be integrable giving rise to a family of string world sheets Y^2 and partonic 2-surfaces X^2 more general than those defined above. X^2 and Y^2 are orthogonal to each other at each point of X^4. One can introduce local light-cone coordinates (u, v) for Y^2 and local E^2 complex coordinate w for X^2.

2. Space-time surface itself would be a deformation of M^4 with Hamilton-Jacobi structure in CP_2 direction. w coordinate as function $w(z)$ of CP_2 complex coordinate z or vice versa would define the string world sheet. This would be a transversal deformation of the basic string world sheet Y^2: stringy dynamics is indeed transversal.

3. The idea about maximal cognitive representation suggests that $w \leftrightarrow z$ correspondence defines elliptic curve. One would have $y^2 = P(x) = x^3 + ax + b$ with either $(y = w, x = z)$ or $(y = z, x = w)$. A natural conjecture is that for the space-time surface corresponding to a given extension K of rationals the coefficients a an b belong to K so that the algebraic

complexity of string world sheet would increase in number theoretic evolution [L10]. The orbit of a algebraic point at string world sheet would be lattice made finite by the size of CD. Elliptic curves would define very special deformed string world sheets in space-time.

4. It is interesting to consider the pre-image of given point y ($y = w$ or $y = z$) covering point x. One has $y = \pm\sqrt{u}$, $u = P(x)$ corresponding to group element and its negative: there are two points of covering given value of u. $u = P(x)$ covers 3 values of x. The values of x would belong to 6-fold covering of rationals. The number theoretic interpretation for the effective Planck constant $h_{eff} = nh_0$ states that n is the number of sheets for space-time surface as covering.

   There is evidence that $h_{eff} = h$ corresponds to $n = 6$ [L1]. Could 6-fold covering of rationals be fundamental since it gives very large cognitive representation at the level of string world sheets?

   For extensions K of rationals the x coordinates for the points of cognitive representation would belong to 6-D extension of K.

5. Ellipticity condition would apply on the string world sheets themselves. In the number theoretic vision string world sheets would correspond at M^8 level to singularities at which the quaternionic tangent space degenerates to 2-D complex space. Are these conditions consistent with each other? It would seem that the two conditions would select cognitively very special string world sheets and partonic 2-surfaces defining by strong form of holography (SH) space-time surface as a hologram in SH. Consciousness theorist interested in mathematical cognition might ask whether the notion of elliptic surfaces have been discovered just because it is cognitively very special. In the case of partonic 2-surfaces geodesic sphere of CP_2 is similar object.

2.4 String like objects and elliptic curves

String like objects - cosmic strings - and their deformations, are fundamental entities in TGD based cosmology and astrophysics and also in TGD inspired quantum biology. One can assign elliptic curves also to string like objects.

1. Quite generally, the products $X^2 \times Y^2 \subset M^4$ of string world sheets X^2 and complex surfaces Y^2 of CP_2 define extremals that I have called cosmic strings [K3].

2. Elliptic curves allow a standard imbedding to CP_2 as complex surfaces constructible in terms of Weierstrass elliptic function $\mathcal{P}(z)$ (`http://tinyurl.com/ycu8oa4r`) satisfying the identity

$$[\mathcal{P}'(z)]^2 = [\mathcal{P}(z)]^3 - g_2\mathcal{P}(z) - g_3 \ . \tag{2.2}$$

   Here g_2 and g_3 are modular invariants. This identity is of the same form as the condition $y^2 = x^3 + ax + b$ with identifications $y = \mathcal{P}'(z), x = \mathcal{P}(z)$ and $(a = -g_2, b = -g_3)$. From the expression

$$y^2 = x(x-1)(x-\lambda) \tag{2.3}$$

   in terms of the modular invariant $\lambda = \omega_1/\omega_2$ of torus one obtains

$$g_2 = \frac{4^{1/3}}{3}(\lambda^2 - \lambda + 1 \ , \quad g_3 = \frac{1}{27}(\lambda+1)(2\lambda^2 - 5\lambda + 2) \ . \tag{2.4}$$

   Note that third root of a appears in the formula. The so called modular discriminant

$$\Delta = g_2^3 - 27g_3^2 = \lambda^2(\lambda-1)^2 \ . \tag{2.5}$$

   vanishes for $\lambda = 0$ and $\lambda = 1$ for which the lattice degenerates.

3. The imbedding of the elliptic curve to CP_2 can be expressed in projective coordinates of CP_2 as

$$(z^1, z^2, z^3) = (\xi^1, \xi^2, 1) = (\frac{\mathcal{P}'(w)}{2}, \mathcal{P}(w), 1) \quad . \tag{2.6}$$

REFERENCES

Books related to TGD

[K1] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html#elvafu, 2006.

[K2] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#wcwnew, 2014.

[K3] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdclass.html#prext, 2015.

Articles about TGD

[L1] Pitkänen M. Hydrinos again. Available at: http://tgdtheory.fi/public_html/articles/Millsagain.pdf, 2016.

[L2] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: part I. Available at: http://tgdtheory.fi/public_html/articles/ratpoints1.pdf, 2017.

[L3] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: part II. Available at: http://tgdtheory.fi/public_html/articles/ratpoints2.pdf, 2017.

[L4] Pitkänen M. Re-examination of the basic notions of TGD inspired theory of consciousness. Available at: http://tgdtheory.fi/public_html/articles/conscrit.pdf, 2017.

[L5] Pitkänen M. Are fundamental entities discrete or continous and what discretization at fundamental level could mean? Available at: http://tgdtheory.fi/public_html/articles/zetaspecu.pdf, 2019.

[L6] Pitkänen M. $M^8 - H$ duality and consciousness. Available at: http://tgdtheory.fi/public_html/articles/M8Hconsc.pdf, 2019.

[L7] Pitkänen M. $M^8 - H$ duality and the two manners to describe particles. Available at: http://tgdtheory.fi/public_html/articles/susysupertwistor.pdf, 2019.

[L8] Pitkänen M. New results related to $M^8 - H$ duality. Available at: http://tgdtheory.fi/public_html/articles/M8Hduality.pdf, 2019.

[L9] Pitkänen M. Scattering amplitudes and orbits of cognitive representations under subgroup of symplectic group respecting the extension of rationals . Available at: http://tgdtheory.fi/public_html/articles/symplorbsm.pdf, 2019.

[L10] Pitkänen M. TGD inspired theory of consciousness and living systems. Available at: http://tgdtheory.fi/public_html/articles/badenbaden.pdf, 2019.

[L11] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, and Inclusions of Hyperfinite Factors. Available at: `http://tgdtheory.fi/public_html/articles/McKay.pdf`, 2019.

[L12] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors. Available at: `http://tgdtheory.fi/public_html/articles/McKaygeneral.pdf`, 2019.

[L13] Pitkänen M. Trying to understand why ramified primes are so special physically. Available at: `http://tgdtheory.fi/public_html/articles/ramified.pdf`, 2019.