Topological Geometrodynamics: Three Visions

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Abstract

In this article I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics.
2. Second vision is number theoretical and involves three threads. The first thread relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept. Second thread is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically. The third threadrelies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units.
3. The third vision is based on TGD inspired theory of consciousness, which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics.

Contents

1 Introduction

Originally Topological Geometrodynamics (TGD) was proposed as a solution of the problems related to the definition of conserved four-momentum in General Relativity. It was assumed that physical space-times are representable as 4-D surfaces in certain higher-dimensional space-time having symmetries of the empty Minkowski space of Special Relativity. This is guaranteed by the decomposition $H = M^4 \times S$, where S is some compact internal space. It turned out that the choice $S = CP_2$ is unique in the sense that it predicts the symmetries of the standard model and provides a realization for Einstein's dream of geometrizing of fundamental interactions at classical level. TGD can be also regarded as a generalization of super string models obtained by replacing strings with light-like 3-surfaces or equivalently with space-like 3-surfaces: the equivalence of these identification implies quantum holography.

The construction of quantum TGD turned out to be much more than mere technical problem of deriving S-matrix from path integral formalism. A new ontology of physics (many-sheeted space-time, zero energy ontology, generalization of the notion of number, and generalization of quantum theory based on spectrum of Planck constants giving hopes to understand what dark matter and dark energy are) and also a generalization of quantum measurement theory leading to a theory of consciousness and model for quantum biology providing new insights to the mysterious ability of living matter to circumvent the constraints posed by the second law of thermodynamics were needed. The construction of quantum TGD involves a handful of different approaches consistent with a similar overall view, and one can say that the construction of M-matrix, which generalizes the S-matrix of quantum field theories, is understood to a satisfactory degree although it is not possible to write even in principle explicit Feynman rules except at quantum field theory limit [26, 27].

In this article I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics [2].

2. Second vision is number theoretical [5] and involves three threads.

   (a) The first thread [35]relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept.

   (b) Second thread [36] is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically.

   (c) The third thread [37]relies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units. This implies number theoretical Brahman=Atman identity or number theoretical holography when one consider hyper-octonionic infinite primes.

   (d) The third vision is based on TGD inspired theory of consciousness [10], which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics. The basic notions at quantum jump identified as as a moment of consciousness and self. Negentropy Maximization Principle (NMP) defines the fundamental variational principle and reproduces standard quantum measurement theory and predicts second law but also some totally new physics in the intersection of real and p-adic worlds where it is possible to define a hierarchy of number theoretical variants of Shannon entropy which can be also negative. In this case NMP favors the generation of entanglement and state function reduction does not mean generation of randomness anymore. This vision has obvious almost applications to biological self-organization.

My aim is to provide a bird's eye of view and my hope is that reader would take the attitude that details which cannot be explained in this kind of representation are not essential for the purpose of getting a feeling about the great dream behind TGD.

2 Quantum physics as infinite-dimensional geometry

The first vision in its original form is a the generalization of Einstein's program for the geometrization of physics by replacing space-time with the WCW identified roughly as the space of 4-surfaces in $H = M^4 \times CP_2$. Later generalization due to replacement of H with book like structures from by real and p-adic variants of H emerged. A further book like structure of imbedding space emerged via the introduction of the hierarchy of Planck constants. These generalizations do not however add anything new to the basic geometric vision.

2.1 World of the classical worlds as the arena of quantum physics

Physics as the classical spinor field geometry of WCW consisting of light-like 3-surfaces in 8-D imbedding space $H = M^4 \times CP_2$ (to be referred as configuration space CH or WCW in the sequel) is the oldest and best developed approach to TGD and means generalization of Einstein's program of geometrizing classical physics so that it applies to entire quantum physics [2]. There are two natural identifications for the 3-surfaces.

1. By general coordinate invariance light-like 3-surfaces can be identified as wormhole throats at which the signature of the induced metric changes from a Minkowskian signature of space-time

sheet to that of deformed CP_2 type vacuum extremal representing elementary particle. One can interpret so called CP_2 type vacuum extremals as lines of generalized Feynman diagrams so that geometrization and generalization of the notion of Feynman diagram emerges.

2. In zero energy ontology causal diamonds (CDs) of M^4 defined as intersection of future and past directed light-cones become define basic building bricks of WCW. The space-time surfaces belonging to CD having their 3-D future and past ends at the light-like boundaries of CD become the basic objects. The ends are 3-surfaces are space-like and come in pairs. WCW decomposes into a union over sub-WCWs associated with various CDs and their unions and the space-like ends of the space-time sheets at future and past boundaries of CD become very natural fundamental objects.

The condition that the two identifications of 3-surfaces are equivalent implies that all information about the geometry of WCW and quantum physics is coded by the 2-dimensional intersections of the space-like and light-like 3-surfaces at the boundaries of CDs plus the information about the distribution of 4-D tangent spaces of the space-time sheet at these surfaces. I have christened partonic 2-surfaces since they are carriers of various quantum numbers. Therefore 4-D General Coordinate invariance implies efffective 2-dimensionality and quantum holography. The effective two-dimensionality is implies also by general consistency conditions related to conformal symmetries: this became obvious much before the emergence of zero energy ontology and led to interpretational difficulties at that time. The non-determinism of Kähler action defining space-time dynamics in the standard sense of the worl implies that effective 2-dimensionality holds only locally.

WCW is endowed with Kähler metric guaranteing the geometrization of hermitian conjugation of quantum theory.

1. The conjecture inspired by the geometry of loop spaces [49] is that H is fixed from the mere requirement that the infinite-dimensional Kähler geometry exists. WCW must reduce to a union of symmetric spaces having infinite-dimensional isometry groups and labeled by zero modes having interpretation as classical dynamical variables. This requires infinite-dimensional symmetry groups. At space-time level super-conformal symmetries are possible only if the basic dynamical objects can be identified as light-like or space-like 3-surfaces. At imbedding space level there are extended super-conformal symmetries assignable to the light-cone of H if the Minkowski space factor is four-dimensional.

   The recent progress in the understanding of the representations of super-conformal symmetries leads to a beautiful generalization of Equivalence Principle in terms of Super Virasoro conditions for the coset construction involving the super-symplectic algebras associated with conformal symmetries of the light-cone of Minkowski space and super Kac-Moody symmetries associated with light-like 3-surfaces [24]. Einstein's equations result at long length scale limit [30]. A string model type description emerges in a finite measurement resolution when light-like 3-surfaces are replaced by braids. This means also quantum holography. General Coordinate Invariance implies that classical space-time physics becomes an exact part of quantum theory in the sense that space-time sheets are analogous to Bohr orbits.

2. The condition that the symmetries of standard model are realized geometrically and that one can understand the known quantum numbers characterizing elementary particles in terms of the geometry of the imbedding space, leads to a unique choice for the imbedding space as $H = M^4 \times CP_2$. The challenge is to understand what makes this choice so special and number theoretic approach based on classical number fields allows to interpret this choice number theoretically so that the standard model symmetries find a number theoretical interpretation.

2.2 Geometrization of fermionic statistics in terms of configuration space spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the configuration space spinor structure in the sense that the anti-commutation relations for configuration space gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields defined at space-time surface.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. Ramond model [48] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin $3/2$ fermionic field rather than a vector field. This suggests that the are analogous to spin $3/2$ fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   Configuration space spinor fields can have arbitrary fermion number and there are good hopes of describing the whole physics in terms of configuration space spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the 'orbital' degrees of freedom of the ordinary spinor field. One non-trivial implication is bosonic emergence: elementary bosons correspond to fermion antifermion bound states associated with the wormhole contacts (pieces of CP_2 type vacuum extremals) with throats carrying fermion and antifermion numbers. Fermions correspond to single throats associated with topologically condensed CP_2 type vacuum extremals.

2. The classical theory for the bosonic fields is an essential part of the configuration space geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the configuration space spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the configuration space geometry. This is indeed true if the complexified configuration space gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the configuration space to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \quad ,$$

   where J_{AB} denotes the matrix elements of the Kähler form of the configuration space. The presence of the Hermitian conjugation is necessary because configuration space gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly.

2.3 Construction of the configuration space Clifford algebra in terms of second quantized induced spinor fields

The construction of WCW spinor structure must have a direct relationship to quantum physics as it is usually understood. The second quantization of the space-time spinor fields is needed to define the anticommutative gamma matrices of WCW: this means a geometrization of Fermi statistics [18] in the sense that free fermionic quantum fields at space-time surface correspond to purely classical Clifford algebra of WCW. This is in accordance with the idea that physics at WCW level is purely classical apart from the notion of quantum jump.

The identification of the correct variational principle for the dynamics of space-time spinor fields identified as induced spinor fields has involved many trials. Ironically, the final outcome was almost the most obvious gess. The so called modified Dirac action (the obvious guess) with measurement interaction term (required by quantum classical correspondence) added defines the fundamental dynamics providing space-time representation of quantum physics via classical space-time physics [19]. One can identify the vacuum functional -exponent of Kähler function of WCW- as a Dirac determinant. The conjecture is that Kähler function equals to Kähler action for a preferred extrema, which by internal consistency conditions must be critical in the sense that it allows infinite number of vanishing second variations. This realizes the notion of quantum criticality-one of guiding principles of quantum TGD-at space-time level.

Number theoretical approach in turn leads to the conclusion that space-time surfaces are either associative or co-associative in the sense that the modified gamma matrices at each point of space-time surface in their octonionic representation reduces to a quaternionic or co-quaternionic algebra and therefore have matrix representation. The conjecture is that these identifications of space-time dynamics are consistent or even equivalent.

The recent understanding of the modified Dirac action has emerged through a painful process and has strong physical implications.

1. Stringy propagators and emerge naturally thanks to the measurement interaction term in the modified Dirac action coupling to four-momentum and color hyper-charge and isospin.

2. The space-time super-symmetry generalizes to what might be called $\mathcal{N} = \infty$ supersymmetry which however effectively reduces to $\mathcal{N} = 1$ broken super-symmetry []susy. The generators of the super-symmetry correspond to the modes of the induced spinor field at space-time sheet. Bosonic emergence means dramatic simplications in the formulation of QFT limit of TGD. This formulation should generalize also to the level of the fundamental theory.

3. It is also possible to generalize the twistor program to TGD framework if one accepts the use ofoctonionic representation of the gamma matrices of imbedding space and hyper-quaternionicity of space-time surfaces [25].

2.4 Zero energy ontology and WCW geometry

In the zero energy ontology quantum states have vanishing net values of conserved quantum numbers and decompose to superposition of pairs of positive and negative energy states defining counterparts of initial and final states of a physical event in standard ontology.

2.4.1 Zero energy ontology

Zero energy ontology was forced by the interpretational problems created by the vacuum extremal property of Robertson-Walker cosmologies imbedded as 4-surfaces in $M^4 \times CP_2$ meaning that the density of inertial mass (but not gravitational mass) for these cosmologies was vanishing meaning a conflict with Equivalence Principle. In zero energy ontology physical states are replaced by pairs of positive and negative energy states assigned to the past *resp.* future boundaries of causal diamonds defined as pairs of future and past directed light-cones ($\delta M^4_\pm \times CP_2$). The net values of all conserved quantum numbers of zero energy states vanish. Zero energy states are interpreted as pairs of initial and final states of a physical event such as particle scattering so that only events appear in the new ontology. It is possible to speak about the energy of the system if one identifies it as the average positive energy for the positive energy part of the system. Same applies to other quantum numbers.

The matrix ("M-matrix") representing time-like entanglement coefficients between positive and negative energy states unifies the notions of S-matrix and density matrix since it can be regarded as a complex square root of density matrix expressible as a product of real squared of density matrix and unitary S-matrix. The system can be also in thermal equilibrium so that thermodynamics becomes a genuine part of quantum theory and thermodynamical ensembles cease to be practical fictions of the theorist. In this case M-matrix represents a superposition of zero energy states for which positive energy state has thermal density matrix.

Zero energy ontology combined with the notion of quantum jump resolves several problems. For instance, the troublesome questions about the initial state of universe and about the values of conserved quantum numbers of the Universe can be avoided since everything is in principle creatable from vacuum. Communication with the geometric past using negative energy signals and time-like entanglement are crucial for the TGD inspired quantum model of memory and both make sense in zero energy ontology. Zero energy ontology leads to a precise mathematical characterization of the finite resolution of both quantum measurement and sensory and cognitive representations in terms of inclusions of von Neumann algebras known as hyperfinite factors of type II_1. The space-time correlate for the finite resolution is discretization which appears also in the formulation of quantum TGD.

2.4.2 Causal diamonds

The imbedding space correlates for zero energy ontology are causal diamonds (CDs) CD serves as the correlate zero energy state at imbedding space-level whereas space-time sheets having their ends at the light-like boundaries of CD are the correlates of the system at the level of 4-D space-time. Zero energy state can be regarded as a quantum superposition of space-time sheets with fermionic and other quantum numbers assignable to the partonic 2-surfaces at the ends of the space-time sheets.

1. The basic construct in the zero energy ontology is the space $CD \times CP_2$, where the causal diamond CD is defined as an intersection of future and past directed light-cones with time-like separation between their tips regarded as points of the underlying universal Minkowski space M^4. In zero energy ontology physical states correspond to pairs of positive and negative energy states located at the boundaries of the future and past directed light-cones of a particular CD.

2. CDs form a fractal hierarchy and one can glue smaller CDs within larger CDs. Also unions of CDs are possible.

3. Without any restrictions CDs would be parametrized by the position of say lower tip of CD and by the relative M^4 coordinates of the upper tip with respect to the lower one so that the moduli space would be $M^4 \times M^4_+$. p-Adic length scale hypothesis follows if the values of temporal distance T between tips of CD come in powers of 2^n: $T = 2^n T_0$. This would reduce the future light-cone M^4_+ reduces to a union of hyperboloids with quantized value of light-cone proper time. A possible interepretation of this distance is as a quantized cosmic time. Also the quantization of the hyperboloids to a lattices of discrete points classified by discrete sub-groups of Lorentz group is an attractive proposal and the quantization of cosmic redshifts provides some support for it.

Zero energy ontology forces to replaced the original WCW by a union of WCWs associated with CDs and their unions. This does not however mean any problems of principle since Clifford algebras are simply tensor products of the Clifford algebras of CDs for the unions of CDs.

2.5 Hierarchy of Planck constants and WCW geometry

The motivations for introducing the hierarchy of Planck constants interpreted in terms of phases of dark matter came from astrophysics [70, 32, 33] and biology [46] and led to a generalization of the imbedding space to a book like structure [29]. This implies additional richness of structure at the level of geometry of WCW. In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized.

2.5.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years [29, 22, 32].

1. The starting point was the proposal of Nottale [70] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [32] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [31]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [45].

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [22]. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [70] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr} G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and aminoacids, and to the mysteries of bio-catalysis. This leads also a model for EEG

interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [34, 45].

2.5.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.
   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.
   (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tedrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

2.5.3 About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

2.5.4 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$.

Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

2.5.5 Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [47].

2.5.6 How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anticommutation relations of various super-conformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants [29]. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

2.5.7 Implications for the construction WCW geometry

1. In the realization of the hierarchy of Planck constants $CD \times CP_2$ is replaced with a Cartesian product of book like structures formed by almost copies of CDs and CP_2s defined by singular coverings and factors spaces of CD and CP_2 with singularities corresponding to intersection $M^2 \cap CD$ and homologically trivial geodesic sphere S^2 of CP_2 for which the induced Kähler form vanishes. The coverings and factor spaces of CDs are glued together along common $M^2 \cap CD$. The coverings and factors spaces of CP_2 are glued together along common homologically non-trivial geodesic sphere S^2. The choice of preferred M^2 as subspace of tangent space of X^4 at all its points and interpreted as space of non-physical polarizations, brings M^2 into the theory also in different manner. S^2 in turn defines a subspace of the much larger space of vacuum extremals as surfaces inside $M^4 \times S^2$.

2. Configuration space (the world of classical worlds, WCW) decomposes into a union of sub-WCWs corresponding to different choices of M^2 and S^2 and also to different choices of the quantization axes of spin and energy, color isospin and hyper-charge for each choice of this kind. This means breaking down of the isometries to a subgroup. This can be compensated by the fact that the union can be taken over the different choices of this subgroup.

3. This means extension of the moduli space of CDs from $M^4 \times X$, where $X \subset M^4_+$ is suggested to be identifiable as a discrete lattice for the relative positions of the tips of CD. What is added is the space characterizing the choice of the quantization axes for energy and spin

on one hand and color hypercharge and isospin on the other hand. This choice is part of a statefunction reduction process and means localization in this space. In the case of color charges the moduli space is the flag-manifold $SU(3)/U(1) \times U(1)$.

2.6 Hyper-finite factors and the notion of measurement resolution

The work with TGD inspired model [43, 44] for topological quantum computation [65] led to the realization that von Neumann algebras [54], in particular so called hyper-finite factors of type II_1 [57], seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. Later came the realization that the Clifford algebra of WCW defines a canonical representation of hyper-finite factors of type II_1 and that WCW spinor fields give rise to HFFs of type III_1 encountered also in relativistically invariant quantum field theories [28].

2.6.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [57].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD.

2.6.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [19] based on the notion of delta function, plus the emergence of s [52], the possibility to formulate the notion of delta function rigorously in terms of distributions [50, 51], and the emergence of path integral approach [53] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [68, 66] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [58, 60] relate closely to type II_1 factors. In topological quantum computation [65] based on braid groups [56] modular S-matrices they play an especially important role.

In algebraic quantum field theory [62] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [63, 64].

2.6.3 Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type II_1 and III_1- the latter appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of WCW is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [31]. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

2.6.4 Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix [28].

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem [61], which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

The concrete construction of M-matrix utilizing the idea of bosonic emergence (bosons as fermion anti-fermion pairs at opposite throats of wormhole contact) meaning that bosonic propagators reduce to fermionic loops identifiable as wormhole contacts leads to generalized Feynman rules for M-matrix in which modified Dirac action containing measurement interaction term defines stringy propagators [24]. This M-matrix should be consistent with the above proposal.

2.6.5 Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product [67] but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

2.6.6 Number theoretical braids as space-time correlates for finite measurement resolution

Finite measurement resolution has discretization as a space-time counterpart. In the intersection of real and p-adic worlds defines as partonic 2-surfaces with a mathematical representation allowing interpretation in terms of real or p-adic number fields one can identify points common to real and p-adic worlds as rational points and common algebraic points (in preferred coordinates dictated by symmetries of imbedding space). Quite generally, one can identify rational points and algebraic points in some extension of rationals as points defining the initial points of what might be called number theoretical braid beginning from the partonic 2-surface at the past boundary of CD and connecting it with the future boundary of CD. The detailed definition of the braid inside light-like 3-surface is not relevant if only the information at partonic 2-surface is relevant for quantum physics.

Number theoretical braids are especially relevant for topological QFT aspect of quantum TGD. The topological QFT associated with braids accompanying light-like 3-surfaces having interpretation as lines of generalied Feynman diagrams should be important part of the definition of amplitudes assigned to generalized Feynman diagrams. The number theoretic braids relate also closely to a symplectic variant of conformal field theory emerges very naturally in TGD framework (symplectic symmetries acting on $\delta M^4_\pm \times CP_2$ are in question) and this leads to a concrete proposal for how to to construct n-point functions needed to calculate M-matrix [24]. The mechanism guaranteing the predicted absence of divergences in M-matrix elements can be understood in terms of vanishing of symplectic invariants as two arguments of n-point function coincide.

2.6.7 Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities [28]. For quantum spinors state function reduction to spin eigenstates cannot be performed unless quantum deformation parameter $q = exp(i2\pi/n)$ equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. Therefore the probability for either spin state becomes a quantized observable. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

3 Physics as a generalized number theory

Physics as a generalized number theory vision involves actually three threads: p-adic ideas [35], the ideas related to classical number fields [36], and the ideas related to the notion of infinite prime [37].

3.1 Fusion of real and p-adic physics to a coherent whole

p-Adic number fields were not present in the original approach to TGD. The success of the p-adic mass calculations (summarized in the first part of [6]) made however clear that one must generalize the notion of topology also at the infinitesimal level from that defined by real numbers so that the attribute "topological" in TGD gains much more profound meaning than intended originally. It took a decade to get convinced that the identification of p-adic physics as a correlate of cognition and intentionality is the most plausible interpretation discovered hitherto [40], and that p-adic topology of p-adic space-time sheets somehow induces the effective p-adic topology of real space-time sheets. The discovery of the properties of number theoretic variants of Shannon entropy led to the idea that living matter could be seen as as something in the intersection of real and p-adic worlds and gave additional support for this interpretation. If even elementary particles reside in this intersection and effective p-adic topology applies for real partonic 2-surfaces, the success of p-adic mass calculations can be understood.

The original view about physics as the geometry of WCW is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [5].

1. The first element is a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common rationals and algebraics to form a larger structure.

2. At the level of imbedding space this gluing corresponds to a gluing of real and p-adic variants of the imbedding space together along rational and common algebraic points (the number of which depends on algebraic extension of p-adic numbers used) to what could be seen as a book like structure. General Coordinate Invariance restricted to rationals or their extension requires preferred coordinates for $CD \times CP_2$ and this kind coordinates can be fixed by isometries of H. The coordinates are however not completely unique since non-rational isometries produce new equally good choices. Whether this can be seen as an objection against the approach is not clear.

3. The analogous gluing of real and various p-adic physics to a larger structure forces to ask what are the common points of WCWs associated with real and various p-adic worlds. What it is to be a partonic 2-surface belonging to the intersection of real and p-adic variants of WCW? The natural answer is that partonic 2-surfaces which have a mathematical representation making sense both for real numbers and p-adic numbers or their algebraic extensions can be regarded as "common points" or identifiable points of p-adicity and reality. This of course applies also to partonic 2-surfaces corresponding to two different p-adic number fields. This mathematical property means a representability in terms of ratios of polynomials with rational (or possibily even algebraic) coefficients in the preferred imbedding space coordinates.

4. The intersections of WCWs and partonic 2-surfaces in different number fields are involved. An attractive idea is that only the information about common points of surfaces belonging to different number fields code for physics so that number-theoretically universal part of physics is number theoretical physics relying only on rationals and their algebraic extensions. For instance, the transition amplitudes between p-adic and real variants of partonic 2-surface can involve only the data at these points. This suggests the existence of what might be called number theoretical QFT. At space-time level this extension of introduce a discretization at space-time level in terms of rational and algebraic points common to real space-time sheets and their p-adic variants. The number of these points is in general finite for a given CD and the proposed interpretation is in terms of cognitive representations. The discrete intersections would define the initial and final points of number theoretical braids central for the formulation of the theory in finite measurement resolution.

5. Much later came the realization that living matter or what makes living matter living could be interpreted as something in this intersection of real and p-adic worlds so that number theoretic QFT might apply to crucial aspects of living matter.

The interpretation for discretization could be in terms of cognitive, sensory, and measurement resolutions rather than fundamental discreteness of the space-time. What looks rather counter intuitive first is that transcendental points of p-adic space-time sheets are at spatiotemporal infinity in real sense so that the correlates of cognition and intentionality cannot be localized to any finite spatiotemporal volume unlike those of sensory experience. The description of intentionality and cognition in this manner predicts p-adic fractality of real physics meaning chaos in short scales combined with long range correlations: p-adic mass calculations represent one example of p-adic fractality.

The realization of this program at the level of WCW is far from trivial. Modified Dirac equation and classical field equations make sense but quantities expressible as space-time integrals - in particular Kähler action- do not make sense p-adically. Therefore one can ask whether only the partonic surfaces in the intersection of real and p-adic worlds should be allowed. Also this restricted theory would be highly non-trivial physically.

3.2 Classical number fields and associativity and commutativity as fundamental law of physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [69]) are involved [36] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the imbedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [36]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of modified Dirac action the identification of space-time surface as a hyper-quaternionic submanifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a hyper-quaternionic sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [19, 25].

3.3 Infinite primes and quantum physics

The hierarchy of infinite primes (and of integers and rationals) [37] was the first mathematical notion stimulated by TGD inspired theory of consciousness. The construction recipe is equivalent with a repeated second quantization of a super-symmetric arithmetic quantum field theory with bosons and fermions labeled by primes such that the many-particle states of previous level become the elementary particles of new level. At a given level there are free many particles states plus counterparts of many particle states. There is strong structural analogy with polynomial primes. For polynomials with rational coefficients free many-particle states would correspond to products of first order polynomials and bound states to irreducible polynomials with non-rational roots.

The hierarchy of space-time sheets with many particle states of space-time sheet becoming elementary particles at the next level of hierarchy. For instance, the description of proton as an elementary fermion would be in a well defined sense exact in TGD Universe. Also the hierarchy of n:th order logics are possible correlates for this hierarchy.

This construction leads also to a number theoretic generalization of space-time point since a given real number has infinitely rich number theoretical structure not visible at the level of the real norm of the number a due to the existence of real units expressible in terms of ratios of infinite integers. This number theoretical anatomy suggest a kind of number theoretical Brahman=Atman identity stating that the set consisting of number theoretic variants of single point of the imbedding space (equivalent in real sense) is able to represent the points of WCW or maybe even quantum states assignable to causal diamond. One could also speak about algebraic holography.

The correspondence between the quantum states defined by WCW spinor fields and wave functions in the infinite-dimensional discrete space of hyper-octonionic units can be made more concrete [37]. These wave functions must transforming irreducibly under discrete subgroup SU(3) of octonion automorpisms transforming ordinary hyper-octonionic prime to a new hyper-octonionic prime. SU(3) has interpretation as color group. One can assign standard model quantum numbers to these wave functions and prime property in principle fixes the spectrum of possible quantum states- in particular the spectrum of masses. Therefore the extremely esoteric looking notion of infinite prime might turn out to be very practical calculational tool.

4 Physics as extension of quantum measurement theory to a theory of consciousness

TGD inspired theory of consciousness could be seen as a generalization of quantum measurement theory to make observer, which in standard quantum measurement theory remains an outsider, a genuine part of physical system subject to laws of quantum physics. The basic notions are quantum jump identified as moment of consciousness and the notion of self [38]: in zero energy ontology these notions might however reduce to each other. Negentropy Maximization Principle [39] defines the dynamics of consciousness and as a special case reproduces standard quantum measurement theory.

4.1 Quantum jump as moment of consciousness

TGD suggests that the quantum jump between quantum histories could identified as moment of consciousness and could therefore be for consciousness theory what elementary particle is for physics [38].

This means that subjective time evolution corresponds to the sequence of quantum jumps $\Psi_i \to U\Psi_i \to \Psi_f$ consisting of unitary process followed by state function process. Originally U was thought to be the TGD counterpart of the unitary time evolution operator $U(-t,t)$, $t \to \infty$, associated with the scattering solutions of Schrödinger equation. It seems however impossible to assign any real Scrödinger time evolution with U. In zero energy ontology U defines a unitary matrix between zero energy states and is naturally assignable to intentional actions whereas the ordinary S-matrix telling what happens in particle physics experiment (for instance) generalizes to M-matrix defining time-like entanglement between positive and negative energy parts of zero energy states. One might say that U process corresponds to a fundamental act of creation creating a quantum superposition of possibilities and the remaining steps generalizing state function reduction process select between them.

4.2 Negentropy Maximization Principle and the notion of self

U-process is followed by a sequence of state function reductions. Negentropy Maximization Principle (NMP [39]) states that in a given quantum state the most quantum entangled subsystem-complement pair can perform the quantum jump. More precisely: the reduction of the entanglement entropy in the quantum jump is as large as possible. This selects the pair in question and in case of ordinary entanglement entropy leads the selected pair to a product state. The interpretation of the reduction of the entanglement entropy as conscious information gain makes sense. The sequence of state function reductions decomposes at first step the entire system to two parts in such a manner that the reduction entanglement entropy is maximal. This process repeats itself for subsystems. If the subsystem in question cannot be divided into a pair of entangled free system the process stops since energy conservation does not allow it to occur (binding energy).

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. Everything is consciousness but consciousness can be lost if self develops bound state entanglement during U process so that state function reduction to smaller un-entangled pieces is impossible.

The existence of number theoretical entanglement entropies in the intersection of real and various p-adic worlds force to modify this picture. The reduction process can stop also if the self in question allows only decompositions to pairs systems with negentropic entanglement. This does not require that that the system forms a bound state for any pair of subsystems so that the systems decomposing it can be free (no binding energy). This defines a new kind of bound state not describable as a jail defined by the bottom of a potential well. Subsystems are free but remain correlated by negentropic entanglement.

The ordinary state function reductions imply dissipation crucial for self organization and quantum jump could be regarded as the basic step of an iteration like process leading to the asympotic self-organization patterns. One could regard dissipation as a Darwinian selector as in standard theories of self-organization. NMP thus predicts that self organization and hence presumably also fractalization can occur inside selves. NMP would favor the generation of negentropic entanglement. This notion is highly attractive since it could allow to understand how quantum selforganization generates larger coherent structures. Note that state function reduction for negentropic entanglement is highly deterministic since the number of degenerate states with same negative entanglement entropy is expected to be small. This could allow to understand how living matter is able to develop almost deterministic cellular automaton like behaviors.

4.3 Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

The observation that Shannon entropy allows an infinite number of number theoretic variants for which the entropy can be negative in the case that probabilities are algebraic numbers leads to the idea that living matter in a well-defined sense corresponds to the intersection of real and p-adic worlds. This would mean that the mathematical expressions for the space-time surfaces (or at least 3-surfaces or partonic 2-surfaces and their 4-D tangent planes) make sense in both real and p-adic sense for some primes p. Same would apply to the expressions defining quantum states. In particular, entanglement probabilities would be rationals or algebraic numbers so that entanglement can be negentropic and the formation of bound states in the intersection of real and p-adic worlds generates information and is thus favored by NMP.

This picture has also a direct connection with consciousness [39].

1. Algebraic entanglement is a prerequisite for the realization of intentions as transformations of p-adic space-time sheets to real space-time sheets representing actions. Essentially a leakage between p-adic and real worlds is in question and makes sense only in zero energy ontology. Since various quantum numbers in real and p-adic sectors are not in general comparable in positive energy ontology so that conservation laws would be broken in positive energy ontology. Algebraic entanglement could be also called cognitiv-that is between real and p-adic worlds. The transformation can occur if the partonic 2-surfaces and their 4-D tangent space-distributions are representable using rational functions with rational coefficients in preferred coordinates for the imbedding space dictated by symmetry considerations. Intentional systems must live in the intersection of real and p-adic worlds. For the minimal option life would be also effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests.

2. The generation of non-rational (non-algebraic) bound state entanglement between the system and external world means that the system loses consciousness during the state function reduction process following the U-process generating the entanglement. What happens that the Universe corresponding to given CD decomposes to two un-entangled subsystems, which in turn decompose, and the process continues until all subsystems have only entropic bound state entanglement or negentropic algebraic entanglement with the external world.

3. If the sub-system generates entropic bound state entanglement in the the process, it loses consciousness. Note that the entanglement entropy of the sub-system is a sum over entanglement entropies over all subsystems involved. This hierarchy of subsystems corresponds to the hierarchy if sub-CDs so that the survival without a loss of consciousness depends on what happens at all levels below the highest level for a given self. In more concrete terms, ability to stay conscious depends on what happens at cellular level too. The stable evolution of systems having algebraic entanglement is expected to be a process proceeding from short to long length scales as the evolution of life indeed is.

4. U-process generates a superposition of states in which any sub-system can have both real and algebraic entanglement with the external world. This would suggest that the choice of the type of entanglement is a volitional selection. A possible interpretation is as a choice between good and evil. The hedonistic complete freedom resulting as the entanglement entropy is reduced to zero on one hand, and the algebraic bound state entanglement implying correlations

with the external world and meaning giving up the maximal freedom on the other hand. The hedonistic option is risky since it can lead to non-algebraic bound state entanglement implying a loss of consciousness. The second option means expansion of consciousness - a fusion to the ocean of consciousness as described by spiritual practices.

5. This formulation means a sharpening of the earlier statement "Everything is conscious and consciousness can be only lost" with the additional statement "This happens when non-algebraic bound state entanglement is generated". Clearly, the quantum criticality of TGD Universe seems has very many aspects and life as a critical phenomenon in the number theoretical sense is only one of them besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [42].

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more approriate term being quantum problem solving. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua.

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

4.4 Two times

The basic implication of the proposed view is that subjective time and geometric time of physicist are not the same [38]. This is not a news actually. Geometric time is reversible, subjective time irreversible. Geometric future and past are in completely democratic position, subject future does not exist at all yet. One can say that the non-determinism of quantum jump is completely outside space-time and Hilbert space since quantum jumps replaces entire 4-D time evolution (or rather, their quantum superposition) with a new one, re-creates it. Also conscious existence defies any geometric description. This new view resolves the basic problem of quantum measurement theory due to the conflict between determinism of Schödinger equation and randomness of quantum jump. The challenge is to understand how these two times correlate so closely as to lead to their erratic identification.

With respect to geometric time the contents of conscious experience is naturally determined by the space-time region inside CD in zero energy ontology. This geometro-temporal integration should have subjecto-temporal counterpart. The experiences of self are determined by the mental images assignable to subselves (having sub-CDs as imbedding space correlates) and the quantum jump sequences associated with sub-selves define a sequence of mental images. The hypothesis is that self experiences these sequences of mental images as a continuous time flow. In absence of mental images self would have experience of "timelessness" in accordance with the reports of practitioners of various spiritual practices. Self would lose consciousness in quantum jump generating entropic entangelement and experience expansion of consciousness if the resulting entanglement is negentropic. The assumption that the integration of experiences of self involves a kind of averaging over sub-selves of sub-selves guarantees that the sensory experiences are reliable despite the fact that quantum nondeterminism is involved with each quantum jump.

Thus the measurement of density matrix defined by the $MM^\dagger$, where M is the M-matrix between positive and negative energy parts of the zero energy state would correspond to the passive aspects of consciousness such as sensory experiencing. U would represent at the fundamental level volition as a creation of a quantum superposition of possibilities. What follows it would be a selection between them. The volitional choice between macroscopically differing space-time sheets representing different maxima of Kähler function could be basically responsible for the active aspect of consciousness. The fundamental perception-reaction feedback loop of biosystems would result from the combination of the active and passive aspects of consciousness represented by U and M.

4.5 General view about psychological time and intentionality

The recent TGD inspired attempts to understand the arrow of psychological time and the localization of the contents of conscious sensory experience and experienced volition to a rather narrow time interval of .1 seconds rely on zero energy ontology. The most argument below summarizes the most recent view [41].

4.5.1 Why sensory experience is about so short time interval?

The picture based on CDs implies automatically the 4-D character of conscious experience and memories form part of conscious experience even at elementary particle level. Amazingly, the secondary p-adic time scale of electron characterizing the time scale of electronic CD is $T = 0.1$ seconds defining a fundamental time scale in living matter. The problem is to understand why the sensory experience is about a short time interval of geometric time rather than about the entire personal CD with temporal size of order life-time. The explanation would be that sensory input corresponds to subselves (mental images) with $T \simeq .1$ s at the upper light-like boundary of CD in question. This requires a strong asymmetry between upper and lower light-like boundaries of CDs.

The localization of the contents of the sensory experience to the upper light-cone boundary and local arrow of time could emerge as a consequence of self-organization process involving conscious intentional action. Sub-CDs would be in the interior of CD and self-organization process would lead to a distribution of CDs concentrated near the upper or lower boundary of CD. The local arrow of geometric time would depend on CD and even differ for CD and sub-CDs.

1. The localization of contents of sensory experience to a narrow time interval would be due to the concentration of sub-CDs representing mental images near the either boundary of CD representing self.

2. Phase conjugate signals identifiable as negative energy signals to geometric past are important when the arrow of time differs from the standard one in some time scale. If the arrow of time establishes itself as a phase transition, this kind of situations are rare. Negative energy signals as a basic mechanism of intentional action and transfer of metabolic energy would explain why living matter is so special.

3. Geometric memories would correspond to subselves in the interior of CD, the oldest of them to the regions near "lower" boundaries of CD. Since the density of sub-CDs is small there geometric memories would be rare and not sharp. A temporal sequence of mental images, say the sequence of digits of a phone number, would correspond to a temporal sequence of sub-CDs.

4. Sharing of mental images corresponds to a fusion of sub-selves/mental images to single sub-self by quantum entanglement: the space-time correlate could be flux tubes connecting space-time sheets associated with sub-selves represented also by space-time sheets inside their CDs.

4.5.2 Arrow of time

TGD forces a new view about the relationship between experienced and geometric time. Although the basic paradox of quantum measurement theory disappears the question about the arrow of geometric time remains. There are actually two times involved. The geometric time assignable to the space-time sheets and the M^4 time assignable to the imbedding space.

Consider first the the geometric time assignable to the space-time sheets.

1. Selves correspond to CDs. The CDs and their projections to the imbedding space do not move anywhere. Therefore the standard explanation for the arrow of geometric time cannot work.

2. The only plausible interpretation at classical level relies on quantum classical correspondence and the fact that space-times are 4-surfaces of the imbedding space. If quantum jump corresponds to a shift for a quantum superposition of space-time sheets towards geometric past in the first approximation (as quantum classical correspondence suggests), one can understand the arrow of time. Space-time surfaces simply shift backwards with respect to the geometric time of the imbedding space and therefore to the 8-D perceptive field defined by the CD. This creates in the materialistic mind a temporal variant of train illusion. Space-time as 4-surface and macroscopic and macro-temporal quantum coherence are absolutely essential for this interpretation to make sense.

Why this shifting should always take place to the direction of geometric past of the imbedding space? Does it so always? The proposed mechanism for the localization of sensory experience to a short time interval suggests an explanation in terms of intentional action.

1. CD defines the perceptive field for self. Selves are curious about the space-time sheets outside their perceptive field and perform quantum jumps tending to shift the superposition of the space-time sheets so that unknown regions of space-time sheets emerge to the perceptive field. Either the upper or lower boundary of CD wins in the competition and the arrow of time results as a spontaneous symmetry breaking. The arrow of time can depend on CD but tends to be the same for CD and its sub-CDs. Global arrow of time could establish itself by a phase transitions establishing the same arrow of time globally by a mechanism analogous to percolation phase transition.

2. Since the news come from the upper boundary of CD, self concentrates its attention to this region and improves the resolution of sensory experience. The sub-CDs generated in this manner correspond to mental images with contents about this region. Hence the contents of conscious experience, in particular sensory experience, tends to be about the region near the upper boundary.

The emergence of the arrow of time at the level of imbedding space reduces to a modification of the oldest TGD based argument for the arrow of time which is wrong as such. If physical objects correspond to 3-surfaces inside future directed light-cone then the sequence of quantum jumps implies a diffusion to the direction of increasing value of light-cone propert time. The modification of the argument goes as follows.

1. CDs are characterized by their moduli. In particular, the relative coordinate for the tips of CD has values in past light cone M^4_- if the future tip is taken as the reference point. An attractive interpretation for the proper time of M^4_- is as cosmic time having quantized values. Quantum states correspond to wave functions in the modular degrees of freedom and each U process creates a non-localized wave function of this kind. Suppose that state function reduction implies a localization in the modular degrees of freedom so that CD is fixed completely apart from its center of mass position to which zero four-momentum constant plane wave is assigned. One can expect that in average sense diffuction occurs in M^4_- so that the size of CD tends to increase and that the most distant geometric past defined by the past boundary of CD recedes. This is nothing but cosmic expansion. This provides a formulation for the flow of time in terms of a cosmic redshift. This argument applies also to the positions of the sub-CDs inside CD. Also their proper time distance from the tip of CD is expected to increase.

2. One can argue that one ends up with contradiction by changing the roles of upper and lower tips. In the case of CD itself is only the proper time distance between the tips which increases and speaking about "future" and "past" tips is only a convention. For sub-CDs of CD the argument would imply that the sub-CDs drifting from the opposite tips tend to concentrate in the middle region of CD unless either tip is in a preferred position. This requires a spontaneous selection of the arrow of time. One could say that the cosmic expansion implied by the drift in M^4_- "draws" the space-time sheet with it to the geometric past. The spontaneous generation of the asymmetry between the tips might require the "curious" conscious entities.

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