

Quaternionic planes of octonions

The space of quaternionic planes of octonion space containing preferred complex plane is CP_2 . One can see this in following manner.

1. Decompose the tangent space unit vectors of complexified octonions O_c to $1, e_7$ spanning preferred complex plane $E_1 = e_1 + ie_4, E_2 = e_2 + ie_5,$ and $E_3 = e_3 + ie_6$ and their complex conjugates transforming as $SU(3)$ triplet (3) and its antitriplet (3bar) meaning that all 6planes orthogonal to E_2 correspond to these choices and are parametrized by $SU(3)$ leaving e_7 and the spaces spanned by the triplets invariant meaning all 6planes orthogonal to E_2 correspond to these choices and are parametrized by $SU(3)$ leaving e_7 and the spaces spanned by the triplets invariant.
2. Important point:
 - (a) complexified octonions can be used in tangent space so that 16D space is in question.
 - (b) This does not mean complexification of $M^4 \times CP_2$ nor of M^8 as I thought for some time
 - (c) Is necessary to avoid problems caused by different signatures of M^8 and real octonion space.
3. Select quaternionic 4-plane containing preferred E^2 by selecting
 - (a) E_2 by fixing e_7
 - (b) E_1, E_2 giving rise to complexified quaternionic plane spanned by multiplication of E_1 and E_2 by e_7 does not lead out from the complexified 4-plane so that complexified quaternionic plane is in question by $1, e_7, E_1, E_2$ since multiplication of E_1 and E_2 by e_7 does not lead out from the complexified 4-plane so that complexified quaternionic plane is in question.
4. Identify the choices E_1, E_2, E_3 which differ by $SU(2)$ rotation of E_1, E_2 and by $U(1)$ rotation of E_3 since they define the same quaternionic plane so that $CP_2 = SU(3)/SU(2) \times U(1)$ parametrizes the choices and has therefore number theoretic meaning raising the hope that standard model quantum numbers could be understood numbertheoretically.
5. The space of quaternionic planes decomposes locally to $S^7 \times CP_2 : S^7$ parametrizes the choices of preferred E^2 . This is essential for M^8H duality since it makes possible to fix CP_2 point characterizing tangent plane uniquely.